Dispersive Analysis of $\eta' \to \eta\pi\pi$ Decays

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Outline

1. Introduction and Motivation
2. Integral equations for $\eta' \to \eta\pi\pi$
3. Fit to Data
4. Summary and Outlook

Introduction and Motivation

Why of interest?

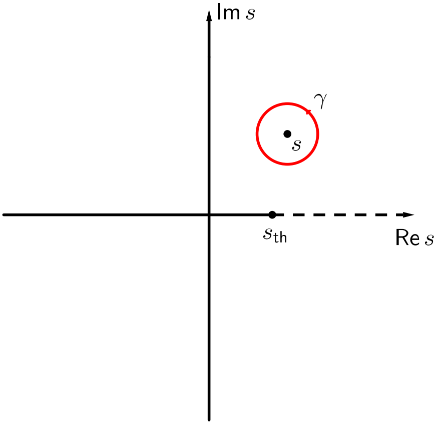
- main (hadronic) contribution to the total decay width $\mathrm{BR}(\eta' \to \eta\pi\pi) = 0.652(11)$ [PDG 2016]
- due to $U(1)_A$ anomaly of QCD η' is not a Goldstone boson: $\Rightarrow$ ChPT breakdown for processes involving an η'
- relatively small phase space, but FSI seem to play an important role already seen in the study of $\eta \to 3\pi$ [talk by B. Moussallam on friday]
- potentially clean access to constrain $\eta\pi$ scattering (energy far below $K\bar{K}$ inelastic threshold)
- measurements of the Dalitz plot available from BES-III and VES (both $\eta' \to \eta\pi^+\pi^-$) and upcoming data from A2 ($\eta' \to \eta\pi^0\pi^0$)
- $\eta' \to \eta\pi^0\pi^0$ expected to show a cusp effect at $\pi^+\pi^-$-threshold

Advantages of dispersion relations

- based on fundamental properties of analyticity, unitarity and crossing
  $\Rightarrow$ model independence
- in contrast to effective field theories: dispersive methods describe the resummation of rescattering effects for considered particles

Dispersion relation for a single-variable function

complex-valued function $f(s)$: analytic in the entire complex plane apart from a branch cut on the real axis for $s \geq s_{\text{th}}$



- Cauchy's theorem:

$$f(s) = \frac{1}{2\pi \mathrm{i}} \oint_\gamma \frac{f(s')}{s' - s} \mathrm{d}s'$$

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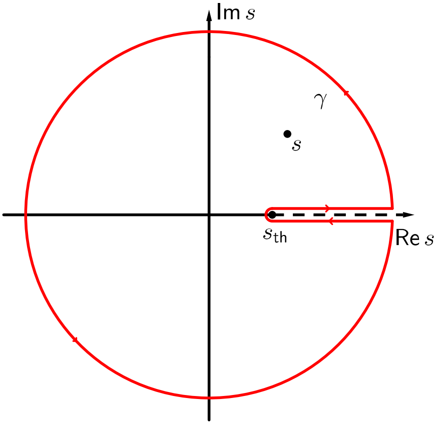

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- extend γ around the cut

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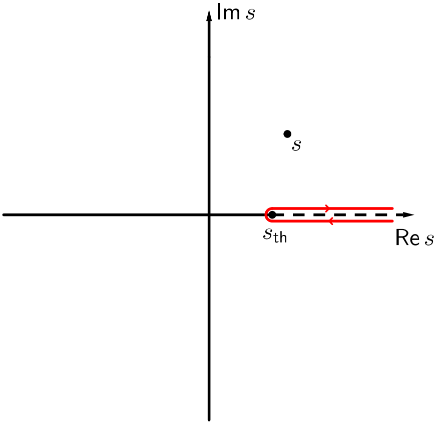

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- extend γ around the cut
- $\lim_{|s| \to \infty} f(s) \to 0$: forces the integral along complex arc to vanish

$$f(s) = \frac{1}{2\pi \mathrm{i}} \int_{s_{\text{th}}}^{\infty} \frac{\text{disc} f(s')}{s' - s} \mathrm{d}s'$$

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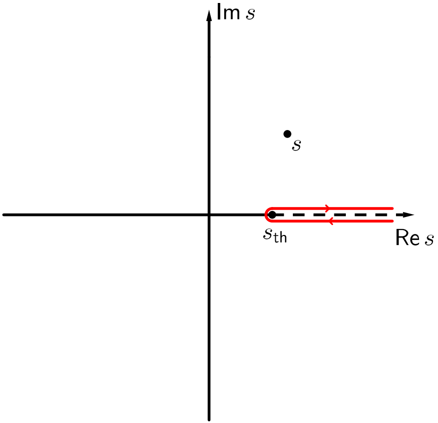

- Cauchy's theorem:

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$$f(s) = \frac{1}{2\pi \mathrm{i}} \int_{s_{\text{th}}}^{\infty} \frac{\mathrm{disc} f(s')}{s'-s} \mathrm{d}s'$$

Can be extended to less restrictive high energy behaviour of $f(s)$ by applying subtractions

Integral equations for $\eta' \to \eta\pi\pi$

Kinematics of the $\eta' \to \eta\pi\pi$ decay

transition amplitude:

$$\langle \pi^i(p_1)\pi^j(p_2)\eta(p_3)|T|\eta'(P)\rangle = (2\pi)^4\delta^{(4)}(P-p_1-p_2-p_3)\,\delta^{ij}\,\mathcal{A}(s,t,u)$$

Mandelstam variables:

$$s=(p_1+p_2)^2, \quad t=(p_1+p_3)^2, \quad u=(p_2+p_3)^2$$

(charged decay channel $\eta' \to \eta\pi^+\pi^-$ and neutral channel $\eta' \to \eta\pi^0\pi^0$ differ only by isospin breaking effects)

Analytic properties of the $\eta' \to \eta\pi\pi$ amplitude

- $\mathcal{A}(s,t,u)$ has a right-hand branch cut in the complex s-plane, starting at the $\pi\pi$-threshold
- similar situation in t- and u-planes, branch cuts starting at the $\eta\pi$-threshold
- left-hand cuts present due to crossing

Reconstruction theorem

[Stern et al. 1993, Ananthanarayan et al. 2001, Zdráhal et al. 2008]

- $\mathcal{A}(s,t,u)$ can be decomposed into single-variable functions that possess just a right-hand cut

$$\mathcal{A}(s,t,u) = \mathcal{A}_0(s) + \mathcal{A}_1(t) + \mathcal{A}_1(u)$$

  $\mathcal{A}_0$ contains $\pi\pi$-FSI effects (I=0, S-wave)

  $\mathcal{A}_1$ contains $\eta\pi$-FSI effects (I=1, S-wave)

- neglect discontinuities of P- and higher partial waves:

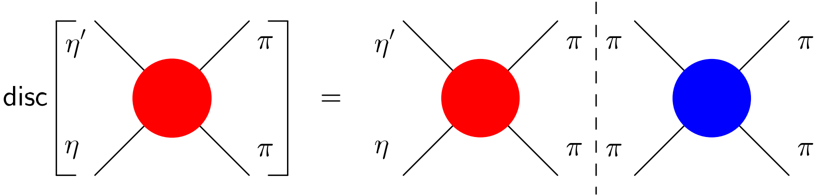
  $\pi\pi$ P-wave forbidden by C-parity

  $\eta\pi$ P-wave has exotic quantum numbers

  D- and higher partial waves neglected due to small phase space

Unitarity condition

s-channel:



(analogous for t- & u-channel)

discontinuity equations for the single-variable functions:

$$\text{disc}\mathcal{A}_0(s) = 2\text{i}\,\theta\big(s - 4M_\pi^2\big)\,\big[\mathcal{A}_0(s) + \hat{\mathcal{A}}_0(s)\big]\,e^{-\text{i}\delta_0(s)}\,\sin\delta_0(s)$$

$$\text{disc}\mathcal{A}_1(t) = 2\text{i}\,\theta\big(t - (M_\eta + M_\pi)^2\big)\,\big[\mathcal{A}_1(t) + \hat{\mathcal{A}}_1(t)\big]\,e^{-\text{i}\delta_1(t)}\,\sin\delta_1(t)$$

$\Rightarrow$ inhomogeneous Omnès problem

$\delta_0(s)$, $\delta_1(t)$: S-wave $\pi\pi$ and $\eta\pi$ scattering phase shifts

$\hat{\mathcal{A}}_I$: inhomogeneities, angular averages of crossed-channel $\mathcal{A}_I$ functions

Omnès representation

dispersive representation for the functions $\mathcal{A}_I$ in Omnès form:

$$\mathcal{A}_0(s) = \Omega_0(s)\left\{\alpha + \beta s + \frac{s^2}{\pi}\int_{4M_\pi^2}^{\infty}\frac{\mathrm{d}s'}{s'^2}\frac{\hat{\mathcal{A}}_0(s)\,\sin\delta_0(s')}{|\Omega_0(s')|(s'-s)}\right\}$$

$$\mathcal{A}_1(t) = \Omega_1(t)\left\{\gamma t + \frac{t^2}{\pi}\int_{(M_\eta+M_\pi)^2}^{\infty}\frac{\mathrm{d}t'}{t'^2}\frac{\hat{\mathcal{A}}_1(s)\,\sin\delta_1(t')}{|\Omega_1(t')|(t'-t)}\right\}$$

Omnès function [Omnès 1958]:

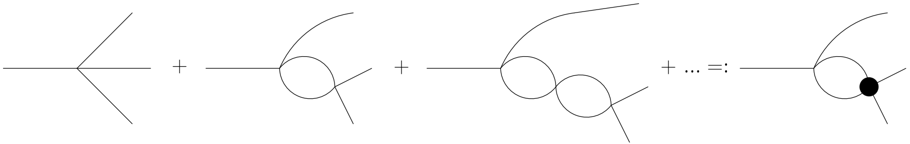
$$\Omega_I(s) = \exp\left\{\frac{s}{\pi}\int_{s_{\mathrm{th}}}^{\infty}\frac{\mathrm{d}s'}{s}\frac{\delta_I(s')}{(s'-s)}\right\}$$

asymptotics: $\delta_0(s)\to\pi$, $\delta_1(t)\to\pi$ and $\mathcal{A}_0(s)=\mathcal{O}(s^0)$, $\mathcal{A}_1(t)=\mathcal{O}(t^0)$

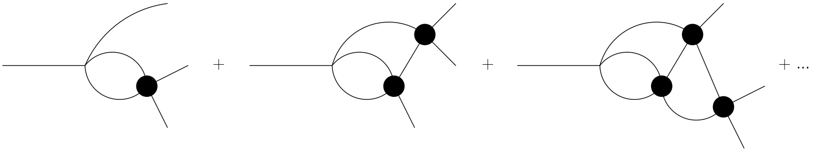
$\Rightarrow$ 3 (real) subtraction constants α,β,γ needed

if $\hat{\mathcal{A}}_I = 0$: back to form factor relations [talk by J. Daub]

Physical interpretation

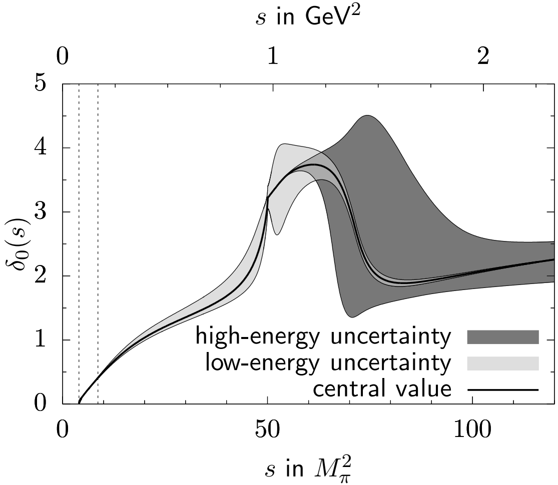


- Omnès function: iteration of two-particle bubble diagrams



- dispersion integral: accounts for crossed-channel interactions
- subtraction constants: free parameters in the dispersion relation, not fixed by unitarity

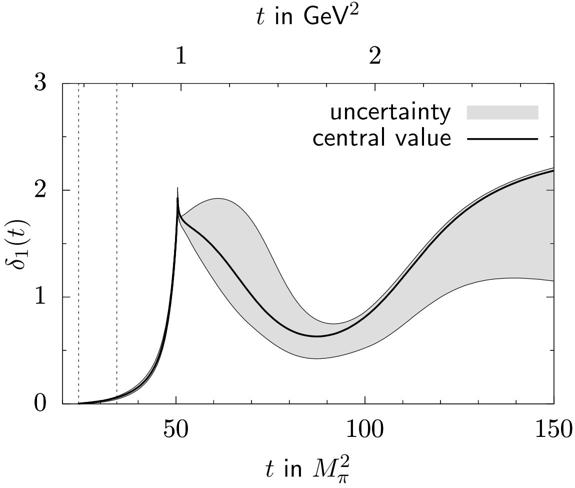
$\pi\pi$-scattering phase shift (S-wave, $I = 0$)



elastic regime: Roy equation analyses [Caprini et al. 2012]

inelastic regime: study of a coupled channel Omnès matrix [Daub et al. 2016] and large-N_c ChPT constraints on $\eta'\eta \to (\pi\pi/K\bar{K})$

$\eta\pi$-scattering phase shift (S-wave, $I = 1$)



phase of the scalar form factor $F_S^{\eta\pi}(t)$ calculated out of a coupled channel T-matrix ($\eta\pi/K\bar{K}$) [Albaladejo, Moussallam 2015]

Intermediate summary

- set of coupled integral equations:

  $\Rightarrow \mathcal{A}_0(s)$, $\mathcal{A}_1(t)$: DR involving $\hat{\mathcal{A}}_0(s)$, $\hat{\mathcal{A}}_1(t)$

  $\Rightarrow \hat{\mathcal{A}}_0(s)$, $\hat{\mathcal{A}}_1(t)$: angular integrals over $\mathcal{A}_0(s)$, $\mathcal{A}_1(t)$

- input: $\pi\pi$- and $\eta\pi$-scattering phase shifts

- problem linear in the 3 subtraction constants

  $\Rightarrow$ construct 3 basis solutions

- system solved numerically by iteration

- determination of the subtraction constants by fit to experimental data or matching to chiral EFT's

Fit to Data

Experimental status on $\eta' \to \eta\pi^+\pi^-$

- partial decay width [PDG 2016]:

$$\Gamma(\eta' \to \eta\pi^+\pi^-) = (84.5 \pm 4.1) \times 10^{-6}\,\text{GeV}$$

- most recent measurements of the charged Dalitz-plot parameters

$$|\mathcal{A}(x,y)|^2 \approx |\mathcal{N}|^2(1 + ay + by^2 + cx + dx^2 + \ldots),$$
$$x \propto (t-u), \qquad y \propto -s$$

- Dalitz plot extremely flat: $a, b, d \ll 1$

in 10^{-3}	BES-III [Ablikim et al. 2011]	VES [Dorofeev et al. 2007]
a	$-47 \pm 11 \pm 3$	$-127 \pm 16 \pm 8$
b	$-69 \pm 19 \pm 9$	$-106 \pm 28 \pm 14$
c	$+19 \pm 11 \pm 3$	$+15 \pm 11 \pm 14$
d	$-73 \pm 12 \pm 3$	$-82 \pm 17 \pm 8$

terms odd in x violate C-parity (not considered in DR)

Fit setup

- perform transformation of the subtraction constants

$$\alpha = \bar{\mathcal{N}}\bar{\alpha}, \quad \beta = \bar{\mathcal{N}}\bar{\beta}, \quad \gamma = \bar{\mathcal{N}}\bar{\gamma} \quad \Rightarrow \mathcal{A}(x,y) = \bar{\mathcal{N}}\bar{\mathcal{A}}(x,y)$$

- fix arbitrary normalisation of $\bar{\mathcal{A}}(x,y)$ to be

$$\int \mathrm{d}x\,\mathrm{d}y\,|\bar{\mathcal{A}}(x,y)|^2 = 1$$

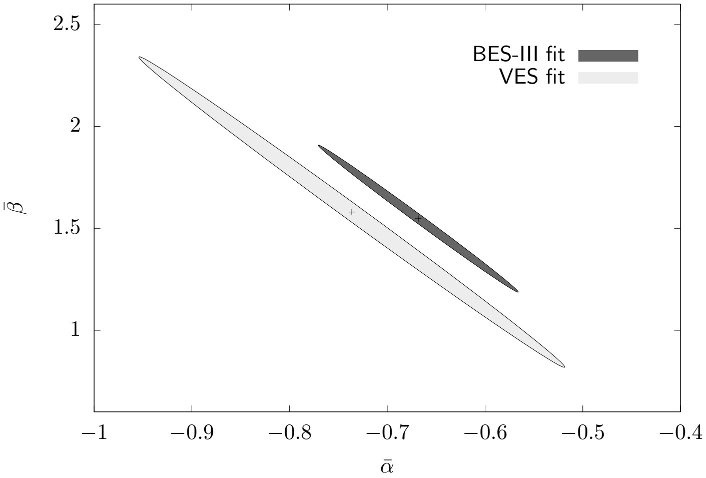
- decouples the Dalitz-plot distribution from the partial decay width
- use condition on $\bar{\mathcal{A}}(x,y)$ to express $\bar{\gamma}$ as function of $\bar{\alpha}$, $\bar{\beta}$
- 4 experimental constraints (Γ, a, b, d), but just 3 degrees of freedom from the DR $(\bar{\mathcal{N}}, \bar{\alpha}, \bar{\beta})$

Fit results

- χ^2/ndof is close to 1 for both fits
- DR needs one parameter less than the phenomenological parameterization
- we observe a strong anticorrelation between $\bar{\alpha}$ and $\bar{\beta}$
- fit error is dominated by the experimental uncertainty from the Dalitz-plot data (first error)
- apart from the error on γ the uncertainty coming from the phase input is small (second error)

	BES-III	VES
χ^2/ndof	$459/435 \approx 1.06$	$44.5/47 \approx 0.95$
α	$-9.3 \pm 1.0 \pm 0.3$	$-10.2 \pm 2.0 \pm 0.4$
β	$21.5 \pm 3.3 \pm 1.9$	$21.9 \pm 7.0 \pm 2.6$
γ	$0.55 \pm 0.24 \pm 0.26$	$1.10 \pm 0.52 \pm 0.32$

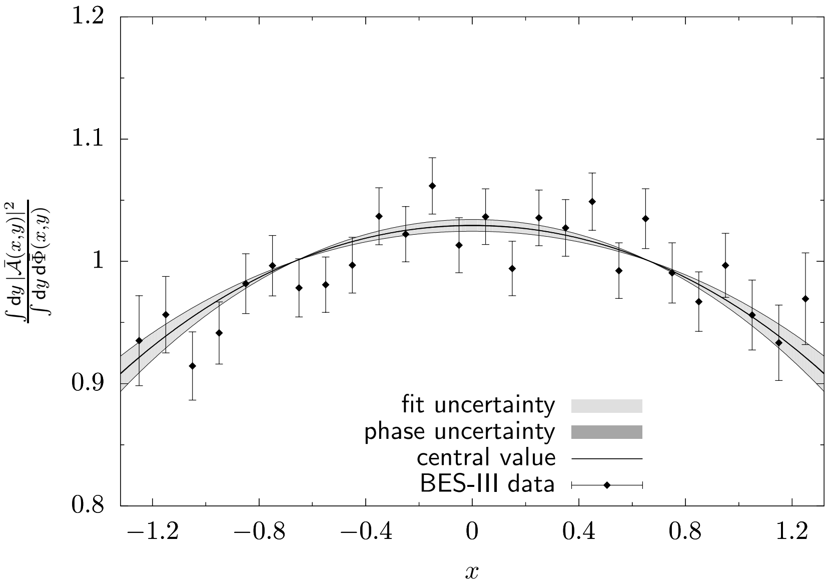
Error ellipse ($\bar{\alpha}$,$\bar{\beta}$)-plane



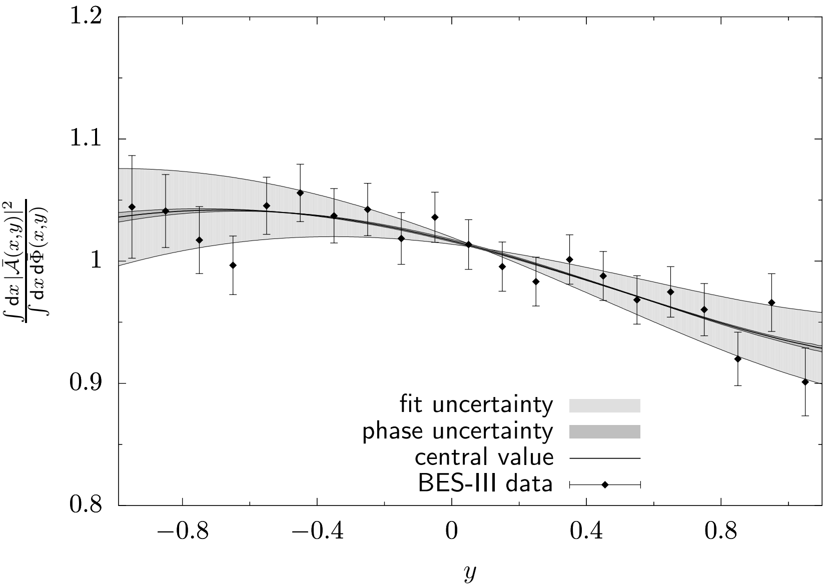
strong tension between BES-III and VES data sets

⇒ fit results are not compatible with each other

Dalitz-plot x-projection



Dalitz-plot y-projection



Dalitz-plot parameters

- extract Dalitz-plot parameters from the Taylor expansion of our amplitude
- apart from VES b, all parameters are well reproduced
- allows us to extract even higher coefficients of the expansion
- higher coefficients extremely tiny

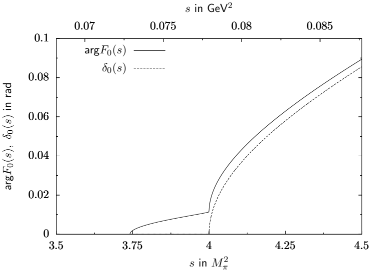
in 10^{-3}	BES-III fit	VES fit
a	$-46 \pm 9 \pm 2$ (-47 ± 11)	$-154 \pm 18 \pm 2$ (-127 ± 18)
b	$-66 \pm 4 \pm 4$ (-69 ± 21)	$-56 \pm 9 \pm 4$ (-106 ± 31)
d	$-71 \pm 11 \pm 2$ (-73 ± 12)	$-85 \pm 24 \pm 3$ (-82 ± 19)
$\kappa_{03}[y^3]$	$4 \pm 1 \pm 1$	$10 \pm 2 \pm 2$
$\kappa_{21}[yx^2]$	$-2 \pm 1 \pm 7$	$3 \pm 2 \pm 9$
$\kappa_{04}[y^4]$	$3 \pm 1 \pm 1$	$3 \pm 1 \pm 1$
$\kappa_{22}[y^2x^2]$	$5 \pm 1 \pm 2$	$7 \pm 3 \pm 3$
$\kappa_{40}[x^4]$	$0 \pm 1 \pm 3$	$0 \pm 1 \pm 4$

Isospin breaking effects in $\eta' \to \eta\pi^0\pi^0$: the $\pi^+\pi^-$ cusp

isospin breaking due to the π mass difference:

- correction for phase space is straightforward
- amplitude must have all thresholds at the right places

  $\Rightarrow$ difficult: $\pi\pi$-phase shifts derived in formalism relying on isospin symmetry

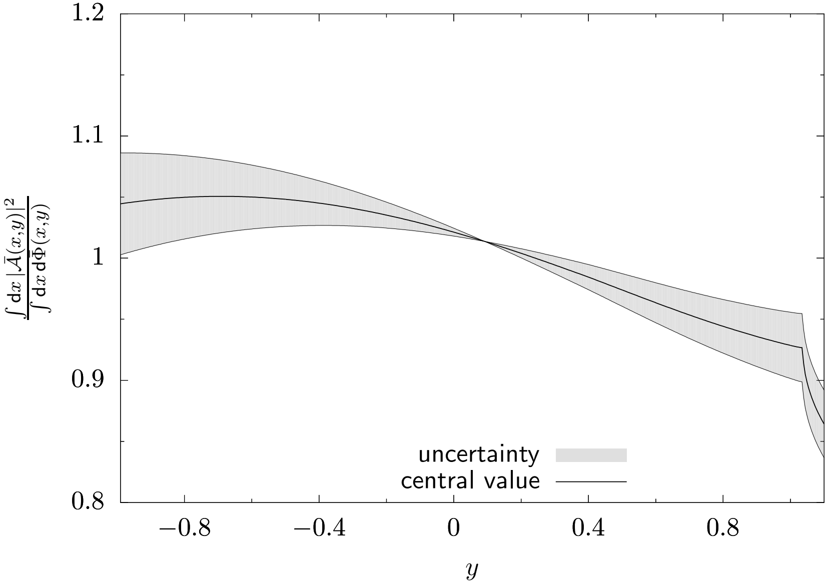


constructing an effective $\pi^0\pi^0$-phase shift based on the neutral-pion scalar form factor $F_0(s)$ [Colangelo et al. 2009]

correct analytic structure near the $\pi\pi$-thresholds:

- isospin breaking $\propto \sqrt{M_{\pi^+}^2 - M_{\pi^0}^2}$ (nonanalytic) retained
- isospin breaking $\mathcal{O}\big(M_{\pi^+}^2 - M_{\pi^0}^2\big)$ (analytic) neglected

Prediction: Dalitz-plot y-projection for $\eta' \to \eta\pi^0\pi^0$



(α, β, γ taken from BES-III fit)

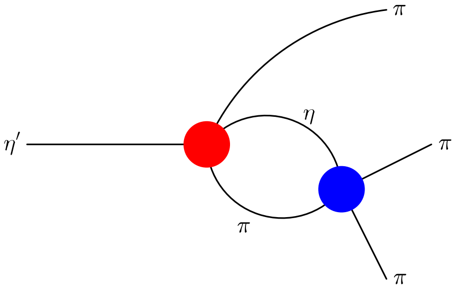
Summary and Outlook

Summary

- derived a dispersive representation for $\eta' \to \eta\pi\pi$ to describe the 3-particle FSI
  - based on analyticity, unitarity and crossing
  - input: S-wave $\pi\pi$- and $\eta\pi$-scattering phase shifts
  - 3 subtraction constants (predictive power)
- experimental data is well described by our representation
- able to extract higher order Dalitz-plot parameters

Outlook

- upcoming high statistic Dalitz-plot data from A2 ($\eta' \to \eta\pi^0\pi^0$) to test our representation
- use chiral EFT's to constrain the subtraction constants from matching
- will serve as input for a dispersive analysis of $\eta' \to 3\pi$ proceeding via $\eta' \to \eta\pi\pi$ decay and an isospin breaking rescattering $\eta\pi \to \pi\pi$



Backup

Soft-pion theorem for $\eta' \to \eta\pi\pi$

[Riazuddin and Oneda 1971, Adler 1965]

current algebra statement for amplitudes involving π's in limit of $p_\pi \to 0$:

- suggests 2 zeros (crossing) in $\mathcal{A}(s,t,u)$ at

$$s_1 = 0, \quad t_1 = M_{\eta'}^2, \quad u_1 = M_\eta^2 \quad \& \quad s_2 = 0, \quad t_2 = M_\eta^2, \quad u_2 = M_{\eta'}^2$$

- protected by chiral symmetry: Adler zeros

removed in models with explicit inclusion of scalar resonance $a_0(980)$
[Deshpande and Truong 1978]

study $\mathcal{A}(s,t,u)$ in our dispersive framework:

- real part: vanishes close to soft-π points
- imaginary part: peaks very close to soft-π points

corrections at soft-π points $\mathcal{O}\big(M_\pi^2/(M_{\eta'}^2 - M_{a_0}^2)\big) \Rightarrow$ not small

Adler-"non"-zeros for BES-III fit