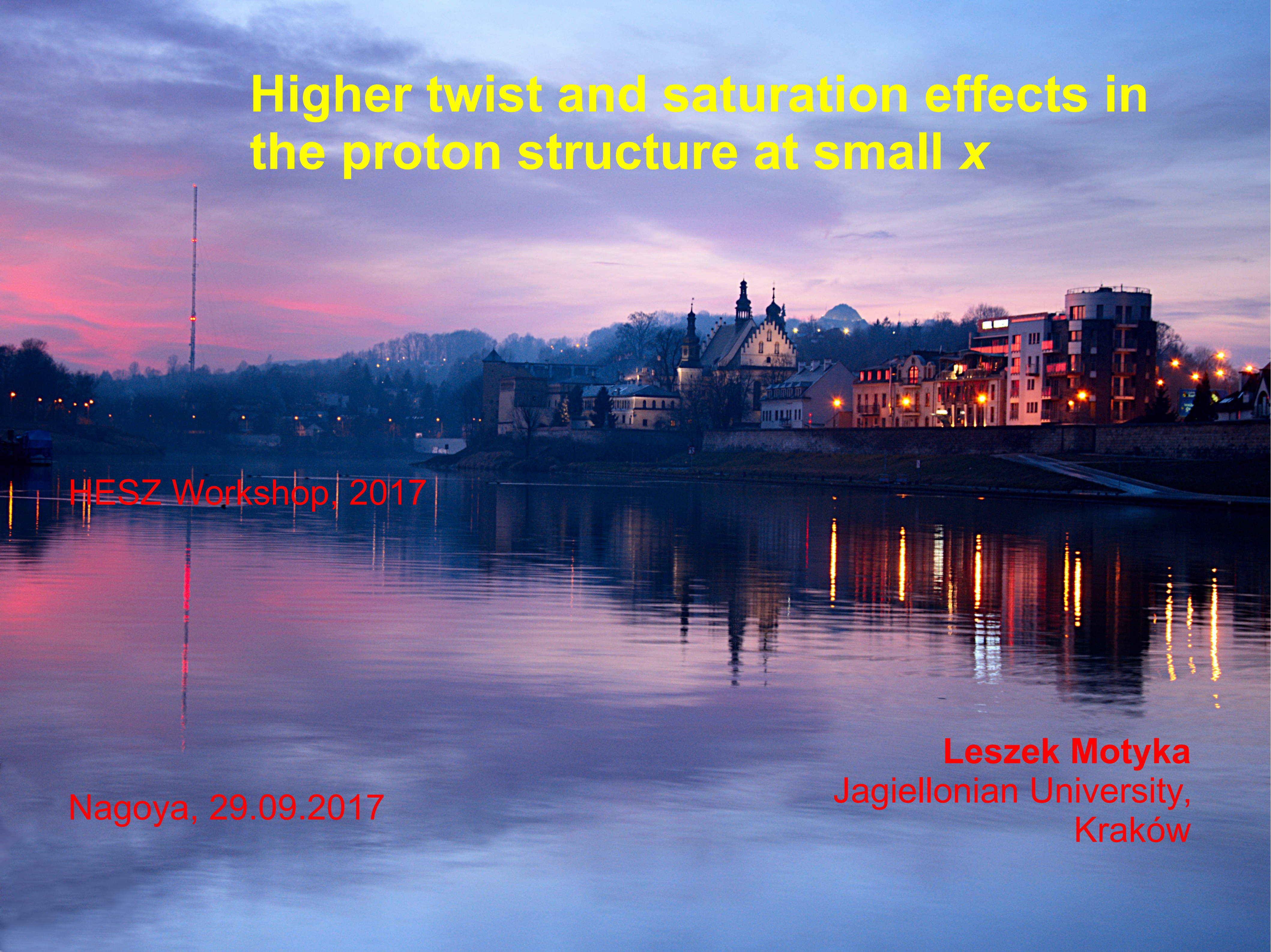

Higher twist and saturation effects in the proton structure at small *x*

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Outline:

- Higher twists in proton structure: motivation
- Higher twists at high energy: QCD picture
- Higher twists corrections to DIS from GBW
- Breakdown of DGLAP in DIS at low Q^2 and interpretation in terms of higher twists
- Combined DGLAP + GBW inspired HT fits to HERA data
- Conclusions

Based on work done by
M. Sadzikowski, W. Słomiński, K. Wichmann and LM,
arXiv:1707.05992

General motivation for higher twist investigation program

- Standard QCD descriptions based on leading-twist DGLAP is very successful and precise
- However, theory of twist-related issue of multiple scattering is not yet satisfactory and higher twist corrections to DGLAP are unknown
- Good understanding of higher twists →
  – broadening of QCD applicability
  – better precision, qualitative determination of DGLAP limitations
  – better determination of parton densities
  – novel observables in proton structure

Deeply Inelastic Scattering: how?

- Unpolarised structure functions
  F_1, F_2 or F_2, F_L

$$\frac{d\sigma}{d\Omega dE'} = \frac{\alpha_{\rm em}}{Q^4} L_{\mu\nu} W^{\mu\nu}(p,q)$$

$$W^{\mu\nu} = -F_1\, g^{\mu\nu} + F_2 \frac{p^\mu p^\nu}{\nu}$$

- OPE: product of local operators in separated points

$$W^{\mu\nu} = \sum_\tau \left(\frac{\Lambda}{Q}\right)^{\tau-2} \sum_i C^{\mu\nu}_{\tau,i} \otimes f_{\tau,i}(Q^2/\Lambda^2)$$

- Twist = dimension – spin: gives the Q dependence
- Leading twist = 2: DGLAP evolution (high precision)

$$\frac{\partial f_i(Q^2)}{\partial \log(Q^2)} = \alpha_s(Q^2) P_{ji} \otimes f_j(Q^2)$$

- \`Easy', efficient but... limited at moderate Q^2

Twists at small x in a nutshell (1)

- Higher twists effects: power suppressed by hard scale:

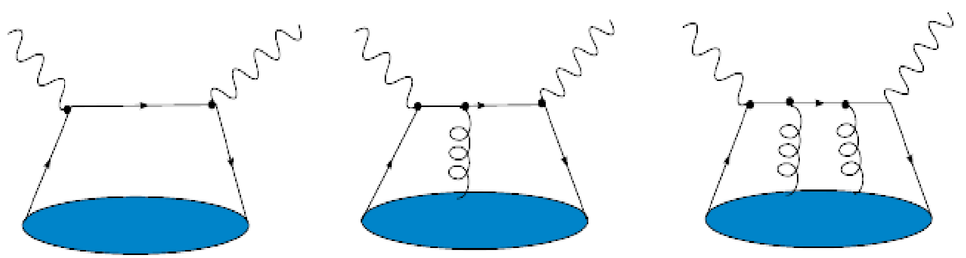
$$W^{\mu\nu} = \sum_\tau \left(\frac{\Lambda}{Q}\right)^{\tau-2} \sum_i C^{\mu\nu}_{\tau,i} \otimes f_{\tau,i}(Q^2/\Lambda^2)$$

- Typical operators:

$$\langle p|\bar{q}\gamma_{\{\mu_1} D_{\mu_2} \ldots D_{\mu_n\}} q|p\rangle = \langle x^n \rangle_q \, p_{\mu_1} \cdots p_{\mu_n}$$

- What is known on higher twists in proton?

Complete twist 4 analysis o $q\bar{q}gg$ evolution [Ellis, Furmanski and Petronzio, 1983]



- Understanding of twist-4 gluonic (gggg) operators – still on the way
- However – dominant contribution should come from **quasipartonic operators**

$$(\partial_. A^\perp_\alpha)^2 (\partial_. A^\perp_\beta)^2,\ \bar\psi\psi\bar\psi\psi$$

(for which: twist = number of free partons in t-channel)

Twists at low x in a nutshell (2)

- DGLAP-like evolution of quasi-partonic operators for twist n: n t-channel partons + pairwise (non-forward) DGLAP interactions [Bokhvostov, Frolov, Lipatov, Kuraev, 1985]

- More rapid QCD evolution of higher twists with x

$$\frac{\text{Twist } 4}{\text{Twist } 2} \sim \frac{1}{Q^2 R^2} \exp\left(\sqrt{b \log(Q^2) \log(1/x)}\right)$$

- Significant corrections to precise parton determination, dependent on x and Q^2

- Quasi-partonic operators: relation of higher twists to multiple scattering, multiple parton densities and parton correlations

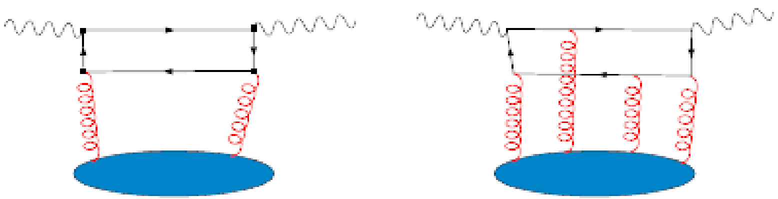
- Higher twists: expected to affect some LHC measurements that reach much lower x than HERA: important to control them

Difficulties in rigorous treatment of higher twists in DIS

- First-principle theory of higher twists: highly involved, few studies done within decades, not complete
- To provide reliable predictions: a lot of input from measurements is necessary – missing so far
- So $\rightarrow$ adopt at first a simplified picture: QCD guided model of rescattering with unitarity constraints
- Most advanced studies within QCD of the rescattering provided so far in the high energy limit, in k_T-factorisation approach and small-x resummations (of logs(1/x))
- Efficient tool to address the problem of multiple scattering: QCD guided saturation model

QCD insight: 4-gluon evolution at twist 4

At small the dominant contribution should come from diagrams of the type:

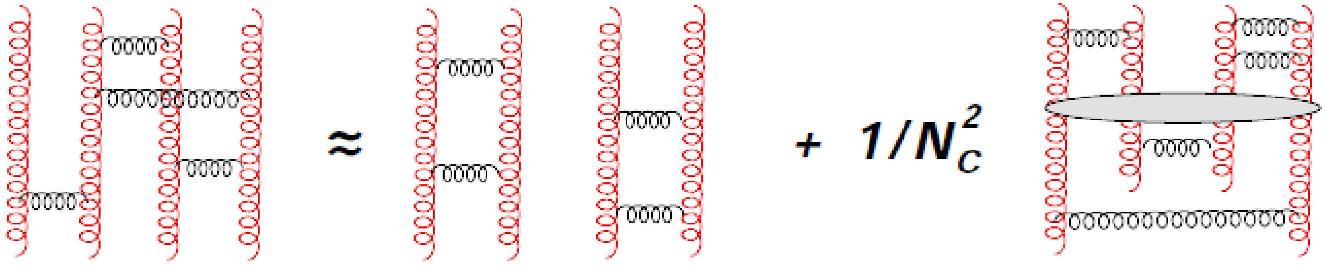


For twist-4, $N_c \to \infty$, in the leading $\alpha_s \log(Q^2) \log(1/x)$ approximation dominant singularity:

$$\gamma = \frac{4N_c\alpha}{\pi}\frac{1}{\omega}$$

coming from two independent DGLAP evolutions

Corrections — color reconnections between ladders supressed by $\sim 1/N_c^2$ [Bartels, Ryskin, 1993]



QCD interpretation of saturation model: eikonal multiple scattering of single gluonic ladder

Taking factorized and symmetric form of unintegrated multi-gluon density

$$G_{2n}^{\{a_i\}}(x,\{k_i^2\}) \sim \sum_{\sigma} \delta^{a_{\sigma(1)}a_{\sigma(2)}} \ldots \delta^{a_{\sigma(2n-1)}a_{\sigma(2n)}} f(x,\boldsymbol{k}_{\sigma(1)},\boldsymbol{k}_{\sigma(2)}) \ldots f(x,\boldsymbol{k}_{\sigma(2n-1)},\boldsymbol{k}_{\sigma(2n)})$$

Invoking AGK rules one obtains the Glauber-Mueller form used by GBW

$$\Delta^{(2n)}\sigma \;\sim\; \frac{(-1)^{n+1}}{n!} R^2 \int d^2r\, dz\, |\Psi(z,\boldsymbol{r})|^2 \prod_{i=1}^{n} \underbrace{\left\{ \int \frac{d^2k_i}{k_i^4}\, \frac{\alpha_s\, f(x,\boldsymbol{k}_i^2)}{R^2} \left[2 - e^{i\boldsymbol{k}_i\boldsymbol{r}} - e^{-i\boldsymbol{k}_i\boldsymbol{r}}\right] \right\}}_{\text{single dipole scattering xs: } \sigma_1(x,r^2)/R^2}$$

In collinear limit ($k^2 \ll C/r^2$) dipole cross section coincides with DGLAP improved saturation model [Bartels, Golec-Biernat, Kowalski]

$$\sigma_1(x,r^2) \simeq \alpha_s(C/r^2) \int^{C/r^2} \frac{dk^2}{k^4}\, f(x,k^2)\,(k^2r^2) \;\simeq\; r^2\,\alpha_s(C/r^2)\, xg(x,C/r^2)$$

Resummed cross section:

$$\sigma_d(x,r^2) \;\simeq\; R^2\,[1-\exp(-\sigma_1(x,r^2)/R^2)]$$

Higher twist extraction from original GBW

Simple Q^2-Mellin structure of the GBW model – simple poles of γ^*-impact factor * simple poles of the dipole cross-section – analytic twist decomposition of saturation [Bartels, Golec-Biernat, Peters, 2000]:

- Twist 2

$$\sigma_T^{(\tau=2)} = \frac{\alpha_{em}\sigma_0}{\pi} \langle e^2 \rangle \frac{Q^2_{\rm sat}}{Q^2} \{\log(Q^2/Q^2_{\rm sat}) + \gamma_E + 1/6\}$$

$$\sigma_L^{(\tau=2)} = \frac{\alpha_{em}\sigma_0}{\pi} \langle e^2 \rangle \frac{Q^2_{\rm sat}}{Q^2}$$

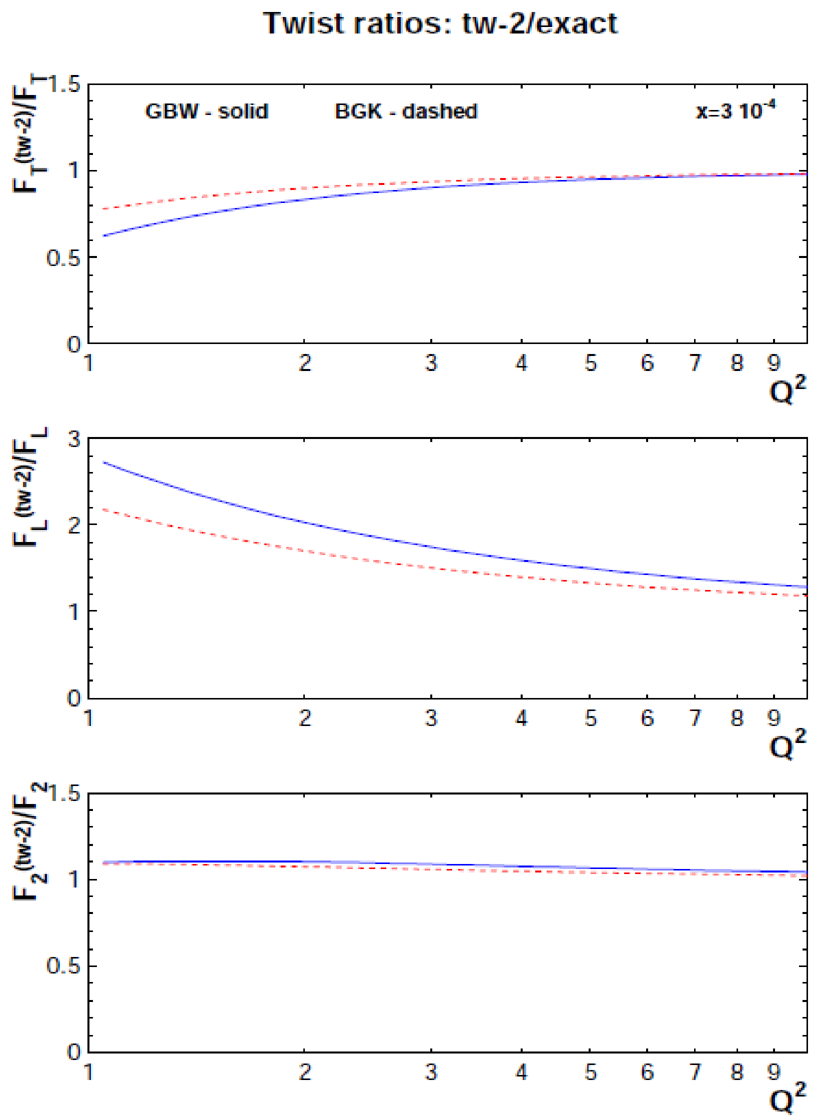
- Twist 4

$$\sigma_T^{(\tau=4)} = \frac{3}{5}\frac{\alpha_{em}\sigma_0}{\pi} \langle e^2 \rangle \frac{Q^4_{\rm sat}}{Q^4}$$

$$\sigma_L^{(\tau=4)} = -\frac{4}{5}\frac{\alpha_{em}\sigma_0}{\pi} \langle e^2 \rangle \frac{Q^4_{\rm sat}}{Q^4} \{\log(Q^2/Q^2_{\rm sat}) + \gamma_E + 1/15\}$$

Pattern of HT corrections from GBW

[Bartels, Golec-Biernat, LM 2009]



Higher twist contribution at $x = 3 \cdot 10^{-4}$ and $Q^2 = 10\ \mathrm{GeV}^2$:

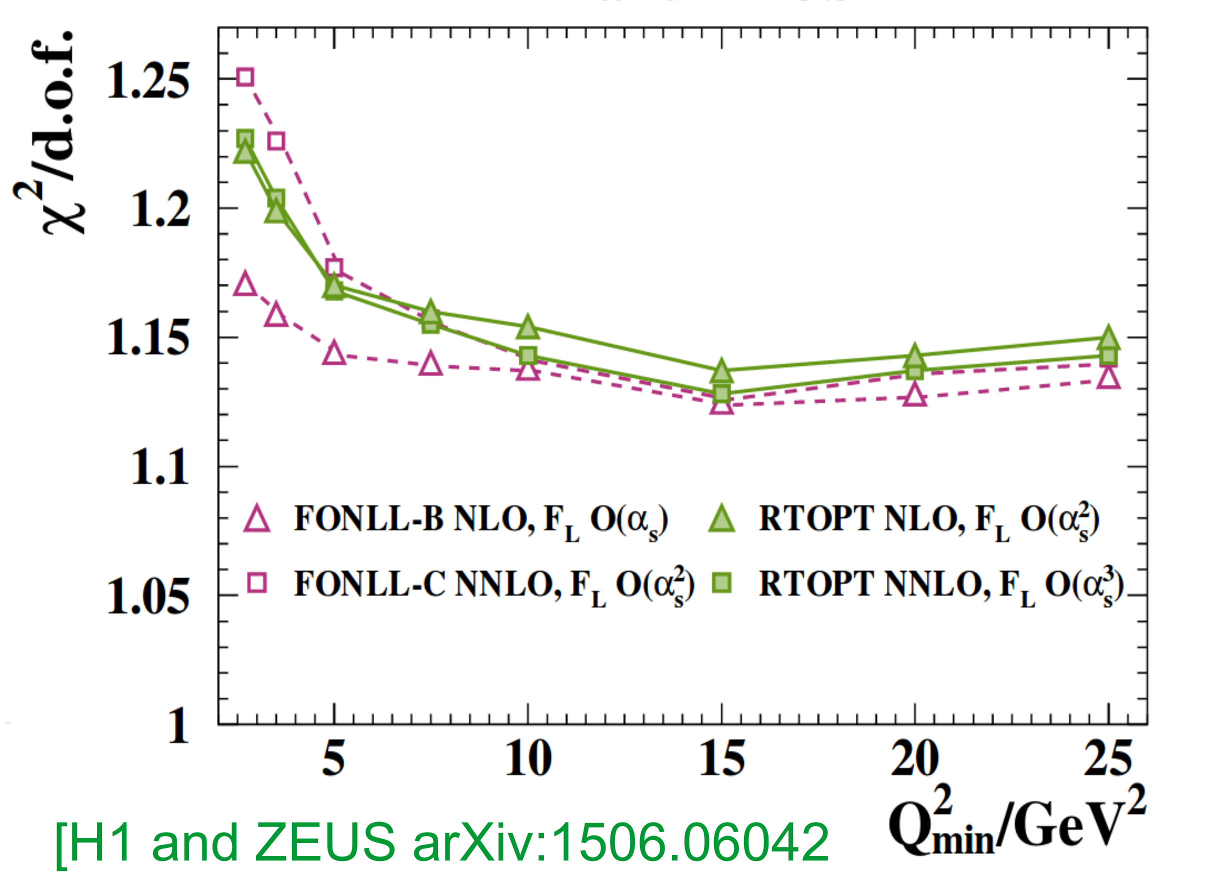
F_T: $\sim 1\%$

F_L: $\sim 20\%$

F_2: $\sim 1\%$

Fresh new twist in higher twists at small x – global fit of combined HERA DIS data (2015)

- Key source of progress: combination of all HERA DIS data



[H1 and ZEUS arXiv:1506.06042 [hep-ex]]

Test of DGLAP description quality:

→ start evolution from Q_0 ($Q_0^2 = 1.9\ GeV^2$)

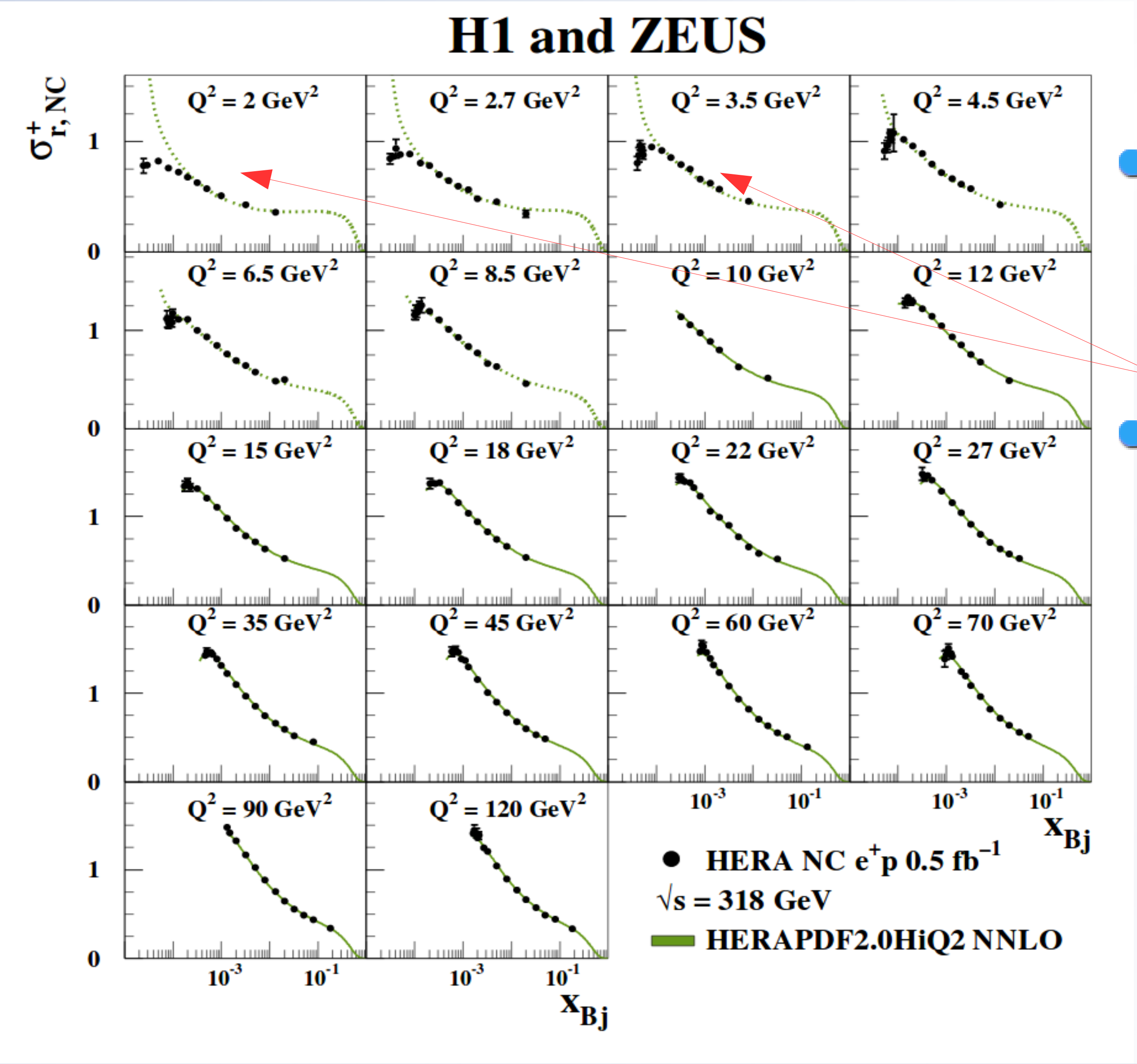
→ cut the data $Q^2 > Q_{min}^2$, fit and compute χ^2

→ check variation of χ^2/d.o.f. as function of Q_{min}^2

→ **found: deterioration of DGLAP fits quality below ~ 5 GeV^2**

DGLAP fit problems at low x and low Q^2

[H1 and ZEUS arXiv:1506.06042 [hep-ex]]



- The source of DGLAP problems: region of low Q^2 and low x
- Exactly where higher twist corrections become important because of their very steep rise with decreasing x

Strategy for higher twist analysis in low *x* HERA data

- Use full range of precision data: dominance of the DGLAP regime
  → **need to maintain the highest quality of twist-2 DGLAP analysis**
- **Provide QCD inspired model of twist-4 correction**

- **Perform combined fit of DGLAP (input parameters) and the model of higher twists** and analyse results in terms of χ^2 / d.o.f.
  for data set with $Q^2 > Q^2_{\mathrm{min}}$

- Earlier successful application to higher-twist analysis in Diffractive DIS
  [M. Sadzikowski, W.Słomiński, LM, arXiv:1203.5461]

- Applied to HT analysis of combined HERA data with a simple model of higher twists
  [L. A. Harland-Lang, A. D. Martin, P. Motylinski, R. S. Thorne, arXiv:1601.03413]
  [I. Abt, A.M. Cooper-Sarkar, B. Foster, V. Myronenko, K. Wichmann and M. Wing, arXiv:1604.02299]

Higher twist model inspired by GBW (new analysis)

- Twist 4 from GBW: [Bartels, Golec-Biernat, Peters, 2000], note the geometric scaling property

$$\sigma_T^{(\tau=4)} = \frac{3}{5}\frac{\alpha_{em}\sigma_0}{\pi}\left\langle e^2\right\rangle \frac{Q_{\rm sat}^4}{Q^4}$$

$$\sigma_L^{(\tau=4)} = -\frac{4}{5}\frac{\alpha_{em}\sigma_0}{\pi}\left\langle e^2\right\rangle \frac{Q_{\rm sat}^4}{Q^4}\left\{\log(Q^2/Q_{\rm sat}^2) + \gamma_E + 1/15\right\}$$

- More flexible parameterisation of the twist-4 effects with geometric scaling:

$$F_{T/L}^{(\tau=4)} = \frac{Q_0^2(x)}{Q^2}\, x^{-2\lambda}\left[c_{T/L}^{(0)} + c_{T/L}^{(log)}\left(\log\frac{Q_0^2}{Q^2} + \lambda \log\frac{1}{x}\right)\right]$$

- Note steep x-dependence of twist-4 corrections
- Constraint from photon impact factor: $c_T^{(\log)} = 0$

Parton saturation and DGLAP input

- In collinear QCD saturation effects manifest themselves in two ways:
  – by higher twist corrections (rescattering above factorisation scale) &
  – by modification of inputs at all twists (rescattering below factoris-n scale)

Twist-2: properties of solutions of Balitsky-Kovchegov equation below Q_{sat} : (the same behavior in saturation model)

$$f_g(x,k_T^2) = \frac{3\sigma_0}{4\pi^2}\frac{k_T^4}{Q_s^2(x)}\exp(-Q^2/Q_s^2(x))$$

$$xg(x,Q^2) = \int^{Q^2}\frac{dk^2}{k^2}f_g(x,k^2) \sim \sigma_0 Q^4/Q_s^2(x) \sim x^{\lambda} \ \text{ for } Q \ll Q_s(x)$$

$$xg(x,Q^2) \sim \sigma_0 Q_s^2(x) \sim x^{-\lambda} \ \text{ for } Q \gg Q_s(x)$$

For Q below $Q_s(x)$: **$xg(x,Q^2) \sim x^{\lambda}$**

Gluon input saturation `damping' at very small x

Parton saturation input damping

- Standard DGLAP input without saturation effects:
  – gluon small-x rise tamed by a negative additive correction
  – sea quark small-x rise not tamed

- We assume that sea quarks are generated from gluons already at input scale :

$$q_{sea}(x, Q_0^2) \simeq \int^{Q_0^2} \frac{d\mu^2}{\mu^2} \frac{\alpha_s(\mu^2)}{\pi} \int_x^1 \frac{dz}{z} P_{qg}(z) g(x/z, \mu^2)$$

- Our approach with saturation damping of PDF inputs:

$$xq_{sea}(x, Q_0^2) \sim xg(x, Q_0^2) \sim x^\lambda$$

Data fitting scheme

- Take all combined HERA data with Q > 1 GeV (n>1200 points)

$$\sigma_{\rm red} = F_{\rm T} + \frac{2(1-y)}{1+(1-y)^2} F_{\rm L} \equiv F_2 - \frac{y^2}{1+(1-y)^2} F_{\rm L}$$

- Fit with standard DGLAP: NLO and NNLO and with
  DGLAP NLO and NNLO with twist-4 corrections and parton freezing

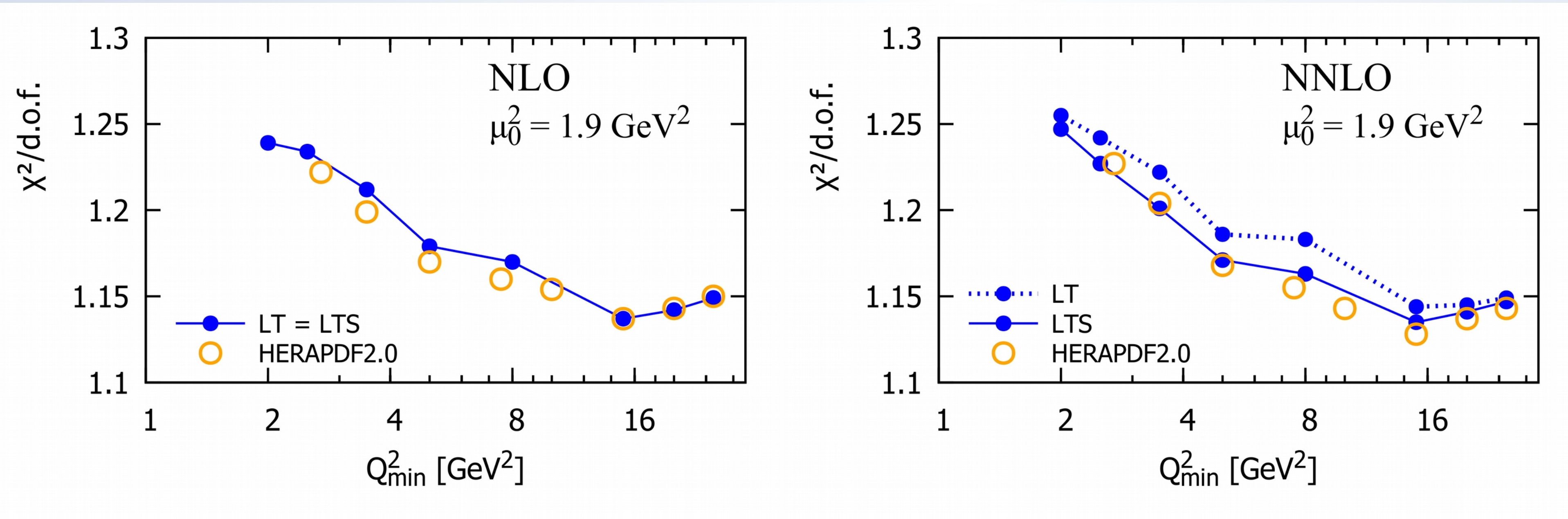
- RTOPT heavy flavour treatment

- Systematic,statistical, correlated and uncorrelated data uncertainties treated with HERA-FITTER

- Vary lower cut-off Q_{min} on Q values and check dependence of χ^2 and fit parameters

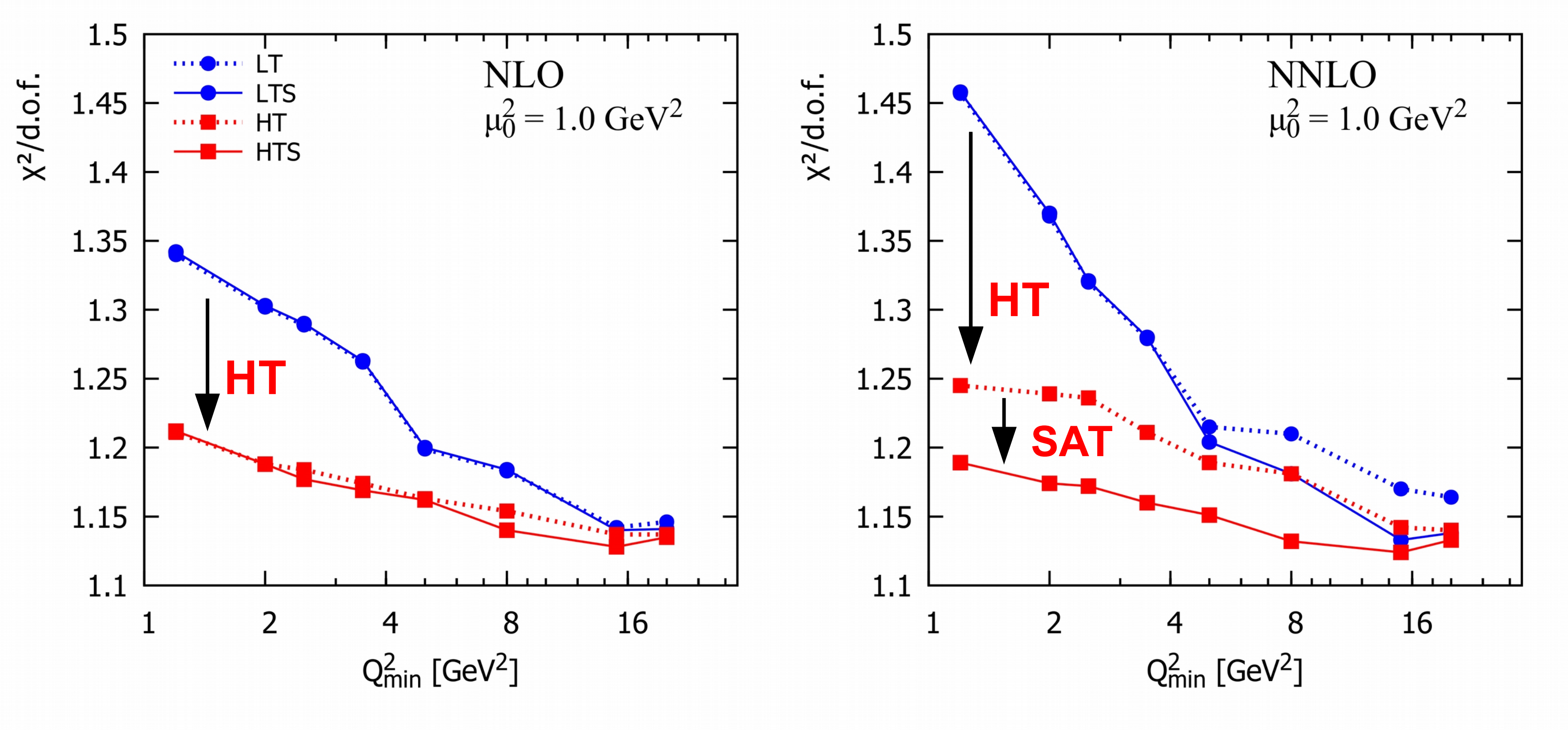
Checking quality of our DGLAP input:

- Our leading-twist DGLAP fit (with 11 parameters) fit results compared to HERAPDF2.0 fit (with 14 parameters)



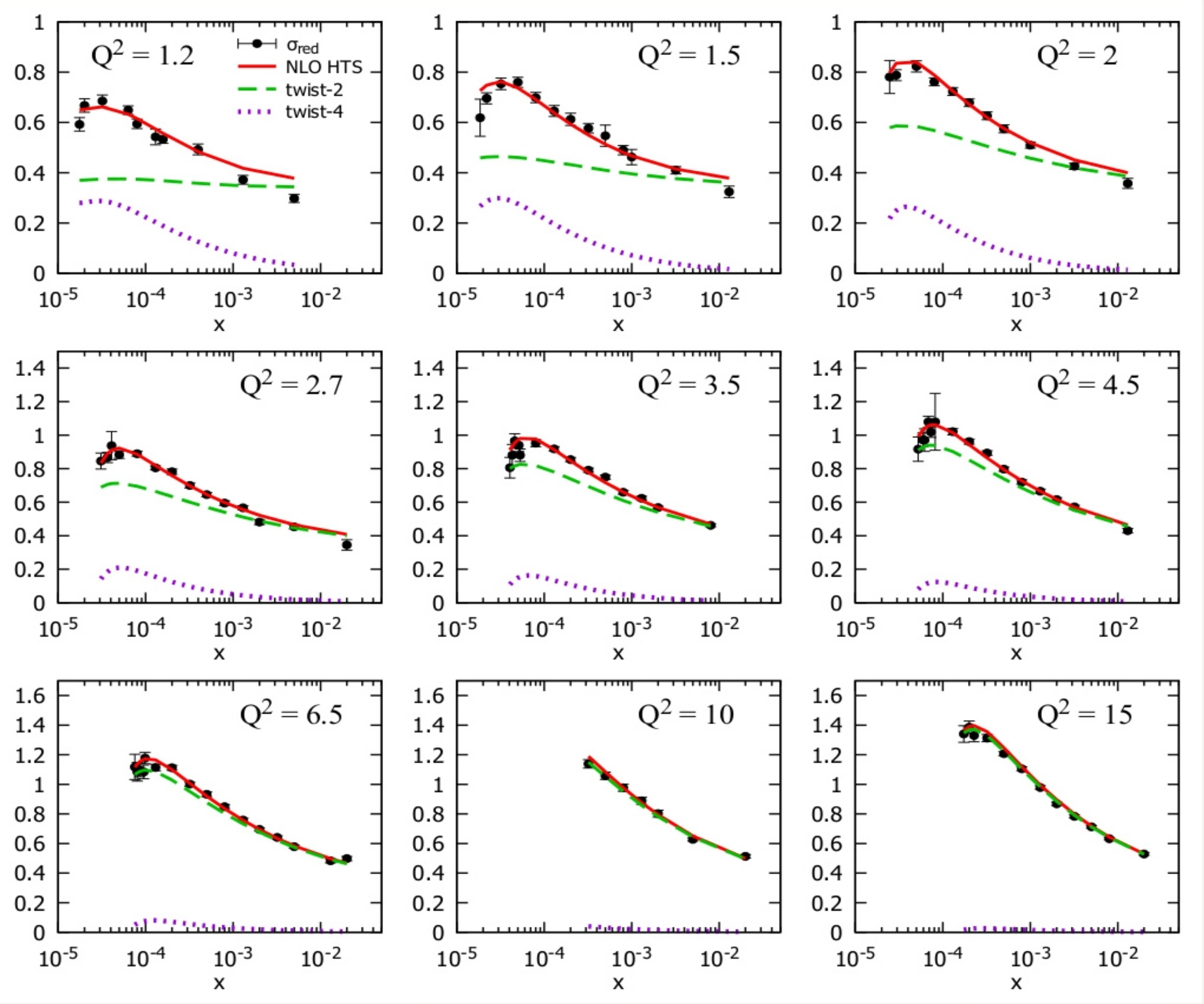
- The two different fits have equivalent quality → DGLAP part well described

Fit results with and without higher twist corrections:



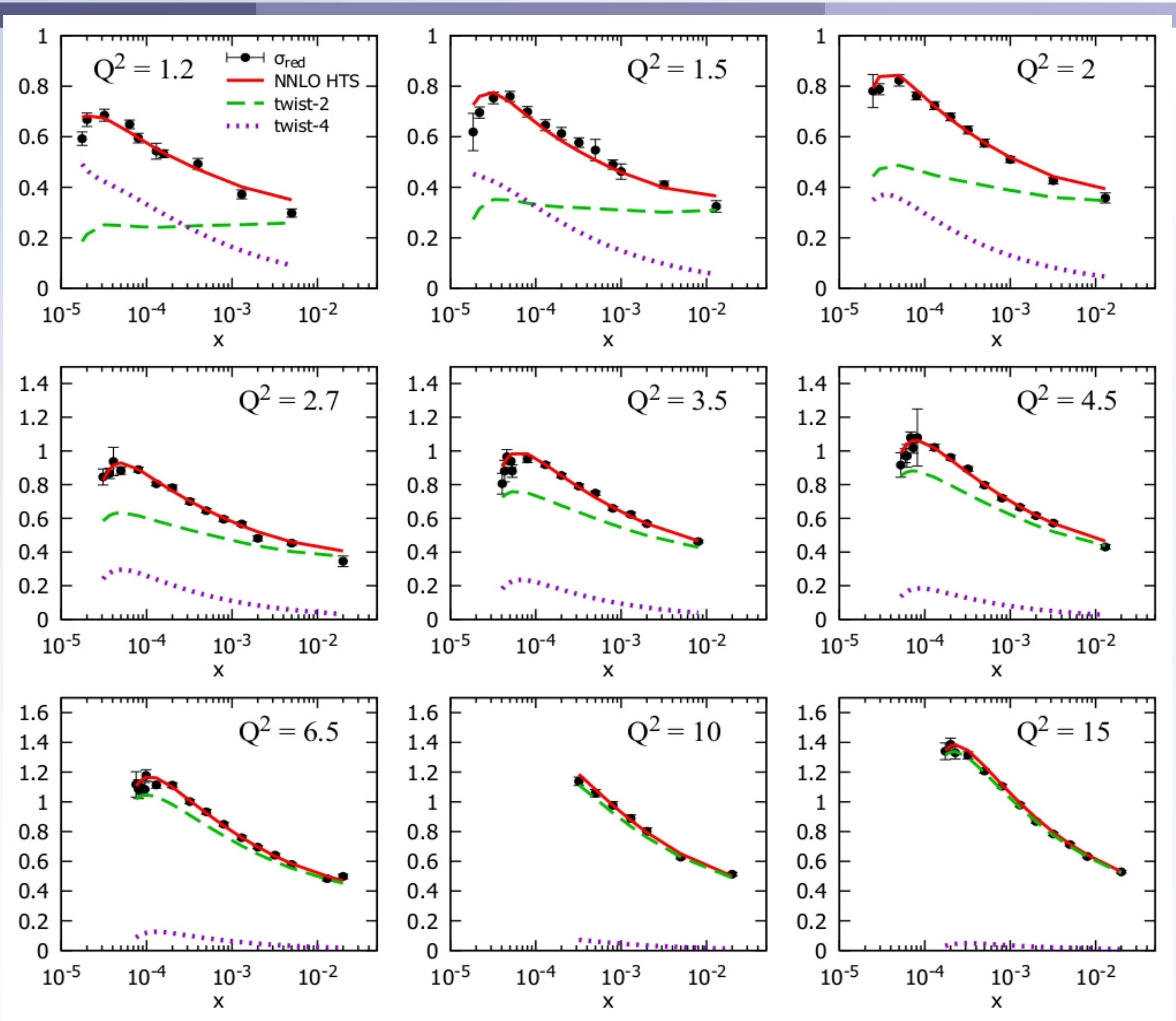
- Clear improvement of fit quality by inclusion higher twist corrections down to $Q^2 = 1$ GeV^2
- Effect of improvement much stronger for NNLO DGLAP fits
- Saturation input damping of the quark sea improves fits significantly for NNLO fit with higher twists, but has little effect in NLO HT fit

Higher twists in sigma reduced at NLO



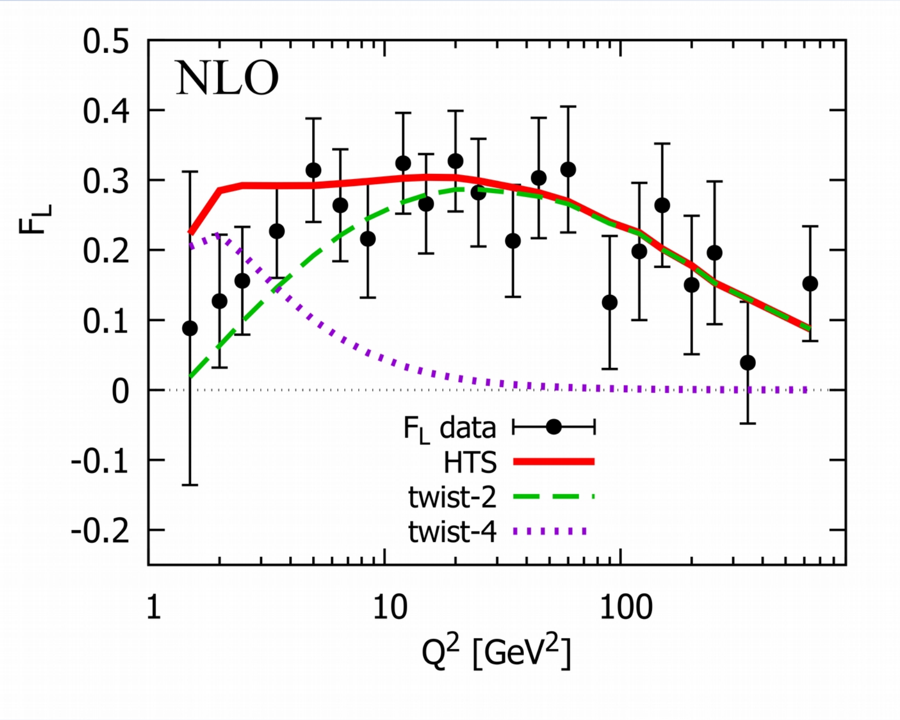
- Effects of higher twists at NLO may reach ~40% of the total reduced cross section at small x and moderate Q^2
- 10% HT effects around $Q^2 = 5\ GeV^2$

Higher twists in sigma reduced at NNLO



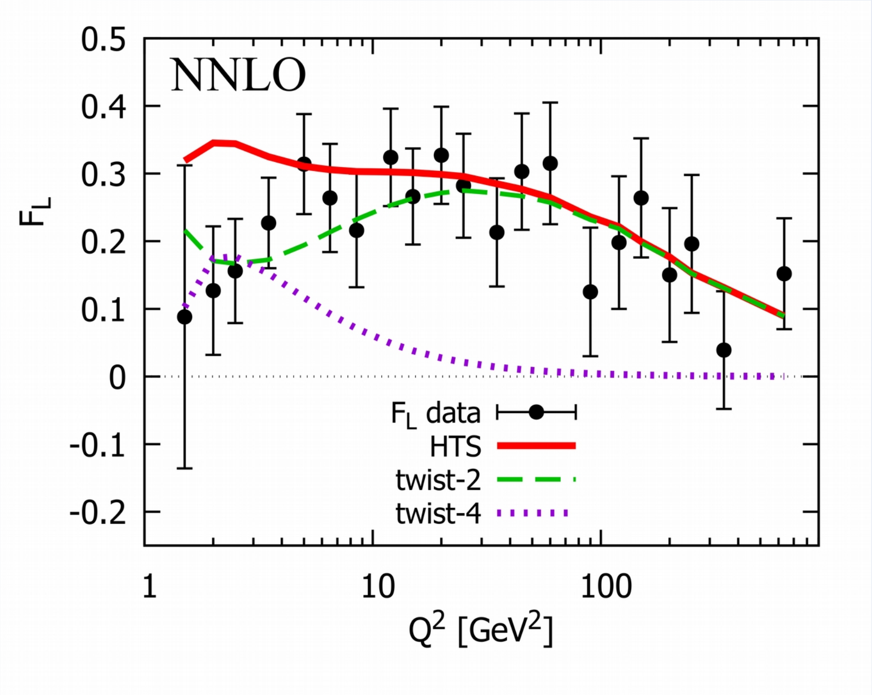
- Effects of higher twists at NLO may reach ~70% of the total reduced cross section at small x and moderate Q^2
- 10% HT effects around Q^2 = 10 GeV^2

Longitudinal structure function F_L



- Reasonably good description of F_L obtained down to lowest Q^2
- NLO DGLAP + HT fit: big contribution of HT below 5 GeV^2
- 10% HT contribution at ~10 GeV^2

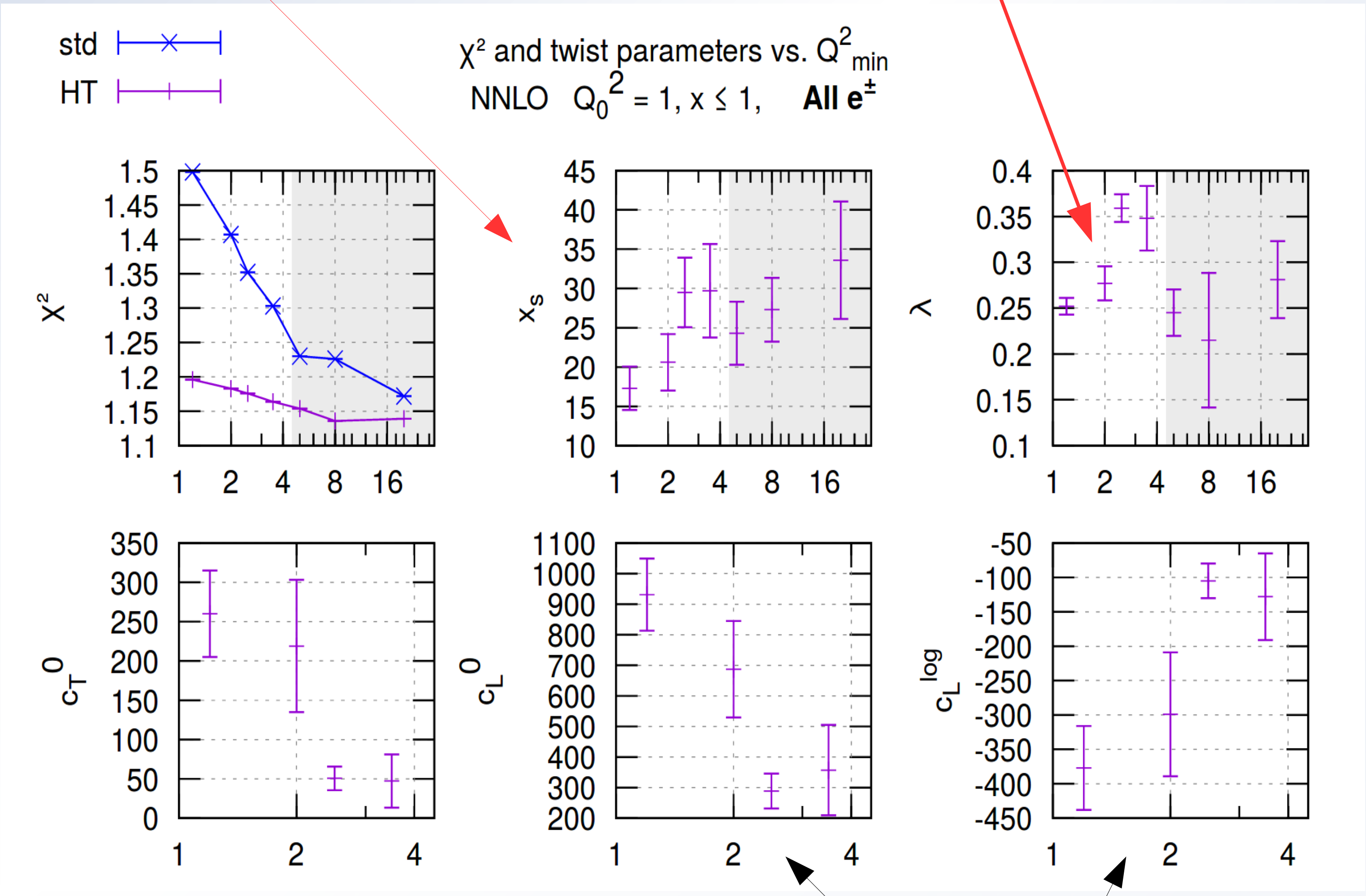
- At NNLO a similar pattern found



Resulting higher twist parameters

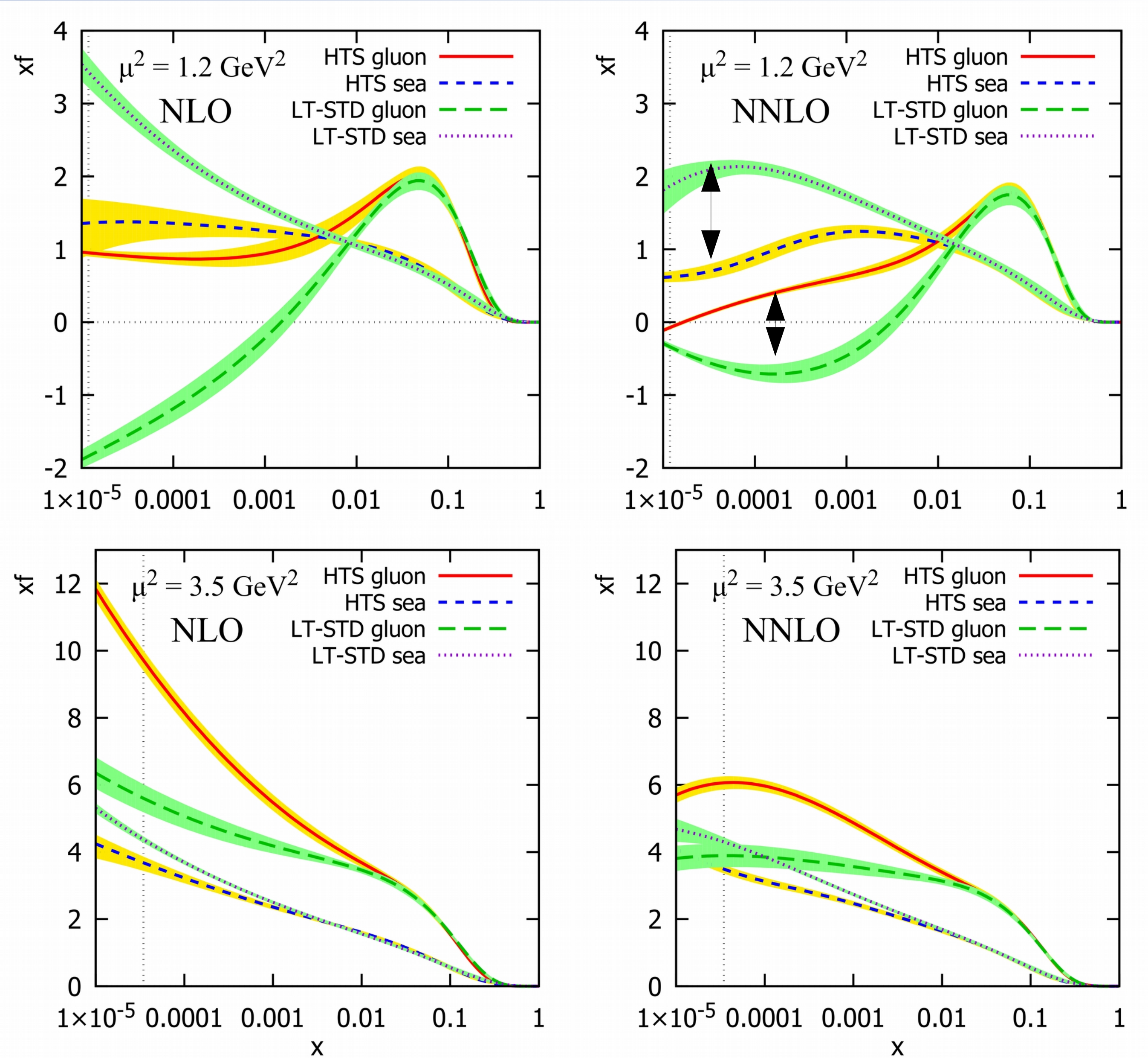
$X_s \sim 1.5 * 10^{-4}$

Correct exponent!



`Wrong' Sign!

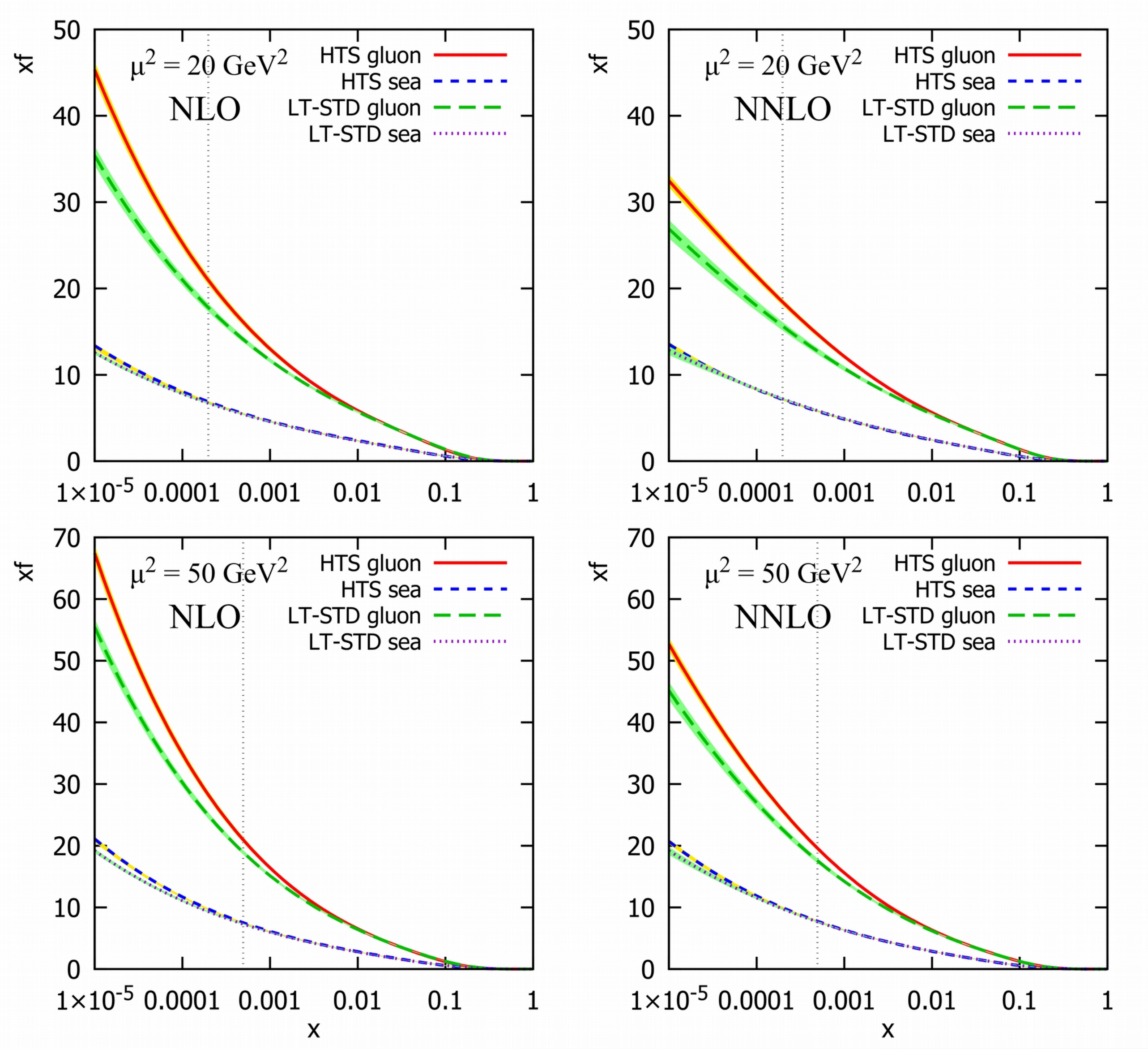
Impact of higher twist inclusion on PDFs – low Q^2



Note that standard PDFs for $Q^2 = 1.2$ GeV^2 are extrapolated

Sizable differences in PDFs found at low Q^2 due to inclusion of higher twist corrections

Impact of HT inclusion on PDFs – intermediate Q^2



Significant differences in PDFs due to inclusion of higher twist corrections still present at higher Q^2

Interpretation and conclusions:

- DGLAP + GBW inspired model of twist-4 works very well for the combined HERA data
- A good description of precision DIS data down to Q=1GeV
- The "saturation parameters": x_0 and λ are consistent with expectations, "double ladder" x-dependence found of HT
- Theoretical puzzle twist 4 correction to F_L found with the opposite sign! → go back to more sophisticated QCD HT analysis by Bartels and Bontus?
- Possible need for saturation effects in the DGLAP input