Rare Radiative and Semileptonic B-Decays and Implications

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Plan of Talk

- The Standard Candle: $B \to X_s\gamma$
- Exclusive Radiative Decay $B \to K^*\gamma$
- Cabibbo-Suppressed Decays $B \to (\rho, \omega)\gamma$
- Inclusive Decays $B \to X_s\ell^+\ell^-$
- Dilepton Mass Spectrum and Leptonic Forward-Backward Asymmetry in $B \to K^*\ell^+\ell^-$
- Summary

The Standard Candle: $B \longrightarrow X_s\gamma$

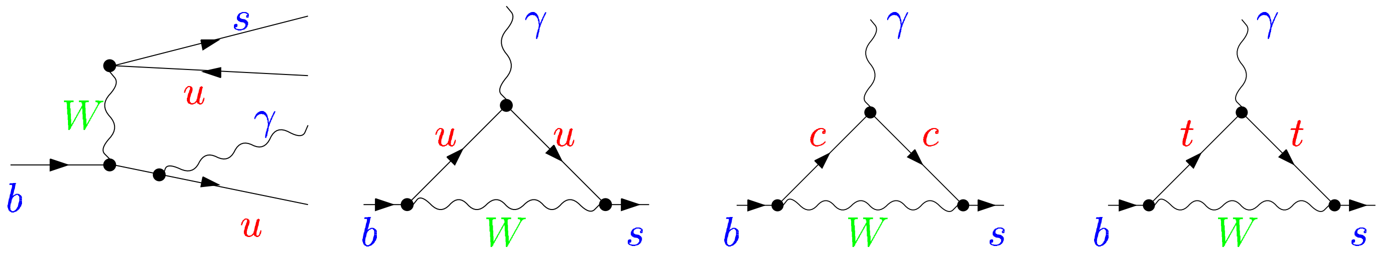
Interest in the rare decay $B \longrightarrow X_s\gamma$ transcends B Physics!

- Already well measured; more precise measurements anticipated at B- and SuperB-factories

Theoretical Interest:

- A monumental theoretical effort has gone in improving the perturbative precision; $B \longrightarrow X_s\gamma$ in NNLO completed last year
  - First estimate of $\mathcal{B}(B \longrightarrow X_s\gamma)$: Misiak et al. (17 authors), hep-ph/0609232
  - Analysis of $\mathcal{B}(B \longrightarrow X_s\gamma)$ at NNLO with a cut on Photon energy, T. Becher and M. Neubert, hep-ph/0610067
- Non-perturbative effects under control thanks to HQET
- Sensitivity to new physics; hence constrains parameters of the BSM models such as Supersymmetry
- A crucial input in a large number of precsion tests of the SM in $b \longrightarrow s$ processes, such as $B \longrightarrow X_s\ell^+\ell^-$

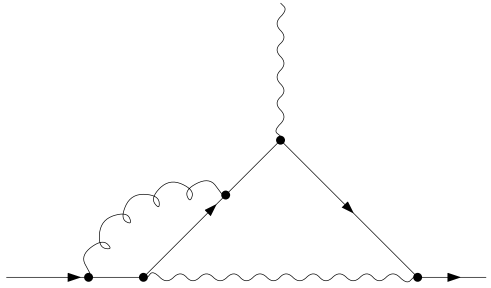
Examples of the leading electroweak diagrams for $\bar{B} \to X_s \gamma$:



$$\left|\frac{V_{ub}V_{us}}{V_{cb}}\right| \simeq \left|\frac{V_{ub}V_{us}}{V_{ts}}\right| \simeq 2\%$$

$\sim +200\%$ $\sim -100\%$

In the amplitude, after including LO QCD effects.



- QCD logarithms $\alpha_s \ln \frac{M_W^2}{m_b^2}$ enhance $\text{BR}(\bar{B} \to X_s\gamma)$ more than twice
- Effective field theory (obtained by integrating out heavy fields) used for resummation of such large logarithms

The effective Lagrangian for $B \to X_s\gamma$ and $B \to X_s\ell^+\ell^-$

$$\mathcal{L} = \mathcal{L}_{QCD\times QED}(q,l) + \frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{10} C_i(\mu) O_i$$

$(q = u, d, s, c, b, \ l = e, \mu)$

$$O_i = \begin{cases} (\bar{s}\Gamma_i c)(\bar{c}\Gamma_i' b), & i = 1, 2, \quad |C_i(m_b)| \sim 1 \\ (\bar{s}\Gamma_i b)\Sigma_q(\bar{q}\Gamma_i' q), & i = 3, 4, 5, 6, \quad |C_i(m_b)| < 0.07 \\ \frac{e m_b}{16\pi^2} \bar{s}_L \sigma^{\mu\nu} b_R F_{\mu\nu}, & i = 7, \quad C_7(m_b) \sim -0.3 \\ \frac{g m_b}{16\pi^2} \bar{s}_L \sigma^{\mu\nu} T^a b_R G^a_{\mu\nu}, & i = 8, \quad C_8(m_b) \sim -0.15 \\ \frac{e^2}{16\pi^2} (\bar{s}_L \gamma_\mu b_L)(\bar{l}\gamma^\mu \gamma_5 l), & i = 9, 10 \quad |C_i(m_b)| \sim 4 \end{cases}$$

Three steps of the calculation:

Matching: Evaluating $C_i(\mu_0)$ at $\mu_0 \sim M_W$ by requiring equality of the SM and the effective theory Green functions

Mixing: Deriving the effective theory RGE and evolving $C_i(\mu)$ from μ_0 to $\mu_b \sim m_b$

Matrix elements: Evaluating the on-shell amplitudes at $\mu_b \sim m_b$

Wilson Coefficients

Wilson Coefficients of Four-Quark Operators

	$C_1(\mu_b)$	$C_2(\mu_b)$	$C_3(\mu_b)$	$C_4(\mu_b)$	$C_5(\mu_b)$	$C_6(\mu_b)$
LL	-0.257	1.112	0.012	-0.026	0.008	-0.033
NLL	-0.151	1.059	0.012	-0.034	0.010	-0.040

Wilson Coefficients of Other Operators

	$C_7^{\text{eff}}(\mu_b)$	$C_8^{\text{eff}}(\mu_b)$	$C_9(\mu_b)$	$C_{10}(\mu_b)$
LL	-0.314	-0.149	2.007	0
NLL	-0.308	-0.169	4.154	-4.261
NNLL			4.214	-4.312

- Obtained for the following input:

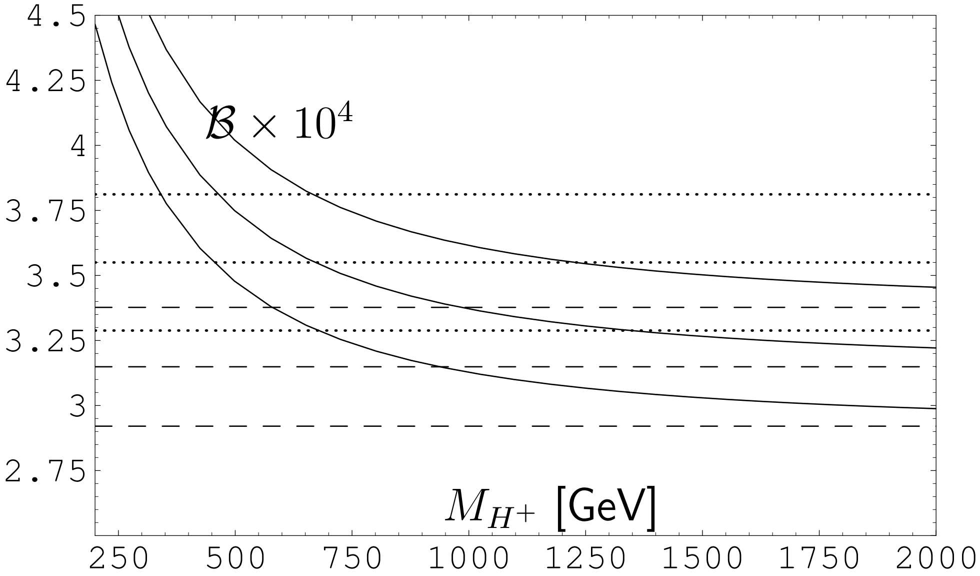
$$\mu_b = 4.6 \text{ GeV} \qquad \bar{m}_t(\bar{m}_t) = 167 \text{ GeV}$$

$$M_W = 80.4 \text{ GeV} \qquad \sin^2\theta_W = 0.23$$

- Three-loop running is used for α_s coupling with $\Lambda^{(5)}_{\overline{\text{MS}}} = 220$ GeV

$\mathcal{B}(\bar{B} \to X_s \gamma)$: Experiment vs. SM & 2HDM

[Misiak et al., hep-ph/0609232]

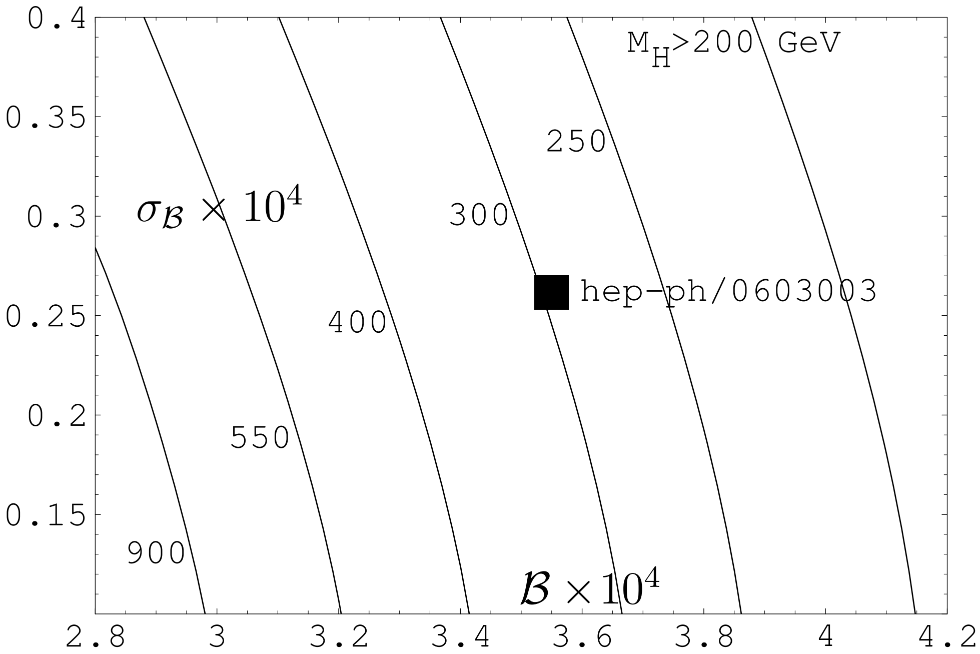


[$\cdot\cdot\cdot\cdot$ (exp); $- - - -$ (SM); solid (2HDM)]

- Experiment ($E_\gamma > 1.6$ GeV):
  $\mathcal{B}(\bar{B} \to X_s \gamma) = (3.55 \pm 0.24^{+0.09}_{-0.10} \pm 0.03) \times 10^{-4}$
- NNLO SM: $\mathcal{B}(\bar{B} \to X_s \gamma) = (3.15 \pm 0.23) \times 10^{-4}$
- SM is below the experiments by about 1 σ
- In 2HDM, preferred value is $M_{H^+} \simeq 650$ GeV
- 95% C.L. lower bound is around 295 GeV

95% C.L. Lower Bound on M_{H^+} in 2HDM from $\mathcal{B}(\bar{B} \to X_s \gamma)$

[Misiak et al., hep-ph/0609232]



BR ($B \to X_s\gamma$) with the cut $E_\gamma > E_0$

- Theory and experiment compared for $E_\gamma > E_0$; need to evaluate the fraction $F(E_0)$ of the events surviving this cut to get full BR; usually calculated using shape functions [Kagan, Neubert; Benson, Bigi, Uraltsev,...]
- $F(E_0)$ can be calculated without reference to shape functions using a multi-scale OPE [Neubert, hep-ph/0408179]
- Theoretical framework: Soft Collinear Effective Theory (SCET) involving several scales: m_b, $m_b\Delta$, and Δ, with $\Delta = m_b - 2E_0$
- Large logarithms associated with these scales are summed at NLL order; sensitivity to the scale $\Delta \simeq 1.4$ GeV (for $E_0 = 1.6$ GeV) introduces additional uncertainties [Becher & Neubert hep-ph/0610067]:

$$T \equiv F(1.6\ \mathrm{GeV})/F(1.0\ \mathrm{GeV}) = (93^{+3}_{-5}(\mathrm{pert}) \pm 2(\mathrm{hadr}) \pm 2(\mathrm{param}))\%$$

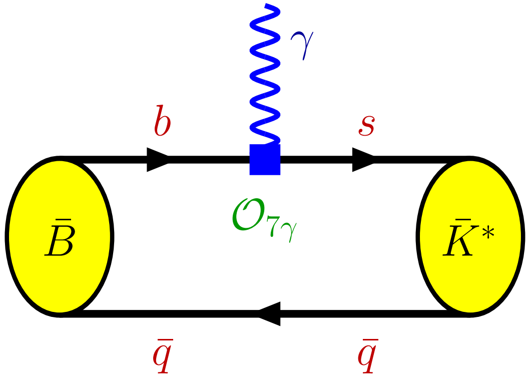
$$\Longrightarrow \mathcal{B}(B \to X_s\gamma) = (2.98 \pm 0.26) \times 10^{-4}$$

- $\mathcal{B}(B \to X_s\gamma)$ in the multi-scale Approach is about 1.4σ below Experiment

$$\frac{\mathcal{B}(B \to X_s\gamma)(\mathrm{exp})}{\mathcal{B}(B \to X_s\gamma)(\mathrm{SM})} = 1.19 \pm 0.09(\mathrm{exp}) \pm 0.10(\mathrm{th})$$

$B \to K^*\gamma$ Branching Fraction in LO

- In LO, only the electromagnetic penguin operator $\mathcal{O}_{7\gamma}$ contributes to the $B \to K^*\gamma$ amplitude; involves the form factor $T_1^{(K^*)}(0)$



$$\mathcal{M}^{\rm LO} = -\frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* C_7^{(0)\rm eff} \frac{e \bar{m}_b}{4\pi^2} T_1^{(K^*)}(0) \left[(Pq)(e^* \varepsilon^*) - (e^* P)(\varepsilon^* q) + i \, {\rm eps}(e^*, \varepsilon^*, P, q) \right]$$

Here, $P^\mu = p_B^\mu + p_K^\mu$; $q^\mu = p_B^\mu - p_K^\mu$ is the photon four-momentum; e^μ is its polarization vector; ε^μ is the K^*-meson polarization vector

- Branching ratio:

$$\mathcal{B}^{\rm LO}(B \to K^*\gamma) = \tau_B \frac{G_F^2 |V_{tb} V_{ts}^*|^2 \alpha M^3}{32\pi^4} \bar{m}_b^2(\mu_b) \, |C_7^{(0)\rm eff}(\mu_b)|^2 \, |T_1^{(K^*)}(0, \mu_b)|^2$$

$B \to K^* \gamma$ decay rates in NLO

- Perturbative improvements undertaken in the Large $E_{K^*} \sim m_B/2$ limit (QCD-F; PQCD; SCET)

Factorization Ansatz (QCDF):

[Beneke, Buchalla, Neubert, Sachrajda; Beneke & Feldmann]

$$f_k(q^2) = C_{\perp k}\xi_\perp(q^2) + \Phi_B \otimes T_k \otimes \Phi_V$$

Perturbative Corrections

- **Wilson Coefficients; Vertex Correction**

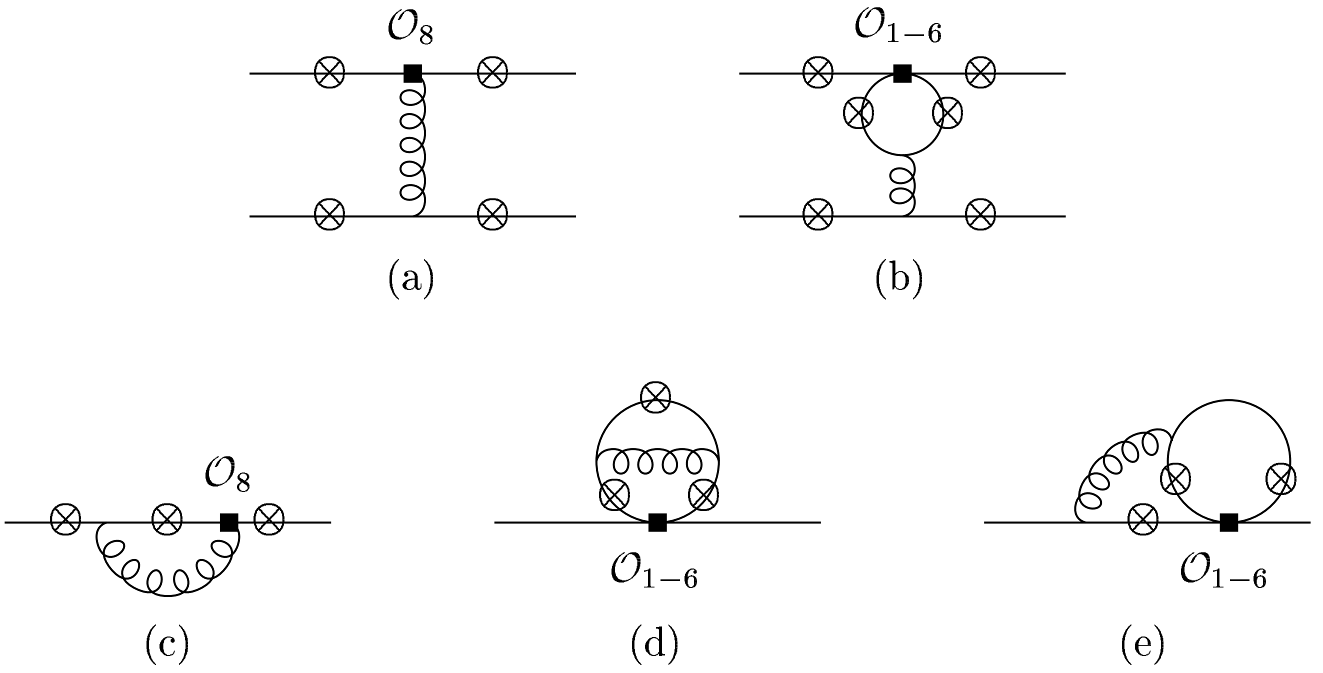
$$C_i = C_i^{(0)} + \frac{\alpha_s}{\pi} C_i^{(1)} + \ldots$$

- T_k: **Hard Spectator Corrections**

$$\Delta\mathcal{M}^{(\mathrm{HSC})} \propto \int_0^1 du \int_0^\infty dl_+ \, M^{(B)} M^{(V)} \, T_k$$

- $M^{(B)}$ and $M^{(V)}$ B-Meson & V-Meson Projection Operators

Nonfactorizable α_s Corrections



- First line: hard-spectator corrections
- Second line: $b \to s\gamma$ vertex corrections

$B \to K^*\gamma$ Branching Fraction in NLO

[Parkhomenko, A.A.; Beneke, Feldmann, Seidel; Bosch, Buchalla]

$$\mathcal{B}(B \to K^*\gamma) = \tau_B \frac{G_F^2 \alpha |V_{tb} V_{ts}^*|^2}{32\,\pi^4}\, m_{b,\mathrm{pole}}^2\, M^3 \left[\xi_\perp^{(K^*)}(0)\right]^2 \left|C_7^{(0)\mathrm{eff}}(\mu) + A^{(1)}(\mu)\right|^2$$

The function $A^{(1)}(\mu)$ includes all the NLO corrections

$$A^{(1)}(\mu) = A_{C_7}^{(1)}(\mu) + A_{\mathrm{ver}}^{(1)}(\mu) + A_{\mathrm{sp}}^{(1)K^*}(\mu_{\mathrm{sp}})$$

- $A_{C_7}^{(1)}(\mu)$: $\mathcal{O}(\alpha_s)$ corrections from WC $C_7^{\mathrm{eff}}(\mu)$ and $\bar{m}_b(\mu)$
- $A_{\mathrm{ver}}^{(1)}(\mu)$: $\mathcal{O}(\alpha_s)$ correction to the $b \to s\gamma$ vertex
- $A_{\mathrm{sp}}^{(1)K^*}(\mu_{\mathrm{sp}})$: $\mathcal{O}(\alpha_s)$ hard-spectator contribution evaluated at intermediate scale $\mu_{\mathrm{sp}} = \sqrt{\mu\Lambda_{\mathrm{H}}}$ with $\Lambda_{\mathrm{H}} \simeq 0.5$ GeV

- $\xi_\perp^{(K^*)}(0)$ determined from data to be compared with NP estimates

Experimental Data on $B \to K^*\gamma$ Decays

Branching ratios (in units of 10^{-6})

Quantity	BABAR	BELLE	CLEO	Average
$\mathcal{B}(B^+ \to K^{*+}\gamma)$	$38.7 \pm 2.8 \pm 2.6$	$42.5 \pm 3.1 \pm 2.4$	$37.6^{+8.9}_{-8.3} \pm 2.8$	40.3 ± 2.6
$\mathcal{B}(B^0 \to K^{*0}\gamma)$	$39.2 \pm 2.0 \pm 2.4$	$40.1 \pm 2.1 \pm 1.7$	$45.5^{+7.2}_{-6.8} \pm 3.4$	40.1 ± 2.0
$\mathcal{B}(B \to K^*\gamma)$	40.4 ± 2.5	42.8 ± 2.4	43.3 ± 6.2	41.8 ± 1.7
$\mathcal{B}(B \to X_s\gamma)$	$327 \pm 18^{+55}_{-41}$	$355 \pm 32^{+30+11}_{-31-7}$	$321 \pm 43^{+32}_{-29}$	$355 \pm 24^{+9}_{-10} \pm 3$
$R(K^*\gamma/X_s\gamma)$	$0.124^{+0.024}_{-0.019}$	$0.121^{+0.018}_{-0.015}$	$0.135^{+0.033}_{-0.027}$	0.124 ± 0.012

$$\bar{\mathcal{B}}(B \to K^*\gamma) \equiv \frac{1}{2}\left[\mathcal{B}(B^+ \to K^{*+}\gamma) + \frac{\tau_{B^+}}{\tau_{B^0}}\mathcal{B}(B^0 \to K^{*0}\gamma)\right]$$

$$R(K^*\gamma/X_s\gamma) \equiv \frac{\bar{\mathcal{B}}(B \to K^*\gamma)}{\mathcal{B}(B \to X_s\gamma)}$$

Life-time ratio $\tau_{B^+}/\tau_{B^0} = 1.076 \pm 0.008$

Comparison with data

$$\mathcal{B}_{\rm th}(B \to K^*\gamma) = \tau_B \frac{G_F^2 \alpha |V_{tb}V_{ts}^*|^2}{32\pi^4} m_{b,\rm pole}^2 M^3 \left[\xi_\perp^{(K^*)}\right]^2 \left(1 - \frac{m_{K^*}^2}{M^2}\right)^3 K_{\rm NLO} \left|C_7^{(0)\rm eff}\right|^2$$

$$K_{\rm NLO} = \frac{\left|C_7^{(0)\rm eff} + A^{(1)}(\mu)\right|^2}{\left|C_7^{(0)\rm eff}\right|^2} \quad \text{with} \quad 1.5 \leq K \leq 1.7$$

$$\mathcal{B}_{\rm th}(B^0 \to K^{*0}\gamma) \simeq (6.9 \pm 1.1) \times 10^{-5} \left(\frac{m_{b,\rm pole}}{4.65\ \rm GeV}\right)^2 \left(\frac{\xi_\perp^{(K^*)}}{0.35}\right)^2$$

$$\mathcal{B}_{\rm th}(B^\pm \to K^{*\pm}\gamma) \simeq (7.4 \pm 1.2) \times 10^{-5} \left(\frac{m_{b,\rm pole}}{4.65\ \rm GeV}\right)^2 \left(\frac{\xi_\perp^{(K^*)}}{0.35}\right)^2$$

- $T_1^{K^*}(0) = (1 + O(\alpha_s))\xi_\perp^{(K^*)}(0)$
  [Beneke, Feldmann]

Current Experimental Average

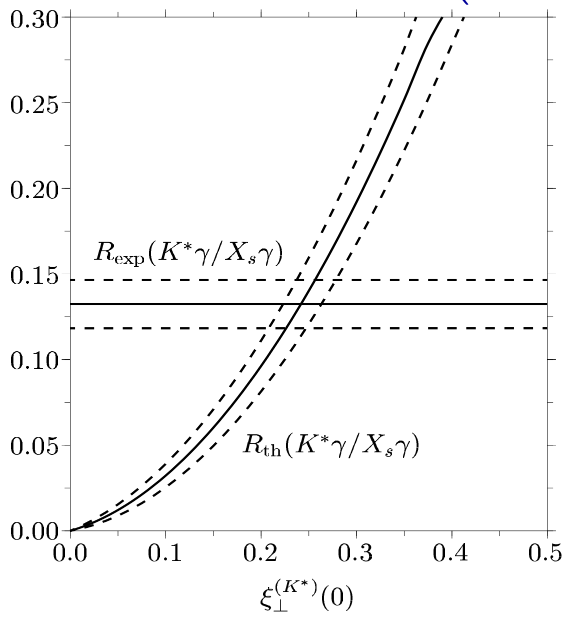
$\mathcal{B}(B^0 \to K^{*0}\gamma) = (4.01 \pm 0.20) \times 10^{-5}$

$\mathcal{B}(B^\pm \to K^{*\pm}\gamma) = (4.03 \pm 0.26) \times 10^{-5}$

- Using the ratio

$R_{\rm exp}(K^*\gamma/X_s\gamma) = 0.124 \pm 0.012$

$\Longrightarrow \quad T_1^{K^*}(0) = 0.27 \pm 0.02$



$B \to K^*\gamma$ *Decays*

Phenomenological Evaluation of $\xi_\perp^{(K^*)}(0)$

- $\mathcal{B}(B^+ \to K^{*+}\gamma)$: $\xi_\perp^{(K^*)}(0) = 0.262 \pm 0.023$
- $\mathcal{B}(B^0 \to K^{*0}\gamma)$: $\xi_\perp^{(K^*)}(0) = 0.272 \pm 0.021$
- $R(K^*\gamma/X_s\gamma)$: $\xi_\perp^{(K^*)}(0) = 0.253 \pm 0.018$

$$\langle \xi_\perp^{(K^*)}(0) \rangle = 0.26 \pm 0.02$$

Relation with the full QCD form factor $T_1^{(K^*)}(0)$ [Beneke, Feldmann]

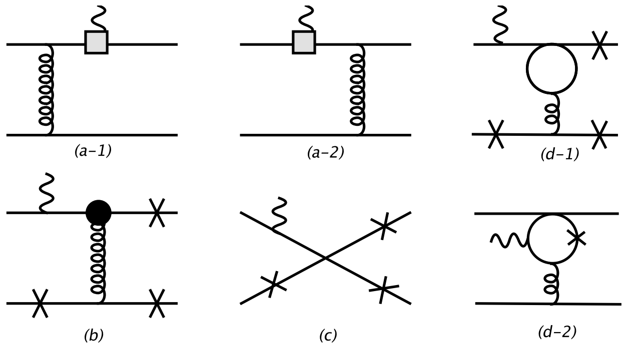
$$T_1^{(K^*)}(0,\mu) = \xi_\perp^{(K^*)}(0) \left\{ 1 + \frac{\alpha_s(\mu)}{4\pi} C_F \left[\ln \frac{m_b^2}{\mu^2} - 1 \right] \right\} + \frac{\alpha_s(\mu_{\rm sp})}{8\pi} C_F \, \Delta F_\perp^{(V)}(\mu_{\rm sp})$$

$$\Longrightarrow \quad \bar{T}_1^{(K^*)}(0, \bar{m}_b) \simeq 1.04 \, \bar{\xi}_\perp^{(K^*)}(0) = 0.27 \pm 0.02$$

- LCSR [Ball and Zwicky (2005)] $T_1^{(K^*)}(0) = 0.33 \pm 0.04$
- Need to calculate NNLO corrections (under way) [Pecjak, Greub, Parkhomenko, AA]

$B \to K^*\gamma$ in PQCD

[Keum, Matsumori, Sanda]



$$\begin{aligned} Br(B^0 \to K^{*0}\gamma) &= (4.9 \pm 2.5) \times 10^{-5} \\ Br(B^\pm \to K^{*\pm}\gamma) &= (5.0 \pm 2.5) \times 10^{-5} \end{aligned}$$

$\Longrightarrow$ Form factor: $T_1^{K^*}(0) = 0.23 \pm 0.06$

in agreement with QCDF-based estimates of the same and data

- Isospin Symmetry Breaking :

$$\Delta_{0-} = \frac{\frac{\tau_{B^+}}{\tau_{B^0}} Br(B^0 \to \bar{K}^{0*}\gamma) - Br(B^- \to K^{*-}\gamma)}{\frac{\tau_{B^+}}{\tau_{B^0}} Br(B^0 \to \bar{K}^{0*}\gamma) + Br(B^- \to K^{*-}\gamma)} = (3.0 \pm 0.9)\%$$

[Cf: $\Delta_{0-} = (8 \pm 4)\%$ [Kagan, Neubert (QCDF)]]

- $\Delta_{0-}(K^*\gamma)^{exp} = (3.9 \pm 4.8)\%$

$B \to \rho\gamma$ Branching Fraction

- Ignoring the perturbative QCD corrections to the penguin amplitudes, one has

$$\frac{\mathcal{B}(B^- \to \rho^-\gamma)}{2\mathcal{B}(B^0 \to \rho^0\gamma)} \simeq \left| 1 + \epsilon_A e^{i\phi_A} \frac{V_{ub}V_{ud}^*}{V_{tb}V_{td}^*} \right|^2$$

- $\epsilon_A e^{i\phi_A}$ includes dominant W-annihilation and possible sub-dominant long-distance contributions
- The strong interaction phase ϕ_A is vanishingly small in $\mathcal{O}(\alpha_s)$ in the chiral limit, $\phi_A \simeq 0$
- Isospin-violating corrections depend on the unitarity triangle angle α

$$\frac{V_{ub}V_{ud}^*}{V_{tb}V_{td}^*} = - \left| \frac{V_{ub}V_{ud}^*}{V_{tb}V_{td}^*} \right| e^{i\alpha} = F_1 + iF_2$$

$B \to \rho\gamma$ Branching Fraction in NLO

- Including the annihilation contribution, the charged-conjugate averaged branching ratio in the NLO is

$$\begin{aligned}
\bar{\mathcal{B}}_{\rm th}(B^\pm \to \rho^\pm\gamma) &= \tau_{B^+} \frac{G_F^2 \alpha |V_{tb}V_{td}^*|^2}{32\pi^4} m_{b,\rm pole}^2 M^3 \left[\xi_\perp^{(\rho)}(0)\right]^2 \\
&\times \left\{ (C_7^{(0)\rm eff} + A_R^{(1)t})^2 + (F_1^2 + F_2^2)(A_R^u + L_R^u)^2 \right. \\
&+ \left. 2F_1 \left[C_7^{(0)\rm eff}(A_R^u + L_R^u) + A_R^{(1)t} L_R^u \right] \right\}
\end{aligned}$$

- The amplitude $A^{(1)t}(\mu)$ can be decomposed into three parts

$$A^{(1)t}(\mu) = A_{C_7}^{(1)}(\mu) + A_{\rm ver}^{(1)}(\mu) + A_{\rm sp}^{(1)\rho}(\mu_{\rm sp})$$

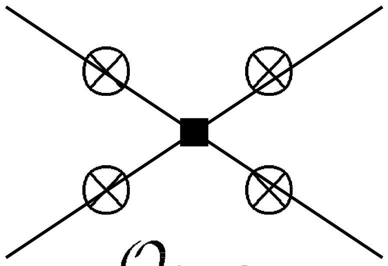
- In addition to $A^{(1)t}(\mu)$, the u-quark contribution $A^u(\mu)$ from penguins can no longer be ignored
- $A^u(\mu)$ also contains vertex and hard-spectator contributions

Weak-Annihilation Contribution

- $L_R^u = \epsilon_A\, C_7^{(0)\text{eff}}$ detemines the strength of the annihilation contribution
- Can be calculated in QCD Factorization

$$L_R^u(\rho^\pm) = \frac{4\pi^2\, f_B\, f_\rho\, m_\rho}{M^2\,\lambda_B\,\xi_\perp^{(\rho)}(0)} \left(C_1 + \frac{C_2}{N_c}\right)$$

$$L_R^u(\rho^0) = \frac{2\pi^2\, f_B\, f_\rho\, m_\rho}{M^2\,\lambda_B\,\xi_\perp^{(\rho)}(0)} \left(C_2 + \frac{C_1}{N_c}\right)$$



- $L_R^u(\omega) = -L_R^u(\rho^0)$ with obvious replacement $f_\rho\, m_\rho \to f_\omega\, m_\omega$
- Weak annihilation counts as Λ_{QCD}/m_b in the heavy-quark limit; power suppressed contribution
- LCSR estimates [AA, V. Braun, PLB 359 (1995) 223]: $\epsilon_A^{(\pm)} = +0.30 \pm 0.07$ and $\epsilon_A^{(0)} = -\epsilon_A^{(\omega)} = +0.03 \pm 0.01$

$B \to (\rho, \omega)\gamma$ Branching Fractions in NLO

- Preferable to work with the ratio of the $B \to \rho\gamma$ and $B \to K^*\gamma$ decay widths; normalize using the experimental value of $B \to K^*\gamma$ branching fraction ($S_\rho = 1$ for the $\rho^\pm$-meson and $S_\rho = 1/2$ for the ρ^0-meson)

$$\frac{\overline{\mathcal{B}}_{\rm th}(B \to \rho\gamma)}{\overline{\mathcal{B}}_{\rm th}(B \to K^*\gamma)} = S_\rho \left|\frac{V_{td}}{V_{ts}}\right|^2 \frac{(M^2 - m_\rho^2)^3}{(M^2 - m_{K^*}^2)^3}\,\zeta^2\,[1 + \Delta R(\rho/K^*)]$$

- Likewise for $B^0 \to \omega\gamma$

$$\frac{\overline{\mathcal{B}}_{\rm th}(B^0 \to \omega\gamma)}{\overline{\mathcal{B}}_{\rm th}(B^0 \to K^{*0}\gamma)} = \frac{1}{2}\left|\frac{V_{td}}{V_{ts}}\right|^2 \frac{(M^2 - m_\omega^2)^3}{(M^2 - m_{K^*}^2)^3}\,\zeta^2\,\zeta_{\omega/\rho}^2\,[1 + \Delta R(\omega/K^*)]$$

- ζ and $\zeta_{\omega/\rho}$ are the ratios of the Effective theory form factors; in the $SU(3)_{\rm F}$-symmetry limit, $\zeta = 1$ and $\zeta_{\omega/\rho} = 1$
- $SU(3)_{\rm F}$-breaking effects in the QCD form factors $T_1^{(K^*)}(0)$, $T_1^{(\rho)}(0)$, and $T_1^{(\omega)}(0)$ have been evaluated within the QCD sum-rules

$$\zeta = \frac{T_1^{(\rho)}(0)}{T_1^{(K^*)}(0)} \simeq \frac{\xi_\perp^{(\rho)}(0)}{\xi_\perp^{(K^*)}(0)} = 0.86 \pm 0.07 \qquad \zeta_{\omega/\rho} = \frac{T_1^{(\omega)}(0)}{T_1^{(\rho)}(0)} \simeq \frac{\xi_\perp^{(\omega)}(0)}{\xi_\perp^{(\rho)}(0)} = 0.9 \pm 0.1$$

$B \to (\rho,\omega)\gamma$ Branching Fractions

- Taking into account the ratio of the CKM matrix elements

$$|V_{td}/V_{ts}| = 0.201 \pm 0.008 \qquad [\Delta M_s;\ \text{CDF Collab. (2006)}]$$

the branching ratios can be estimated as

$$\bar{\mathcal{B}}_{\rm th}(B^\pm \to \rho^\pm\gamma) = (1.37 \pm 0.26[\rm th] \pm 0.09[\rm exp]) \times 10^{-6}$$
$$\bar{\mathcal{B}}_{\rm th}(B^0 \to \rho^0\gamma) = (0.65 \pm 0.12[\rm th] \pm 0.03[\rm exp]) \times 10^{-6}$$
$$\bar{\mathcal{B}}_{\rm th}(B^0 \to \omega\gamma) = (0.53 \pm 0.12[\rm th] \pm 0.02[\rm exp]) \times 10^{-6}$$

Branching ratios (in units of 10^{-6}) [HFAG, April 2007]

Mode	BABAR	BELLE	CLEO	Average [HFAG]
$B^+ \to \rho^+\gamma$	$1.10^{+0.37}_{-0.33} \pm 0.09$	$0.55^{+0.42+0.09}_{-0.36-0.08}$	< 13.0	$0.88^{+0.28}_{-0.26}$
$B^0 \to \rho^0\gamma$	$0.79^{+0.22}_{-0.20} \pm 0.06$	$1.25^{+0.37+0.07}_{-0.33-0.06}$	< 17.0	$0.93^{+0.19}_{-0.18}$
$B^0 \to \omega\gamma$	$0.40^{+0.24}_{-0.20} \pm 0.05$	$0.56^{+0.34+0.05}_{-0.27-0.10}$	< 9.2	$0.46^{+0.20}_{-0.17}$

- Large isospin-violation in Data curious, in all likelihood statistical

$SU(3)_{\rm F}$-averaged $B \to (\rho,\omega)\gamma$ Branching Ratio

- This averaging procedure is defined as

$$\bar{\mathcal{B}}[B \to (\rho,\omega)\,\gamma] \equiv \frac{1}{2}\left\{\mathcal{B}(B^+ \to \rho^+\gamma) + \frac{\tau_{B^+}}{\tau_{B^0}}\left[\mathcal{B}(B_d^0 \to \rho^0\gamma) + \mathcal{B}(B_d^0 \to \omega\gamma)\right]\right\}$$

- Combining all the branching fractions together, such an estimate gives

$$\bar{\mathcal{B}}_{\rm th}[B \to (\rho,\omega)\gamma] = (1.32 \pm 0.26) \times 10^{-6}$$

- Good agreement with experimental measurements within current errors

Branching ratios (in units of 10^{-6}) [HFAG, April 2007]

Mode	BABAR	BELLE	CLEO	Average [HFAG]
$B \to (\rho,\omega)\gamma$	$1.25 \pm 0.25 \pm 0.09$	$1.32^{+0.34+0.10}_{-0.31-0.09}$	< 14.0	$1.28^{+0.21}_{-0.20}$

Determination of $|V_{td}/V_{ts}|$ from $\bar{R}_{\rm exp}[(\rho,\omega)\,\gamma/K^*\gamma]$

- To extract the value of $|V_{td}/V_{ts}|$ from the $B \to (K^*,\rho,\omega)\,\gamma$ decays, one can use the ratio

$$\bar{R}_{\rm exp}[(\rho,\omega)\gamma/K^*\gamma] = \frac{\bar{\mathcal{B}}_{\rm exp}[B \to (\rho,\omega)\,\gamma]}{\bar{\mathcal{B}}_{\rm exp}(B \to K^*\gamma)} = r_{\rm th}^{(\rho/\omega)} \left|\frac{V_{td}}{V_{ts}}\right|^2 \zeta^2$$

- ζ and $|V_{td}/V_{ts}|$ are treated as free variables
- All other parametric uncertainties are combined in $r_{\rm th}^{(\rho/\omega)}$ error

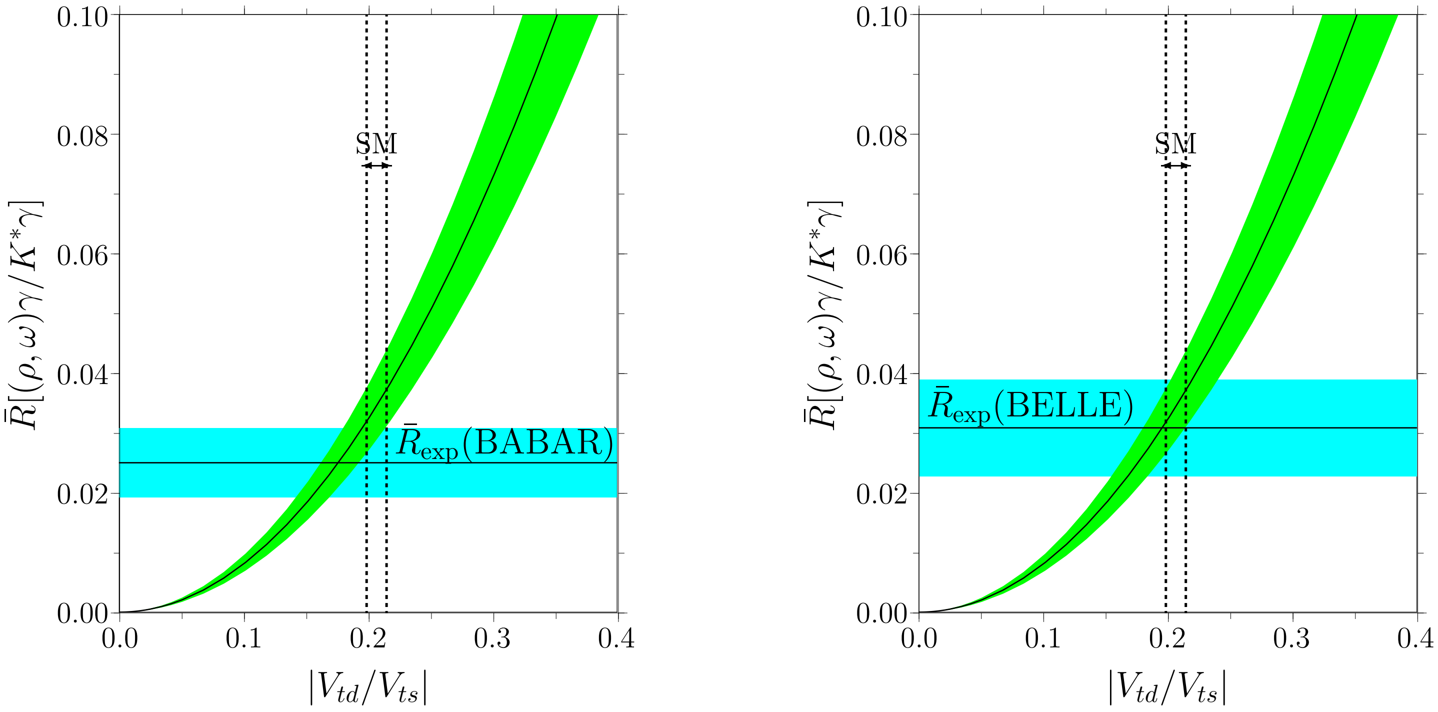
$$r_{\rm th}^{(\rho/\omega)} = 1.09 \pm 0.06$$

- Using recent estimates $\zeta = 0.86 \pm 0.07$ by Ball and Zwicky

Quantity	BABAR	BELLE	Average [HFAG]
$\bar{R}_{\rm exp}[(\rho,\omega)\gamma/K^*\gamma]$	0.025 ± 0.006	$0.032 \pm 0.008 \pm 0.002$	0.027 ± 0.005
$\|V_{td}/V_{ts}\|\,\zeta$	$0.151^{+0.017}_{-0.019}$	$0.171^{+0.021}_{-0.024}$	0.156 ± 0.014
$\|V_{td}/V_{ts}\|$	0.176 ± 0.026	0.199 ± 0.031	0.181 ± 0.022

- From global CKM fits: $|V_{td}/V_{ts}| = 0.2003^{+0.0146}_{-0.0059}$ [CKMfitter]
  $|V_{td}/V_{ts}| = 0.208 \pm 0.007$ [UTfit]

Determination of $|V_{td}/V_{ts}|$ from $\bar{R}_{\rm exp}[(\rho,\omega)\,\gamma/K^*\gamma]$



$|V_{td}/V_{ts}| = 0.171^{+0.018}_{-0.021}(\mathrm{exp})^{+0.017}_{-0.014}(\mathrm{th})$ $\quad$ $|V_{td}/V_{ts}| = 0.199^{+0.026}_{-0.025}(\mathrm{exp})^{+0.018}_{-0.015}(\mathrm{th})$

$$|V_{td}/V_{ts}|_{\rm SM} = 0.2003^{+0.0146}_{-0.0059}$$

$\bar{B} \to X_s l^+ l^-$

- The NNLO calculation of $\bar{B} \to X_s l^+ l^-$ corresponds to the NLO calculation of $\bar{B} \to X_s \gamma$, as far as the number of loops in the diagrams is concerned.

- Wilson Coefficients of the two additional operators

$$O_i = \frac{e^2}{16\pi^2} (\bar{s}_L \gamma_\mu b_L)(\bar{l}\gamma^\mu \gamma_5 l), \qquad i = 9, 10$$

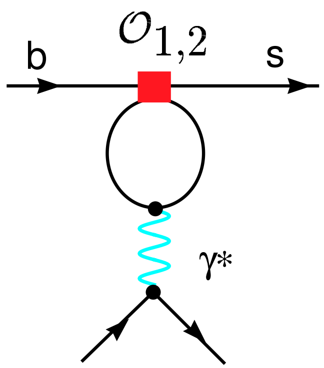
have the following perturbative expansion:

$$C_9(\mu) = \frac{4\pi}{\alpha_s(\mu)} C_9^{(-1)}(\mu) + C_9^{(0)}(\mu) + \frac{\alpha_s(\mu)}{4\pi} C_9^{(1)}(\mu) + \ldots$$

$$C_{10} = C_{10}^{(0)} + \frac{\alpha_s(M_W)}{4\pi} C_{10}^{(1)} + \ldots$$

- After an expansion in α_s, the term $C_9^{(-1)}(\mu)$ reproduces (the dominant part of) the electro-weak logarithm that originates from photonic penguins with charm quark loops:

$$\frac{4\pi}{\alpha_s(m_b)} C_9^{(-1)}(m_b) = \frac{4}{9} \ln \frac{M_W^2}{m_b^2} + \mathcal{O}(\alpha_s)$$

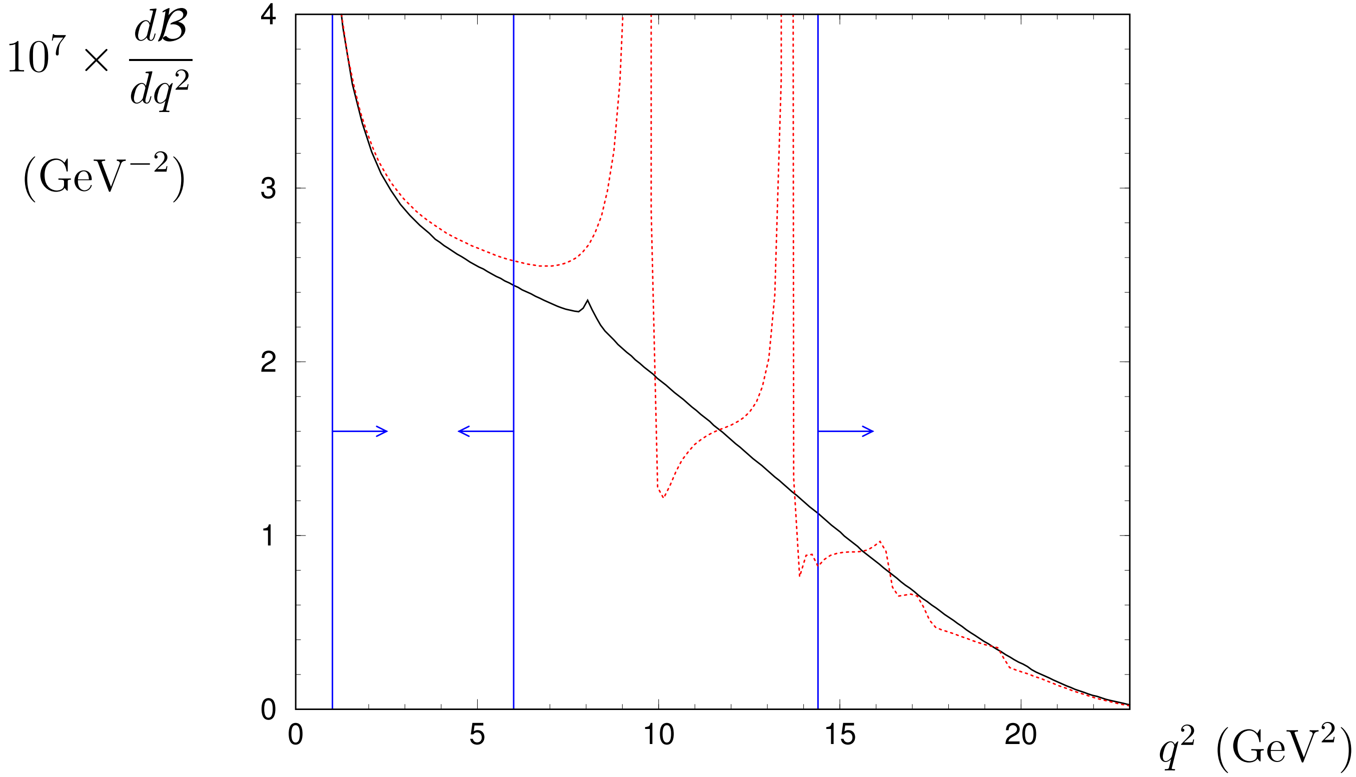


$C_9^{(-1)}(m_b) \simeq 0.033 \ll 1 \quad \Longrightarrow \quad \frac{4\pi}{\alpha_s(m_b)} C_9^{(-1)}(m_b) \simeq 2$

On the other hand: $C_9^{(0)}(m_b) \simeq 2.2$; need to calculate NNLO

Dilepton invariant mass distribution in $\bar{B} \to X_s \ell^+ \ell^-$:

[Ghinculov, Hurth, Isidori, Yao 2004]



- $\mathbf{BR}(\bar{B} \to X_s \ell^+ \ell^-);\quad q^2 \in\ [1,6]\ \mathrm{GeV}^2 = (1.63 \pm 0.20) \times 10^{-6}$
- $\mathbf{BR}(\bar{B} \to X_s \ell^+ \ell^-);\quad q^2 > 14\ \mathrm{GeV}^2 = (4.04 \pm 0.78) \times 10^{-7}$
- $\mathbf{BR}(\bar{B} \to X_s \ell^+ \ell^-);\quad q^2 > 4m_\mu^2 = (4.6 \pm 0.8) \times 10^{-6}$,
  in agreement with the other NNLO analysis [AA, Greub, Hiller, Lunghi 2001; Bobeth, Gambino, Gorbahn, Haisch, 2003; Huber, Lunghi, Misiak, Wyler 2005]

Comparison of $B \to X_s \ell^+ \ell^-$ with Data

- $B \to X_s \ell^+ \ell^-$ decay rate

$$\mathcal{B}(B \to X_s \ell^+ \ell^-)(M_{\ell\ell} > 0.2\ \mathrm{GeV}) = (4.50^{+1.03}_{-1.02}) \times 10^{-6} \quad [\mathrm{HFAG'07}]$$

$$SM : (4.2 \pm 0.7) \times 10^{-6} \quad [\mathrm{AGHL'01}]; \quad (4.6 \pm 0.8) \times 10^{-6} \quad [\mathrm{GHIY'04}]$$

- Differential distributions in $B \to X_s \ell^+ \ell^-$

• $M(X_s)$-distribution: tests $s \to X_s$ fragmentation model; current FMs provide reasonable fit to data

• $q^2 = M^2_{\ell^+\ell^-}$-distribution away from the $J/\psi, \psi', ...$ resonances is sensitive to short-distance physics; current data in agreement with the SM estimates but the precision is not better than 25%

• Forward-Backward Asymmetry (FBA) is likewise sensitive to the SM and BSM effects, in particular encoded in the Wilson coefficients C_7, C_9 and C_{10}

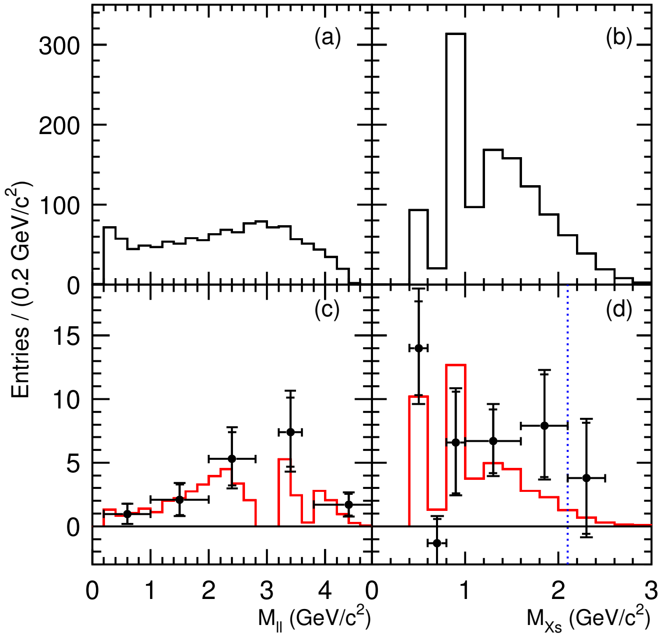
$$A_{\mathrm{FB}}(\hat{s} \sim C_{10}(2C_7 + C_9(\hat{s})\hat{s}); \quad \hat{s} = q^2/M_B^2$$

- $A_{\mathrm{FB}}(\hat{s})$ not yet measured; possible only in experiments at B factories

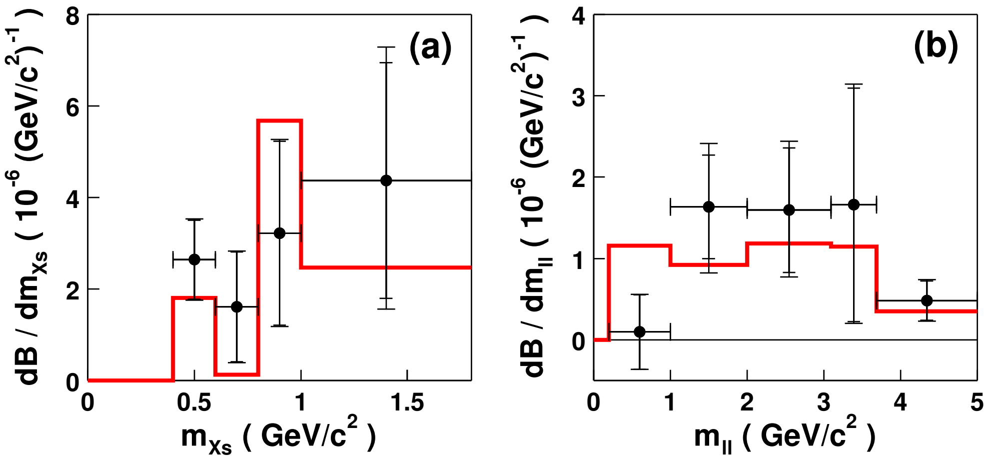
Decay distributions in $\bar{B} \to X_s \ell^+ \ell^-$

$M_{\ell\ell}$ and M_{X_s} Spectra

[BELLE]



[BABAR]



- In agreement with the NNLO SM calculations

NNLL-Forward-Backward Asymmetry in $B \to X_s \ell^+ \ell^-$

[Asatrian, Bieri, Greub, Hovhannisyan; Ghinculov, Hurth, Isidori, Yao]

Normalized FB Asymmetry

$$\overline{A}_{\mathrm{FB}}(\hat{s}) = \frac{\int_{-1}^{1} \frac{d^2\Gamma(b \to X_s\, \ell^+\ell^-)}{d\hat{s}\, dz}\, \mathrm{sgn}(z)\, dz}{\int_{-1}^{1} \frac{d^2\Gamma(b \to X_s\, \ell^+\ell^-)}{d\hat{s}\, dz}\, dz}$$

Unnormalized FB Asymmetry

$$A_{\mathrm{FB}}(\hat{s}) = \frac{\int_{-1}^{1} \frac{d^2\Gamma(b \to X_s\, \ell^+\ell^-)}{d\hat{s}\, dz}\, \mathrm{sgn}(z)\, dz}{\Gamma(B \to X_c e \bar{\nu}_e)}\, \mathrm{BR}_{\mathrm{sl}}$$

$$\int_{-1}^{1} \frac{d^2\Gamma(b \to X_s\, \ell^+\ell^-)}{d\hat{s}\, dz}\, \mathrm{sgn}(z)\, dz = \left(\frac{\alpha_{\mathrm{em}}}{4\,\pi}\right)^2 \frac{G_F^2\, m_{b,\mathrm{pole}}^5 \left|V_{ts}^* V_{tb}\right|^2}{48\,\pi^3} (1-\hat{s})^2$$
$$\times \left[-3\,\hat{s}\, \mathrm{Re}(\widetilde{C}_9^{\mathrm{eff}} \widetilde{C}_{10}^{\mathrm{eff}*}) \left(1 + \frac{2\alpha_s}{\pi} f_{910}(\hat{s})\right) - 6\, \mathrm{Re}(\widetilde{C}_7^{\mathrm{eff}} \widetilde{C}_{10}^{\mathrm{eff}*}) \left(1 + \frac{2\alpha_s}{\pi} f_{710}(\hat{s})\right)\right]$$

- NNLL Contributions stabilize the scale ($= \mu$) dependence of the FB Asymmetry

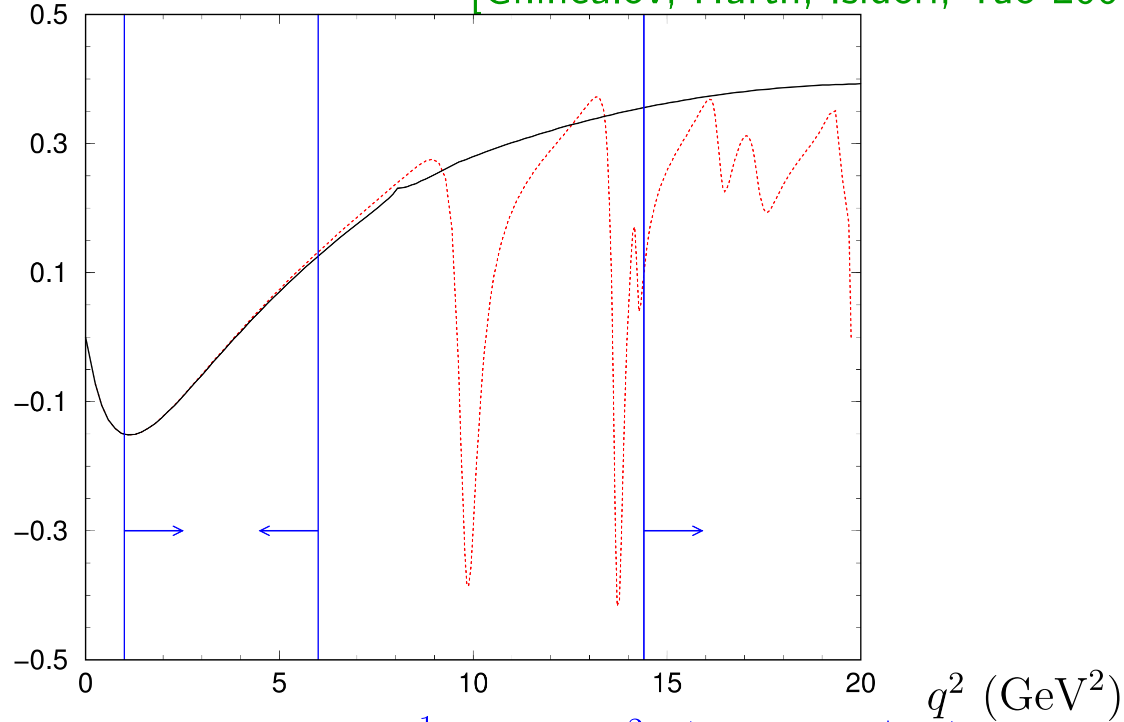
  $A_{\mathrm{FB}}^{\mathrm{NLL}}(0) = -(2.51 \pm 0.28) \times 10^{-6}$; $\quad A_{\mathrm{FB}}^{\mathrm{NNLL}}(0) = -(2.30 \pm 0.10) \times 10^{-6}$

- Zero of the FB Asymmetry is a precise test of the SM, correlating $\widetilde{C}_7^{\mathrm{eff}}$ and $\widetilde{C}_9^{\mathrm{eff}}$

  $\hat{s}_0^{\mathrm{NLL}} = 0.144 \pm 0.020$; $\quad \hat{s}_0^{\mathrm{NNLL}} = 0.162 \pm 0.008$

Normalized FB-Asymmetry in $\bar{B} \to X_s \ell^+ \ell^-$:

[Ghinculov, Hurth, Isidori, Yao 2004]



$$\bar{\mathcal{A}}_{\rm FB}(q^2) = \frac{1}{d\mathcal{B}(B \to X_s \ell^+ \ell^-)/dq^2} \int_{-1}^{1} d\cos\theta_\ell \, \frac{d^2\mathcal{B}(B \to X_s \ell^+ \ell^-)}{dq^2 \, d\cos\theta_\ell} \mathrm{sgn}(\cos\theta_\ell)$$

- Zero of the FB-Asymmetry is a precision test of the SM

$$q_0^2 = (3.90 \pm 0.25) \ \mathrm{GeV}^2 \qquad \text{[Ghinculov, Hurth, Isidori, Yao 2004]}$$
$$q_0^2 = (3.76 \pm 0.22_{\rm theory} \pm 0.24_{m_b}) \ \mathrm{GeV}^2 \qquad \text{[Bobeth, Gambino, Gorbahn, Haisch 2003]}$$

Exclusive Decays $B \to (K, K^*)\ell^+\ell^-$

- $B \to K$ (pseudoscalar P); $B \to K^*$ (Vector V) Transitions involve the currents:

$$\Gamma^1_\mu = \bar{s}\gamma_\mu(1-\gamma_5)b, \quad \Gamma^2_\mu = \bar{s}\sigma_{\mu\nu}q^\nu(1+\gamma_5)b$$

$$\langle P|\Gamma^1_\mu|B\rangle \supset f_+(q^2), f_-(q^2)$$

$$\langle P|\Gamma^2_\mu|B\rangle \supset f_T(q^2)$$

$$\langle V|\Gamma^1_\mu|B\rangle \supset V(q^2), A_1(q^2), A_2(q^2), A_3(q^2)$$

$$\langle V|\Gamma^2_\mu|B\rangle \supset T_1(q^2), T_2(q^2), T_3(q^2)$$

- 10 non-perturbative q^2-dependent functions (Form factors) $\Longrightarrow$ model-dependence
- Data on $B \to K^*\gamma$ provides normalization of $T_1(0) = T_2(0) \simeq 0.28$
- HQET/SCET-Approach allows to reduce the number of independent form factors from 10 to 3; perturbative symmetry-breaking corrections [Beneke, Feldmann, Seidel; Beneke, feldmann]
- HQET & SU(3) relate $B \to (\pi, \rho)\ell\nu_\ell$ and $B \to (K, K^*)\ell^+\ell^-$ to determine the remaining FF's

Electroweak Penguins $b \to s\ell^+\ell^-$

- $B \to (K, K^*)\ell^+\ell^-$ decay rates

  • Decay rates and distributions depend on the form factors; estimates given below based on Light-cone QCD Sum Rules [AA, Ball, Hiller, Handoko; hep-ph/9910221]

  $\mathcal{B}(B \to K\ell^+\ell^-) = (0.45 \pm 0.05) \times 10^{-6}$ [HFAG'06]; $(0.35 \pm 0.12) \times 10^{-6}$ [SM]

  $\mathcal{B}(B \to K^*e^+e^-) = (1.26 \pm 0.28) \times 10^{-6}$ [HFAG'06]; $(1.6 \pm 0.5) \times 10^{-6}$ [SM]

  $\mathcal{B}(B \to K^*\mu^+\mu^-) = (1.45 \pm 0.23) \times 10^{-6}$ [HFAG'06]; $(1.2 \pm 0.5) \times 10^{-6}$[SM]

- Differential distributions in $B \to (K, K^*)\ell^+\ell^-$

  • $q^2 = M^2_{\ell^+\ell^-}$-distribution away from the $J/\psi, \psi', \ldots$ resonances is sensitive to short-distance physics; first measurements available from BABAR and BELLE; will be greatly improved at the LHC and Super-B Factory

- The ratio $R_{K^*} \equiv \mathcal{B}(B \to K^*\mu^+\mu^-)/\mathcal{B}(B \to K^*e^+e^-)$ (likewise R_K) sensitive to Higgs contribution in the large-$\tan\beta$ region [Hiller & Krüger;...]

  • $A_{\rm FB}(\hat{s})[B \to K\ell^+\ell^-] \simeq 0$ in the SM and most BSM extensions; in agreement with data which is used as a control sample to measure $A_{\rm FB}(\hat{s})[B \to K^*\ell^+\ell^-]$

  • $A_{\rm FB}(\hat{s})$ in $B \to K^*\ell^+\ell^-$ - A precision test of SM in $b \to s$ sector; in particular, the zero-point of the FBA-function; calculated in NLO QCD-Factorization [Beneke, Feldmann, Seidel] and SCET [AA, Kramer, Zhu]; first measurements from BELLE and BABAR at hand; Super-B and LHC-B will measure $A_{\rm FB}(\hat{s})$ precisely

Experimental data vs. SM in $B \to (X_s, K, K^*)\ell^+\ell^-$ Decays

Branching ratios (in units of 10^{-6}) [HFAG: April 2007]

SM: [A.A., Lunghi, Greub, Hiller, hep-ph/0112300]

Decay Mode	Expt. (BELLE & BABAR)	Theory (SM)
$B \to K\ell^+\ell^-$	0.39 ± 0.06	0.35 ± 0.12
$B \to K^* e^+e^-$	$1.13^{+0.28}_{-0.26}$	1.58 ± 0.49
$B \to K^* \mu^+\mu^-$	$1.03^{+0.26}_{-0.23}$	1.19 ± 0.39
$B \to X_s \mu^+\mu^-$	$4.3^{+1.3}_{-1.2}$	4.2 ± 0.7
$B \to X_s e^+e^-$	4.7 ± 1.3	4.2 ± 0.7
$B \to X_s \ell^+\ell^-$	$4.50^{+1.03}_{-1.01}$	4.2 ± 0.7

- Inclusive measurements and the SM rates include a cut $M_{\ell^+\ell^-} > 0.2$ GeV
- SM & Data agree within $\mathbf{25\%}$

[AA, Kramer, Zhou; hep-ph/0601034]

The factorization formula in SCET

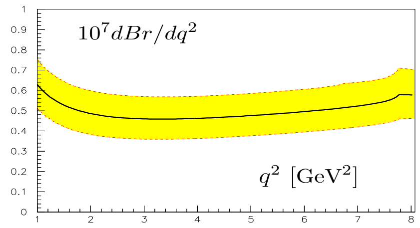
$$\langle K_a^* \ell^+ \ell^- | H_{eff} | B \rangle = T_a^I(q^2) \zeta_a(q^2) + \sum_{\pm} \int_0^\infty \frac{d\omega}{\omega} \phi_\pm^B(\omega) \int_0^1 du\, \phi_{K^*}^a(u) T_{a,\pm}^{II}(\omega, u, q^2)$$

where $a = \parallel, \perp$ denotes the polarization of the K^* meson

- formally coincides with the formula in QCD Factorization [Beneke/Feldmann/Seidel 2001], but valid to all orders of α_s
- for T^{II}, the logarithms are summed from $\mu = m_b$ to $\sqrt{m_b \Lambda_h}$
- Compared with BFS, the definition of $\zeta_{\perp,\parallel}$ is also different here

Comparison with Data

Numerical results

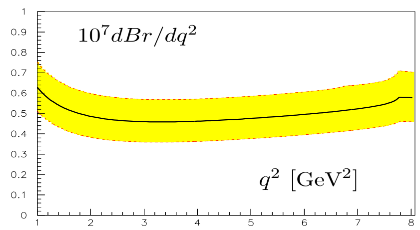


Theor. vs. Belle

$$Br|_{q^2 \in [4,8]\ \mathrm{GeV}^2} = (1.94^{+0.44}_{-0.40}) \times 10^{-7}$$
$$= (4.8^{+1.4}_{-1.2}|_{\mathrm{stat}} \pm 0.3|_{\mathrm{syst}} \pm 0.3|_{\mathrm{model}}) \times 10^{-7}$$

Comparison with experiments

Numerical results



Form factor determination

LCSRs $\zeta_{||}(0) = 0.40 \pm 0.05$, $\zeta_{\perp}(0) = 0.40 \pm 0.04$, their q^2 dependencies

LCSRs + $B \to K^* \gamma$ $\zeta_{\perp}(0) = 0.32 \pm 0.02$

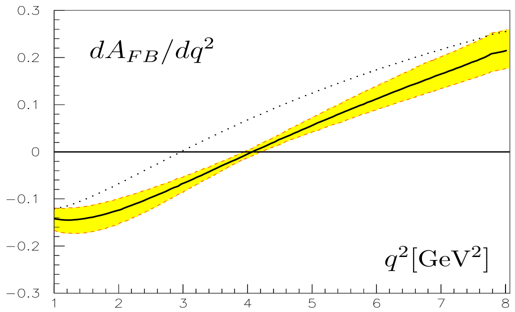
Theor. vs. BaBar

$$Br|_{q^2 \in [1,7]\ \mathrm{GeV}^2} = (2.92^{+0.57}_{-0.50}|_{\zeta_{||}}\ {}^{+0.30}_{-0.28}|_{\mathrm{CKM}}\ {}^{+0.18}_{-0.20}) \times 10^{-7}$$

$$Br|_{q^2 \in [0.1,8.4]\ \mathrm{GeV}^2} = (2.7^{+1.2}_{-1.0}|_{\mathrm{stat}} \pm 0.5|_{\mathrm{syst}}) \times 10^{-7}$$

Reduction of Scale Uncertainty in SCET

Forward-backward asymmetry



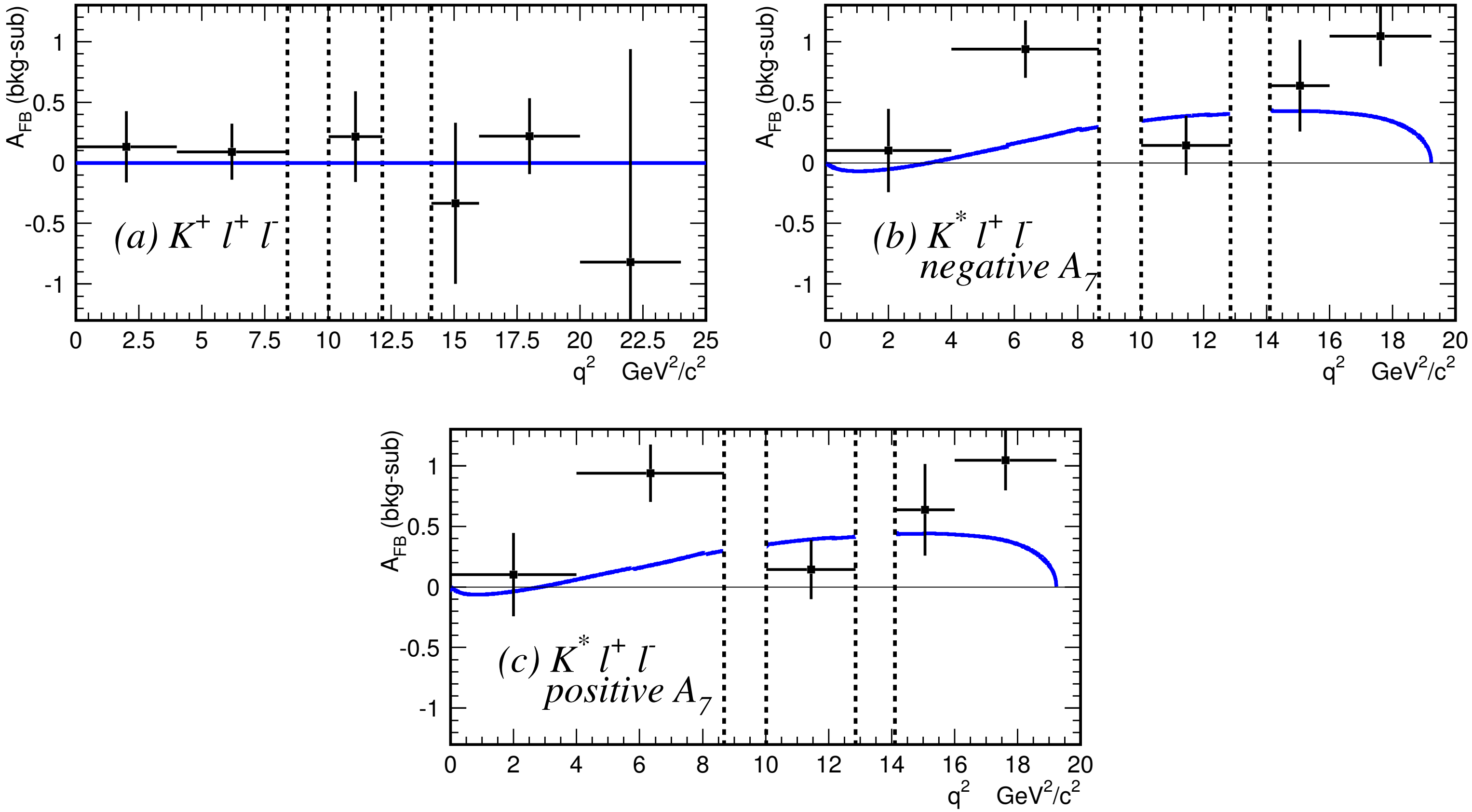
$A_{FB}(q_0^2) = 0$ free of hadronic uncertainties [Burdman1998, Ali et al., 2000]

$q_0^2 = (4.07^{+0.16}_{-0.13})$ GeV^2 with $\Delta(q_0^2)_{\text{scale}} = {}^{+0.08}_{-0.05}$ GeV^2

QCD-F [Beneke/Feldmann/Seidel 2001]

$q_0^2 = (4.39^{+0.38}_{-0.35})$ GeV^2 with $\Delta(q_0^2)_{\text{scale}} = \pm 0.25$ GeV^2

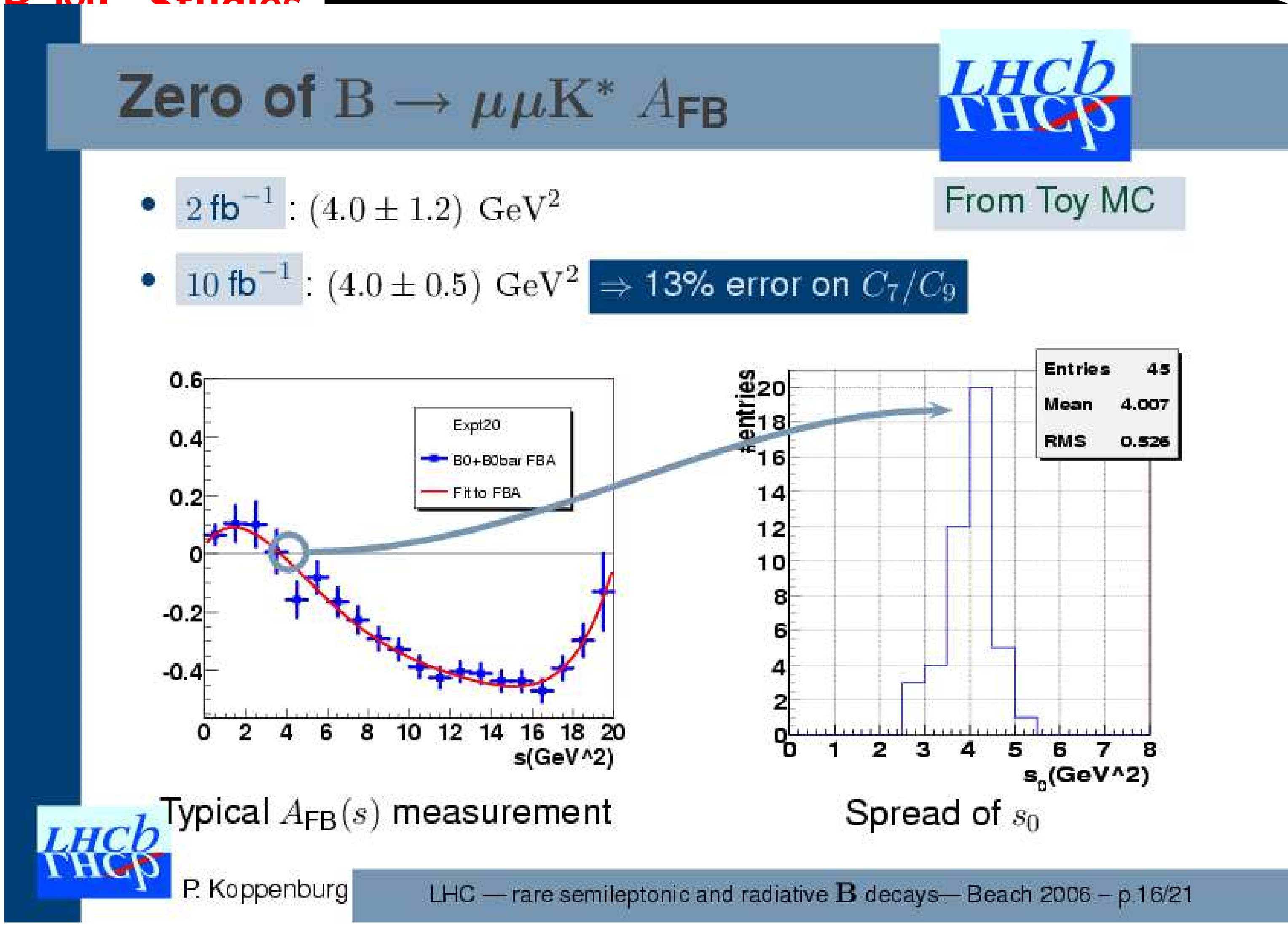
Belle FB Asymmetry Distributions (EPS 2005)



Best Fits

- $A_7 = -0.33$: $A_9/A_7 = -15.3^{+3.4}_{-4.8}$; $A_{10}/A_7 = 10.3^{+5.2}_{-3.5}$
- $A_7 = +0.33$: $A_9/A_7 = -16.3^{+3.7}_{-5.7}$; $A_{10}/A_7 = 11.1^{+6.0}_{-3.9}$
- SM: $A_7 = -0.33$; $A_9/A_7 = -12.3$; $A_{10}/A_7 = 12.8$

LHC-B MC Studies



Summary

- A tremendous experimental and theoretical effort is under way to quantitatively understand the rare B-decays; NNLO calculations undertaken for $\mathcal{B}(B \to X_s \gamma)$ and $\mathcal{B}(B \to X_s \ell^+ \ell^-)$; Similar theoretical precision underway for the exclusive decays using SCET

- Agreement with the SM is not completely quantitative; some benchmark measurements remain to be done; current attention is on the quantities $\mathcal{B}(B \to X_s \gamma)$; $\mathcal{B}(B \to X_s \ell^+ \ell^-)$, in particular the Forward-Backward Asymmetry of the leptons; $\mathcal{B}(B \to (K^*, \rho, \omega)\gamma)$; $\mathcal{B}(B_s \to \mu^+ \mu^-)$

- Rare B-Decays (and CP Asymmetries) have the potential to discover Physics Beyond-the-SM in the flavour sector; In all likelihood this would require the statistical power of a Super-B factory and LHC

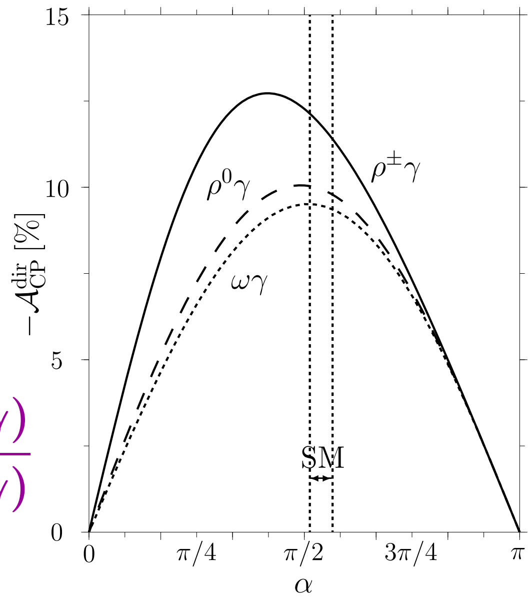
- Hope: The synergy of high energy frontier and low energy precision physics, which worked so well in piecing together the SM yielding precise knowledge of the CKM matrix, will continue to hold sway in the LHC-era, providing valuable information on the flavour aspects of the next Paradigm!

Backup Slides

Direct CP-Asymmetry

- CP-asymmetry arises from interference of penguin operator $\mathcal{O}_{7\gamma}$ and four-quark operators $\mathcal{O}_1$ and $\mathcal{O}_2$
- Direct CP-asymmetry in the $B^\pm \to \rho^\pm\gamma$ decay rates

$$\mathcal{A}_{\rm CP}(\rho^\pm\gamma) = \frac{\mathcal{B}(B^- \to \rho^-\gamma) - \mathcal{B}(B^+ \to \rho^+\gamma)}{\mathcal{B}(B^- \to \rho^-\gamma) + \mathcal{B}(B^+ \to \rho^+\gamma)}$$



- Similar defenitions in other two modes $B^0 \to \rho^0\gamma$ and $B^0 \to \omega\gamma$

$$\mathcal{A}_{\rm CP}(\rho^\pm\gamma) = (-11.8 \pm 2.9)\%$$

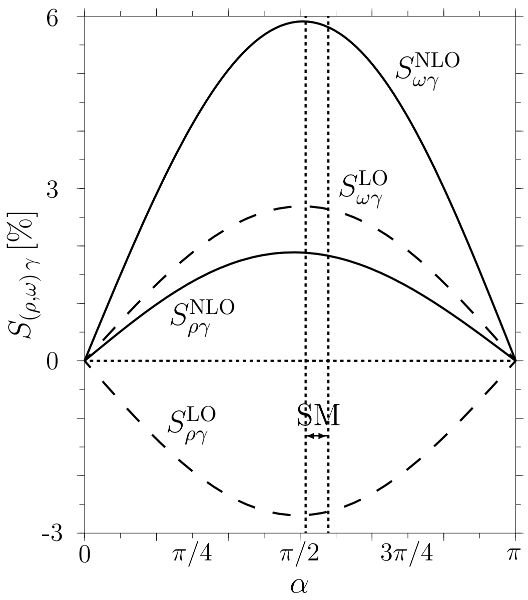
$$\mathcal{A}_{\rm CP}(\rho^0\gamma) = (-9.9^{+3.8}_{-3.4})\%$$

$$\mathcal{A}_{\rm CP}(\omega\gamma) = (-9.5^{+4.0}_{-3.6})\%$$

Mixing-Induced CP-Asymmetry

- Time-dependent CP-asymmetry in neutral B-meson decays involves interference of $B^0 - \bar{B}^0$ mixing and decay amplitudes

$$a_{\rm CP}^{\rho\gamma}(t) = -C_{\rho\gamma}\cos(\Delta M_d\, t) + S_{\rho\gamma}\sin(\Delta M_d\, t)$$



- $C_{\rho\gamma} = -\mathcal{A}_{\rm CP}^{\rm dir}(\rho^0\gamma)$
- Mixing-induced CP-asymmetry is

$$S_{\rho\gamma} = \frac{2\,{\rm Im}(\lambda_{\rho\gamma})}{1 + |\lambda_{\rho\gamma}|^2}, \quad \lambda_{\rho\gamma} \equiv \frac{q}{p}\,\frac{A(\bar{B}_d^0 \to \rho^0\gamma)}{A(B_d^0 \to \rho^0\gamma)}$$

- In the SM, $q/p = {\rm e}^{-2i\beta}$ is pure phase factor to a good approximation
- Similar defenitions can be written for the $B^0 \to \omega\gamma$ decay mode

$$S_{\rho\gamma}^{\rm LO} = (-2.7 \pm 0.9)\%, \qquad S_{\rho\gamma}^{\rm NLO} = (1.9^{+3.8}_{-3.2})\%$$

$$S_{\omega\gamma}^{\rm LO} = (+2.7 \pm 0.9)\%, \qquad S_{\omega\gamma}^{\rm NLO} = (5.9^{+4.1}_{-3.5})\%$$

Isospin-Violating Ratio

- Isospin-violating ratios in $B \to \rho\gamma$ decays are defined as

$$\Delta^{+0} = \frac{\Gamma(B^+ \to \rho^+\gamma)}{2\Gamma(B^0 \to \rho^0\gamma)} - 1, \qquad \Delta^{-0} = \frac{\Gamma(B^- \to \rho^-\gamma)}{2\Gamma(\bar{B}^0 \to \rho^0\gamma)} - 1$$

- Charged-conjugate averaged ratio

$$\Delta = \frac{1}{2}\left[\Delta^{+0} + \Delta^{-0}\right]$$

- In the leading order in α_s

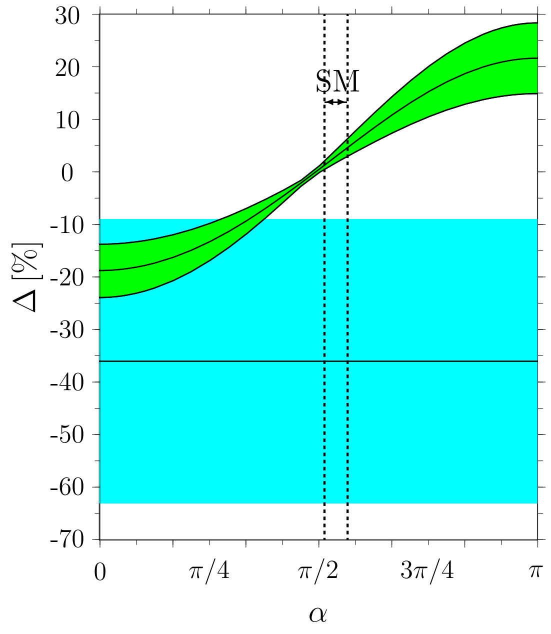
$$\Delta_{\rm LO} = -2\epsilon_A^{(\pm)}\,|\lambda_u|\cos\alpha + (\epsilon_A^{(\pm)})^2\,|\lambda_u|^2$$

- SM value is small $\quad [\epsilon_A^{(\pm)} = 0.30 \pm 0.07]$

$$\Delta = (2.9 \pm 2.1)\%$$

- Current BABAR measurement

$$\Delta_{\rm exp} = (-36 \pm 27)\%$$



$SU(3)_{\mathrm{F}}$-Violating Ratio

- $SU(3)_{\mathrm{F}}$-violating ratio $\Delta^{(\rho/\omega)}$ is based on neutral B-meson modes

$$\Delta^{(\rho/\omega)} \equiv \frac{1}{2}\left[\Delta_B^{(\rho/\omega)} + \Delta_{\bar{B}}^{(\rho/\omega)}\right]$$

$$\Delta_B^{(\rho/\omega)} \equiv \frac{(M_B^2 - m_\omega^2)^3\,\mathcal{B}(B_d^0 \to \rho^0\gamma) - (M_B^2 - m_\rho^2)^3\,\mathcal{B}(B_d^0 \to \omega\gamma)}{(M_B^2 - m_\omega^2)^3\,\mathcal{B}(B_d^0 \to \rho^0\gamma) + (M_B^2 - m_\rho^2)^3\,\mathcal{B}(B_d^0 \to \omega\gamma)}$$

- Main dependence on $\zeta_{\omega/\rho} = \bar{\xi}_\perp^{(\omega)}(0)/\bar{\xi}_\perp^{(\rho)}(0)$

$$\Delta^{(\rho/\omega)} = \frac{1 - \zeta_{\omega/\rho}^2}{1 + \zeta_{\omega/\rho}^2} + \frac{4\zeta_{\omega/\rho}^2}{(1 + \zeta_{\omega/\rho}^2)^2}\,\Delta_{SU(3)}^{(\rho/\omega)}$$

- In $SU(3)_{\mathrm{F}}$-symmetry limit, $\zeta_{\omega/\rho} = 1$

$$\Delta^{(\rho/\omega)} = \Delta_{SU(3)}^{(\rho/\omega)} = (2.0 \pm 1.9) \times 10^{-3}$$

- For $\zeta_{\omega/\rho} = 0.9 \pm 0.1$ $\qquad \Delta^{(\rho/\omega)} = (11 \pm 11)\%$

- Estimate based on the HFAG averages

$$\Delta_{\mathrm{exp}}^{(\rho/\omega)} = (34 \pm 20)\%$$