Parametrisation for near-threshold states

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F.K. Guo, C. Hanhart, QW, Q. Zhao., PRD91(2015)051504

C. Hanhart, Y.S. Kalashnikova, P. Matuschek, R.V. Mizuk, A.V. Nefediev, QW,
PRL115(2015)202001, J.Phys.Conf. Ser. 675(2016)022016

F.K. Guo, C. Hanhart, Y.S. Kalashnikova, P. Matuschek, R.V. Mizuk, A.V. Nefediev, QW,

J. L. Wynen, PRD93(2016)074031

Outline

Properties of near-threshold states

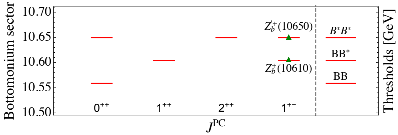
Cannot be a cusp alone

Parametrisation for near-threshold states

Apply to the two Z_b cases

Summary

Properties of near-threshold states



★ Molecular scenario

$E^{\text{Exp}}_{Z_b} \sim 3$ MeV

⇒ Extended object

Size $\frac{\hbar c}{\gamma} \sim 1.56$ fm $\gg R_0$

$R_0 \ll 1$ fm confinement radius

$\gamma = \sqrt{2\mu E}$ binding momentum

μ reduced mass

⇒ Probability to find Z_b

M.Cleven et al., EPJA47(2011)120

$$1 - \left[1 + \frac{\mu^2 g^2_{\text{bare}}}{8\pi\gamma}\right]^{-1} \leq 1|_{g_{\text{bare}}\to\infty}$$

Large coupling $g_{Z_b B B^*}$

$$g^2_{\text{eff}} = \frac{g^2_{\text{bare}}}{1+\frac{\mu^2 g^2_{\text{bare}}}{8\pi\gamma}} \leq \frac{8\pi\gamma}{\mu^2}\Big|_{g_{\text{bare}}\to\infty}$$

Exp: Belle, PRL116(2016)212001

$\mathcal{BR}(Z_b \to B\bar{B}^* + c.c.) \sim 85.6\%$

★ Cusp alone ?

★ BW does not work,

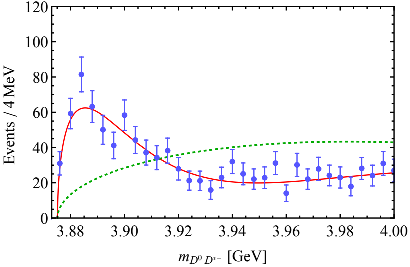
⇒ $E^{\text{Exp}}_{Z_b^{(\prime)}} \ll \Gamma_{Z_b^{(\prime)}}$

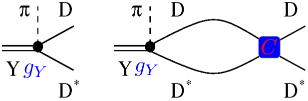
⇒ two Z_b states

Cannot be a cusp alone

$$Y(4260) \to \underbrace{D\bar{D}^*}_{Z_c(3900)?} \; \pi$$

F.K. Guo, et al., PRD91(2015)051504, BESIII, PRL112(2014)022001



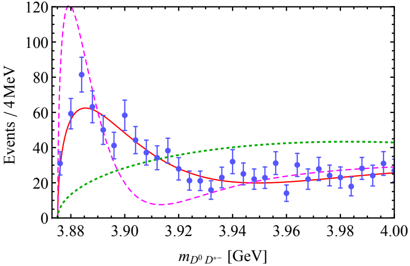


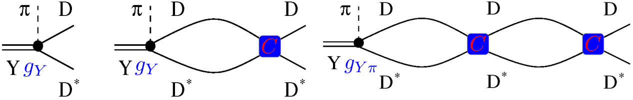
★$g_Y[1 - G_\Lambda(E)C]$ to fit the data

Cannot be a cusp alone

$$Y(4260) \to \underbrace{D\bar{D}^*}_{Z_c(3900)?} \; \pi$$

F.K. Guo, et al., PRD91(2015)051504, BESIII, PRL112(2014)022001



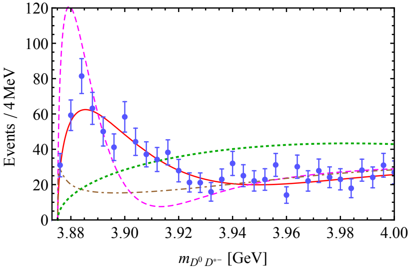


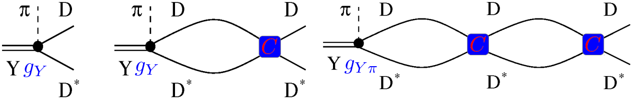
★ $\left|G_\Lambda(th)C\right| > 1$ non-perturbative

Cannot be a cusp alone

$$Y(4260) \to \underbrace{D\bar{D}^*}_{Z_c(3900)?} \pi$$

F.K. Guo, et al., PRD91(2015)051504, BESIII, PRL112(2014)022001





★Summing all the bubble loops $\frac{g_Y}{1+G_\Lambda(E)C}$ gives a bound state pole very close to threshold.

Parametrisation for near-threshold states

Multichannel LSE

Potential $\hat{V}$

$$\begin{matrix} & b=\overline{1,N_{\rm p}} & \beta=\overline{1,N_{\rm e}} & i=\overline{1,N_{\rm in}} & \\ \hat{V} = & \begin{pmatrix} v_{ab} & v_{a\beta}(\boldsymbol{p}') & v_{ai}(\boldsymbol{k}) \\ v_{\alpha b}(\boldsymbol{p}) & v_{\alpha\beta}(\boldsymbol{p},\boldsymbol{p}') & v_{\alpha i}(\boldsymbol{p},\boldsymbol{k}) \\ v_{ja}(\boldsymbol{k}') & v_{j\beta}(\boldsymbol{k}',\boldsymbol{p}') & v_{ji}(\boldsymbol{k}',\boldsymbol{k}) \end{pmatrix} & & \begin{matrix} a=\overline{1,N_{\rm p}} \\ \alpha=\overline{1,N_{\rm e}} \\ j=\overline{1,N_{\rm in}}. \end{matrix} \end{matrix}$$

Bare pole terms: with indices a, b, ...

Inelastic channels: hidden-flavor channels with indices i, j, ...

Elastic channels: open-flavor channels with indices α, β, ...

C. Hanhart et al., PRL115(2015)202001, F.K. Guo et al., PRD93(2016)074031

Note: $v_{ij}(\boldsymbol{k},\boldsymbol{k}') \equiv 0$, $\pi - Q\bar{Q}$ scattering length ≤ 0.02 fm

L. Liu et al., Proc. Sci., LATTICE2008(2008)112, W. Detmold, et al., PRD87(2013)094504

Parametrisation for near-threshold states

Multichannel LSE

Potential $\hat{V}$

$$\hat{V} = \begin{pmatrix} v_{AB}(\boldsymbol{p},\boldsymbol{p}') & v_{Ai}(\boldsymbol{p},\boldsymbol{k}) \\ v_{jB}(\boldsymbol{k}',\boldsymbol{p}') & 0 \end{pmatrix} \qquad B = \overline{1,\, N_{\rm e}+N_{\rm p}} \quad i = \overline{1, N_{\rm in}} \quad A = \overline{1\, N_{\rm e}+N_{\rm p}} \quad j = \overline{1, N_{\rm in}},$$

Capital greek letters A, B for elastic channels and bare poles.

Bare pole terms: with indices a, b, ...

Inelastic channels: hidden-flavor channels with indices i, j, ...

Elastic channels: open-flavor channels with indices α, β, ...

C. Hanhart et al., PRL115(2015)202001, F.K. Guo et al., PRD93(2016)074031

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L. Liu et al., Proc. Sci., LATTICE2008(2008)112, W. Detmold, et al., PRD87(2013)094504

Parametrisation for near-threshold states

LSE $t = v - vSt$ can be decomposed into two sub sets

$$\begin{cases} t_{iB} = v_{iB} - \sum_A v_{iA} S_A t_{AB} \\ t_{AB} = v_{AB} - \sum_C v_{AC} S_C t_{CB} - \sum_i v_{Ai} S_i t_{iB} \end{cases}$$

$$\begin{cases} t_{Aj} = v_{Aj} - \sum_B t_{AB} S_B v_{Bj} \\ t_{ij} = -\sum_A v_{iA} S_A v_{Aj} + \sum_{AB} v_{iA} S_A t_{AB} S_B v_{Bj} \end{cases}$$

Define effective potential V

$$\begin{aligned} t_{AB} &= v_{AB} - \sum_C v_{AC} S_C t_{CB} - \sum_i v_{Ai} S_i \left(v_{iB} - \sum_C v_{iC} S_C t_{CB} \right) \\ &= \underbrace{v_{AB} - \sum_i v_{Ai} S_i v_{iB}}_{V_{AB}} - \sum_C \underbrace{\left(v_{AC} - \sum_i v_{Ai} S_i v_{iC} \right)}_{V_{AC}} S_C t_{CB} \end{aligned}$$

Parametrisation for near-threshold states

LSE $t = v - vSt$ can be decomposed into two sub sets

$$\begin{cases} t_{iB} = v_{iB} - \sum_A v_{iA} S_A t_{AB} \\ t_{AB} = v_{AB} - \sum_C v_{AC} S_C t_{CB} - \sum_i v_{Ai} S_i t_{iB} \end{cases}$$

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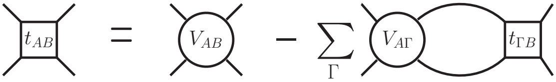


$$t_{AB} = \underbrace{v_{AB} - \sum_i v_{Ai} S_i v_{iB}}_{V_{AB}} - \sum_C \underbrace{\left(v_{AC} - \sum_i v_{Ai} S_i v_{iC} \right)}_{V_{AC}} S_C t_{CB}$$

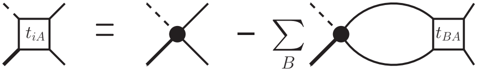
Parametrisation for near-threshold states

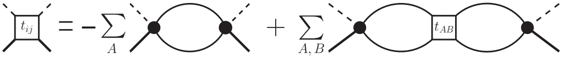
⇒ S_i the ith $(Q\bar{Q})(q\bar{q})$ channel, enters V additively

⇒ Arbitrary number of inelastic channels non-perturbatively

⇒ The dimension of LSE is from $N_e + N_p + N_{\rm in}$ to $N_e + N_p$

⇒ Other components of t matrix can be obtained algebraically



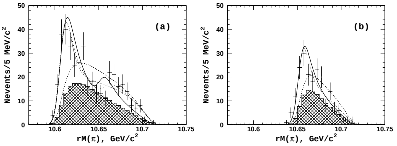


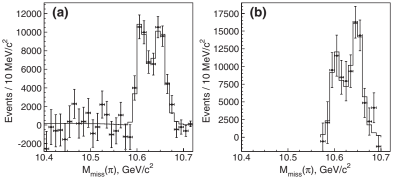
⇒ Satisfy unitarity and analyticity

Apply to the two Z_b cases

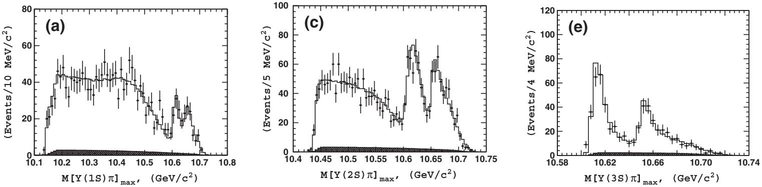
All the available data for the two Z_b states

$\Upsilon(5S) \to Z_b^{(\prime)\pm}\pi^\mp \to (B^{(*)}\bar{B}^*)\pm\pi^\mp$, $\Upsilon(5S) \to Z_b^{(\prime)\pm}\pi^\mp \to h_b(mP)\pi^\pm\pi^\mp$ with $m = 1, 2$





$\Upsilon(5S) \to Z_b^{(\prime)\pm}\pi^\mp \to \Upsilon(nS)\pi^\pm\pi^\mp$ with $n = 1, 2, 3$



In total: 7 channels, i.e. $B^{(*)}\bar{B}^*$, $h_b(mP)\pi$ and $\Upsilon(nS)\pi$

Belle, PRL108(2012)122001, PRL116(2016)212001

Heavy quark symmetry and light quark symmetry

- $m_Q \gg \Lambda_{QCD} \rightarrow$ physics at the m_Q scale is perturbative
- Heavy quark limit $\rightarrow$ spin symmetry & flavor symmetry

To the leading order,

$$\mathcal{L}_{QCD} = \bar{h}_v iv \cdot D h_v + \mathcal{O}(\Lambda_{QCD}/m_Q)$$

No Dirac matrix:

$\rightarrow$ spin symmetry (HQSS)

$\rightarrow s_Q$ and light degrees of freedom conserved individually

$\rightarrow$spin doublet: $s_l = \frac{1}{2}^-$ (B, B^*) with $m_{B^*} - m_B \sim \Lambda_{QCD}$

No heavy quark mass:

$\rightarrow$ flavor symmetry (HQFS)

★ $\langle HL|\hat{H}_I|H'L'\rangle \equiv V_{HL}\delta_{HH'}\delta_{LL'} \rightarrow V_{H'L} \overset{HQSS}{=} V_{HL} \overset{LQSS}{=} V_{HL'}$

M.B. Voloshin, PRD93(2016)074011

Apply to the two Z_b cases

The wave functions of $B\bar{B}^* + c.c.$ and $B^*\bar{B}^*$ with $J^{PC} = 1^{+-}$

$$
\begin{aligned}
|B\bar{B}^*\rangle &= -\frac{1}{\sqrt{2}}|1_H \otimes 0_L\rangle - \frac{1}{\sqrt{2}}|0_H \otimes 1_L\rangle \\
|B^*\bar{B}^*\rangle &= \frac{1}{\sqrt{2}}|1_H \otimes 0_L\rangle - \frac{1}{\sqrt{2}}|0_H \otimes 1_L\rangle
\end{aligned}
$$

A.E. Bondar et al., PRD84(2011)054010

Direct potential between elastic channels

$$V_0 \equiv \langle 1_H \otimes 0_L|\hat{H}_{\mathrm{I}}|1_H \otimes 0_L\rangle, \quad V_1 \equiv \langle 0_H \otimes 1_L|\hat{H}_{\mathrm{I}}|0_H \otimes 1_L\rangle$$

with redefined parameters

$${\gamma_t}^{-1} \equiv (2\pi)^2\mu V_0, \quad {\gamma_s}^{-1} \equiv (2\pi)^2\mu V_1$$

$$v = \frac{(2\pi)^2\mu}{2}\begin{pmatrix} {\gamma_s}^{-1} + {\gamma_t}^{-1} & {\gamma_s}^{-1} - {\gamma_t}^{-1} \\ {\gamma_s}^{-1} - {\gamma_t}^{-1} & {\gamma_s}^{-1} + {\gamma_t}^{-1} \end{pmatrix}$$

$\Rightarrow t^v \Rightarrow \Delta = \gamma_s\gamma_t - k_1k_2 + \frac{i}{2}(\gamma_s + \gamma_t)(k_1 + k_2)$

Apply to the two Z_b cases

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\end{aligned}
$$

A.E. Bondar et al., PRD84(2011)054010

$\Rightarrow$ Total spin of $b\bar{b}$, $s_{b\bar{b}}^{\Upsilon} = 1$, $s_{b\bar{b}}^{h_b} = 0$

$\Rightarrow$Potential between elastic channels and inelastic channels

$$
v^{ei} = \begin{pmatrix} g_{1P} & g_{2P} & g_{1S} & g_{2S} & g_{3S} \\ g_{1P}\xi_{1P} & g_{2P}\xi_{2P} & g_{1S}\xi_{1S} & g_{2S}\xi_{2S} & g_{3S}\xi_{3S} \end{pmatrix}
$$

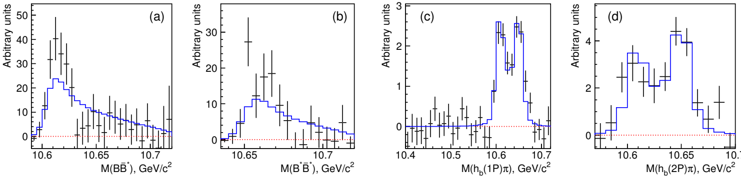
with $\xi_i \equiv g_{iB^*\bar{B}^*}/g_{iB\bar{B}^*}$. In HQSS, $\xi_{nS} = -1$ and $\xi_{mP} = 1$.

Apply to the two Z_b cases

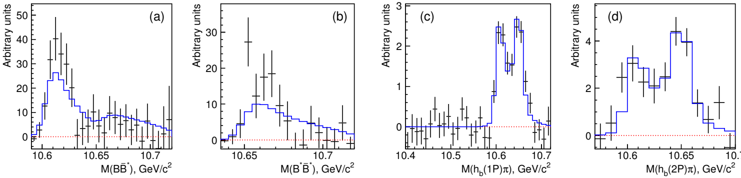
Hadronic molecular picture

HQSS limit $\Rightarrow \gamma_s \approx \gamma_t \Rightarrow$ light-quark spin symmetry (LQSS)

M.B. Voloshin, PRD93(2016)074011



HQSS breaking $\Rightarrow \gamma_s \neq \gamma_t \Rightarrow$ sizable LQSS breaking



Belle, PRD91(2015)072003, PRL108(2012)122001, PRL116(2016)212001

Apply to the two Z_b cases

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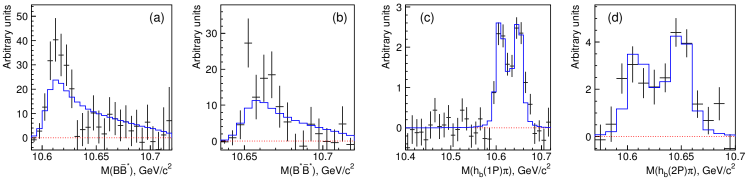
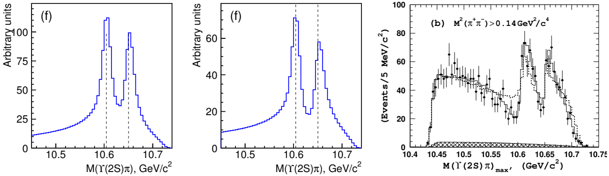


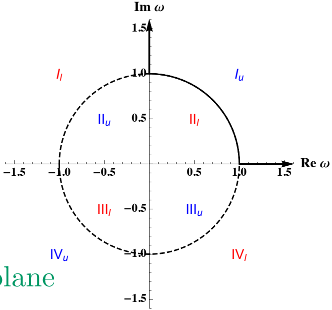
Can accommodate $\Upsilon(nS)\pi$ channels



Belle, PRD91(2015)072003, PRL108(2012)122001, PRL116(2016)212001

Apply to the two Z_b cases

Nature of Z_b and Z_b' (2^7 RS to 2^2 RS)



RS-I: $\text{Im } k_1 > 0, \quad \text{Im } k_2 > 0,$

RS-II: $\text{Im } k_1 < 0, \quad \text{Im } k_2 > 0,$

RS-III: $\text{Im } k_1 > 0, \quad \text{Im } k_2 < 0,$

RS-IV: $\text{Im } k_1 < 0, \quad \text{Im } k_2 < 0,$

Conformal mapping from k-plane to ω-plane

$$k_1 = \sqrt{\frac{\mu\delta}{2}}\left(\omega + \frac{1}{\omega}\right), \quad k_2 = \sqrt{\frac{\mu\delta}{2}}\left(\omega - \frac{1}{\omega}\right).$$

Energy relative to the $B\bar{B}^*$ threshold

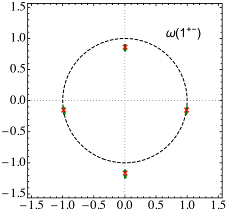
$$E = \frac{k_1^2}{2\mu} = \frac{k_2^2}{2\mu} + \delta = \frac{\delta}{4}\left(\omega^2 + \frac{1}{\omega^2} + 2\right)$$

with $\delta = m_{B^*} - m_B$.

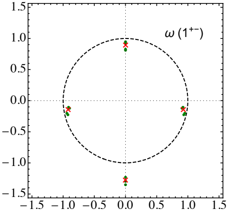
Apply to the two Z_b cases

Pole positions of Z_b and Z_b'

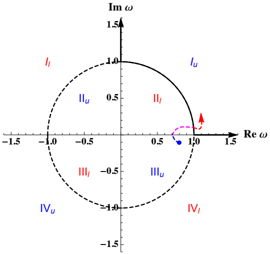
HQSS limit



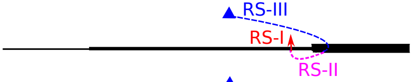
HQSS breaking



Path of the RS-III pole to RS-I



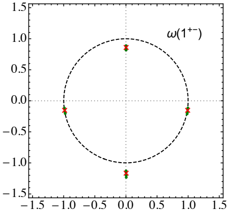
Energy plane



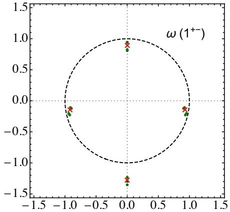
Apply to the two Z_b cases

Pole positions of Z_b and Z_b'

HQSS limit



HQSS breaking



Energies of Z_b and Z_b' (below the respective thresholds)

MeV	HQSS limit	HQSS breaking
$\varepsilon_B(Z_b)$	$1.10^{+0.79}_{-0.54} \pm i0.06^{+0.02}_{-0.02}$	$0.60^{+1.40}_{-0.49} \pm i0.02^{+0.02}_{-0.01}$
$\varepsilon_B(Z_b')$	$1.10^{+0.79}_{-0.53} \pm i0.08^{+0.03}_{-0.05}$	$0.97^{+1.42}_{-0.68} \pm i0.84^{+0.22}_{-0.34}$

Apply to the two Z_b cases

Z_b as a virtual state

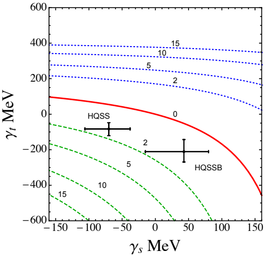
Determinant of t^v

$$\Delta = \gamma_s\gamma_t - k_1k_2 + \frac{i}{2}(\gamma_s + \gamma_t)(k_1 + k_2)$$

Bound state vs. Virtual state

$$\xrightarrow[k_2=\sqrt{-2\mu\delta}]{k_1=0} \quad \gamma_t = \left(\gamma_s^{-1} - \sqrt{2/(\mu\delta)}\right)^{-1}$$

Parameter space γ_s-γ_t



⇒ Inelastic channels do not change the virtual state Z_b

⇒ Indicate the hadronic molecular nature of Z_b

⇒ A similar conclusion holds for Z_b'

Summary

- Narrow near-threshold structure in elastic channels calls for a pole
- Practical parametrization for the line shape of near-threshold states compatible with all requirements of unitarity and analyticity
- Can include bare poles and an arbitrary number of elastic and inelastic channels nonperturbatively
- A good description of Z_b and Z_b' as virtual state and resonance, respectively

Thank you very much for your attention!

BackUp

Apply to the two Z_b cases

LSE for a given momentum-independent direct interaction

$$t^v_{\alpha\beta} = v_{\alpha\beta} - \sum_\gamma v_{\alpha\gamma} J_\gamma t^v_{\gamma\beta},$$

with loop integral $J_\gamma = R_\gamma + iI_\gamma$. The real part R can be absorbed into the renormalization of the direct potential

$$(t^v)^{-1} = v^{-1} + (R + iI) = v^{-1}_{\rm ren} + iI,$$

with $v_{\rm ren} = Z^{-1}v$ and $Z = 1 + vR$. The t matrix is

$$t^v = \frac{1}{(2\pi)^2\mu}\frac{1}{\Delta}\begin{pmatrix} \frac{1}{2}(\gamma_s + \gamma_t) + ik_2 & \frac{1}{2}(\gamma_t - \gamma_s) \\ \frac{1}{2}(\gamma_t - \gamma_s) & \frac{1}{2}(\gamma_s + \gamma_t) + ik_1 \end{pmatrix},$$

with

$$\Delta = \gamma_s\gamma_t - k_1k_2 + \frac{i}{2}(\gamma_s + \gamma_t)(k_1 + k_2).$$

Apply to the two Z_b cases

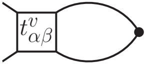
Switch on the $h_b(mP)\pi$ and $\Upsilon(nS)\pi$ channels

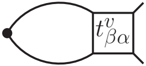
The t matrix is (separable interaction)

$$t = t^v + \psi[\mathcal{G} - G^{-1}]^{-1}\bar{\psi},$$

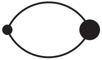
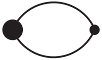
$\Rightarrow$ dressed incoming form factor $\psi_{\alpha\beta} = \delta_{\alpha\beta} - t^v_{\alpha\beta} J_\beta$

$\Rightarrow$ dressed outgoing form factor $\bar{\psi}_{\alpha\beta} = \delta_{\alpha\beta} - J_\alpha t^v_{\alpha\beta}$

$\psi_{\alpha\beta} =$  $-$ 

, $\bar{\psi}_{\alpha\beta} =$  $-$ 

$$\Rightarrow \mathcal{G}_{\alpha\beta} = J_\alpha \underbrace{(\delta_{\alpha\beta} - t^v_{\alpha\beta} J_\beta)}_{\psi_{\alpha\beta}} = \underbrace{(\delta_{\alpha\beta} - J_\alpha t^v_{\alpha\beta})}_{\bar{\psi}_{\alpha\beta}} J_\beta$$

$\mathcal{G}_{\alpha\beta} =$  $=$ 

Apply to the two Z_b cases

⇒ inelastic bubble loop reads as

$$
\begin{aligned}
G_{\alpha\beta} &= \sum_i \int \varphi_{i\alpha}(\boldsymbol{q}) S_i(\boldsymbol{q}) \varphi_{i\beta}(\boldsymbol{q}) d^3 q \\
&\to \frac{i(2\pi)^2}{\sqrt{s}} \sum_i m_{\mathrm{th}_i^{\mathrm{in}}} \mu_i^{\mathrm{in}} g_{i\alpha} g_{i\beta} (k_i^{\mathrm{in}})^{2l_i+1}, \\
&= \sum_i
\end{aligned}
$$

The production amplitudes

$$
\begin{aligned}
\mathcal{M}_\alpha^{\mathrm{e}}(\boldsymbol{p}) &= \mathcal{F}_\alpha(\boldsymbol{p}) - \sum_\beta \int \mathcal{F}_\beta(\boldsymbol{q}) S_\beta(\boldsymbol{q}) t_{\beta\alpha}(\boldsymbol{q}, \boldsymbol{p}) d^3 q, \\
\mathcal{M}_i^{\mathrm{in}}(\boldsymbol{k}) &= -\sum_\alpha \int \mathcal{F}_\alpha(\boldsymbol{q}) S_\alpha(\boldsymbol{q}) t_{\alpha i}(\boldsymbol{q}, \boldsymbol{k}) d^3 q
\end{aligned}
$$

⇒ Elastic bare production amplitude

⇒ Interaction between spectator and other particles is neglected