Small x and multiple scattering effects in forward Drell-Yan scattering

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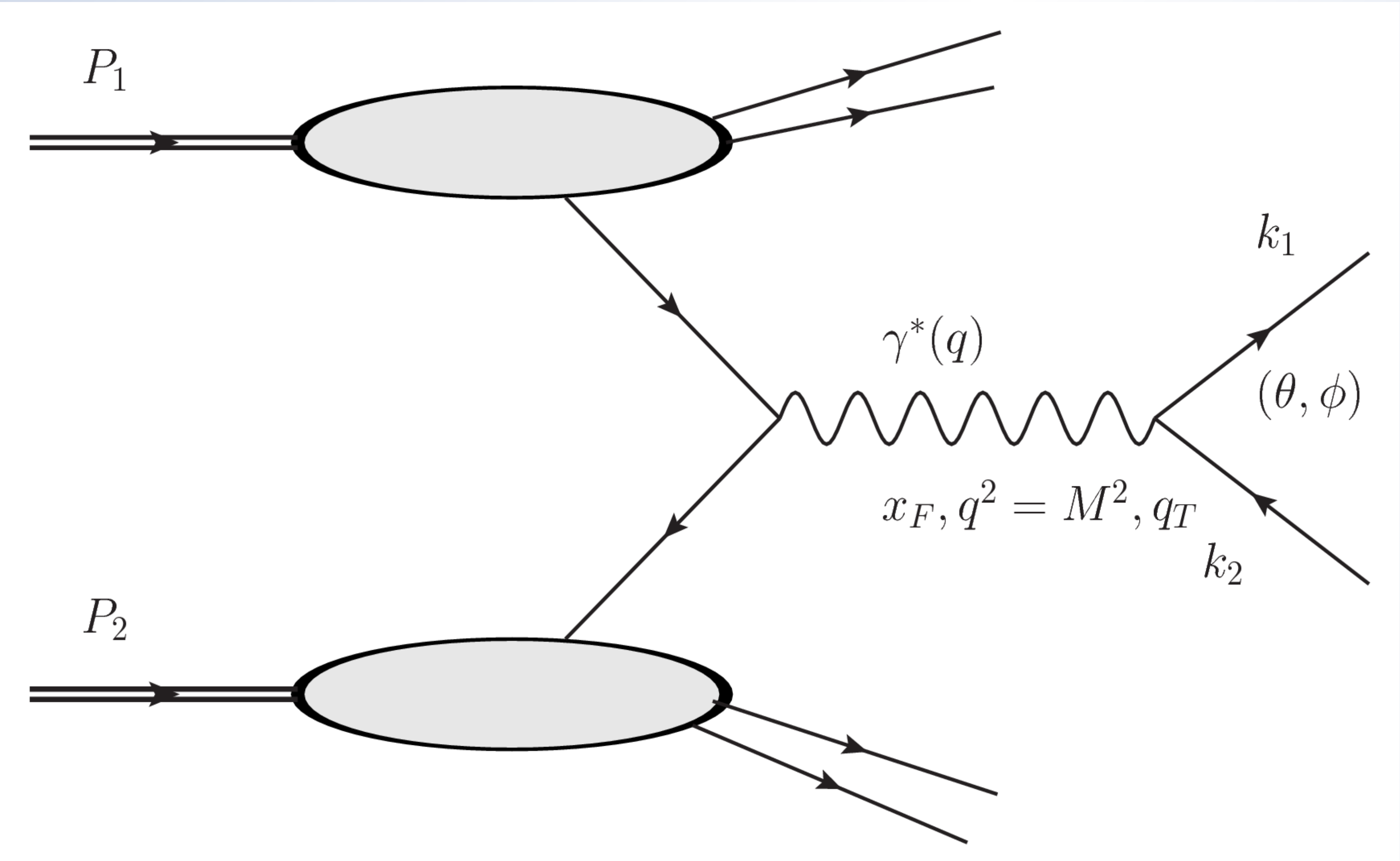
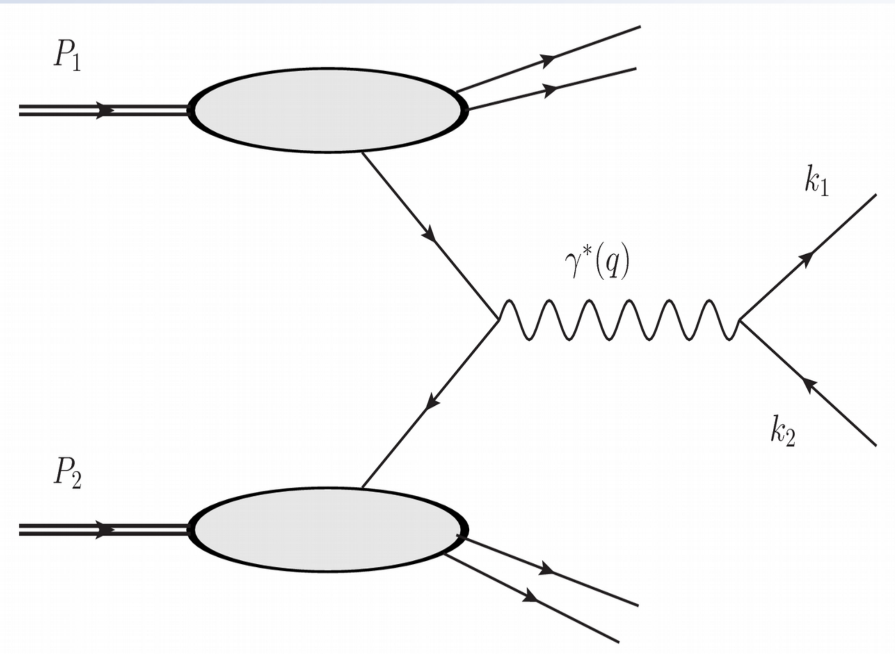
Drell-Yan process and kinematics

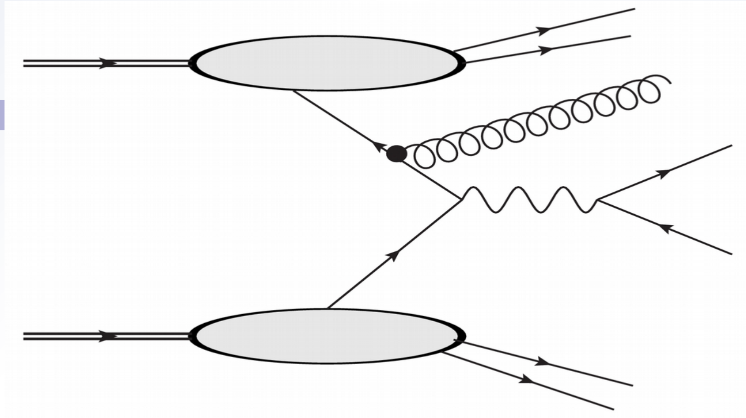


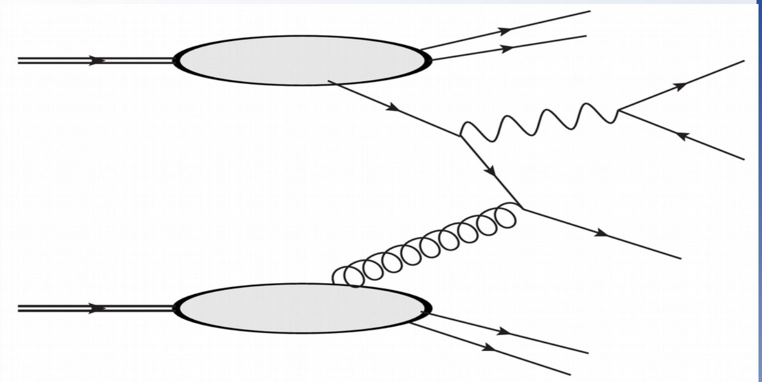
Partonic diagrams of Drell-Yan

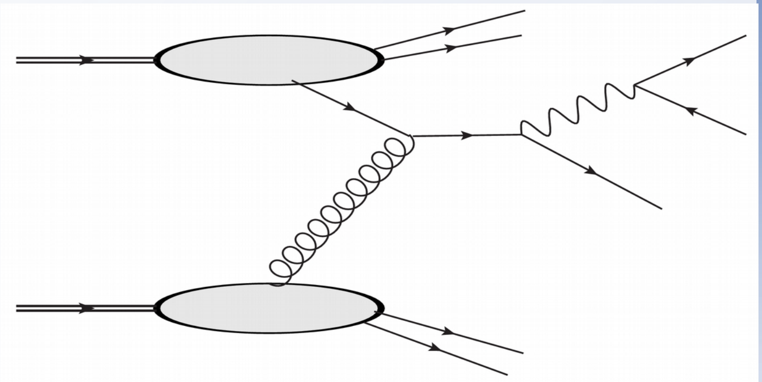
- Leading Order



- NLO



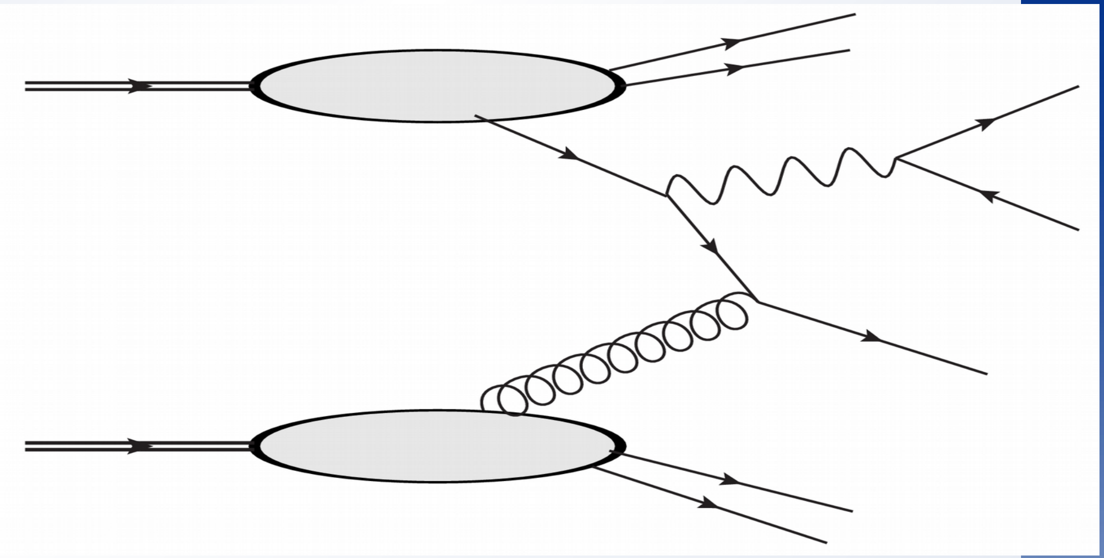




Gluon splitting to antiquark

Forward Drell-Yan scattering

- Asymmetric kinematics
  → large $x_1 >> x_2$

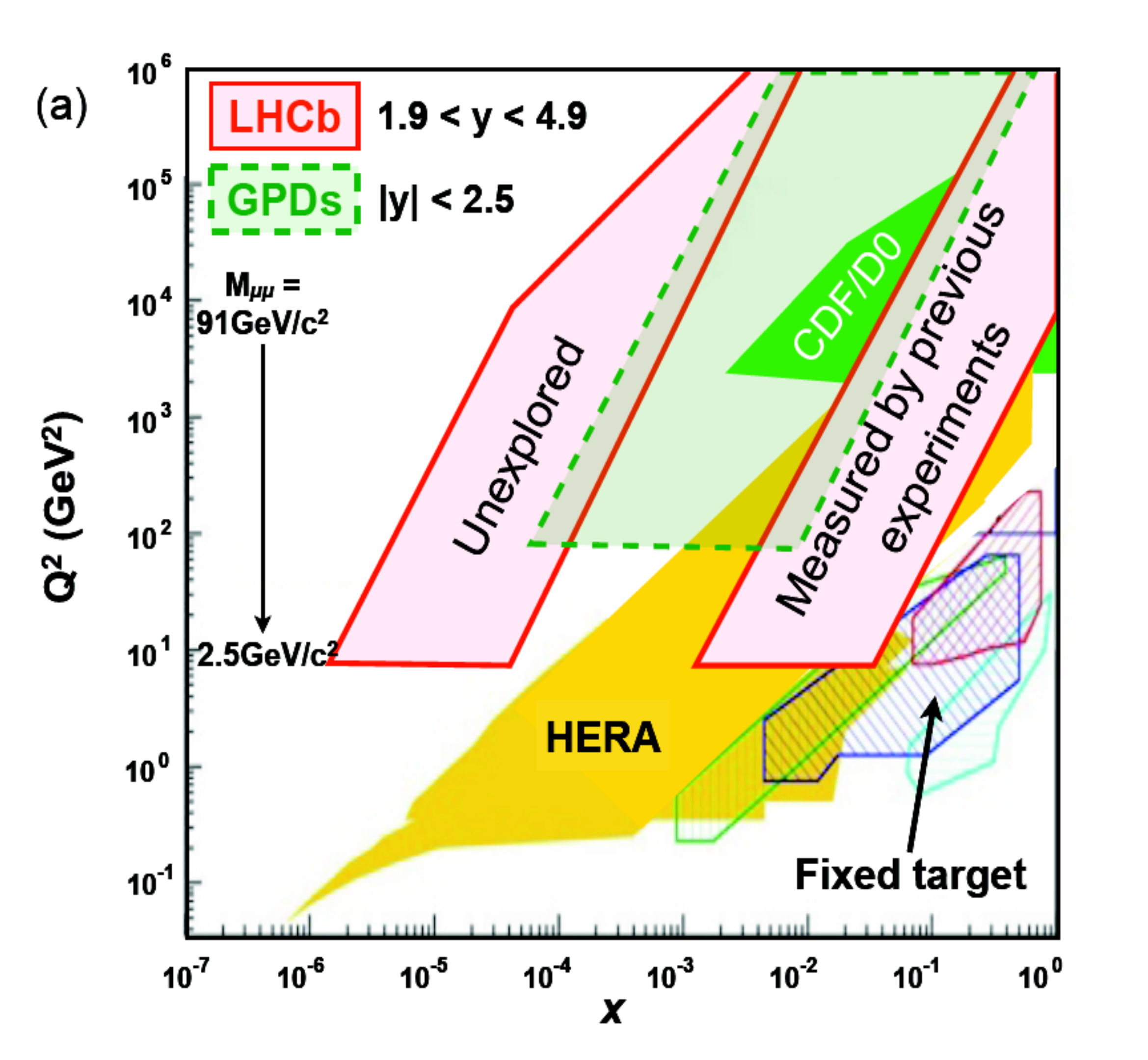


- Dominance of gluons at small x_2 and valence quarks at $x_1 \sim 1$ => dominance of valence quark – gluon fusion channel

- Interesting to measure → probe of gluon distribution at very small x

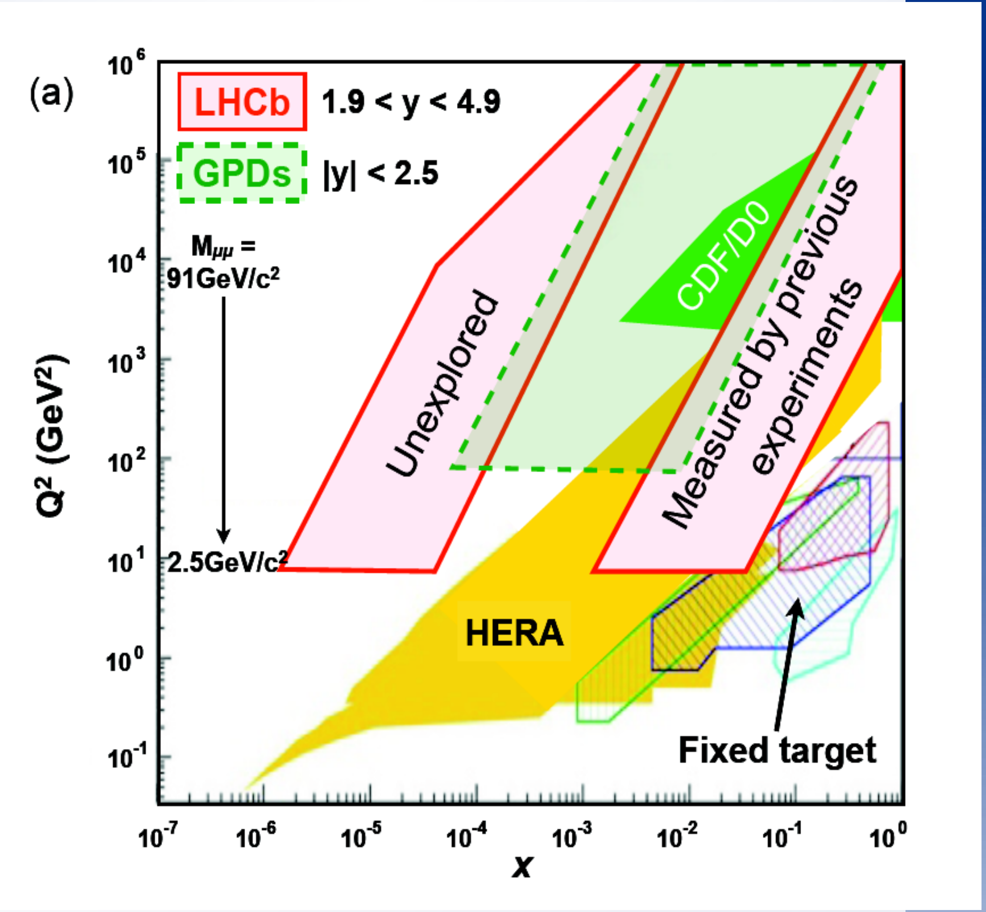
Forward Drell-Yan at LHC: kinematical reach

- At LHC forward Drell-Yan may be used to measure parton densties down to $x \sim 10^{-6}$ at $M^2 \sim 10\ GeV^2$
- Unique opportunity to explore this kinematic region and extend measurements of parton density functions



Theoretical interest in the forward Drell-Yan at LHC

- Kinematical range:
  $x < 10^{-6}$ at $M^2 \sim 10$ GeV^2
- Expected strong effects of small x resummation
- If the mass is sufficiently small, multiple scattering and higher twists effects are expected to turn on:
  higher twist are suppressed by ***$1/M^2$***
  but enhanced by *$x^{-\lambda}$*



- Higher twist effects should be determined to avoid systematic errors of pdf determination, they are also interesting for deeper understanding of proton structure and dynamics of strong interactions

Plan: to make full use of forward Drell-Yan process at the LHC as a probe of high energy scattering in QCD

- Introduce Drell-Yan structure functions
- Lam-Tung relation in QCD
- Dipole picture of forward Drell-Yan scattering
- Small x resummation and twist decomposition in forward DY scattering *(technical)*
- Results
- Conclusions

Work done with

Dawid Brzemiński, Mariusz Sadzikowski and Tomasz Stebel, JHEP 2015, 2017

Drell-Yan structure functions:

- Lepton angular distributions: 4 Drell-Yan structure functions (W_a – frame dependent)

$$\frac{d\sigma}{dx_F dM^2 d\Omega d^2 q_T} = \frac{\alpha_{\rm em}^2}{2(2\pi)^4 M^4}\Big[(1-\cos^2\theta)W_L + (1+\cos^2\theta)W_T + (\sin^2\theta\cos 2\phi)W_{TT} + (\sin 2\theta\cos\phi)W_{LT}\Big]$$

- Helicity structure functions → elements of virtual photon production helicity density matrix

- Photon decays into leptons → interference between different virtual photon polarisations possible:

W_T: $T(+) \to T(+)$ W_L : $L \to L$

W_{LT} : $T \to L$, $L \to T$ W_{TT} : $T(+) \to T(-)$

Lam-Tung relation

- Hence: DY helicity structure functions: projections of DY amplitudes on virtual photon polarization states

- Lam-Tung relation (1980, 1982): vanishing combination of DY structure functions at leading
twist up to NNLO in ollinear QCD

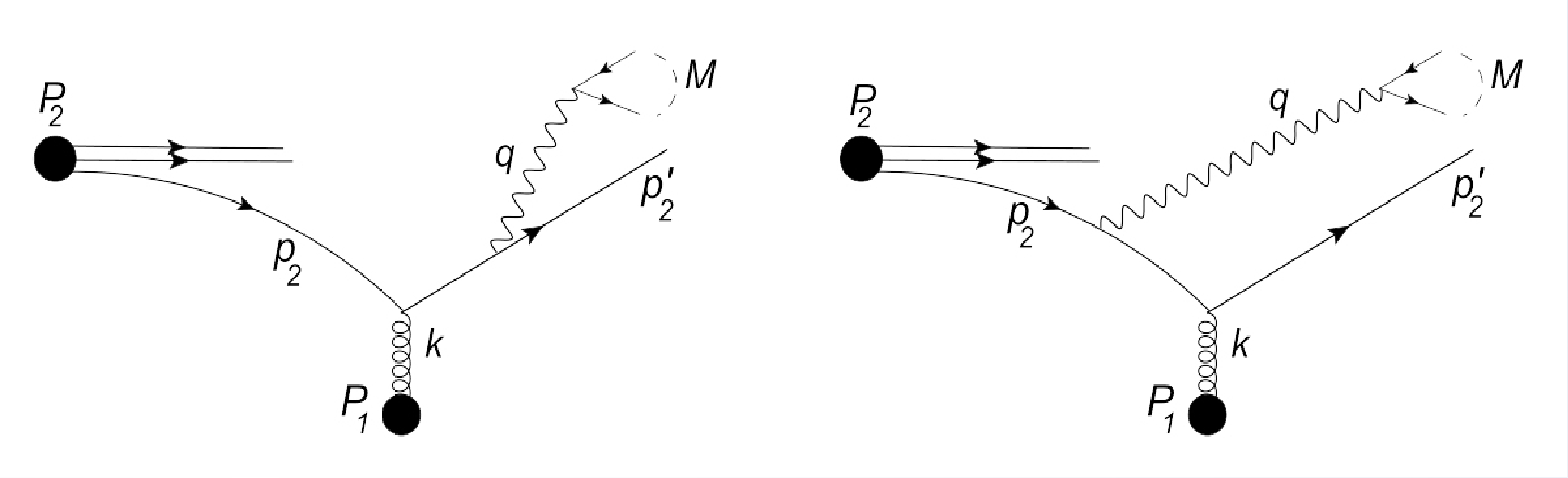
$$W_L - 2W_{TT} = 0$$

- Advantage of Lam-Tung relation: it is invariant under frame rotations w.r.t. axis perpendicular to reaction plane

- Lam-Tung relation breaking by higher order QCD effects related to parton k_T
- At twist 4 – non-zero contribution → enhanced higher twist contributions

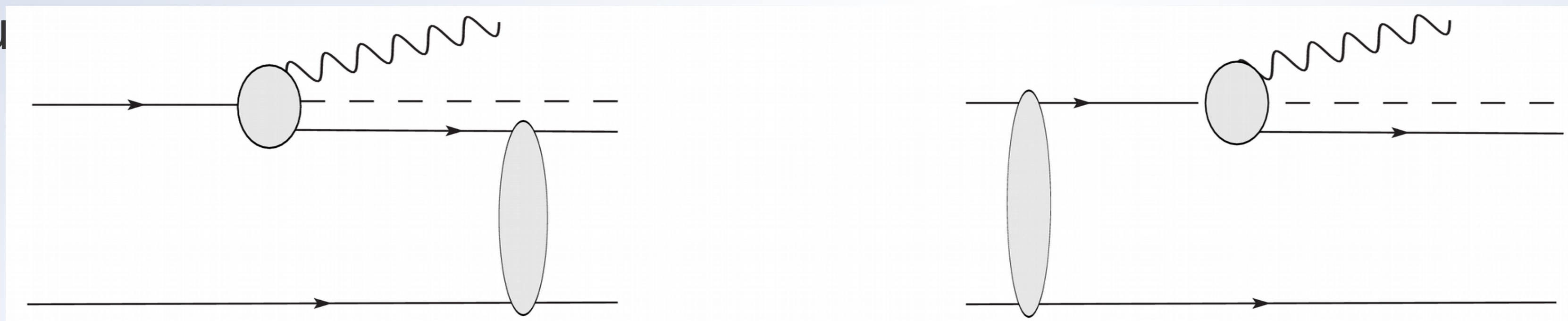
Leading diagrams of forward Drell-Yan

- Asymmetric kinematics: $x_2 >> x_1$
- Dominance of the quark sea → driven by gluon evolution
- Good approximation: gluon evolution followed by splitting to quark (anti-quark) in the last step

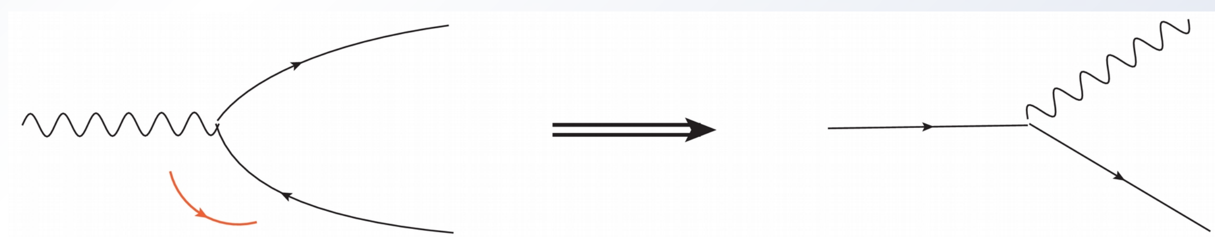


Forward Drell-Yan in dipole formulation

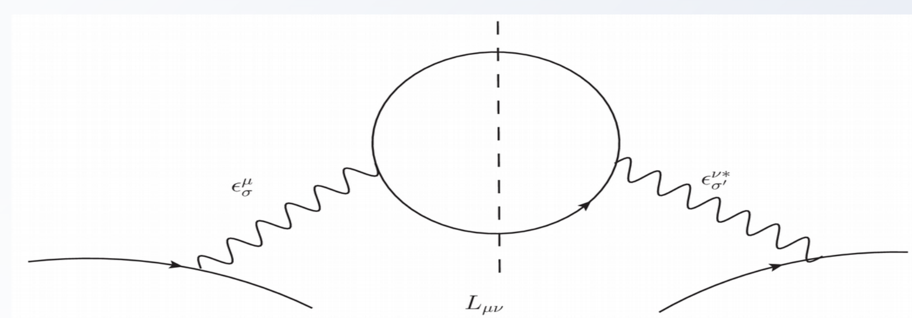
- Large energy limit: conservation of transverse positions in scattering
- "Effective color dipole" emerges from interference of photon emission before and after scattering, γ^* carries fraction z of p^+ of incident qu



- "Crossed" photon wave function:



- Interference of photon helicity states through leptonic tensor



Forward Drell-Yan in dipole formulation

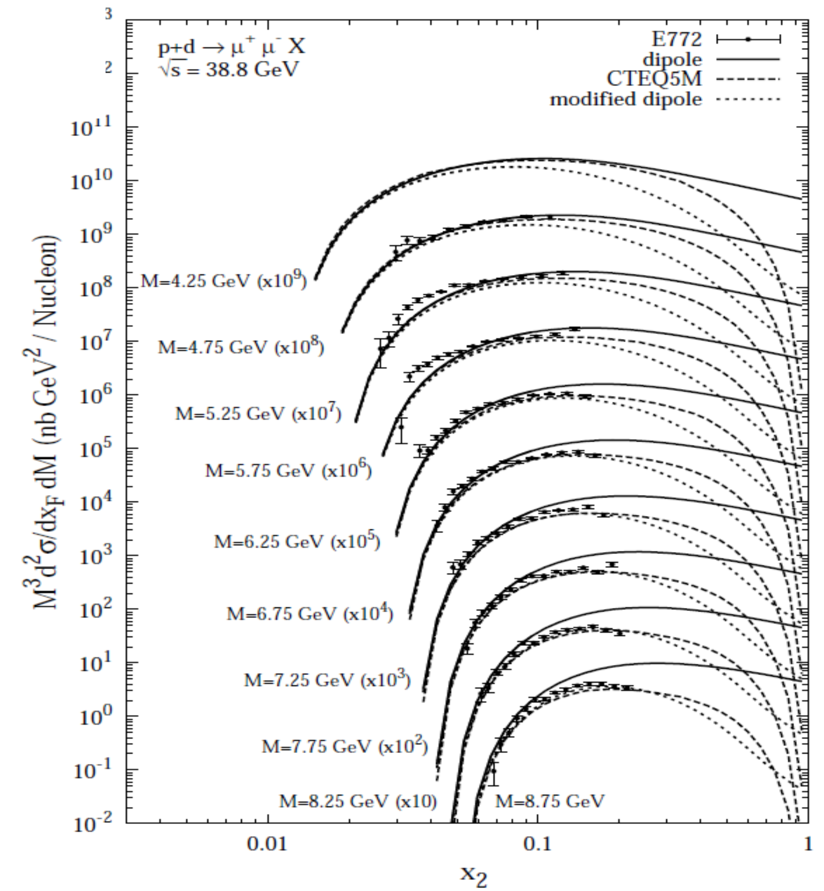
$$\sigma^f_{T,L}(qp \to \gamma^* X) = \int d^2r\, W^f_{T,L}(z, r, M^2, m_f)\, \sigma_{q\bar{q}}(x_2, zr)$$

$$W^f_T = \frac{\alpha_{em}}{\pi^2} \{[1 + (1 - z)^2]\, \eta^2 K_1^2(\eta r) + m_f^2\, z^4 K_0^2(\eta r)\}$$

$$W^f_L = \frac{2\alpha_{em}}{\pi^2} M^2 (1 - z)^2 K_0^2(\eta r)\ ,$$

Formalism proposed and developed by:

- Brodsky, Hebecker, Quack (1997)
- B. Z. Kopeliovich, J. Raufeisen, A. V. Tarasov (2001)
- Gelis,Jalilian-Marian (2002)
- Raufeisen, Peng, Nayak (2002): plot →
- V. Goncalves et al, A. Szczurek et al.

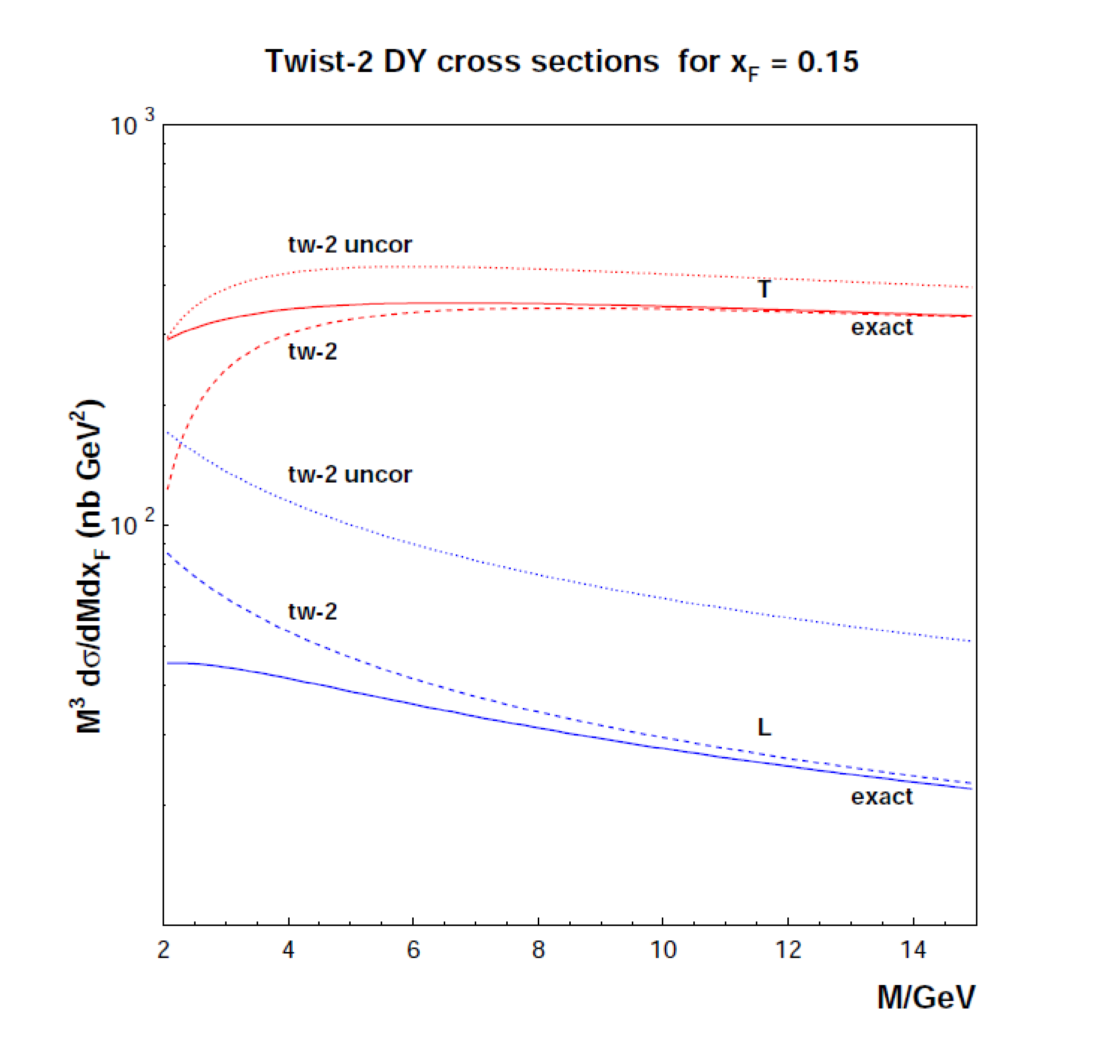


Inclusive forward Drell-Yan at LHC: higher twist corrections

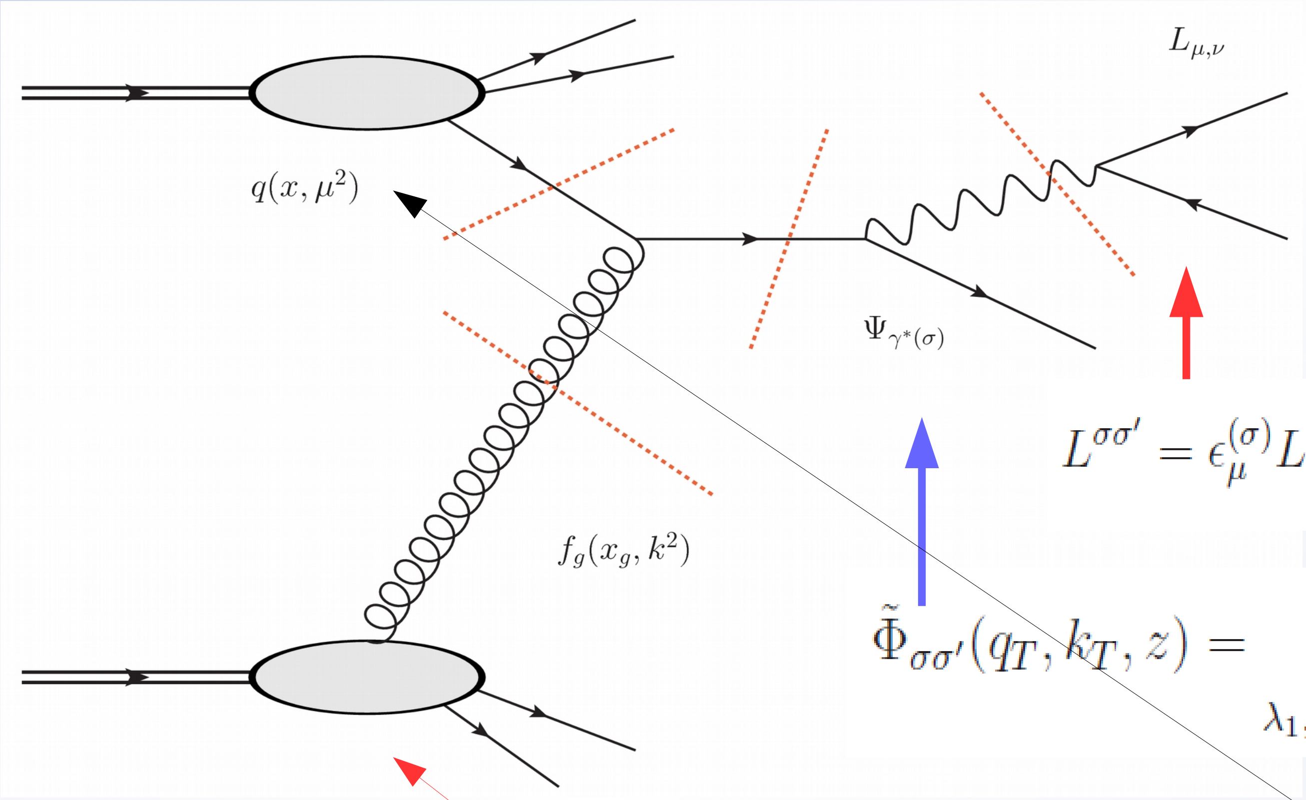
- Golec-Biernat, Lewandowska, Staśto, 2010 (plot):

  twist content of forward Drell-Yan within the GBW saturation model for dipole cross-section, done for the inclusive cross-section (in qT and the lepton azimuthal angle)

- Predictions for the LHC (plot) large higher twist corrections within kinematical range of LHC (LHCb)



Forward Drell-Yan cross-section in kT factorisation



$$L^{\sigma\sigma'} = \epsilon_\mu^{(\sigma)} L^{\mu\nu}\, \epsilon_\nu^{(\sigma')\dagger}, \quad L^{\mu\nu} = -g^{\mu\nu} + \frac{\kappa^\mu \kappa^\nu}{\kappa^2}$$

$$\tilde{\Phi}_{\sigma\sigma'}(q_T, k_T, z) = \sum_{\lambda_1,\lambda_2=+,-} A^{(\sigma)}_{\lambda_1,\lambda_2}(\vec{q}_T)^\dagger A^{(\sigma')}_{\lambda_1,\lambda_2}(\vec{q}_T)$$

$$\frac{d\sigma}{dx_F dM^2 d\Omega d^2 q_T} = \frac{\alpha_{\rm em}}{(2\pi)^2 (P_1\cdot P_2)^2\ M^2\ x_F^2(1-z)} L^{\sigma\sigma'}(\Omega) \int_{x_F}^1 dz\ \wp(x_F/z)$$
$$\times \int d^2k_T \frac{2\pi\alpha_s}{3} \frac{f(x_g, k_T^2)}{k_T^4} \tilde{\Phi}_{\sigma\sigma'}(q_T, k_T, z)$$

Mellin representation of forward Drell-Yan structure functions:

- Standard procedure: position-space → Mellin moments space

$$W_i = \int_{x_F}^{1} dz\, \wp(x_F/z) \int_C \frac{ds}{2\pi i}\, \tilde{\sigma}(-s) \left(\frac{z^2 Q_0^2}{\eta_z^2}\right)^s \hat{\Phi}_i(q_T, s, z)$$

- Convolution of Mellin transform of dipole cross section and impact factor

$$\eta_z^2 = M^2(1-z)$$

$$\hat{\Phi}_i(q_T, s, z) = \frac{2(2\pi)^4 M^4}{\alpha_{\rm em}^2} \int d^2 r \left(\frac{\eta_z^2}{4z^2}\, r\right)^s \Phi_i(q_T, r, z)$$

- Dipole cross section encodes QCD dynamics, e.g. small x evolution, higher twists

$$\tilde{\sigma}(-s) = \int_0^\infty \frac{d\rho^2}{\rho^2} \left(\rho^2\right)^{-s}\, \hat{\sigma}(\vec{\rho})$$

Two descriptions of dipole cross-section in kT factorisation:

- Phenomenological: eikonal multiple gluon ladder exchange – GBW model → twist 2n contribution enhanced by $x^{-n\lambda}$ at small x
- BFKL description: based on LL BFKL cross-section and its twist decomposition
- Higher twist effects extracted from singularities of the BFKL kernel in Mellin space, $\chi(\gamma)$, at integer values of anomalous dimensions s → γ

$$\sigma(\gamma) \sim \exp(c \log(1/x)\, \alpha_s\, \chi(\gamma))$$

$$\chi(\gamma) = 2\psi(1) - \psi(\gamma) - \psi(1-\gamma) \sim 1/(\gamma - n)$$

→ essential singularities of Mellin cross-section

- Saddle point treatment of γ-integral of BFKL amplitudes

→ power suppression x^{δ} of higher twist terms in LL BFKL amplitudes

Previous findins: Mellin representation of DY impact factors

- Mellin transforms of impact factors for all DY structure functions found, e.g.:

$$\hat{\Phi}_L(q_T,s,z) = \frac{2}{z^2}\left\{\frac{2\Gamma^2(s+1)}{1+q_T^2/\eta_z^2}\ {}_2F_1\left(s+1,s+1,1,-\frac{q_T^2}{\eta_z^2}\right)\right.$$
$$\left. -\Gamma(s+1)\Gamma(s+2)\ {}_2F_1\left(s+1,s+2,1,-\frac{q_T^2}{\eta_z^2}\right)\right\}$$

$$\hat{\Phi}_{TT}(q_T,s,z) = \frac{1}{2z^2}\left\{\frac{2\pi}{\Gamma(1-s)\sin\pi s\ q_T^2/\eta_z^2}\left(1+\frac{q_T^2}{\eta_z^2}\right)^{-s-3}\Gamma(s+2)\right.$$
$$\left[\left(1+\frac{q_T^2}{\eta_z^2}\right)\left(1+\frac{q_T^2}{\eta_z^2}(s+2)\right)\ {}_2F_1\left(-s+1,s+1,1,\frac{q_T^2}{q_T^2+\eta_z^2}\right)\right.$$
$$\left.-\left(1+2\frac{q_T^2}{\eta_z^2}(s+1)\right)\ {}_2F_1\left(-s+1,s+2,1,\frac{q_T^2}{q_T^2+\eta_z^2}\right)\right]$$
$$\left.-\frac{4q_T^2/\eta_z^2}{1+q_T^2/\eta_z^2}\Gamma(s+1)\Gamma(s+2)\ {}_2F_1\left(s+1,s+2,2,-\frac{q_T^2}{\eta_z^2}\right)\right\}$$

- Useful in BFKL approach and for twist analysis

Mellin treatment of BFKL DY amplitudes

- BFKL essential singularities at twist poles via Laurent expansion

$$W_j = \int_{x_F}^{1} dz\, \wp(x_F/z)\, \sigma_j(q_T, z, Y)$$

$$\sigma_j(q_T, z, Y) = \int_{\mathcal{C}} \frac{ds}{2\pi i} \left(\frac{z^2 \bar{Q}_0^2}{M^2(1-z)} \right)^{-s} \tilde{\sigma}(s, Y) \hat{\Phi}_j(q_T, -s, z)$$

$$\sigma_j(q_T, z, Y) = \sum_{n=1}^{\infty} \sigma_j^{(2n)}(q_T, z, Y)$$

$$\sigma_j^{(2n)}(q_T, z, Y) = -R_p^2 e^{-nt} \int_0^{2\pi} d\theta h_j^{(2n)}(\epsilon e^{i\theta}, q_T, z, Y) \exp\left(\epsilon e^{i\theta}\, t + \frac{\bar{\alpha}_s Y}{\epsilon} e^{-i\theta} \right)$$

$$\chi_{reg}^{(n)} = \chi\left(-n + \epsilon e^{i\theta} \right) - \frac{e^{-i\theta}}{\epsilon}$$

Mellin treatment of BFKL DY amplitudes

$$h_j^{(2n)}(\epsilon e^{i\theta}, q_T, z, Y) = \epsilon e^{i\theta} \left(\frac{z^2}{1-z}\right)^{n-\epsilon \exp i\theta} \hat{\Phi}_j(q_T, n-\epsilon e^{i\theta}, z)\, \Gamma(-n+\epsilon e^{i\theta})\, e^{\bar{\alpha}_s Y \chi_{reg}^{(n)}}$$

$$h_j^{(2n)}(\epsilon e^{i\theta}, q_T, z, Y) = \sum_{m=0}^{\infty} a_m^{(2n)j} \left(\epsilon e^{i\theta}\right)^m$$

$$\sigma_j^{(2n)}(q_T, z, Y) = -2\pi R_p^2 \left(\frac{\bar{Q}_0^2}{M^2}\right)^n \sum_{m=0}^{\infty} a_m^{(2n)j} \left(\frac{\bar{\alpha}_s Y}{t}\right)^{\frac{m}{2}} I_m\left(2\sqrt{\bar{\alpha}_s Y t}\right)$$

$$W_j^{(2n)} = -\sigma_0' \left(\frac{\bar{Q}_0^2}{M^2}\right)^n \sum_{m=0}^{\infty} \int_{x_F}^{1} dz\, a_m^{(2n)j} \wp(x_F/z) \left(\frac{\bar{\alpha}_s Y}{t}\right)^{\frac{m}{2}} I_m\left(2\sqrt{\bar{\alpha}_{sS} Y t}\right)$$

where $t \sim \log(M^2)$

- Integrals over the contour angle parameter θ performed and expansion coefficients $a_m^{(2n)\,j}$ evaluated analytically → analytic form of expansions of BFKL forward Drell-Yan structure functions

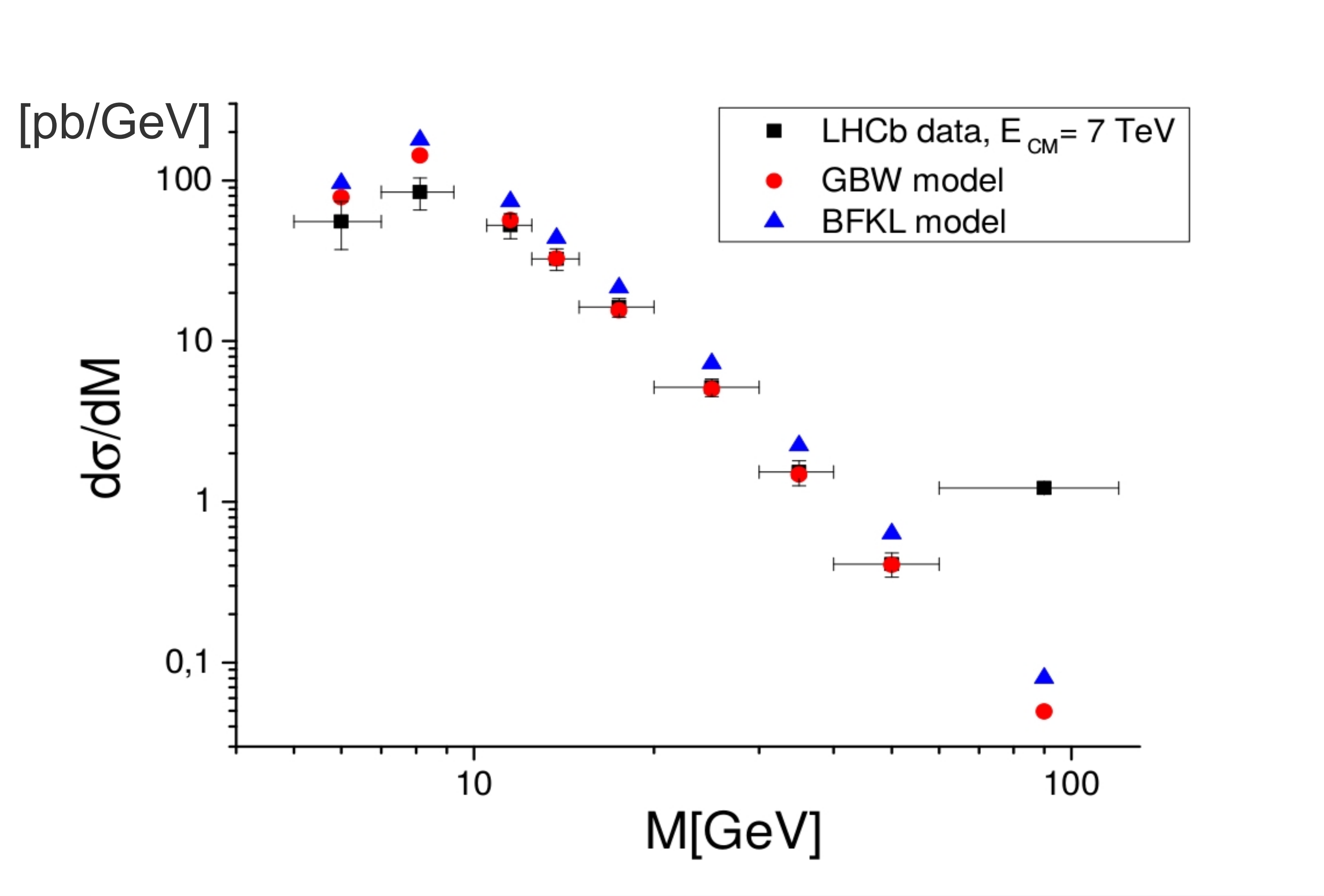
Results for forward DY BFKL structure functions

- In LL BFKL subleading twist contributions found to decrease exponentially with rapidity!
- Exponential increase in rapidity found only for the leading twist
- Example of results for q_T -integrated structure functions
- Lam-Tung relation fulfilled in double logarithmic limit, broken beyond

$$
\begin{aligned}
\tilde{W}_j^{(2)} &= -\sigma_0' \left(\frac{\bar{Q}_0^2}{4M^2}\right) \int_{x_F}^{1} dz\ f_j(z) \frac{z^2}{1-z}\ \wp(x_F/z) \\
&\quad \times \sum_{m=0}^{\infty} \tilde{a}_m^{(2)j} \left(\frac{\bar{\alpha}_s Y}{\ln(4M^2/\bar{Q}_0^2)}\right)^{\frac{m}{2}} I_{|m|}\left(2\sqrt{\bar{\alpha}_s Y \ln\frac{4M^2}{\bar{Q}_0^2}}\right)
\end{aligned}
$$

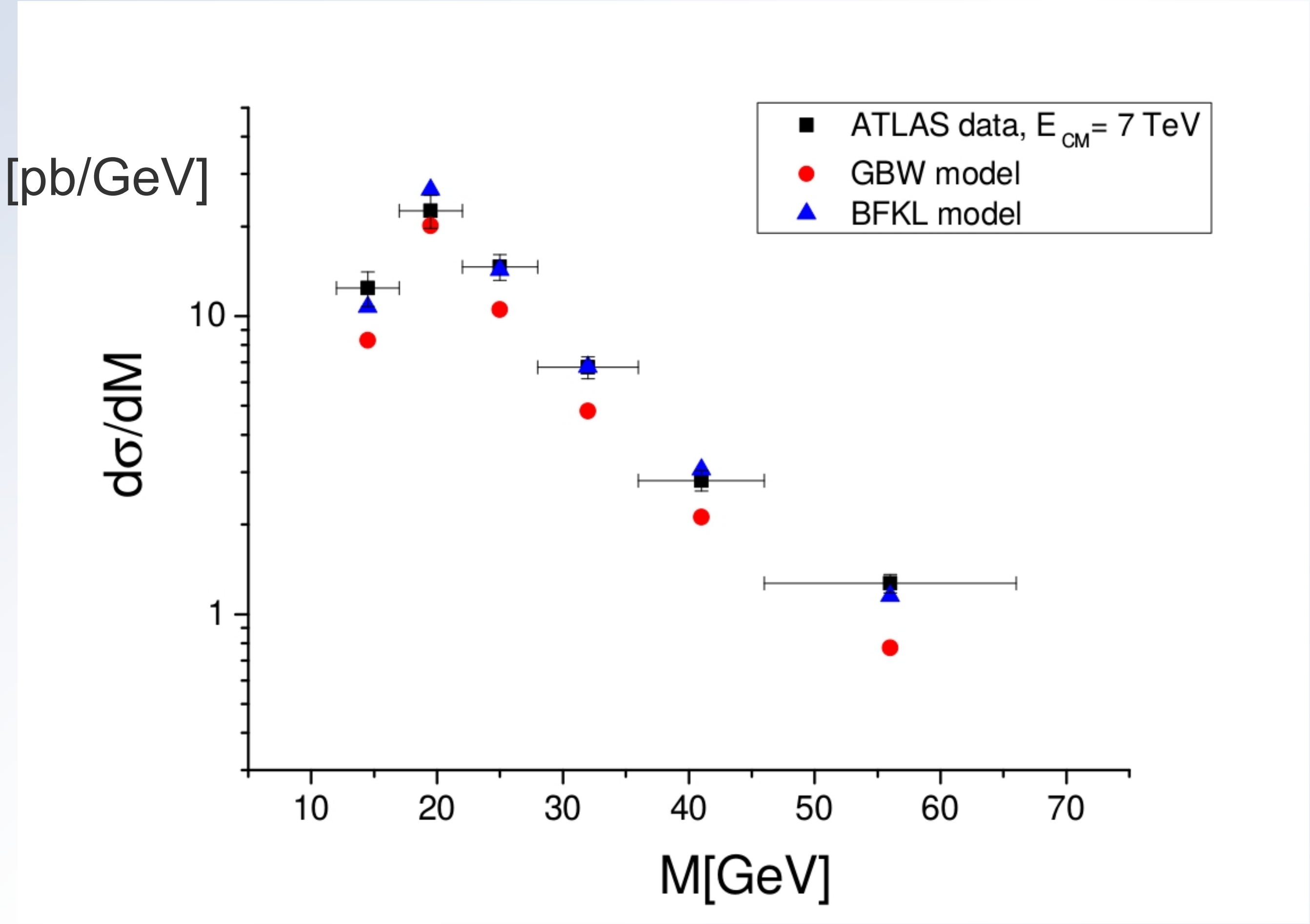
$$
\begin{aligned}
\tilde{a}_0^{(2)L} &= -\frac{4}{3}, \quad \tilde{a}_1^{(2)L} = -\frac{4}{3}\left(-2 + 2\gamma_E + \ln\frac{1-z}{z^2} + \psi(5/2)\right) \\
\tilde{a}_0^{(2)TT} &= -\frac{2}{3}, \quad \tilde{a}_1^{(2)TT} = \frac{2}{3}\left(-3 + \gamma_E - \ln\frac{1-z}{z^2} + \ln(64) + 2\psi(5/2)\right) \\
\tilde{a}_0^{(2)LT} &= 0, \quad \tilde{a}_1^{(2)LT} = 0.5236
\end{aligned}
$$

Results: inclusive Drell-Yan at LHC: LHCb data from BFKL and GBW dipole cross sections



- Good description in terms of GBW, BFKL somewhat above data

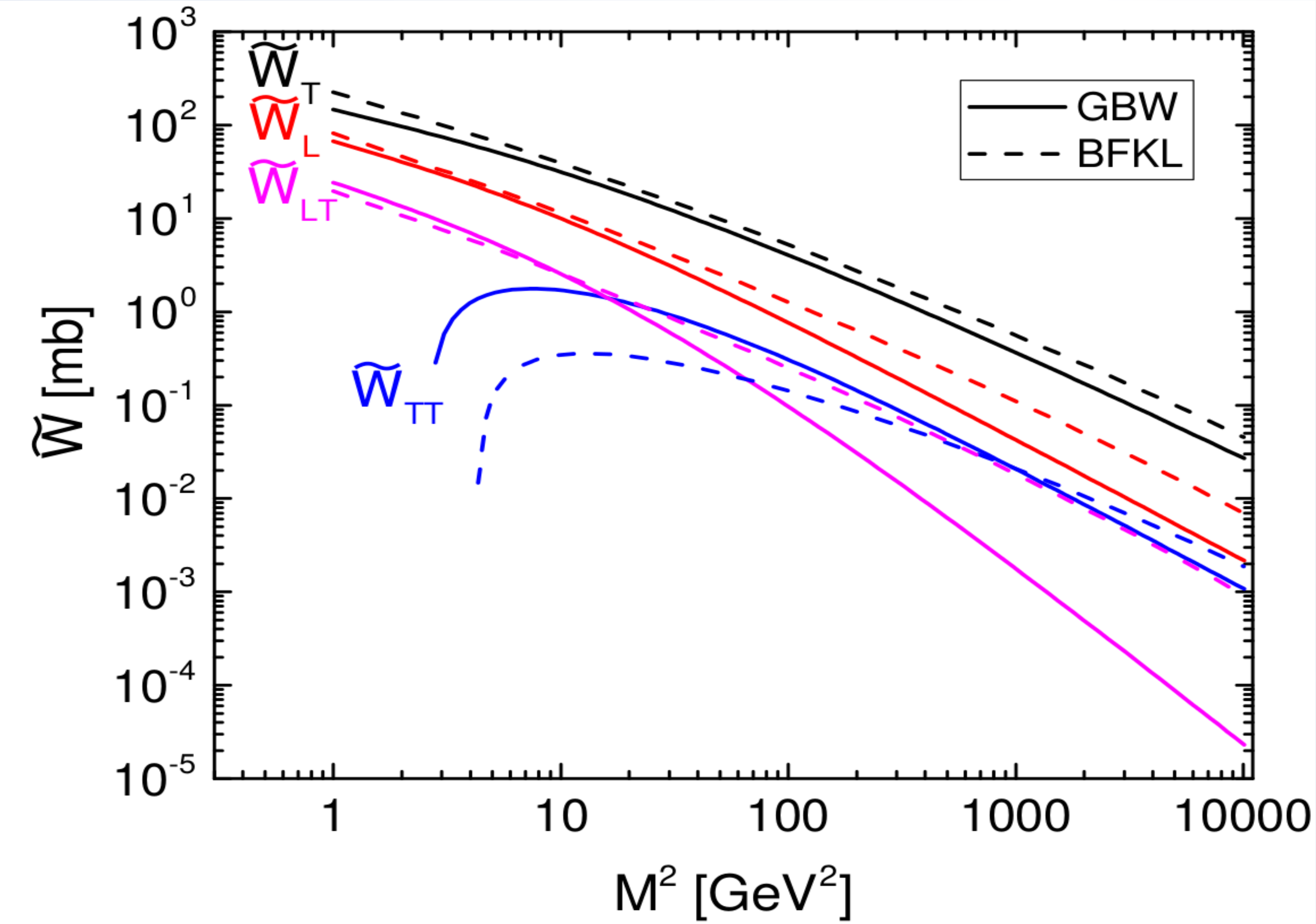
Results: inclusive Drell-Yan at the LHC: ATLAS data



- Good description in terms of BFKL, GBW somewhat below data
- However: central kinematics → expected terms beyond forward DY approximation

DY at LHC: structure functions from BFKL vs GBW

- Similar behavior of W_T and W_L in BFKL and GWB, butsignificant differences in “inteference” structure functions W_{TT} and W_{LT}



Lam-Tung relation in GBW model

- Lam-Tung relation: at leading twist up to NNLO:

$$W_L - 2W_{TT} = 0$$

- Holds in the GBW model at twist 2
- At twist 4 – non-zero contribution → enhanced higher twist contributions

$$W_L^{(4)} - 2W_{TT}^{(4)} = \sigma_0 \frac{Q_0^4}{M^4} \int_{x_F}^1 dz\ \wp(x_F/z) z^2 \frac{4M^8(1-z)^2}{\left[q_T^2 + M^2(1-z)\right]^4}$$

- qT-integrated cross-section also shows breaking of Lam-Tung relation

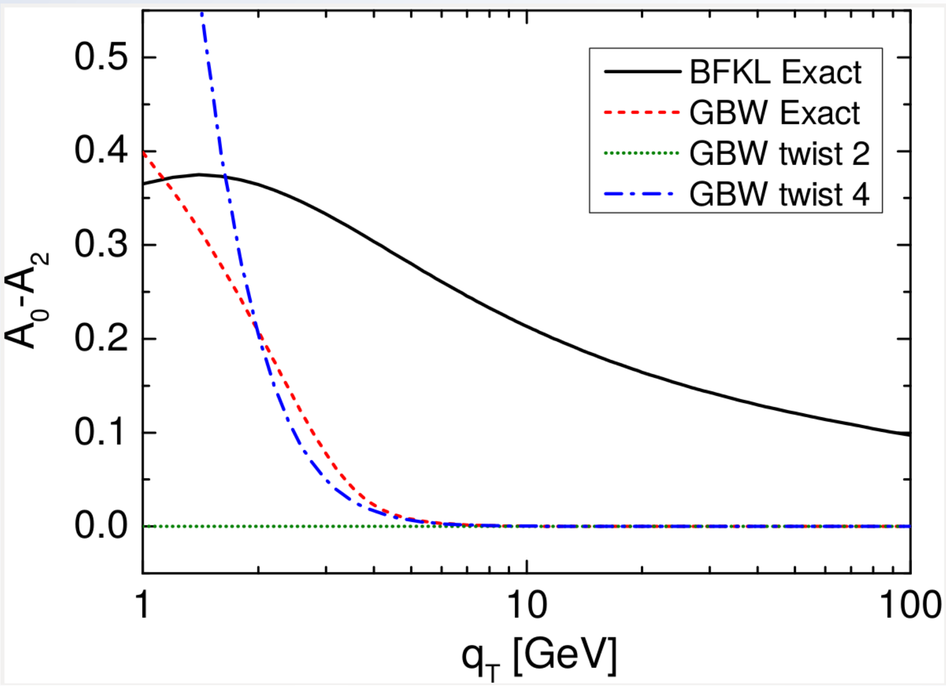
$$\int \left(W_L^{(4)} - 2W_{TT}^{(4)}\right) d^2q_T = 2\pi\sigma_0 M^2 \left(\tilde{W}_L^{(4)} - 2\tilde{W}_{TT}^{(4)}\right)$$
$$= 2\pi\sigma_0 \frac{Q_0^4}{M^2} \left\{ \frac{1}{18}\wp(x_F)\left[-19 + 12\gamma_E + 12\ln\left(\frac{M^2(1-x_F)}{Q_0^2}\right)\right] + \right.$$
$$\left. + \frac{2}{3}\int_{x_F}^1 dz\ \frac{\wp(x_F/z)z^2 - \wp(x_F)}{1-z} \right\}$$

Lam-Tung relation from GBW and BFKL

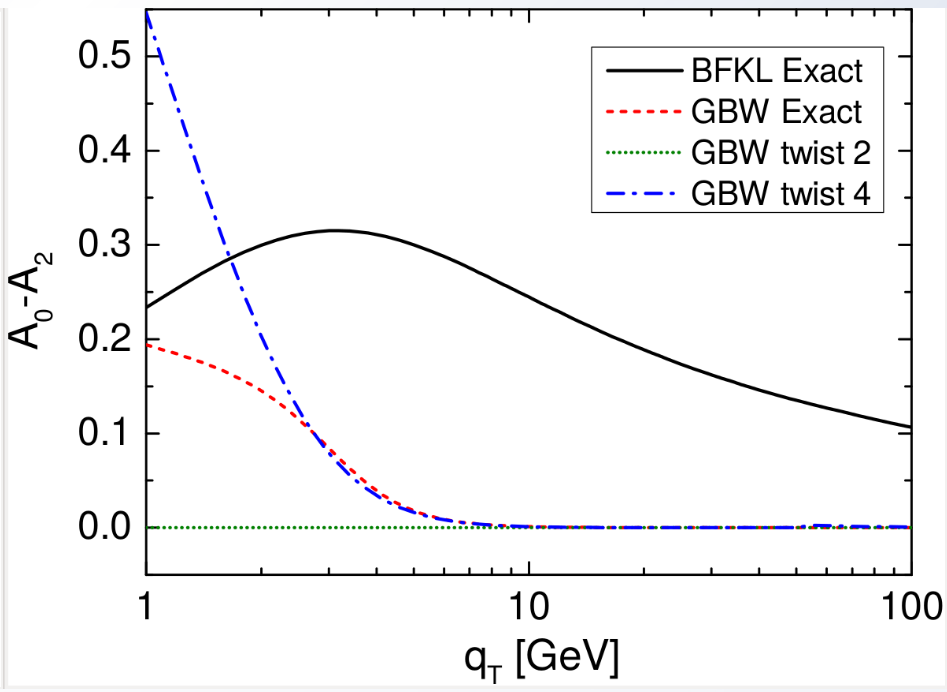
$\sqrt{s} = 14$ TeV

- Striking difference in Lam – Tung relation breaking
- Subleading twist effects in GBW vs leading twist in BFKL
- **Importance of parton k_T effects in BFKL**

$M^2 = 5$ GeV^2



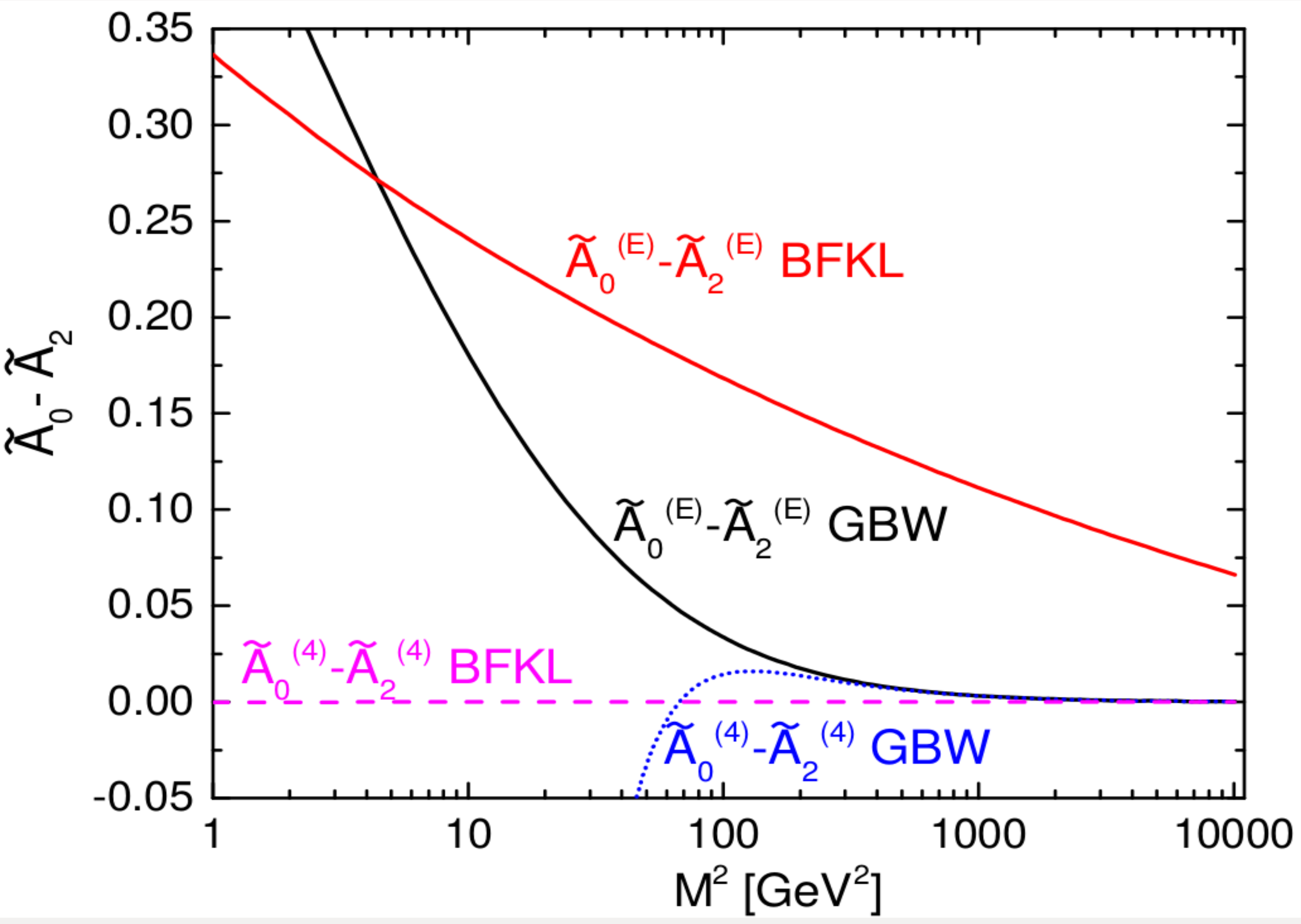
$M^2 = 20$ GeV^2



Lam-Tung relation from GBW and BFKL in q_T integrated structure functions

$\sqrt{s} = 14$ TeV

- Lam – Tung relation breaking in BFKL occurs at any mass



- Lam – Tung relation breaking at larges masses is an excellent probe of gluon k_T

Conclusions

- Forward Drell Yan process is of great interest for determination parton density functions at small x, studies of small x resummation and higher twist effects
- Angular distributions of DY leptons give access to four independent DY structure functions
- We provide estimates of both small x and higher twist effects in forward DY scattering using BFKL and GBW approach
- The models were tested against LHC data: good description of angular averaged cross-sections was found
- Lam-Tung relation is particularly interesting observable with enhanced small x and higher twist sensitivity, it also probes parton k_T .
- Essentially different predictions obtained for higher twists and Lam-Tung relation breaking from GBW and BFKL

THANKS!