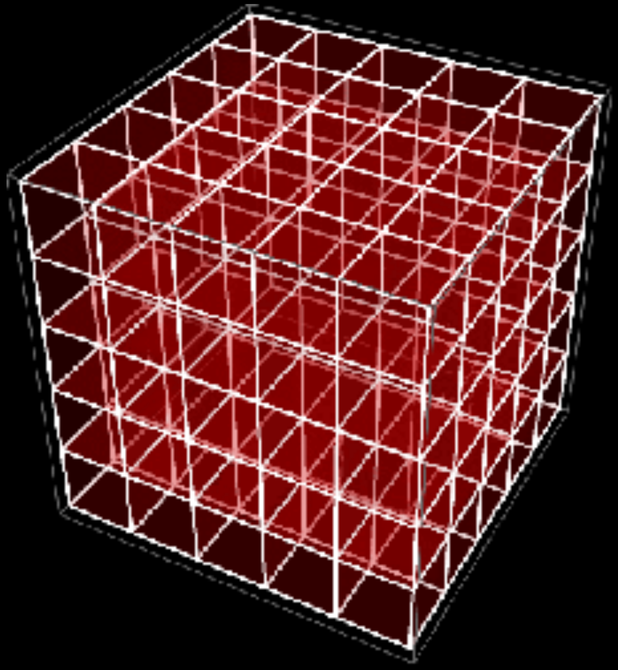
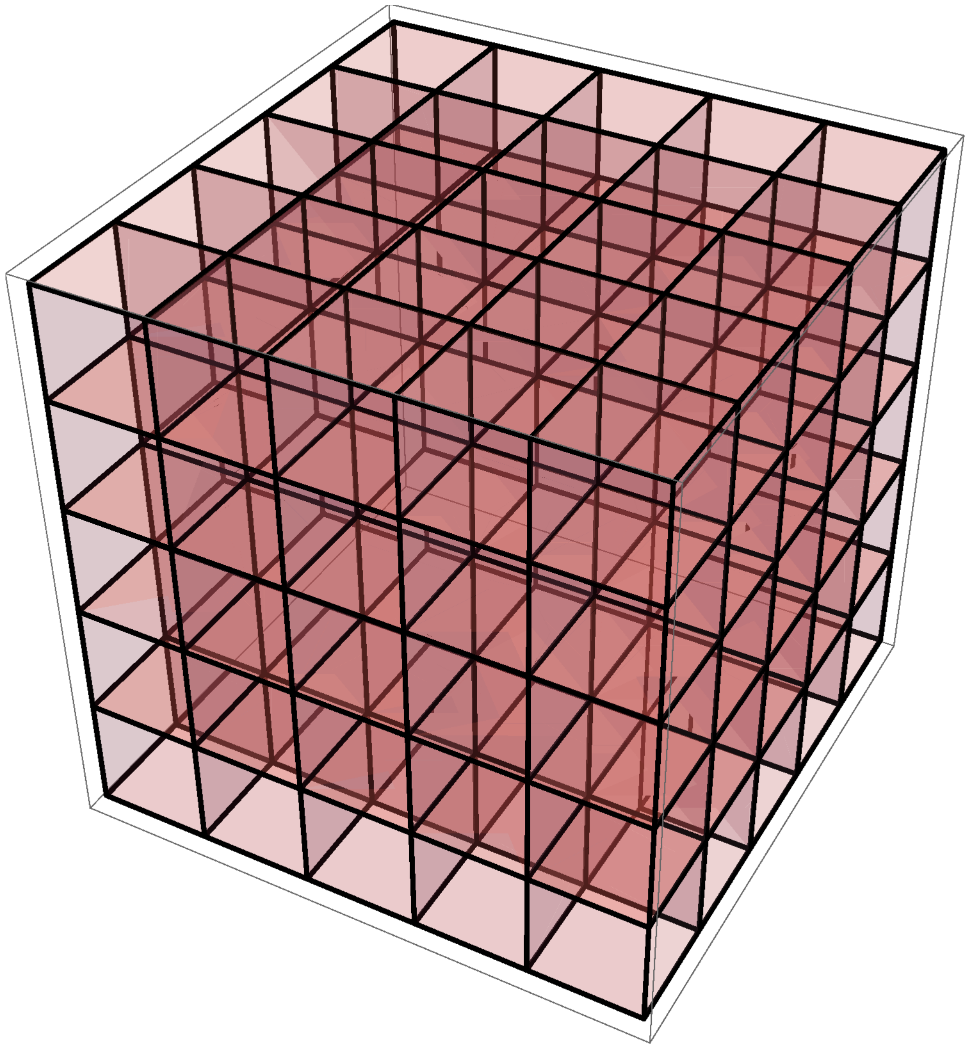
Excited Hadrons on the Lattice



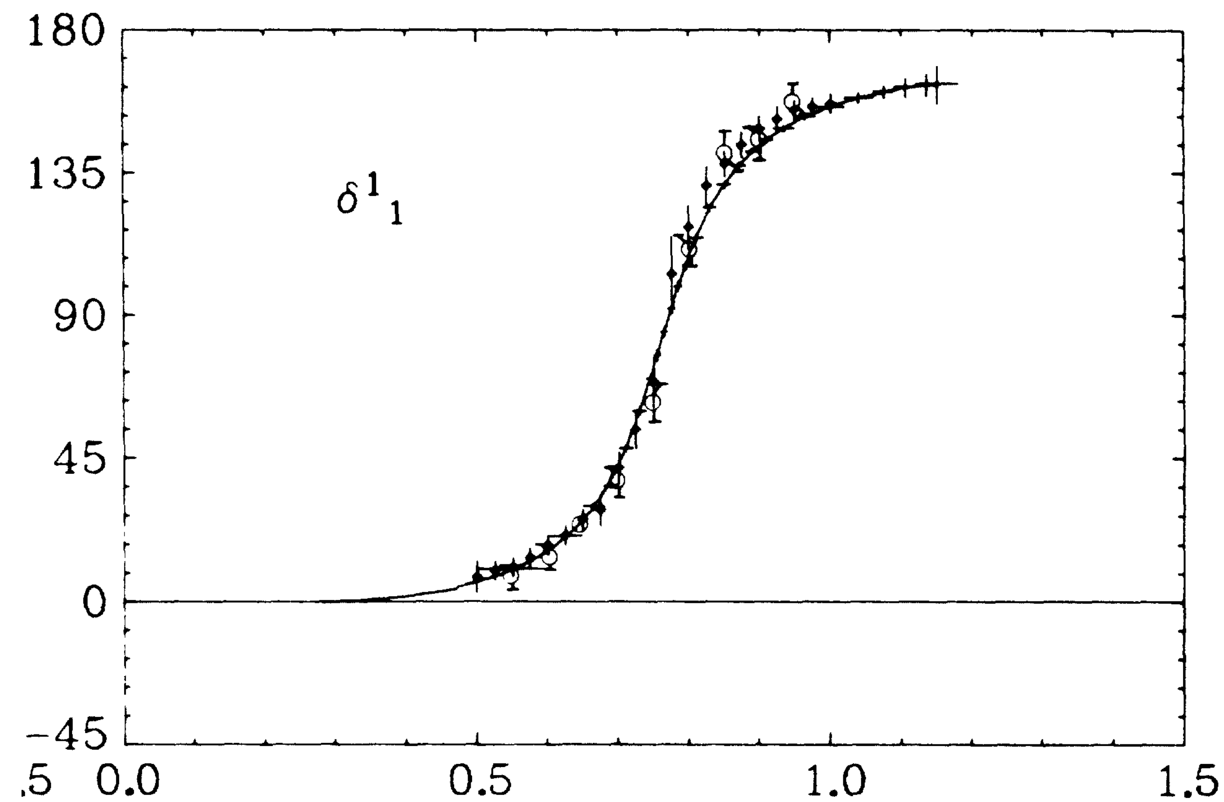
Christian B. Lang
University of Graz, Austria

Lattice QCD ➛ Analysis ➛ Phase shifts



Partial wave scattering amplitudes

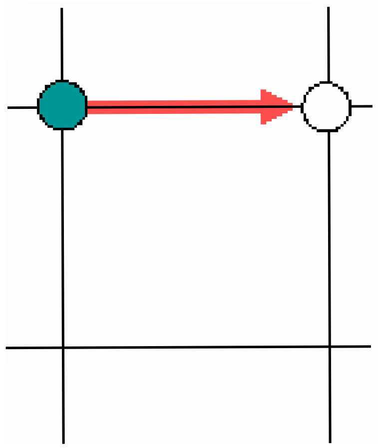
Phase shifts



The lattice approach

Regularization: Lattice QCD *(1974)*

Ken Wilson

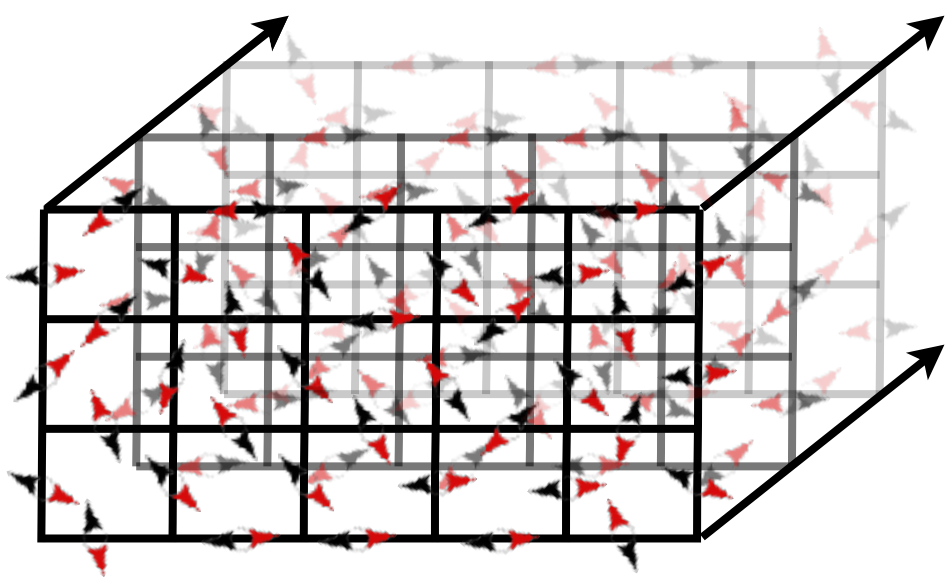


lattice spacing a

Gluon $U_\mu = e^{i\,A_\mu} \in \mathrm{SU}(3)$

Quark ψ

Antiquark $\overline{\psi}$



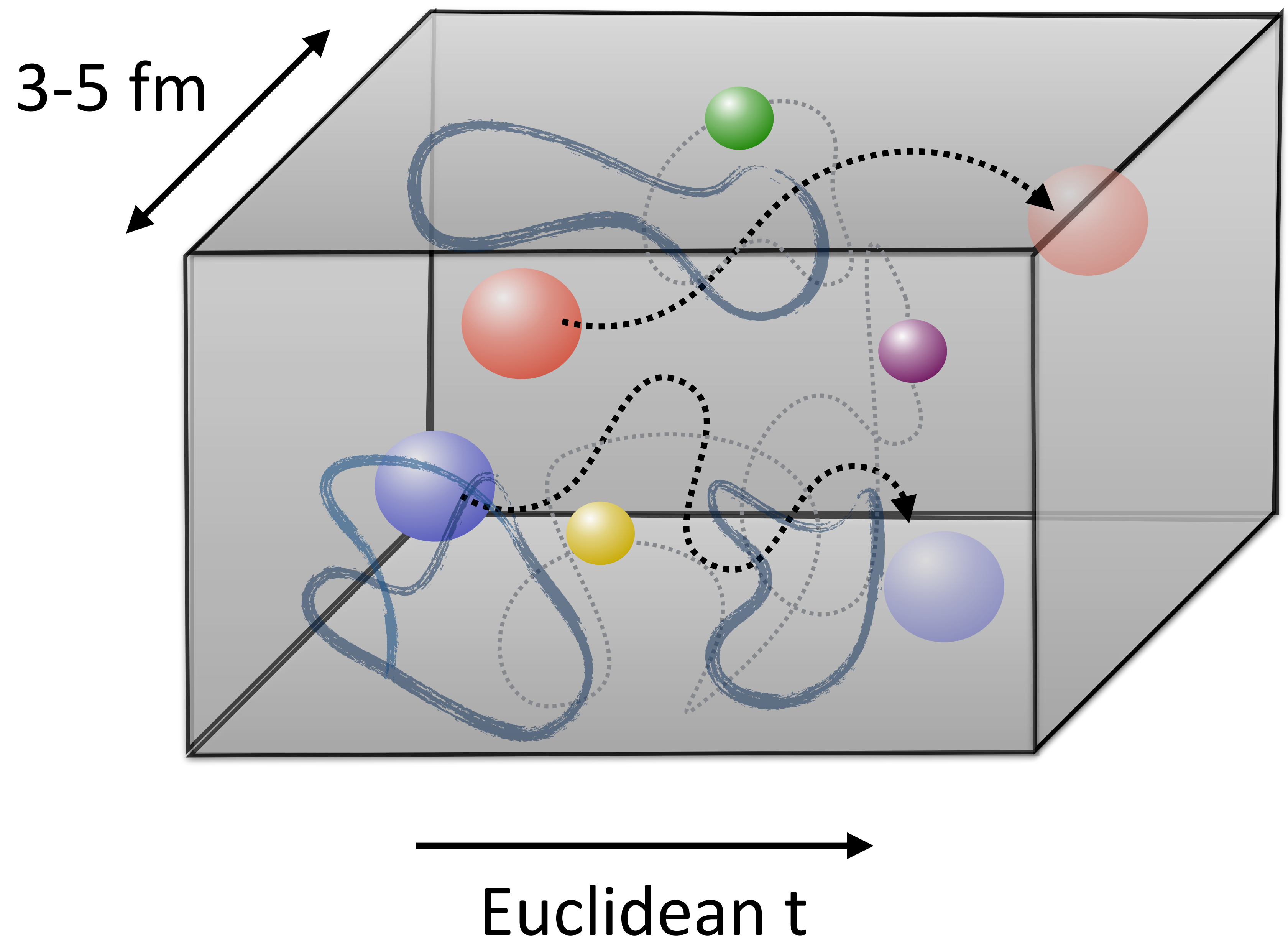
$U_\mu(x, y, z, \tau)$

Quantization:

$$\int [dU][d\psi][d\bar\psi] \rightarrow \sum_{\{U.\psi,\bar\psi\}}$$

The path integral becomes a well-defined (very large) sum over field configurations

Femto universe



Why the lattice?

- Continuum QFT is singular: regular formulation needed
- Hadrons are non-perturbative compounds: non-pert. method needed
- Gauge invariance is important: gauge inv. formulation preferred
- Limit to “continuum QCD”: should be controllable

“Femto universe” - is it systematically controllable?

can be controlled/improved

- dynamical quarks: u,d … s …c …b … ☑
- light quarks (pion mass ↘ 140 MeV) ☑
- lattice spacing (mostly a>0.05 fm) ☑
- lattice size (typically 2-6 fm) ☑
- limited statistics ☑
- analysis of correlators (operator set, asymptotic behavior) ?

Approaches

Lattice QCD

solve lattice
regularised QFT
of quarks and gluons

compute hadron
propagators

discrete energy levels

Hybrid approaches

NLEFT;
hadron EFT on a
lattice

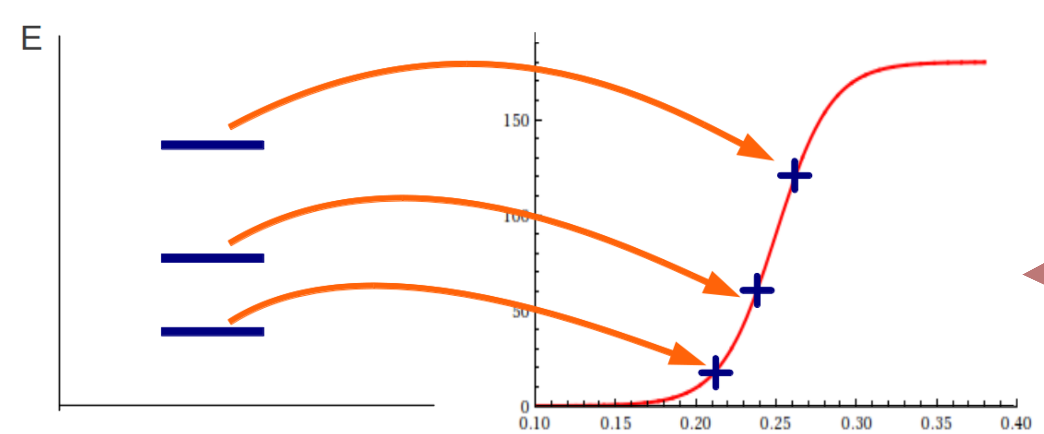
HAL-QCD
Use input from LQCD

Bethe-Salpeter eq.

QCD inspired continuum models

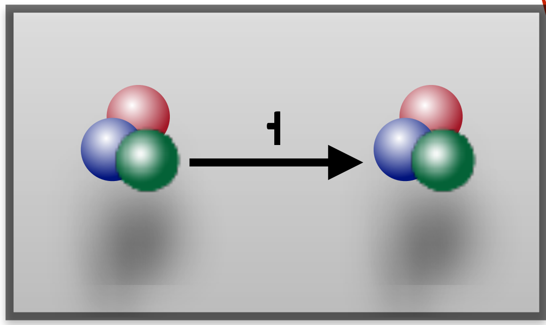
Eff. Hamiltonian
(ChEFT, HEFT,..)
constituent quarks
Unitarized ChPT
Bethe-Salpeter eq.

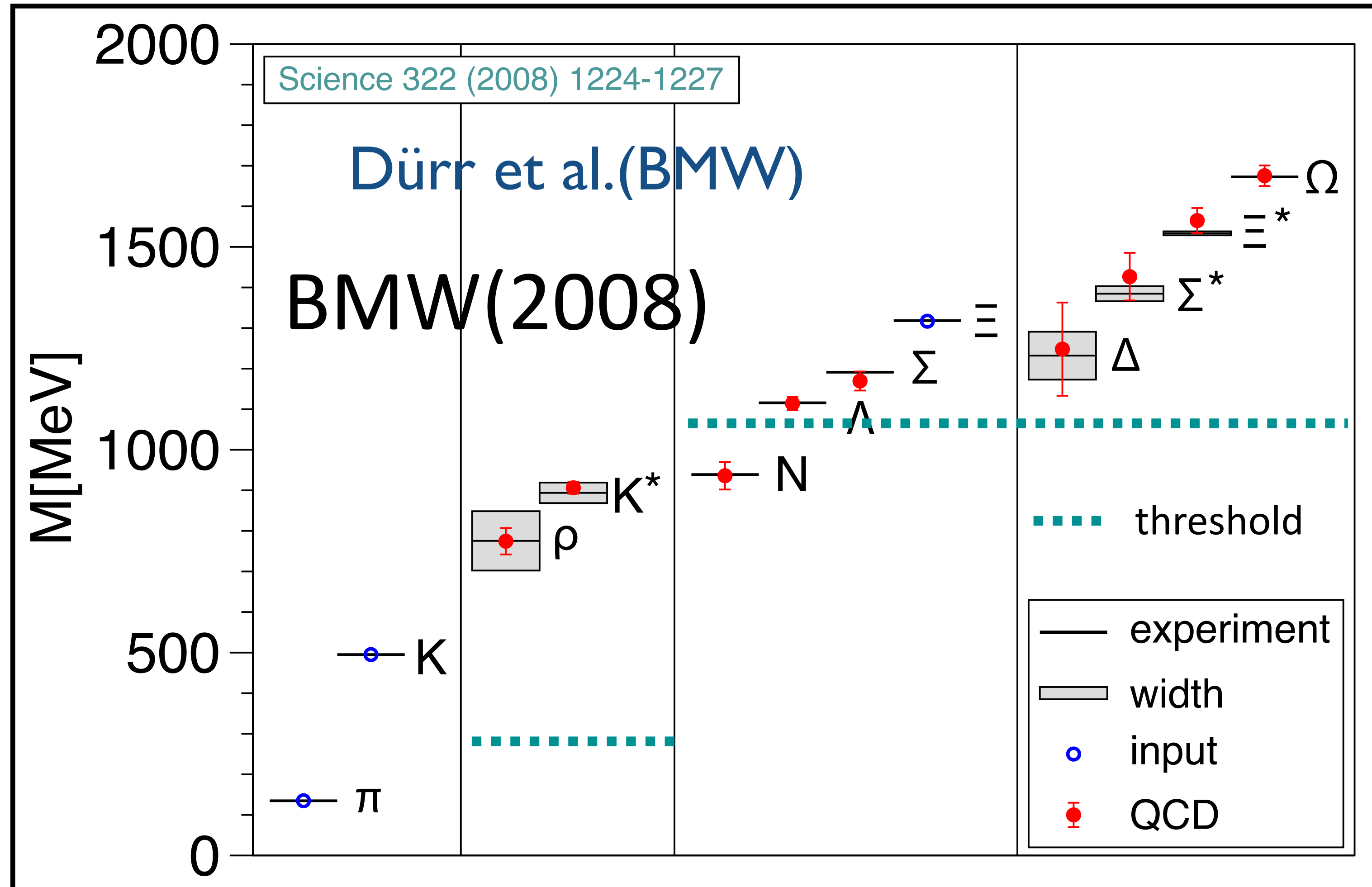
“latticise” by replacing
mom. integration
by lattice sum



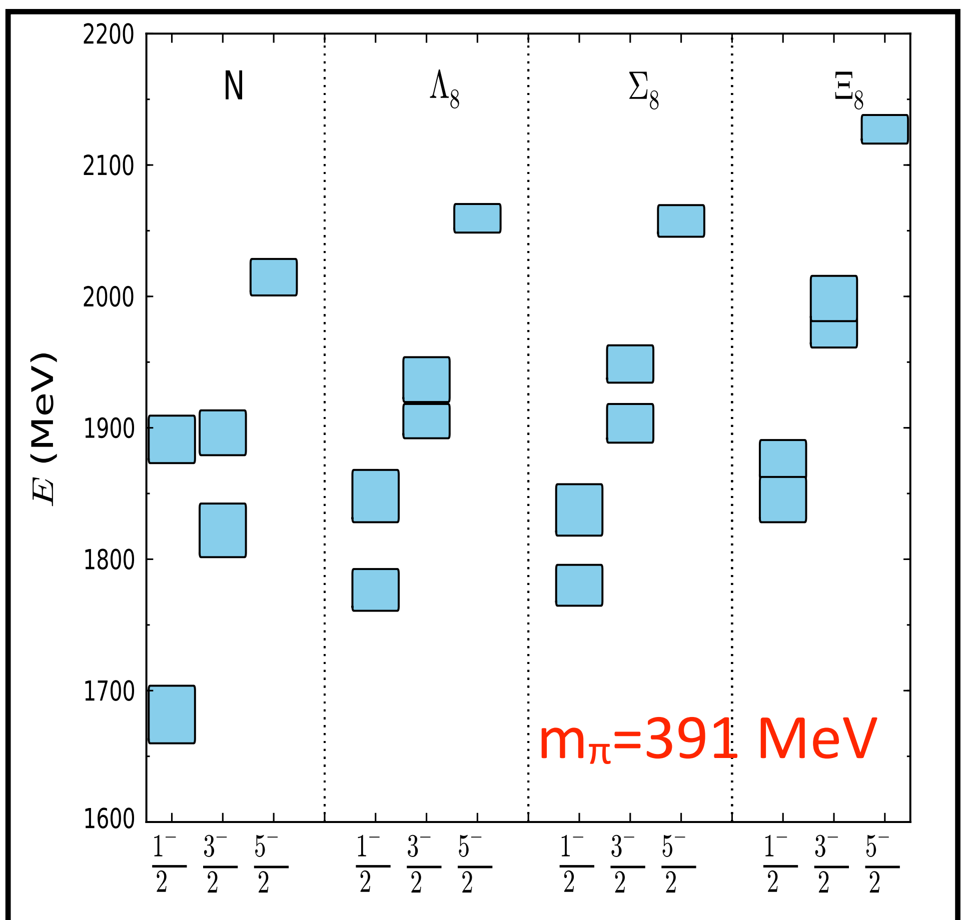
Milestones

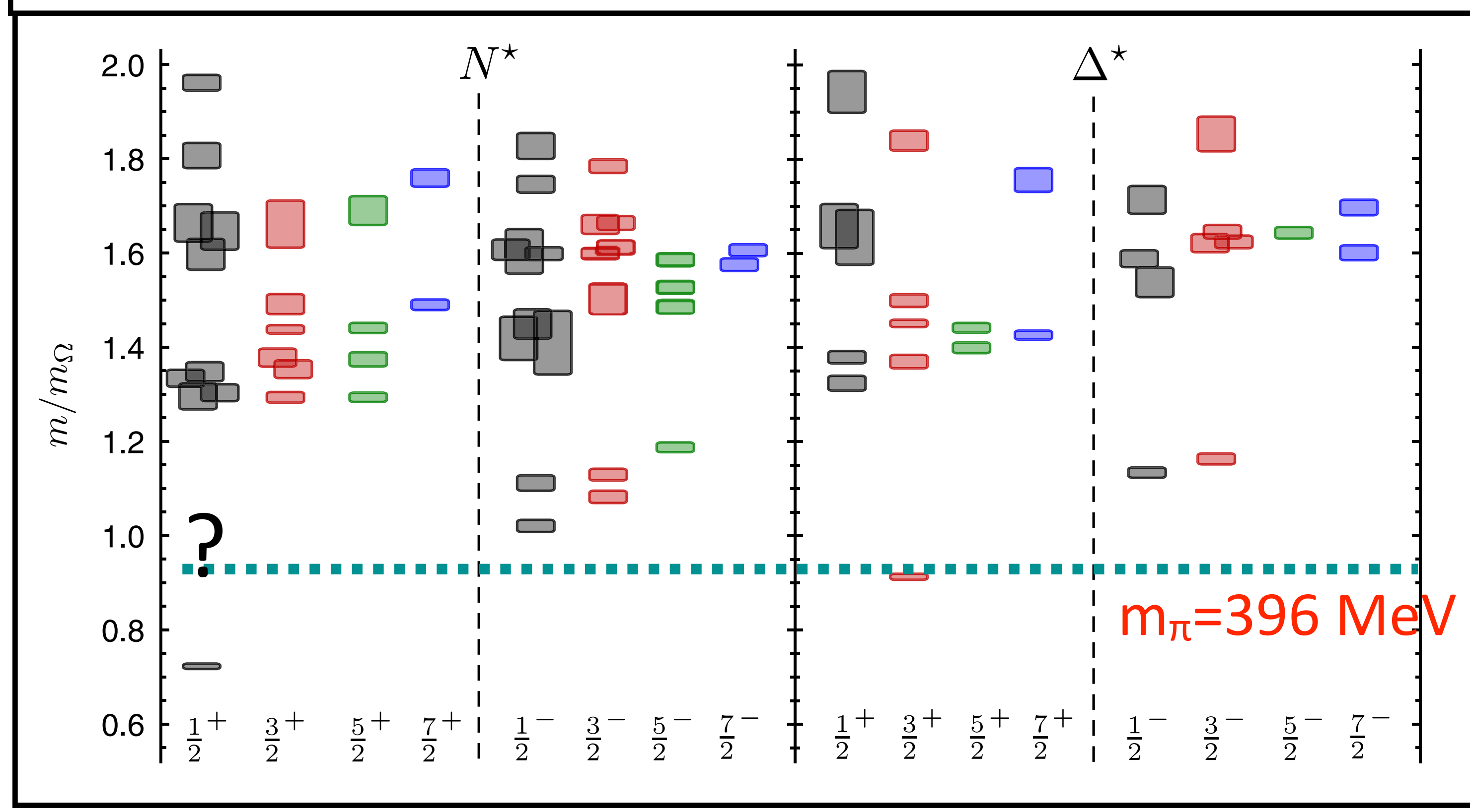
Simple hadron approximation





HSC(2011, 2013)





Edwards et al. (HSC) Phys.Rev. D87, 054506 (2013). Edwards et al. (HSC) PR D 84, 074508 (2011)

Spectroscopy

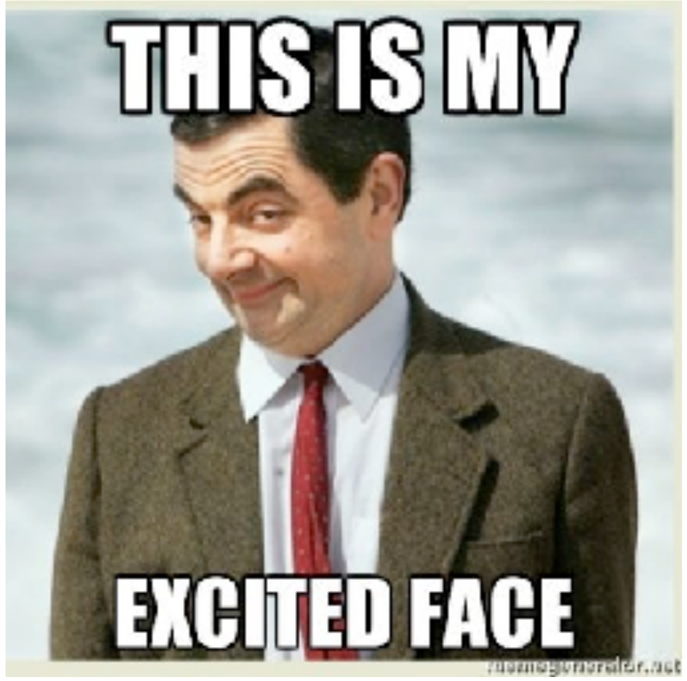
Ground state spectroscopy

Is correct only for stable particles.
Simple hadron approach qqq or qq
is valid only below scattering threshold
("bound states" or "artificial bound states")

Resonances and bound states

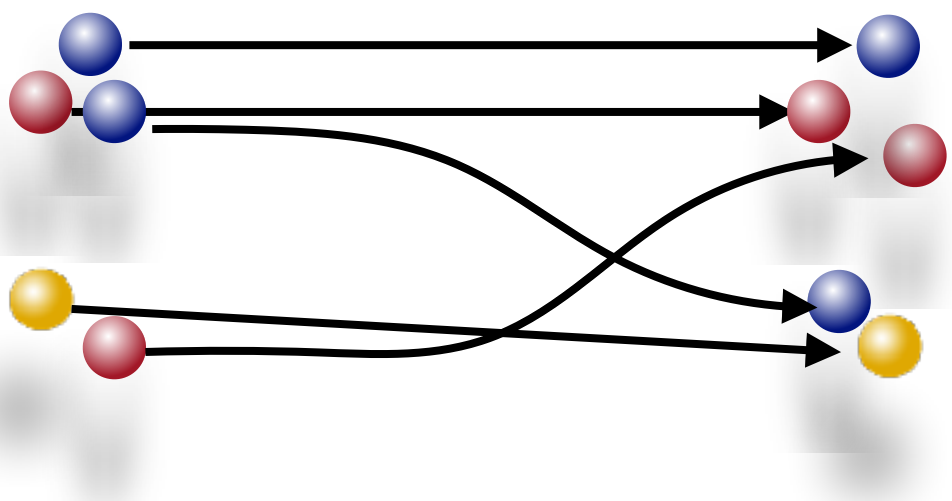
require inclusion of hadron-hadron channels in the calculation.
Multi-hadron approach: we need to extend the space of operators: (q$\underline{q}$)(q$\underline{q}$),(qqq)(q$\underline{q}$), (qqq)($\underline{qqq}$)...

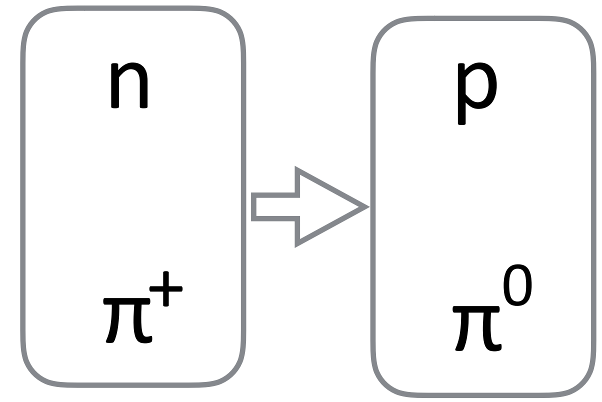
Excited states spectroscopy
and scattering amplitudes

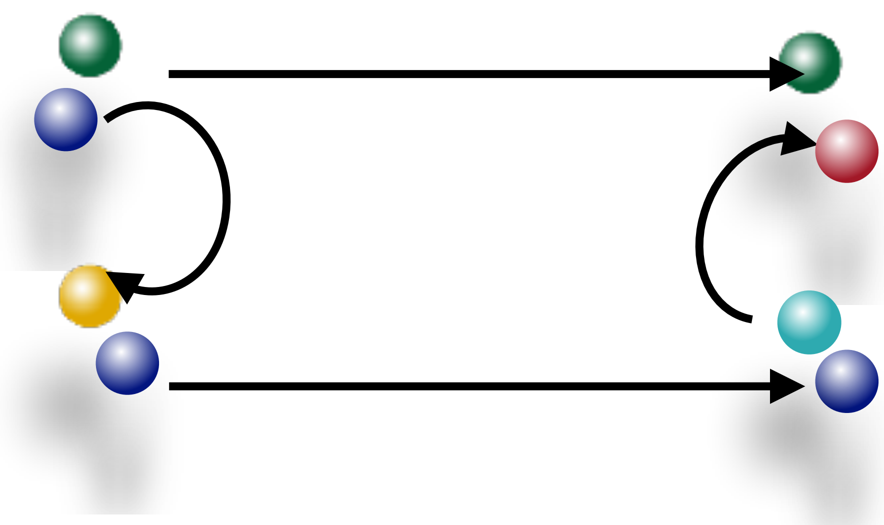


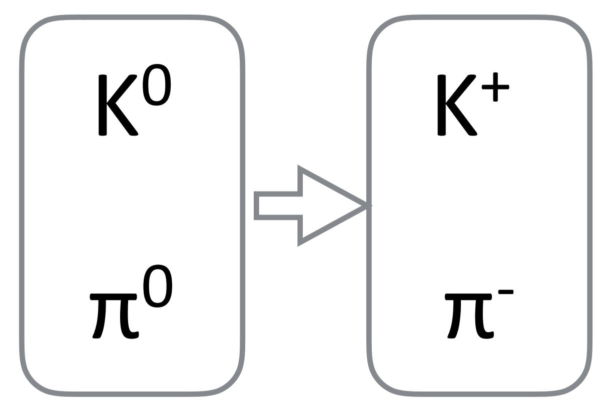
What is the challenge?

BM ➛ BM









MM ➛ MM

More terms

More quark propagators

Backtracking loops are expensive! → Stochastic method and "Distillation method"
Peardon et al. (HSC), PR D 80, 054506 (2009).
Morningstar et al., PR D 83, 114505 (2011).

Continuum vs. lattice

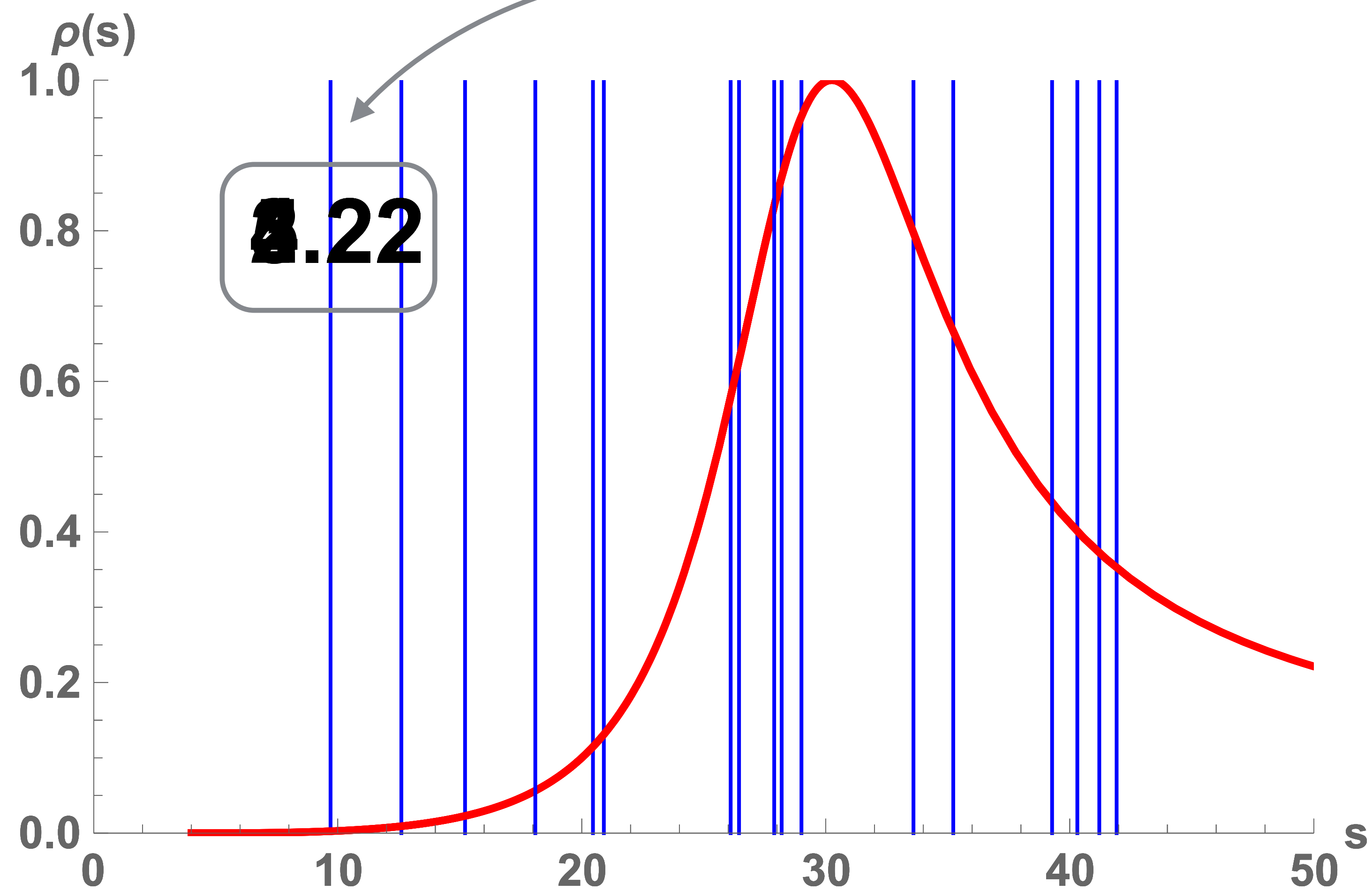
Correlation functions have discrete energy levels!

$m_\pi L$=4 ~ L=5.6 fm

$m_\pi L$

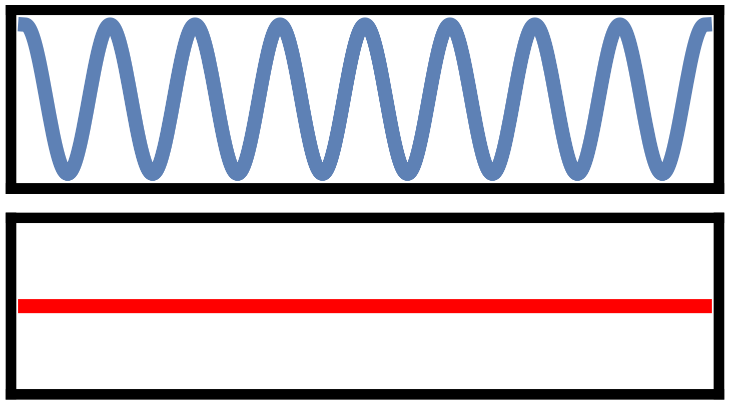
Example:

Spectral density of a simple resonance in continuum and the discrete energies for a lattice volume



One cannot arbitrarily fix the energies: they are eigenvalues depending on the control parameters (volume, couplings,...).

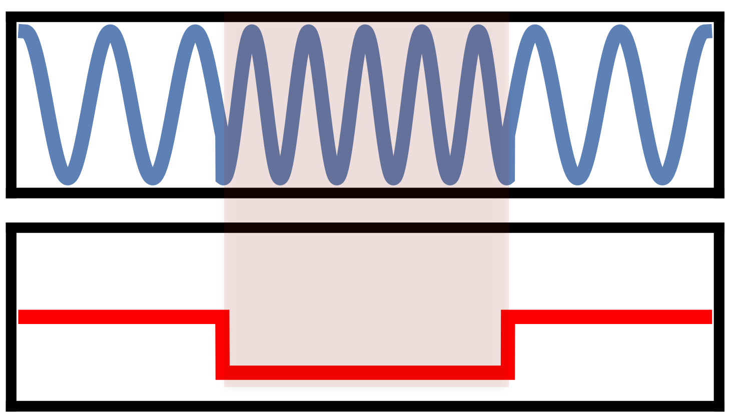
Energy levels ➛ Phase shift points (in the elastic region)



$V = const.$ $\delta = 0$

$$e^{ipL} = 1 \qquad p_n = 2n\pi/L$$

periodic b.c.



$V = localized$ $\delta \neq 0$

$$e^{ipL+2i\delta(p)} = 1 \qquad 2\delta(p_n) = 2n\pi - p_n L$$

$$\cot\delta(p_n) = -\cot(p_n L/2)$$

Solving the Helmholtz eq. for d=3

$$\boxed{p_n \cot\delta(p_n) = c\mathcal{Z}_{00}\left(1;\left(\frac{p_n L}{2\pi}\right)^2\right)}$$

Lüscher, CMP 105(86) 153,
NP B354 (91) 531, NP B 364 (91) 237

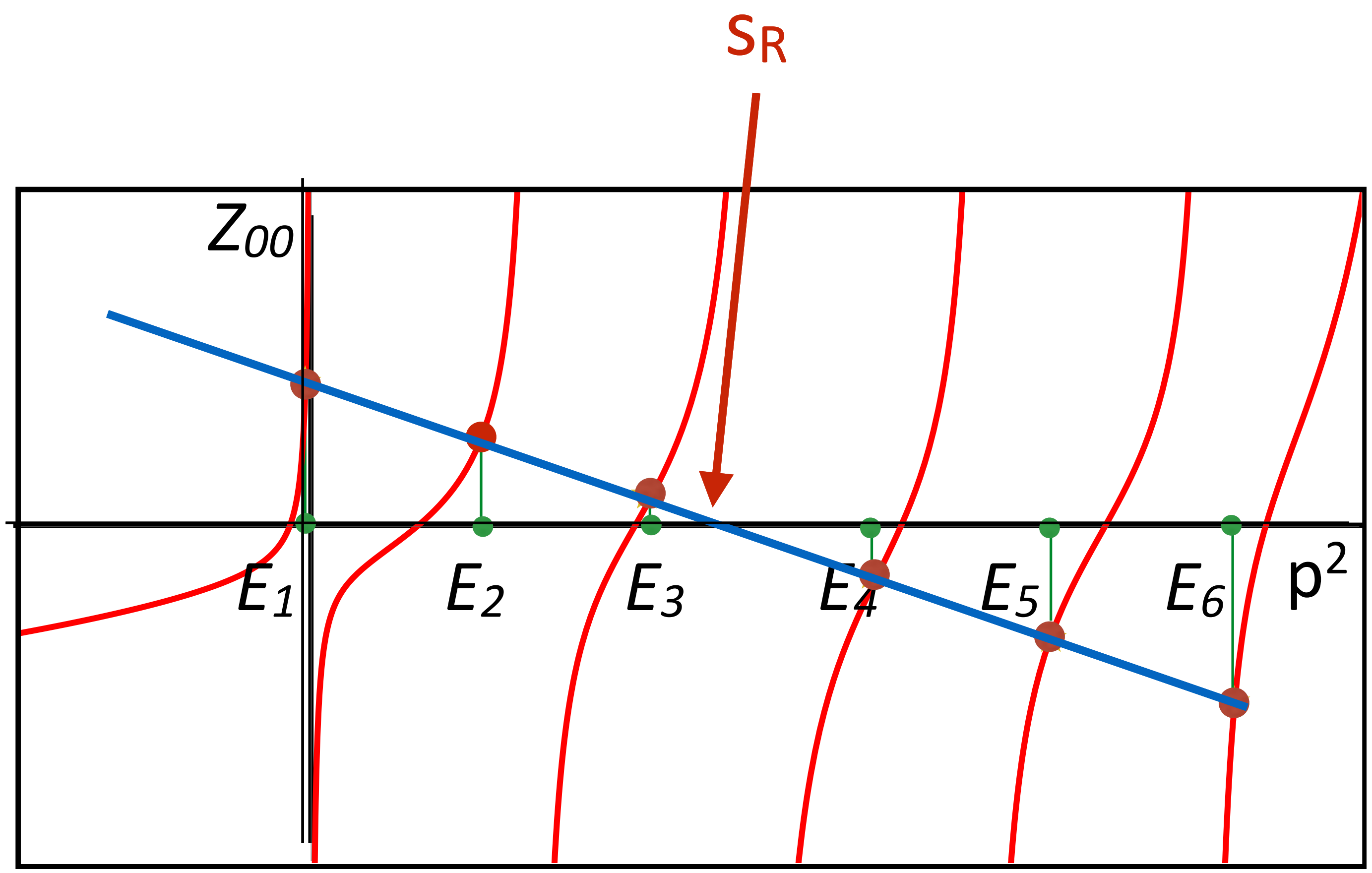
Elastic scattering

Partial wave amplitude t_ℓ

$$t^{-1} = p\cot\delta(p) - \mathrm{i}\,p$$

$$\equiv k^{-1} - \mathrm{i}p$$

e.g. $k^{-1} \propto (s_R - s)$



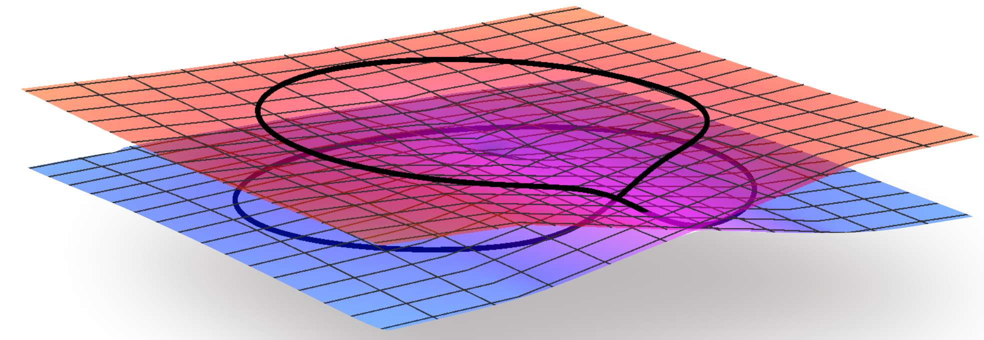
"Lüscher curves"

$$\boxed{p_n \cot\delta(p_n) = c\mathcal{Z}_{00}\left(1;\left(\frac{p_n L}{2\pi}\right)^2\right)}$$

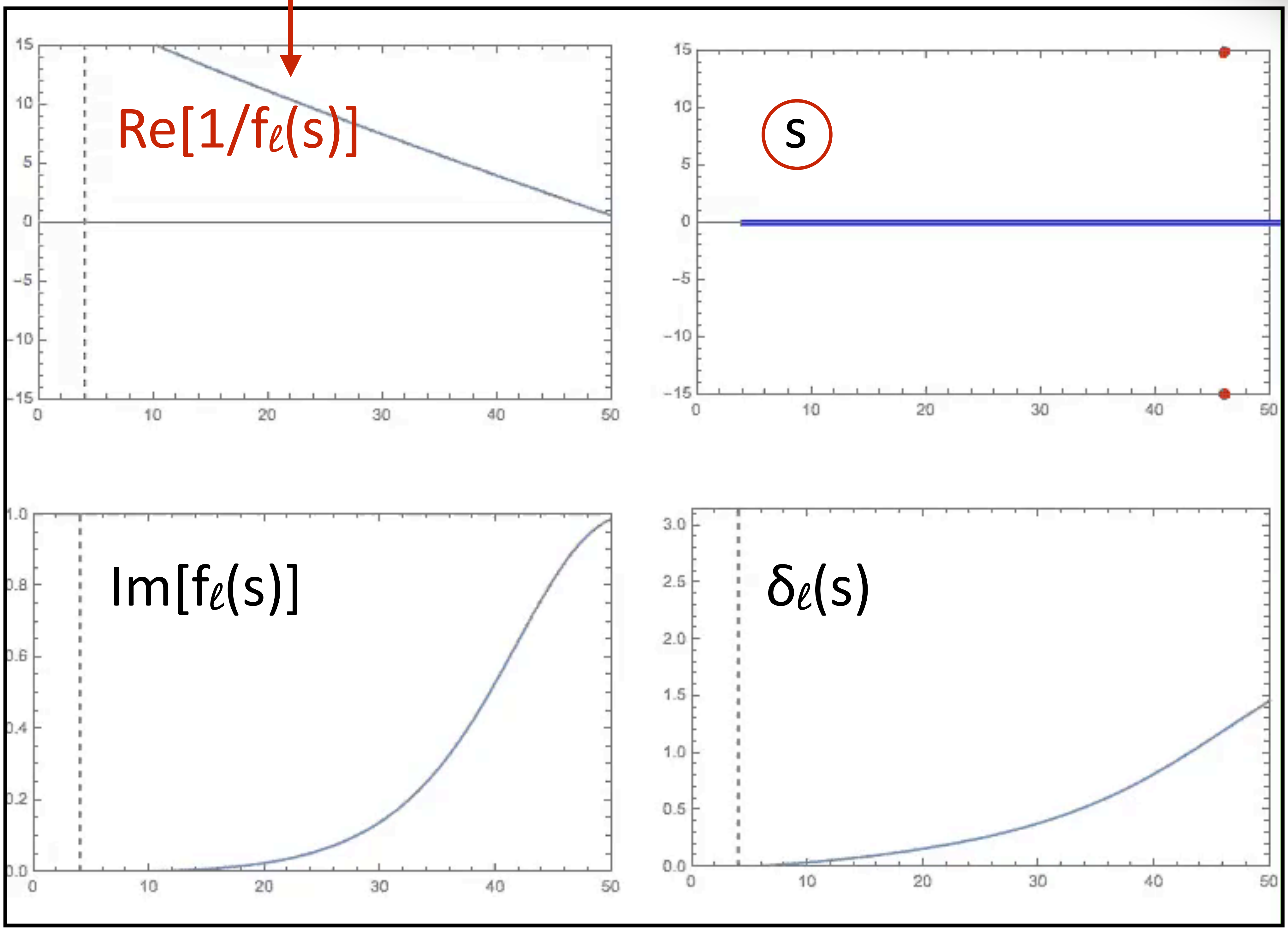
(lattice loop term)

Energy levels $E_n \to p\cot\delta \to \delta(E_n)$

Resonance or bound state



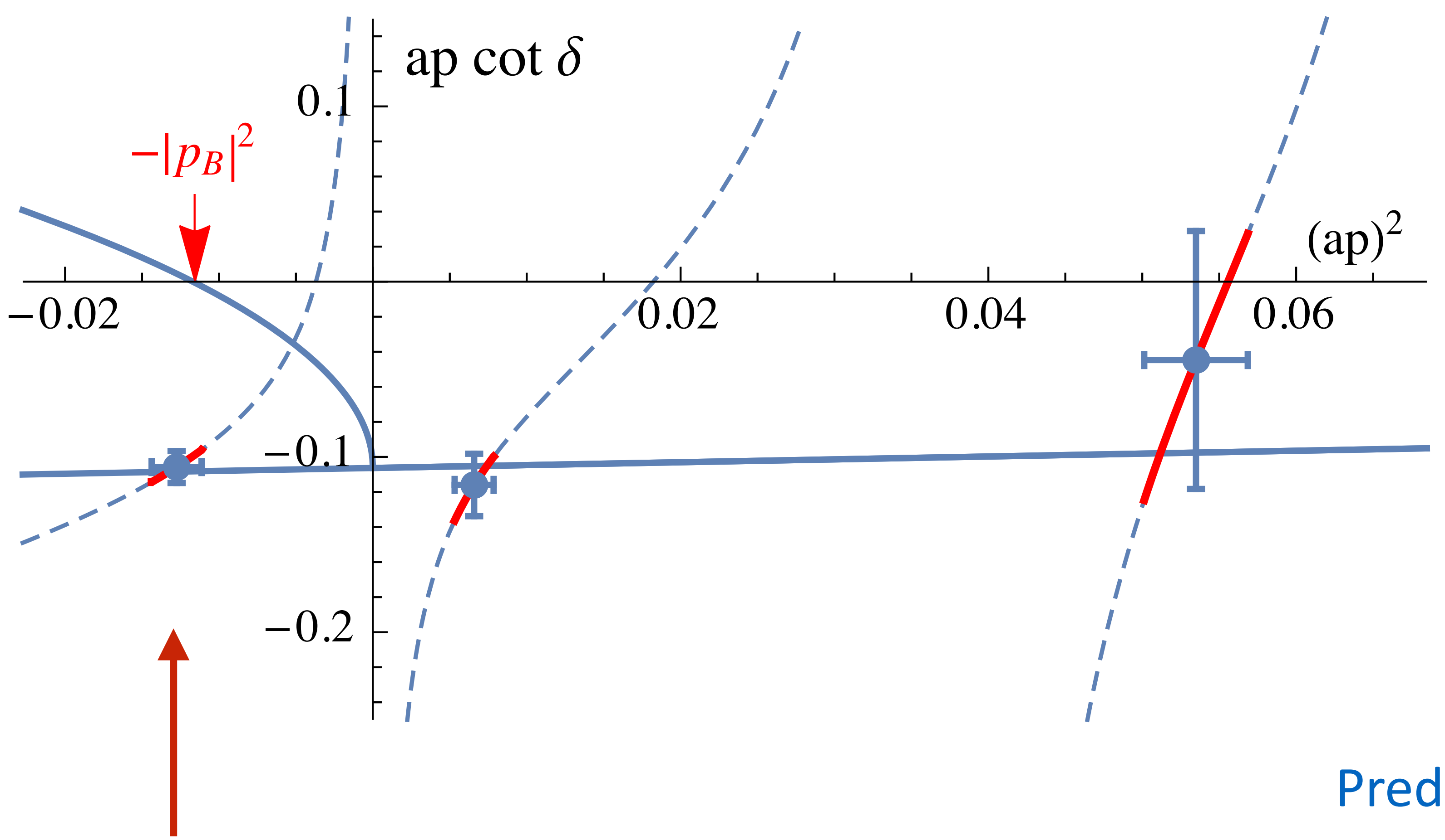
This is K^{-1}



- pole 2[nd] sheet
- pole 1[st] sheet

Continuation below threshold: Bound states

Example: BK scattering in $J^{PC}=0^{++}$ near threshold

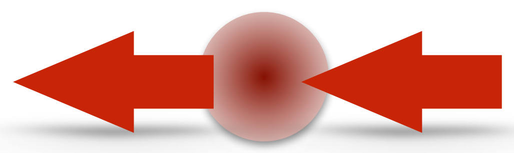


Bound state B_{s0} with
$m(B_{s0})$ = 5.711(13)(19) GeV

(E_B=64(13)(19) MeV)

Predicting positive parity B_S mesons from lattice QCD
CBL, Mohler et al., Physics Letters B 750 (2015) 17

Lattice tools: correlation functions

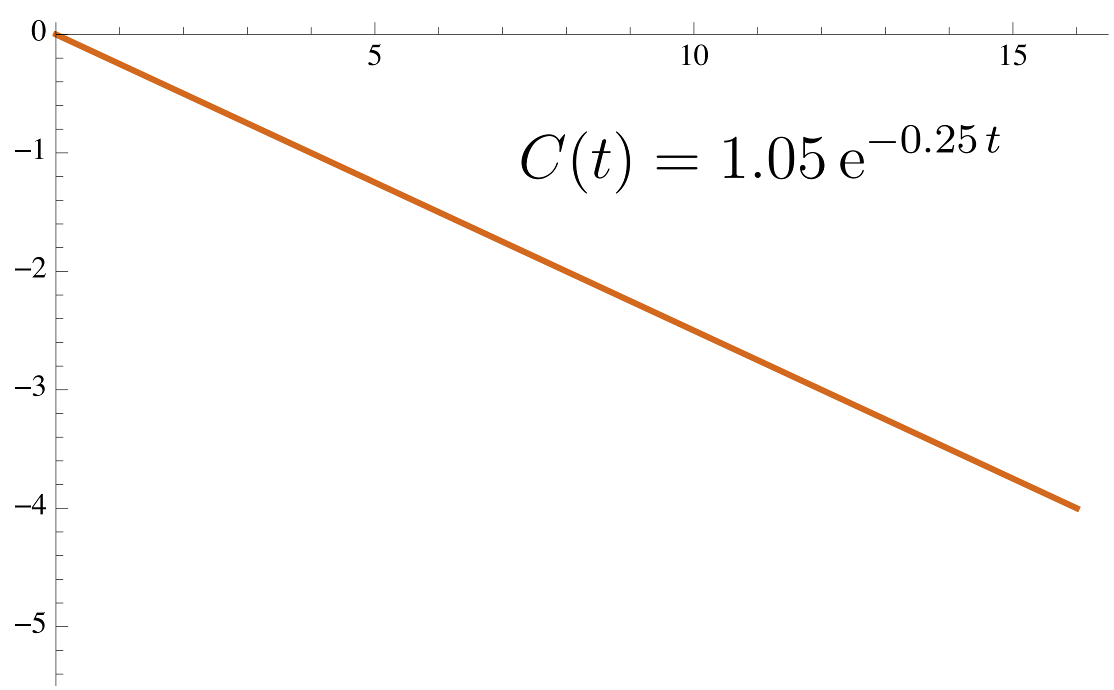
$X_i(t)$  $\overline{X}_j(0)$

$$C_{ij}(t) \equiv \langle X_i(t)\overline{X}_j(0)\rangle = \sum_n \langle X_i|n\rangle\, \mathrm{e}^{-t\,E_n}\,\langle n|\overline{X}_j\rangle$$

X_i lattice operator

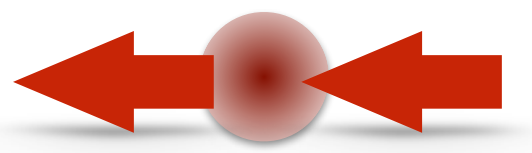
n “physical” eigenstate

$Log\ C(t)$



slope=
energy of
ground state

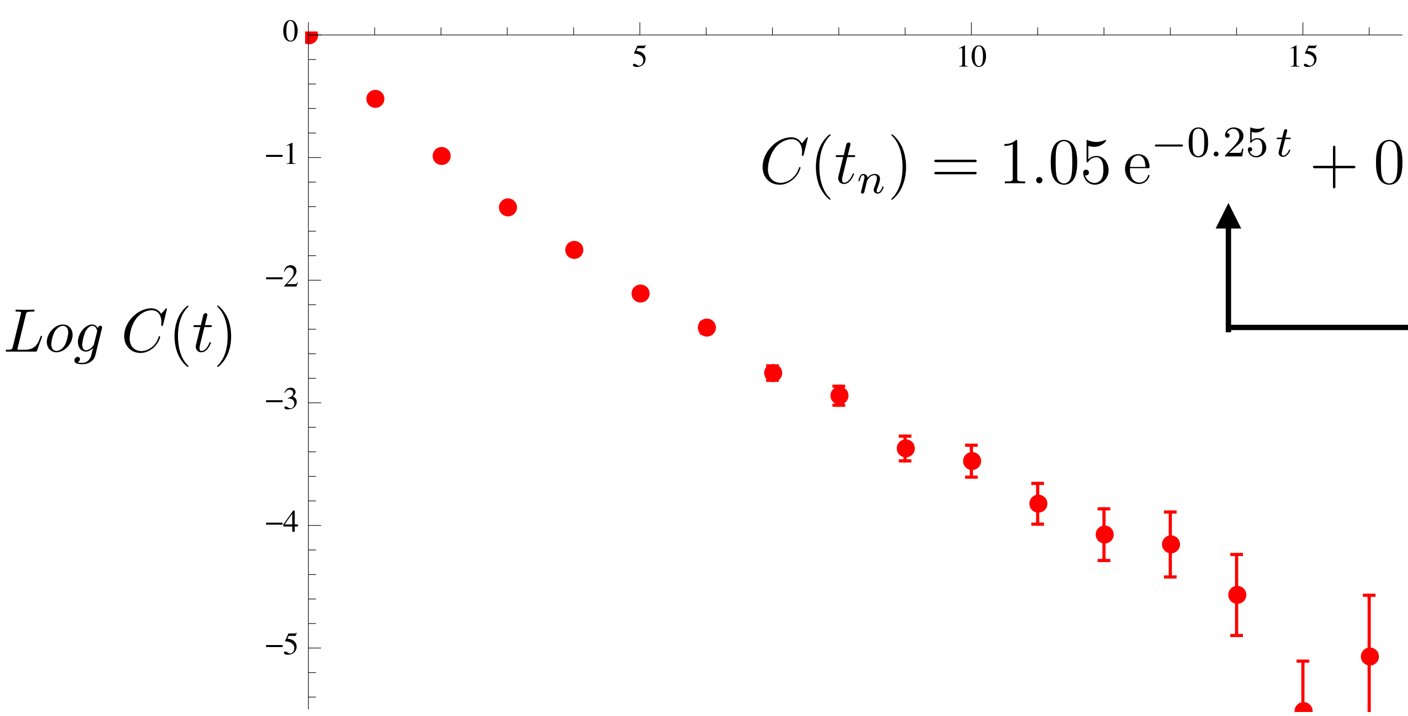
Lattice tools: correlation functions

$X_i(t)$  $\overline{X}_j(0)$

$$C_{ij}(t) \equiv \langle X_i(t)\overline{X}_j(0)\rangle = \sum_n \langle X_i|n\rangle \, \mathrm{e}^{-t\,E_n} \, \langle n|\overline{X}_j\rangle$$

X_i lattice operator

n “physical” eigenstate



$$C(t_n) = 1.05\,\mathrm{e}^{-0.25\,t} + 0.78\,\mathrm{e}^{-0.55\,t} + 0.54\,\mathrm{e}^{-0.85\,t} + \ldots + \text{noise}$$

energy levels

slope=
energy of
ground state

How to get the energy levels?

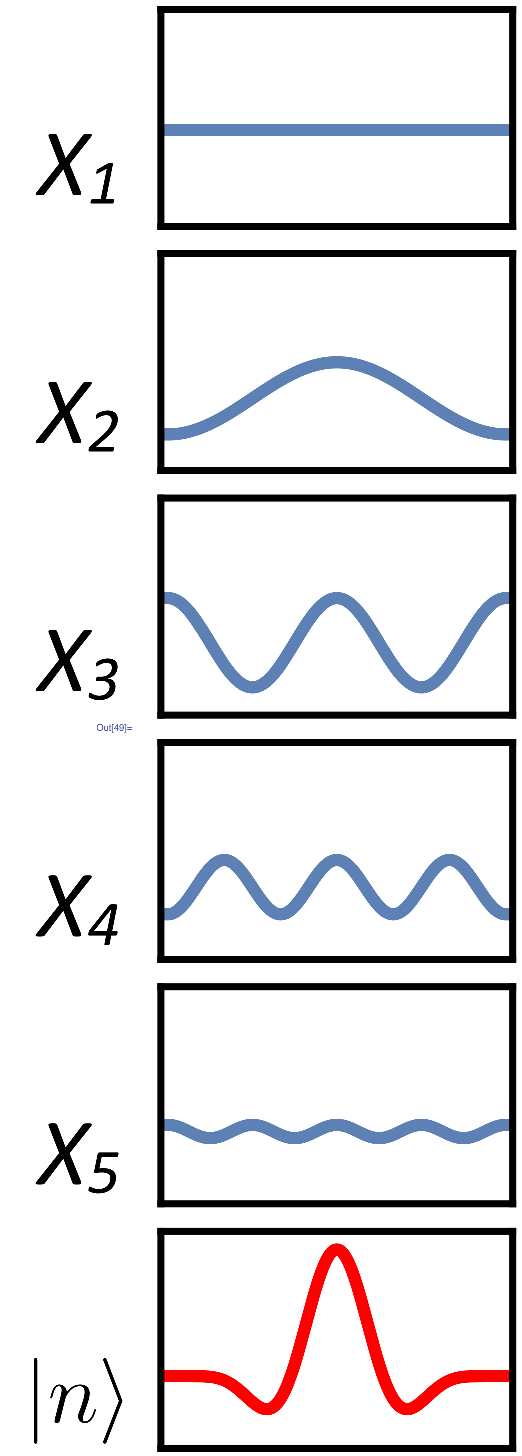
Lüscher, Wolff: NPB339(90)222
Michael, NPB259(85)58
See also Blossier et al., JHEP0904(09)094

- Compute all cross-correlations for several lattice operators with the same quantum numbers:

$$C_{ij}(t) \equiv \langle X_i(t)\overline{X}_j(0)\rangle$$

- Solve the eigenvalue problem. The eigenvalues give the energy levels:

$$\lambda^{(n)}(t) \propto e^{-t\boxed{E_n}}\left(1+\mathcal{O}(e^{-t\Delta E_n})\right)$$



Results

Results

- First steps: Light quark elastic resonance region results

  $\pi\pi$ p-wave (ρ) and s-wave (σ)
  πK p-wave (K^*) and s-wave (κ)
  $\rho\pi(a_1)$, $\omega\pi$ (b_1)...
  phase shifts, scattering lengths

- Coupled channel formalism (beyond the elastic region)

  $(\pi\pi, KK)$, $(\pi K, \eta K)$

Carsten Urbach
David Wilson

- Heavy-heavy and heavy-light quark mesons near thresholds

  $\psi(3770)$,X(3872), Z(3900)
  D_s and DK, D*K
  B_s and BK, B*K

Daniel Mohler

- Meson-baryon resonances

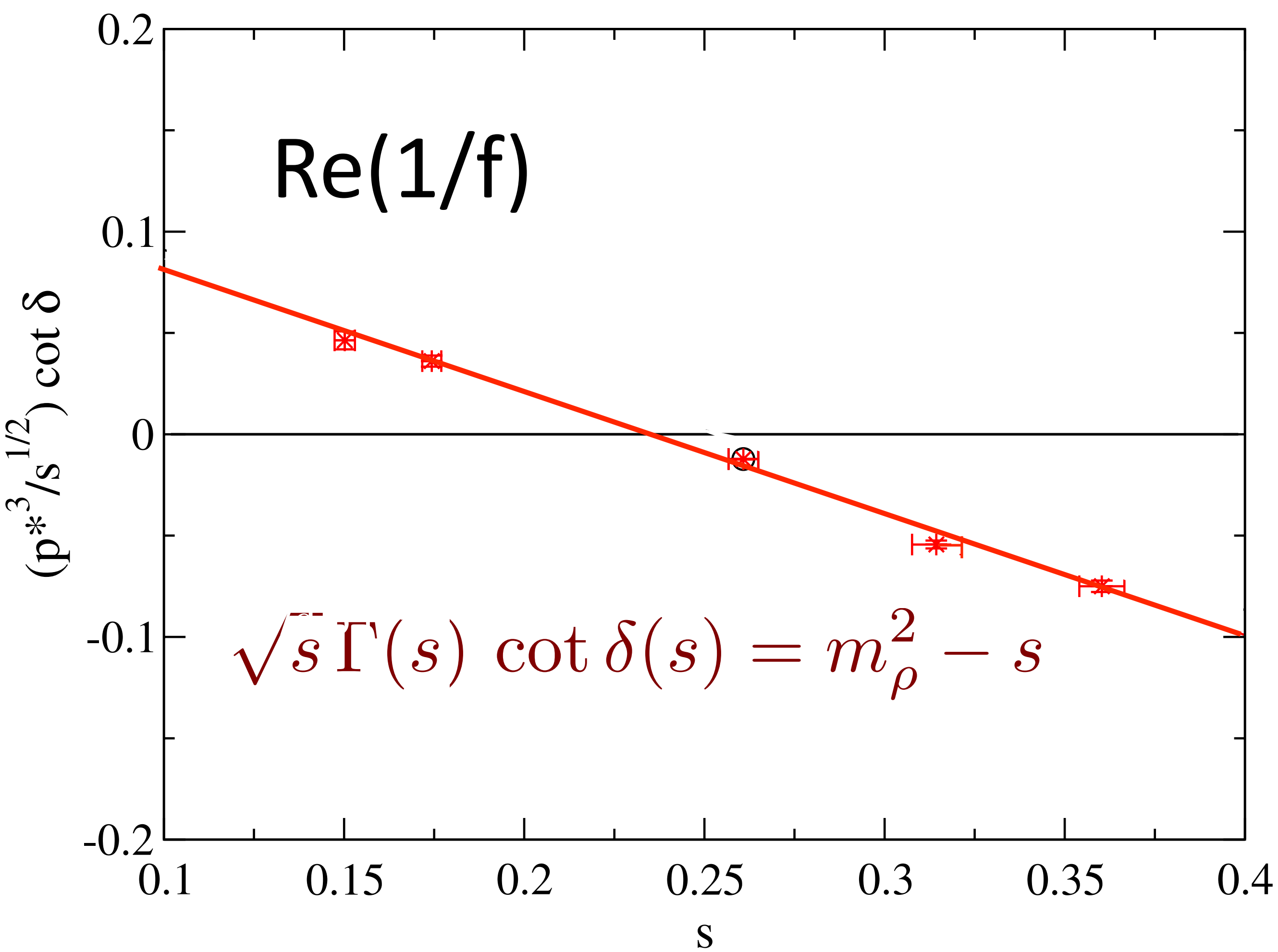
  $N\pi$

Mesons

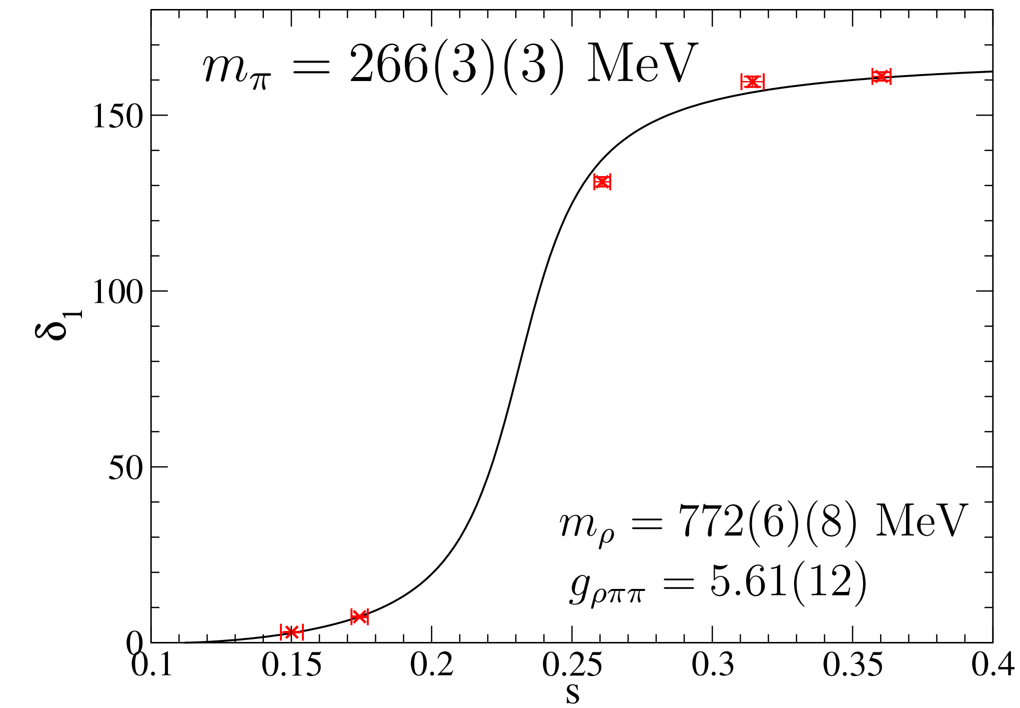
Example: ππ ➝ ρ ➝ ππ

2011 2012 2013 2014 2015

Elastic region



CBL, Mohler, Prelovsek, Vidmar;
PR D **84**, 054503 (2011)
Erratum PR D 89 (2014) 059903(E)



Up to 18 ρ and ππ operators
P=(000), (001), (011)

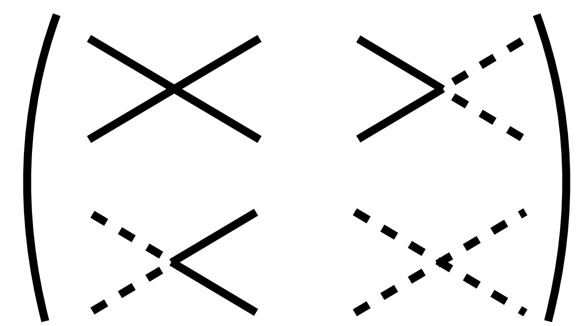
Beyond the elastic region: coupled channels

"..to boldly go, where.."

Extension to several coupled channels

Matrices T, Z:

$$\boxed{\det\left[T^{-1} - Z\right] = 0}$$



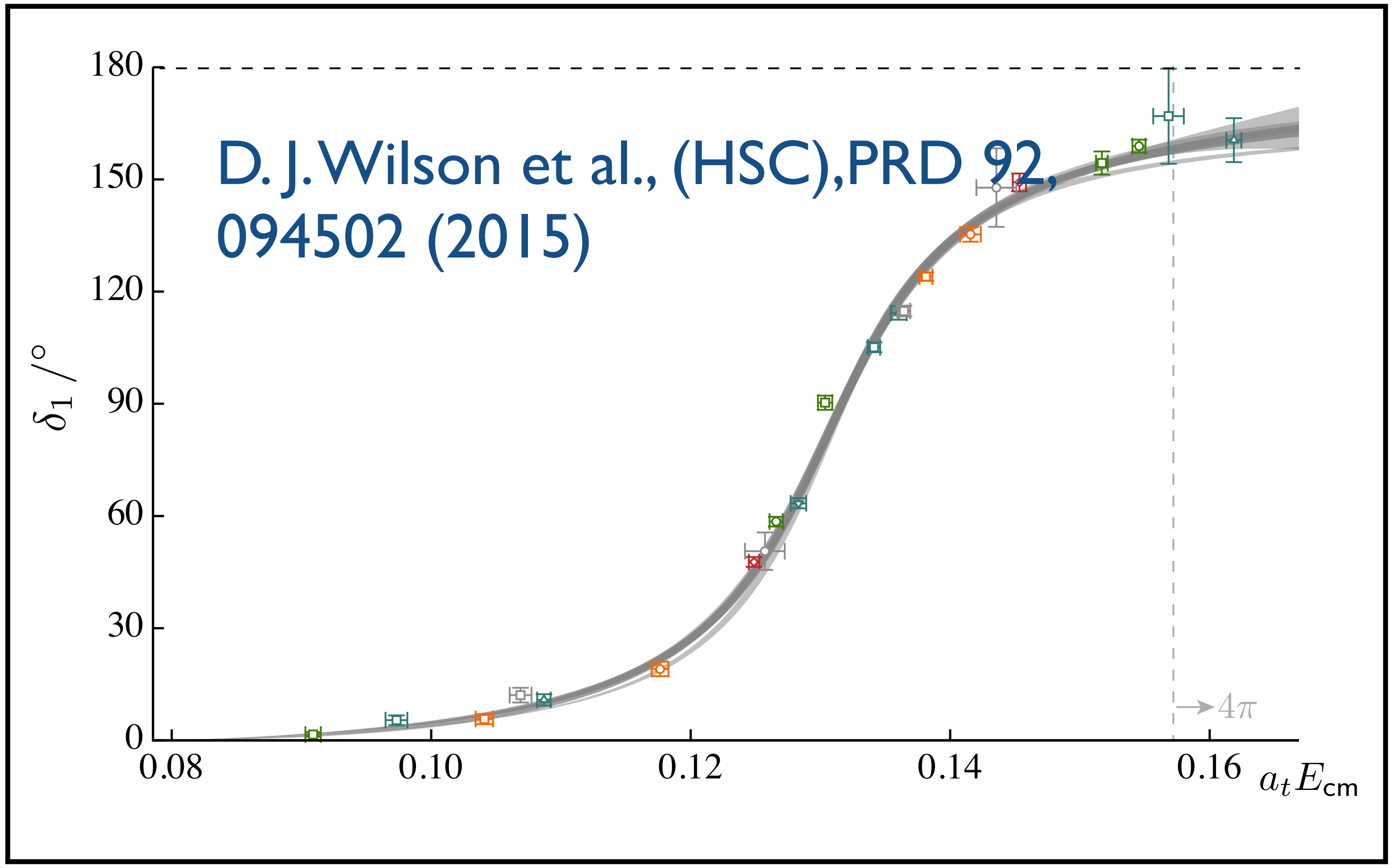
Döring et al., Eur. Phys. J. A 47, 139 (2011)
Bernard et al ., JHEP 1101 (2011) 019
Briceno et al ., PR D 88, 034502 (2013)
Briceno et al , PR D 88, 094507 (2013)
Briceno et al ., PR D 89, 074507 (2014)

coupled channels

two nucleons
moving multichannels
arbitrary spin

Example: (ππ,KK) ➤ ρ ➤ (ππ,KK)

201 201 201 201 2015



$$m_\pi = 236(2)\ \text{MeV}$$
$$m_\rho = 790(2)\ \text{MeV}$$
$$\Gamma_\rho = 87(2)\ \text{MeV}$$
$$g_{\rho\pi\pi} = 5.69(7)(3)$$
$$g_{\rho\pi\pi,exp} = 5.96$$

ππ- s-wave R.A.Briceno et al.,PRL **118**, 022002 (2017)

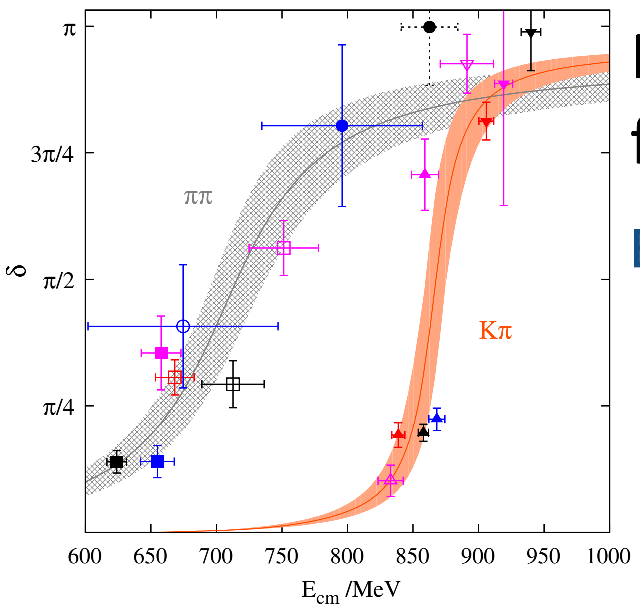
scattering lengths: L. Liu et al., arXiv 1612.02061

see talk by Carsten Urbach

(πK, ηK) D.J. Wilson et al. (HSC), PRD 91, 054008 (2015)

see talk by David Wilson

Extrapolating to physical pion mass

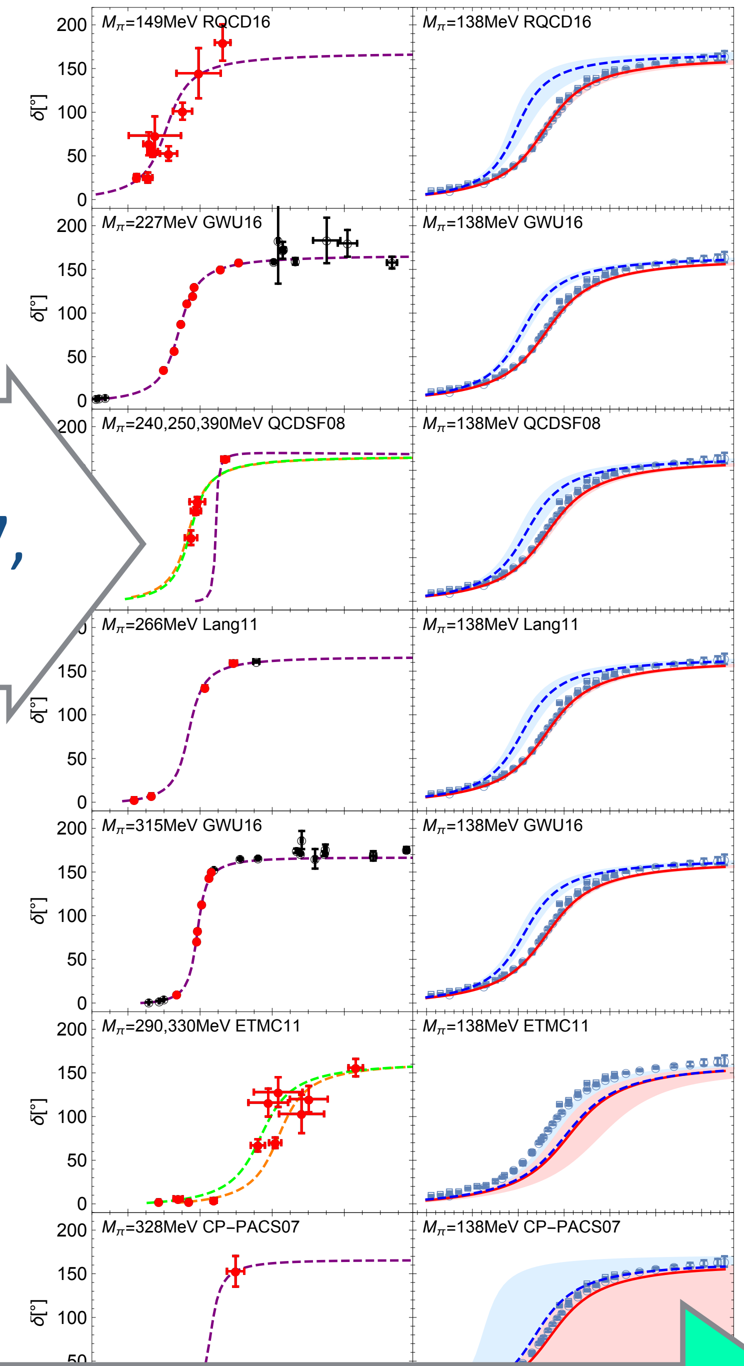


Recent result (n_f=2) for m_π=149 MeV

Bali et al., PRD 93, 054509 (2016)

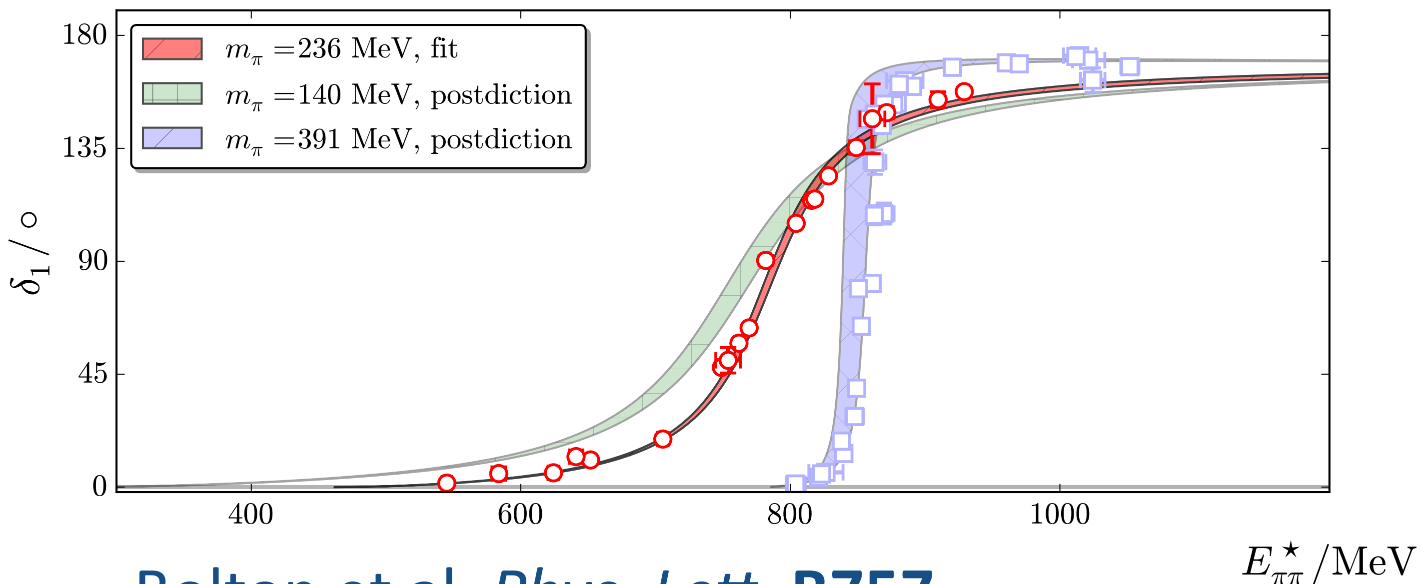
n_f =2

n_f =2 adding KK effects



ρ mass too small?

n_f dependence: Hu et al., PRL **117**, 122001 (2016)



Bolton et al. *Phys. Lett.* **B757**, 50 (2016)

for details ask Michael Döring

Issues

Lattice operator basis: is it sufficiently complete?

Coupled channels: need to combine different measurement, parametrisation, interpolation for K matrix elements

Resonance pole position: analytic continuation (parametrize Re T^{-1}, disp.rel.; left hand cut?)

Unstable decay products:

$a_1(1260) \to \rho\pi \to \pi\pi\pi$

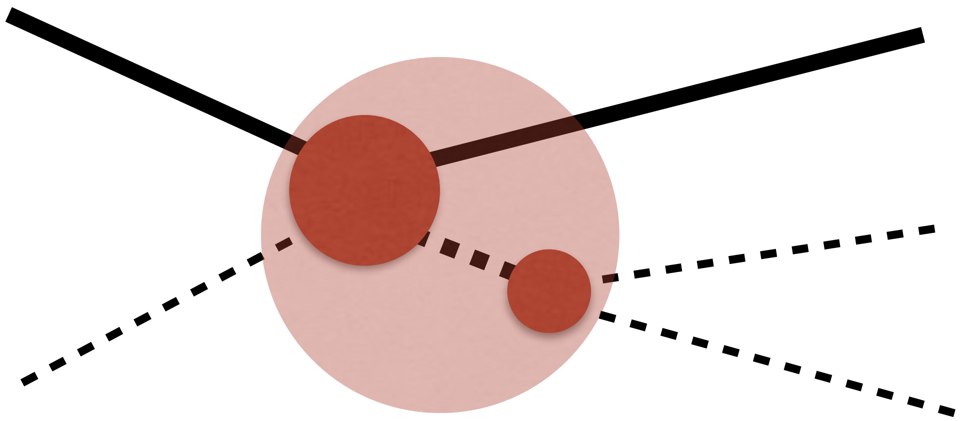
$b_1(1235) \to \omega\pi \to \pi\pi\pi\pi$

$N\pi \to \text{Roper} \to N\Delta \to N\pi\pi$

$\to N\rho \to N\pi\pi$

$\to Nf_0 \to N\pi\pi$

...3-particles



Extension to 3-particle channels

Hansen & Sharpe,PR D 90, 116003 (2014) [arXiv:1408.5933] quantization condition

Meißner et al.,PRL 114, 091602 (2015) [arXiv:1412.4969] shallow bound states

Hansen & Sharpe, Phys. Rev. D92, 114509 (2015) [arXiv:1504.04248]

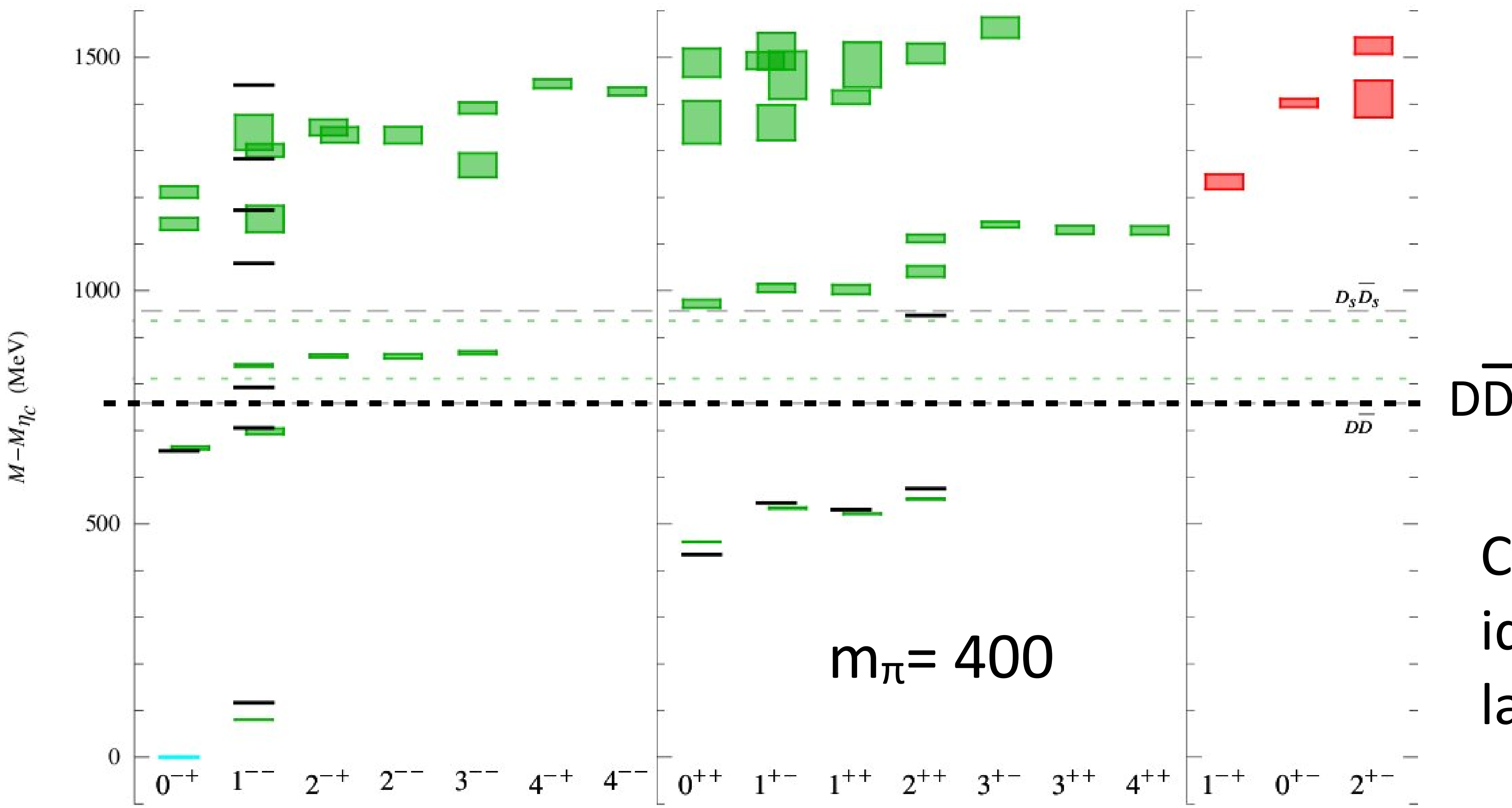
Rusetsky, *PoS* LATTICE2015, 279 (2016) [*arXiv*:1510.01206]

Hansen & Sharpe, PR D 95, 034501 (2017)

But: No numerical lattice results yet!

Heavy quarks: Charmonium

L. Liu et al. (HSC), JHEP 1207, 126 (2012), [1204.5425]

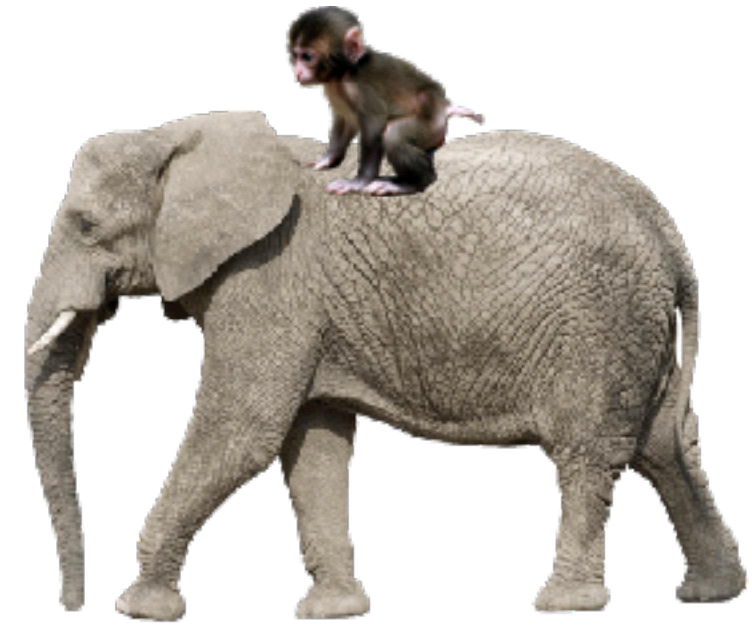


Challenge: Spin identification (from lattice irreps)

See also: Mohler et al., Phys.Rev. D 87, 034501 (2013) [arXiv:1208.4059]
Galloway et al. (HPQCD) PoS (LATTICE2014) 092 [arXiv:1411.1318]
(extrapolation to physical point)

Heavy quark results (with meson-meson operators)

Heavy-Heavy $\mathbf{c\,\bar{c}}$



Heavy-Light $\mathbf{c\,\bar{s},\ b\,\bar{s}}$

What is the effect of nearby thresholds?

Example: $D\bar{D}$ threshold and $\psi(3770)$

Example: $\bar{D}K$ and $\bar{D}^*K$ in $D_{sn}^{(*)}$

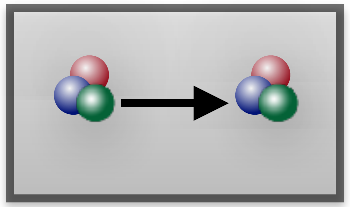
Are there new states?

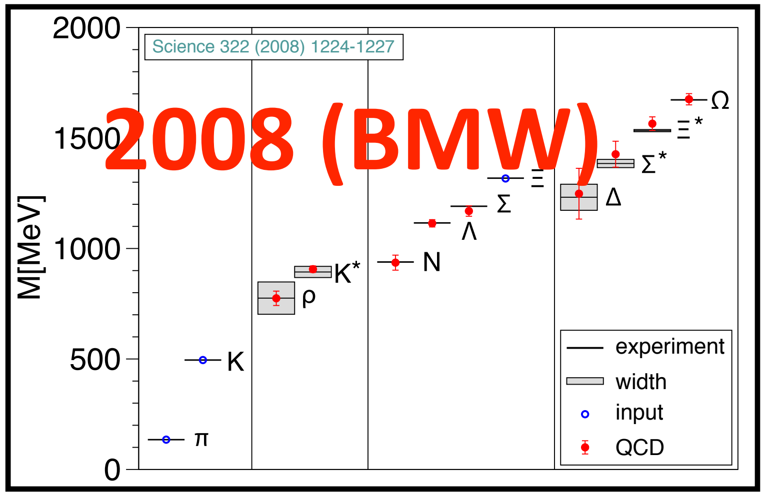
Example: "level hunting": X(3872), Z(3900), X(5568)

Example: $\bar{B}K$ and $\bar{B}^*K$ in $B_{sn}^{(*)}$

see talk by Daniel Mohler

Baryons

Baryons in  approximation



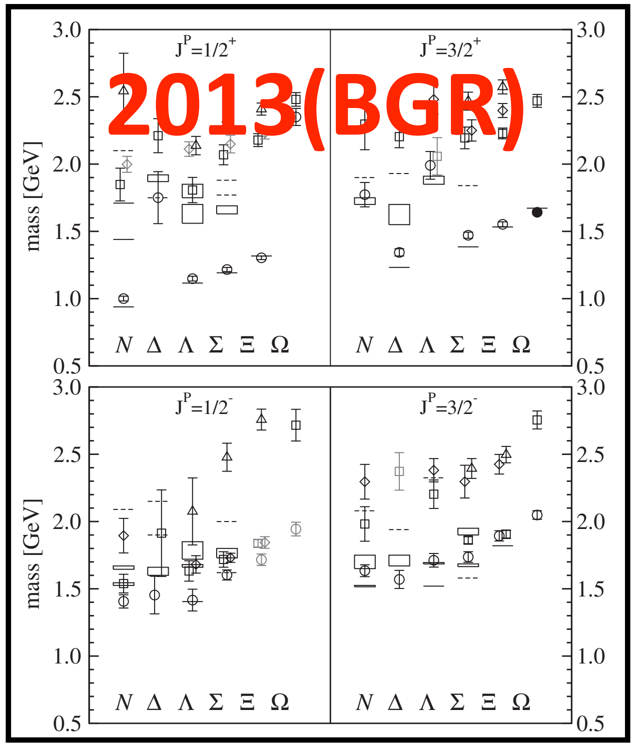
Zillion LQCD papers, mainly ground states

Namekawa et al. (PACS-CS), PRD 87, 094512 (2013)

Alexandrou et al.(ETMC), PRD 90, 074501 (2014).

Brown et al. PRD 90, 094507 (2014).

Rubio et al. (RQCD), PRD 92, 034504 (2015)



Engel et al., PRD 87, 074504 (2013)

Hadron Spectrum Collaboration et al.: Excited levels

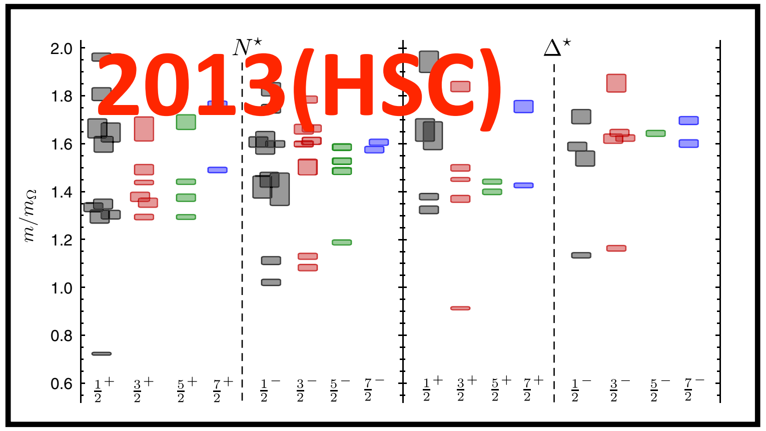
ccx Baryons

Padmanath et al., Phys.Rev. D91 (2015) 094502

Spin identification up to 7/2

ccc Baryons

Padmanath et al., Phys.Rev. D90 (2014) 074504



Nucleon-nucleon scattering

Berkowitz et al. (CalLat),*Phys. Lett.* **B765**, 285-292 (2017)

m_π=800 MeV (u,d,s flavor symmetric limit)
spatial extent up to 4.6 fm
partial-waves: S, P, D, F

no annihilation diagrams
(no backtracking quarks)!

Alarcón, et al. (2017) [*arXiv*:1702.05319]
Nucleons on the lattice:
np scattering with Nuclear Lattice Effective Field Theory

Iritani, T (HAL QCD) (2016) [arXiv:1610.09779]
ΞΞ interactions; criticism on the direct (Lüscher) method
vs. the HAL QCD potential method (Ishii et al., PRL 99 (2007) 022001)

Meson-Baryon without annihilation diagrams

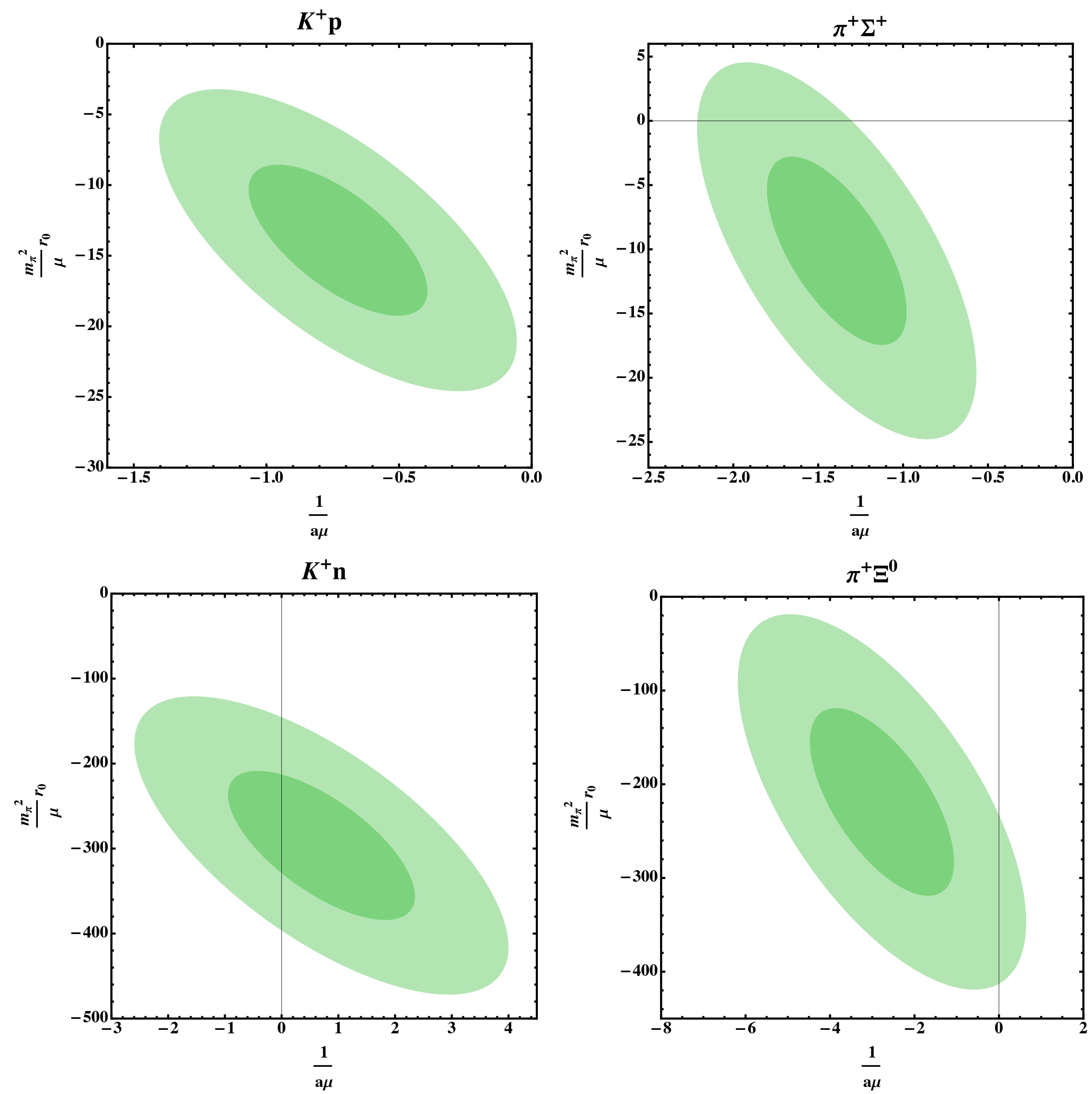
Low energy scattering phase shifts for meson-baryon systems:

Detmold & Nicholson, PRD 93, 114511 (2016) arXiv:1511.02275

$\Xi\pi^+$, $\Sigma^+\pi^+$, pK^+, nK^+

m_π=390 MeV and 230 MeV

Chiral extrapolation:
threshold parameters



$\Delta \rightarrow N\pi$

Alexandrou et al., Phys. Rev. D88, 031501 (2013).[arXiv: 1305.6081]

Alexandrou et al. *PoS* LATTICE2015, 084 (2016); [arXiv:1511.02752]

$$\Delta \to \pi N\,, \quad \Sigma^* \to \pi\Lambda\,, \quad \Sigma^* \to \pi\Sigma\,, \quad \Xi^* \to \pi\Xi$$

Decay parameters from ratio
$$\frac{C^k_{B^*-MB}(t_f - t_i, \vec{q})}{\sqrt{C_{B^*-B^*}(t_f - t_i) \times C_{MB-MB}(t_f - t_i, \vec{k}_f, \vec{k}_i)}}$$

(suggested by McNeile *et al.* [UKQCD], Phys. Rev. D 65 (2002) 094505 [hep-lat/0201006].

No phase shift results!

Pion-nucleon scattering

For a recent disp.rel.approach, see
Hoferichter et al.,Phys. Rev. Lett. 115, 092301 (2015) and Phys. Rept. 625 (2016) 1

πN (1/2⁻): Effect of open 2-hadron channel?

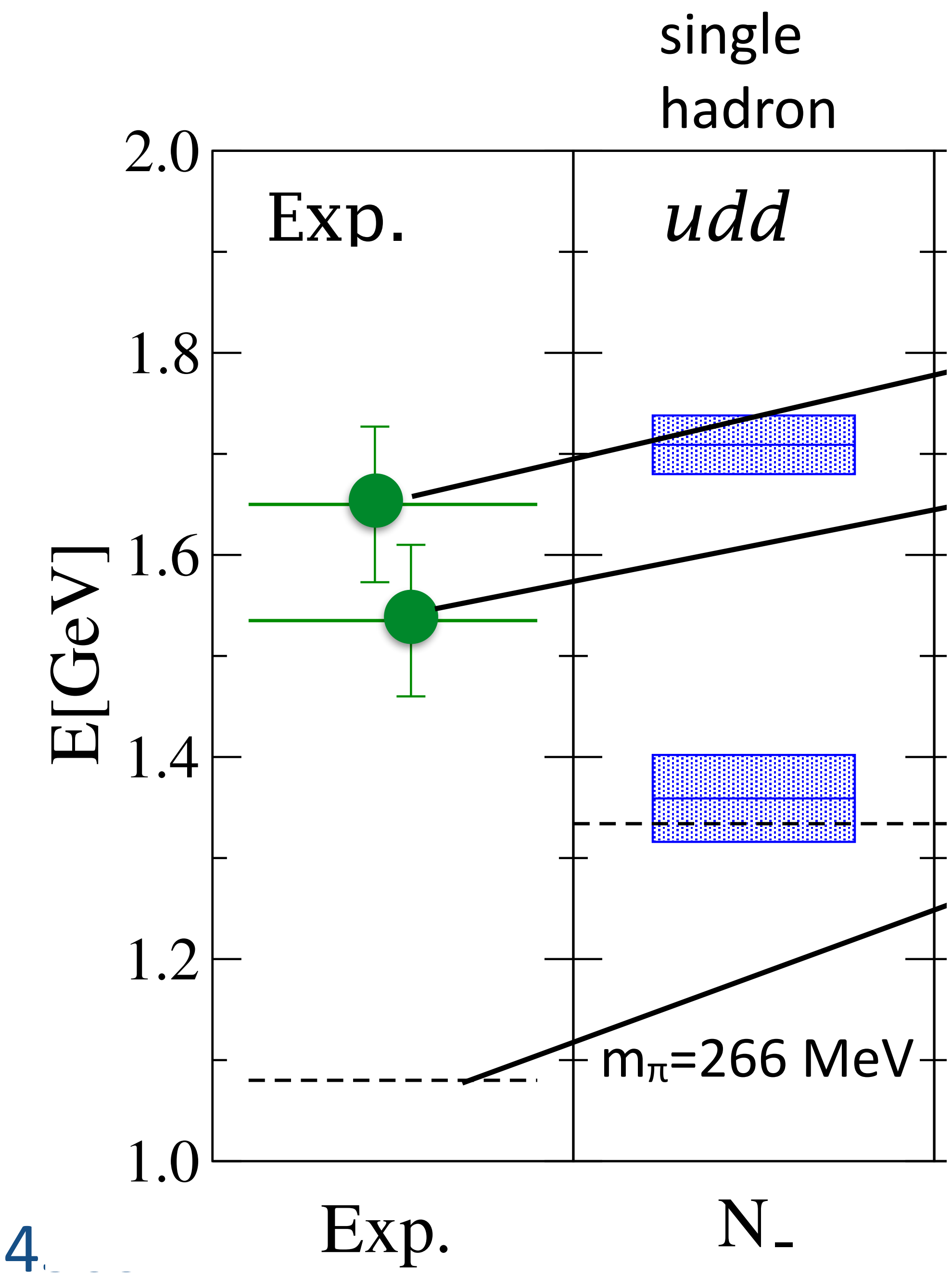
N*(1535),N*(1650)

Nπ negative parity

4502

91, 094

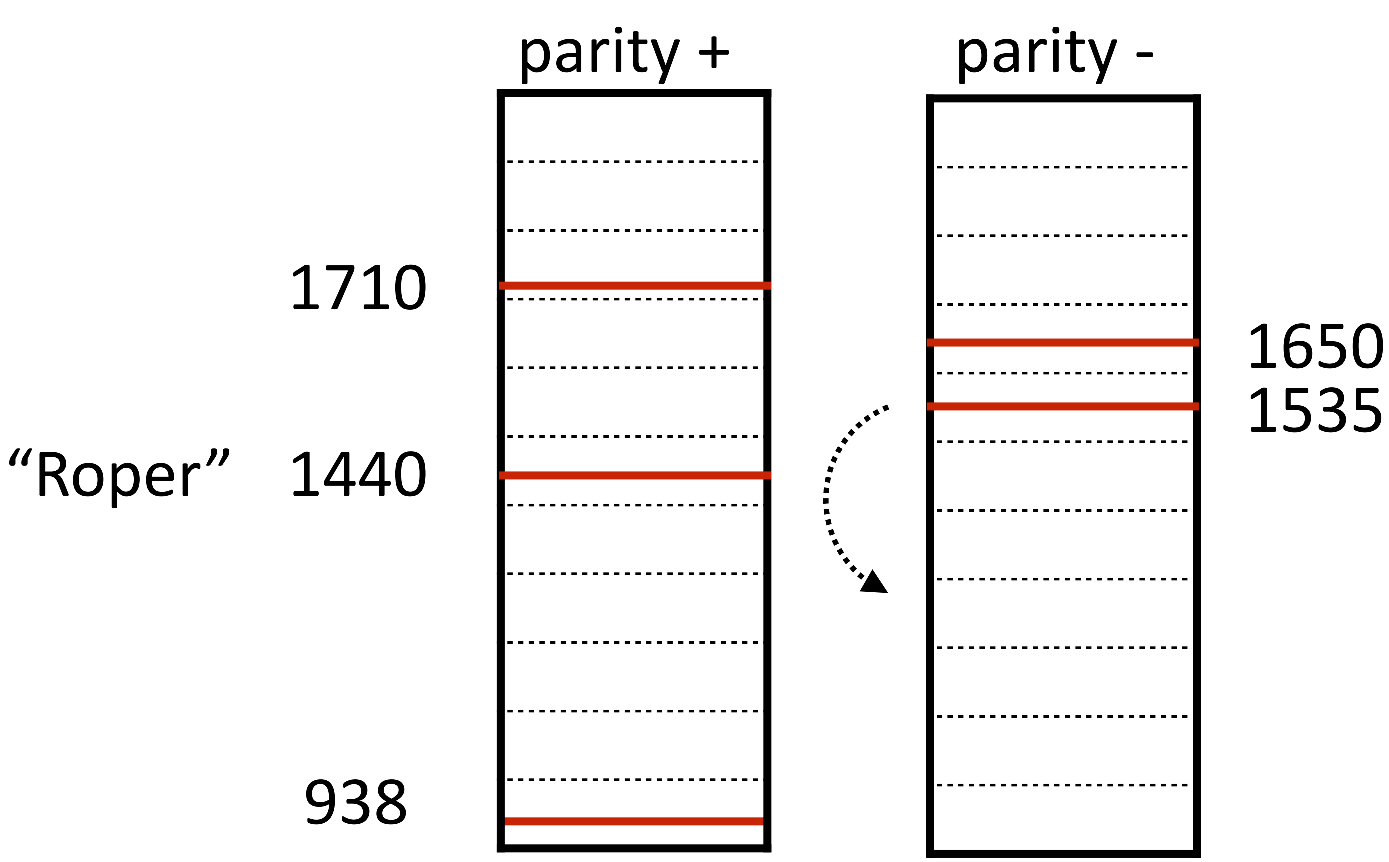
7667]



πN ($1/2^+$): And what about the Roper?

$N\pi$ in p-wave

baryon	*width*	*decay*	*%*
p(938)	-	-	-
N(1440)	250-450	$N\pi$	75–52
		$N\pi\pi$	30-40
		through $\Delta\pi$	20-30
		through $N(\pi\pi)_s$	10-20
N(1710)	50-250	$N\pi$,$N\omega$,$N\eta$,ΛK, $N\pi\pi(\Delta\pi,n\rho,..)$..	



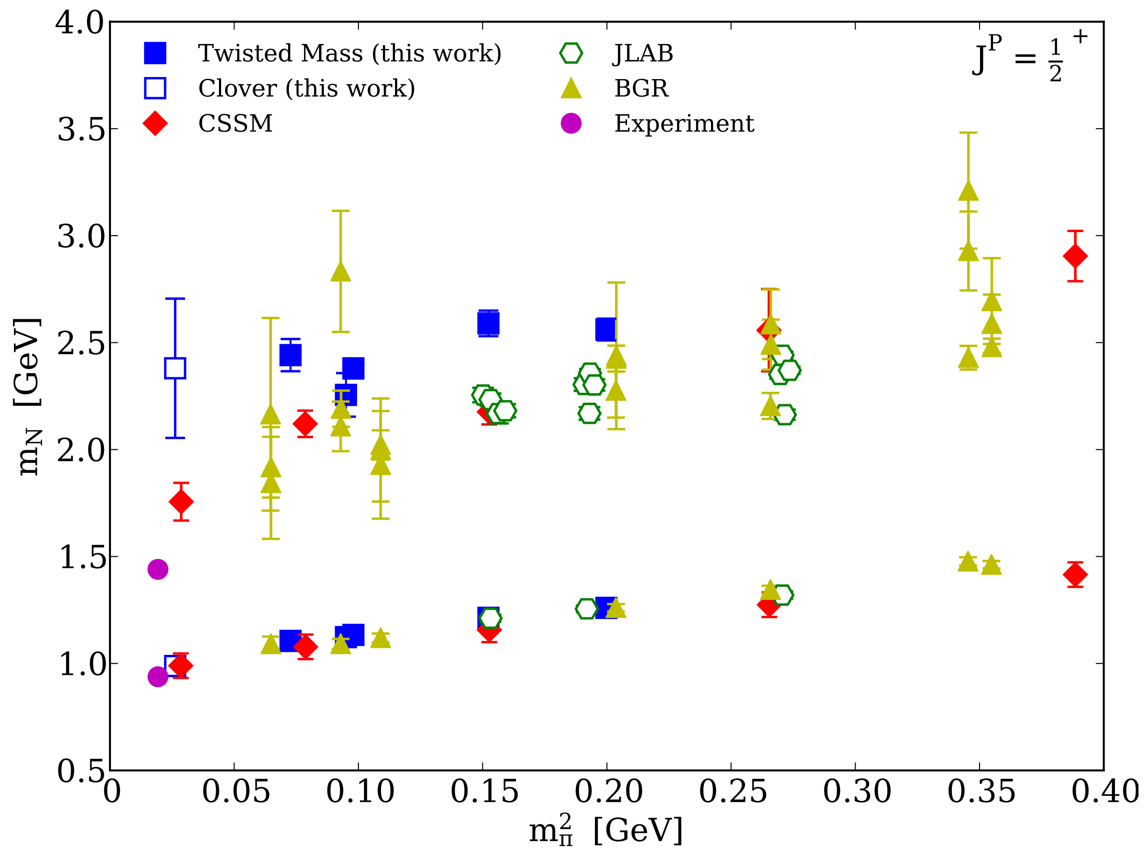
Parity puzzle:

why not + - + - ?

πN (1/2$^+$): And what about the Roper?

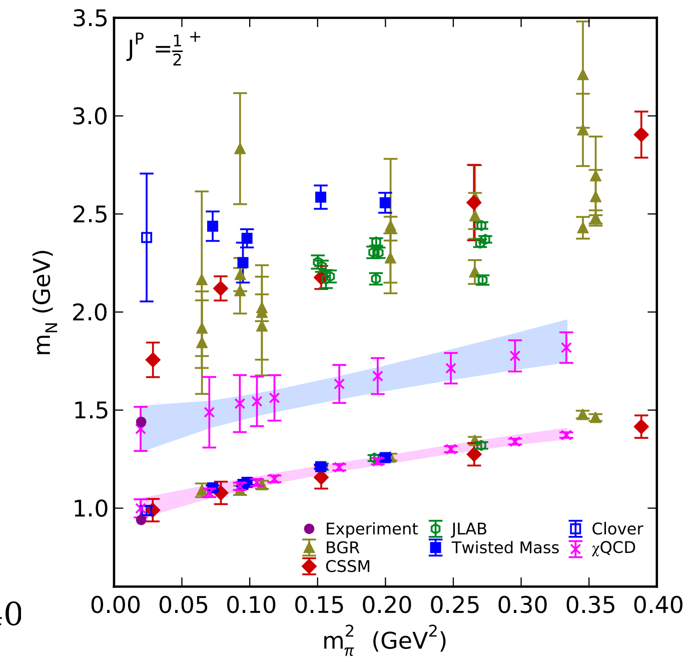
Roper Technologies

Up to now almost all studies only in single hadron (qqq) approximation (e.g. CSSM,χQCD, HSC, ETMC, BGR)

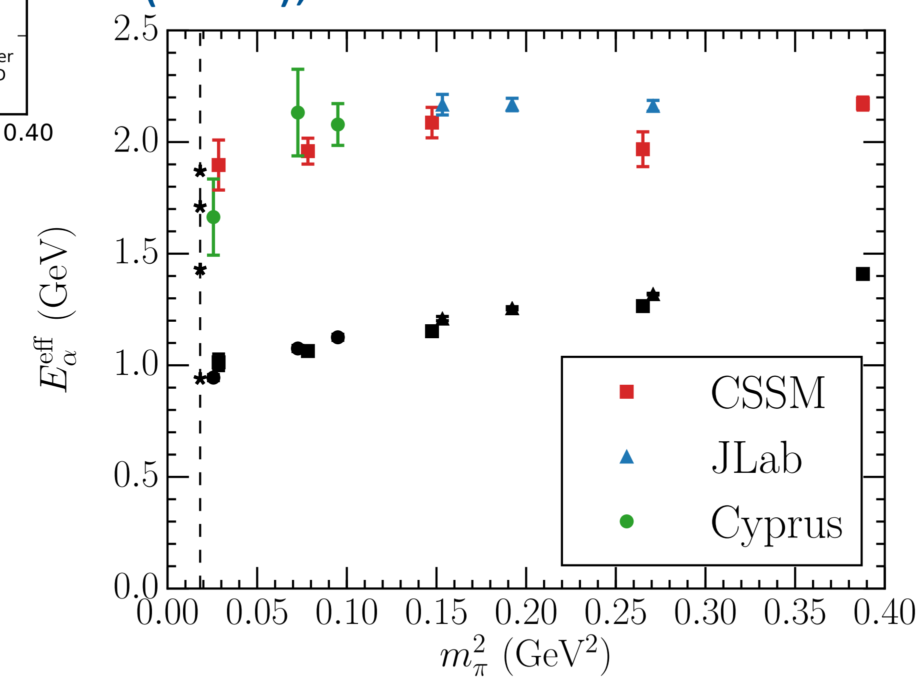


Alexandrou et al.[ETMC],Phys. Rev. D89, 034502 (2014) [arXiv:1302.4410]

Liu et al. [χQCD],*PoS* LATTICE2013 (2014) 507, [arXiv:1403.6847].



Survey:Leinweber et al. (2015), arXiv:1511.09146



Typical result: “Roper” level too high

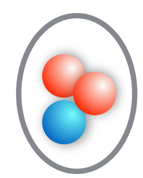
Pion-nucleon scattering with qqqq$\bar{q}$ operators

Lattice baryon spectroscopy with multi-particle interpolators
Kiratidis et al.,[CSSM] Phys. Rev. D 91, 094509 (2015).
arXiv:1501.07667

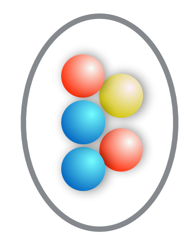
Search for low-lying lattice QCD eigenstates in the Roper regime
Kiratidis, et al.,[CSSM] (2016); arXiv:1608.03051

Local operators used

(Stochastic source method)

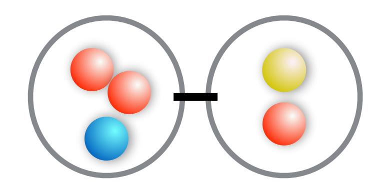


(udu)(x)



(udu$\underline{d}$u)(x)

instead of



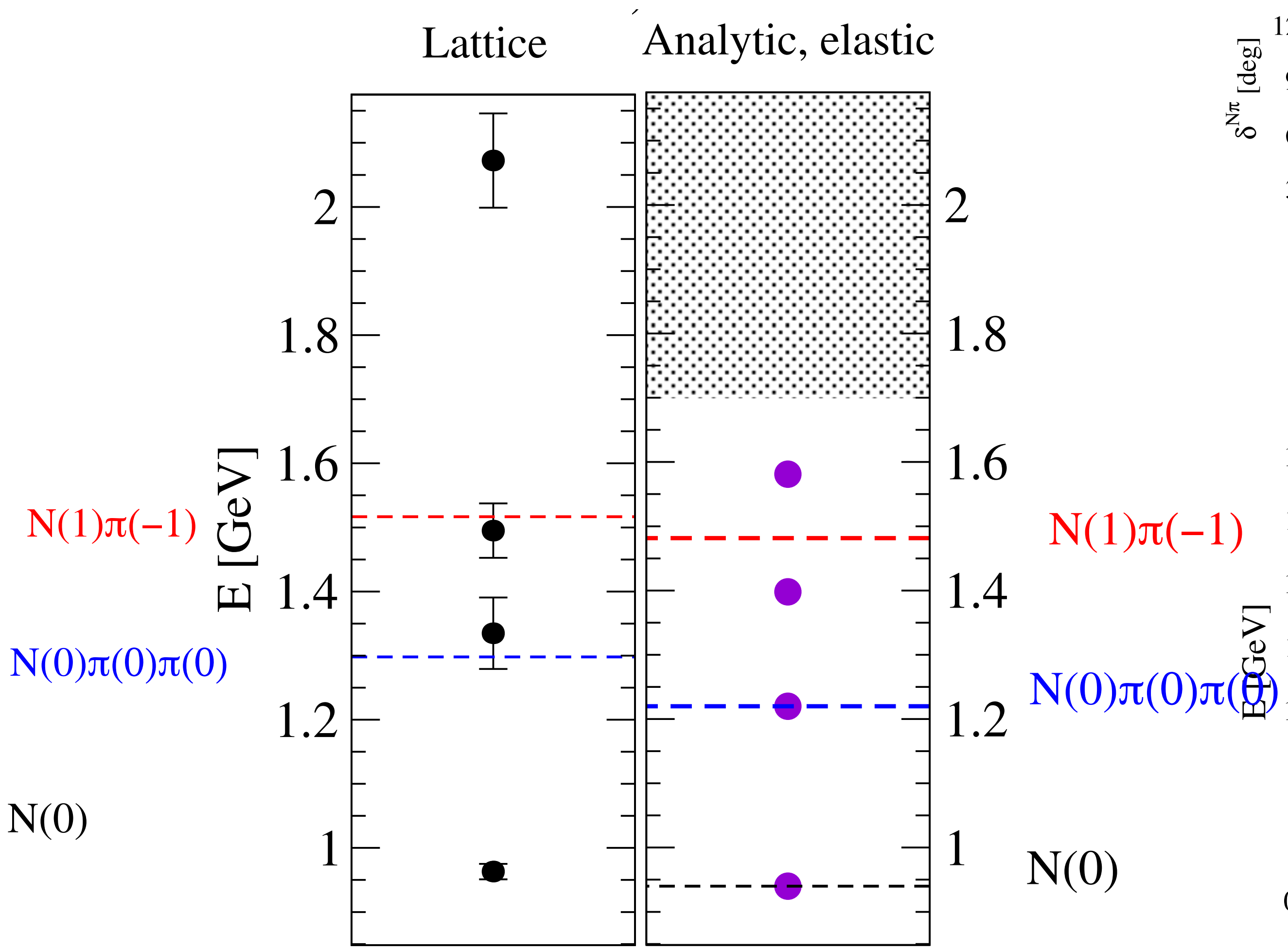
(udu)(p) ($\underline{d}$u)(-p)

$\boxed{\text{No N}\pi\text{ signal, no Roper!}}$

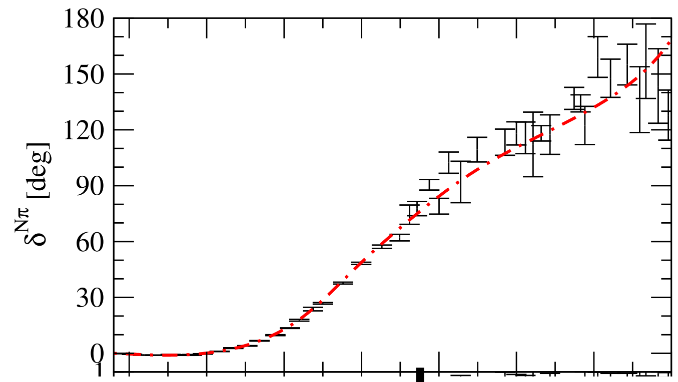
Beyond the single hadron approximation

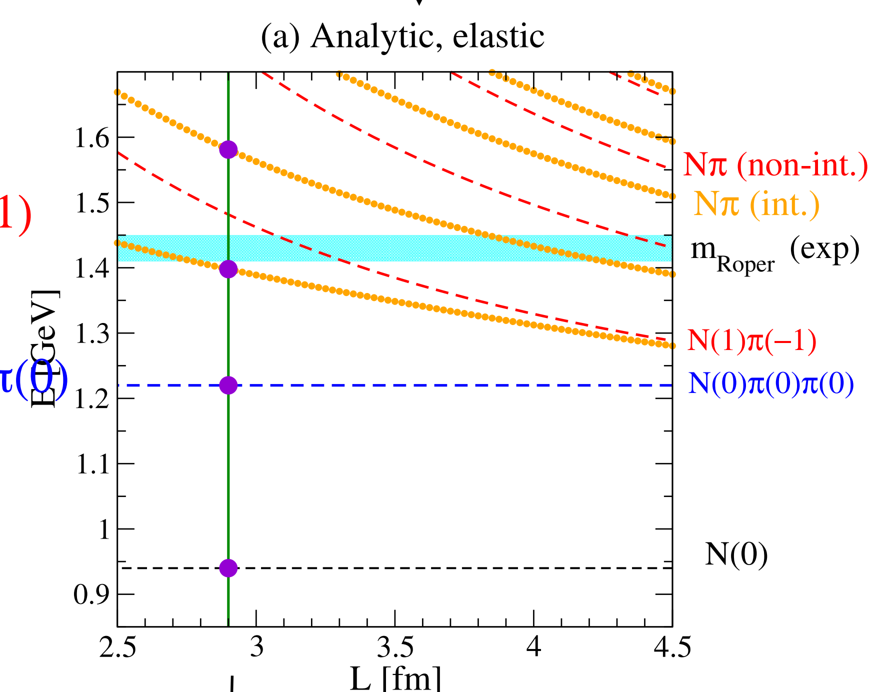
CBL et al. PRD 95 (2017) 014510 ;
arXiv:1610.01422

Operator basis ($1/2^+$) allows for:
N, Nπ (p wave), Nσ (s wave)
m_π=157 MeV



R. L. Workman et al. PRC 86, 035202 (2012)





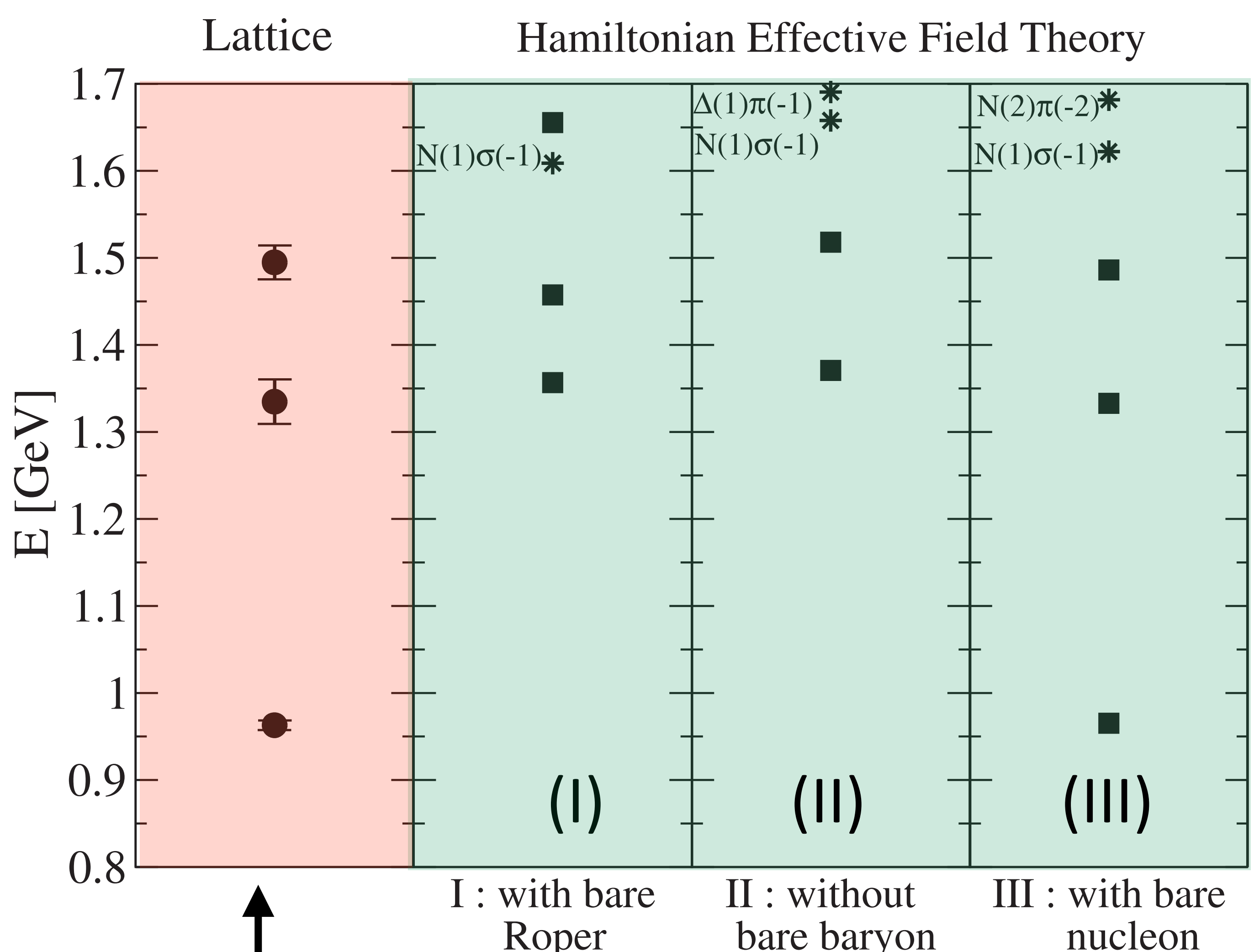
Nσ mimics Nππ

Comparison with HEFT model

Z.-W. Liu et al.(CSSM), PRD 95 034034 (2017); arXiv: 1607.04536.

Hamiltonian EFT (parameters fitted to exp. phase shift)

Coupled channels study



(I) Nπ,Δπ,Nσ+Roper

(II) Nπ,Δπ,Nσ

(III) Nπ,Δπ,Nσ+N

(Δ and σ assumed stable)

CBL et al. PRD 95 (2017) 014510 ; arXiv:1610.01422

Excuses

Why LQCD studies did not (yet) see the Roper?

- Coupled channel phenomenon; some channels missing ($N\pi\pi$,$\Delta\pi$)?
- Spatial size of operators and lattice?
- Pentaquark operator needed?
- Chiral symmetry?

Summary

- Simple hadron propagators give excited level - however, their positions are shifted: a decay channel study is needed.
- The Lüscher formalism (+extensions) becomes standard
- Light quark hadrons: threshold parameters and resonance phase shifts become available for 2→2 processes, approaching physical masses
- 2→3 in progress
- Heavy quarks: work nicely in simple cases but no multi-channel study yet; solves quark model puzzles.
- Baryons: some results for Nπ, but not always conclusive; in the near future I see no hope for trustworthy results > 1.7-2.0 GeV
- Model calculations help understand the LQCD results (and vice versa)


The End