Lepton flavour violation in semileptonic tau decays

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Work in collaboration with
Vincenzo Cirigliano and **Emilie Passemar**.
I use here some of their good slide material

Unterstützt von / Supported by
Alexander von Humboldt
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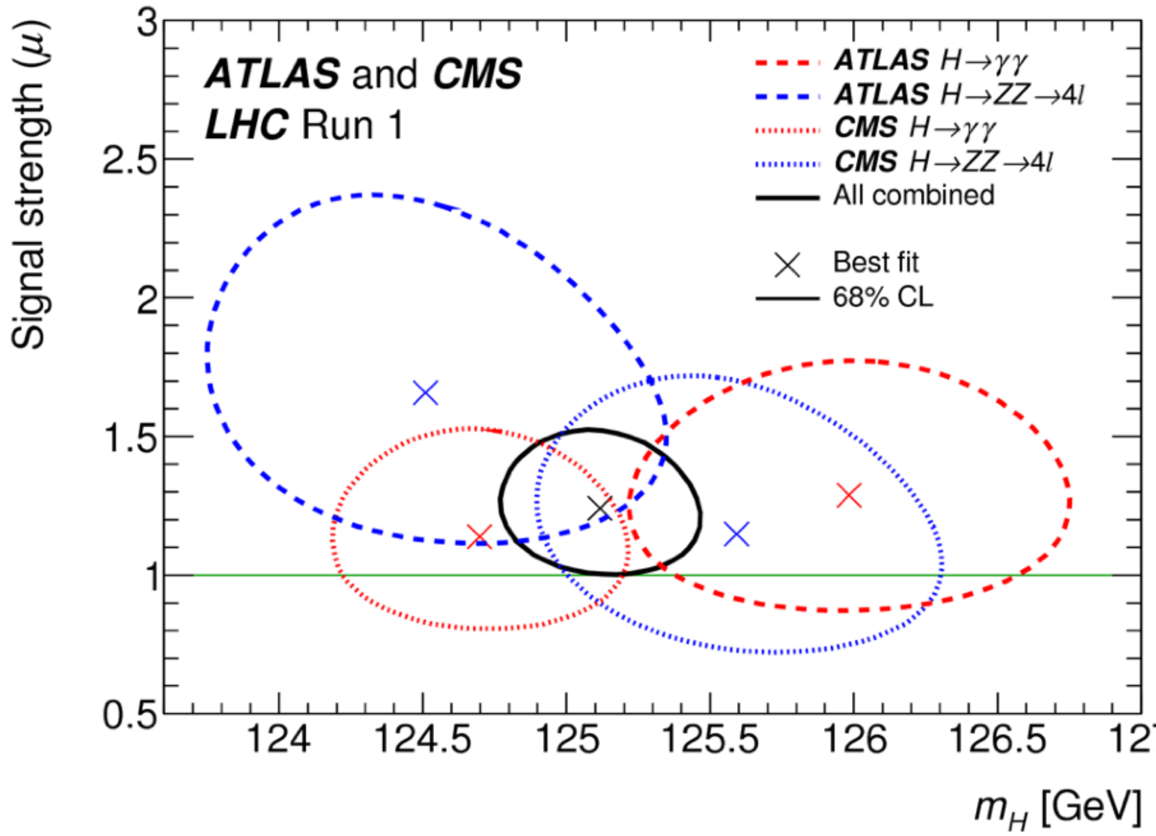
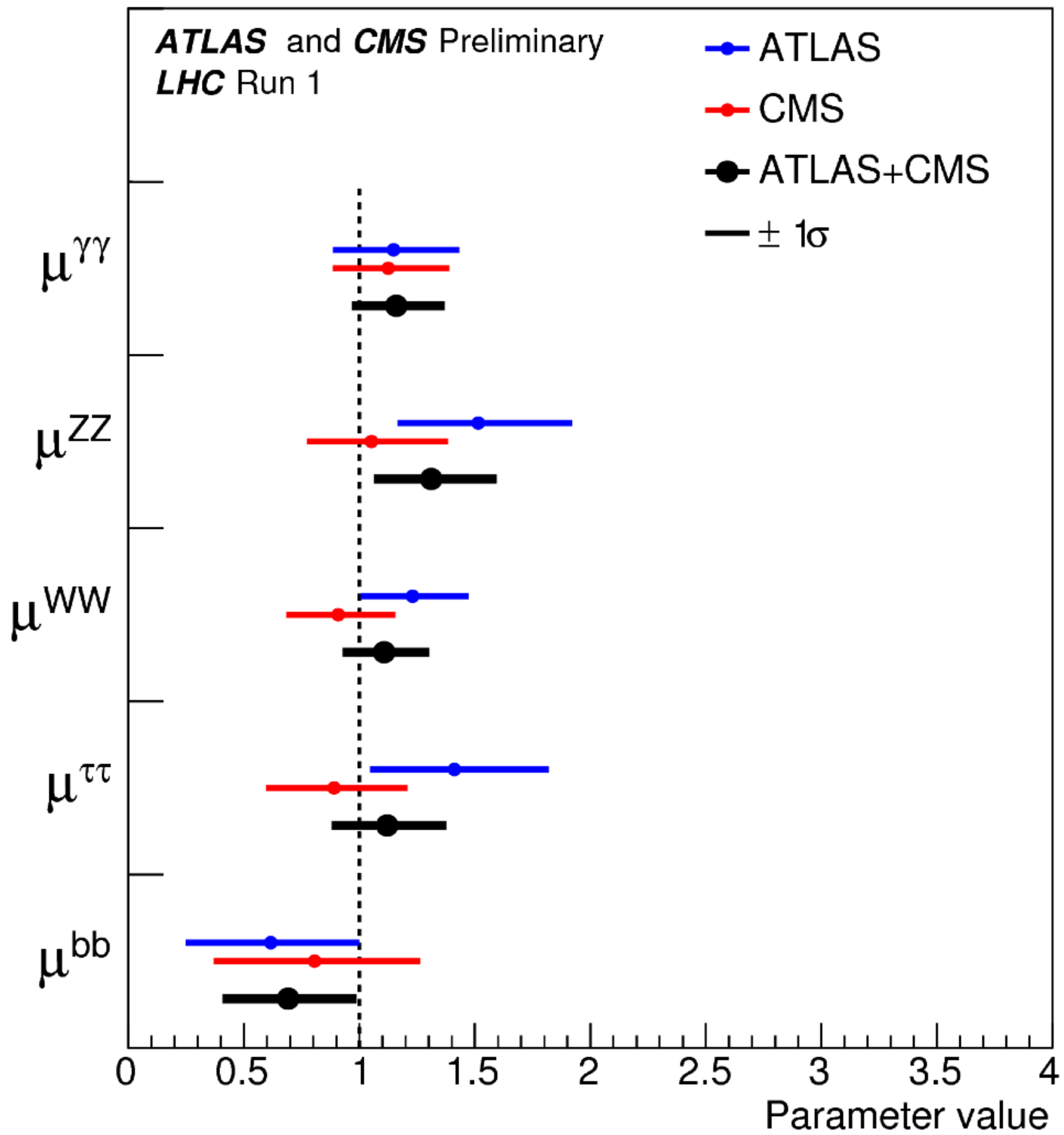
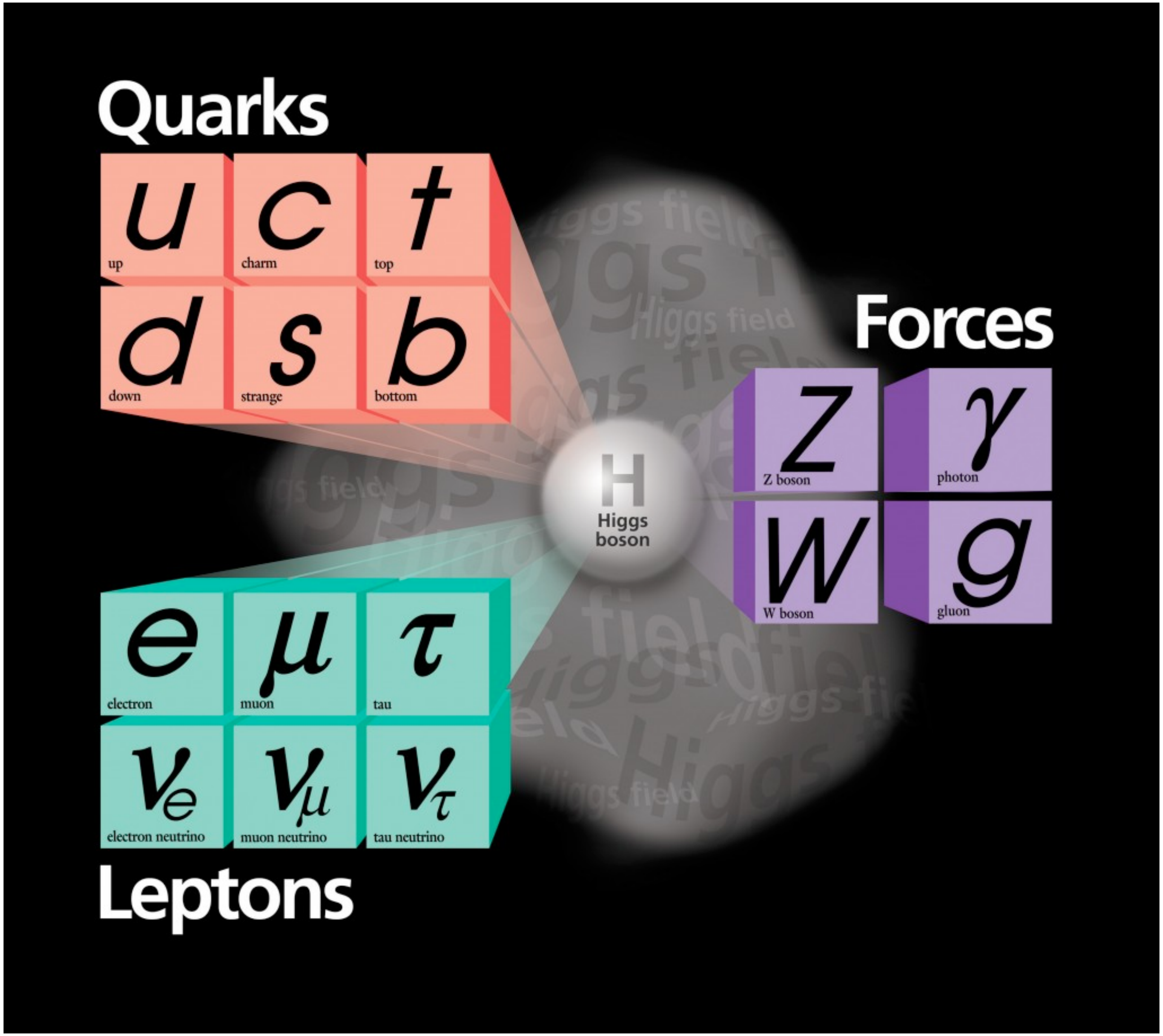
The Higgs discovery
and the success of the Standard Model

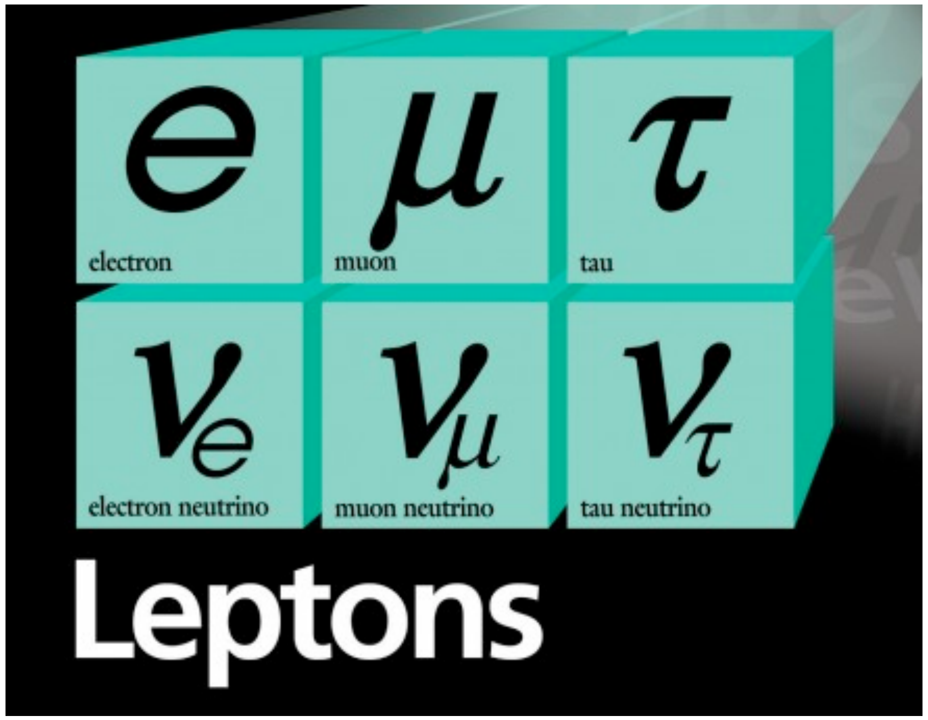




The Standard Model (SM)



Lepton Flavour (LFV) Conservation

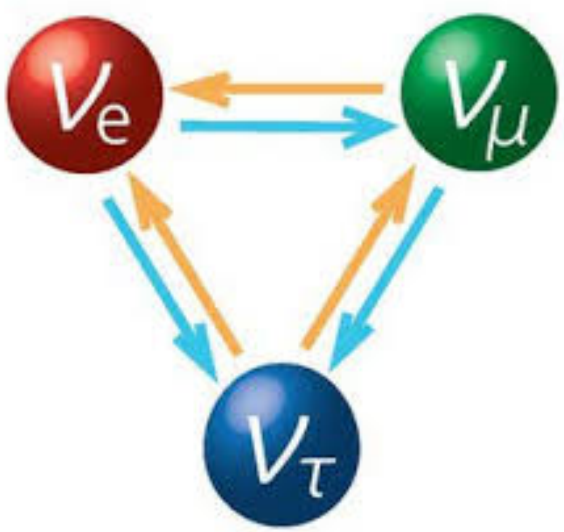


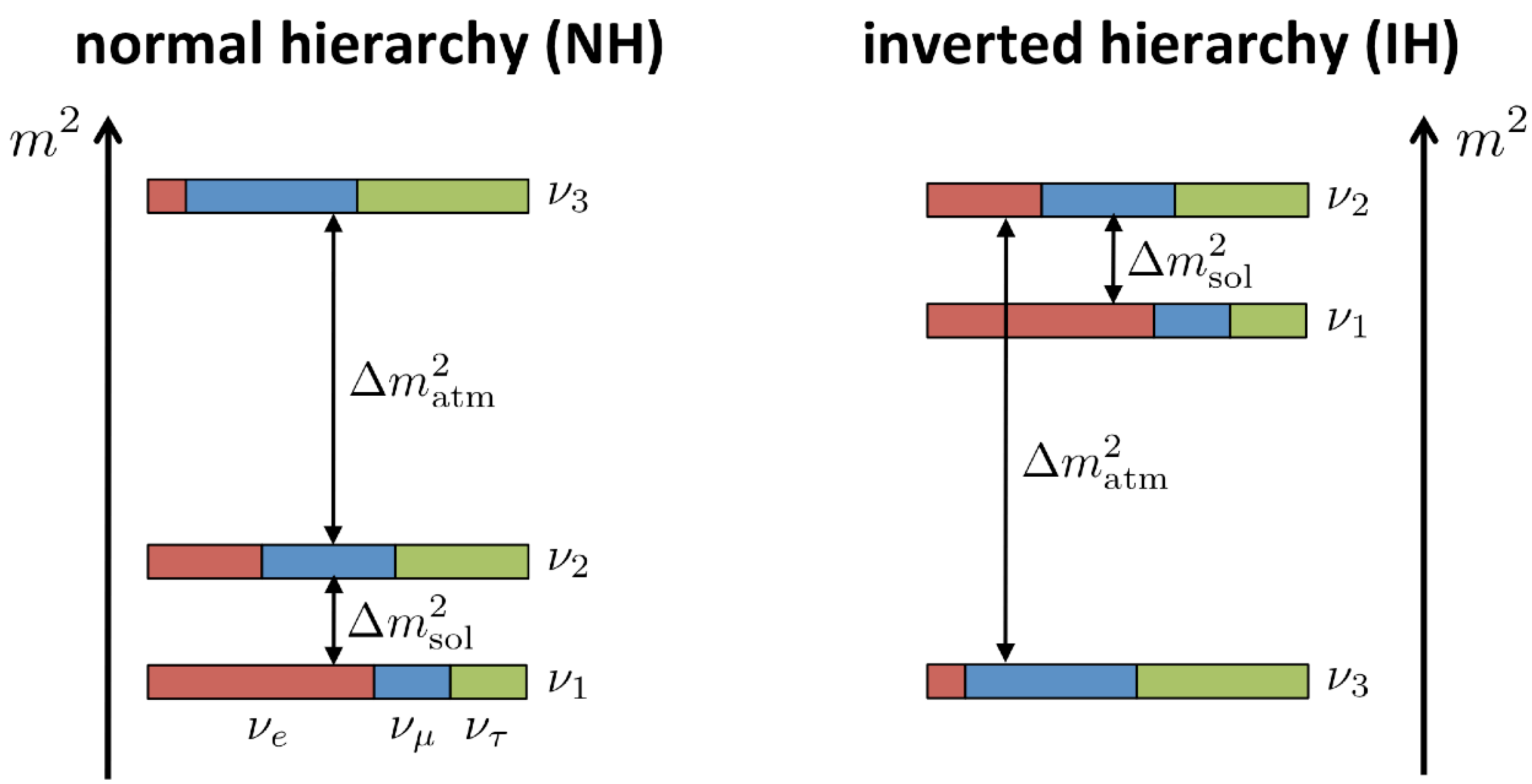
Lepton Flavour Number is an accidental global symmetry of the Standard Model

$$\begin{array}{cccccccc} \pi^+ & \rightarrow & \mu^+ & + & \nu_\mu & & \\ & & \downarrow & & & & \\ & & e^+ & + & \nu_e & + & \bar{\nu}_\mu \end{array}$$

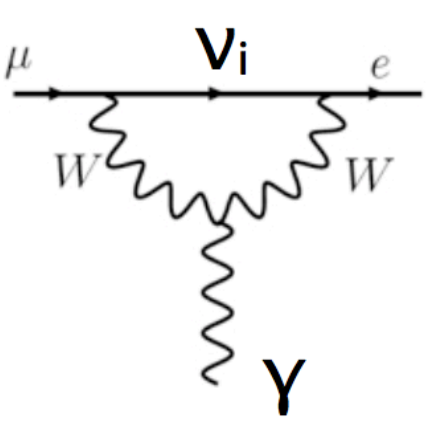
Charged lepton transitions observed so far conserve Lepton Flavour Number

Neutrino masses and Lepton Flavour Violation (LFV)





Lepton Flavour Violation very suppressed in the SM



$$Br(\mu \to e\gamma) = \frac{3\alpha}{32\pi} \left| \sum_{i=2,3} U^*_{\mu i} U_{ei} \frac{\Delta m^2_{1i}}{M^2_W} \right|^2 < 10^{-54}$$

Petcov '77, Marciano-Sanda '77 ....

Bounds on LFV with muons

mode	limit (90% C.L.)	year	Exp./Lab.
$\mu^+ \to e^+\gamma$	1.2×10^{-11}	2002	MEGA / LAMPF
$\mu^+ \to e^+e^+e^-$	1.0×10^{-12}	1988	SINDRUM I / PSI
$\mu^+e^- \leftrightarrow \mu^-e^+$	8.3×10^{-11}	1999	PSI
μ^- Ti $\to e^-$Ti	6.1×10^{-13}	1998	SINDRUM II / PSI
μ^- Ti $\to e^+$Ca*	3.6×10^{-11}	1998	SINDRUM II / PSI
μ^- Pb $\to e^-$Pb	4.6×10^{-11}	1996	SINDRUM II / PSI
μ^- Au $\to e^-$Au	7×10^{-13}	2006	SINDRUM II / PSI

Bounds on LFV tau decays

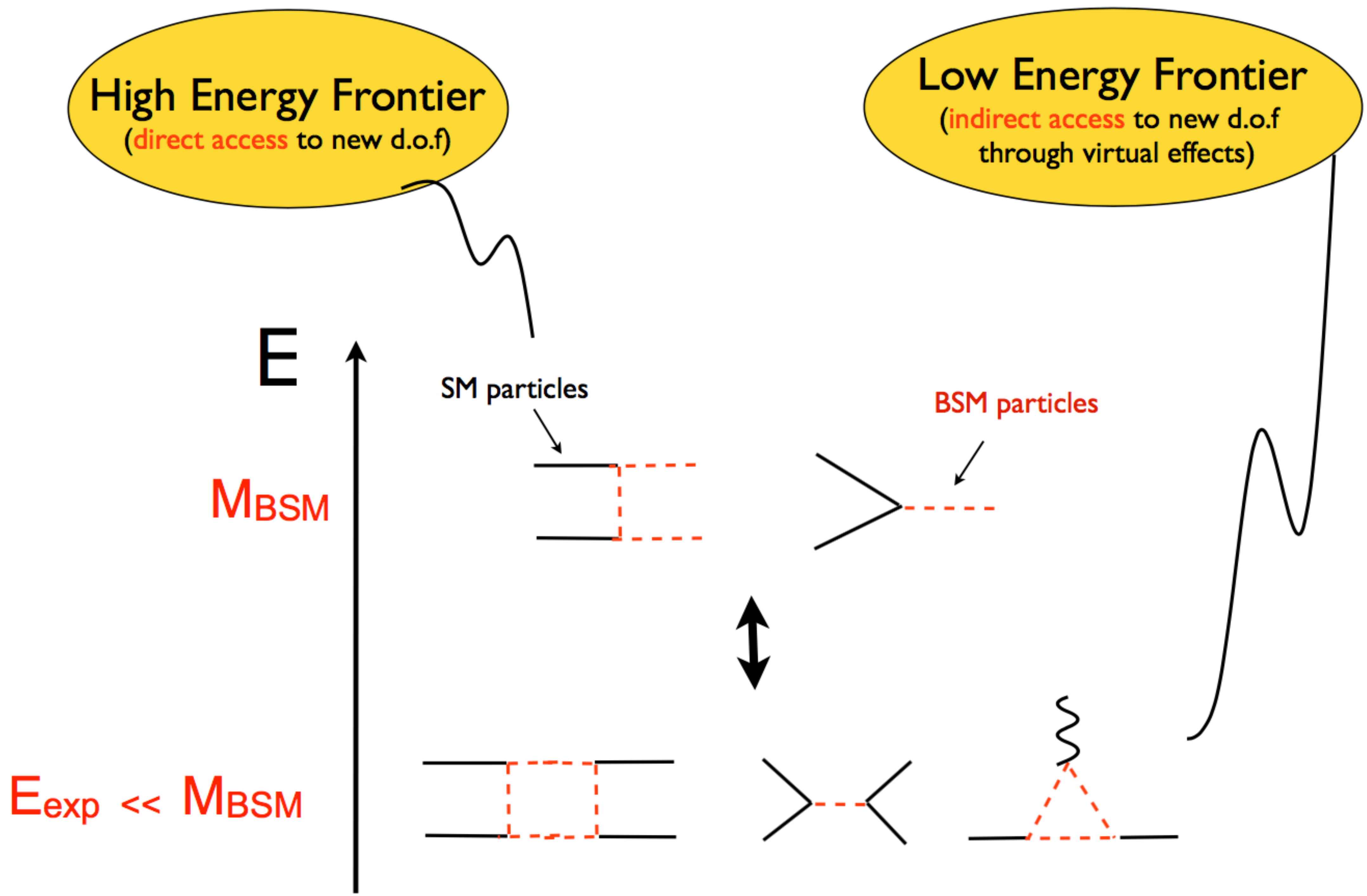
- Experimental status: taus (90% BR limits from PDG)

$e^- \gamma$	LF	< 1.1	$\times 10^{-7}$
$\mu^- \gamma$	LF	< 4.5	$\times 10^{-8}$
$e^- e^+ e^-$	LF	< 3.6	$\times 10^{-8}$
$e^- \mu^+ \mu^-$	LF	< 3.7	$\times 10^{-8}$
$e^+ \mu^- \mu^-$	LF	< 2.3	$\times 10^{-8}$
$\mu^- e^+ e^-$	LF	< 2.7	$\times 10^{-8}$
$\mu^+ e^- e^-$	LF	< 2.0	$\times 10^{-8}$
$\mu^- \mu^+ \mu^-$	LF	< 3.2	$\times 10^{-8}$
$e^- \pi^0$	LF	< 8.0	$\times 10^{-8}$
$\mu^- \pi^0$	LF	< 1.1	$\times 10^{-7}$
$e^- K_S^0$	LF	< 3.3	$\times 10^{-8}$
$\mu^- K_S^0$	LF	< 4.0	$\times 10^{-8}$
$e^- \eta$	LF	< 9.2	$\times 10^{-8}$
$\mu^- \eta$	LF	< 6.5	$\times 10^{-8}$

...

High- and Low-Energy Frontiers

- Two *complementary* strategies to probe BSM physics:



from V. Cirigliano

Weinberg (1979)

Buchmüller, Wyler (1986)
Grzadkowski et al. (2010)

$$\mathcal{L}_{\mathrm{SM}} = \mathcal{L}_{\mathrm{SM}}^{(4)} + \frac{1}{\Lambda}\sum_k C_k^{(5)} Q_k^{(5)} + \frac{1}{\Lambda^2}\sum_k C_k^{(6)} Q_k^{(6)} + \mathcal{O}\left(\frac{1}{\Lambda^3}\right)$$

At dimension six

59 independent operators

(assuming baryon number conservation & barring flavour)

Grzadkowski et al. (2010)

The full one-loop anomalous dimension matrix
for the dimension six basis is already known

Trott et al. (2013)

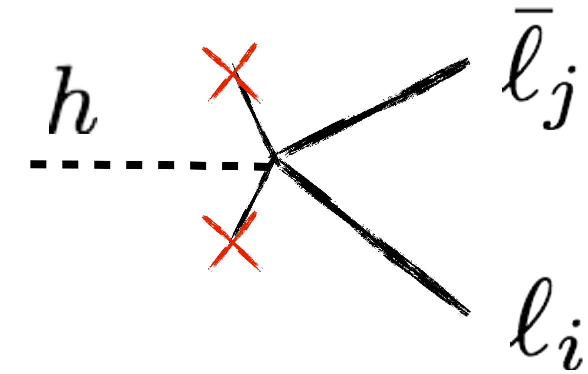
Standard Model Effective Theory

$$\langle 0|\varphi^0|0\rangle = \frac{(v+h)}{\sqrt{2}}$$

$(\varphi^\dagger\varphi)\,(\bar{\ell}e\varphi)$	$h \to l_1\bar{l}_2$
$(\bar{\ell}\gamma_\mu\ell)\,(\bar{\ell}\gamma^\mu\ell)$	$\ell \to \ell_1\bar{\ell}_2\ell_3$
$(\bar{\ell}\gamma_\mu\ell)\,(\bar{q}\gamma^\mu q)$ $(\bar{\ell}\gamma_\mu\tau^I\ell)\,(\bar{q}\gamma^\mu\tau^I q)$	$\ell \to \ell' q\bar{q}$
$\left(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi\right)(\bar{\ell}\gamma^\mu\ell)$ $\left(\varphi^\dagger i\overleftrightarrow{D}^I_\mu\varphi\right)(\bar{\ell}\tau^I\gamma^\mu\ell)$ $\left(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi\right)(\bar{e}\gamma^\mu e)$	$Z \to \ell_1\bar{\ell}_2$ $W \to \ell_1\bar{\nu}_2$
$(\bar{\ell}\sigma^{\mu\nu}e)\,\tau^I\varphi W^I_{\mu\nu}$ $(\bar{\ell}\sigma^{\mu\nu}e)\,\varphi B_{\mu\nu}$	$\ell \to \ell'\gamma$

Lepton Flavour Violating Higgs Couplings

$(\varphi^\dagger\varphi)\,(\bar{\ell}e\varphi)$ $\qquad h \to l_1\bar{l}_2$

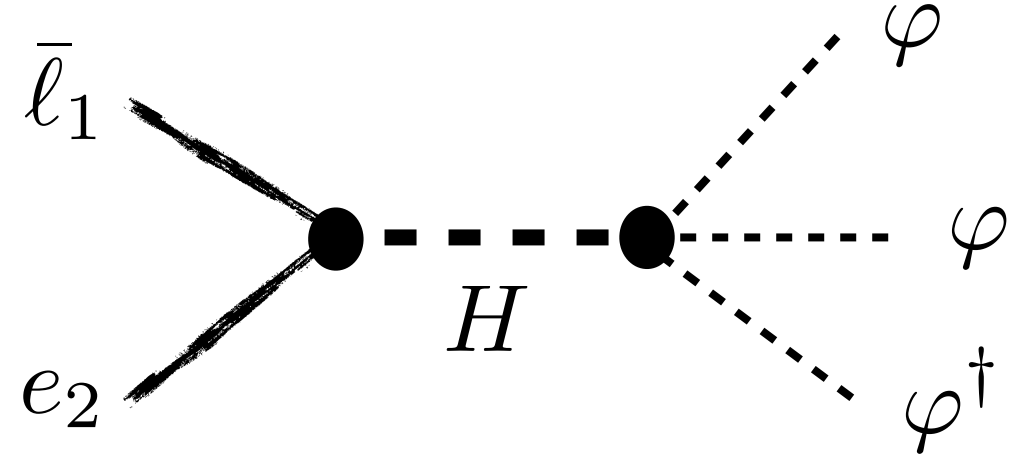


UV examples

Two-Higgs Doublet Model

$$\mathcal{L} \supset H\bar{\ell}_1 e_2 + \lambda(\varphi^\dagger H)(\varphi^\dagger\varphi)$$

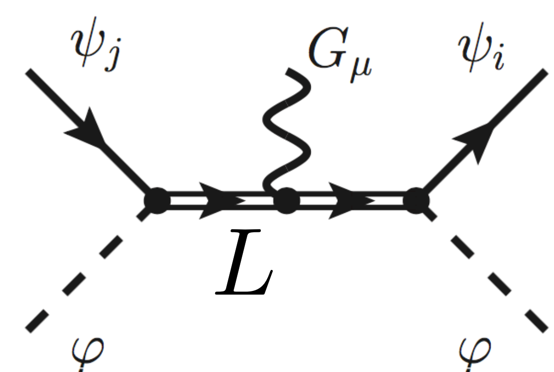
$$M_H \gg M_\varphi$$



Heavy Vector Like Lepton

$$\mathcal{L} \supset \varphi\bar{L}e$$

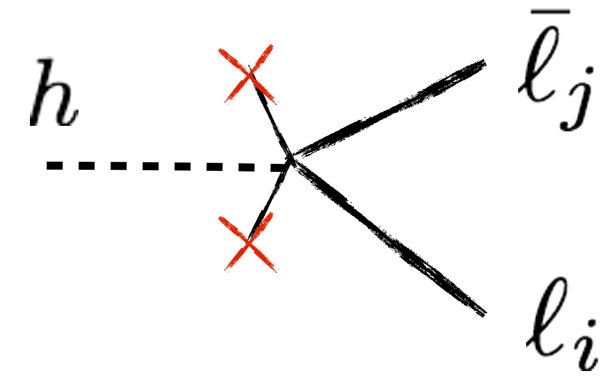
$$M_L \gg M_W$$



+ e.o.m

Lepton Flavour Violating Higgs Couplings

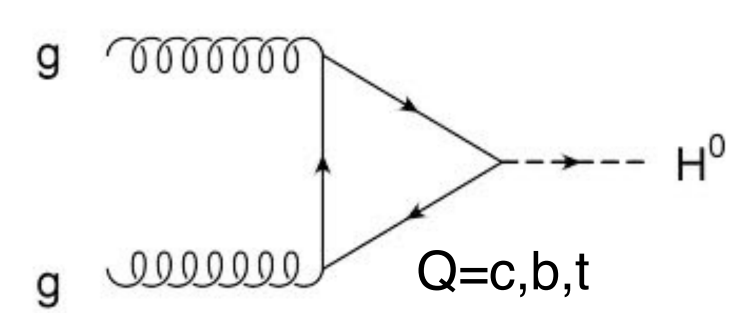
$$(\varphi^\dagger\varphi)\,(\bar{\ell}e\varphi) \longrightarrow$$



$$\mathcal{L}_Y \supset -h\left\{Y^h_{\tau\mu}\,(\bar{\tau}_L\,\mu_R) + Y^h_{\mu\tau}\,(\bar{\mu}_L\,\tau_R) + \text{h.c.}\right\}$$

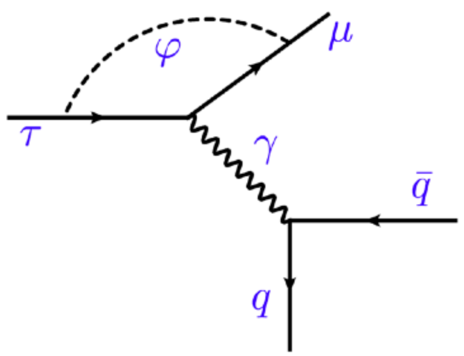
$$\mathcal{L}^h_{eff} \simeq -\frac{h}{v}\left(\sum_{q=u,d,s} y^h_q\, m_q\,\bar{q}\,q - \sum_{q=c,b,t}\frac{\alpha_s}{12\pi}\,y^h_q\,G^a_{\mu\nu}G^{\mu\nu}_a\right)$$

Integrating out heavy quarks

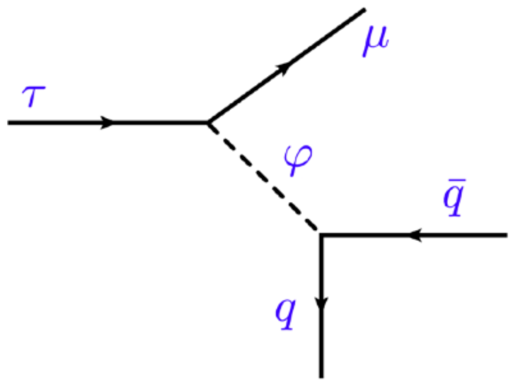


Lepton Flavour Violating Higgs Couplings

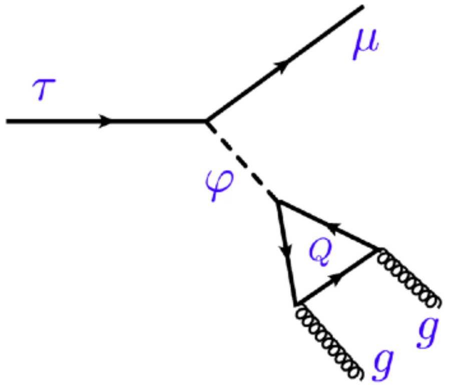
Consider the following operators at the low scale



$$(\bar{\mu}\sigma^{\rho\nu}P_{L,R}\tau)F_{\rho\nu}$$ **Dipole operators**



$$(\bar{\mu}P_{L,R}\tau)(\bar{q}q)$$ **Scalar operators**

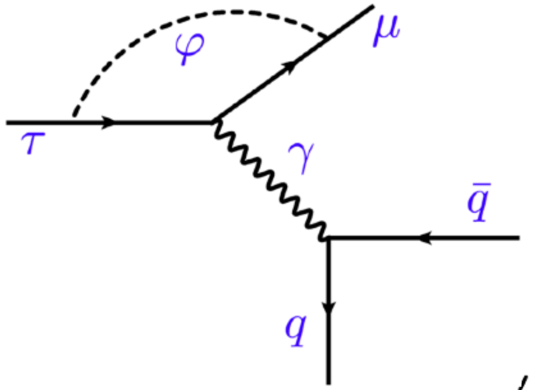


$$(\bar{\mu}P_{L,R}\tau)G^{a}_{\rho\nu}G^{\rho\nu}_{a}$$ **Gluonic operators**

$$\tau \to \mu \pi^+ \pi^-$$

Need a proper description of the hadronic matrix elements up to invariant masses of the pion pair of ~1 GeV

$$s = (p_{\pi^+} + p_{\pi^-})^2 \qquad \sqrt{s} \leq m_\tau - m_\mu$$



Photon mediated contribution requires the pion vector form factor

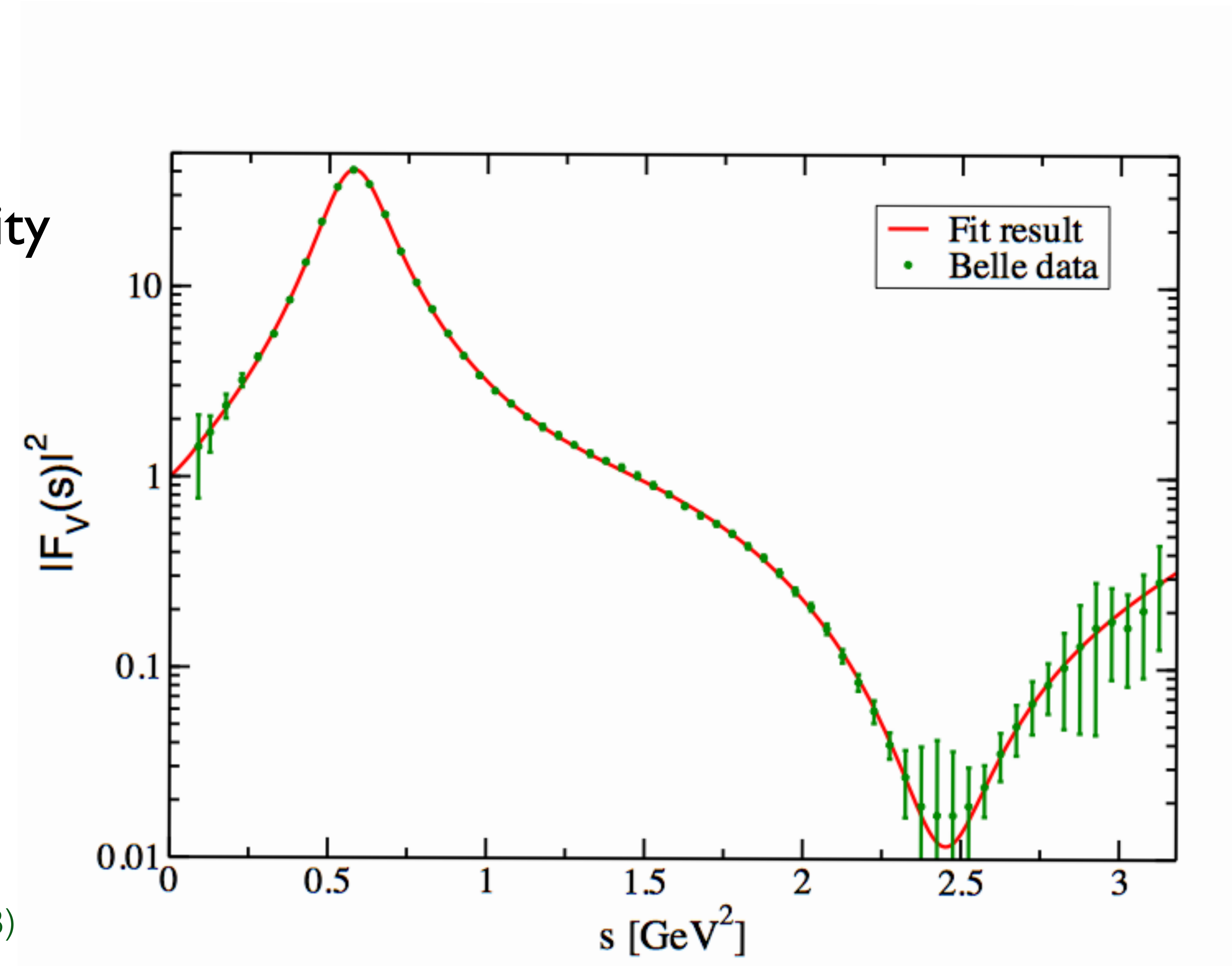
$$\left\langle \pi^+(p_{\pi^+})\pi^-(p_{\pi^-}) \middle| \tfrac{1}{2}(\bar{u}\gamma^\alpha u - \bar{d}\gamma^\alpha d) \middle| 0 \right\rangle \equiv F_V(s)(p_{\pi^+} - p_{\pi^-})^\alpha$$

Dispersive parametrization following the properties of analyticity and unitarity of the Form Factor

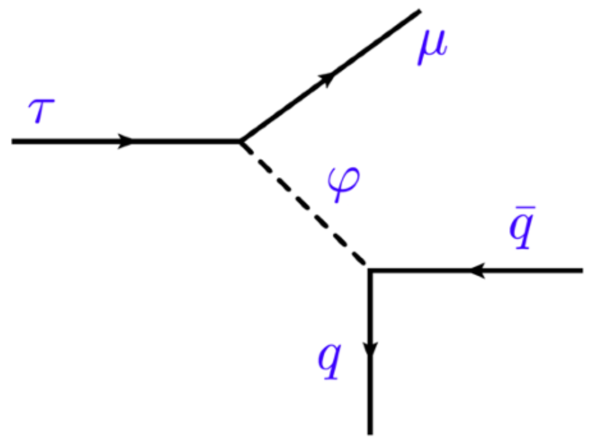
Gasser, Meißner ´91
Guerrero, Pich ´97,
Oller, Oset, Palomar ´01
Pich, Portolés ´08,
…

Determined from a fit to the $\tau^- \to \pi^0\pi^-\nu_\tau$ Belle data

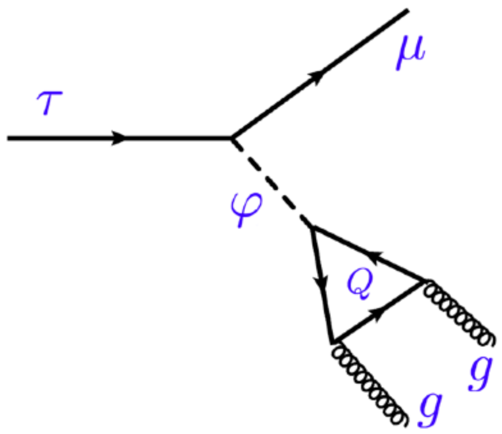
Belle Collaboration, Phys.Rev. D78, 072006 (2008)



AC, Cirigliano, Passemar (1309.3564)



$(\bar{\mu} P_{L,R} \tau)(\bar{q} q)$ Scalar operators



$(\bar{\mu} P_{L,R} \tau) G^{a}_{\rho\nu} G^{\rho\nu}_{a}$ Gluonic operators

It is possible to obtain a reliable estimate of the form factors with dispersion relations matched at low energy to ChPT

J. T. Daub et al. (1212.4408)

AC, Cirigliano, Passemar (1309.3564)

These hadronic matrix elements have been determined in in the context of very light Higgs decays $H \to \pi\pi$

Donoghue, Gasser, Leutwyler (1990)

$$\langle \pi^+(p_{\pi^+})\pi^-(p_{\pi^-}) | m_u \bar{u}u + m_d \bar{d}d | 0 \rangle \equiv \Gamma_\pi(s)$$

$$\langle \pi^+(p_{\pi^+})\pi^-(p_{\pi^-}) | m_s \bar{s}s | 0 \rangle \equiv \Delta_\pi(s)$$

$$\langle \pi^+(p_{\pi^+})\pi^-(p_{\pi^-}) | \theta^\mu_\mu | 0 \rangle \equiv \theta_\pi(s)$$

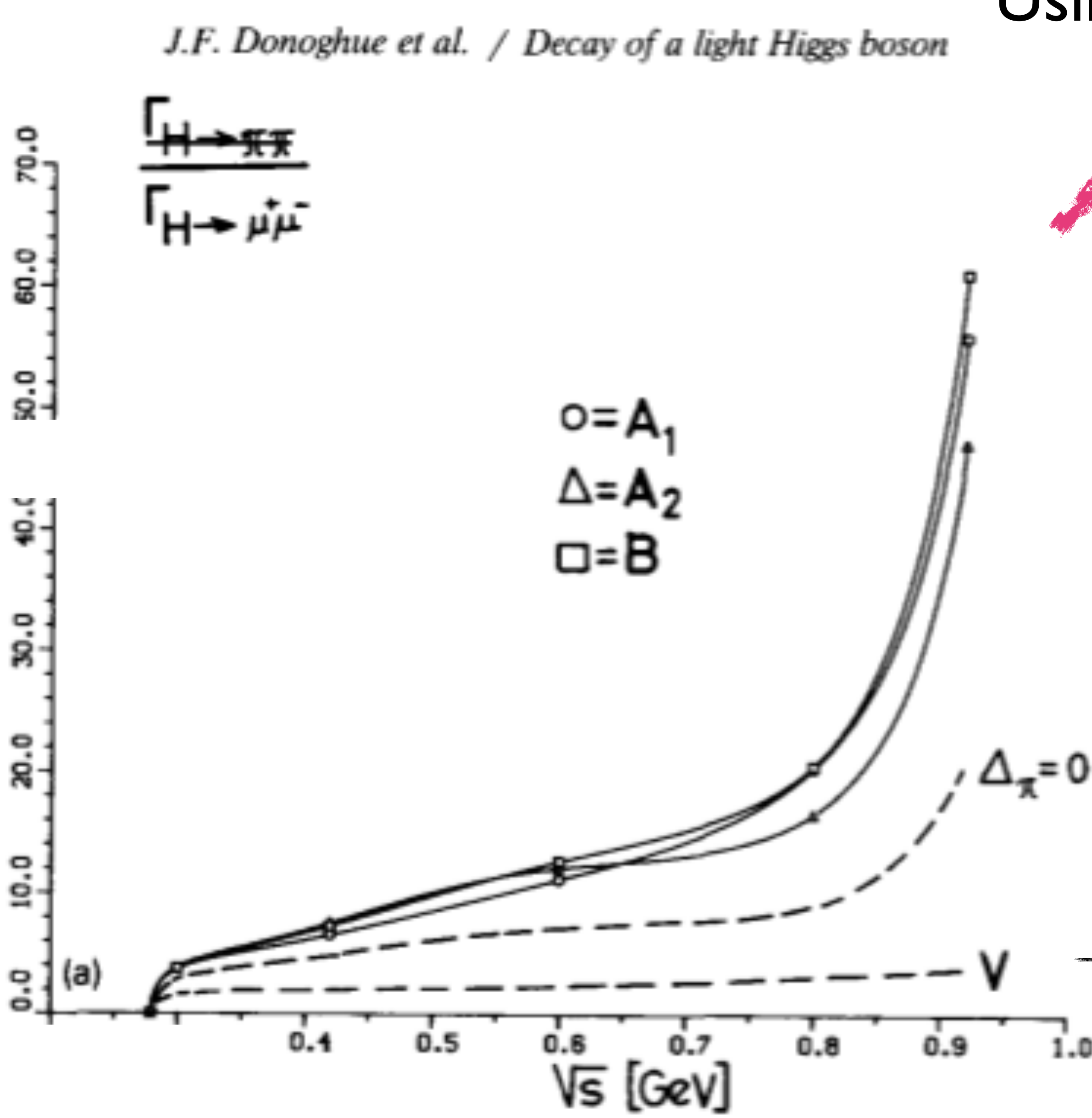
$$\theta^\mu_\mu = -9\frac{\alpha_s}{8\pi} G^a_{\mu\nu} G^{\mu\nu}_a + \sum_{q=u,d,s} m_q \bar{q}q$$

Using LO ChPT

$$\Gamma_\pi(s) = m_\pi^2 \qquad \theta_\pi(s) = s + 2m_\pi^2 + \mathcal{O}(p^4)$$

Voloshin (1985)

extracted from Donoghue, Gasser, Leutwyler (1990)



Using the triple constraints of chiral symmetry, analyticity, and unitarity, together with exp. input from pion scattering

very far from the naive expectation

$$\frac{\Gamma(h \to \mu^+\mu^-)}{\Gamma(h \to \pi^+\pi^-)} \sim \frac{m_\mu^2}{m_\pi^2}$$

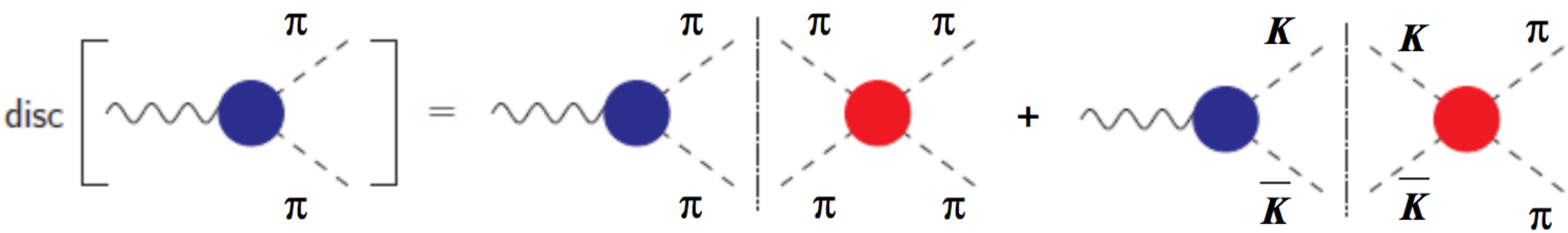
→ Voloshin (1985)

The elastic approximation breaks down for the $\pi\pi$ S-wave at the $K\bar{K}$ threshold due to the strong inelastic coupling involved in the region of $f_0(980)$

need to solve a two-channel Muskhelishvili-Omnès problem

Donoghue, Gasser, Leutwyler (1990)
Osset, Oller (1998)
B. Moussallam (1999)

- Unitarity ⇒ the discontinuity of the form factor is known



$$\mathrm{Im}F_n(s) = \sum_{m=1}^{2} T^*_{nm}(s)\sigma_m(s)F_m(s)$$

$n = \pi\pi, K\bar{K}$

Scattering matrix:

$$\begin{pmatrix} \pi\pi \to \pi\pi, & \pi\pi \to K\bar{K} \\ K\bar{K} \to \pi\pi, & K\bar{K} \to K\bar{K} \end{pmatrix}$$

from E. Passemar

Reconstruct the relevant T-matrix elements

using

$\delta_\pi(s)$

$\delta_K(s)$ **S-wave phase shifts**

$\eta = \cos\gamma$ **inelasticity**

determined from $\pi\pi$ and $K\pi$ scattering data

Buettiker, Descotes-Genon, Moussallam (2004)

above $s_{\rm cut} \lesssim m_\tau$ the T matrix is driven to zero consistently with unitarity

$$s_{\rm cut} \sim (1.4\ {\rm GeV})^2 - (1.8\ {\rm GeV})^2$$

General solution to *Mushkhelishvili-Omnès* problem:

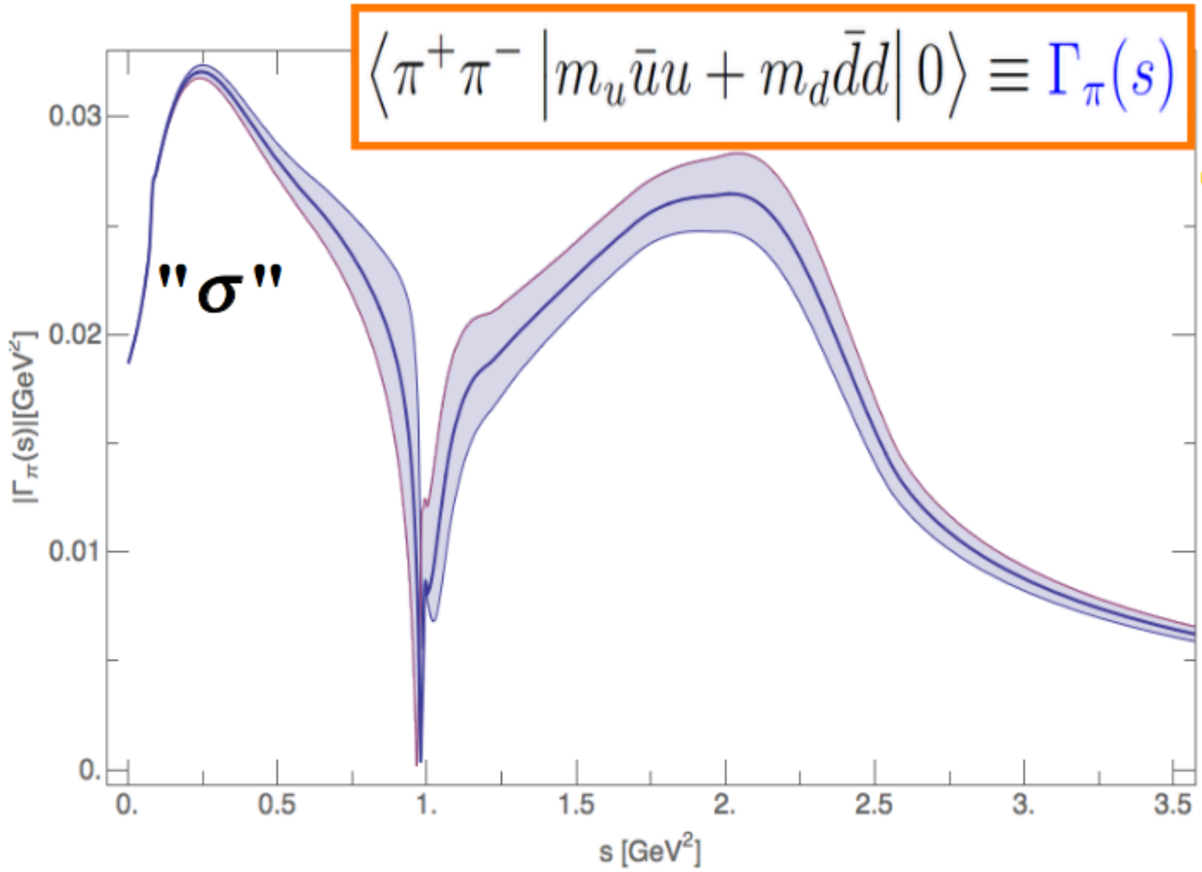
$$\begin{pmatrix} F_\pi(s) \\ \frac{2}{\sqrt{3}} F_K(s) \end{pmatrix} = \begin{pmatrix} C_1(s) & D_1(s) \\ C_2(s) & D_2(s) \end{pmatrix} \begin{pmatrix} P_F(s) \\ Q_F(s) \end{pmatrix}$$

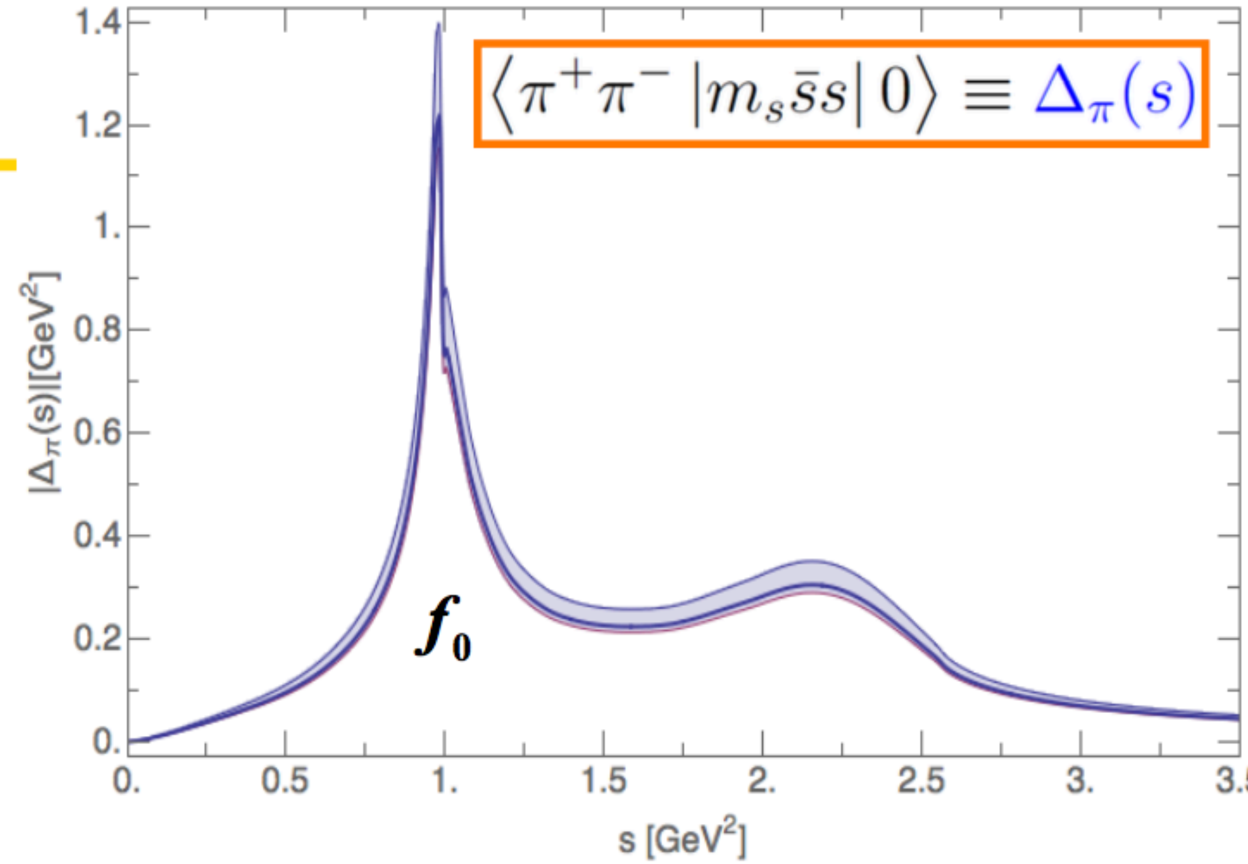
Canonical solution

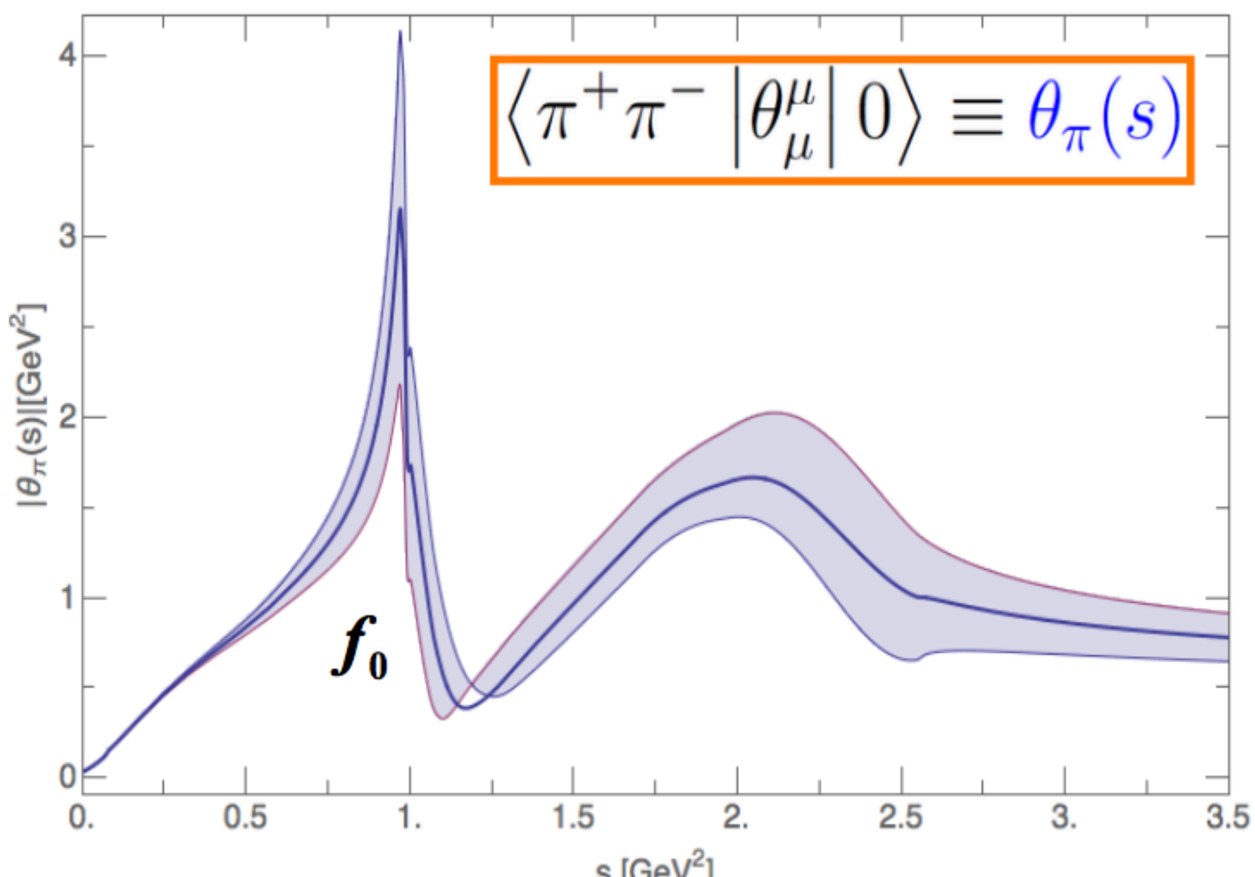
Polynomial determined from a matching to ChPT + lattice

Canonical solution is found by solving dispersive integral equations iteratively starting with Omnès functions that are solutions of the one-channel unitary condition

$$\Omega_{\pi,K}(s) \equiv \exp\left[\frac{s}{\pi}\int_{4M_\pi^2}^{\infty} \frac{dt}{t}\frac{\delta_{\pi,K}(t)}{(t-s)}\right]$$



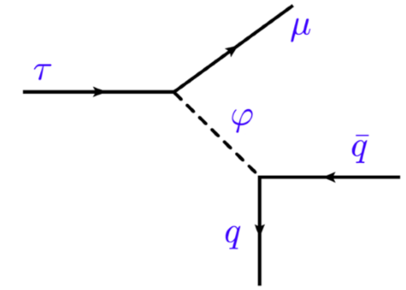


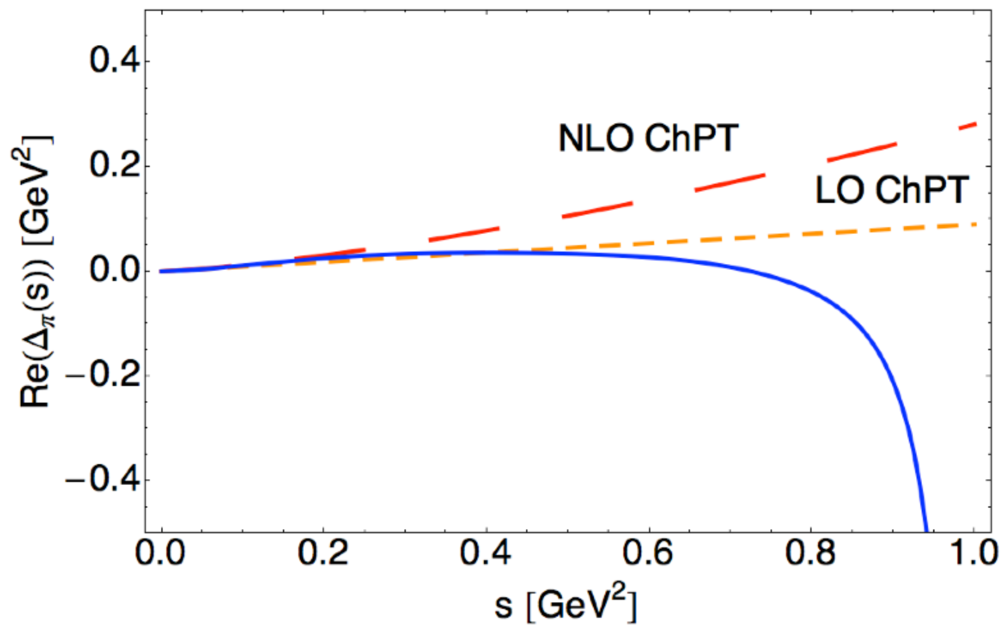


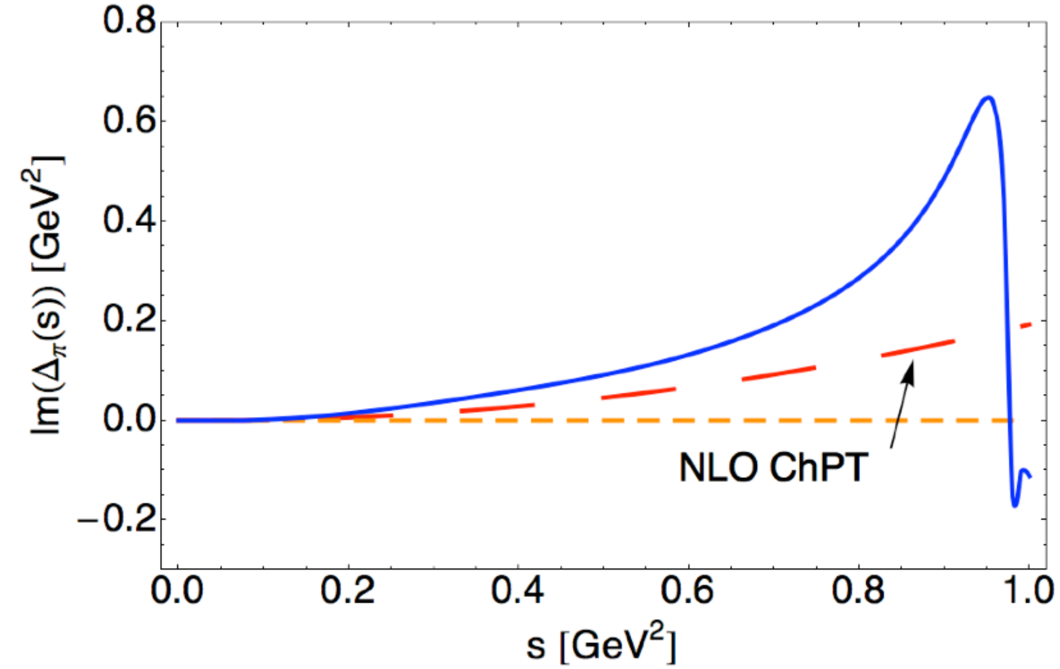
see also J. T. Daub et al. (1212.4408)

- Uncertainties:
  - Varying s_{cut} (1.4 GeV^2 -1.8 GeV^2)
  - Varying the matching conditions
  - T matrix inputs

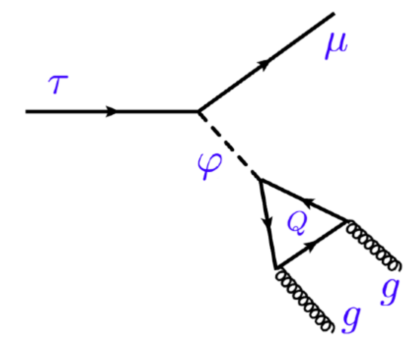
$$\langle \pi^+(p_{\pi^+})\pi^-(p_{\pi^-})|m_s\bar{s}s|0\rangle \equiv \Delta_\pi(s)$$

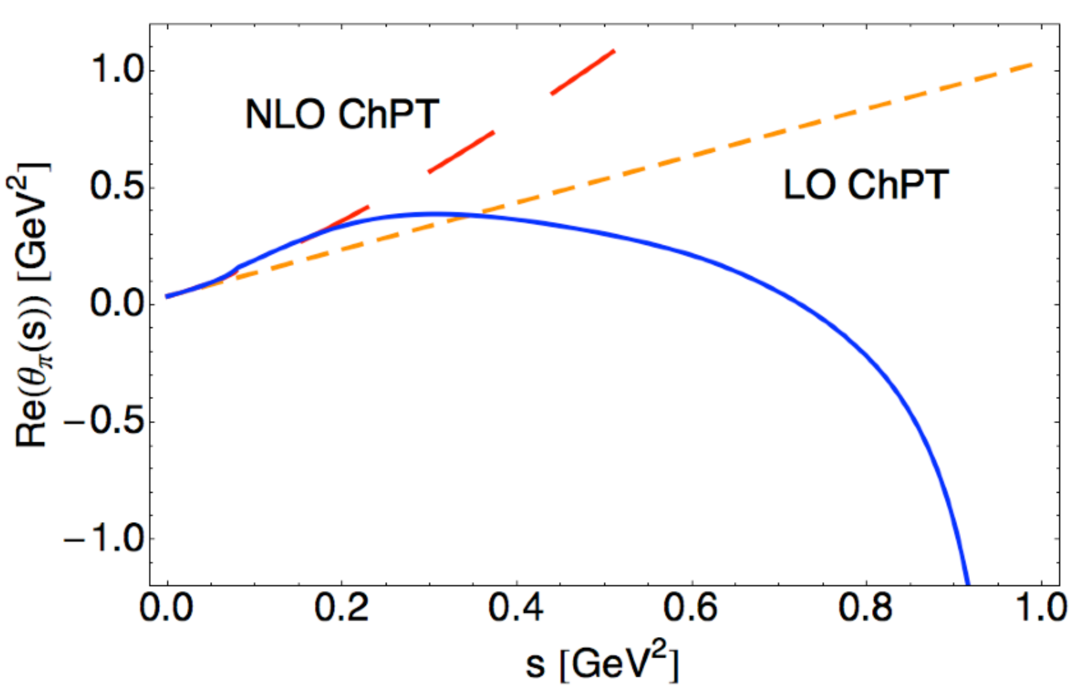


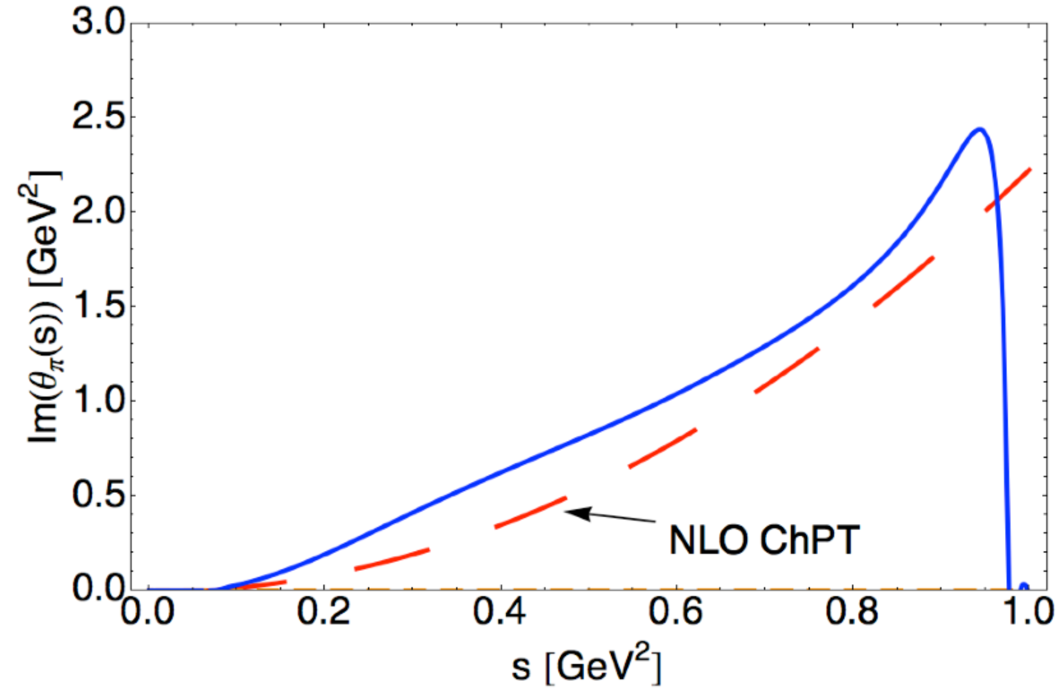




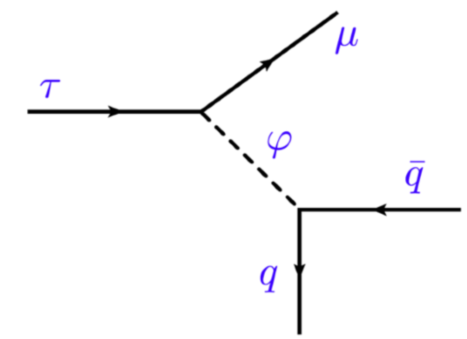
$$\langle \pi^+(p_{\pi^+})\pi^-(p_{\pi^-})|\theta^\mu_\mu|0\rangle \equiv \theta_\pi(s)$$

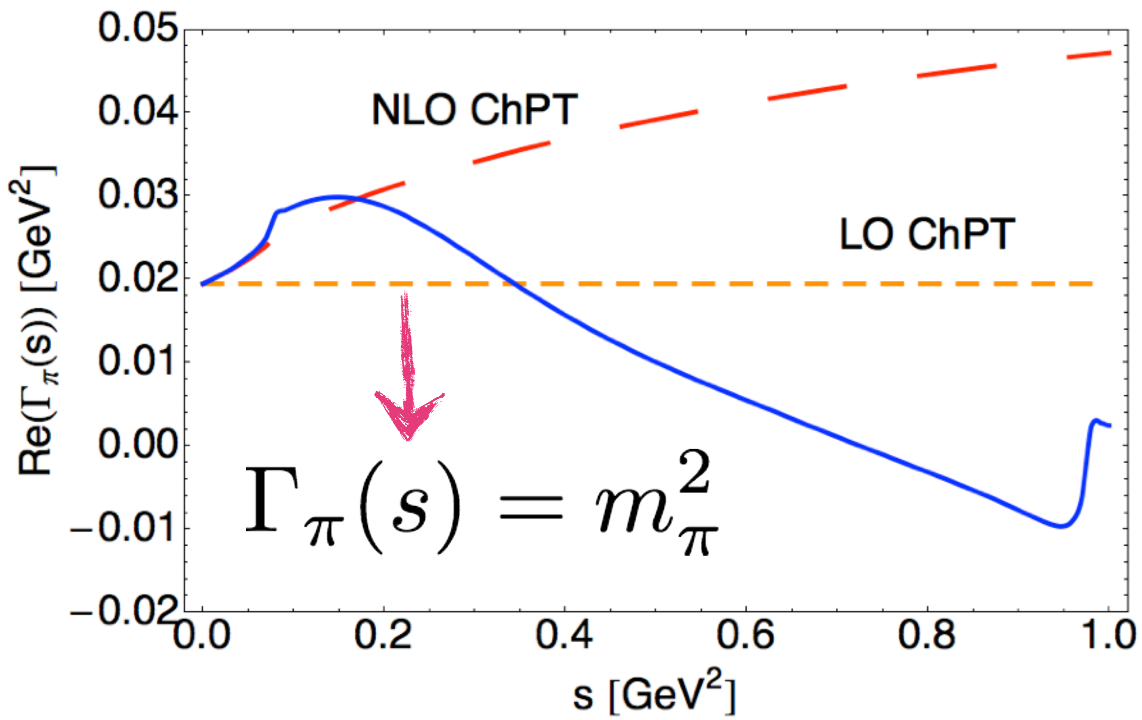


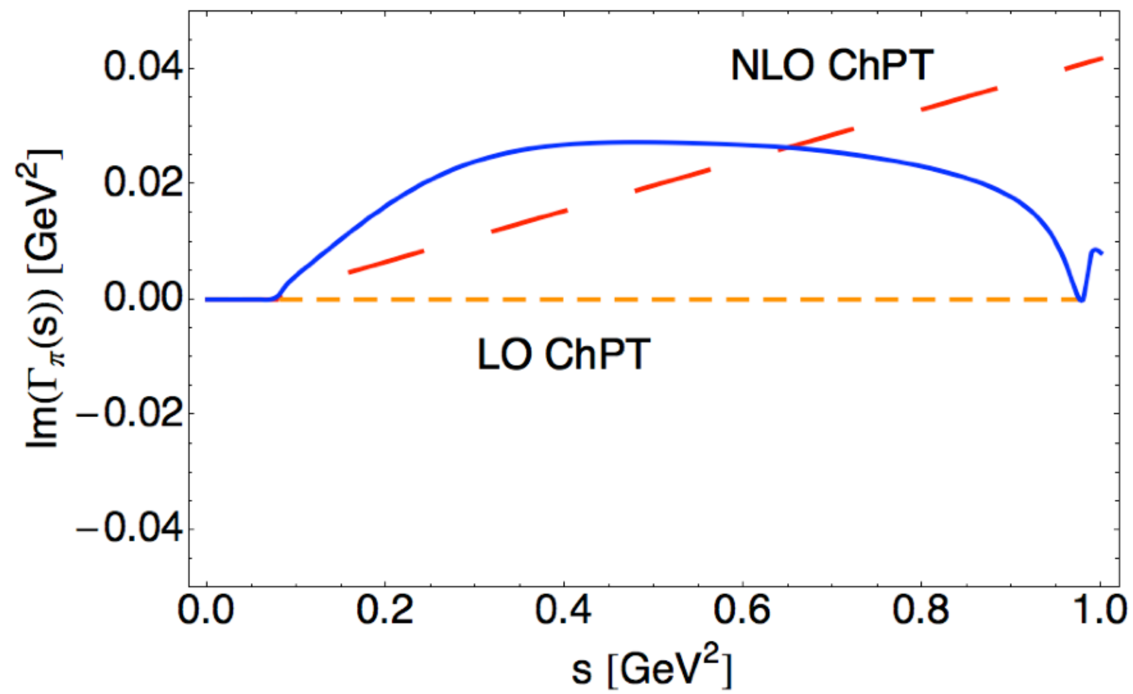




$$\langle \pi^+(p_{\pi^+})\pi^-(p_{\pi^-}) \big| m_u \bar{u}u + m_d \bar{d}d \big| 0 \rangle \equiv \Gamma_\pi(s)$$



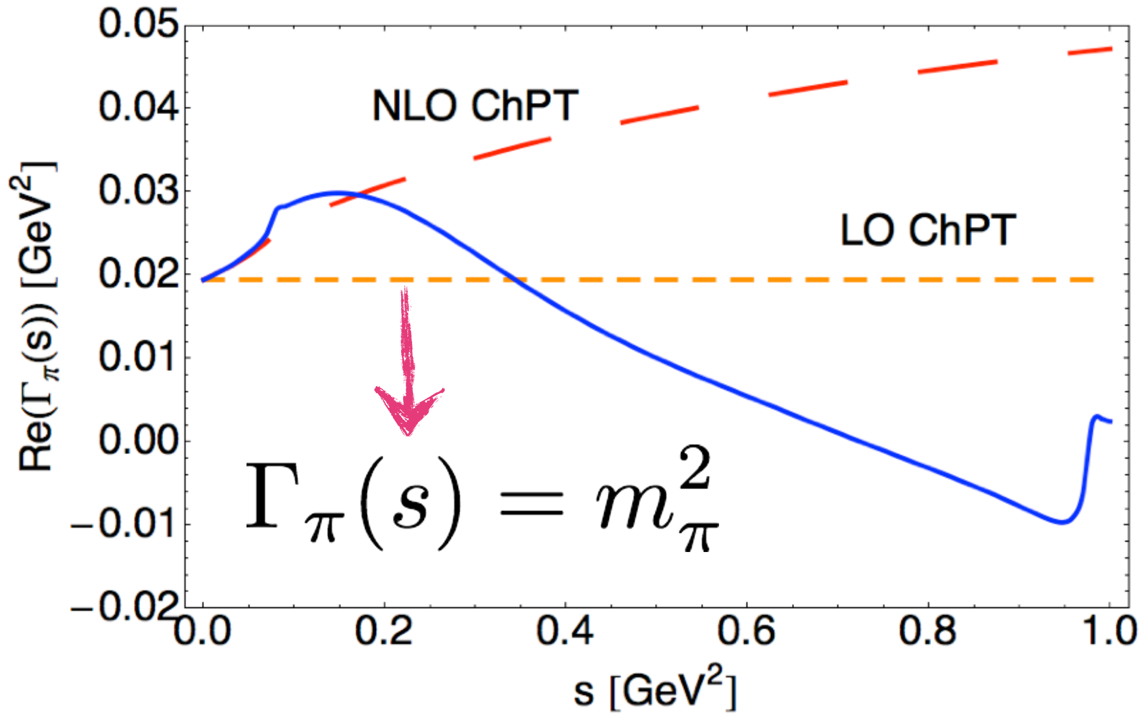




Donoghue, Gasser, Leutwyler (1990)

~1 GeV Higgs decay to two pions proceeds mostly through the Higgs-gluon coupling and the Higgs-strange quark coupling.

$$\langle \pi^+(p_{\pi^+})\pi^-(p_{\pi^-}) | m_u \bar{u}u + m_d \bar{d}d | 0 \rangle \equiv \Gamma_\pi(s)$$



Some previous studies were considering only

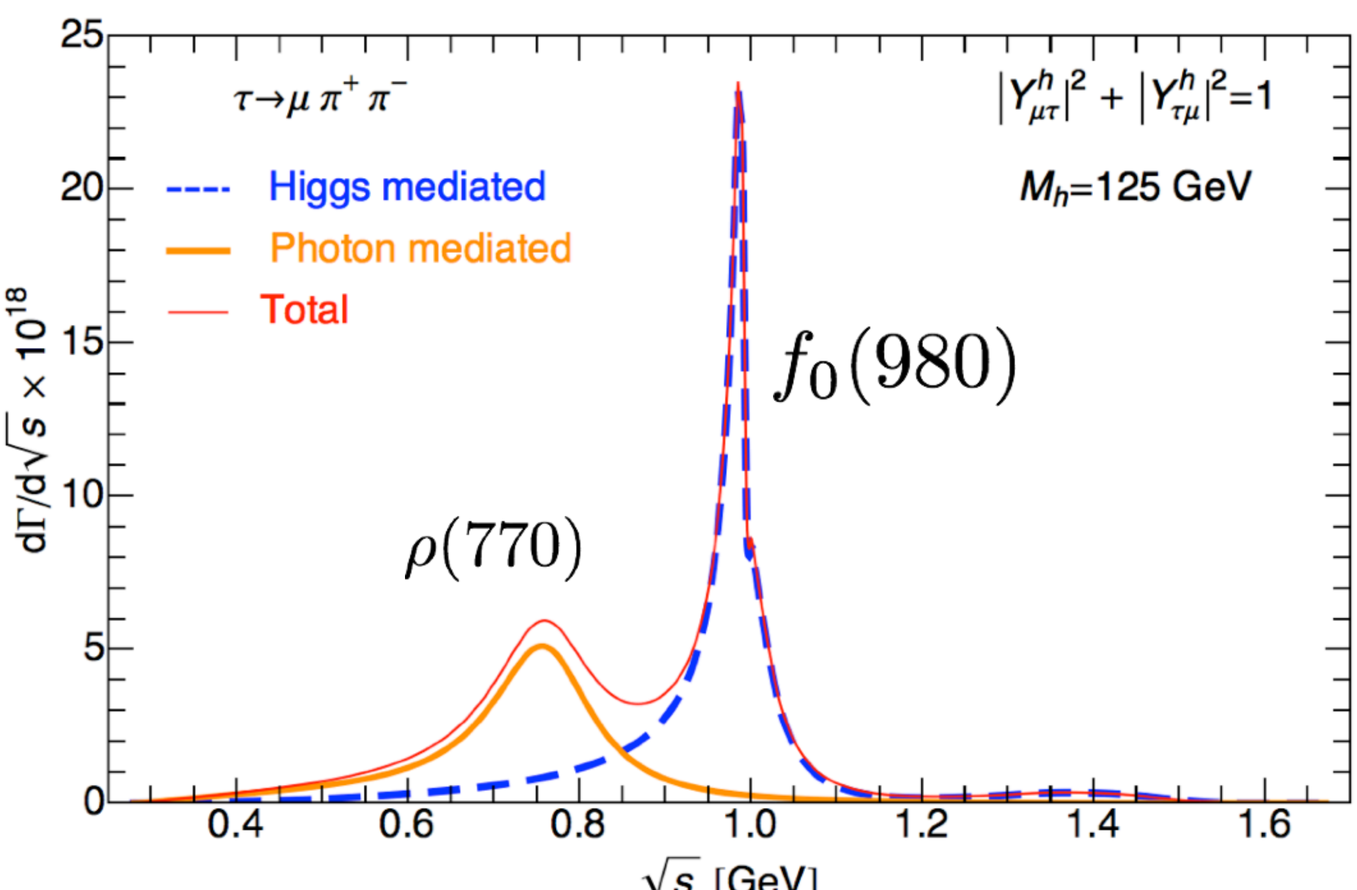
$$\Gamma_\pi(s) = m_\pi^2$$

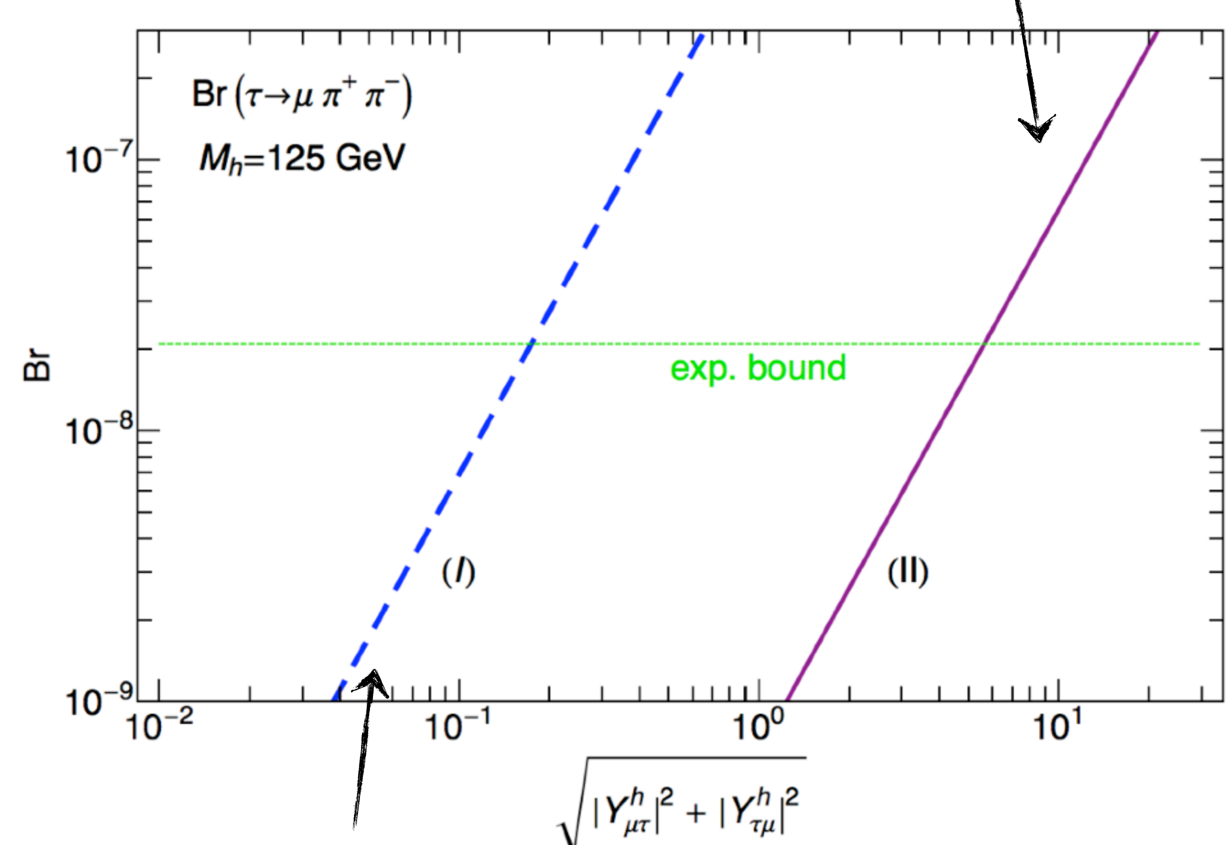
(LO-ChPT)

Equivalent to the naive estimate

$$\frac{\Gamma(h \to \mu^+\mu^-)}{\Gamma(h \to \pi^+\pi^-)} \sim \frac{m_\mu^2}{m_\pi^2}$$

Impact of hadronic matrix elements on $\tau \to \mu\pi^+\pi^-$

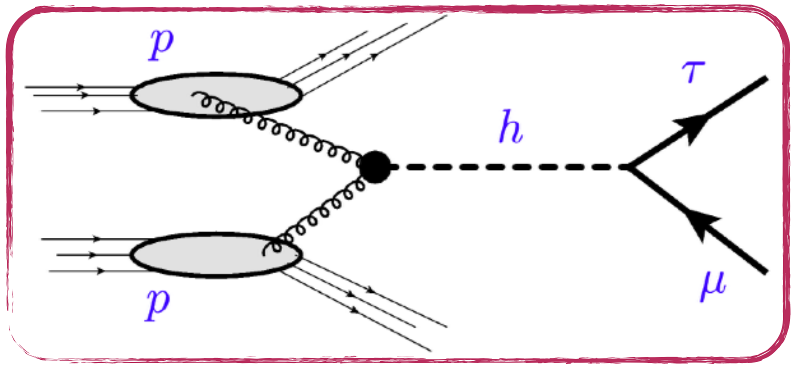


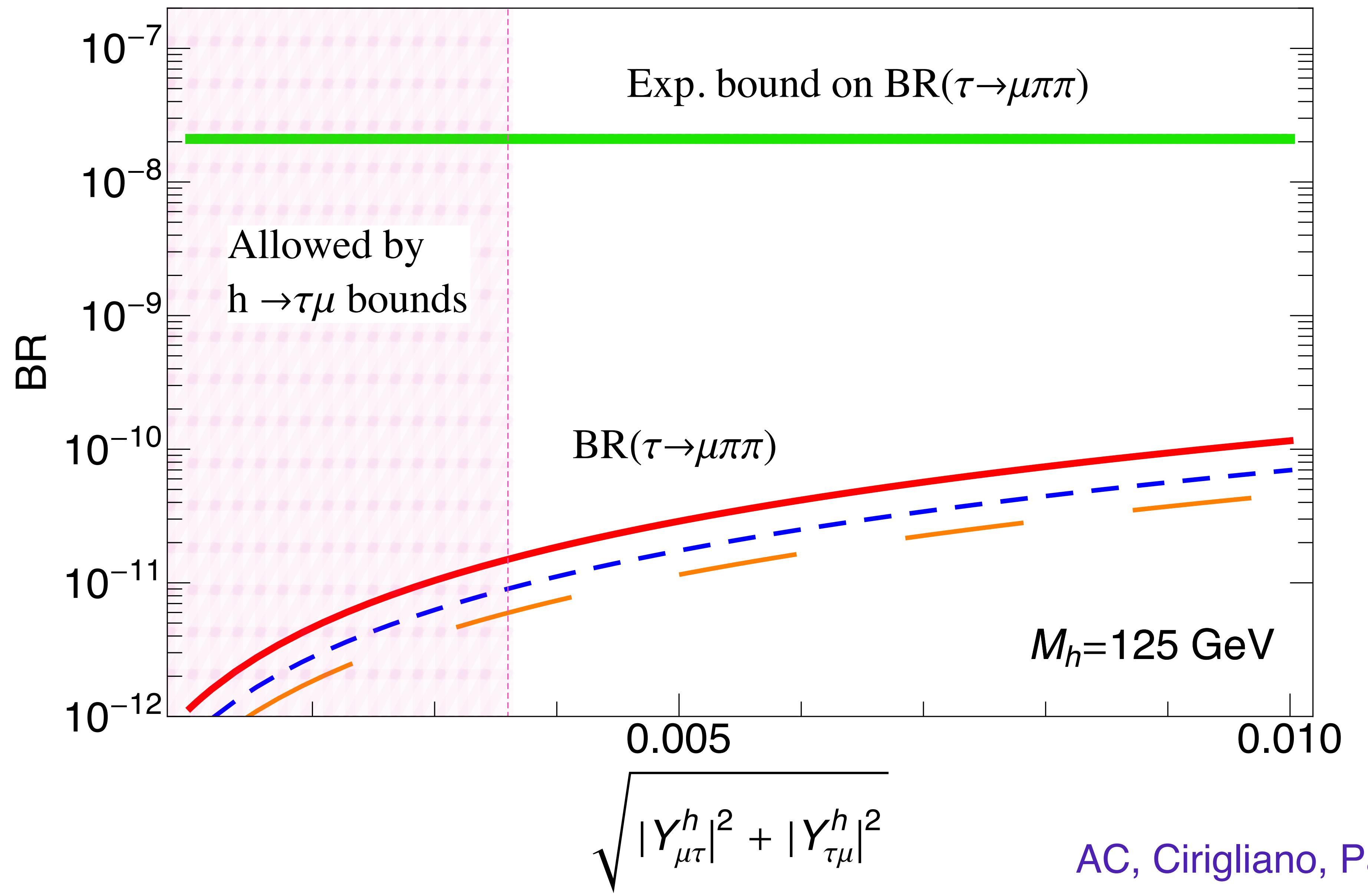


AC, Cirigliano, Passemar (1309.3564)

Interesting relation with LFV Higgs decays being probed at the LHC

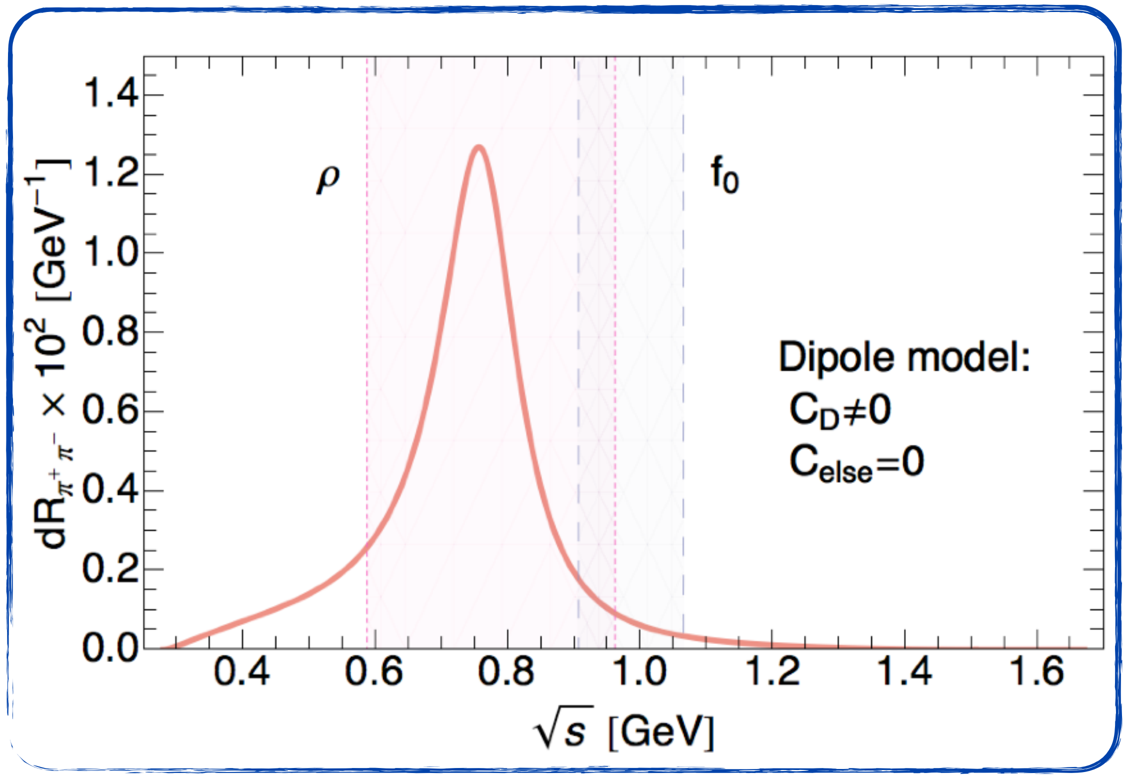
$$pp(gg) \to h \to \tau\mu$$





AC, Cirigliano, Passemar

$$\mathcal{L}_{\rm eff} \supset -\frac{\rm C_D}{\Lambda^2} m_\tau (\bar{\mu}\sigma^{\rho\nu} P_R \tau) F_{\rho\nu}$$



LFV tau decays into the hadronic resonances

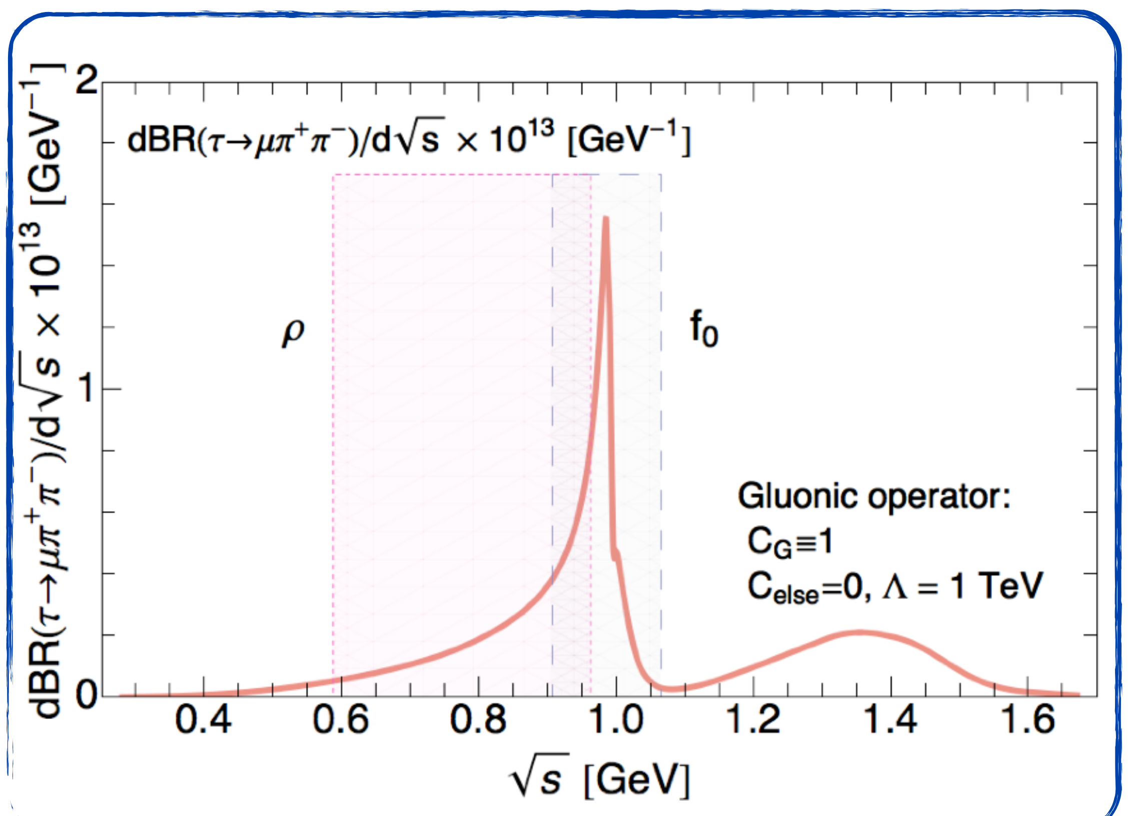
$$M_\rho \sim 770 \text{ MeV}, \text{J}^{\rm PC} = 1^{--}$$

$$M_{f_0} \sim 980 \text{ MeV}, \text{J}^{\rm PC} = 0^{++}$$

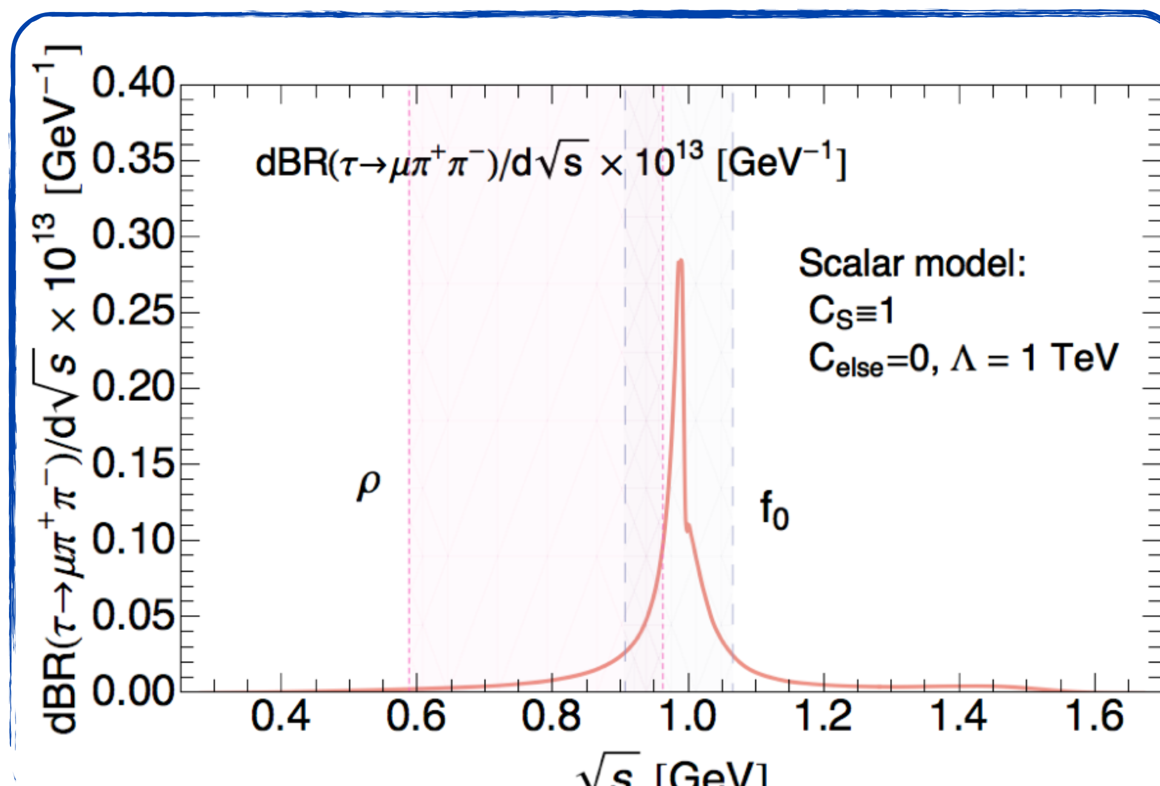
are measured by applying a cut on the $\pi^+\pi^-$ invariant mass

J. T. Daub et al. (1212.4408)

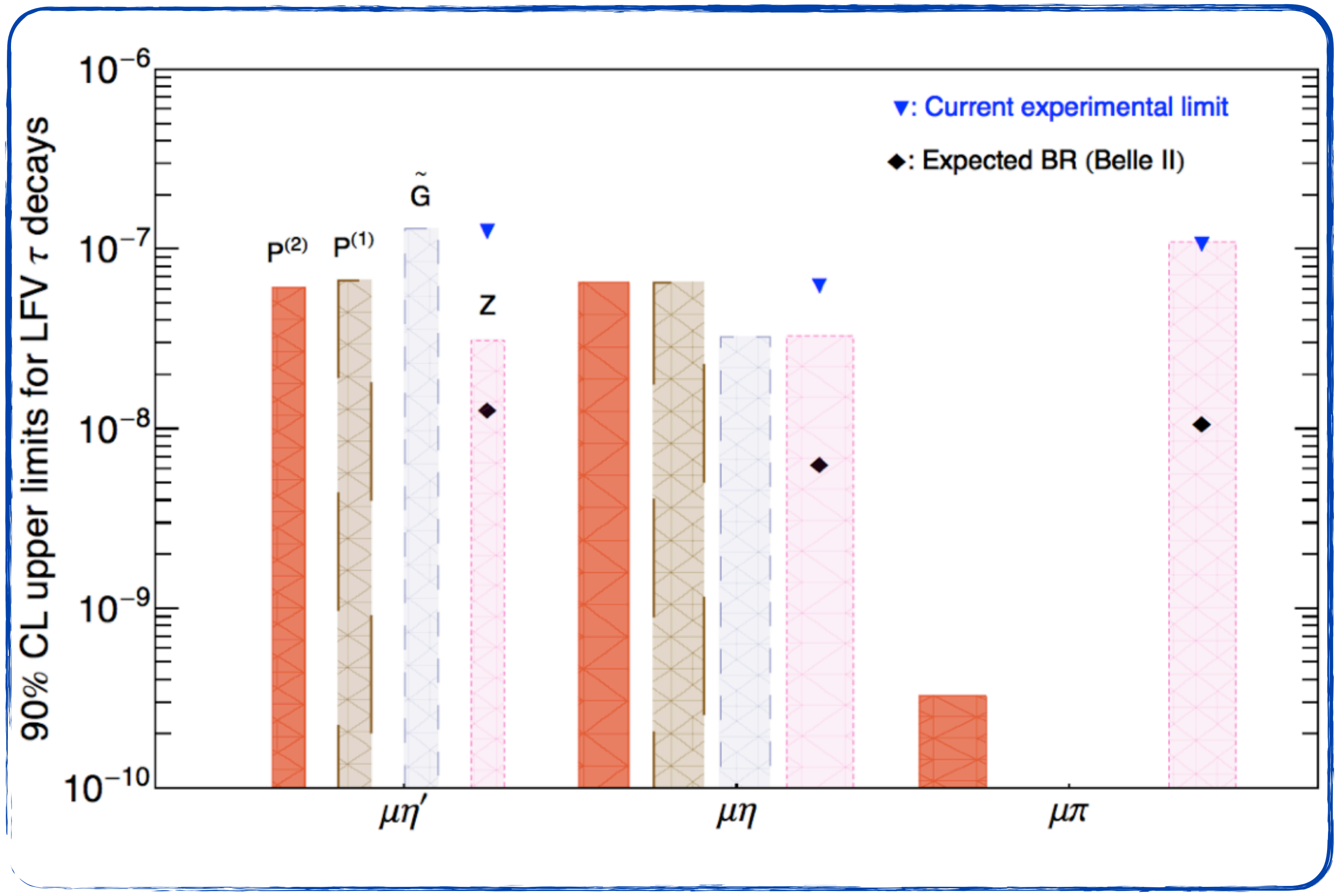
$$\mathcal{L}_{\rm eff} \supset -\frac{\rm C_G}{\Lambda^2} m_\tau G_F (\bar{\mu} P_R \tau) G_a^{\rho\nu} G_{\rho\nu}^a$$



$$\mathcal{L}_{\rm eff} \supset -\frac{\rm C_S}{\Lambda^2} \sum_{q=u,d,s} m_\tau m_q G_F (\bar{\mu} P_R \tau) \bar{q} q$$



AC, Cirigliano, Passemar (2014)



Semileptonic decays into a Pseudoscalar meson

$$\tau \to \mu P \qquad (P = \pi^0, \eta, \eta')$$

Hadronic matrix elements obtained using the Feldmann-Kroll-Stech (FKS) mixing scheme

Feldmann, Kroll, Stech (1998)

axial anomaly of QCD

$$\partial_\mu(\bar{q}\gamma^\mu\gamma_5 q) = 2im_q\bar{q}\gamma_5 q + \frac{\alpha_s}{4\pi}G^a_{\mu\nu}\widetilde{G}^{\mu\nu}_a$$

Benchmark scenarios

CP-odd Higgs → $P^{(1)}$, $\widetilde{G}$

	O_j	
$P^{(1)}$	$\sum_{q=u,d,s} m_\tau m_q G_F(\bar{\mu}P_R\tau)\bar{q}\gamma_5 q$	(Pseudoscalar operator #1)
$P^{(2)}$	$\sum_{q=u,d,s} m_\tau m_q G_F T_3^q(\bar{\mu}P_R\tau)\bar{q}\gamma_5 q$	(Pseudoscalar operator #2) $T_3^{u,d} = \pm 1/2$
$\widetilde{G}$	$m_\tau G_F(\bar{\mu}P_R\tau)G^a_{\mu\nu}\widetilde{G}^{\mu\nu}_a$	(P-odd Gluonic operator)
Z	$\sum_{q=u,d,s} T_3^q\,(\bar{\mu}\gamma^\rho P_L\,\tau)\;\bar{q}\gamma_\rho\gamma_5 q$	(effective Z-penguin LFV vertex) $T_3^{u,d} = \pm 1/2$

Summary

LFV can probe new physics scales much higher than those directly observable at the LHC

It is possible to observe signals of new physics via LFV transitions in the near future, despite the lack of new physics observed so far at the LHC

Different operators expected at low scales, we need to measure as many processes as possible

$$\tau \to \mu\pi^+\pi^-$$

Use a combination of dispersive methods and ChPT/Lattice QCD in order to determine the relevant hadronic matrix elements in a robust way

possible extension

Use of dispersive methods to describe

$$\tau \to \mu K^+ K^-$$

PDG Live

$\Gamma(\tau^- \to \mu^- K^+ K^-)/\Gamma_{\text{total}}$

Test of lepton family number conservation.

VALUE	*CL%*	*DOCUMENT ID*		*TECN*
$< 4.4 \times 10^{-8}$	90	MIYAZAKI	2013	BELL