Inclusive Semileptonic B Decays

Theoretical Tools and Uncertainties

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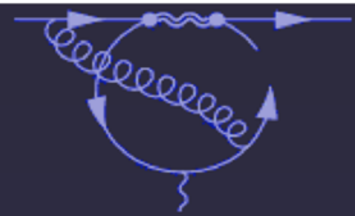
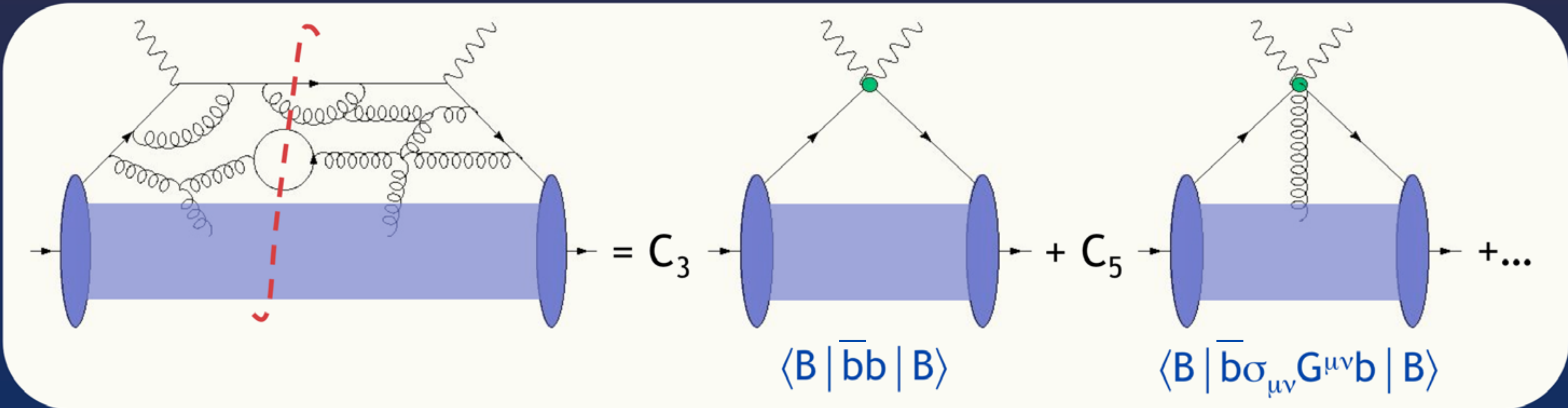


Inclusive $B \rightarrow X_c l\nu$ Decay: $|V_{cb}|$

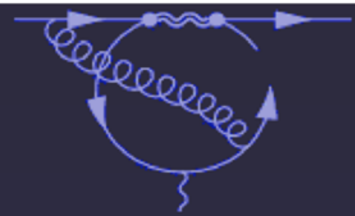
Laboratory for m_b & heavy-quark parameters

Theoretical tool: OPE

- Optical theorem:



- Model-independent predictions!



Theoretical tool: OPE

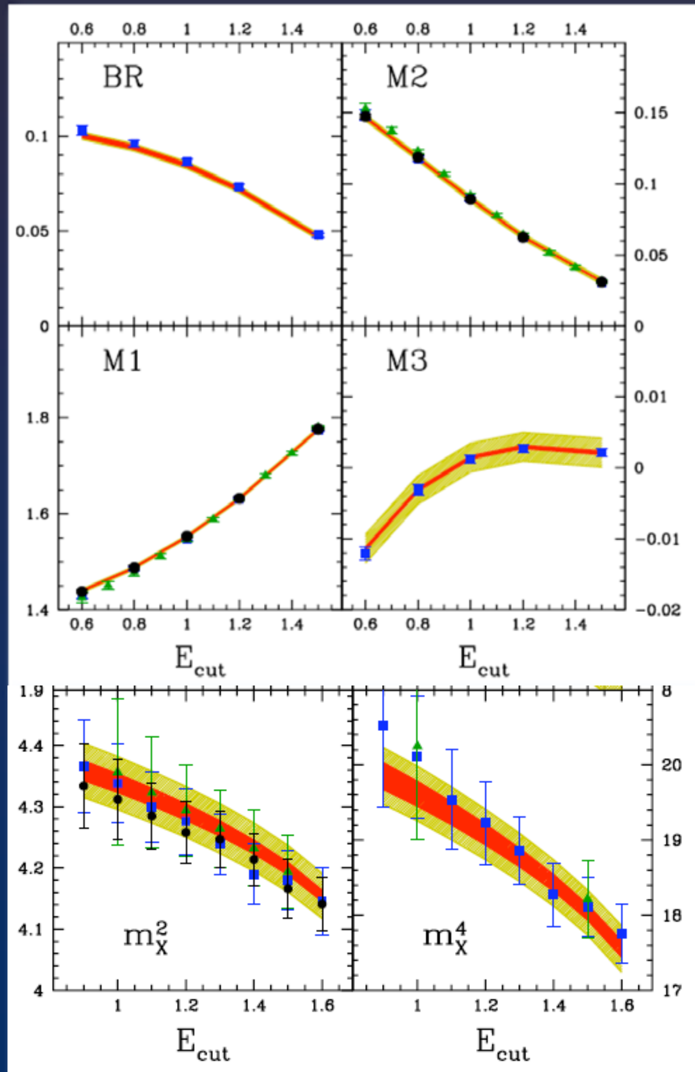
- Hadronic physics encoded in few parameters (forward B-meson matrix elements of local operators):

$$m_Q,\ \mu_\pi^2,\ \mu_G^2,\ \rho_D^3,\ \ldots\ (\text{or: } \bar{\Lambda},\ \lambda_1,\ \lambda_2\ \ldots)$$

- Only assumption: quark-hadron duality (believed to be reliable for $\Delta E = M_B - M_D$)

Global moment fit

- $|V_{cb}|$, m_Q, μ_π^2, μ_G^2 extracted from combined analysis of different decay spectra:
  - $B \to X_c l\nu$ lepton energy moments
  - $B \to X_c l\nu$ hadronic mass moments
  - $B \to X_s\gamma$ photon energy moments (problematic!)
- Data from BaBar, Belle, CLEO, CDF, DELPHI
- Measurements highly correlated



[Bauer, Ligeti, Luke, Manohar, +Trott (2002,2004); Battaglia et al. (2002); Bigi, Uraltsev (2003); Gambino, Uraltsev (2004)]

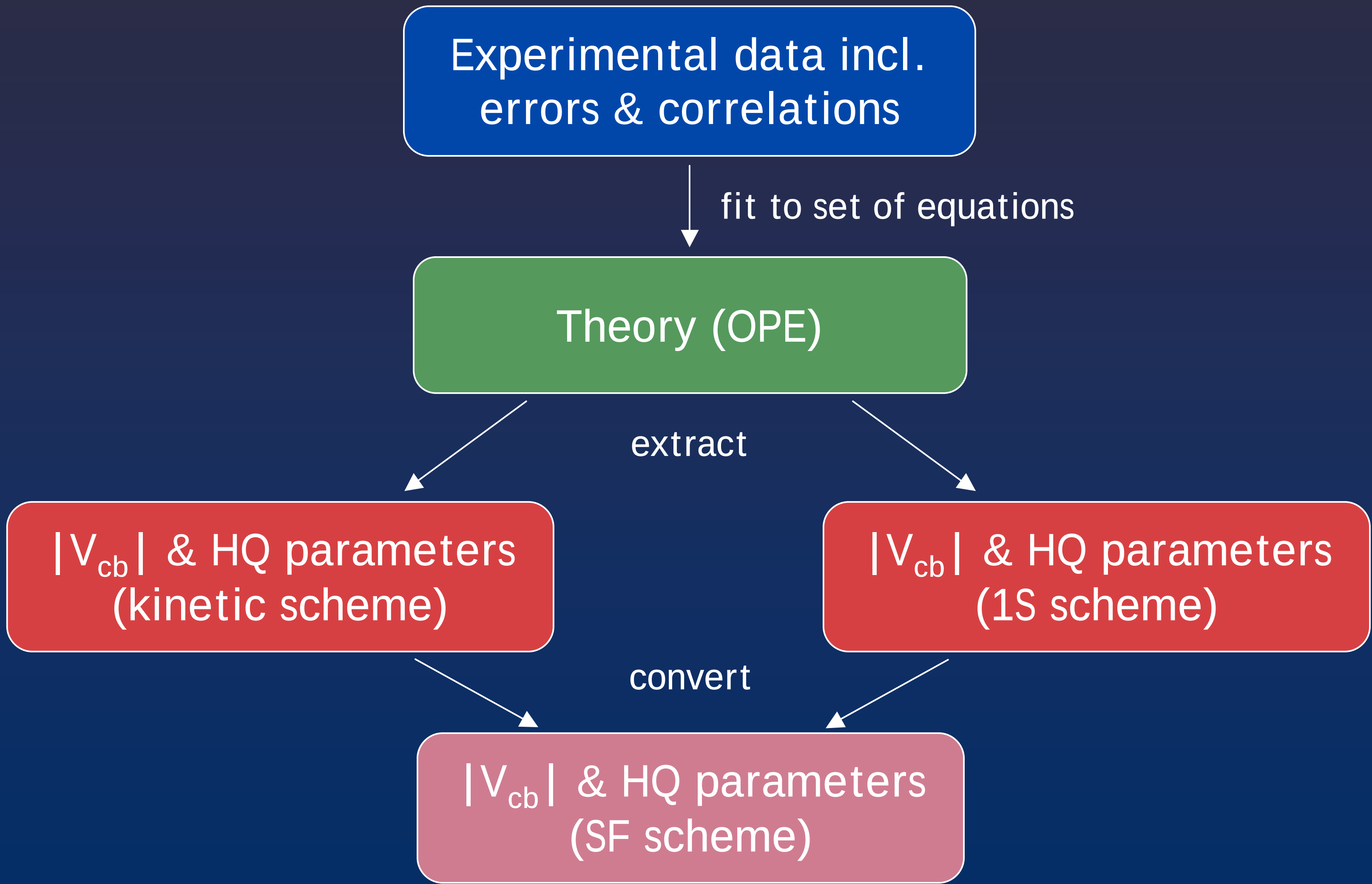
Status of theory

- Leading term at $O(\alpha_s, \alpha_s^2\beta_0)$, but not $O(\alpha_s^2)$
- Power corrections at tree level
- Technology exists for two-loop calculation of decay spectra [Anastasiou, Melnikov, Petriello (2005)]

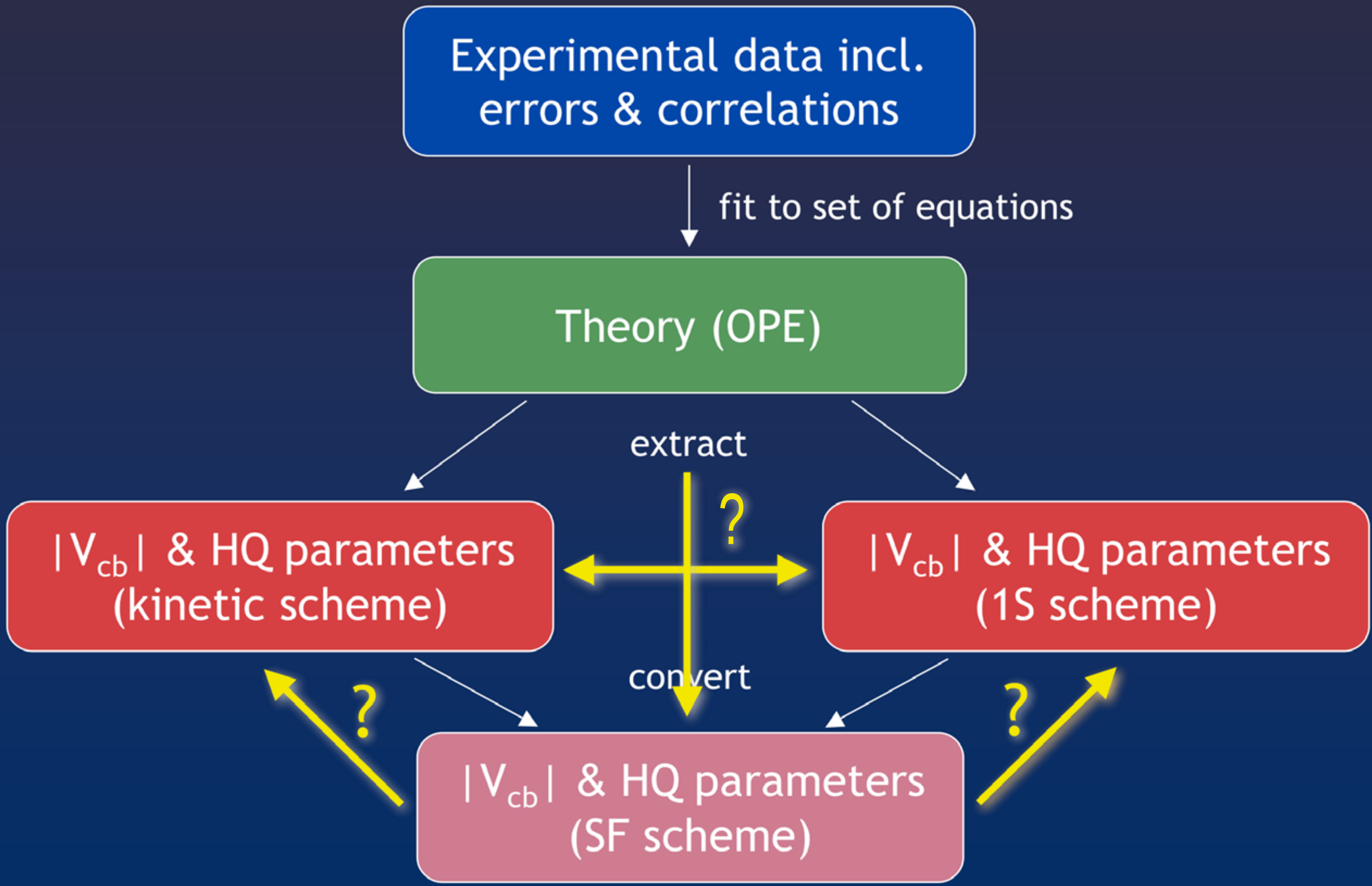
→ work in progress by several groups (also for one-loop corrections to μ_π^2 and μ_G^2 terms)

→ important!

Fit strategy



Fit strategy



Fit strategy

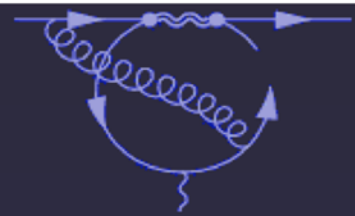
- Without truncation of perturbation theory, any path to a given scheme would lead to same result, e.g.:

[Fit in kinetic scheme]

=

[Fit in 1S scheme] ⊕ [Translation: 1S → kin.]

- In practice, results differ at finite order in α_s
- Presently quoted theory errors do not take this into account → too optimistic!

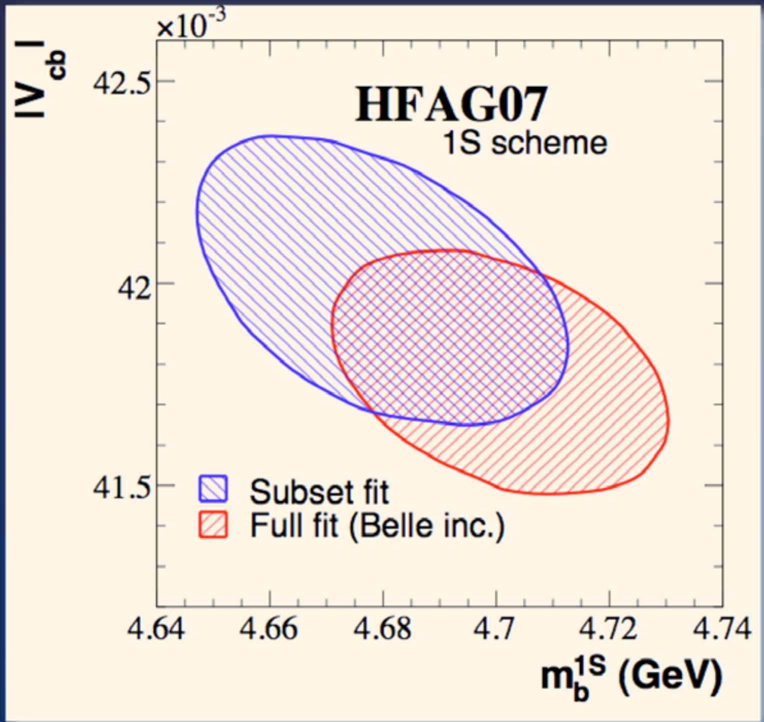


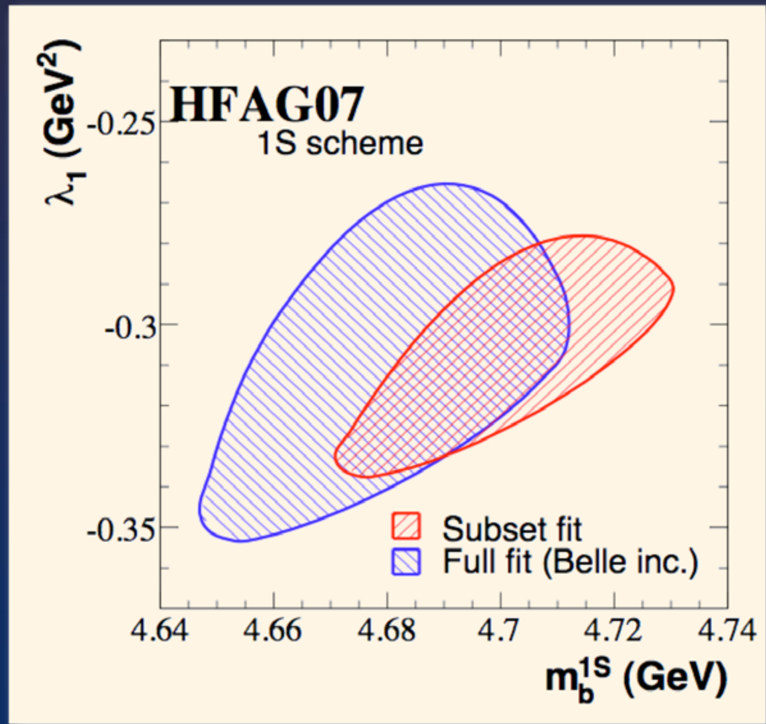
Fit results

Source (Scheme)	Measurements
Battaglia *et al.* (Kinetic) [274]	$\|V_{cb}\| = (41.9 \pm 0.7_{meas} \pm 0.6_{fit} \pm 0.4_{pert}) \times 10^{-3}$ $m_b^{\text{kin}} = 4.59 \pm 0.08_{fit} \pm 0.01_{syst.}$ GeV/c^2
Battaglia *et al.* (Pole) [274]	$\|V_{cb}\| = (41.3 \pm 0.7_{meas} \pm 0.7_{fit} \pm 0.2_{nl} \pm 0.9_{pert}) \times 10^{-3}$ $\overline{\Lambda} = 0.40 \pm 0.10_{fit} \pm 0.02_{syst.}$ GeV/c^2
CLEO (Pole) [275] (1S)	$\|V_{cb}\| = (40.8 \pm 0.5_{\Gamma_{\text{SL}}} \pm 0.4_{\lambda_1,\overline{\Lambda}} \pm 0.9_{theory}) \times 10^{-3}$ $\overline{\Lambda} = 0.39 \pm 0.03_{stat} \pm 0.06_{syst.} \pm 0.12_{theory}$ GeV/c^2 $m_b^{1\text{S}} = 4.82 \pm 0.07_{exp} \pm 0.11_{theory}$ GeV/c^2
BABAR (Kinetic) [276]	$\|V_{cb}\| = (41.4 \pm 0.4_{exp} \pm 0.4_{HQE} \pm 0.6_{theory}) \times 10^{-3}$ $m_b^{\text{kin}} = 4.61 \pm 0.05_{exp} \pm 0.04_{HQE} \pm 0.02_{theory}$ GeV/c^2
Bauer *et al.* (1S) [277]	$\|V_{cb}\| = (41.4 \pm 0.6 \pm 0.1_{\tau B}) \times 10^{-3}$ $m_b^{1\text{S}} = 4.68 \pm 0.03$ GeV/c^2
Buchmüller & Flächer (Kinetic) [261]	$\|V_{cb}\| = (41.96 \pm 0.23_{exp} \pm 0.35_{HQE} \pm 0.59_{\Gamma_{\text{SL}}}) \times 10^{-3}$ $m_b^{\text{kin}} = 4.59 \pm 0.025_{exp} \pm 0.030_{HQE}$ GeV/c^2
Belle (Kinetic) [278]	$\|V_{cb}\| = (41.93 \pm 0.65_{fit} \pm 0.48_{\alpha_s} \pm 0.68_{theory}) \times 10^{-3}$ $m_b^{\text{kin}} = 4.564 \pm 0.076$ GeV/c^2
Belle (1*S*) [278]	$\|V_{cb}\| = (41.5 \pm 0.5_{fit} \pm 0.2_{\tau_B}) \times 10^{-3}$ $m_b^{1\text{S}} = 4.73 \pm 0.05$ GeV/c^2

2007 HFAG fit (prelim.)

[→ thanks to Phillip Urquijo]





$|V_{cb}| = (41.78 \pm 0.36_{fit} \pm 0.08_{\tau B})\cdot 10^{-3}$

$m_b^{1S} = (4.701 \pm 0.030)$ GeV

Perturbative error on $|V_{cb}|$

- Moments insensitive to normalization of decay rate
- $O(\alpha_s^2)$ corrections to $\Gamma(B \to X_c l\nu)$ still unknown (calculation in progress)
- Look at similar processes:
  - $\Gamma(B \to X_u l\nu)$: $1 - 0.77\alpha_s - (2.50_{BLM} - 0.34)\alpha_s^2 + \ldots$

    [van Ritbergen (1999)]
  - $\Gamma(\tau \to X\nu)$: $1 + 0.32\alpha_s + 0.53\alpha_s^2 + 0.85\alpha_s^3 + \ldots$

    (BLM approximation to 3rd-order term poor)

Important: expansion is never in powers of $(\alpha_s/4\pi)$!

Perturbative error on $|V_{cb}|$

- With $\mu = m_b/2$:

$0.34\alpha_s^2 = 0.028$	$0.85\alpha_s^3 = 0.020$

- Add in quadrature and take 1/2 to estimate perturbative error on $|V_{cb}|$:

$$\delta|V_{cb}|_{pert} = \pm 0.72\cdot 10^{-3} \quad (1.7\%)$$

→ twice as large as quoted total theory error!

Important: when $O(\beta_0\alpha_s^2)$ terms are included, scale variation cannot be used to estimate unknown higher-order terms!

Perturbative error on m_b

- Conversion to mass definition scheme introduces irreducible theory uncertainty
- (Gu)estimates:
  - $\delta m_b \sim 100$ MeV (order α_s)
  - $\delta m_b \sim 60$ MeV (order $\beta_0 \alpha_s^2$) ← present
  - $\delta m_b \sim 30$ MeV (order α_s^2)

(Note: Values for m_b^{1S} obtained by different groups differ by 110 MeV!)

- Result: $\delta m_{b,pert} = \pm 60$ MeV (1.3%)

→ twice as large as quoted total theory error!

→ very important for $|V_{ub}|$ determination!

$B \to X_s \gamma$ photon energy moments

- Inclusion in global OPE fit problematic due to sensitivity to very low scales
- Cut $E_\gamma > E_0$ introduces $\Delta = m_b - 2E_0 \approx 1$ GeV much below m_b
- Theoretical treatment requires multi-scale OPE:

[M.N. (2004)]

$$\Gamma \sim H(\mu_h) \;*\; U(\mu_h,\mu_i) \;*\; J(\mu_i) \;*\; U(\mu_i,\mu_0) \;*\; M(\mu_0)$$

QCD → SCET → RG evolution → HQET → RG evolution → Local OPE

$\mu_h \sim m_b$

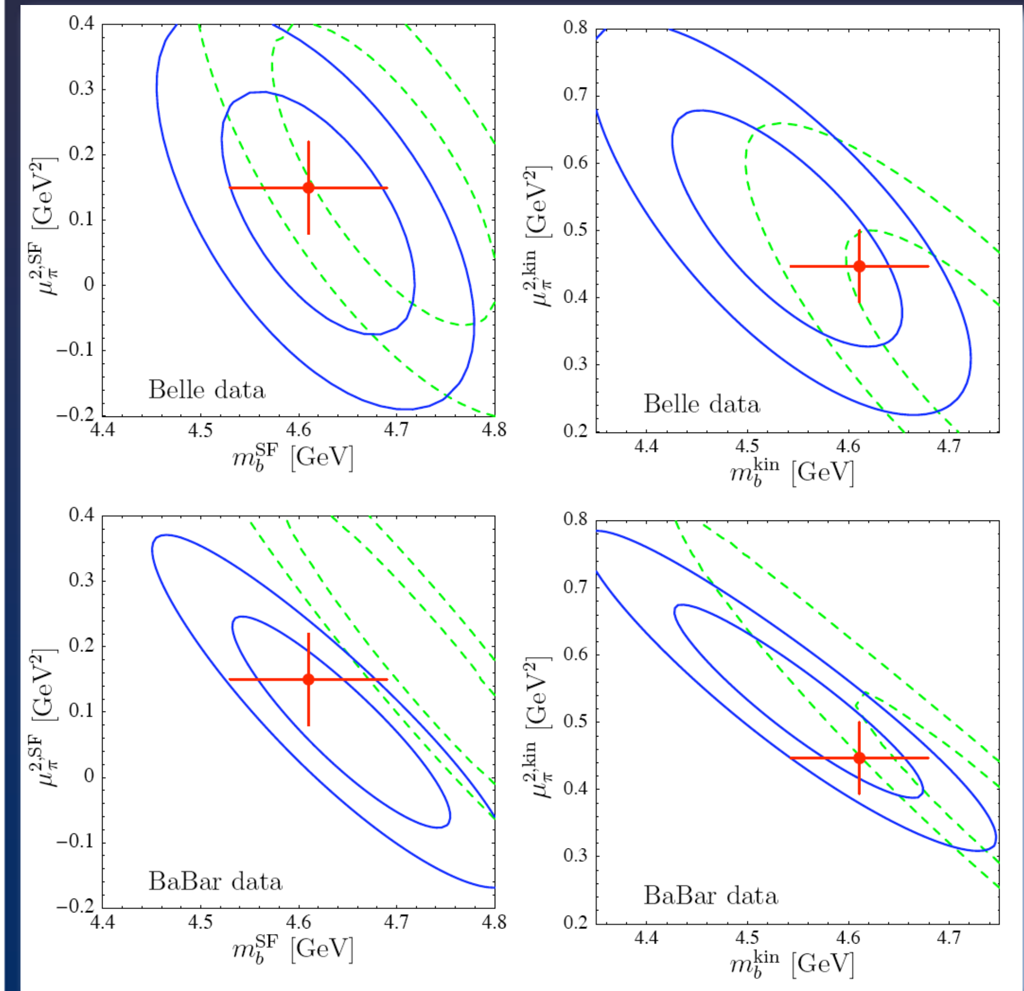
$\mu_i \sim \sqrt{m_b \Delta}$

$\mu_0 \sim \Delta$

Perturbation theory

Hadronic physics

$B \to X_s \gamma$ photon energy moments



- Only complete NNLO calculation ($\sim\alpha_s^2$) available [M.N. (2005)]
- Results (Belle data):

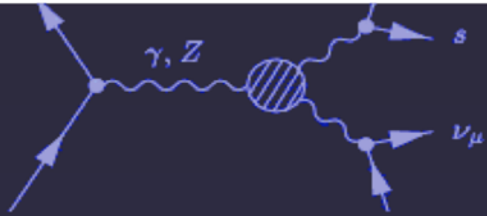
$m_b^{SF} = (4.622 \pm 0.099 \pm 0.030)$ GeV

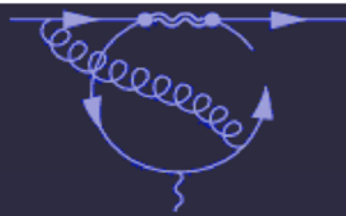
$\mu_\pi^{2,SF} = (0.108 \pm 0.186 \pm 0.077)$ GeV^2

$m_b^{kin} = (4.534 \pm 0.114 \pm 0.041)$ GeV

$\mu_\pi^{2,kin} = (0.495 \pm 0.176 \pm 0.085)$ GeV^2

→ very small theory errors, but not used by HFAG





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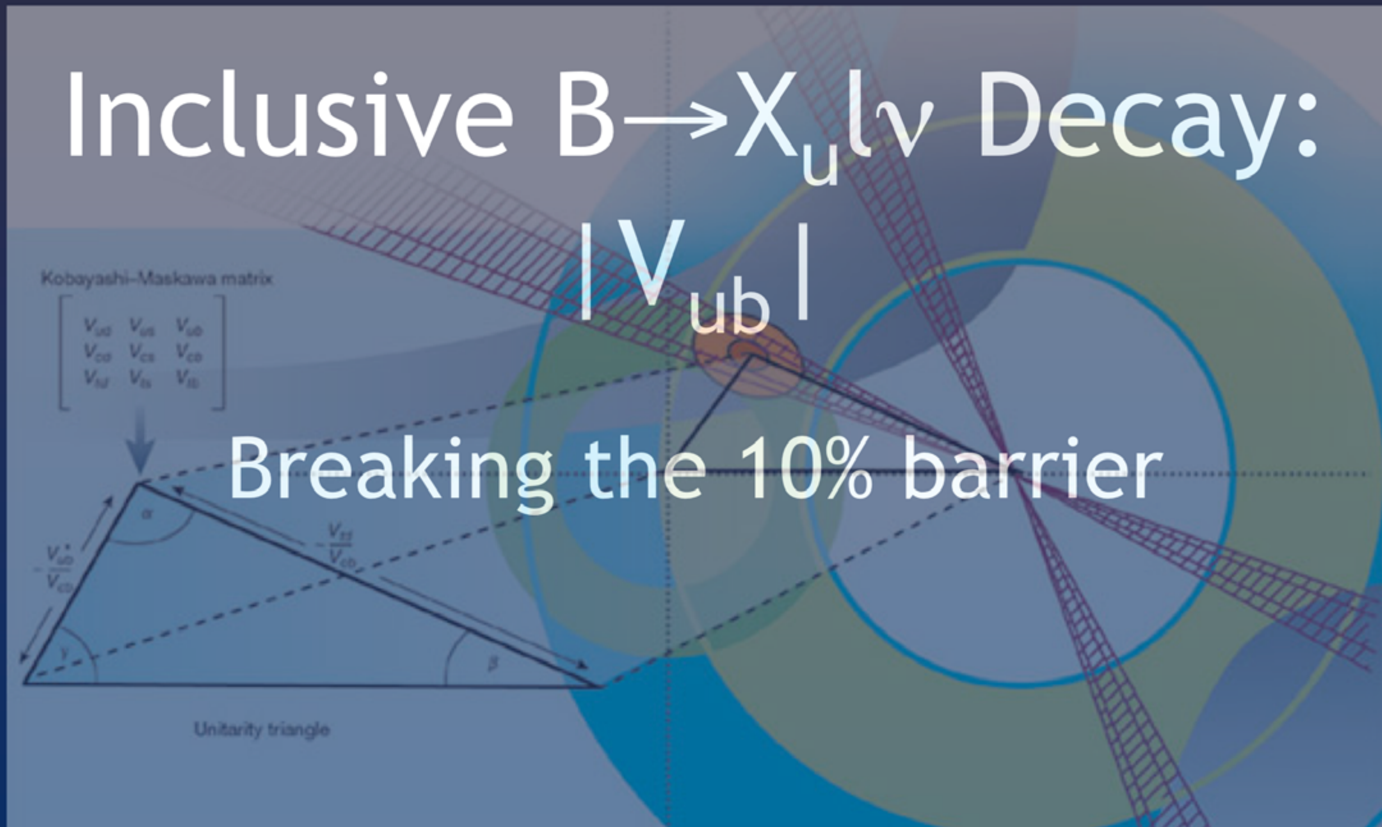
Inclusive $B \rightarrow X_u l \nu$ Decay: $|V_{ub}|$

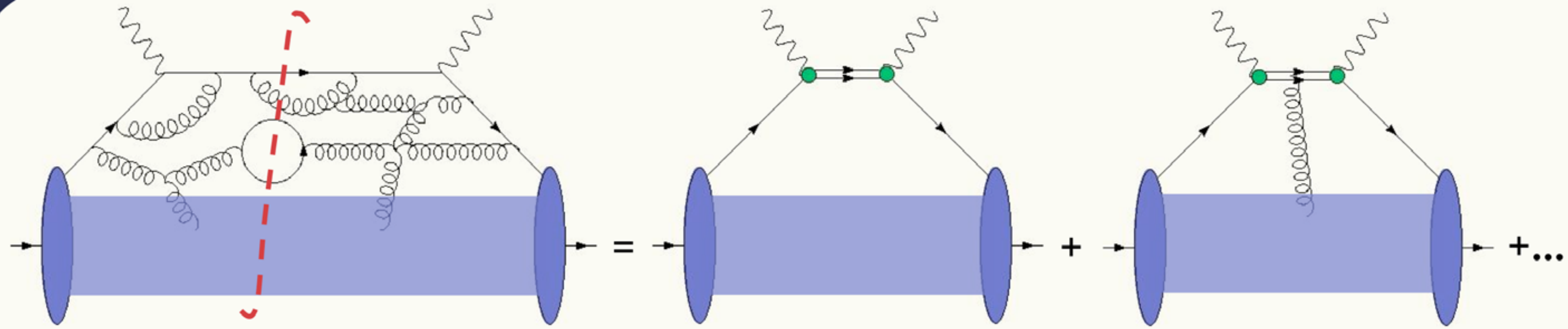
Breaking the 10% barrier



Theoretical tool: LC expansion

[M.N. (1993); Bigi et al. (1993)]

- Expansion in light-cone operators:



$\langle B | \bar{b}(x) b(0) | B \rangle$ several $1/m_b$ ops.

- Hadronic physics encoded in nonperturbative shape functions (generalized PDFs)

Factorization

- Factorization formula: [Korchemsky, Sterman (1994)]

$$d\Gamma(B\to \text{light}) = H\, J \otimes S$$

hard and jet functions (perturbative) → $H\,J$

shape functions (nonperturbative) → S

- Shape functions are universal, process independent

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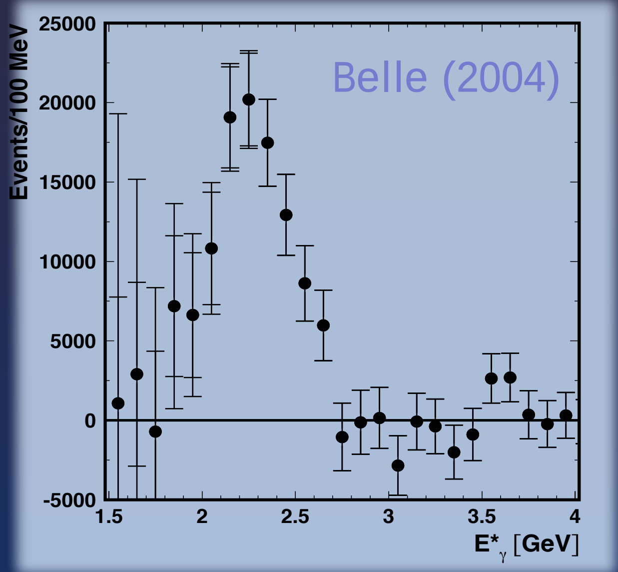
Strategy

- Extract shape function from $B \to X_s\gamma$ photon spectrum, then predict arbitrary $B \to X_u l\nu$ decay distributions

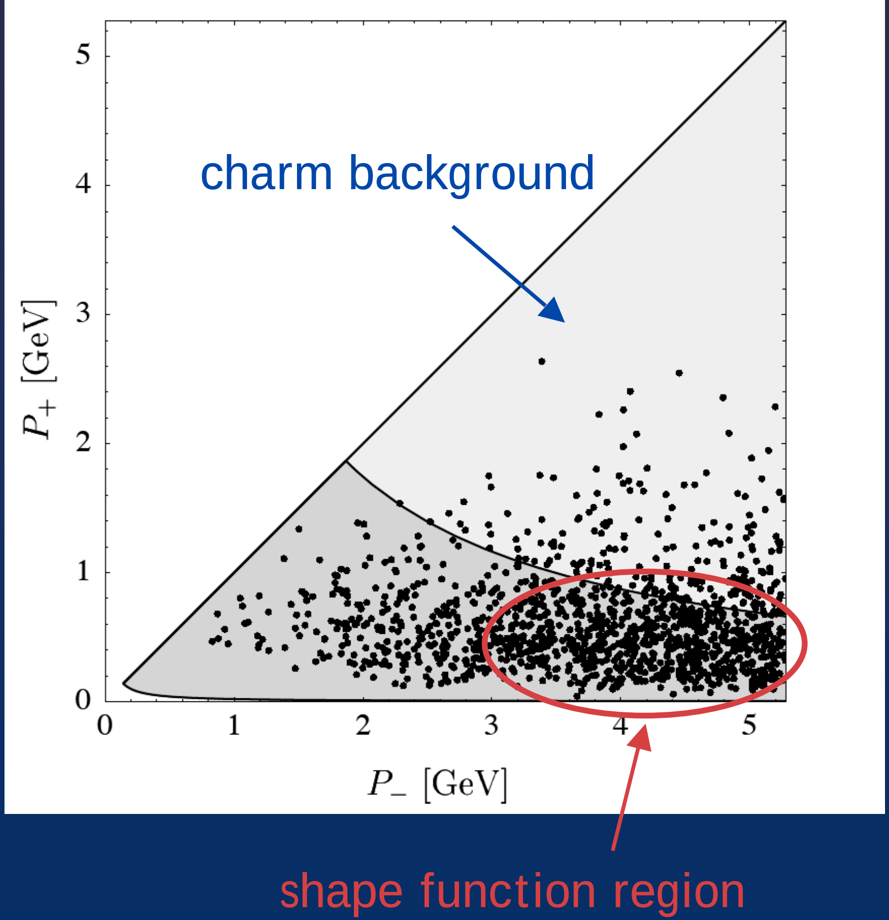
  [Bosch, Lange, M.N., Paz (2004,2005)]



- Functional form constrained moment relations (also for subleading SFs)
- Knowledge of m_b and μ_π^2 helps, but does not eliminate uncertainties

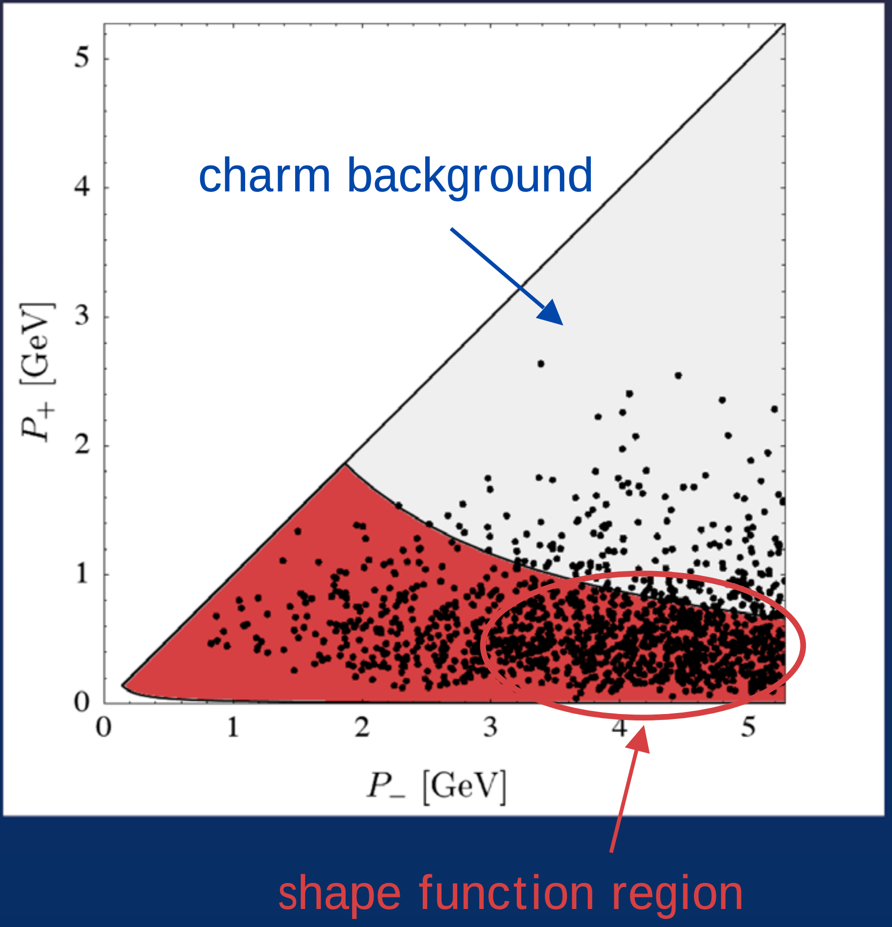
Elimination of charm

- Hadronic phase space is most transparent in variables $P_+=E_X-P_X$ and $P_-=E_X+P_X$
- $P_+ \ll P_-$ for most cuts eliminating charm background
- Collinear kinematics



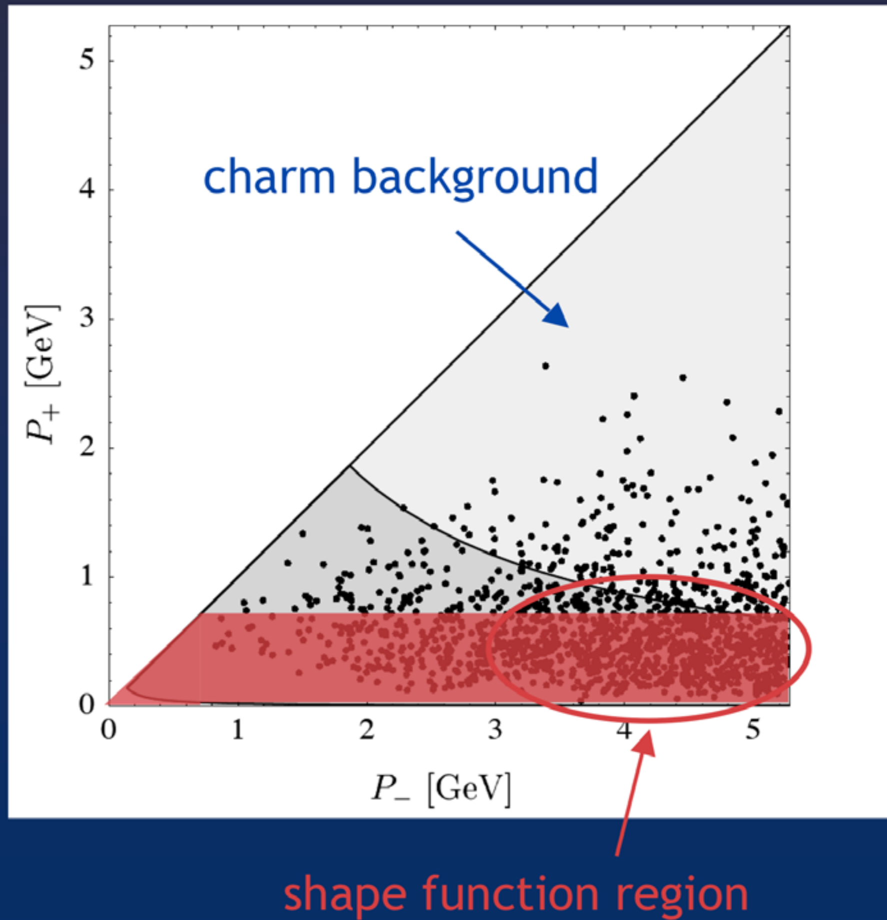
Elimination of charm

- Cut on hadronic invariant mass: $M_X^2 < M_D^2$



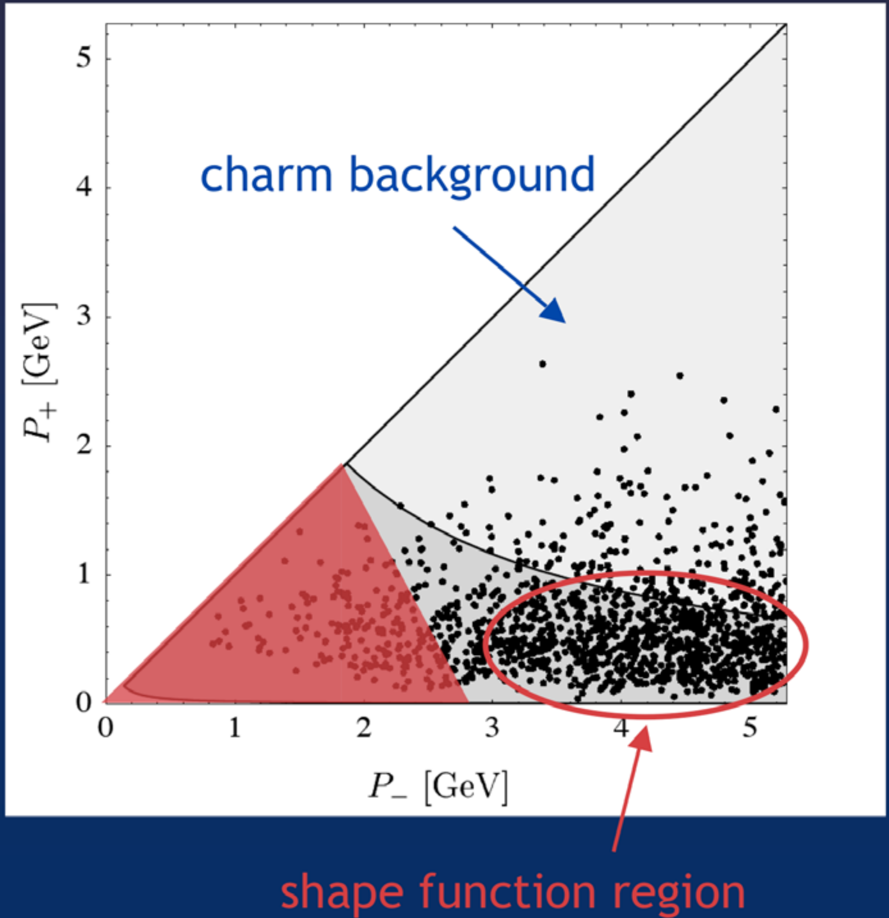
Elimination of charm

- Cut on hadronic invariant mass: $M_X^2 < M_D^2$
- Cut on hadronic $P_+ < M_D^2/M_B$ or lepton $E_l > (M_B^2 - M_D^2)/2M_B$



Elimination of charm

- Cut on hadronic invariant mass: $M_X^2 < M_D^2$
- Cut on hadronic $P_+ < M_D^2/M_B$ or lepton $E_l > (M_B^2 - M_D^2)/2M_B$
- Cut on leptonic invariant mass $q^2 > (M_B - M_D)^2$



Status of theory (BLNP)

- Leading term at O(α_s), partial results at O(α_s^2)
  [M.N. (2004); Becher, M.N. (2005,2006)]
- Large Sudakov logarithms resummed to all orders in perturbation theory (at NLO)
- Subleading shape functions included at tree level
  $\to$ $1/m_b$ terms integrate to zero in inclusive rates
  [Lee, Stewart (2004); Bosch, M.N., Paz (2004); Beneke et al. (2005)]
- Kinematical power corrections included at O(α_s)
- Residual $\mu_{\pi,G}^2/m_b^2$ corrections included at tree level
- Sensitivity to m_b and heavy-quark parameters only via shape-function moments!

Status of theory (BLNP)

- Error budget:
  - perturbative uncertainty estimated by scale variation (three scales)
  - power corrections estimated by sampling over 729 different sets of subleading shape functions
  - weak annihilation (±1.8% on total rate)
- Sensitivity to leading shape function is treated as an experimental error!

Predictions for various cuts

	m_b [GeV]	4.50	4.55	4.60	4.65	4.70	**Theory Error**
$M_X \le M_D$ Eff = 84%	a Functional Form	9.5 1.4%	8.8 1.1%	8.2 0.8%	7.7 0.5%	7.3 0.4%	7%
$M_X \le 1.7\,\text{GeV}$ Eff = 75%	a Functional Form	12.5 2.9%	11.5 2.6%	10.5 2.2%	9.7 1.9%	8.9 1.6%	7%
$M_X \le 1.7\,\text{GeV}$ $q^2 \ge 8\,\text{GeV}^2$ 35%	a Functional Form	10.3 2.0%	9.8 1.7%	9.3 1.5%	9.0 1.4%	8.7 1.4%	10%
$q^2 \ge (M_B - M_D)^2$ Eff = 18%	a Functional Form	11.4 5.0%	11.1 4.4%	10.9 4.0%	10.8 3.6%	10.6 3.2%	15%
$P_+ \le M_D^2/M_B$ Eff = 65%	a Functional Form	16.7 5.3%	15.0 4.8%	13.6 4.4%	12.2 4.0%	11.1 3.6%	7%
$E_l \ge 2.2\,\text{GeV}$ Eff = 11%	a Functional Form	22.6 16.2%	21.0 13.1%	19.7 11.0%	18.5 9.3%	17.4 7.9%	19%

Rate $\Gamma \sim (m_b)^a$

[Lange, M.N., Paz (2005)]

Results for various cuts [HFAG (2007)]

	accepted region	f_u	$\lvert V_{ub}\rvert[10^{-3}]$
CLEO [313]	$E_e > 2.1\,\text{GeV}$	0.13	$4.09 \pm 0.48 \pm 0.37$
BELLE [316]	$E_e > 1.9\,\text{GeV}$	0.24	$4.82 \pm 0.45 \pm 0.30$
BABAR [315]	$E_e > 2.0\,\text{GeV}$	0.19	$4.39 \pm 0.25 \pm 0.32$
BABAR [314]	$E_e > 2.0\,\text{GeV},\ s_{\text{h}}^{\text{max}} < 3.5\,\text{GeV}^2$	0.13	$4.57 \pm 0.31 \pm 0.42$
BELLE [309]	$M_X < 1.7\,\text{GeV}/c^2$	0.47	$4.06 \pm 0.27 \pm 0.24$ ←
BELLE [318]	$M_X < 1.7\,\text{GeV}/c^2, q^2 > 8\,\text{GeV}^2/c^2$	0.24	$4.37 \pm 0.46 \pm 0.29$
BABAR [317]	$M_X < 1.7\,\text{GeV}/c^2, q^2 > 8\,\text{GeV}^2/c^2$	0.24	$4.75 \pm 0.35 \pm 0.31$
Average	$\chi^2 = 6/6$, CL $= \mathbf{0.41}$		$\mathbf{4.52 \pm 0.19 \pm 0.27}$
BELLE (?)	$P_+ < 0.66$ GeV	0.57	$4.14 \pm 0.35 \pm 0.29$ ←

- Measurements with higher efficiency give lower $|V_{ub}|$!
- Small shape-function uncertainty (in exp. error) due to overly optimistic use of moment relations!

Results for various cuts [HFAG (2007)]

	accepted region	f_u	$\lvert V_{ub}\rvert [10^{-3}]$
CL[illegible]	[illegible]	[illegible]	[illegible]
BELLE [316]	$E_e > 1.9$ GeV	0.24	$4.82 \pm 0.45 \pm 0.$[illegible]0
BABAR [illegible]	[illegible]	[illegible]	[illegible]5 $\pm 0.$[illegible]2
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BELLE [31[illegible]]	[illegible]	[illegible]	[illegible]$.46 \pm 0.$[illegible]9
BABAR [317]	$M_X < 1.7$ GeV$/c^2, q^2 > 8$ GeV$^2/c^2$	0.24	$4.75 \pm 0.35 \pm 0.$[illegible]1
Average	$\chi^2 = 6/6$, CL$= 0.41$		**$4.52 \pm 0.19 \pm 0.27$**
BELLE (?)	$P_+ < 0.66$ GeV	0.57	$4.14 \pm 0.35 \pm 0.29$

Experimental error includes uncertainty in leading shape function, which is fully correlated between different cuts

→ **Cannot possibly be that small!**

- Measurements with higher efficiency give lower $\lvert V_{ub}\rvert$!
- Small shape-function uncertainty (in exp. error) due to overly optimistic use of moment relations!

Alternative schemes

- Dressed Gluon Exponentiation (DGE):

  [Gardi (2004); Anderson, Gardi (2005)]

  – renormalon-inspired model for the leading shape function (parameter m_b)
  – no attempt to include subleading shape functions or other power corrections
  – less flexible functional form

→ numerical results similar to BLNP fits

Alternative schemes

- Combined M_X-q^2 cut using OPE (BLL):

  [Bauer, Ligeti, Luke (2000,2001)]

  – cutting on leptonic invariant mass in part eliminates shape-function region
  – low efficiency and enhanced sensitivity to weak annihilation
  – OPE approach reintroduces sensitivity to b-quark mass (~10^{th} power!)
- Gives largest $|V_{ub}|$ by far (~ $5.0 \cdot 10^{-3}$)!

Shape-function free relations

- At leading power (only), possible to construct shape-function free relations between weighted spectra, e.g.:

$$\left|\frac{V_{ub}}{V_{cb}}\right|^2 \simeq \left|\frac{V_{ub}}{V_{tb}V_{ts}^*}\right|^2 = \frac{3\alpha}{\pi}\,|c_7(m_b)|^2\,\eta_{\rm QCD}\,\frac{\widehat{\Gamma}_u(E_0)}{\widehat{\Gamma}_s(E_0)} + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}}{m_b}\right)$$

leading logs

with:

$$\widehat{\Gamma}_u(E_0) \equiv \int_{E_0}^{\infty} dE_\ell\,\frac{d\Gamma(B\to X_u\,\ell\,\bar\nu)}{dE_\ell}$$

$$\widehat{\Gamma}_s(E_0) \equiv \frac{2}{m_B}\int_{E_0}^{\infty} dE_\gamma\,(E_\gamma - E_0)\,\frac{d\Gamma(B\to X_s\,\gamma)}{dE_\gamma}$$

[M.N. (1993)]

Shape-function free relations

- Refinements:
  - resummation of subleading logs (but introducing Landau pole!) and extension to hadronic mass distribution [Leibovich, Low, Rothstein (1999,2000)]
  - inclusion of NLO QCD corrections [M.N. (2001)]
  - generalization to arbitrary cuts, inclusion of subleading shape functions and higher power corrections, removal of Landau pole singularity, …

$\rightarrow$ first systematic error estimates!

[Lange, M.N., Paz (2005); Lange (2005)]

Shape-function free relations

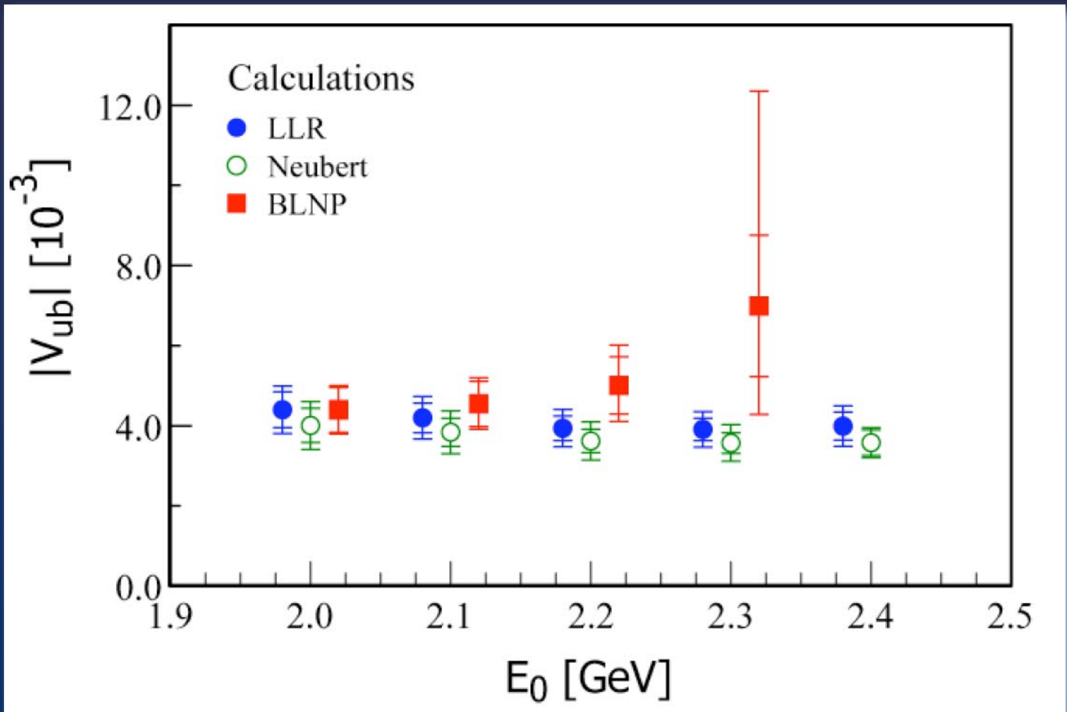
- Example:

$$\Gamma_u(\Delta) = \underbrace{\int_0^\Delta dP_+ \frac{d\Gamma_u}{dP_+}}_{\text{exp. input}} = |V_{ub}|^2 \int_0^\Delta dP_+ \underbrace{W(\Delta, P_+)}_{\text{theory}} \underbrace{\frac{1}{\Gamma_s(E_*)} \frac{d\Gamma_s}{dP_+}}_{\text{exp. input}}$$

  - weight function perturbatively calculable; leading $O(\alpha_s^2)$ terms included!
  - hadronic uncertainties enter at $O(1/m_b)$
  - error analysis like in BLNP

Shape-function free relations

- BaBar analysis of lepton spectrum:



- good lesson on treatment of theory errors in exp. analyses
- only BLNP includes power corrections and complete error analysis
- errors must blow up at large E_0!

Result: $|V_{ub}| = (4.40 \pm 0.30 \pm 0.41_{th} \pm 0.23) \cdot 10^{-3}$


Summary

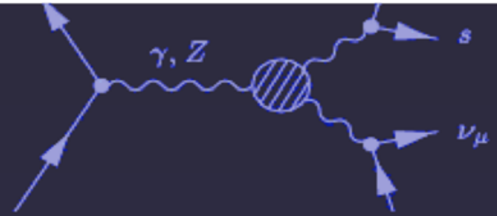
- $B \to X_c l\nu$ decays: $\delta|V_{cb}|_{th} = \pm 0.8 \cdot 10^{-3}$ (2%)
  $\delta m_{b,th} = \pm 70$ MeV (1.5%)

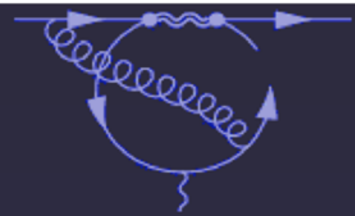
- $B \to X_u l\nu$ decays: $\delta|V_{ub}|_{th} \geq \pm 0.3 \cdot 10^{-3}$ (7%)
  depending on cut

→ best determinations (highest efficiency, best theoretical control) yield:

$$|V_{ub}| = (4.10 \pm 0.30_{exp}(?) \pm 0.29_{th}) \cdot 10^{-3}$$

Consistent with recent exclusive values! → talk by P. Ball





Summary

- General remarks:
  - makes no sense to average theory approaches referring to different approximations (LO vs. NLO, inclusion of power corrections, etc.)
  - makes no sense to quote small theory errors from approaches that do not include error analysis
- Closer interaction with theorists required in HFAG (should revive V_{xb} workshops)!