The $a_1(1260)$ lineshape measured in $D^0 \to \pi^+\pi^-\pi^+\pi^-$ decays

PWA9/ATHOS4, Bad Honnef

P. d'Argent[1], J. Benton[2], J. Dalseno[2], E. Gersabeck[1], S.T. Harnew[2], P. Naik[2], C. Prouve[2], J. Rademacker[2], N. Skidmore[2]

[1]Physikalisches Institut Heidelberg

[2]University of Bristol

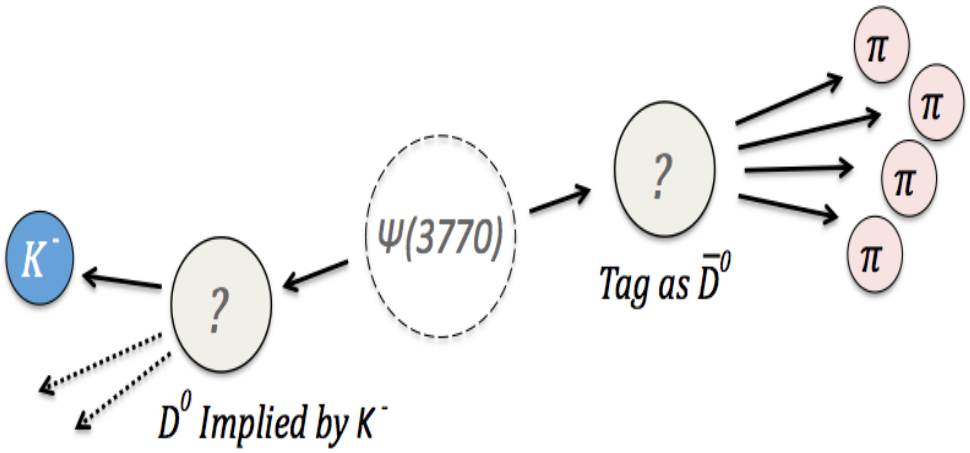
14.03.2017

Amplitude analysis of $D \to \pi^+\pi^-\pi^+\pi^-$ [arXiv:1611.09253]

- Provides insights into strong phase difference between D and $\overline{D}$ decays
  $\to$ Input for measurement of CKM angle γ in $B^- \to D\,K^-$
- Understanding of hadron dynamics
- Excellent environment to study properties of the $a_1(1260)$ meson

PDG	$m(\,\mathrm{MeV})$	$\Gamma(\,\mathrm{MeV})$	$BF(a_1 \to \rho\pi)$
1978	≈ 1100	≈ 300	$\approx 100\%$
1980	1100 - 1300	≈ 300	dominant
1982	1275 ± 30	315 ± 45	dominant
1988	1260 ± 30	300 - 600	dominant
1992	1260 ± 30	≈ 400	dominant
2000	1230 ± 40	250 - 600	seen

Data Set

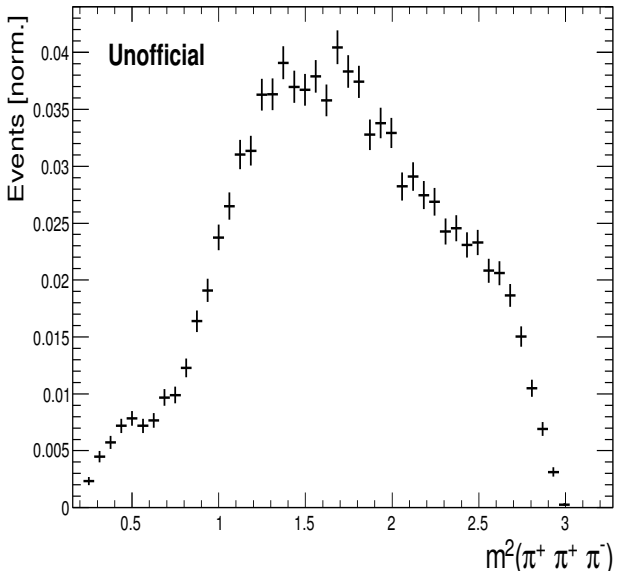


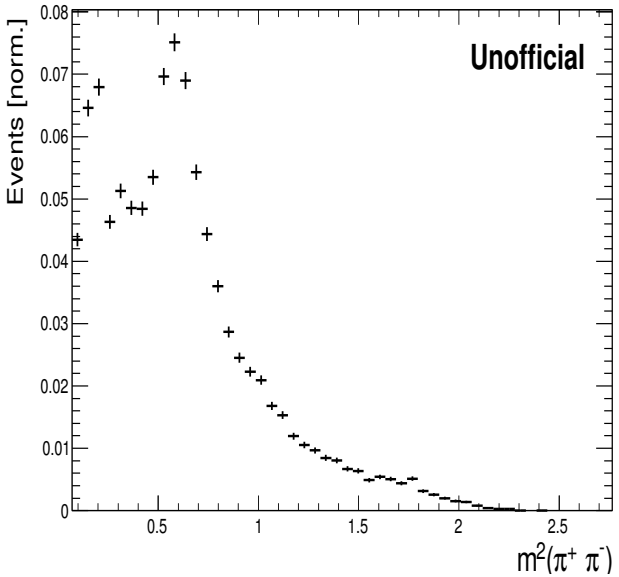
CLEO-c

- Detector at the Cornell Electron Storage Ring (CESR)
- $e^+ e^- \rightarrow \psi(3770) \rightarrow D_a D_b$
- **Flavour tag:**
  $D_b \rightarrow K^- \pi^+ \Rightarrow D_a = \overline{D^0}$

Look at CLEO-c data

$\approx 7k$ $D \to 4\pi$ flavor-tagged signal candidates





Resonances ?

- $a_1(1260) \to \pi\,\pi\,\pi$
- $\rho(770) \to \pi\,\pi$

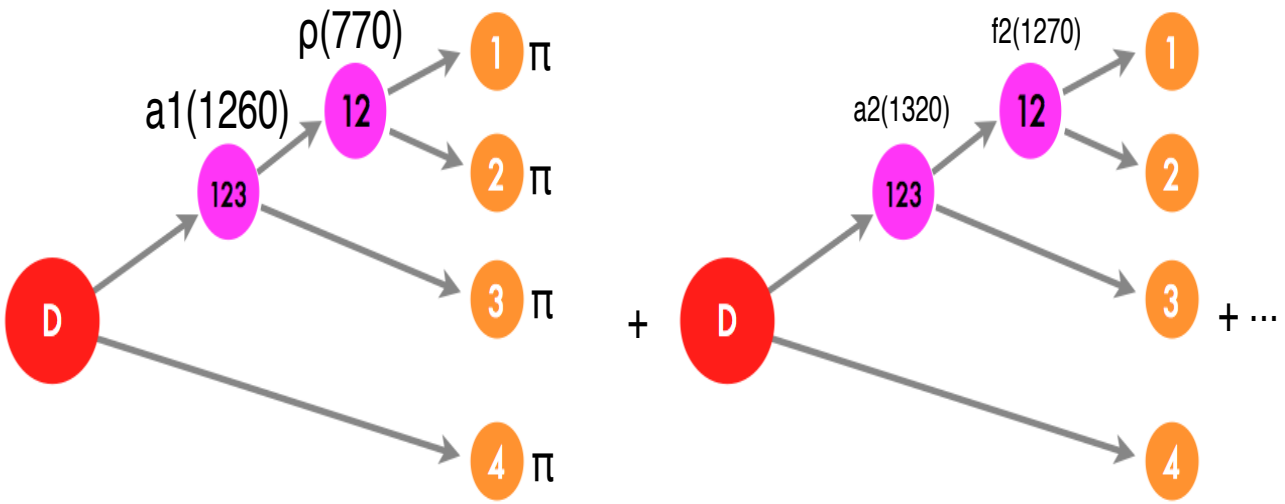
Decay channels

Plenty of possible decay channels

- cascade decays
  $D \to \pi^- \left[a_1(1270)^+[S,D] \to \rho(770)\,\pi^+\right]$
  $D \to \pi^- \left[a_1(1270)^+ \to \sigma\,\pi^+\right]$
- quasi-two-body
  $D \to \sigma\,\sigma$
  $D[S,P,D] \to \rho(770)\,\rho(770)$
- single resonance
  $D \to \sigma\,(\pi\pi)_S$
  $D \to \rho(770)\,(\pi\pi)_S$
- non-resonant
  $D \to (\pi\pi)_S\,(\pi\pi)_S$
  $D[P] \to (\pi\pi)_V\,(\pi\pi)_S$

$\Rightarrow$**Need amplitude analysis !**

Amplitude analysis



Isobar formalism

- Amplitude $\mathcal{A}_i \approx \mathcal{B}W_{a1}(m^2_{123}) \cdot \mathcal{B}W_{\rho}(m^2_{12}) \cdot S_f$
- $PDF \approx |\sum_i a_i \mathcal{A}_i|^2$
- Complex coefficients a_i

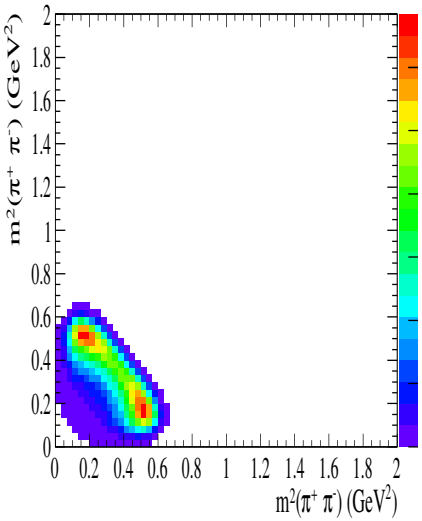
Lineshapes

BW Resonance

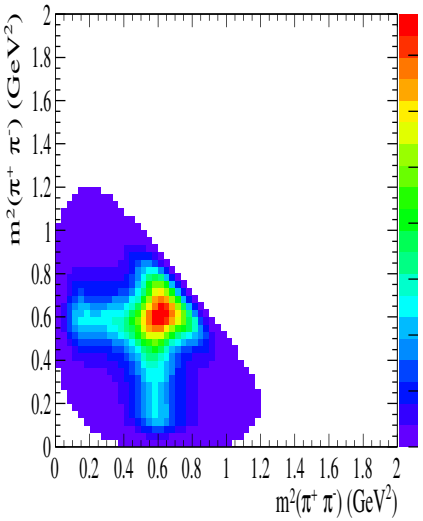
- Most general form:
  $BW(s) = 1/(s - m^2(s) - i\, m_0\, \Gamma(s))$
- Simple BW:
  $m^2(s) = m_0^2,\ \Gamma(s) = \Gamma_0$
- Relativistic BW:
  $m^2(s) = m_0^2,\ \Gamma(s) = \Gamma_0 \frac{m_0\, q}{\sqrt{s}\, q_0} \frac{B_L(q)^2}{B_L(q_0)^2}$
  Only valid for decay into two **stable** particles

Lineshapes

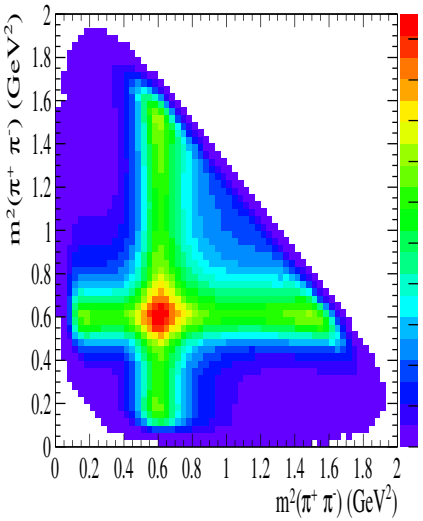
- What about cascade decays ?
  $D^0 \to \pi^-[a_1(1270)^+ \to \pi^+(\rho(770) \to \pi^+\pi^-)]$
- Need to integrate over a_1 Dalitz plot to obtain running width:
  $\Gamma_{a_1}(s) = \frac{1}{s}\int |A_{a_1}(m_{12}^2, m_{23}^2)|^2 dm_{12}^2\, dm_{23}^2$
- $A_{a_1}(m_{12}^2, m_{23}^2) = BW_\rho(m_{12}^2) \cdot S_f$



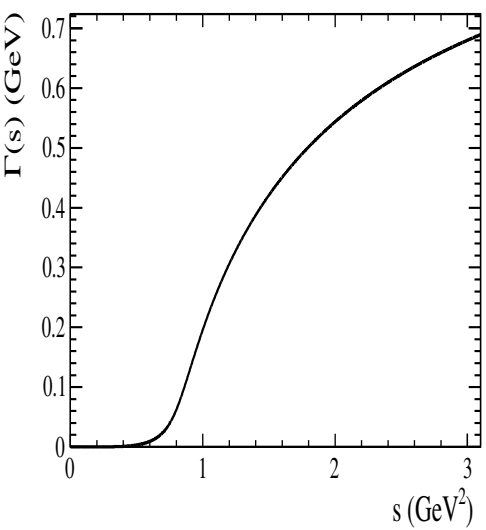
$s = 0.8\,\mathrm{GeV}^2$



$s = 1.4\,\mathrm{GeV}^2$

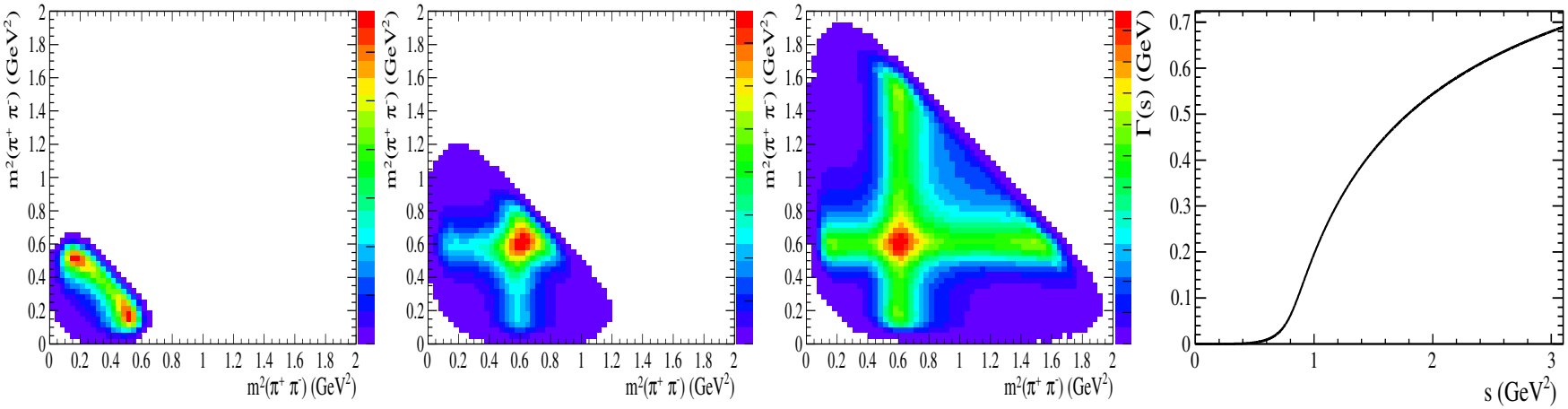


$s = 2.3\,\mathrm{GeV}^2$

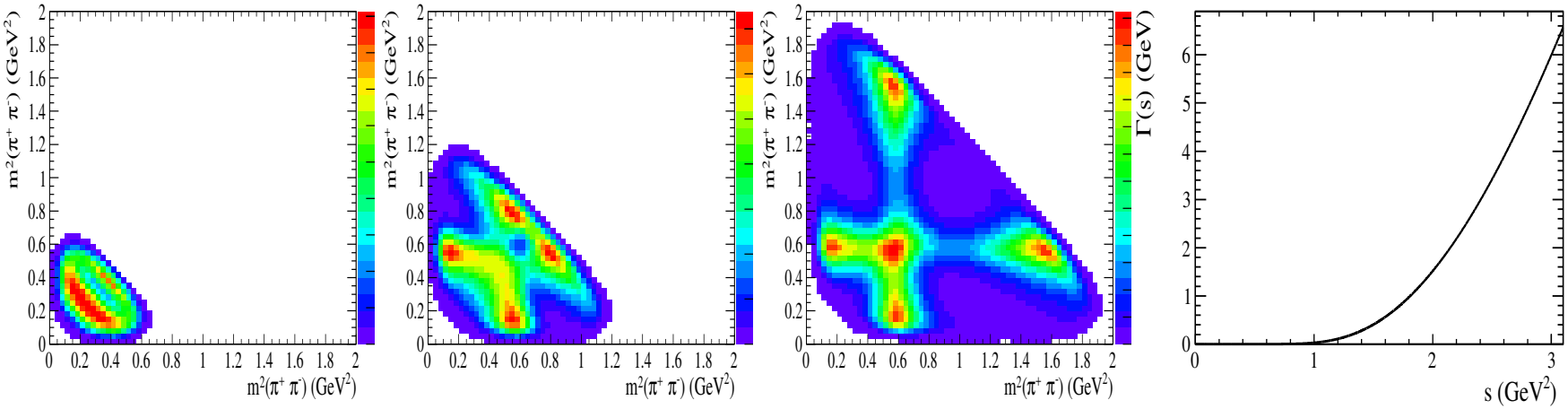


Lineshapes

$a_1(1270)^+[S] \to \pi^+(\rho(770) \to \pi^+\pi^-)$



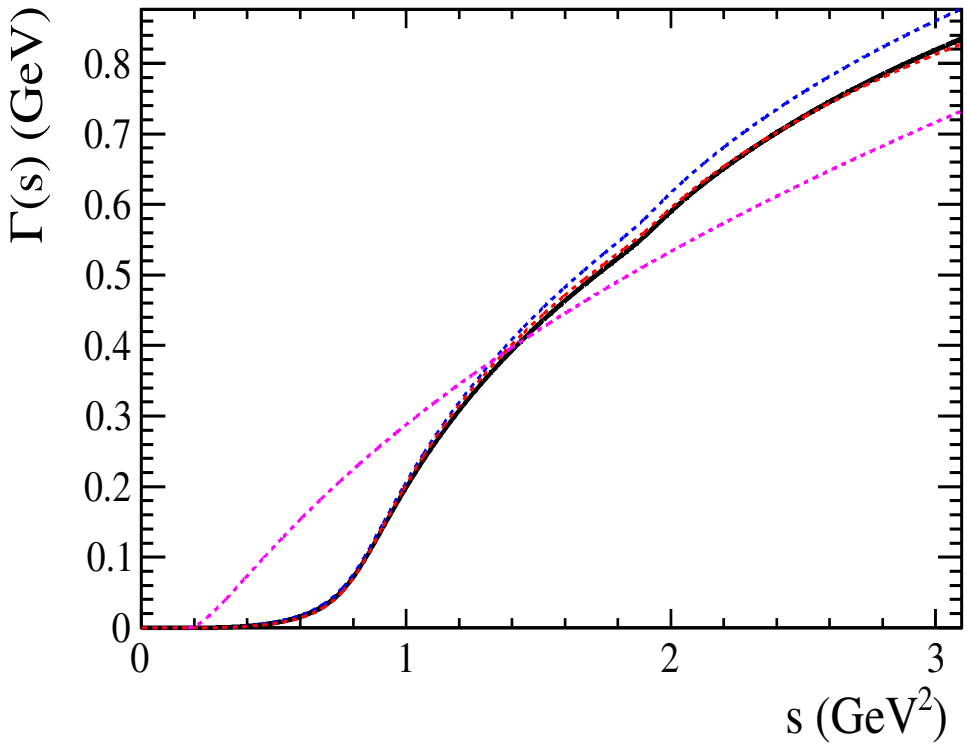
$a_1(1270)^+[D] \to \pi^+(\rho(770) \to \pi^+\pi^-)$



Lineshape Iterations

- Lineshape requires knowledge of resonant substructure
- Amplitude fit requires lineshape
- Normalized to $\Gamma_{a_1}(s = m_0^2) = \Gamma_0$

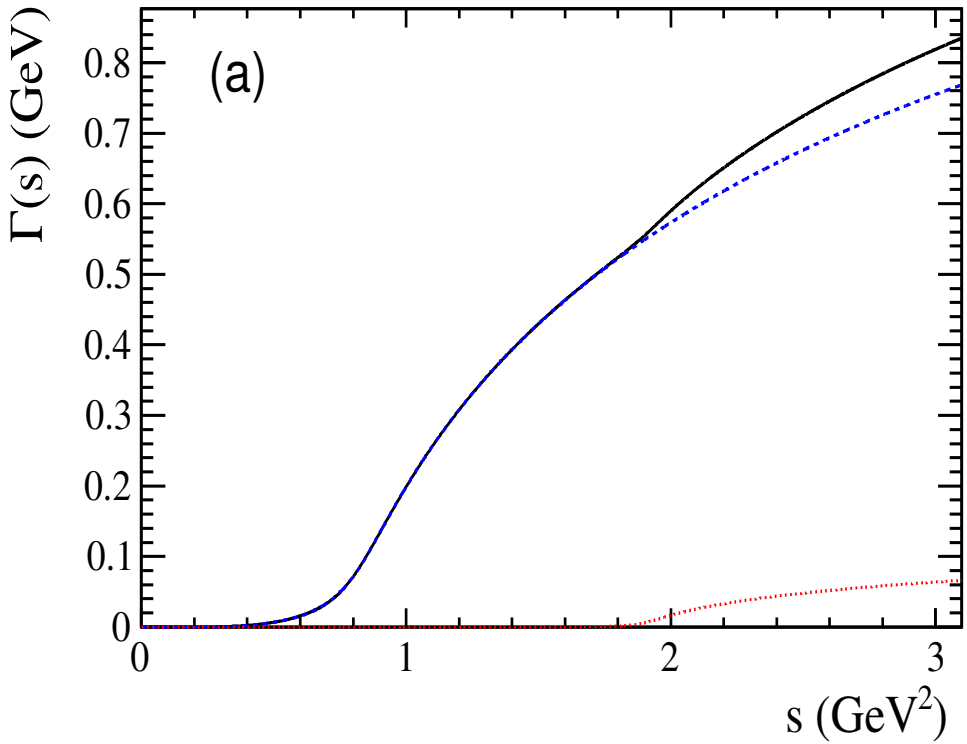
Phasespace
1. iteration
2. iteration
Final iteration



Running width

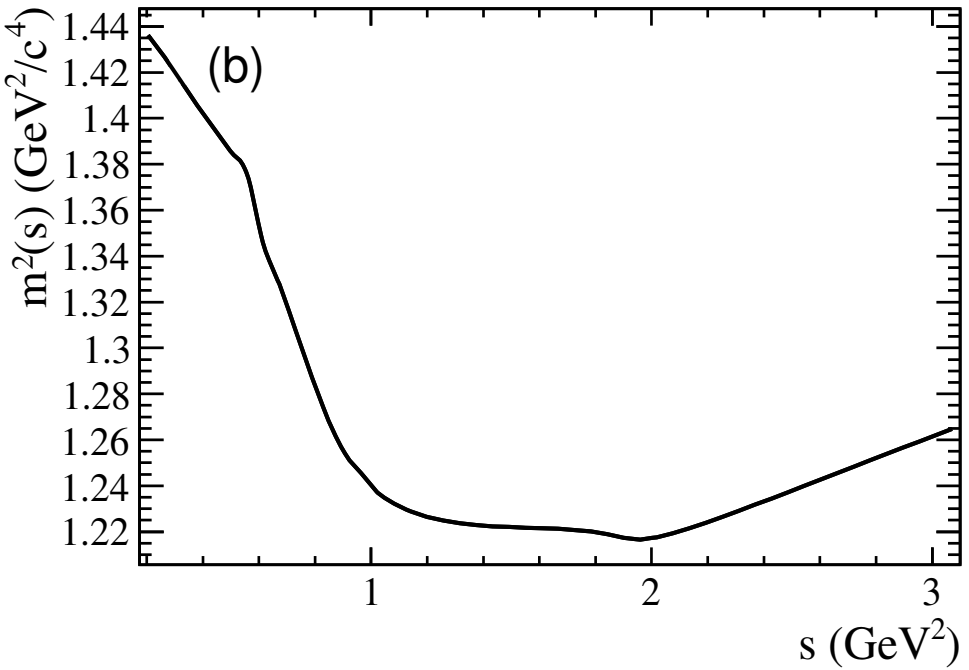
- $\Gamma_{a_1}(s) = \Gamma_{\pi^+\pi^+\pi^-}(s) + \Gamma_{\pi^+\pi^0\pi^0}(s) + \Gamma_{KK^*}(s)$
- $\Gamma_{\pi^+\pi^0\pi^0}(s) \approx \Gamma_{\pi^+\pi^+\pi^-}(s)$

97 % : $a_1(1260) \to \pi\pi\pi$
3 % : $a_1(1260) \to K^*(892)K$



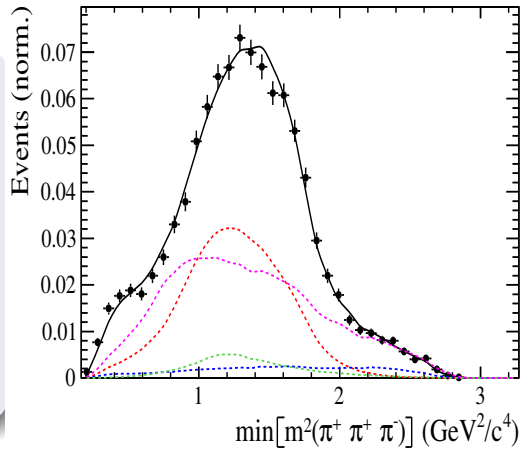
Running mass

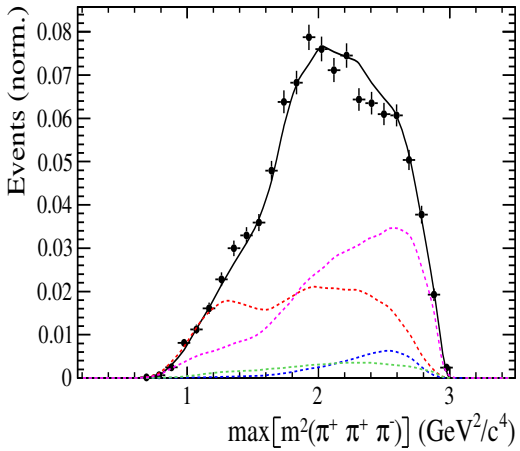
- $m^2(s) = m_0^2 + \frac{1}{\pi} \int_{s_{min}}^{\infty} \frac{m_0 \Gamma(s)}{s - s'} ds'$
- Integrand contains singularity
- Integral is logarithmically divergent
- Renormalized to $m^2(s = m_0^2) = m_0^2$

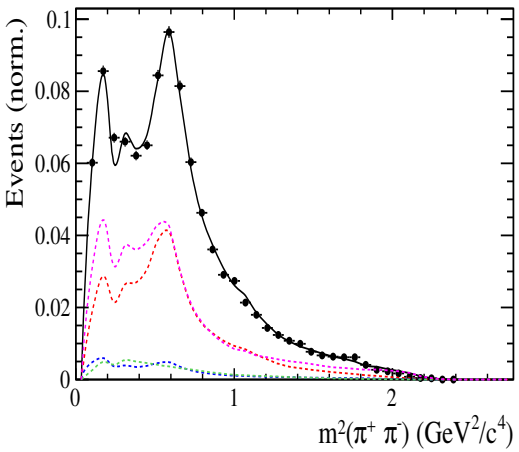


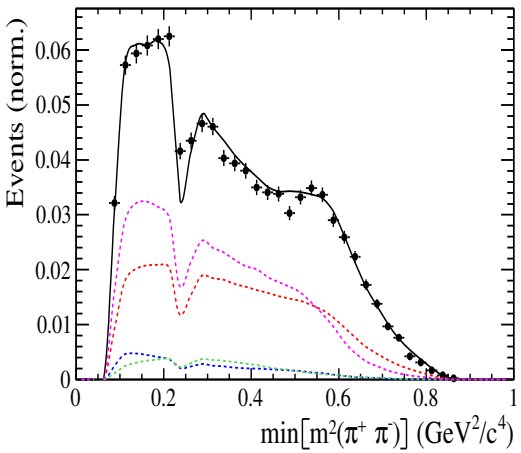
Amplitude fit

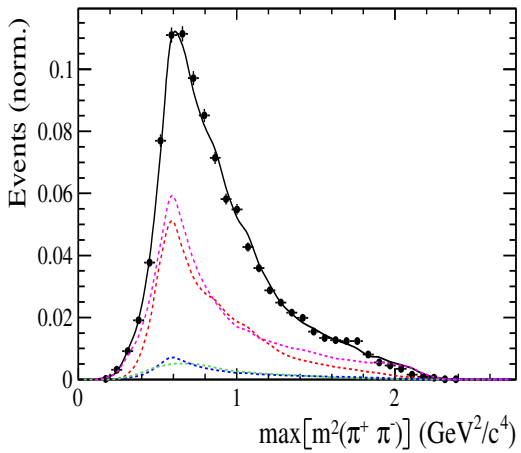
$D \to \pi^- a_1(1260)^+$
$D \to \pi^- \pi(1300)^+$
$D \to \pi^- a_1(1640)^+$
others
$\chi^2/\nu = 1.4$











Preliminary results

Fit fractions

Decay channel	F_i (%)
$D^0 \to \pi^- \left[a_1(1260)^+ \to \pi^+ \rho(770)^0\right]$	$38.1 \pm 2.3 \pm 3.2 \pm 1.7$
$D^0 \to \pi^- \left[a_1(1260)^+ \to \pi^+ \sigma\right]$	$10.2 \pm 1.4 \pm 2.1 \pm 2.5$
$D^0 \to \pi^+ \left[a_1(1260)^- \to \pi^- \rho(770)^0\right]$	$3.1 \pm 0.6 \pm 0.5 \pm 0.9$
$D^0 \to \pi^+ \left[a_1(1260)^- \to \pi^- \sigma\right]$	$0.8 \pm 0.2 \pm 0.1 \pm 0.4$
$D^0 \to \pi^- \left[\pi(1300)^+ \to \pi^+ \sigma\right]$	$6.8 \pm 0.9 \pm 1.5 \pm 3.1$
$D^0 \to \pi^+ \left[\pi(1300)^- \to \pi^- \sigma\right]$	$3.0 \pm 0.6 \pm 2.0 \pm 2.0$
$D^0 \to \pi^- \left[a_1(1640)^+[D] \to \pi^+ \rho(770)^0\right]$	$4.2 \pm 0.6 \pm 0.9 \pm 1.8$
$D^0 \to \pi^- \left[a_1(1640)^+ \to \pi^+ \sigma\right]$	$2.4 \pm 0.7 \pm 1.1 \pm 1.3$
$D^0 \to \pi^- \left[\pi_2(1670)^+ \to \pi^+ f_2(1270)\right]$	$2.7 \pm 0.6 \pm 0.7 \pm 0.9$
$D^0 \to \pi^- \left[\pi_2(1670)^+ \to \pi^+ \sigma\right]$	$3.5 \pm 0.6 \pm 0.8 \pm 0.9$
$D^0 \to \sigma\, f_0(1370)$	$21.2 \pm 1.8 \pm 4.2 \pm 5.2$
$D^0 \to \sigma\, \rho(770)^0$	$6.6 \pm 1.0 \pm 1.2 \pm 3.0$
$D^0[S] \to \rho(770)^0\, \rho(770)^0$	$2.4 \pm 0.7 \pm 1.1 \pm 1.0$
$D^0[P] \to \rho(770)^0\, \rho(770)^0$	$7.0 \pm 0.5 \pm 1.6 \pm 0.3$
$D^0[D] \to \rho(770)^0\, \rho(770)^0$	$8.2 \pm 1.0 \pm 1.7 \pm 3.5$
$D^0 \to f_2(1270)\, f_2(1270)$	$2.1 \pm 0.5 \pm 0.3 \pm 2.3$
Sum	$122.0 \pm 4.0 \pm 6.4 \pm 7.6$

Preliminary results

Resonance parameters

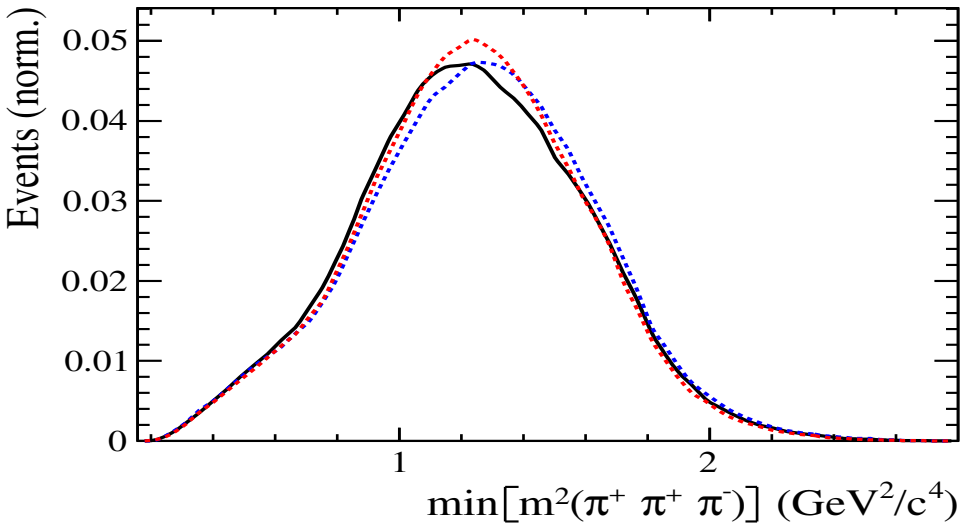
Resonance		Our Result (MeV)	PDG (MeV)
$a_1(1260)$	m_0	$1225 \pm 9 \pm 16 \pm 10$	1230 ± 40
	Γ_0	$430 \pm 24 \pm 13 \pm 18$	$250 - 600$
$\pi(1300)$	m_0	$1128 \pm 16 \pm 56 \pm 37$	1300 ± 100
	Γ_0	$314 \pm 39 \pm 58 \pm 26$	$200 - 600$
$a_1(1640)$	m_0	$1691 \pm 18 \pm 16 \pm 25$	1647 ± 22
	Γ_0	$171 \pm 33 \pm 20 \pm 35$	254 ± 7

Preliminary results

Lineshape parameterizations

$a_1(1260)$ parameter	SBW	RBW	Nominal
m_0	1134 ± 8	1221 ± 8	$1225 \pm 9 \pm 16 \pm 10$
Γ_0	367 ± 15	387 ± 18	$430 \pm 24 \pm 13 \pm 18$

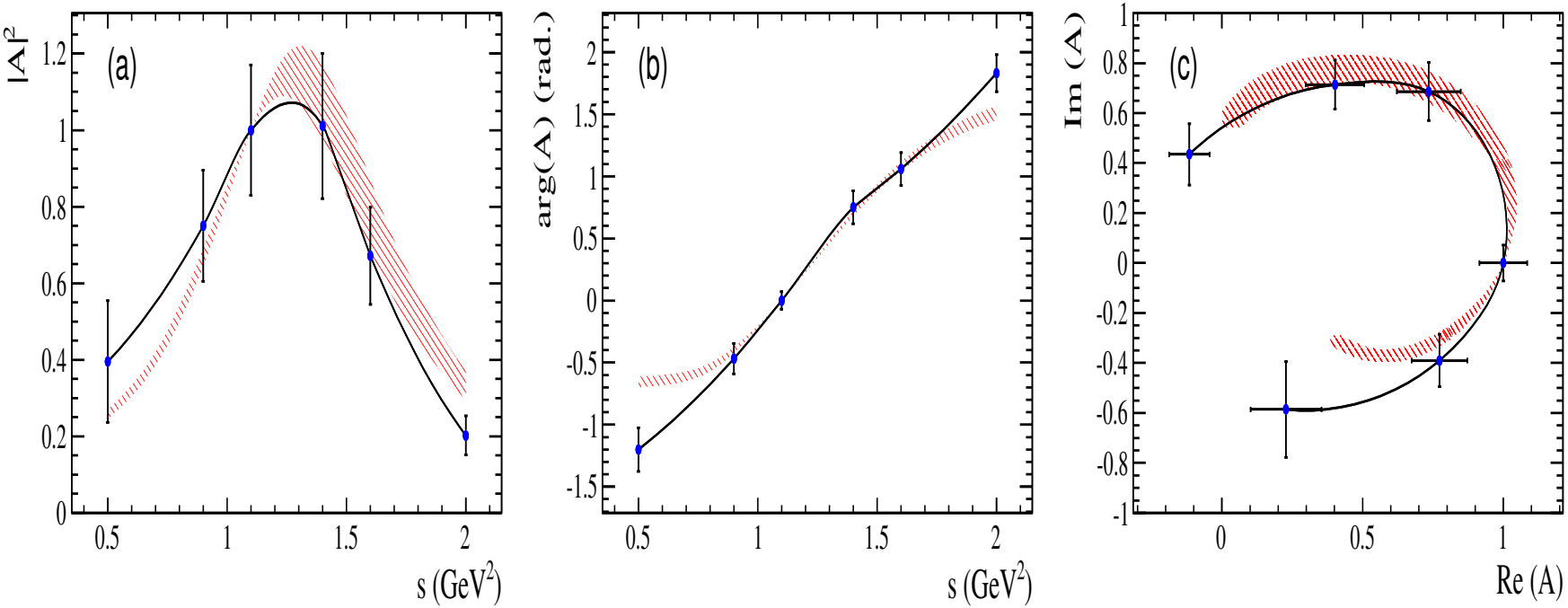
Nominal model preferred over SBW (RBW) with significance of 7 (10) σ



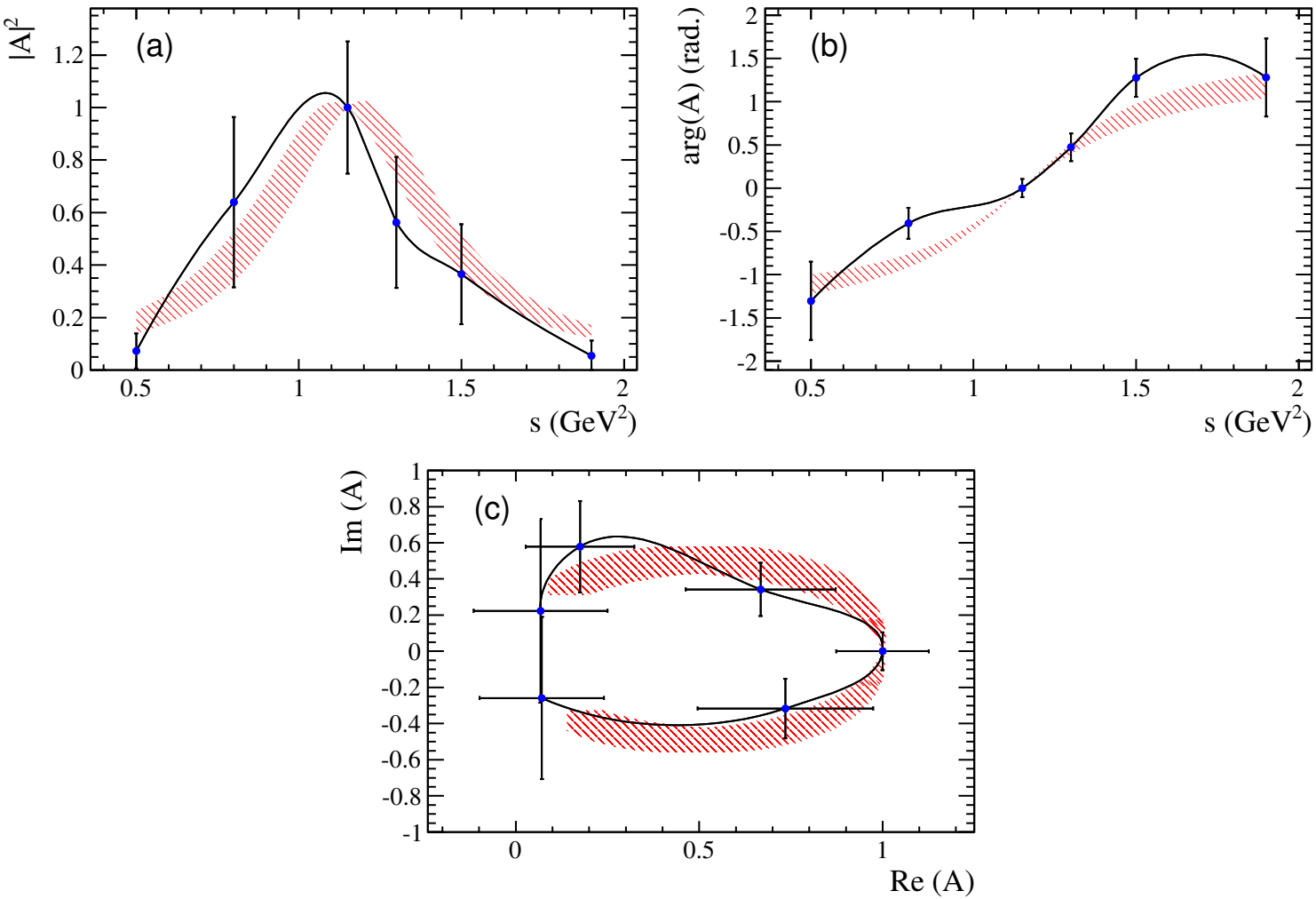
Preliminary results

Model-independent cross-check: $a_1(1260)$

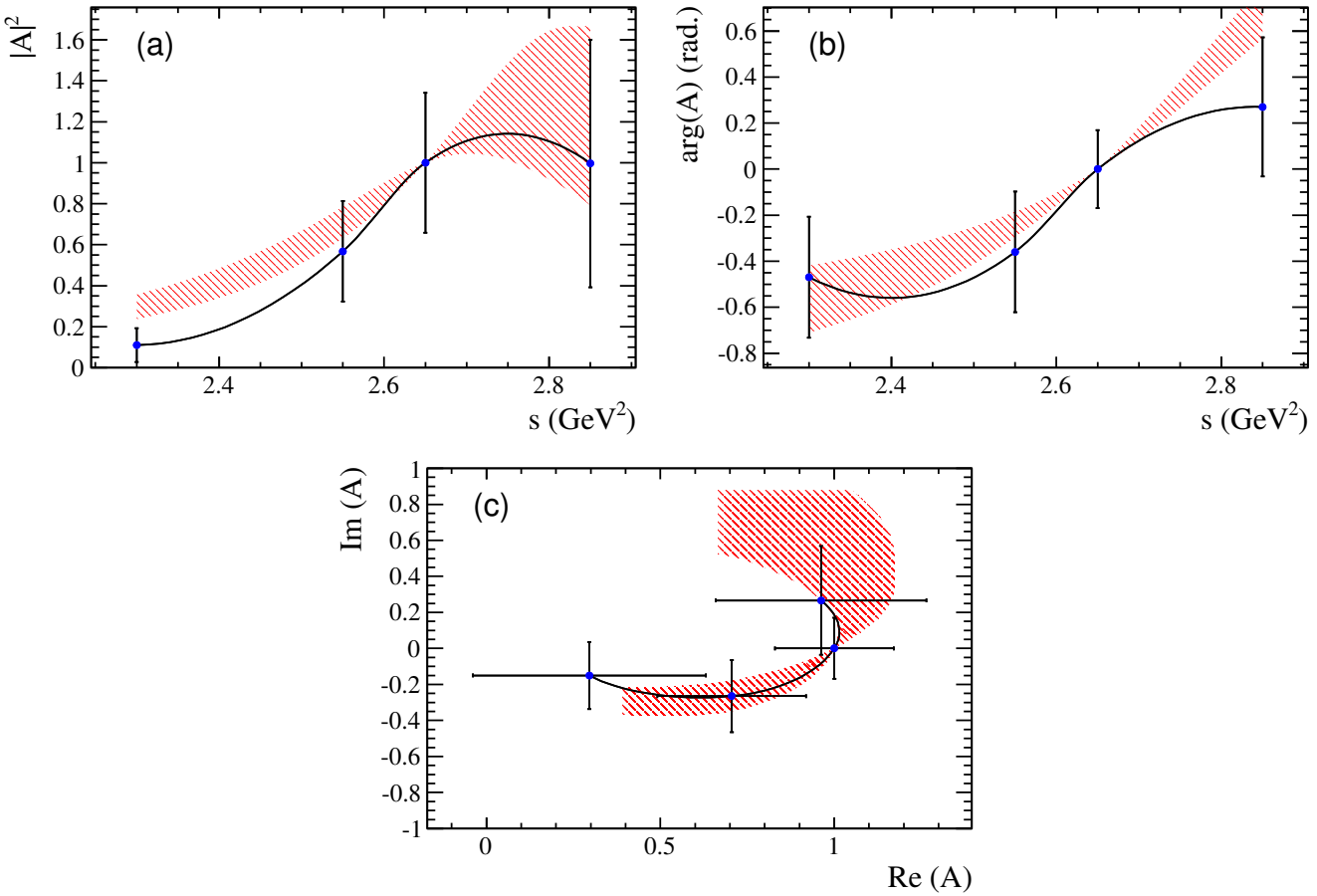
- Replace BW by non-parametric function
- 6 independent complex knots placed in $m^2(\pi\pi\pi)$
- Interpolate between knots with cubic spline
- Compare with BW expectation



Model-independent cross-check: $\pi(1300)$



Model-independent cross-check: $a_1(1640)$



Conclusion

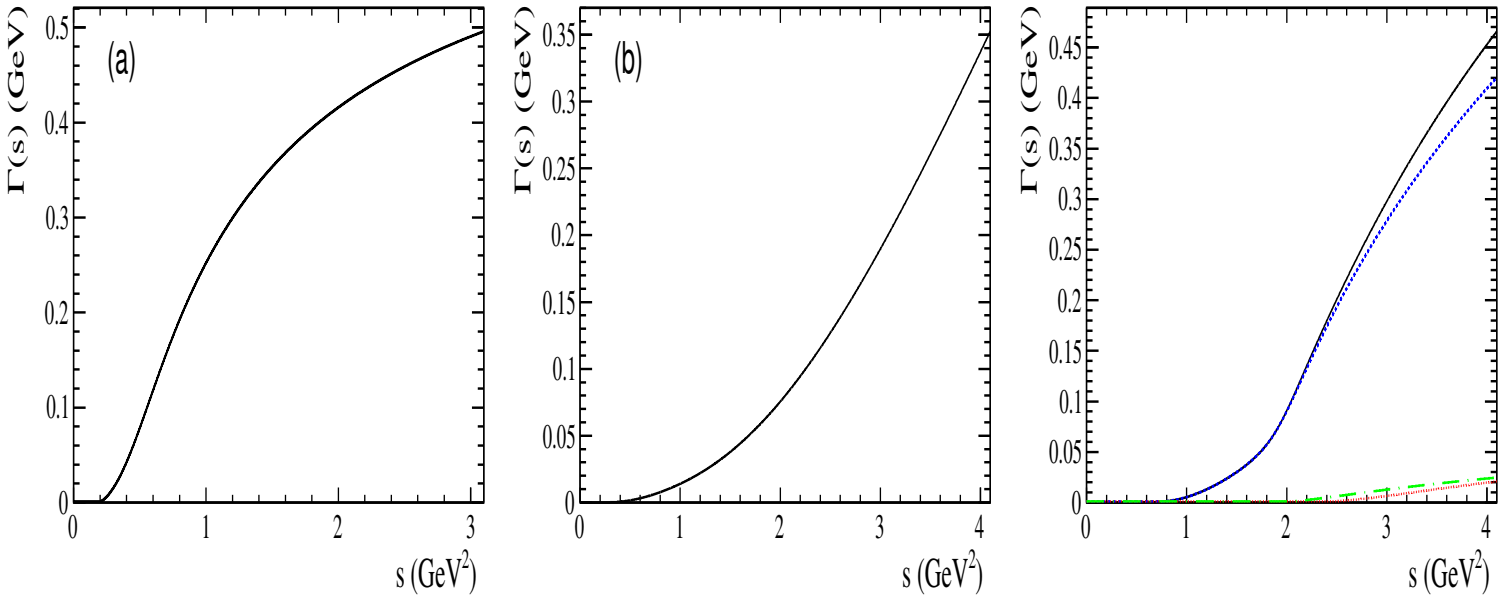
Summary

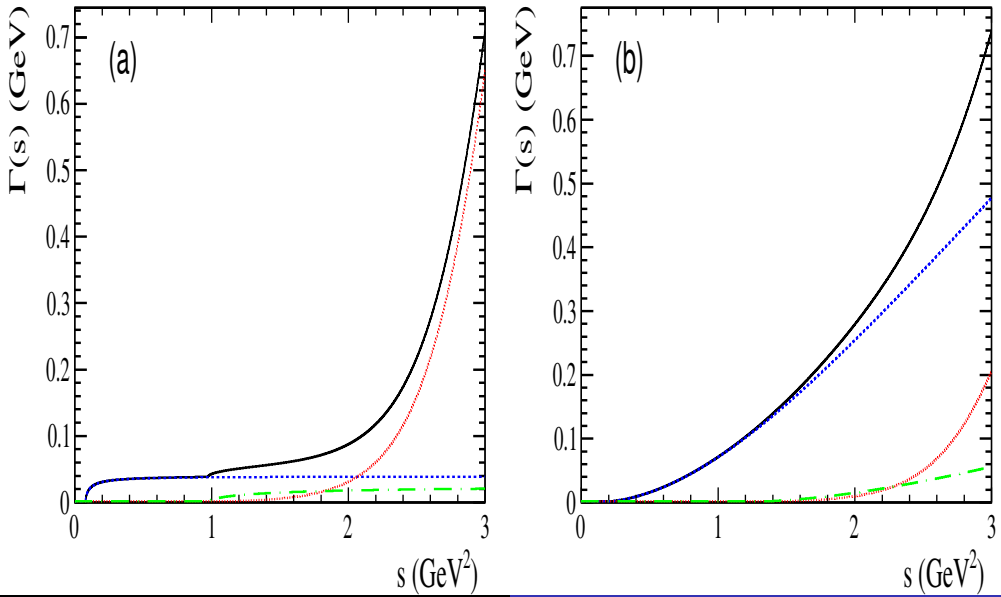
- First flavor-tagged amplitude analysis of $D \to \pi^+\pi^-\pi^+\pi^-$
- Determined mass and width of $a_1(1260)$ meson taking resonant substructure into account
- Observed significant dependence on chosen lineshape paramerization
- Verified resonant phase motion of observed $a_1(1260)$, $\pi(1300)$ and $a_1(1640)$ states

Acknowledgement

We thank the former CLEO collaboration for the privilege to use their precious data !

Backup: Running Widths





Backup: Bugg Model

$$D(s) = M^2 - s - g_1^2 \frac{s - s_A}{M^2 - s_A} z_s - m(s) - iM\Gamma_{tot}(s)$$

$$\begin{aligned}
M\Gamma_1(s) &= g_1^2 \frac{s - s_A}{M^2 - s_A} \rho_1(s) \\
g_1^2 &= M(b_1 + b_2 s) \exp[-(s - M^2)/A] \\
j_1(s) &= \frac{1}{\pi}\left[2 + \rho_1 ln_e\left(\frac{1-\rho_1}{1+\rho_1}\right)\right] \\
z_s &= j_1(s) - j_1(M^2) \\
M\Gamma_2(s) &= 0.6 g_1^2 F F_2^2(s) \\
M\Gamma_3(s) &= 0.19 g_1^2 F F_3^2(s) \\
F F_i(s) &= \exp(-\alpha |k|_i^2) \\
M\Gamma_4(s) &= M g_4 \rho_{4\pi}(s)/\rho_{4\pi}(M^2).
\end{aligned}$$

Backup: LASSO

t

Model selection for amplitude analysis

- Overwhelmingly high number of possible amplitudes
- Adding more fit parameters will describe **this** data better
- **Overfitting:** Large interference effects to produce unphysical features

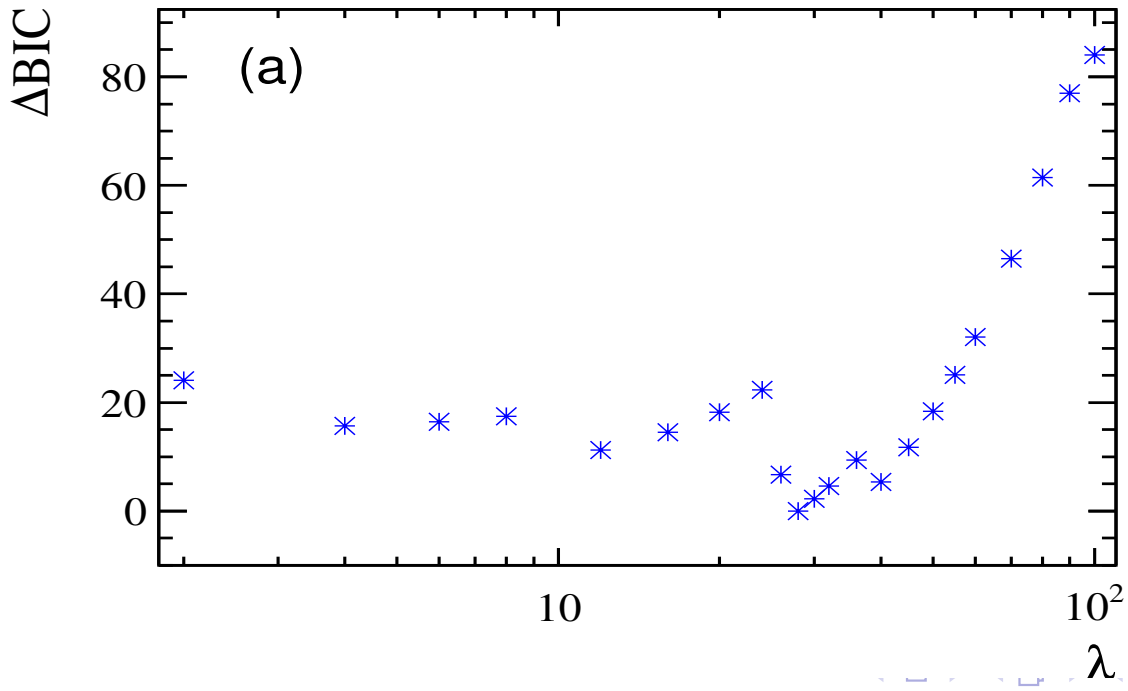
- **LASSO:** Data-driven method for model selection (M. Williams, JINST 10 (2015) no.09, P09002)
- Include "all" amplitudes, but penalize complexity in the likelihood:

$$-2\cdot\log(L) \rightarrow -2\cdot\log(L) + \lambda \cdot \sum_i |a_i|$$

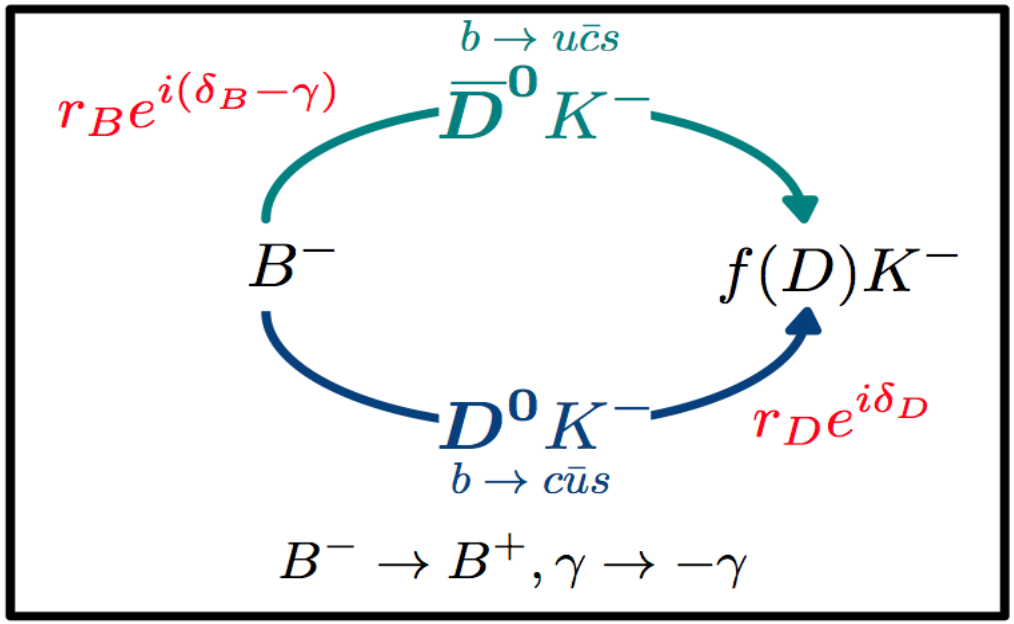
Backup: LASSO

How to choose λ ?

- $\mathrm{BIC}(\lambda) = -2 \cdot \log(L) + r \cdot \log(N_{events})$
  r = amplitudes with fit fraction $>$ threshold (0.5 %)
- Balances **gain in fit quality vs. complexity**
- Optimal value $\lambda \approx 30$



Backup: Measuring γ from $B^- \to D\,K^-$ decays



$$A = A_{\overline{D^0}} + r_B\, e^{i\,(\delta_B + \gamma)}\, A_{D^0}$$

$$\Gamma(B^\pm \to f_D\, K^\pm) \approx 1 + r_B^2 + 2\, r_B\, (2\, F_+ - 1)\, cos(\delta_B \pm \gamma)$$

Backup: *CP* even fraction

$$F_+^{4\pi} = \frac{N_{CP+}}{N_{CP+}+N_{CP-}} = \frac{1}{2}\left(1 + \frac{1}{N}\int |A_D|\,|A_{\overline{D}}|\,cos(\delta_D)\,\mathrm{d}\phi_4\right)$$

- $F_+^{4\pi}$(flavour-tagged, model-dependent) = $(72.9 \pm 0.9 \pm 1.5 \pm 1.0)\%$ (**Preliminary result**)
- $F_+^{4\pi}$(CP-tagged, model-independent) = $(73.7 \pm 2.8)\%$ (Malde et al., PLB 747 (2015) 9)

Backup: Systematics

Error sources

- $f_{bkg} = 18.5 \pm 0.8\%$
- background shape
- mistag prob. $\omega = 4.5 \pm 0.5\%$
- efficiency
- fitter bias
- resonance parameters
- form factors

$$\sigma_{sys} \approx 1 \text{ to } 3\,\sigma_{stat}$$