Progress in Lattice QCD

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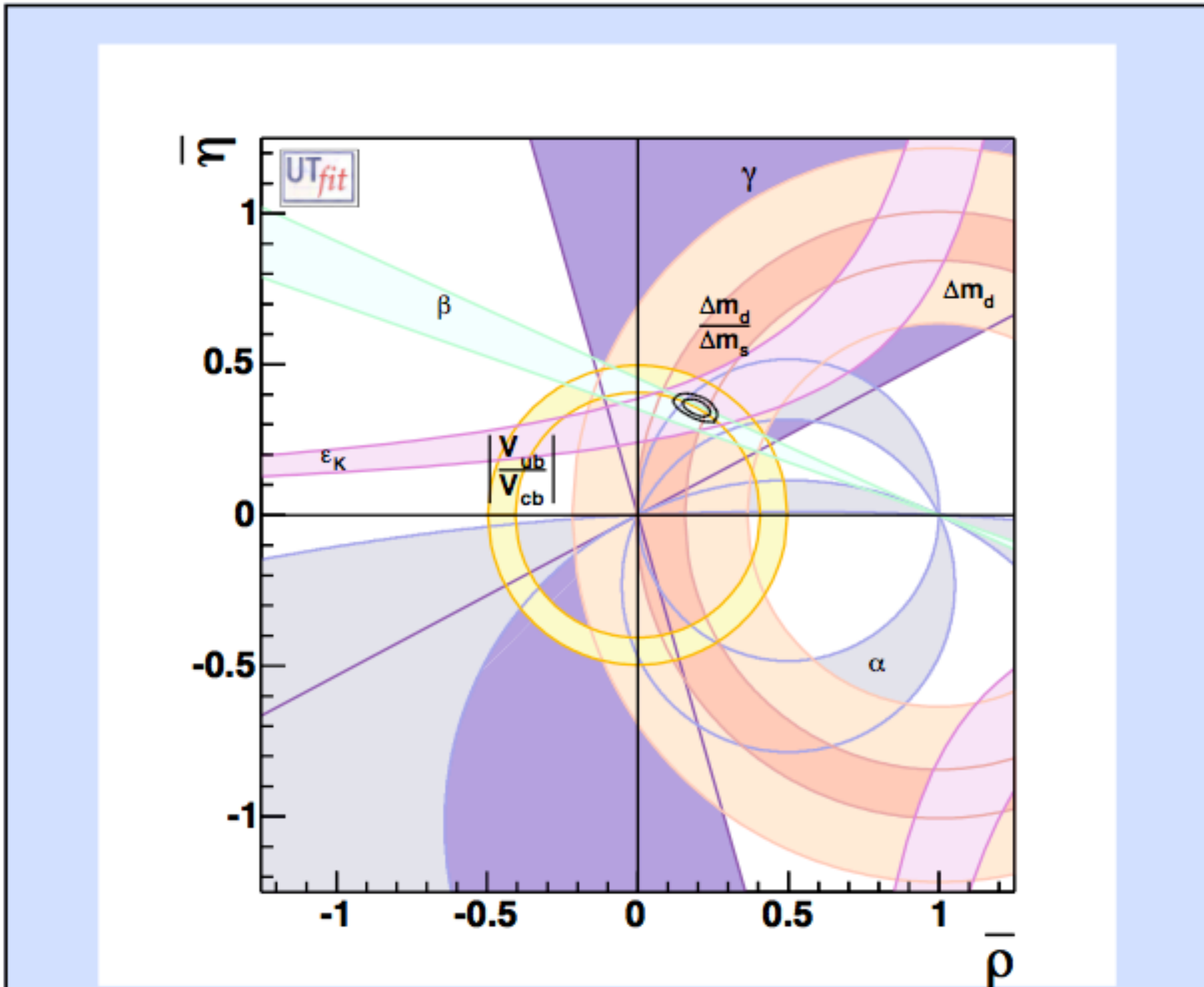
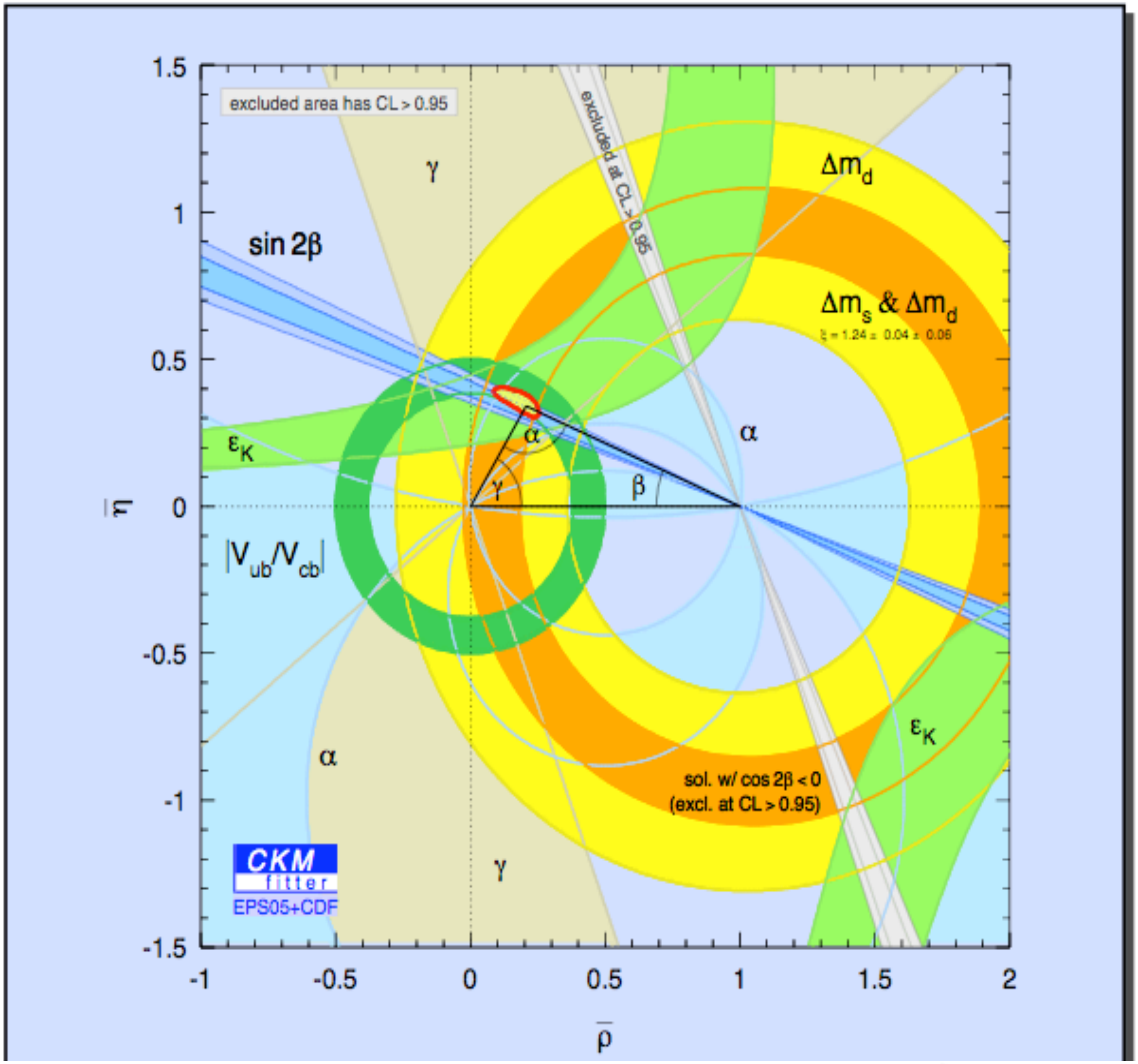
CKM unitarity triangle analysis by UTfit (Bayesian)



CKM unitarity triangle analysis by CKM-fitter (frequentist)



CP violation studies and search for physics beyond Standard Model:

- $|V_{us}|$ — $K \to \ell\nu, K \to \pi\ell\nu$
- $|V_{cs}|$ — $D_s \to \ell\nu, D \to K\ell\nu$; $D \to K^*\ell\nu, D_s \to \phi\ell\nu$
- $|V_{cd}|$ — $D \to \ell\nu, D \to \pi\ell\nu$; $D \to \rho\ell\nu, D_s \to K^{(*)}\ell\nu$
- $|V_{cb}|$ — $B_c \to \ell\nu, B \to D^{(*)}\ell\nu$; $B_s \to D_s^{(*)}\ell\nu$
- $|V_{ub}|$ — $B \to \ell\nu, B \to \pi\ell\nu$; $B \to \rho\ell\nu, B_s \to K^{(*)}\ell\nu$
- $|V_{ts}/V_{td}|$ — $B \to K^*\gamma/B \to \rho\gamma$
- $K^0 - \bar{K}^0$ & $B^0 - \bar{B}^0$ mixing amplitudes

Non-perturbative QCD input needed

e.g.

$$\underbrace{\frac{d\Gamma(B \to \pi e \nu)}{dq^2}}_{\text{measure exp.}} = \overbrace{|V_{ub}|^2}^{\text{CKM}} \underbrace{\frac{G_F^2}{192\pi^3 m_B^3} \lambda^{3/2}(q^2)}_{\text{kinematics}} \underbrace{|F_+(q^2)|^2}_{\text{compute th.}}$$

- *Impressive exp. statistics $\Rightarrow$ experimental input*
- *Theory input: quantities that carry info on hadrons*

(decay constants, form factors, bag parameters etc.)

> *Do keep in mind that in spite of appearances*
> *we do not understand the non-perturbative QCD dynamics*

High precision results...

Main goal is to compute...

$$\langle 0|\, \phi(x)\phi(x')\phi(x'')...\,|0\rangle = \frac{1}{Z}\int \mathcal{D}[\psi,\bar{\psi},A_\mu]\phi(x)\phi(x')\phi(x'')...e^{iS_{QCD}}$$

with $\quad Z = \int \mathcal{D}[\psi,\bar{\psi},A_\mu]e^{iS_{QCD}}$

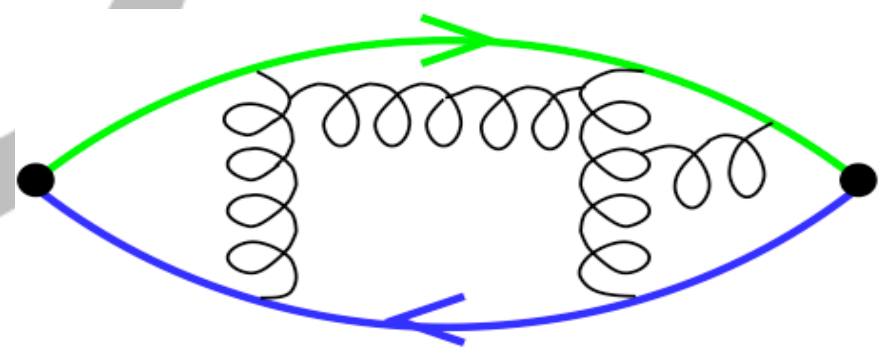
→ *Minkowski to Euclidean space* $e^{iS} = e^{-S}$ *etc.*

→ *Functional integral handled by MC on discrete space-time*

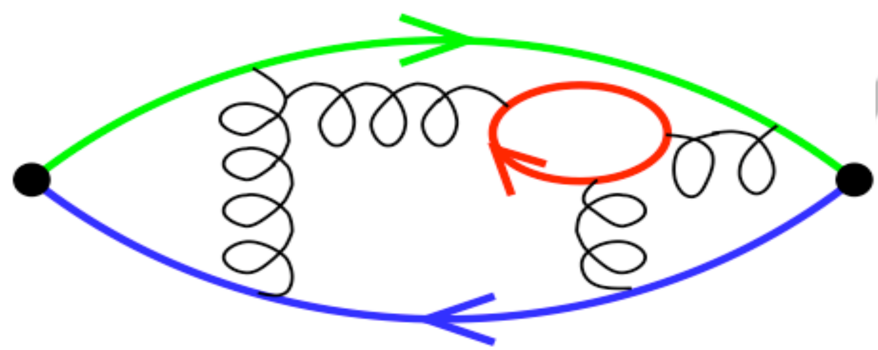
→ *Suitable choice of* $\phi(x)\phi(x')...$ → *desired physics info*

In practice we are interested in two- and three-point functions

- MC : snapshots of QCD vacuum through which we propagate quarks
- 2pt correlation functions :
  hadron masses and decay constants
- 3pt correlation functions :
  form factors and bag parameters
- Methodology developed in QQCD, nowadays unquenching



(A) Quenched QCD: quark loops neglected



(B) Full QCD

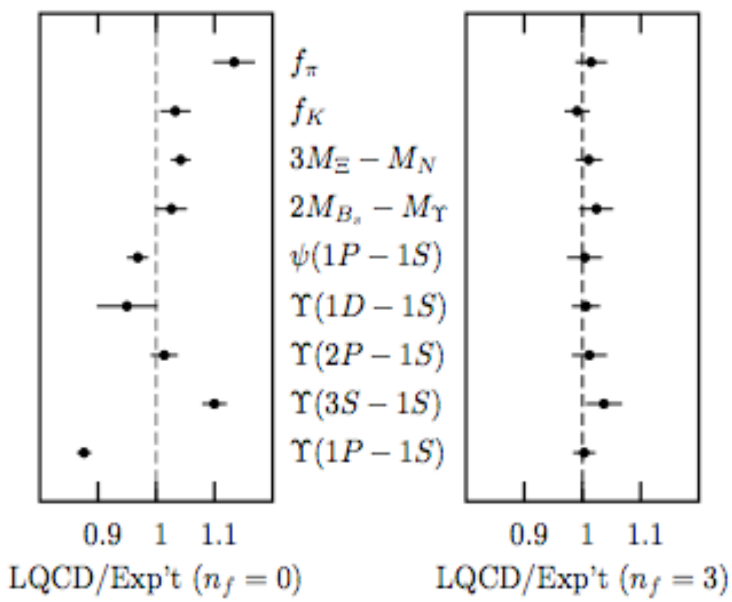
Unquenched progress

quark action	cost	chiral sym.	flavor sym.	$N_f = 2$ or $N_f = 2{+}1$
Overlap	VERY!	exact	OK	JLQCD
Domain Wall	Yes	$m_{\rm res}$	OK	RBC,UKQCD
Wilson (WI)	Moderate	No ($\mathcal{O}(a^2)$)	OK	JLQCD, QCDSF, ALPHA, CERN
tmQCD	Moderate	No ($\mathcal{O}(a^2)$)	No ($\mathcal{O}(a^2)$)	ETMC
Staggered	No	$U(1) \times U(1)$	No ($\mathcal{O}(a^2)$)	MILC,HPQCD, F-lab

- *Algorithmic progress (improvement on HMC)*
  Now with Wilson quarks down to $m_q/m_s = 1/4$, staggered at m_q/m_s=$1/8$
- *Machines (PCA, Clusters)*
  e.g. In KEK (IBM BlueGene/L 57.3 Tf peak + Hitachi SL11000 (2 Tf peak)) $\longrightarrow$ Overlap unq.
  e.g. PC clusters at Flab : 6.2 Tf-years in 2006
- *Hadronic matrix elements studied only with staggered quarks (MILC conf.)*
  others are being tested : work in progress
- *"Pragmatic View" : mixed actions (sea $\neq$ valence quarks)*
  But swamp : unitarity violation (?? case-by-case)

Before continuing...

- Use or not to use staggered fermions?
- Fact 1: $\mathrm{Det}^{1/4}$ is made by hand and there is no proof that it is correct!
- Fact 2: circumstantial evidence from SLQCD/expt.



Before continuing...

- Lattice 2005: *"Is staggered QCD really QCD or just a model of QCD?"*-S.Dür
- Lattice 2006: *"Not bad, just ugly"*-S.Sharpe
- Bernard et al 2006: *The fourth root trick corresponds to a non-local theory at $a \neq 0$, but argue that the non-local behavior is likely to go away in the continuum limit."*
- So what do you do about the non-locality and renormalisability?
- SChPT is used to extract f_π and f_K: fit contains more than 50 parameters!
- renormalisation is only perturbative (c.f. lesson from m_s!)
- MILC and HPQCD made a great effort to make the best out of SQCD, BUT the use of other actions is indispensable!

On 2-pt functions

$$C_2(t) = \int d^3x e^{i\vec{p}\vec{x}} < 0|\Phi(\vec{x},t)\Phi^\dagger(\vec{0},0)|0 >$$

$\Phi(x)$- *interpolating operator for the hadron state (h) which we want to study*

$$\begin{aligned} C_2(t) &= \sum_n \int d^3x e^{i\vec{p}\vec{x}} \langle 0|\Phi(\vec{x},t)|n\rangle\langle n|\Phi^\dagger(\vec{0},0)|0\rangle \\ &= \int d^3x e^{i\vec{p}\vec{x}} \langle 0|\Phi(\vec{x},t)|h\rangle\langle h|\Phi^\dagger(\vec{0},0)|0\rangle + \dots \\ &= \frac{1}{2E} e^{-iEt} \left|\langle 0|\Phi(\vec{0},0)|h\rangle\right|^2_{E=\sqrt{\vec{p}^2+m_h^2}} + \dots \end{aligned}$$

→ *Minkowski to Euclidean space:* $iEt \rightarrow Et$

→ *Fit* $C_2(t)$ *to extract matrix element and hadron mass* ($|\vec{p}| = 0$)

→ *e.g.* $\Phi = \bar{u}\gamma_\mu\gamma_5 s \Rightarrow m_K$ *and* $\langle 0|\bar{u}\gamma_\mu\gamma_5 s|K\rangle = m_K f_K$

On 3-pt functions

$$C_3(t,t_x) = \int d^3x d^3y e^{i(\vec{p}.\vec{x}+\vec{q}.\vec{y})} \langle 0|\Phi'(\vec{x},t_x)\mathcal{O}(\vec{y},t)\Phi^\dagger(\vec{0},0)|0\rangle$$

$$\simeq \frac{e^{-Et}}{2E}\frac{e^{-E'(t_x-t)}}{2E'}\langle 0|\Phi'(\vec{0},0)|h_2(\vec{p})\rangle \times$$

$$\langle h_2(\vec{p})|\mathcal{O}(\vec{0},0)|h_1(\vec{p}+\vec{q})\rangle\langle h_1(\vec{p}+\vec{q})|\Phi^\dagger(\vec{0},0)|0\rangle$$

$E' = \sqrt{\vec{p}^2 + m_{h_2}^2}$ and $E = \sqrt{(\vec{p}+\vec{q})^2 + m_{h_1}^2}$

$\longrightarrow$ *Combining with 2-pt functions* $\Rightarrow$ *extract the transition matrix elements*

$\longrightarrow$ *Matrix elements of* $\Delta F = 2$ *operators*

$\longrightarrow$ *e.g. set* $|\vec{p}| = |\vec{q}| = 0$

$\Phi = \bar{d}\gamma_5 s$, $\Phi' = \bar{s}\gamma_5 d$, and $\mathcal{O} = (\bar{s}d)_{V-A}(\bar{s}d)_{V-A}$

$\Rightarrow \langle \bar{K}^0|\bar{s}\gamma_\mu(1-\gamma_5)d\bar{s}\gamma_\mu(1-\gamma_5)d|K^0\rangle = \frac{8}{3}f_K^2 m_K^2 B_K$

Sources of errors -1-

→ **Statistical**

- *can be kept at the percent level although the time to decorrelate the configurations might increase as the (sea) quark mass and lattice spacing is lowered while the lattice box is kept sufficiently large*

→ **Systematics**

- *UV cut-off* $\to \infty \Leftrightarrow$ *continuum limit* $a \to 0$
  controllable and depends on the discretised QCD action (Wilson, staggered, GW, DWF, FLIC)

$$F(a) = F^{\text{cont.}}(1 + c \times a^2 + \ldots)$$

- *IR cut-off* $\to 0 \Leftrightarrow$ *physical limit* $L \to \infty$

$$F(L) = F^{\text{cont.}}[1 + \widetilde{c} \times \exp(-m_\pi L) + \ldots]$$

Sources of syst. errors -2-

$\rightarrow$ *Finite a*

- *Actual $a \simeq (0.06 \div 0.10)$ fm : charm YES, beauty NO*

  need $a \ll 0.04$ to make precision b-physics

  help from effective theories: HQET, NRQCD, but...

$\rightarrow$ *Finite L*

- *Actual $L \simeq (2 \div 3)$ fm : light pions mutilated*

  need $L \simeq 4$ fm to keep FV effects at the percent level

  Plus, physical and algorithmic impediments to get close to $m_{u,d}^{\mathrm{phys.}}$

  strange YES, up/down NO

  help from effective theory: ChPT with $N_f = 2$ and $N_f = 3$, but...

"but" : Effective theories are non-renormalisable: Matching to QCD (and LQCD in particular) is not fully controlled: $1/a \lesssim m_Q$ for heavy quark theories generates the extra-scale problem in perturbative series
Matching of ChPT to QCD is unknown. ChPT has not been tested and the use of ChPT with $N_f = 3$ to extrapolate lattice data might induce a surprising syst. errors

Sources of syst. errors -3-

→ *Renormalisation :* $\mathcal{O}_R = Z_{\mathcal{O}}(g_0^2, a)\mathcal{O}(a)$

For a 1% *accuracy* $Z_{\mathcal{O}}(g_0^2)$ *must be determined non-perturbatively*

→ *Warning*

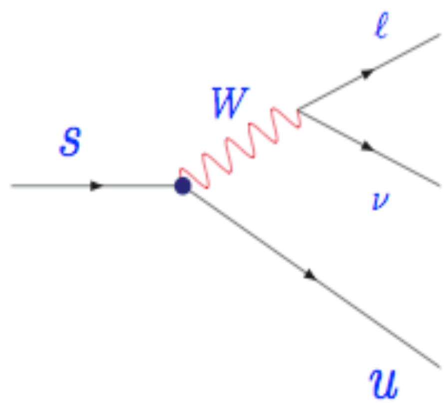
Actual systematic errors may be somewhat missleading because they are combined in quadrature, e.g. $\pm 4\% \pm 5\% \pm 2\% \pm 3\% \to 7\%$

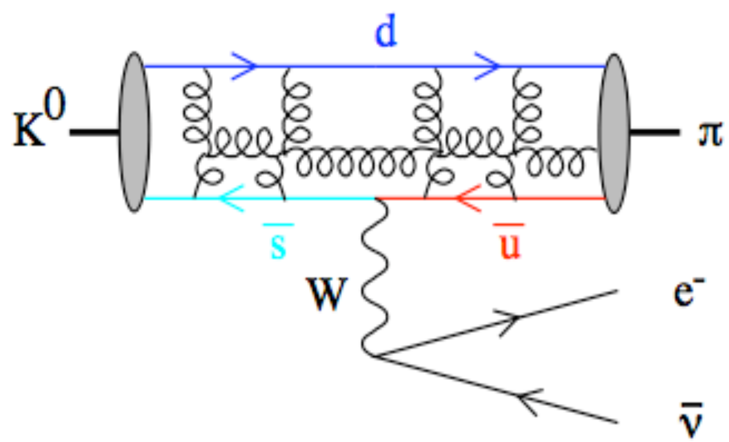
In the future one must keep each of those sources at the 1% *level (and less!)*

Most of the currently quoted errors heavily rely on the ChPT to extrapolate to the physical light quark mass.

Should there be more surprises...
numerical (when the lattice volumes are huge),
physical (when the excited states $P' \to P + \pi$, giving it a fake/euclidean width)

$|V_{us}|\ (\lambda)$





$$\langle \pi^-(p)|\bar{s}\gamma_\mu u|K^0(p_K)\rangle = \left(p_K + p - q\frac{m_K^2 - m_\pi^2}{q^2}\right)_\mu F_+(q^2) + \frac{m_K^2 - m_\pi^2}{q^2} q_\mu F_0(q^2)$$

$F^{K\to\pi}(0) = ?$

- *CVC $\to$ normalisation in SU(3) limit:* $F(0) = 1 \otimes$ *AGT* $\mathcal{O}[(m_s - m_u)^2]$
- *ChPT to* $\mathcal{O}(p^6)$*:* $F^{K^0\to\pi^-}(0) = 1 + f_2 + f_4$ *AND* $f_2 = -0.0227$
- $f_4 = ?$ *: high precision possible from double ratio (Hashimoto et al. 2000)*

$$\frac{\langle\pi|\bar{s}\gamma_0 u|K\rangle\,\langle K|\bar{u}\gamma_0 s|\pi\rangle}{\langle K|\bar{s}\gamma_0 s|K\rangle\,\langle\pi|\bar{u}\gamma_0 u|\pi\rangle} = \frac{(m_K + m_\pi)^2}{4m_\pi m_K}\left(F_0(q^2_{\rm max})\right)^2$$

- *Plus one momentum injection to e.g. pion* $F(0)$
- *Mass dependence (ChPT, QChPT and PQChPT formulas available)*
- *Results:*
  - $F(0) = 0.960(5)(7)$ *SPQcdR (2004) W-*$N_f = 0$
  - $F(0) = 0.962(6)(9)$ *Fermilab (2005) St-*$N_f = 2+1$
  - $F(0) = 0.952(6)$ *JLQCD (2005) W-*$N_f = 2$
  - $F(0) = 0.955(12)$ *RBC (2006)) DWF-*$N_f = 2$
  - $F(0) = 0.968(2)$ *RBC/UKQCD (2006)) DWF-*$N_f = 3$ *prelim.*

Improving strategy

- *Periodic boundary condition* $\psi(x+L)=\psi(x)$: $p_{\min}=2\pi/L$ *(large)*
- *Twisted boundary conditions* $\psi(x+L)=e^{i\theta}\psi(x)$: $p_{\min}=\theta/L$ *(small!)*
- *tBC implement as* $U_\mu^{\vec{\theta}}(x)=e^{2\pi i a\vec{\theta}_\mu/L}U_\mu(x)$

  compute one valence quark with tBC and combine with others (non-twisted)

  $\Rightarrow$ *resulting hadronic state has* $\vec{p}_\theta$
- $K_{\ell 3}$ *UKQCD/RBC use tbc (results soon) DWF-*$N_f=3$
- *Applying it to* $D\to K$ *and* $D\to\pi$ *(DB, Haas, Mescia) WI-*$N_f=2$

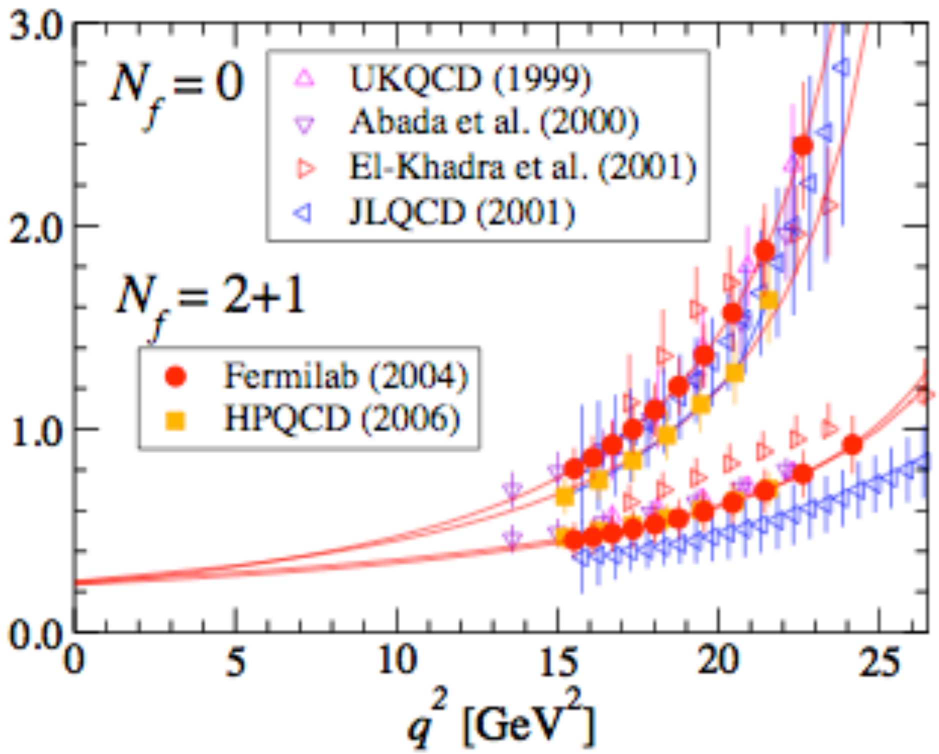
  + two new double-ratio methods, eg.

$$\frac{\langle D(\vec{0})|\bar{c}\gamma_0 q|P(\vec{p})\rangle\,\langle P(\vec{p})|\bar{q}\gamma_0 c|D(\vec{0})\rangle}{\langle P(\vec{p})|\bar{q}\gamma_0 q|P(\vec{p})\rangle\,\langle D(\vec{0})|\bar{c}\gamma_0 c|D(\vec{0})\rangle}=\frac{1}{4E_P m_D}\left[(m_D+E_P)f^+(q^2)+(m_D-E_P)f^-(q^2)\right]^2$$

$$\frac{\langle D(\vec{0})|\bar{c}\gamma_i q|P(\vec{p})\rangle\,\langle P(\vec{p})|\bar{q}\gamma_i c|D(\vec{0})\rangle}{\langle P(\vec{p})|\bar{q}\gamma_i q|P(\vec{p})\rangle\,\langle D(\vec{0})|\bar{q}\gamma_0 q|D(\vec{0})\rangle}=\frac{p_i}{4m_D}\left[f^+(q^2)-f^-(q^2)\right]^2$$

Preliminary results look promising : results for summer$\rightarrow$*september*

Carrying over to $B \to \pi \ell \nu$?



- *too large momenta needed - UNSOLVED PROBLEM!*
- *double ratios strategies - attempts desirable!*
- *only standard strategy implemented so far*
- $N_f = 0$ *various effective approaches agree quite well*
- $N_f = 2 + 1$ *only staggered*
- *Fermilab approach*
- *HPQCD=NRQCD - no* $a \to 0$

Working on Fermilab approach and need Wilson dynamical quarks too.

Fermilab approach revisited

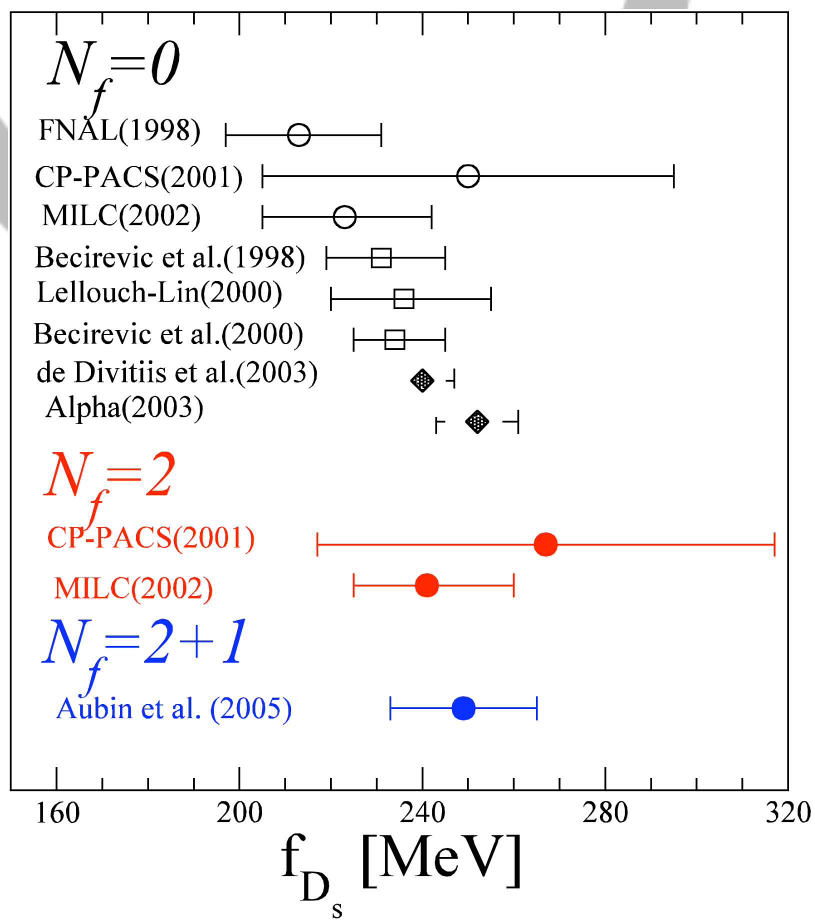
- on-shell improvement à la Symanzik + elliminate $(am_h)^n$
-

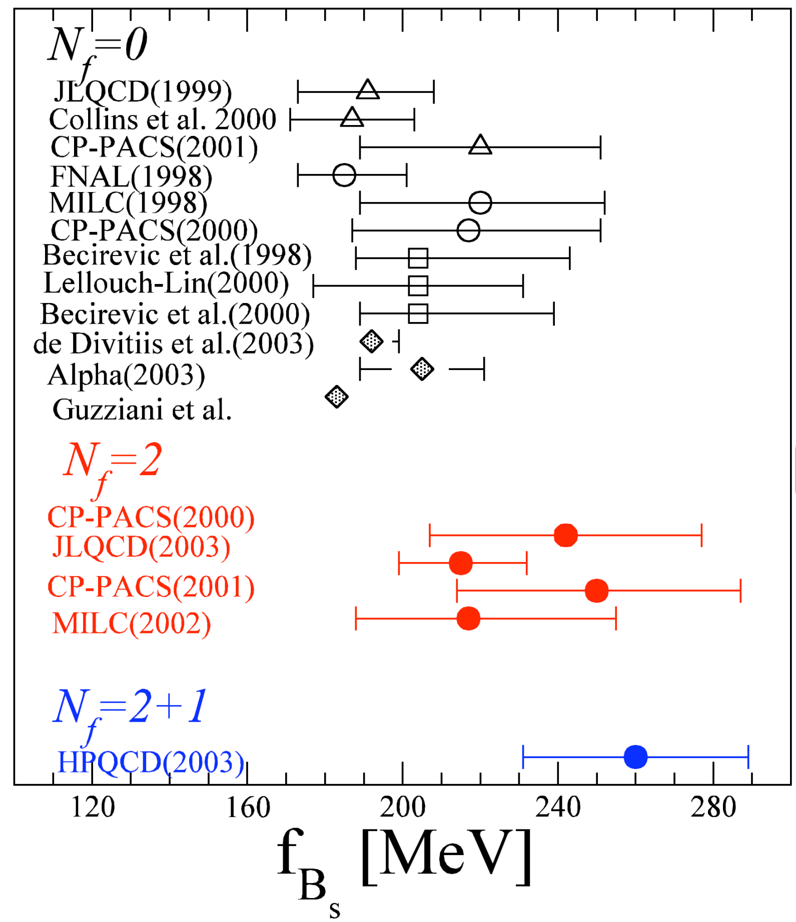
$$S = \sum_{n,m} \bar{\psi}_n \left[\gamma_0 D_0 + \zeta \vec{\gamma} \cdot \vec{D} + m_0 - \frac{r_t}{2} D_0^2 + c_B \frac{i}{4} \sigma_{ij} G_{ij} + c_E \sigma_{i0} G_{i0} \right]_{nm} \psi_m$$

  6 dependent parameters which depend on am_h fixed perturbatively
- Christ and Lin take $r_s = r_t = 1$ and propose a method to fix m_0, ζ and $c_E = c_B$ non-perturbatively $\rightarrow$ way to go $\Rightarrow$ precision b-physics on the lattice
- Similar formulation by Aoki et al.
- more coefficients to fix to improve the operators - preferably non-perturbatively
- currently all those coefficients handled perturbatively

Decay constants

New Result by Ali Khan et al.(2007) WI both charm and light (Nf=2):
fDs = 220(5)(5)(11)MeV





Group	heavy	light	n_f	f_{B_s}/f_{B_d}	visible chiral log?
CP-PACS	NRQCD	clover	2	1.18(2)(2)	NO
CP-PACS	fermilab	clover	2	$1.20(3)(3)(^{+4}_{-0})$	NO
MILC	fermilab	Wilson	2	$1.16(1)(2)(2)(^{+4}_{-0})$	NO
JLQCD	NRQCD	clover	2	$1.13(3)(^{+13}_{0})$	NO
Gadiyak-Loktik	static	DW	2	1.29(4)(6)	NO
HPQCD	NRQCD	Imp Stag	2+1	1.20(3)(1)	YES

Group	heavy	light	n_f	f_{D_s}/f_{D_d}	visible chiral log?
CP-PACS	fermilab	clover	2	$1.18(4)(3)(^{+4}_{-0})$	NO
MILC	fermilab	wilson	2	$1.14(1)(^{+2}_{-3})(3)(^{+5}_{-0})$	NO
FNAL/MILC/HPQCD	fermilab	Imp stag	2+1	1.24(7)	YES

f_{B_s}/f_{B_d} from D-decays

- *Compare:*

$$\frac{\Phi_s(m_b)}{\Phi_{u/d}(m_b)} = \frac{\phi_s^{(0)}}{\phi_{u/d}^{(0)}} + \frac{\phi_s^{(0)}}{\phi_{u/d}^{(0)}} \times \frac{1}{m_B} + \ldots$$

$$\mathcal{R} = \frac{\Phi_s(m_b)/\Phi_d(m_b)}{\Phi_s(m_c)/\Phi_u(m_c)} = 1 + \alpha\left(\frac{1}{m_B} - \frac{1}{m_D}\right) + \ldots$$

- *What has been done on the lattice?*

propagating heavy	$\mathcal{R}^{\mathrm{n_f}=0} = 1.017(17)(??)$
NRQCD heavy	$\mathcal{R}^{\mathrm{n_f}=2} = 1.005(6)\ \left({}^{+29}_{-00}\right)$
Fermilab heavy	$\mathcal{R}^{\mathrm{n_f}=2} = 1.001(6)(10)$

Always Wilson light!

f_{B_s}/f_{B_d} from D-decays

- *THERE IS something "exclusive" that can be computed with $\approx 1\%$ accuracy.*
- *Illustration*

$$\frac{f_{B_s}}{f_{B_d}} = \overbrace{(1.018 \pm 0.006 \pm 0.010)}^{\text{from } \mathcal{R}^{\mathrm{n_f=2}}_{\mathrm{JLQCD}}} \times \overbrace{(1.26 \pm 0.11 \pm 0.03}^{\mathrm{CLEO-c}}$$
$$= 1.28 \pm 0.11 \underbrace{\pm 0.03^{\mathrm{exp.}} \pm 0.01^{\mathrm{latt.}}}_{\mathrm{syst.}}$$

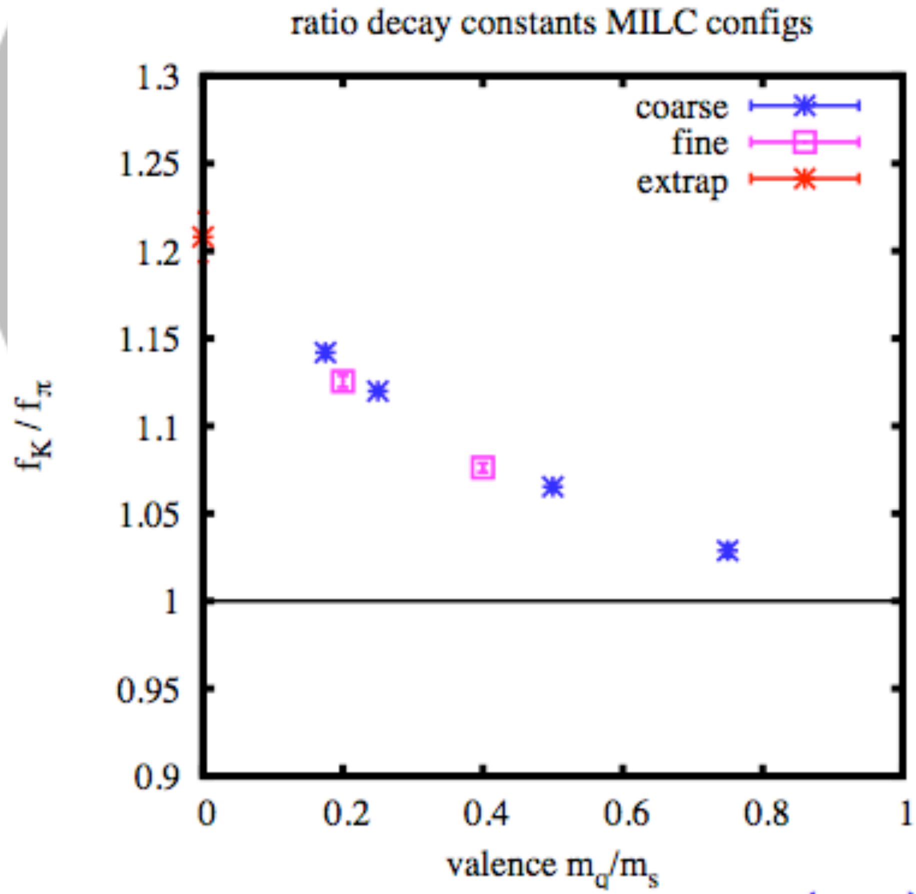
- *What should be done?*
  - *New (better) lattice estimates*

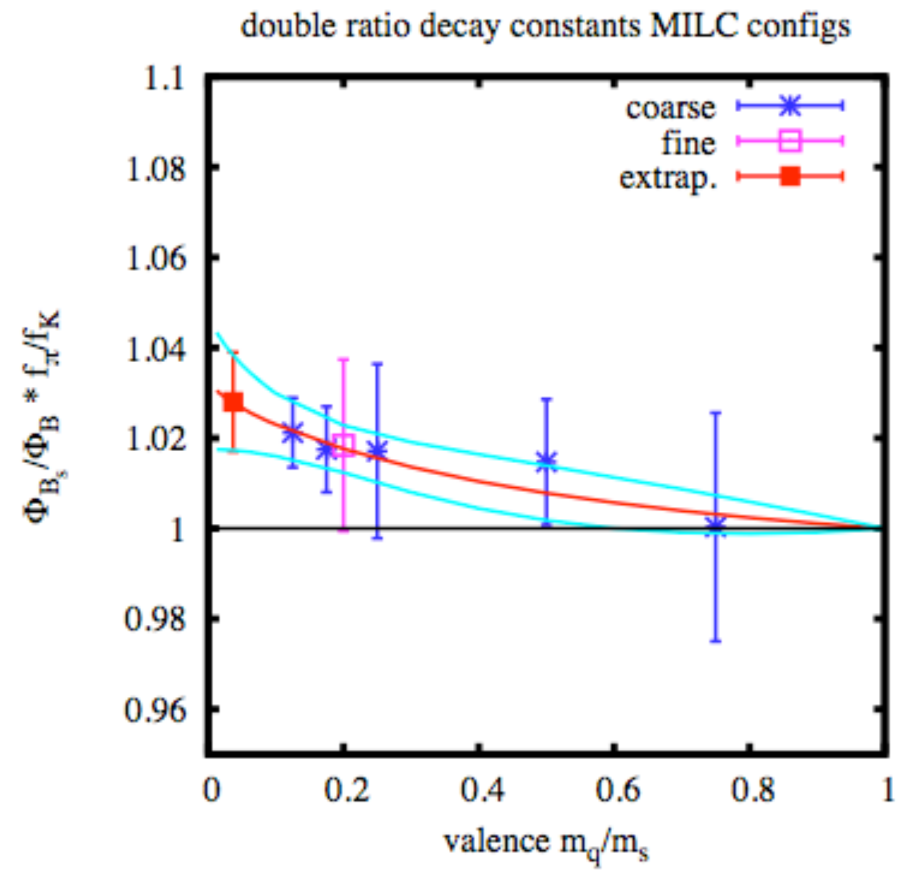
    many lattice groups working
  - *NLO chiral-log correction to "α"*

    almost done

Try double ratio to f_K/f_π

Much flatter chiral extrapoln





MILC results - $f_K/f_\pi = 1.208(2)\begin{pmatrix}+7\\-14\end{pmatrix}$

yield $V_{us} = 0.2223(26)$

Competitive with PDG from SL decay

Sugar, MILC, LAT06

$f_{B_s}/f_B \times f_\pi/f_K = 1.019(11)$

f_{B_s}/f_B Total error 2%

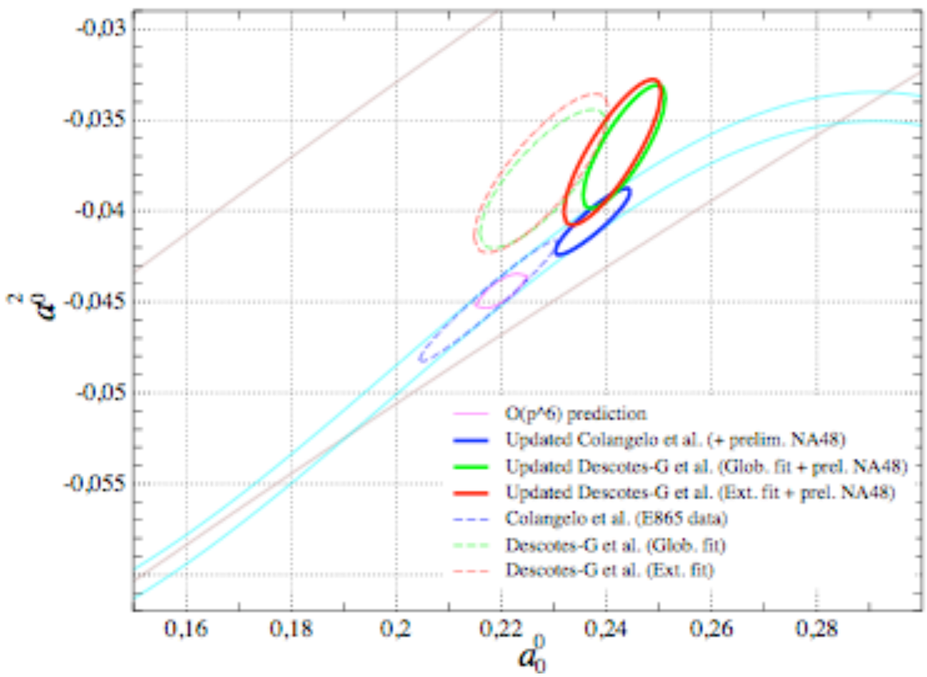
Becirevic et al,hep-ph/0211271

Check on ChPT ($SU(2)$ case - only pions!)

- $\pi\pi$-scattering length: for $kr \ll 1, S = \exp(i\delta_I)$ with $\delta_I = ka_I$

$$\text{Weinberg '66} \qquad a_2 m_{\pi^+} = -0.045, \quad a_0 m_{\pi^+} = 0.159$$
$$\text{CGL2001} \qquad a_2 m_{\pi^+} = -0.044(1), \quad a_0 m_{\pi^+} = 0.220(5)$$

- New $K_{\ell 4}$ data of NA48, difficulties begin? e.g. $m_\pi^2 f_\pi^2 = 4\langle\bar{q}q\rangle m_q + \mathcal{O}(m_q^{n>1})$ Leading term (GMOR) saturated not to $\approx 90\%$ but $\approx 75\%$



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Strange mass in ChPT?

Order parameters:

$$\Sigma(2, m_s) = -\lim_{m_{u,d}\to 0}\langle 0|\bar{u}u|0\rangle \quad f(2, m_s) = \lim_{m_{u,d}\to 0} f_\pi$$

2 scenarios:

- $m_{u,d} \to 0$ *and* $m_s \to 0 \Rightarrow \Sigma(3) \equiv \Sigma(2, 0)$
- $m_{u,d} \to 0$ *and* $m_s \to m_s^{\text{phys}} \Rightarrow \Sigma(2) \equiv \Sigma(2, m_s^{\text{phys.}})$

$$\Sigma(2, m_s^{\text{phys}}) = \Sigma(2, 0) + m_s \frac{\partial \Sigma(2, m_s)}{\partial m_s} + \mathcal{O}(m_s^2)$$

$$\Sigma(2) = \Sigma(3) + m_s \lim_{m_{u,d}\to 0}\left[i \int dx \langle 0|u\bar{u}(x) s\bar{s}(0)|0\rangle\right] + \mathcal{O}(m_s^2)$$

- 2^{nd} *term* ($\sim L_{4,6}$)*: Zweig and* N_c*-suppressed, but how effective this is in the scalar sector? Or, is* $\Sigma(3) = \Sigma(2)$ *or not?*
- *If* L_6 *much different from zero,* $\Sigma(3) < \Sigma(2)$
  If L_4 *much different from zero,* $f(3) < f(2)$
  If so then the leading and NLO in chiral expansion competitive in size! Recent analysis of $K\pi$*-scatt. Descotes and Moussalam:* $L_{4,6}$ *might be large*

Error projections

If you ignore my warnings and assume

1. *The efficiency of numerical algorithms used in LQCD simulations today will remain as such till 2015*
2. *That a 1% accuracy for the physics of light hadrons can be attained with*
   $N_{\rm conf} = 120$ *and* $a = 0.05$ *fm,* $m_\pi = 200$ *MeV,* $L = 4.5$ *fm*
   lattice: $90^3 \times 180$*; requires* ~ 0.1 *PFlop-years with Wilson quarks*
3. *That a 1% accuracy for the b-physics can be attained with*
   $N_{\rm conf} = 120$ *and* $a = 0.033$ *fm,* $m_\pi = 200$ *MeV,* $L = 4.5$ *fm*
   lattice: $136^3 \times 270$*; requires* ~ 1 *PFlop-years with Wilson quarks*

V.Lubicz made the errors projections presented at SuperB workshop (nov. 2006)

Estimates of error for 2015

Hadronic matrix element	Current lattice error	6 TFlop Year	60 TFlop Year	1-10 PFlop Year
$f_+^{K\pi}(0)$	0.9% (22% on $1-f_+$)	0.7% (17% on $1-f_+$)	0.4% (10% on $1-f_+$)	**< 0.1%** **(2.4% on $1-f_+$)**
$\hat{B}_K$	11%	5%	3%	**1%**
f_B	14%	3.5 - 4.5%	2.5 - 4.0%	**1 – 1.5%**
$f_{Bs}B_{Bs}^{1/2}$	13%	4 - 5%	3 - 4%	**1 – 1.5%**
ξ	5% (26% on ξ-1)	3% (18% on ξ-1)	1.5 - 2 % (9-12% on ξ-1)	**0.5 – 0.8 %** **(3-4% on ξ-1)**
$\mathcal{F}_{B\to D/D^*l\nu}$	4% (40% on $1-\mathcal{F}$)	2% (21% on $1-\mathcal{F}$)	1.2% (13% on $1-\mathcal{F}$)	**0.5%** **(5% on $1-\mathcal{F}$)**
$f_+^{B\pi}$,...	11%	5.5 - 6.5%	4 - 5%	**2 – 3%**
$T_1^{B\to K^*/\rho}$	13%	----	----	**3 – 4%**

Instead of conclusions

- *The errors on hadronic quantities (decay constants, form factors) for the charm physics not explored: Many lattice projects underway. A few percent precision (relatively soon) is likely and the "flatness" of the charmed unitarity triangle should be measured/tested too.*
- *Error reduction harder in B-physics but eventually feasible provided the PFlop-Year architectures are built (B_s decays not explored - may be cleaner!)*
- *Theoretical work for better controlling the systematic uncertainties is needed (esp. check on ChPT!) and non-perturbative methods to fix the parameters in Flab heavy quark action*
- *Going more chiral and to larger volumes $\longrightarrow$ dealing with hadronic widths (?)*
- *Going from 10% to 1% is very complicated: you need experts!*

European iniciative and ideas exist; IBM is building "Roadrunner", the PFlop architecture which combines the CELL chips and the standard (LQCD) design of parallel computing architectures, so... CELL processor, built in PlayStation3 is based on the parallel computing strategy which IBM learned from Lattice QCD practitioners!