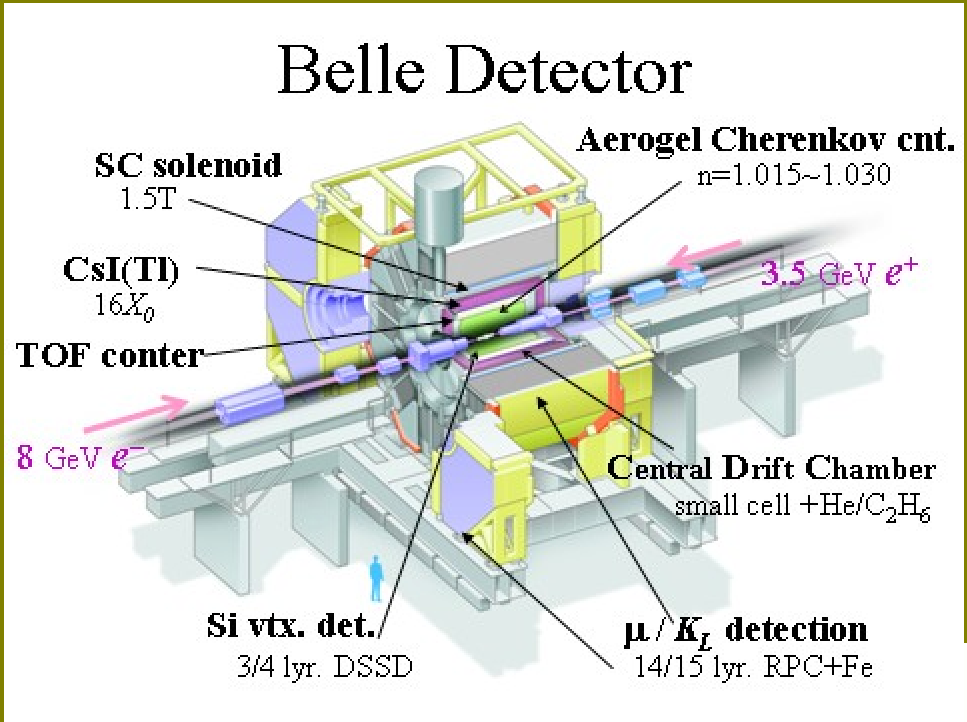
Multi-body amplitude analyses at Belle

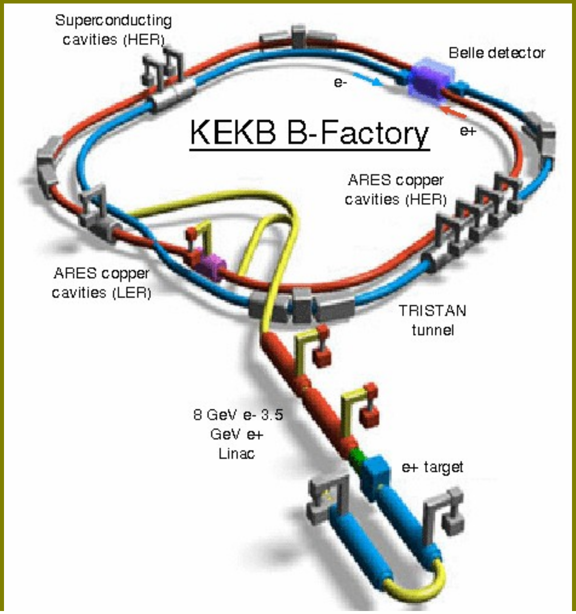
A.Kuzmin

BINP/NSU

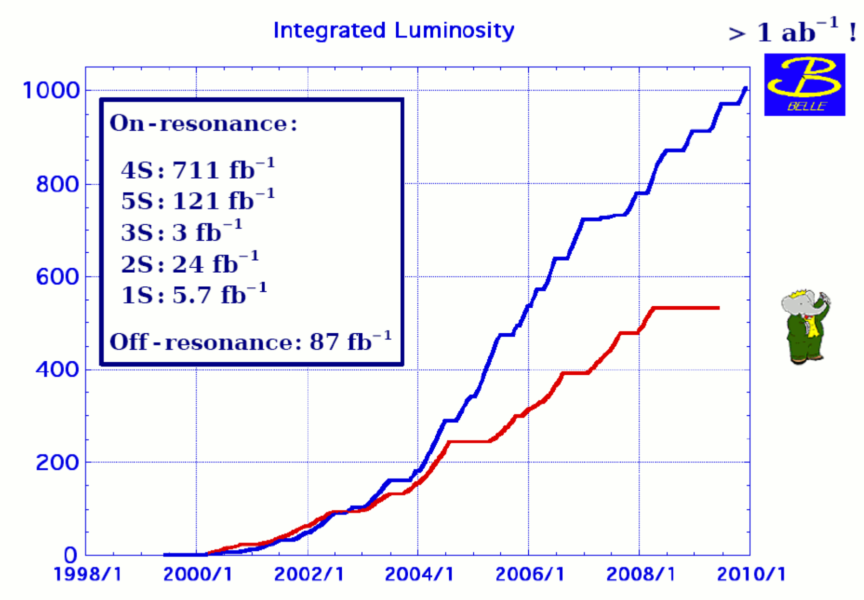
For Belle Collaboration

ATHOS 2017, March 13





- 3.5 GeV $e^+ \times 8.0$ GeV e^-.
- $\mathcal{L}_{\max} = 2.1 \times 10^{34} cm^{-2} s^{-1}$
- Continuous injection $\to 1.1\,\mathrm{fb}^{-1}/\mathrm{day}$.
- $\int \mathcal{L} dt \approx 1\, ab^{-1}$



Multidimensional DP

- 3-body decay amplitude of scalar particle is described by 2 Dalitz variables 3x3-4(4-momentum conservation)-3 (decay angles)=2.

- If initial particle has non-zero spin or number of final particles is more than 3, more variables are required to describe the decay.
- Approach is the same as in usual DP:

$$PDF(x_i)=N_s S(x_i)+N_b B(x_i)$$

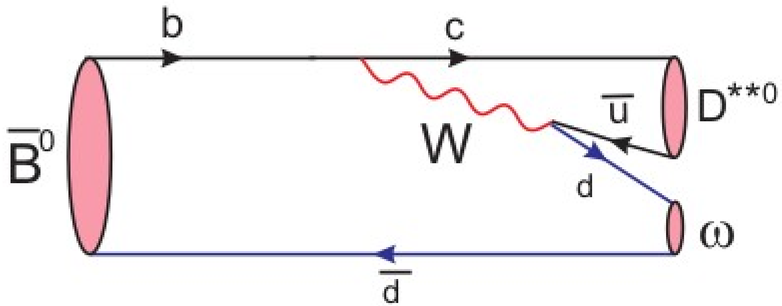
-Both binned or unbinned approach can be used

-More difficult visualization and goodness of fit determination

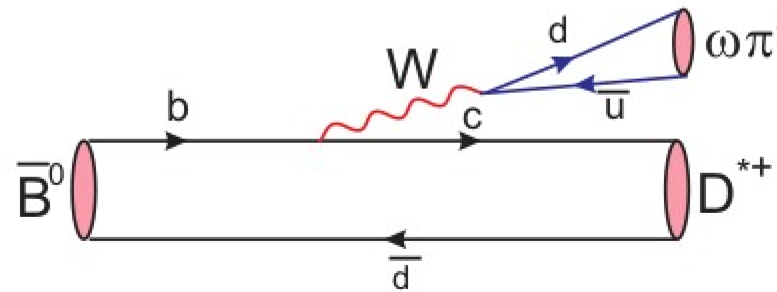
-Very efficient to distinguish different contributions and determine the quantum numbers of intermediate states

$\bar{B}^0 \to D^{*+}\omega\pi^-$

$711 fb^{-1}$

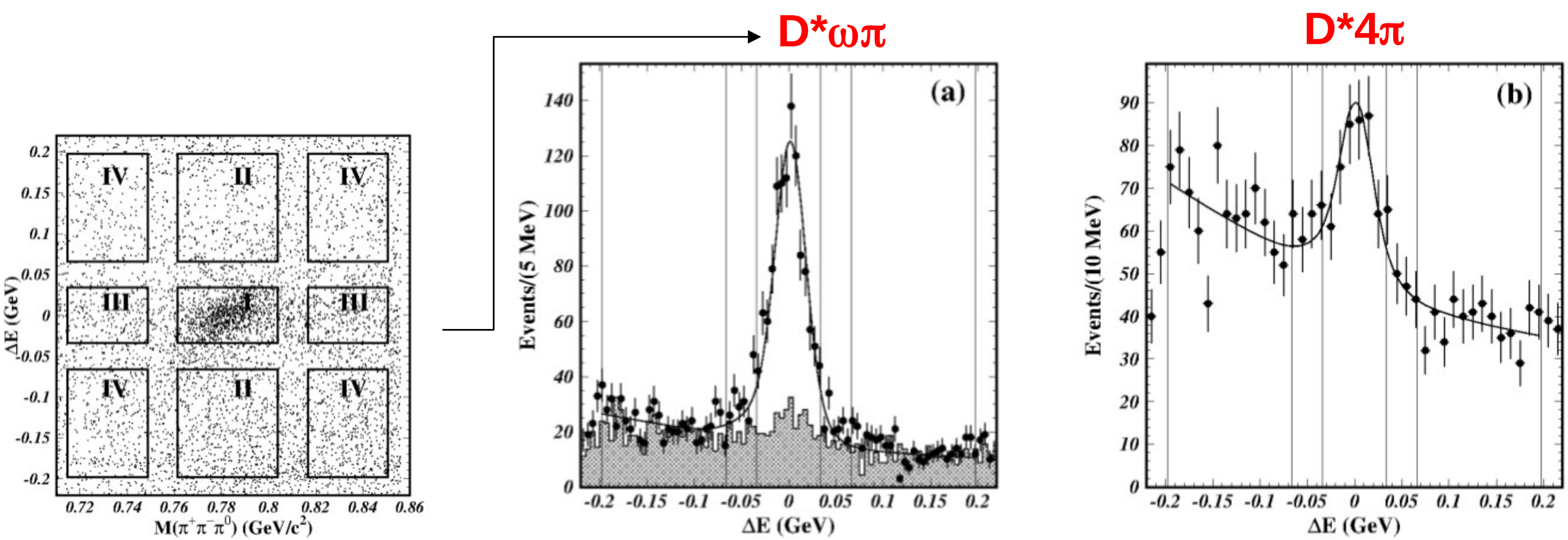


Broad $D_1(2430)\omega$



Light meson $\rho(770),\rho(1450),..,b_1(1235)$

$$\bar{B}^0 \to D^{*+}\omega\pi^-,\ D^{*+} \to D^0\pi^+,\ \omega \to \pi^+\pi^-\pi^0,\ D^0 \to K^-\pi^+,\ \pi^0 \to \gamma\gamma$$



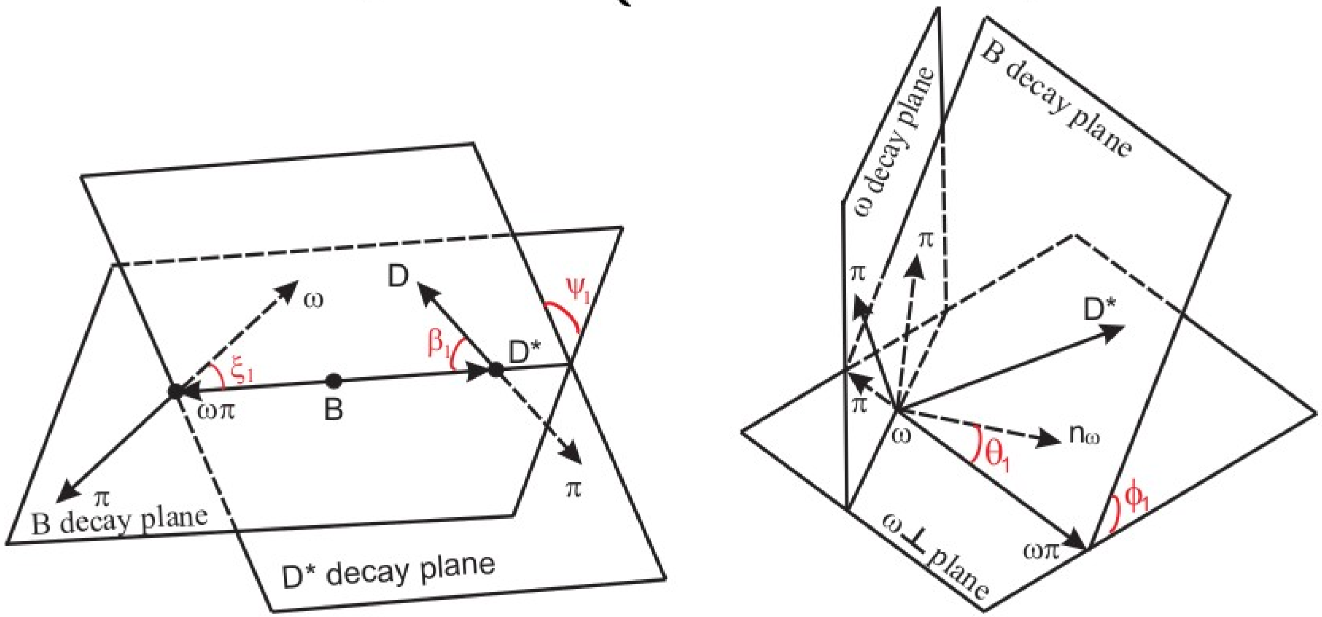
PRD 92,012013 (2015)

$$\mathcal{B}(\bar{B}^0 \to D^{*+}\omega\pi^-) = (2.31 \pm 0.11\,(\text{stat.}) \pm 0.14\,(\text{syst.})) \times 10^{-3}$$

-Unbinned likelihood in 6D phase spase:
$q^2_{\omega\pi}$, helicity angle D*, helicity angle of ωπ, angle beetwin planes of decay, 2 angles of ω decay

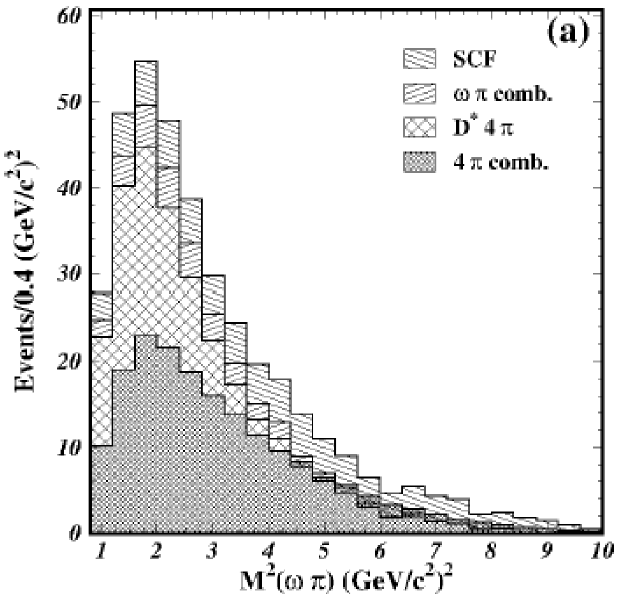
-For D** similar set of 6 variables.

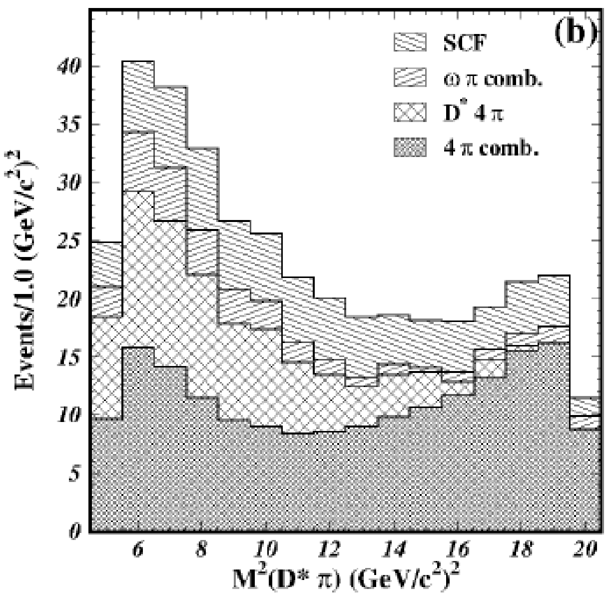
$$\mathrm{PDF}(\vec{x},\vec{a}) = \frac{\epsilon(\vec{x})}{n_s + \sum_j n_{\mathrm{bkg}\,j}} \times \left\{ n_s \frac{|M(\vec{x},\vec{a})|^2}{\epsilon_s(\vec{a})} + \sum_j n_{\mathrm{bkg}\,j} \frac{B_j(\vec{x})}{\epsilon_{\mathrm{bkg}\,j}} \right\}$$



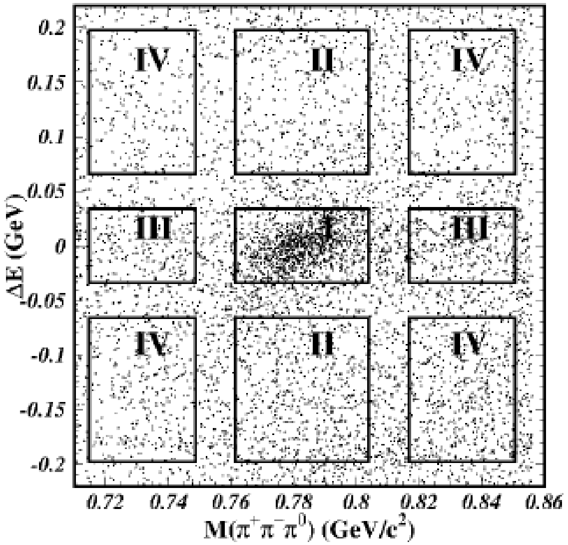
-Background functions obtained for different background regions ΔE,Mω

Background description





- SCF background
- $\omega\pi$ combinatorial background (I and III)
- $D^*4\pi$ signal background (I and II)
- $D^*4\pi$ combinatorial background (I,II,III,IV)



Signal description

$$d\Gamma = \frac{6\mathcal{B}_{D^{*+}\to D^0\pi^+}}{(4\pi)^{10} m_B^2} \frac{|M_\pm|^2 p_{3\pi} p_{D^*,B}}{\sqrt{q^2}} \frac{W(p^2)}{|D_\omega(p^2)|^2} dp^2 \, (d\cos\theta_1 \, d\phi_1)\times$$

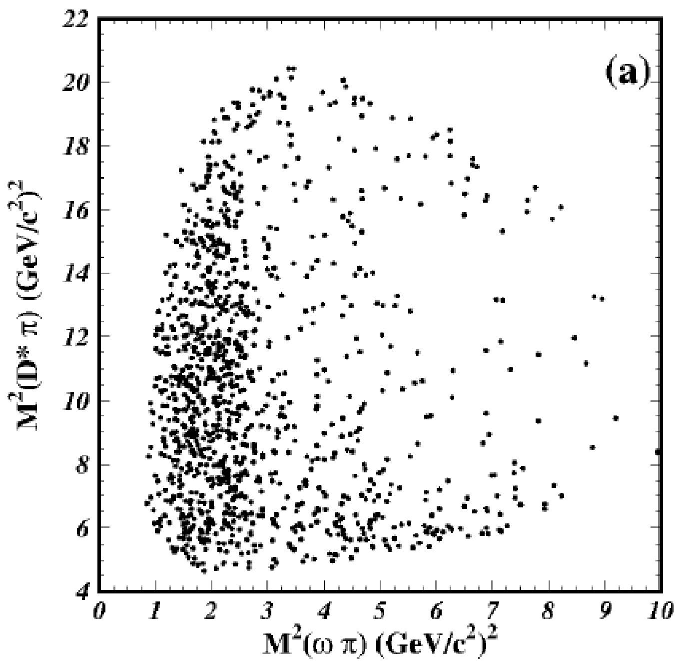
$$\times \, (d\cos\beta_1 \, d\psi_1)\, (dq^2 \, d\cos\xi_1)$$

$$M_\pm = \sum_R a_R e^{i\phi_R} M_{R\pm}$$

-Each resonance contribution is described by reletivistic Breit-Wigner function with width depending on q^2.
-Angular dependences are expressed via the defined angles

-Take into account mixing of pure $D_{1/2}$, $D_{3/2}$

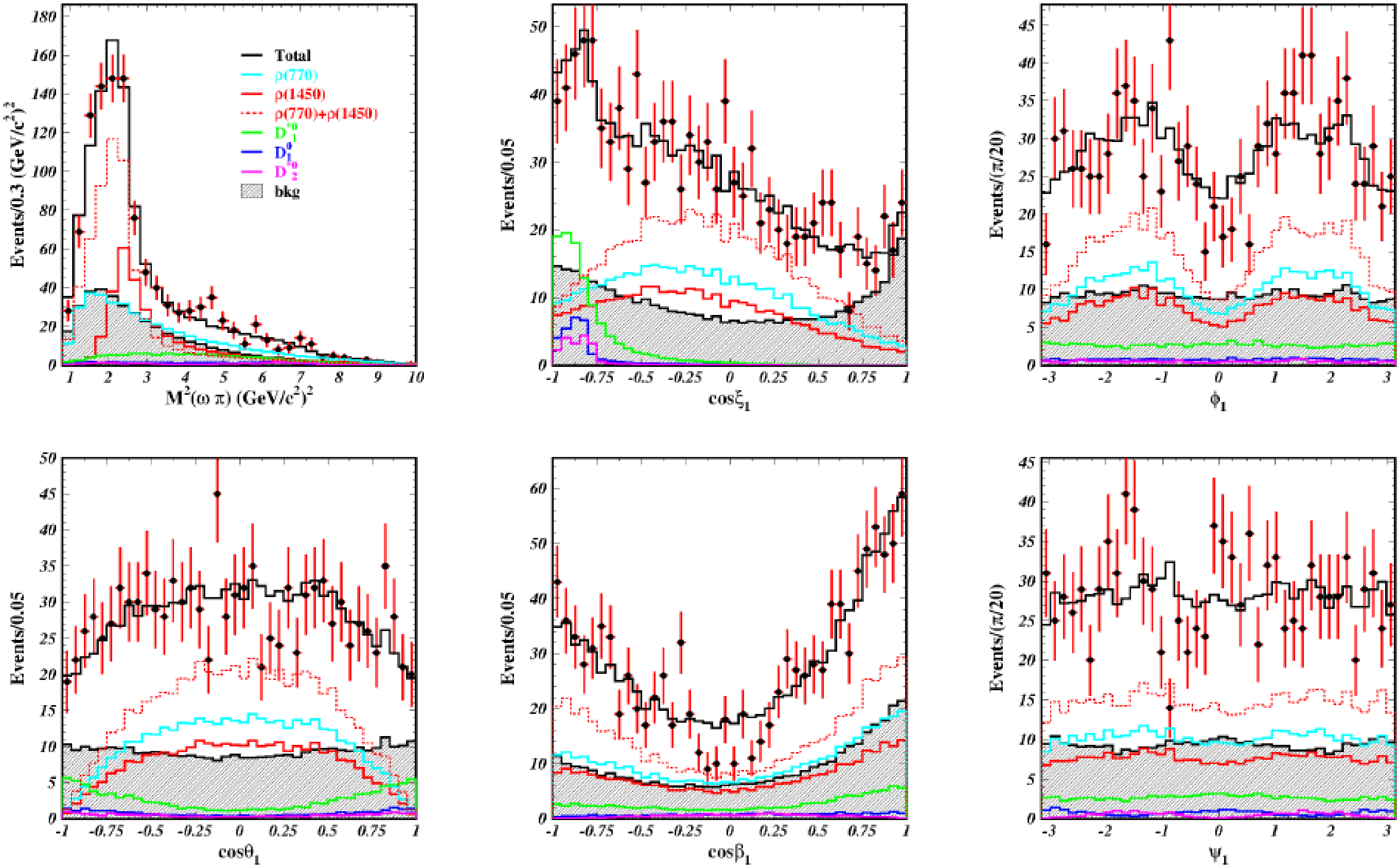
$$\begin{aligned} |D_1(2420)^0> &= \sin\omega\,|j_q = 1/2> + \cos\omega\, e^{-i\varphi}\,|j_q = 3/2> \\ |D_1'(2430)^0> &= \cos\omega\,|j_q = 1/2> - \sin\omega\, e^{i\varphi}\,|j_q = 3/2> \end{aligned}$$



L_1	L_2	$\mathcal{A}_{L_1 L_2}$
S	P	$-s_\theta s_\phi c_\beta s_\xi + s_\theta c_\phi s_\beta s_\psi - s_\theta s_\phi s_\beta c_\psi c_\xi$
P	P	$s_\theta s_\phi s_\beta s_\psi c_\xi + s_\theta c_\phi s_\beta c_\psi$
D	P	$2s_\theta s_\phi c_\beta s_\xi + s_\theta c_\phi s_\beta s_\psi - s_\theta s_\phi s_\beta c_\psi c_\xi$
S	S	$-c_\theta c_\beta c_\xi + s_\theta c_\phi c_\beta s_\xi - s_\theta s_\phi s_\beta s_\psi + $ $+ s_\theta c_\phi s_\beta c_\psi c_\xi + c_\theta s_\beta c_\psi s_\xi$
P	S	$-c_\theta s_\beta s_\psi s_\xi + s_\theta s_\phi s_\beta c_\psi - s_\theta c_\phi s_\beta s_\psi c_\xi$
D	S	$2c_\theta c_\beta c_\xi + s_\theta c_\phi c_\beta s_\xi - s_\theta s_\phi s_\beta s_\psi +$ $+ s_\theta c_\phi s_\beta c_\psi c_\xi - 2c_\theta s_\beta c_\psi s_\xi$
S	D	$2c_\theta c_\beta c_\xi - 2s_\theta c_\phi c_\beta s_\xi - s_\theta s_\phi s_\beta s_\psi +$ $+ s_\theta c_\phi s_\beta c_\psi c_\xi + c_\theta s_\beta c_\psi s_\xi$
P	D	$2c_\theta s_\beta s_\psi s_\xi + s_\theta s_\phi s_\beta c_\psi - s_\theta c_\phi s_\beta s_\psi c_\xi$
D	D	$-4c_\theta c_\beta c_\xi - 2s_\theta c_\phi c_\beta s_\xi - s_\theta s_\phi s_\beta s_\psi +$ $+ s_\theta c_\phi s_\beta c_\psi c_\xi - 2c_\theta s_\beta c_\psi s_\xi$

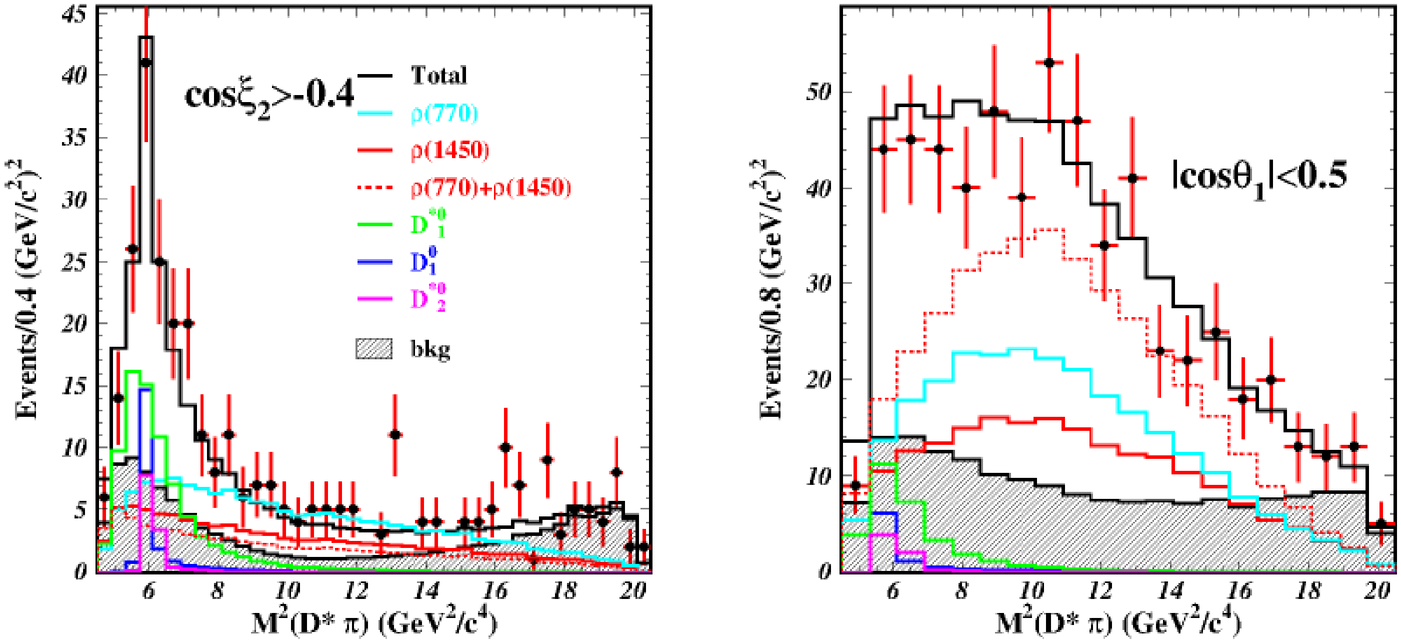
- $\omega\pi$ резонансы $\Longrightarrow$ $\rho(770)$, $\rho(1450)$, $b_1(1235)$
- D^{**} резонансы $\Longrightarrow$ $D_1(2430)$, $D_1(2420)$, $D_2(2460)$

Results of the fit



Results of the fit

PRD 92,012013 (2015)



No significant mixing observed $\omega = -0.03 \pm 0.02$

$$\mathcal{B}_{D_1(2430)^0} = (2.5 \pm 0.4\,(\mathrm{stat.})\,{}^{+0.7}_{-0.2}\,(\mathrm{syst.})\,{}^{+0.4}_{-0.1}\,(\mathrm{model})) \times 10^{-4}$$ $\delta = 8.6\sigma$

$$\mathcal{B}_{D_1(2420)^0} = (0.7 \pm 0.2\,(\mathrm{stat.})\,{}^{+0.1}_{-0.0}\,(\mathrm{syst.}) \pm 0.1\,(\mathrm{model})) \times 10^{-4}$$ $\delta = 5.5\sigma$

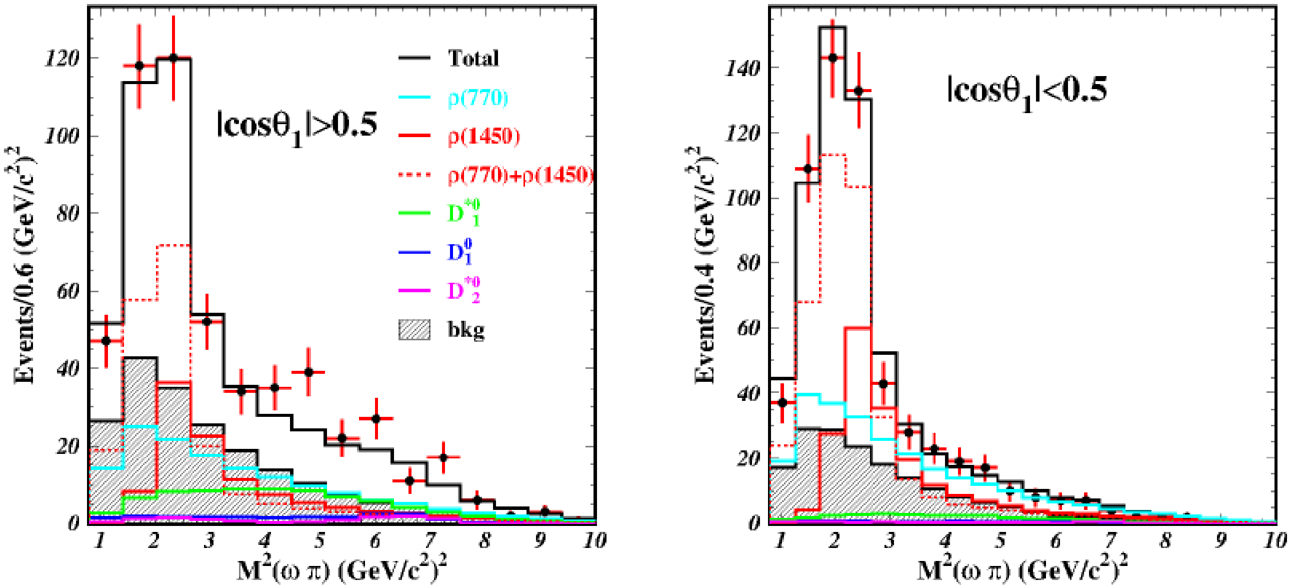
$$\mathcal{B}_{D_2^*(2460)^0} = (0.4 \pm 0.1\,(\mathrm{stat.})\,{}^{+0.0}_{-0.1}\,(\mathrm{syst.}) \pm 0.1\,(\mathrm{model})) \times 10^{-4}$$ $\delta = 5.0\sigma$

D_1(2430) is consistent with estimation in $\mathrm{HQET} \Longrightarrow \mathcal{B}_{D_1(2430)^0} \sim 3.5 \times 10^{-4}$

D_1(2420),D_2^* are suppressed in factorization approach

Results of the fit

PRD 92,012013 (2015)



$$\mathcal{B}_{\rho(770)^-} = (1.48 \pm 0.27\,(\mathrm{stat.})\,^{+0.15}_{-0.09}\,(\mathrm{syst.})\,^{+0.21}_{-0.56}\,(\mathrm{model})) \times 10^{-3} \qquad \delta = 10.5\sigma$$

$$\mathcal{B}_{\rho(1450)^-} = (1.07\,^{+0.15}_{-0.31}\,(\mathrm{stat.})\,^{+0.06}_{-0.13}\,(\mathrm{syst.})\,^{+0.40}_{-0.02}\,(\mathrm{model})) \times 10^{-3} \qquad \delta = 15.0\sigma$$

$$\mathcal{B}_{\mathrm{FCC}} = (1.90 \pm 0.11\,(\mathrm{stat.})\,^{+0.11}_{-0.13}\,(\mathrm{syst.})\,^{+0.02}_{-0.06}\,(\mathrm{model})) \times 10^{-3} \qquad \delta = 29.8\sigma$$

$$\mathcal{B}_{\mathrm{SCC}} < 0.7 \times 10^{-4}\,(90\%\ \mathrm{C.L.})$$

$$g_{\rho'\omega\pi}/g_{\rho\omega\pi} \times f_\rho/f_{\rho'} =$$

$$0.18\,^{+0.02}_{-0.06}(\mathrm{stat.})\,^{+0.00}_{0.02}\,(\mathrm{syst.})\,^{+0.10}_{-0.01}\,(\mathrm{model})$$

0.137 ± 0.006 **(SND PRD 88, 054013(2013))**

ρ(1450) contribution is consistent with e+e- data:
Mass and width of ρ(1450) are consistent with PDG

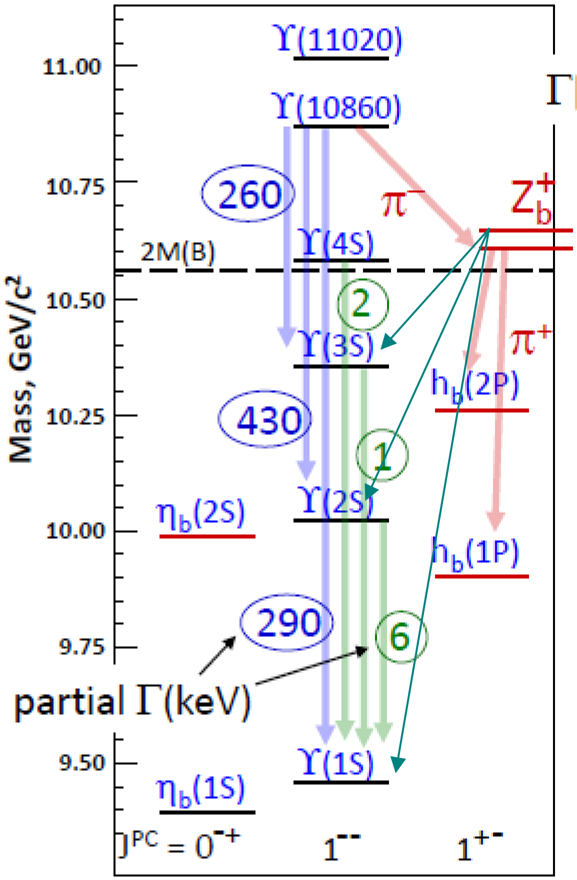
$$m_{\rho'} = (1544 \pm 22\,(\mathrm{stat.})\,^{+11}_{-1}\,(\mathrm{syst.})^{+1}_{-46}\,(\mathrm{model}))\ \mathrm{MeV}/c^2 \Longleftrightarrow$$

$$m_{\rho'} = (1465 \pm 25)\ \mathrm{MeV}/c^2\ \text{(Particle Data Group)}$$

$$\rho(1450)\ \Gamma_{\rho'} = (303\,^{+31}_{-52}\,(\mathrm{stat.})\,^{+3}_{-4}\,(\mathrm{syst.})\,^{+69}_{-6}\,(\mathrm{model}))$$

$$\Gamma_{\rho'} = (400 \pm 60)\ \mathrm{MeV}\ \text{(Particle Data Group)}$$

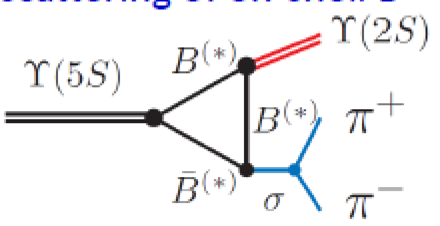
Anomalies in $\Upsilon(5S) \to (b\bar{b})\,\pi^+\pi^-$ transitions



Belle PRL100,112001(2008)

$$\Gamma[\Upsilon(5S) \to \Upsilon(1,2,3S)\,\pi^+\pi^-] \overset{\sim 100}{>>} \Gamma[\Upsilon(2,3,4S) \to \Upsilon(1S)\,\pi^+\pi^-]$$

$\Leftarrow$ Rescattering of on-shell $B^{(*)}\bar{B}^{(*)}$?



Belle PRL108,032001(2012)

$\Upsilon(5S) \to h_b(1,2P)\,\pi^+\pi^-$ are **not suppressed**

~~Heavy Quark Symmetry~~

spin-flip

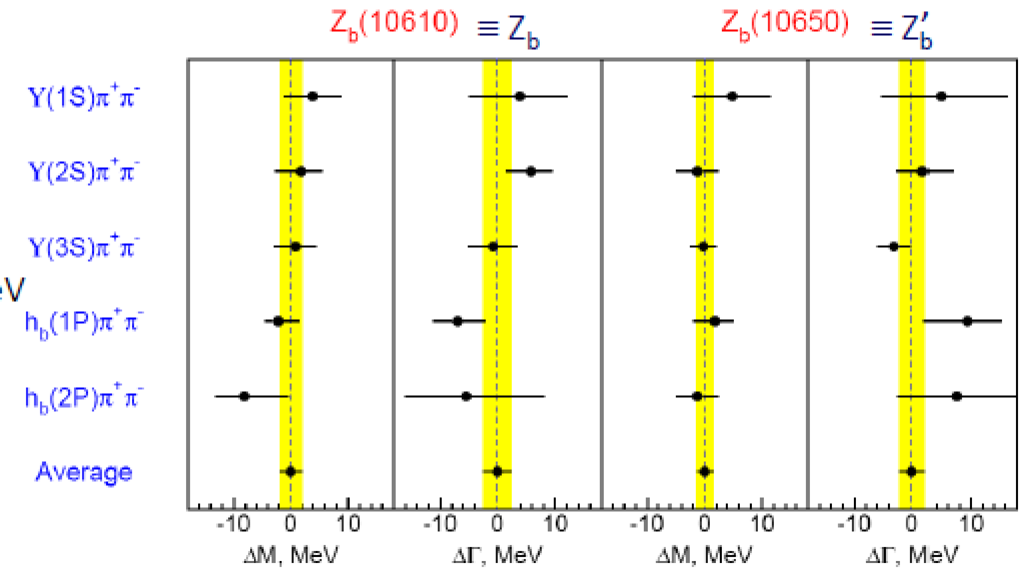
expect suppression $(\Lambda_{QCD}/m_b)^2 \sim 10^{-2}$

Fit results

Average over 5 channels

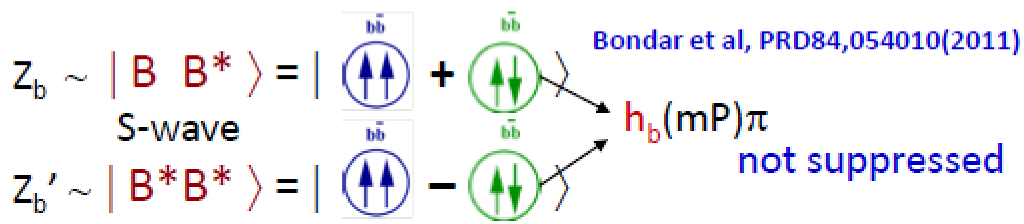
$M_1 = 10607.2 \pm 2.0$ MeV
$\Gamma_1 = 18.4 \pm 2.4$ MeV
$M_{Zb} - (M_B + M_{B*}) = +2.6 \pm 2.1$ MeV

$M_2 = 10652.2 \pm 1.5$ MeV
$\Gamma_2 = 11.5 \pm 2.2$ MeV
$M_{Zb'} - 2M_{B*} = +1.8 \pm 1.7$ MeV



Angular analysis $\Rightarrow$ both states are $J^P = 1^+$ Decays $\Rightarrow$ $I^G = 1^+$ (C= – for Z_b^0)

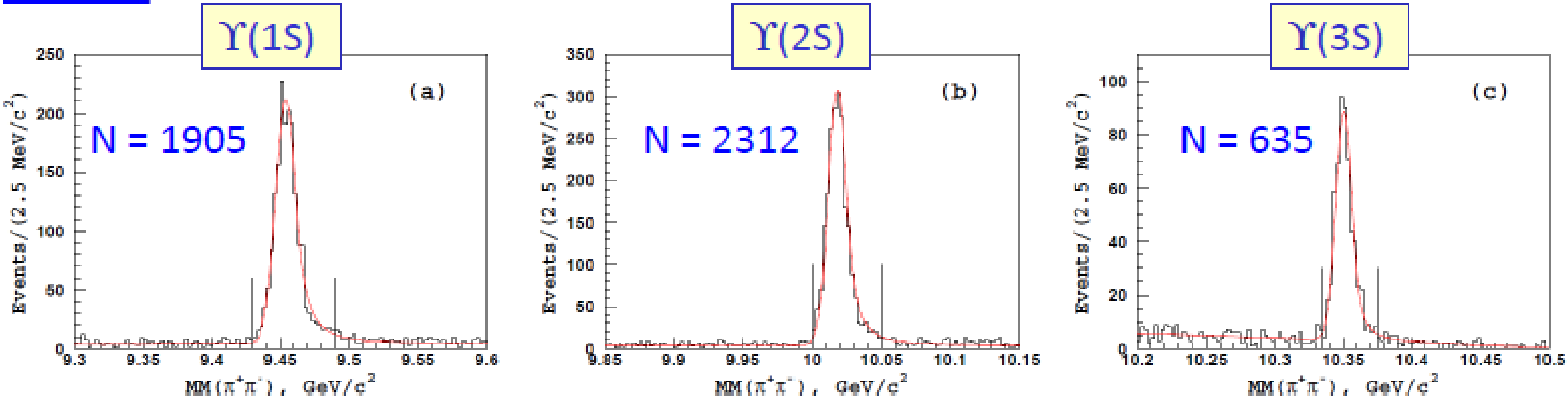
Proximity to thresholds favors molecule over tetraquark



Phase btw Z_b and Z_b' amplitudes is $\sim 0^o$ for $\Upsilon(nS)\pi\pi$ and $\sim 180^o$ for $h_b(mP)\pi\pi$

Properties of Z_b states are consistent with molecular structure.

$\Upsilon(5S) \to \Upsilon(nS)(\to \mu^+\mu^-)\ \pi^+\pi^-$ 6D-analysis



12 – 4 (energy-momentum) – 1 (Υ(nS) mass) – 1 (rotation around beam axis) = 6 d.o.f.

e.g. $M^2(\Upsilon(nS)\pi)$, $M^2(\pi^+\pi^-)$ and 4 angles

Υ(nS) ->$\mu^+\mu^-$

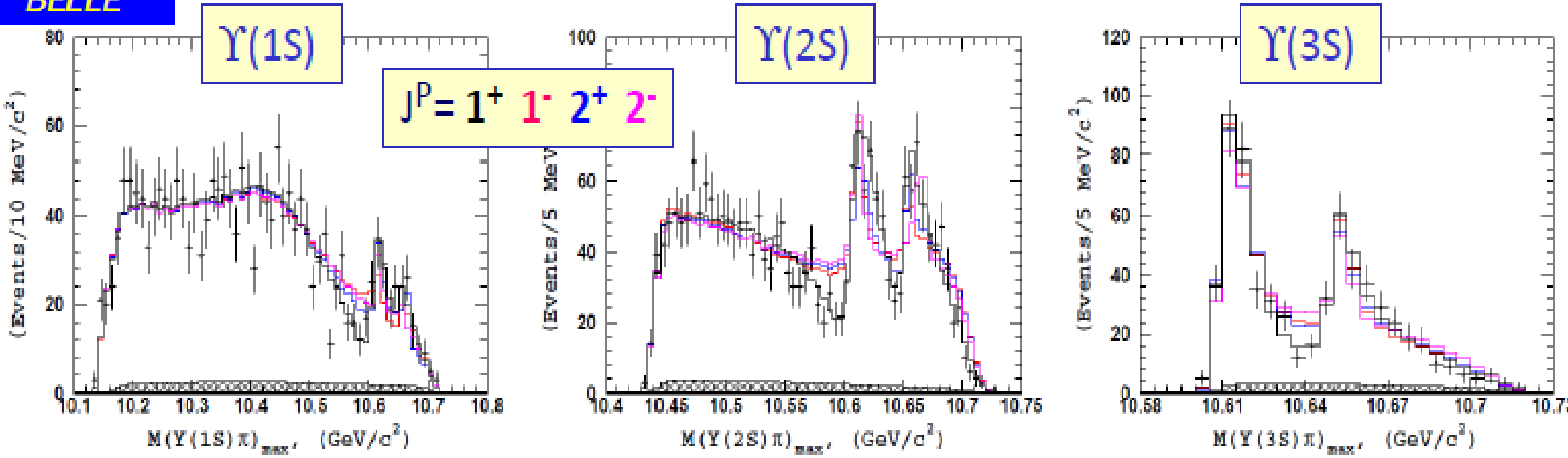
Amplitudes in Lorentz invariant form $|\mathcal{M}|^2 = \delta_{\perp}^{\mu\mu'} O_{\mu\nu} R^{\nu\nu'} O^*_{\nu'\mu'}$

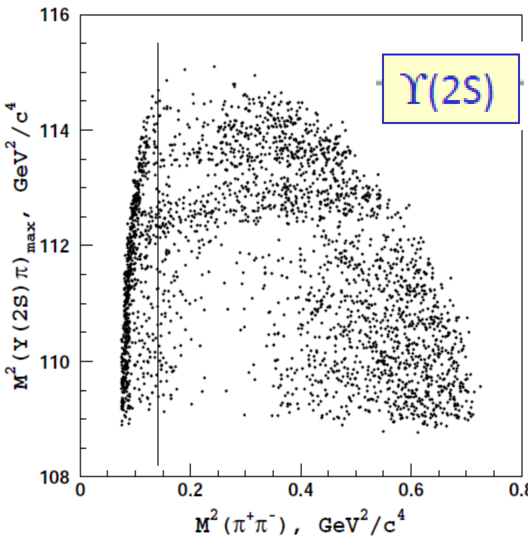
$$Q_{\mu\nu} = A_{Z_{b1}} + A_{Z_{b2}} + A_{NR} + A_{f_0(980)} + A_{f_2(1275)} + A_{\sigma}$$

BW $\quad$ $C_1 + C_2 \cdot m^2(\pi\pi)$ $\quad$ Flatte $\quad$ BW $\quad$ BW

Combinatorial background from Υ(nS) sidebands

Different hypotheses





$Z_b(10610)$ \ $Z_b(10650)$	1^+	1^-	2^+	2^-
1^+	0 (0)	60 (33)	42 (33)	77 (63)
1^-	226 (47)	264 (73)	224 (68)	277 (106)
2^+	205 (33)	235 (104)	207 (87)	223 (128)
2^-	289 (99)	319 (111)	321 (110)	304 (125)

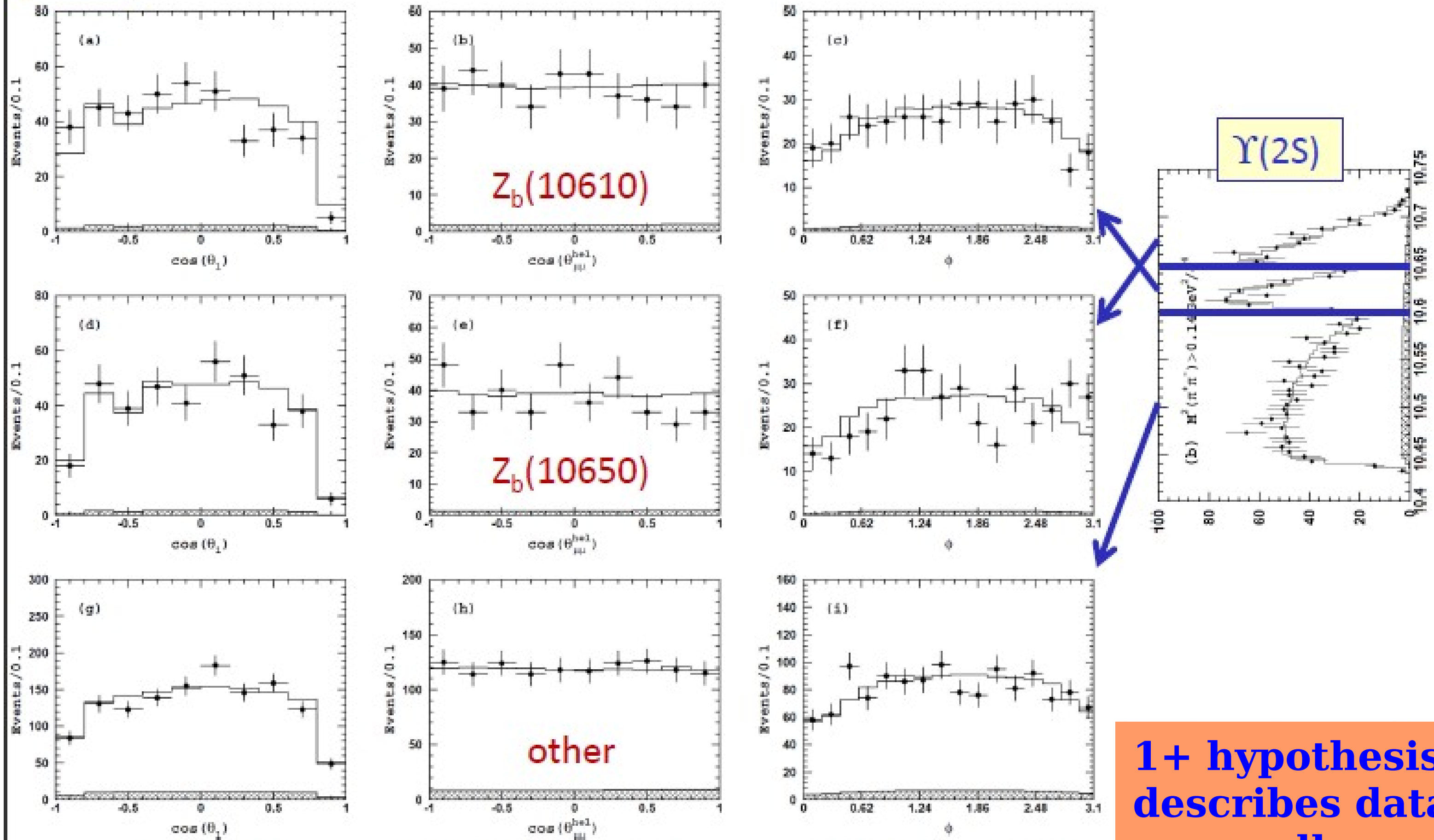
Spin parity of Zb(10610) and Zb(10650) is 1^+.
All other $J^P<3$ are excluded.

PRD 91,072003 (2015)

Consistent with S-wave molecule

Angular projections of 6D fit

$J^P=1^+$



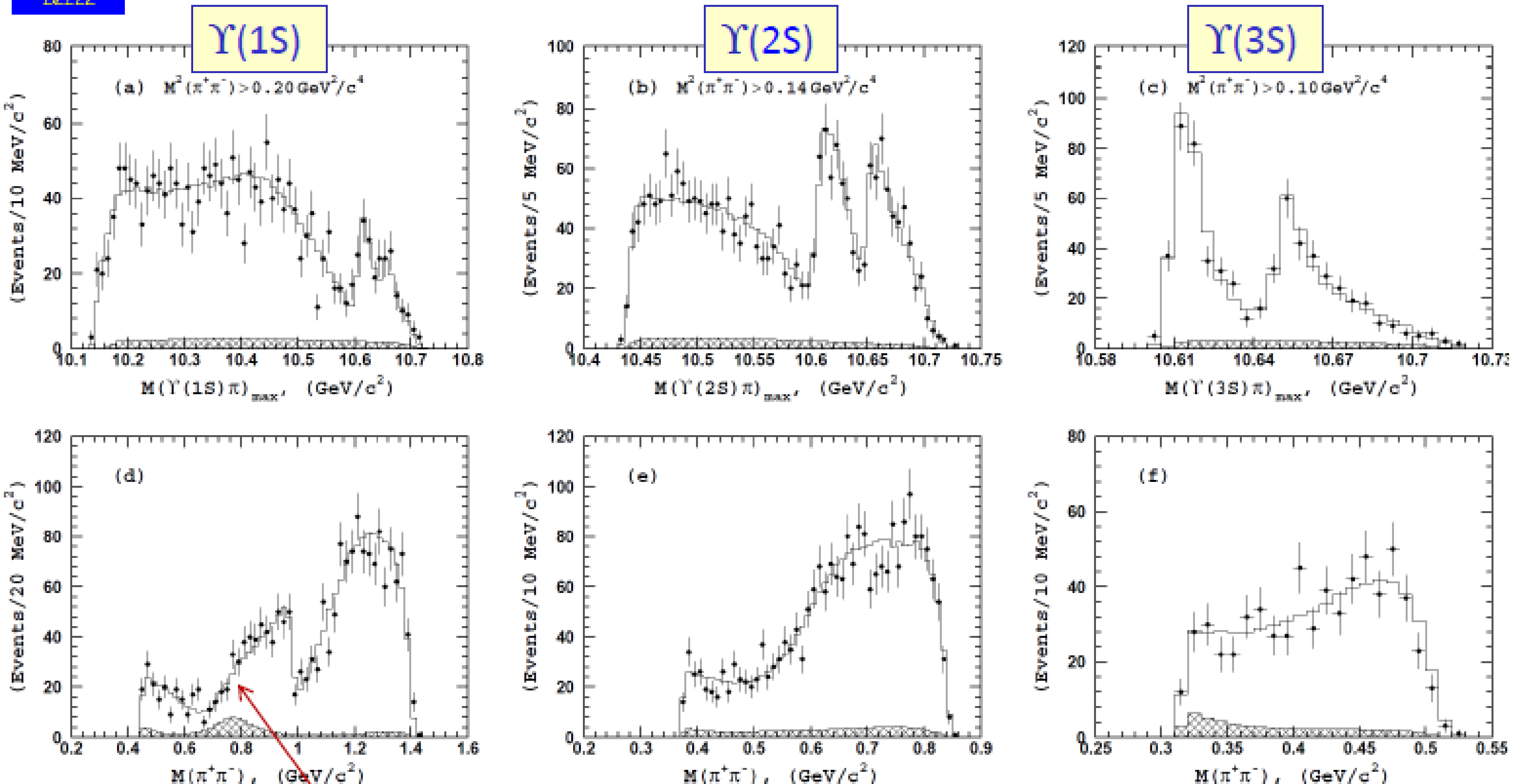
$\angle(\pi_1$, Z-axis) $\Upsilon(2S)$ helicity angle $\angle$[plane(π_1, Z-axis), plane($\pi^+\pi^-$)]

1+ hypothesis describes data very well

PRD 91,072003 (2015)

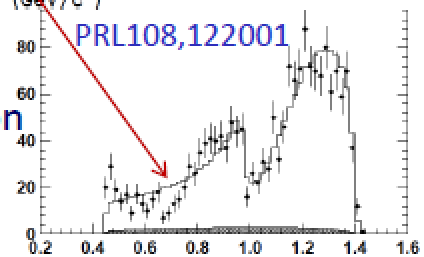
Mass projections of 6D fit

$J^P=1^+$



Improvement due to inclusion of σ state

PRL108,122001

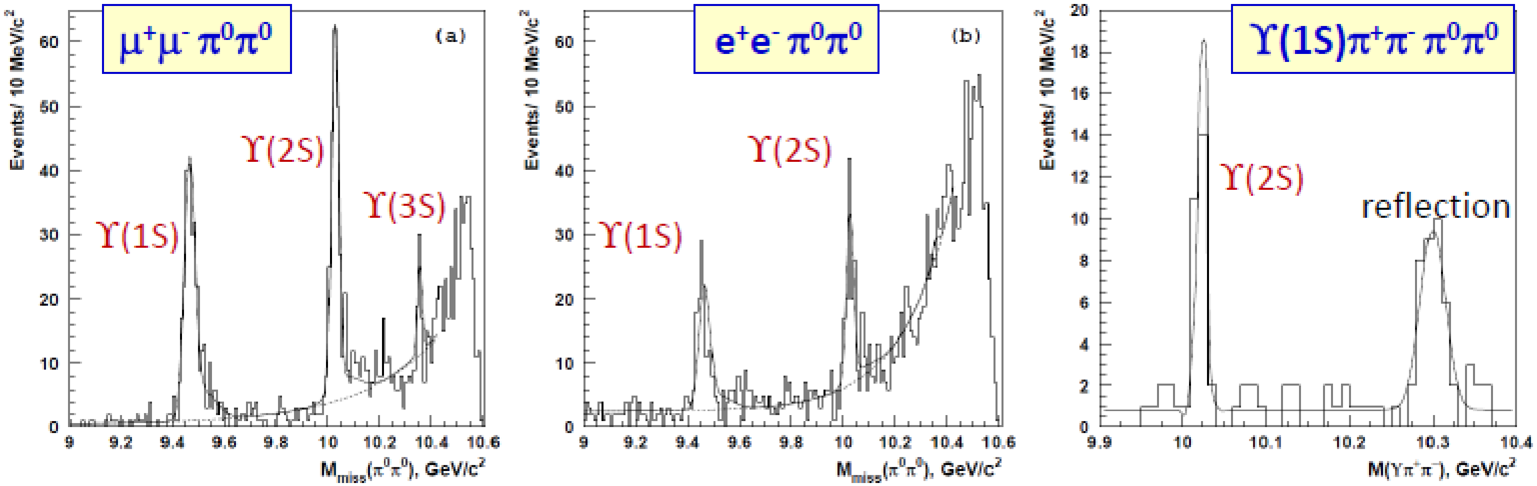


BW amplitudes describe Z_b states very well.

Resonant behavior of Z_b amplitudes (intensity & phase).

PRD 91,072003 (2015)

Observation of $\Upsilon(5S)\to\Upsilon(nS)\pi^0\pi^0$



$\sigma[e^+e^-\to\Upsilon(5S)\to\Upsilon(1S)\pi^0\pi^0] = (1.16\pm0.06\pm0.10)$ pb

$\sigma[e^+e^-\to\Upsilon(5S)\to\Upsilon(2S)\pi^0\pi^0] = (1.87\pm0.11\pm0.23)$ pb

$\sigma[e^+e^-\to\Upsilon(5S)\to\Upsilon(3S)\pi^0\pi^0] = (0.98\pm0.24\pm0.19)$ pb

Consistent with ½ of $\Upsilon(nS)\pi^+\pi^-$

$\Upsilon(1S)\pi^+\pi^-$	$\Upsilon(2S)\pi^+\pi^-$	$\Upsilon(3S)\pi^+\pi^-$
$2.29\pm0.12\pm0.14$	$4.11\pm0.16\pm0.45$	$1.47\pm0.09\pm0.16$

$\sigma_{e^+e^-\to\Upsilon(nS)\pi^+\pi^-}$, pb

PRD 91,072003 (2015)

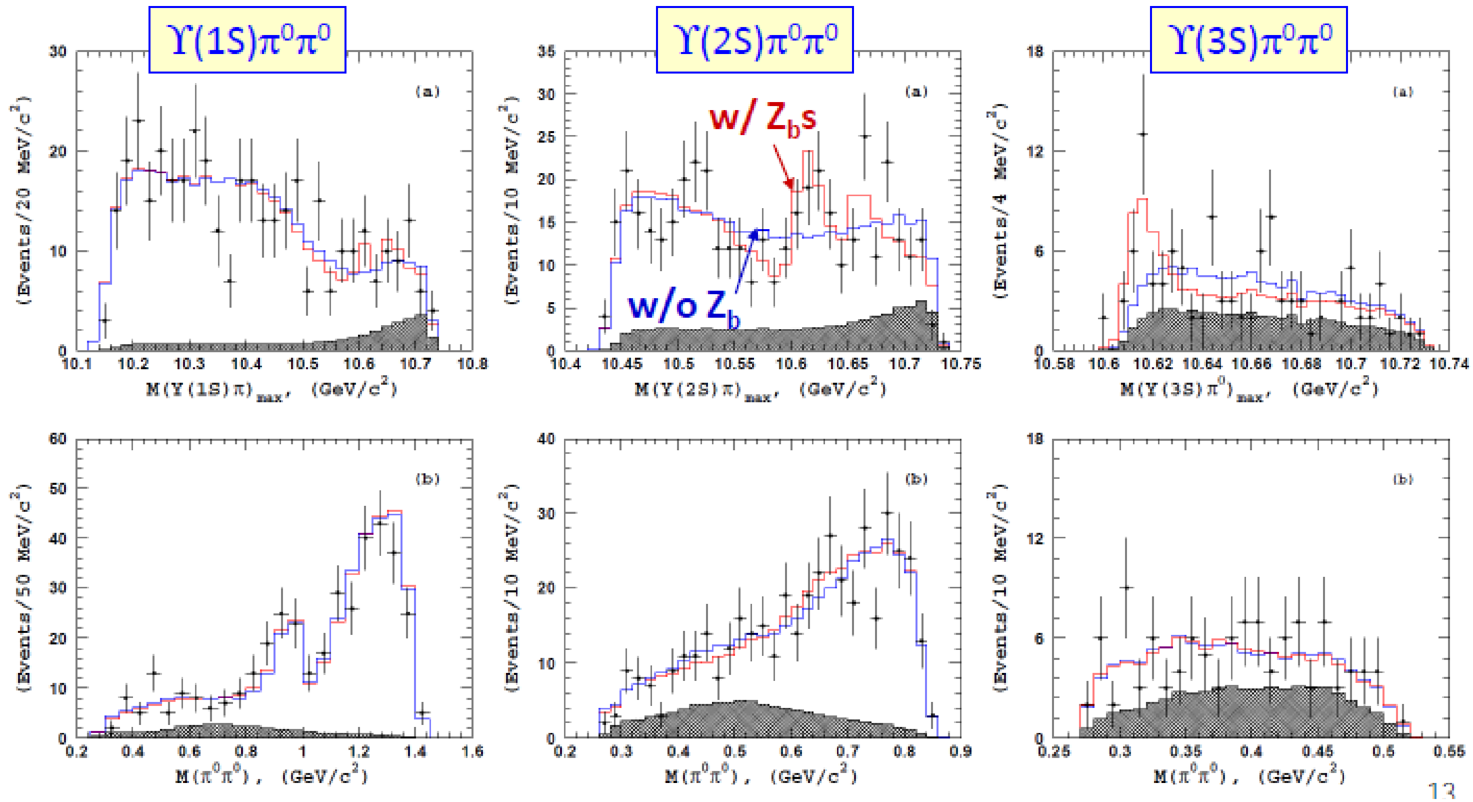
PRD 88,052016 (2013)

Dalitz plot analysis of $\Upsilon(5S) \to \Upsilon(nS)\pi^0\pi^0$

PRD 88,052016 (2013)

2D analysis

$$S(s1,s2) = A(Z_{b1}) + A(Z_{b2}) + A(f_0(980)) + A(f_2(1275)) + A_{NR}$$



- Z_b^0 resonant structure has been observed in $\Upsilon(2S)\pi^0\pi^0$ and $\Upsilon(3S)\pi^0\pi^0$
- Statistical significance of $Z_b^0(10610)$ signal is 6.5σ including systematics
- $Z_b^0(10650)$ signal is not significant (~2σ), not contradicting with its existence
- $Z_b^0(10610)$ mass from the fit $M=10609 \pm 4 \pm 4$ MeV/c^2 $\quad M(Z_b^+)=10607\pm2$ MeV/c^2

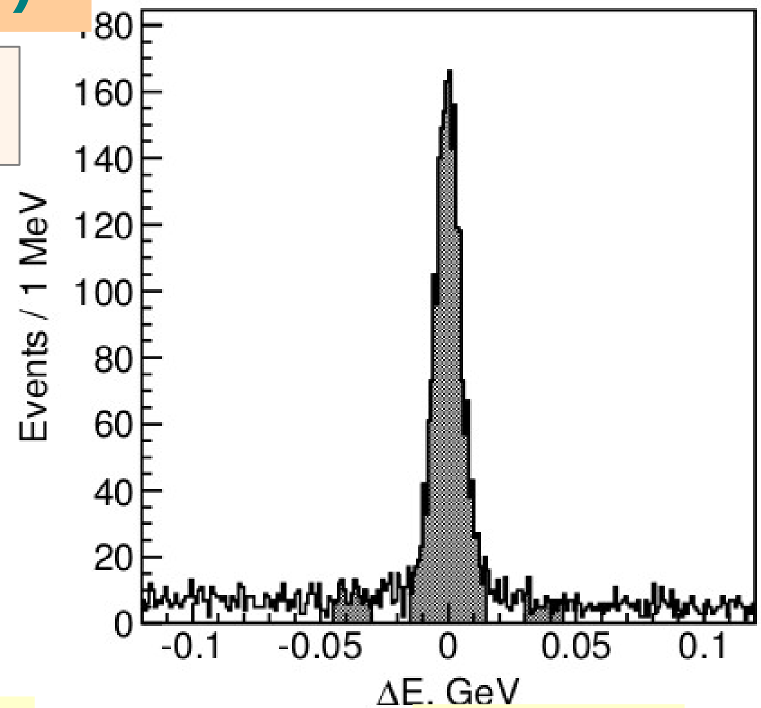
Quantum numbers of Z(4430)

$$B^0 \to \psi' K^+ \pi^-$$

$$\psi' \to e^+ e^-$$

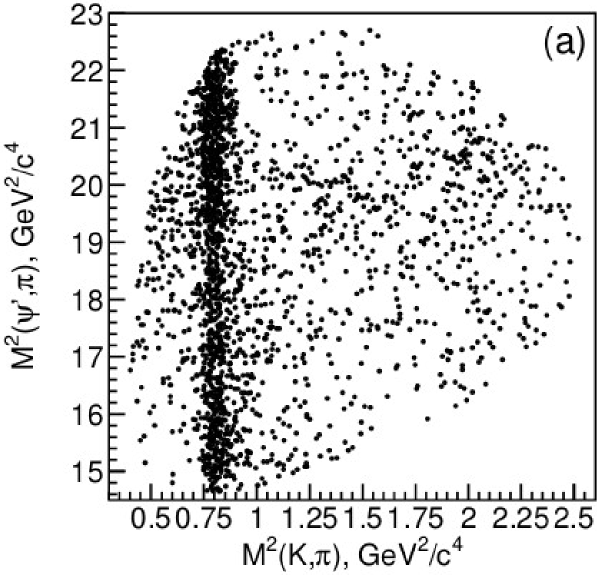
$$\psi' \to \mu^+ \mu^-$$

PRD 88,074026 (2013)

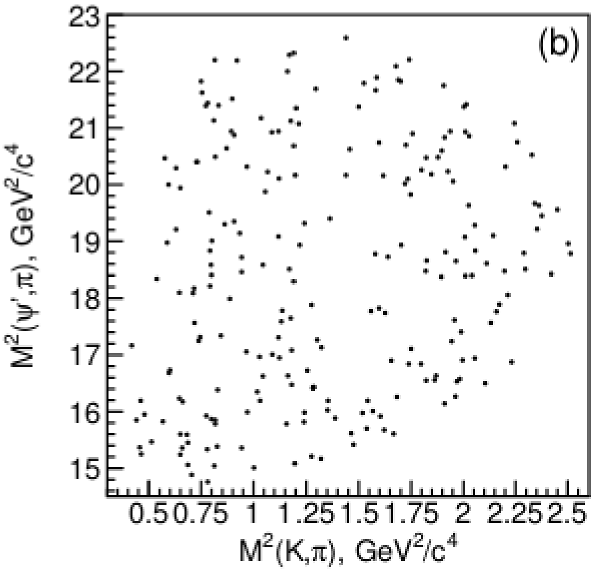


Signal: K π resonances: $K_0^*(800)$, $K^*(892)$, $K^*(1410)$, $K_0^*(1430)$, $K_2^*(1430)$, $K^*(1680)$, And $Z_c(4430)^+$

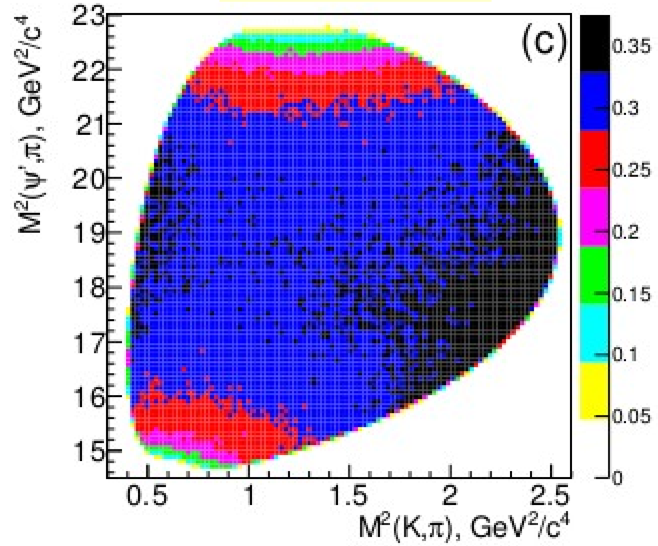
Signal



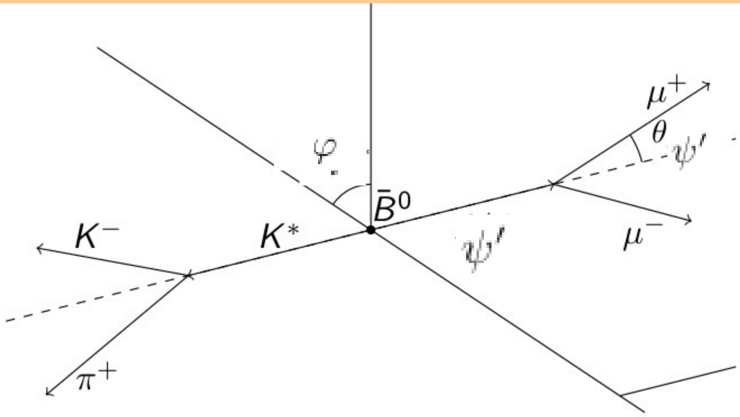
Sidebands



Efficiency



4-dimensional amplitude analysis



$$\Phi = (M^2_{K^-\pi^+},\, M^2_{\psi'\pi^+},\, \theta_{\psi'},\, \varphi)$$

Signal: sum of reliativistic Breit-Wigners

$$A^R(M_R^2) = \frac{F_B^{(L_B)} F_R^{(L_R)} \left(\frac{p_B}{m_B}\right)^{L_B} \left(\frac{p_R}{M_R}\right)^{L_R}}{m_R^2 - M_R^2 - i m_R \Gamma(M_R)},$$

Angular dependences are obtained from helicity formalizm:

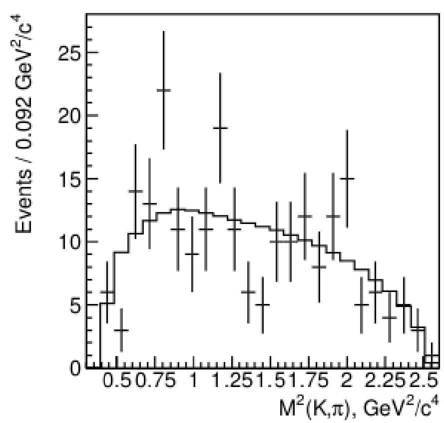
$$S(\Phi) = \sum_{\xi=1,-1} \left| \sum_{K^*} \sum_{\lambda=-1,0,1} A^{K^*}_{\lambda\,\xi} + \sum_{\lambda'=-1,0,1} A^{Z^-}_{\lambda'\,\xi} \right|^2$$

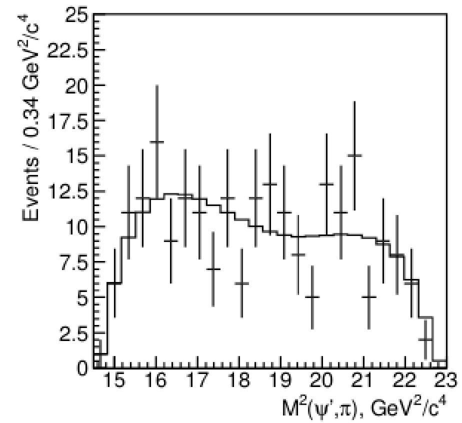
$$A^{K^*}_{\lambda\,\xi}(\Phi) = H^{K^*}_{\lambda} A^{K^*}(M^2_{K^-\pi^+})\, d^{J(K^*)}_{\lambda\,0}(\theta_{K^*}) \times e^{i\lambda\varphi} d^1_{\lambda\,\xi}(\theta_{\psi'}),$$

Unbinned likelihood with smooth background function

$$F = -2 \sum_i \ln\left((1-b) \frac{S(\Phi_i)}{\sum_j S(\Phi_j)} + b \frac{B(\Phi_i)}{\sum_j B(\Phi_j)} \right),$$

Background function obtained from sidebands

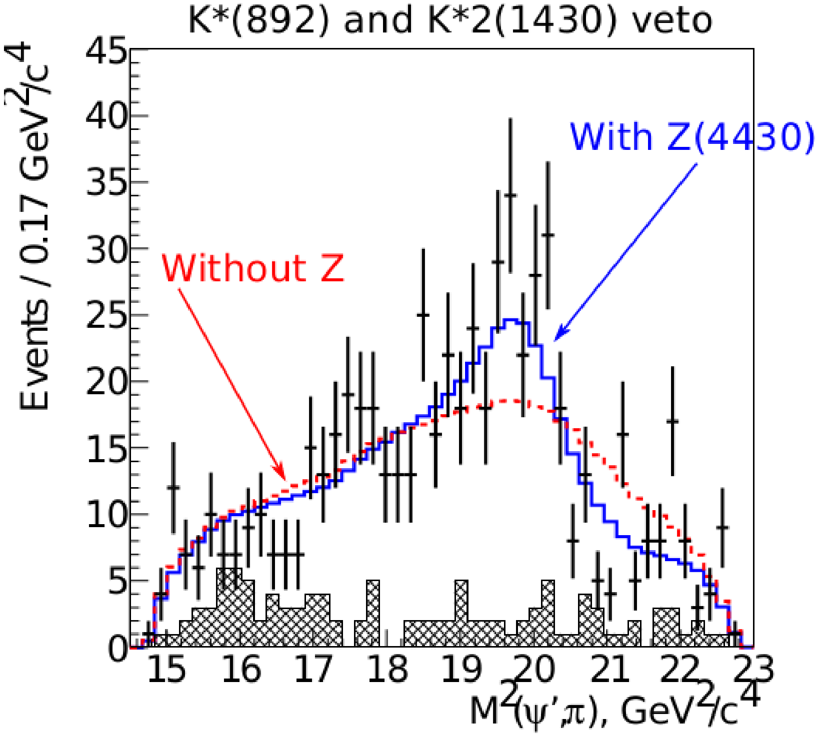




Results of the fit

J^P	0^-	1^-	1^+	2^-	2^+
Mass, MeV/c^2	4479 ± 16	4477 ± 4	4485 ± 20	4478 ± 22	4384 ± 19
Width, MeV	110 ± 50	22 ± 14	200 ± 40	83 ± 25	52 ± 28
Significance	4.5σ	3.6σ	6.4σ	2.2σ	1.8σ

$$\mathcal{B}(B^0 \to \psi' K^+ \pi^-) = (5.80 \pm 0.39) \times 10^{-4}$$



Preferred J^P hypothesis: 1^+.

Exclusion levels (0^-, 1^-, 2^- and 2^+ hypotheses): 3.4σ, 3.7σ, 4.7σ and 5.1σ.

$$M = 4485^{+22+28}_{-22-11}\ \mathrm{MeV}/c^2,$$

$$\Gamma = 200^{+41+26}_{-46-35}\ \mathrm{MeV}.$$

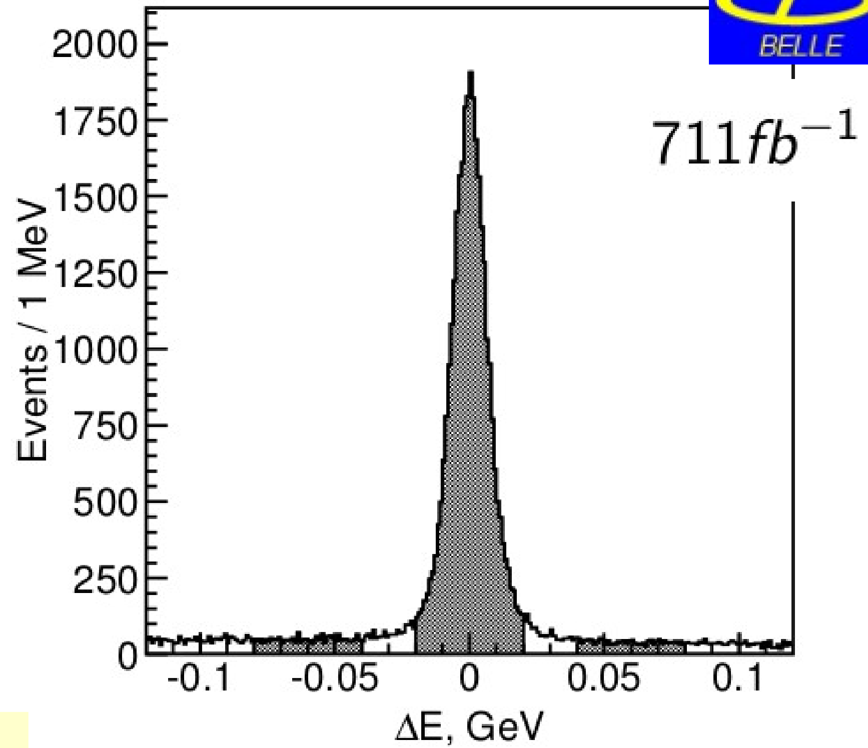
$$\mathcal{B}(B^0 \to Z(4430)^- K^+) \times \mathcal{B}(Z(4430)^- \to \psi' \pi^-) = (6.0^{+1.7+2.5}_{-2.0-1.4}) \times 10^{-5},$$

Observation of Z(4200)

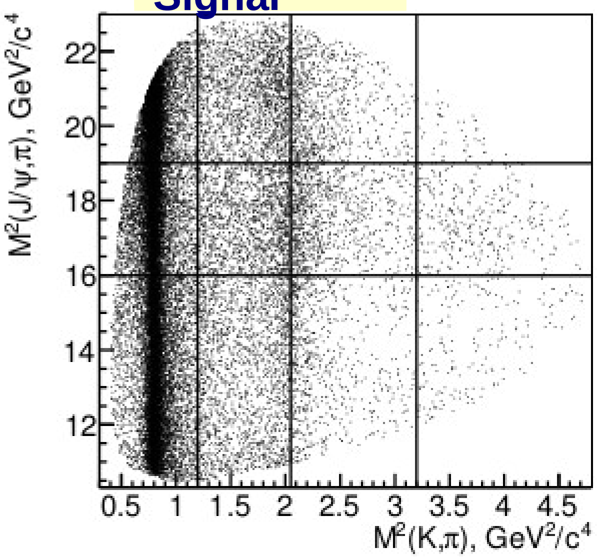
PRD 90,112009 (2014)

$$\bar{B}^0 \to J/\psi K^- \pi^+$$

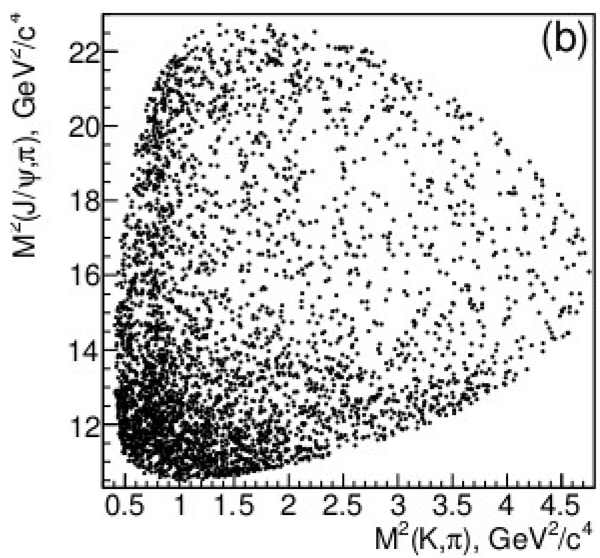
- $J/\psi \to e^+e^-$ and $J/\psi \to \mu^+\mu^-$
- All tracks: $dr < 0.2cm, dz < 2.0cm$
- e: $P(e) > 0.1$; μ: $P(\mu) > 0.1$
- K: $L(K/\pi) > 0.6$, π: $L(\pi/K) > 0.6$, e veto
- $|m_{J/\psi} - m_{J/\psi}^{(PDG)}| < 30 MeV$ for $\mu^\pm$;
  $|m_{J/\psi} - m_{J/\psi}^{(PDG)}| < 60 MeV$ for $e^\pm$
- Mass constrained fit to J/ψ candidates
- $|M_{bc} - m_B^{(PDG)}| < 7 MeV$; $M_{bc} = \sqrt{E_{\text{beam}} - (\sum_i \vec{p}_i)^2}$
- Mass constrained fit to B candidates

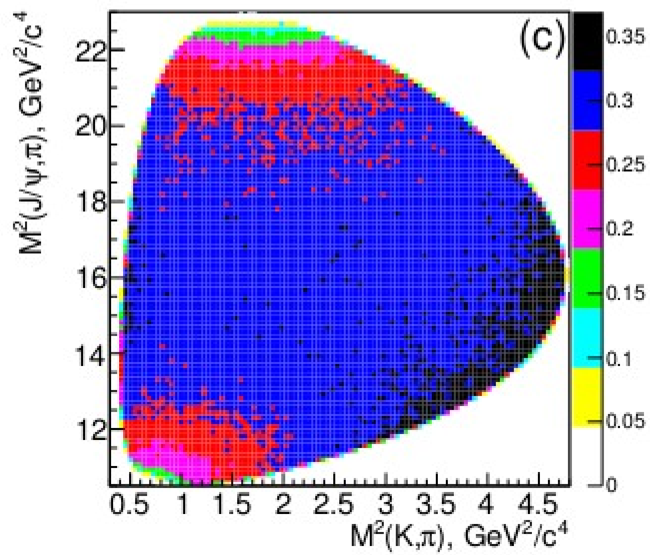


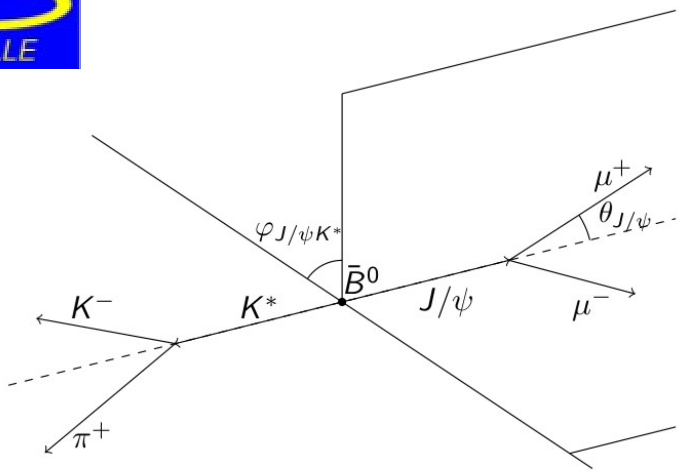
Signal



Sideband







Signal: K π resonances: $K_0^*(800)$, $K^*(892)$, $K^*(1410)$, $K_0^*(1430)$, $K_2^*(1430)$, $K^*(1680)$, $K_3^*(1780)$, $K_0^*(1950)$, $K_2^*(1980)$,$K_4^*(2045)$. $Z_c(4430)^+$, additional Z_c^+

Signal: sum of reliativistic Breit-Wigners

Angular dependences are obtained from helicity formalizm.

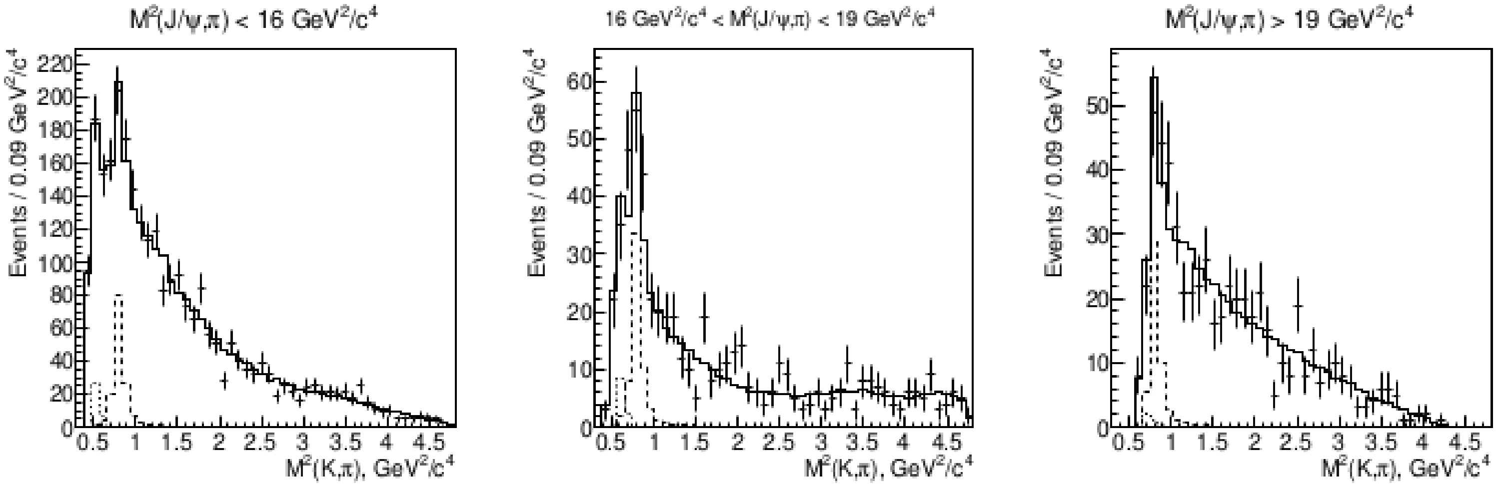
Background description

$$B(\Phi) = (B_{\rm sm}(\Phi) + B_{K^*}(\Phi) + B_{K_S^0}(\Phi)) \times P_{\theta_{J/\psi}}(\cos\theta_{J/\psi}) P_\varphi(\varphi),$$

$$B_{\rm sm}(\Phi) = (\alpha_1 e^{-\beta_1 M^2_{K^-\pi^+}} + \alpha_2 e^{-\beta_2 M^2_{J/\psi K^-}}) \times P_{\rm sm}(M^2_{K\pi}, M^2_{J/\psi\pi}),$$

$$B_{K^*}(\Phi) = |A^{K^*(892)}(M^2_{K\pi})|^2 P_{K^*}(M^2_{J/\psi\pi}),$$

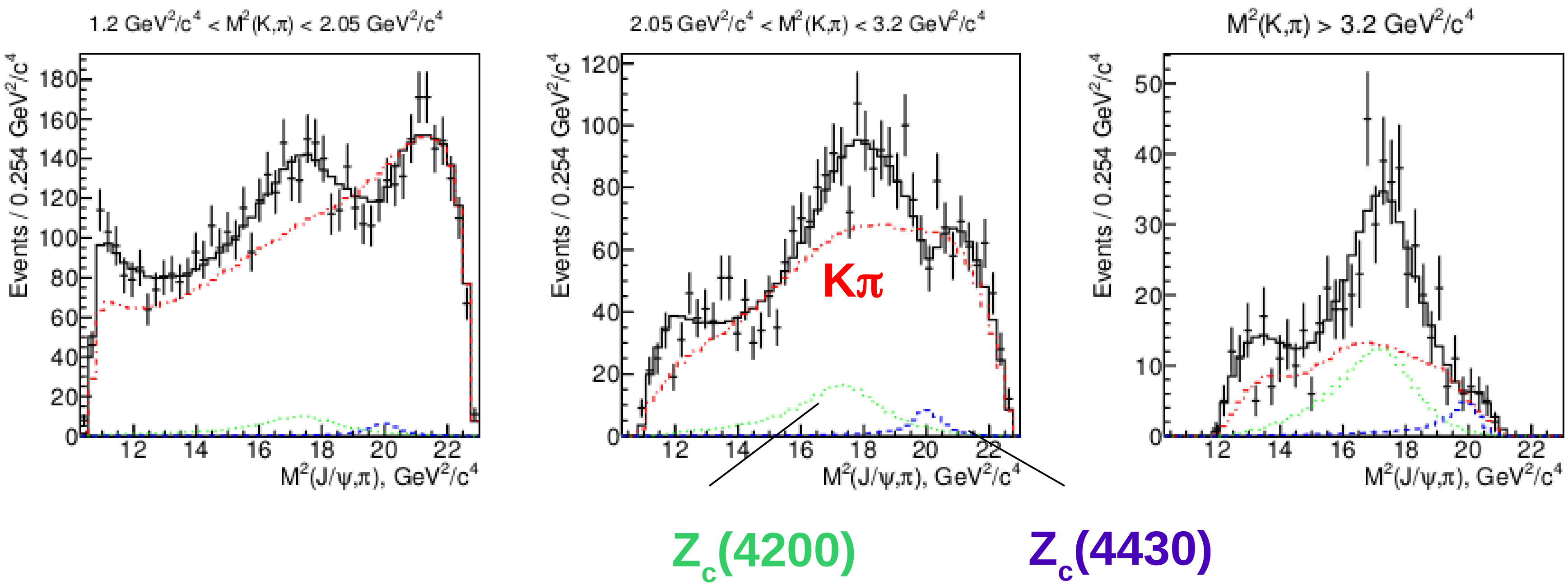
$$B_{K_S^0}(\Phi) = \exp\left[-\frac{(M^2_{K\pi} - M^2_{K\pi}(K_S^0))^2}{2\sigma^2}\right] P_{K_S^0}(M^2_{J/\psi\pi}),$$



Results of the fit

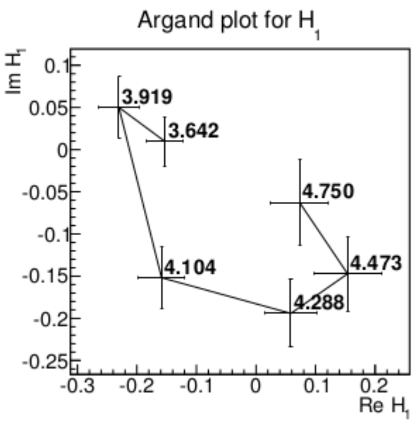
PRD 90,112009 (2014)

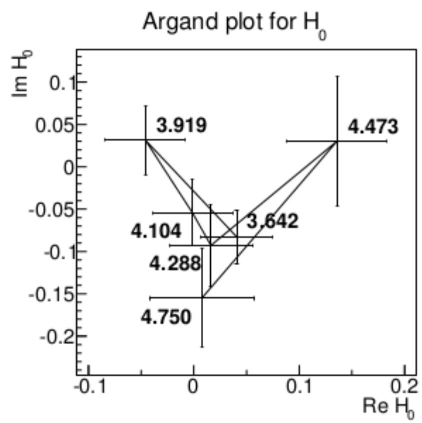
J^P	0^-	1^-	1^+	2^-	2^+
Mass, MeV/c^2	4318 ± 48	4315 ± 40	4196^{+31}_{-29}	4209 ± 14	4203 ± 24
Width, MeV	720 ± 254	220 ± 80	370 ± 70	64 ± 18	121 ± 53
Significance (Wilks)	3.9σ	2.3σ	8.2σ	3.9σ	1.9σ



Results of the fit

$J^P=1^+$, other J^P are excluded





Resonance	Fit fraction	Significance (local)
$K_0^*(800)$	$(7.1^{+0.7}_{-0.5})\%$	22.5σ
$K^*(892)$	$(69.0^{+0.6}_{-0.5})\%$	166.4σ
$K^*(1410)$	$(0.3^{+0.2}_{-0.1})\%$	4.1σ
$K_0^*(1430)$	$(5.9^{+0.6}_{-0.4})\%$	22.0σ
$K_2^*(1430)$	$(6.3^{+0.3}_{-0.4})\%$	23.5σ
$K^*(1680)$	$(0.3^{+0.2}_{-0.1})\%$	2.7σ
$K_3^*(1780)$	$(0.2^{+0.1}_{-0.1})\%$	3.8σ
$K_0^*(1950)$	$(0.1^{+0.1}_{-0.1})\%$	1.2σ
$K_2^*(1980)$	$(0.4^{+0.1}_{-0.1})\%$	5.3σ
$K_4^*(2045)$	$(0.2^{+0.1}_{-0.1})\%$	3.8σ
$Z_c(4430)^+$	$(0.5^{+0.4}_{-0.1})\%$	5.1σ
$Z_c(4200)^+$	$(1.9^{+0.7}_{-0.5})\%$	8.2σ

Exclusion levels of other spin-parity hypothesis

j^P	1^+ over j^P		1^+ C. L.
	MC	$\sqrt{\Delta(-2\ln L)}$	
0^-	8.6σ	7.9σ	26%
1^-	9.8σ	8.7σ	48%
2^-	8.8σ	7.6σ	40%
2^+	10.6σ	8.8σ	42%

$\mathcal{B}(\bar{B}^0 \to J/\psi K^-\pi^+) = (1.15 \pm 0.01 \pm 0.05) \times 10^{-3}$,

$\mathcal{B}(\bar{B}^0 \to J/\psi K^*(892)) = (1.19 \pm 0.01 \pm 0.08) \times 10^{-3}$,

$\mathcal{B}(\bar{B}^0 \to Z_c(4430)^+ K^-) \times \mathcal{B}(Z_c(4430)^+ \to J/\psi\pi^+) = (5.4^{+4.0+1.1}_{-1.0-0.9}) \times 10^{-6}$,

$\mathcal{B}(\bar{B}^0 \to Z_c(4200)^+ K^-) \times \mathcal{B}(Z_c(4200)^+ \to J/\psi\pi^+) = (2.2^{+0.7+1.1}_{-0.5-0.6}) \times 10^{-5}$,

$\mathcal{B}(\bar{B}^0 \to Z_c(3900)^+ K^-) \times \mathcal{B}(Z_c(3900)^+ \to J/\psi\pi^+) < 9 \times 10^{-7}$ (90% CL).

PRD 90,112009 (2014)

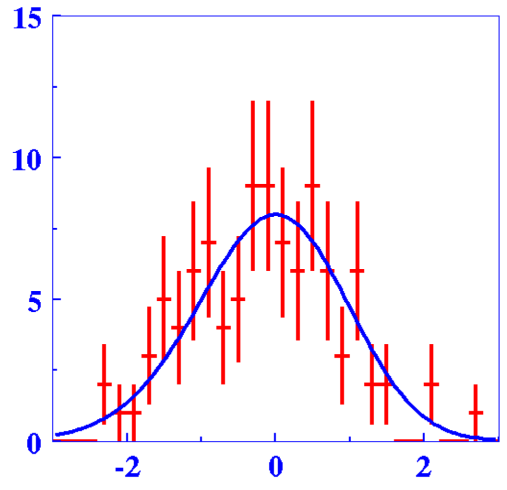
Summary

In spite of relatively small statistics
many results were obtained by Belle using Dalitz plot analysis:

- Relative contributions of known states
- Discovery of new exotic states
- Quantum numbers of known and new states

Backup slides

Binned fit

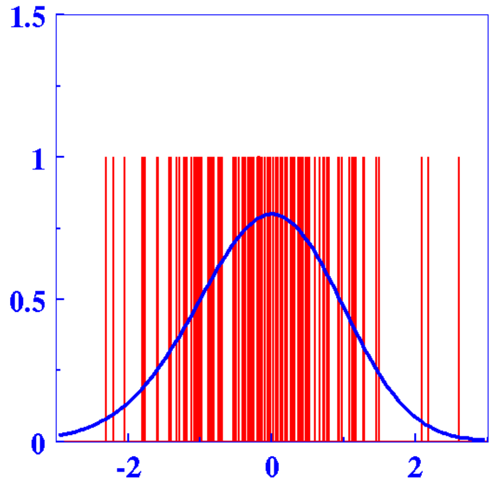


$$\chi^2 = \sum_{i_1,i_2\ldots} \frac{(N_{i_1,i_2\ldots} - AF(\vec{q_i},\vec{\xi}))^2}{\sigma^2_{i_1,i_2\ldots}}$$

Unbinned fit

$$\mathcal{L} = -\sum_k \ln F(\vec{q_k},\vec{\xi}) \to min,$$

$$\int F(\vec{q},\vec{\xi})dQ = 1$$



$$S(\vec{q},\vec{\xi}) = \int \varepsilon(\vec{q},\vec{q_t})|\mathcal{M}(\vec{q_t},\vec{\xi})|^2 dQ_t \approx \varepsilon(\vec{q})|\mathcal{M}(\vec{q},\vec{\xi})|^2.$$

$$F(\vec{q}) = \frac{N_s\varepsilon(\vec{q})|\mathcal{M}(\vec{q},\vec{\xi})|^2 + n_{bg}b(\vec{q})}{N_s\int\varepsilon(\vec{q})|\mathcal{M}(\vec{q},\vec{\xi})|^2 d\vec{q} + n_{bg}}$$

$$\mathcal{L} \;=\; -\sum_{events} \ln\left(n_s\frac{|\mathcal{M}(\vec{q},\vec{\xi})|^2}{\sum_{MC}|\mathcal{M}(\vec{q},\vec{\xi})|^2} + n_{bg}\frac{B(\vec{q})}{\sum_{MC}B(\vec{q})}\right)/(n_s+n_{bg})$$

$$-\cancel{\sum_{events}}\ln\varepsilon(\vec{q}) \quad + \quad \frac{(n_s+n_{bg}-n_{tot})^2}{2\sigma^2_{tot}}$$

D^{**} are p-wave excitations of D mesons.

$B \to D^{**}\pi$, $D^{**} \to D^{(*)}\pi$

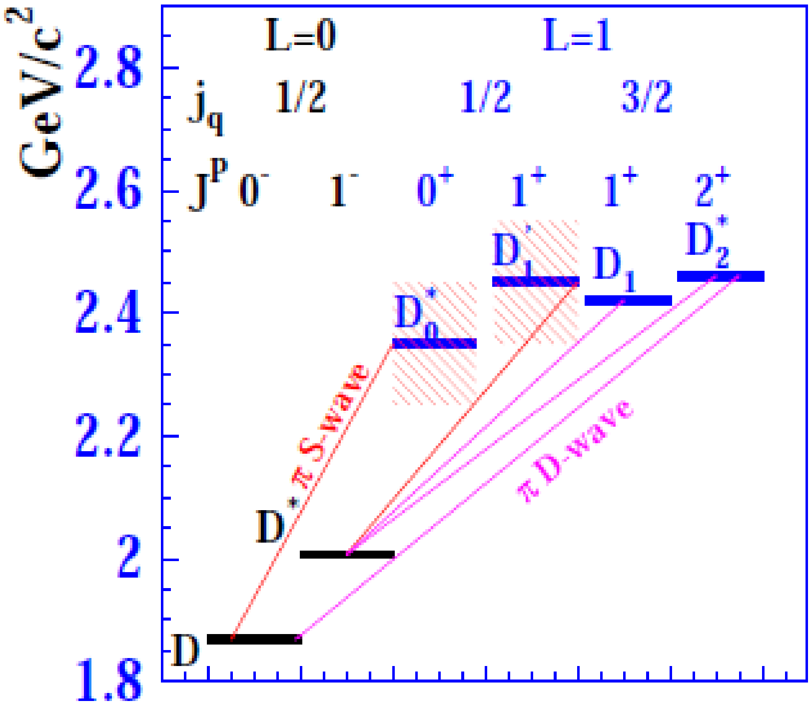
$D^{**} \to D^{(*)}\pi$ have different dependences

$D_2^* \to$	$D\pi$,	$D^*\pi$	D-wave
$D_1 \to$		$D^*\pi$	D-wave
$D_1' \to$		$D^*\pi$	S-wave
$D_0^* \to$	$D\pi$		S-wave

In B decay fixed initial state spin 0.

$B \to D_2^*\pi$	D-wave
$B \to D_1\pi$	P-wave
$B \to D_1'\pi$	P-wave
$B \to D_0^*\pi$	S-wave

All D^{**} states can be distinguished using Dalitz plot analysis



Test of HQET and QCD sum rule predictions.