Triangle singularities in the meson sector and charmonium decay in three mesons

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--Triangle singularities

--The $f_1(1420)$ as a decay mode of the $f_1(1285)$ into π $a_0(980)$ and K* Kbar

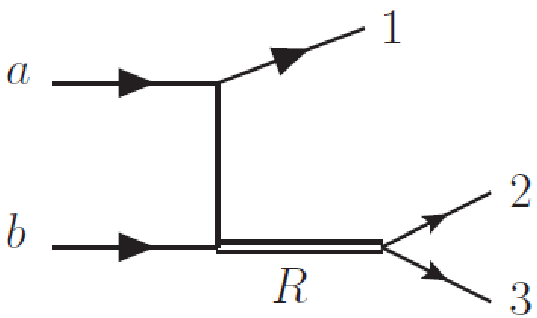
-- The $f_2(1810)$ as a triangle singularity

-Predictions for Bbar -> Kbar π $D_{s0}^*(2317)$ through a TS.

-- The χ_{c1} -> $\pi\pi\eta$ reaction

--Predictions for the η_c -> $\pi\pi\eta$ reaction

Resonance production at high energies



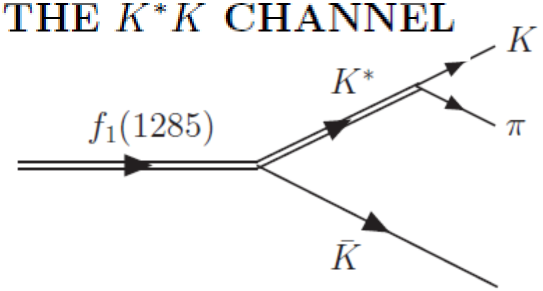
$$\frac{d^2\sigma}{dt\, dM_{\rm inv}} = \frac{\prod(2m_F)}{32\, p_a^2\, s}\, \frac{1}{(2\pi)^3}\, C^2 \frac{8\pi M_{\rm inv}^2 \Gamma_{R,23}}{|M_{\rm inv}^2 - M_R^2 + i M_R \Gamma_R|^2}$$

The $f_1(1420)$: PDG, about 20 experiments. All claim that the decay mode is K* Kbar, but one experiment claims 4% decay to π $a_0(980)$ D. Barberis et al. PLB 1998

However, let us study the decay of $f_1(1285)$ into K* Kbar

Debastiani, Aceti, Liang, E.O, PRD 2017

THE $K^*\bar{K}$ CHANNEL



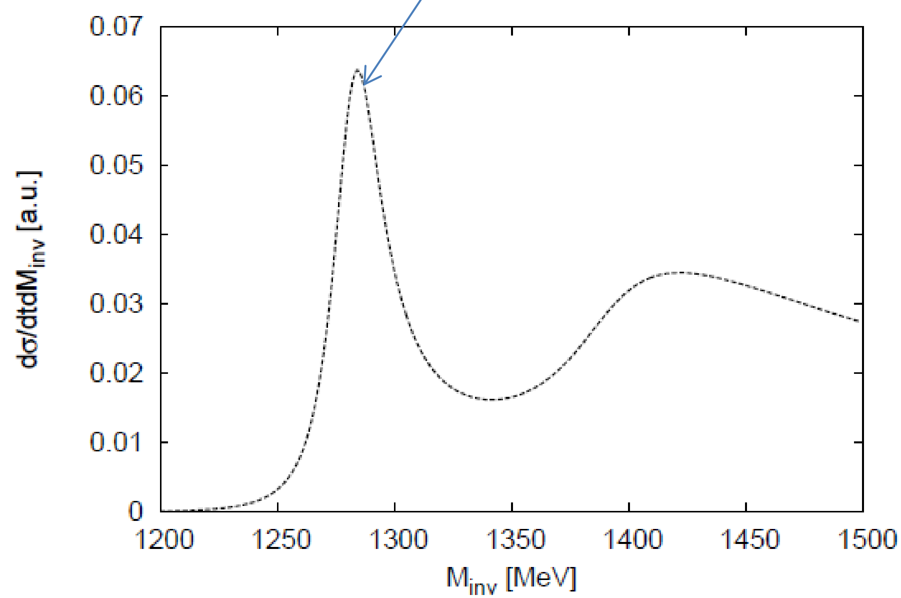
The $f_1(1285)$ is dynamically generated from the K* Kbar - cc channel.

Roca, E. O. Singh, PRD 2005

Lutz, Kolomeitsev, NPA 2004

Strength in agreement with exp. , Aceti, Xie, E.O, PLB 2015

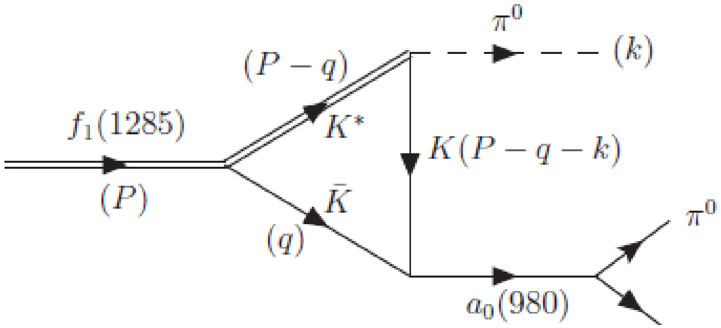
About 8% of f_1 width



Beware: This is quite common.

As a rule, if one finds an enhancement of a channel close above threshold, the chances are large that this indicates a bound state of this component.

THE $\pi a_0(980)$ DECAY MODE OF THE $f_1(1285)$



$$t_T = i\int \frac{d^4q}{(2\pi)^4}\,\vec{\epsilon}_{f_1}\cdot\vec{\epsilon}_{K^*}\;\vec{\epsilon}_{K^*}\cdot(2\vec{k}+\vec{q})\,\frac{1}{q^2-m_K^2+i\epsilon}\,\frac{1}{(P-q)^2-m_{K^*}^2+im_{K^*}\Gamma_{K^*}}\,\frac{1}{(P-q-k)^2-m_K^2+i\epsilon}$$

$$\tilde{t}_T = \int\frac{d^3q}{(2\pi)^3}\left(2+\frac{\vec{k}\cdot\vec{q}}{\vec{k}^2}\right)\frac{1}{8\,\omega(q)\omega'(q)\omega^*(q)}\,\frac{1}{k^0-\omega'(q)-\omega^*(q)+i\epsilon}\,\frac{1}{P^0-\omega^*(q)-\omega(q)+i\epsilon}$$
$$\times\frac{2P^0\omega(q)+2k^0\omega'(q)-2(\omega(q)+\omega'(q))(\omega(q)+\omega'(q)+\omega^*(q))}{(P^0-\omega(q)-\omega'(q)-k^0+i\epsilon)(P^0+\omega(q)+\omega'(q)-k^0-i\epsilon)},$$

$$\omega(q) = \sqrt{\vec{q}^{\,2}+m_K^2},\;\; \omega'(q) = \sqrt{(\vec{q}+\vec{k})^2+m_K^2},\;\; \omega^*(q) = \sqrt{\vec{q}^{\,2}+m_{K^*}^2}$$

Poles in the integration

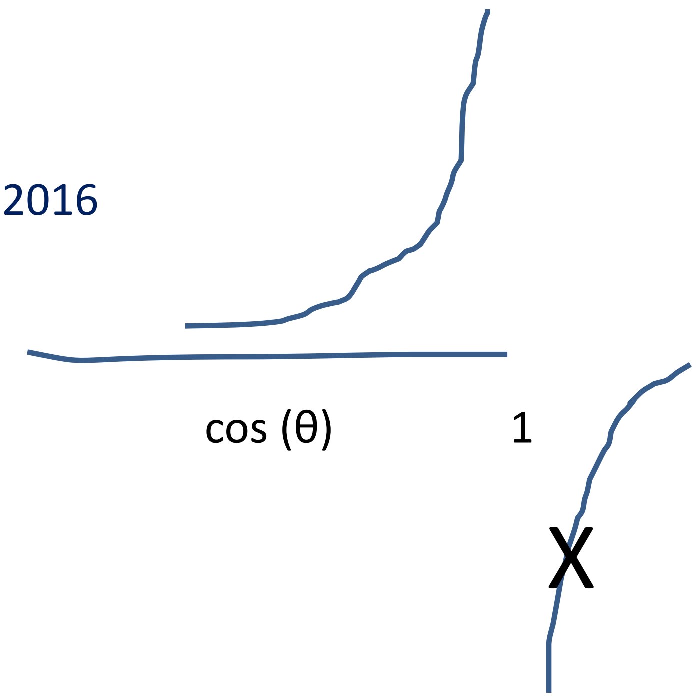
$$P^0-\omega^*(q)-\omega(q)+i\epsilon = 0\,, \qquad q_{\rm on+} = q_{\rm on}+i\,\epsilon \quad \text{with} \quad q_{\rm on} = \frac{1}{2M}\sqrt{\lambda(M^2,m_1^2,m_2^2)}$$

$$P^0-\omega(q)-\omega'(q)-k^0+i\epsilon = 0$$

$$P^0 - \omega(q) - \omega'(q) - k^0 + i\epsilon = 0 \qquad \omega'(q) = \sqrt{(\vec{q} + \vec{k})^2 + m_K^2}$$

If we fix cos (θ)= ± 1 and we make this expression zero, then in the integral of cos(θ) one cannot cancel the divergence with the principal value, and the divergence remains. Θ is the angle between $\vec{k}$ and $\vec{q}$.

Bayar, Aceti, Guo, E. O, PRD 2016



For cos(θ)=-1

$$q_{a+} = \gamma\,(v\,E_2^* + p_2^*) + i\,\epsilon\,, \qquad q_{a-} = \gamma\,(v\,E_2^* - p_2^*) - i\,\epsilon$$

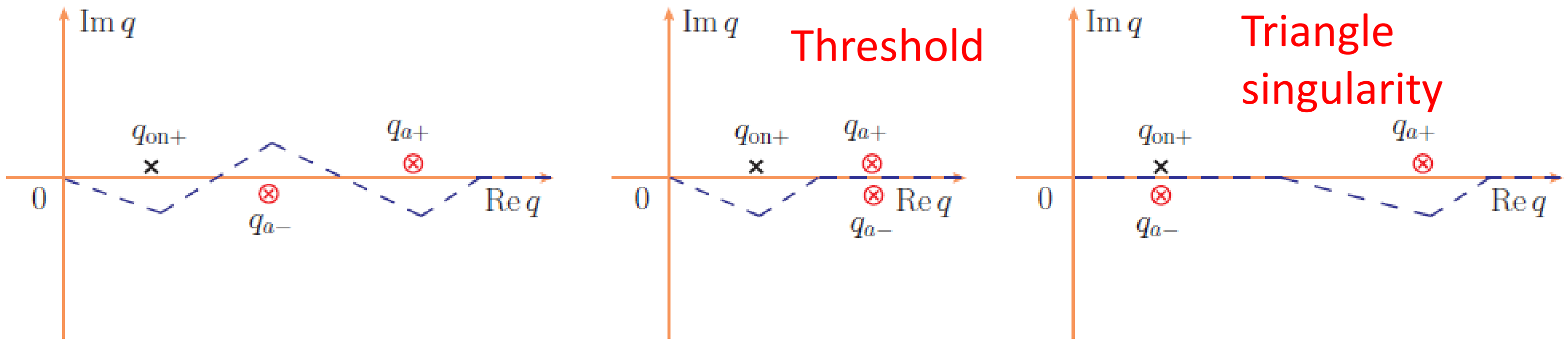
$$v = \frac{k}{E_{23}}\,, \qquad \gamma = \frac{1}{\sqrt{1-v^2}} = \frac{E_{23}}{m_{23}}\,,$$

$$E_2^* = \frac{1}{2m_{23}}\left(m_{23}^2 + m_2^2 - m_3^2\right), \qquad p_2^* = \frac{1}{2m_{23}}\sqrt{\lambda(m_{23}^2, m_2^2, m_3^2)}$$

For cos(θ)=1

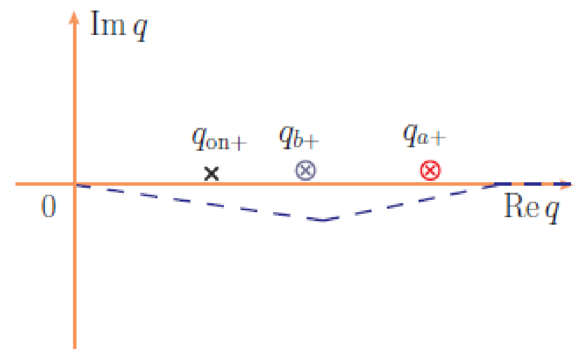
$$q_{b+} = \gamma\,(-v\,E_2^* + p_2^*) + i\,\epsilon\,, \qquad q_{b-} = -\gamma\,(v\,E_2^* + p_2^*) - i\,\epsilon$$

For cos(θ)=-1



(a) (b) (c)

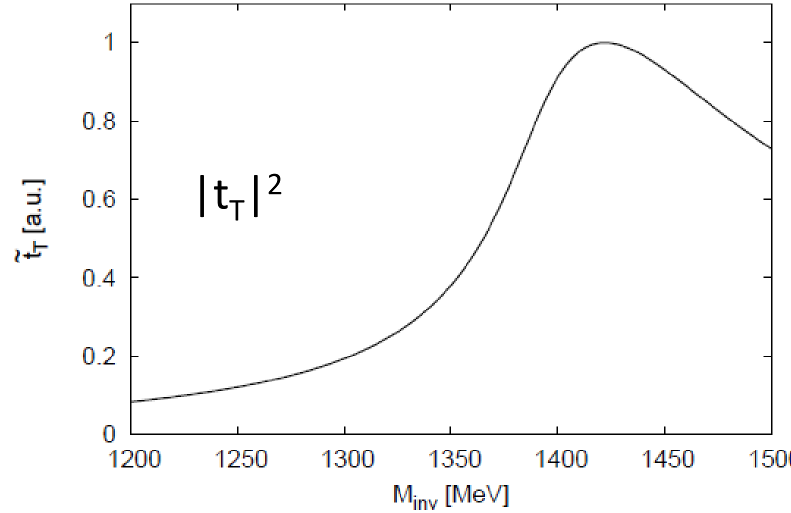
For cos(θ)=1



Triangle singularity

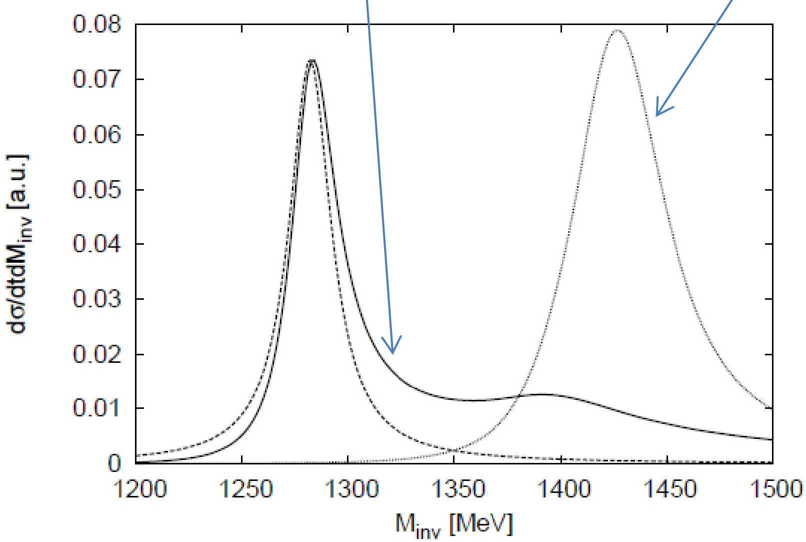
$$\lim_{\epsilon \to 0} (q_{\mathrm{on}+} - q_{a-}) = 0$$

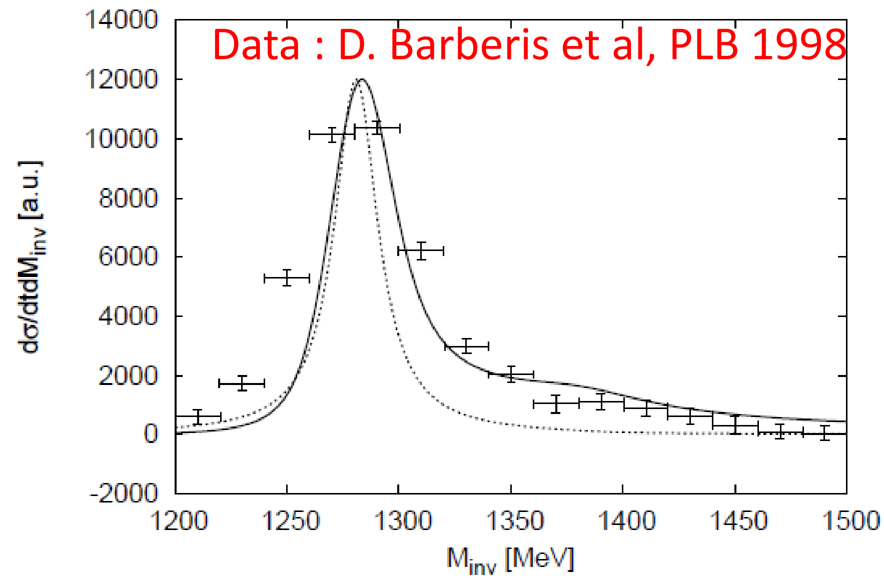
Very simple expression to see where the TS appears , and to explain the Coleman-Norton theorem, Nuovo Cim. 1965, (TS appears when the decays in the loop can occur at the classical level).



π $a_0(980)$ production through $f_1(1285)$

π $a_0(980)$ production assuming that there is a new resonance $f_1(1420)$ that decays to K* Kbar





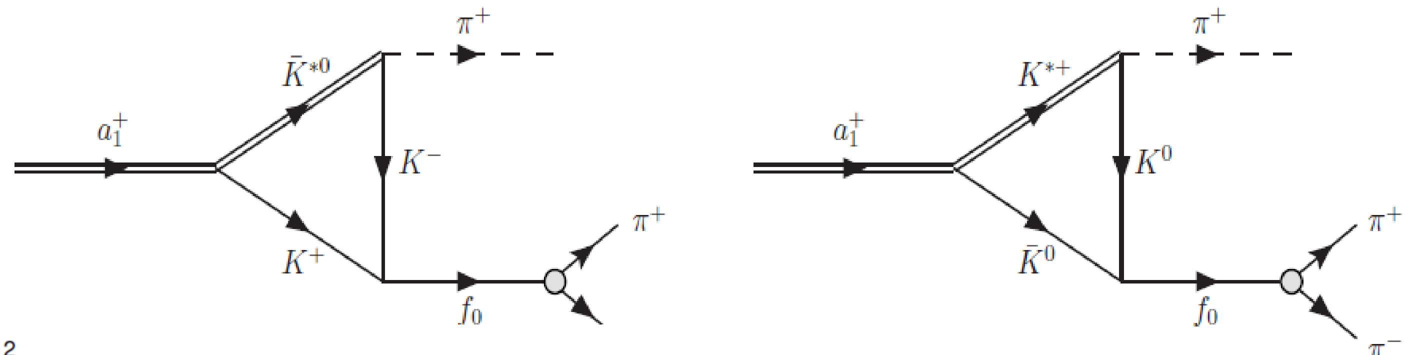
If $f_1(1420)$ were a new resonance, we also find a BR for π $a_0(980)$ decay of 20%, contrary to all experimental findings.

Conclusion: The $f_1(1420)$ is not a new resonance: it is the manifestation of the decay of the $f_1(1285)$ into K* Kbar and π $a_0(980)$

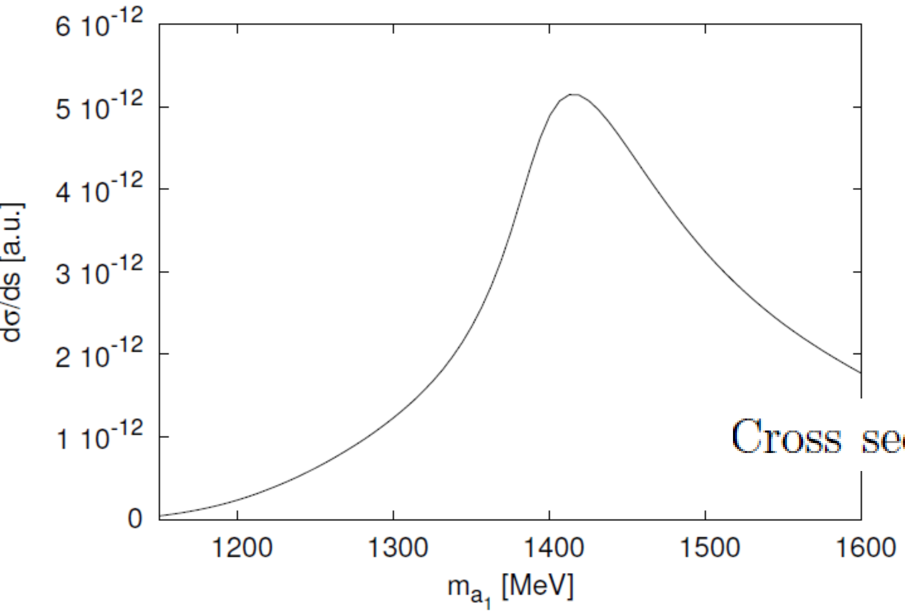
These results are a replica of what was found for the “a_1(1420)”

In M. Mikhasenko, B. Ketzer, A. Sarantsev, PRD 2015
F. Aceti, L.R. Dai, E. O., PRD 2016
suggested in X.H. Liu, M. Oka, Q. Zhao, PLB 2016

The “a1(1420)” peak is a manifestation of the decay of the a_1(1260) into π f_0(980)



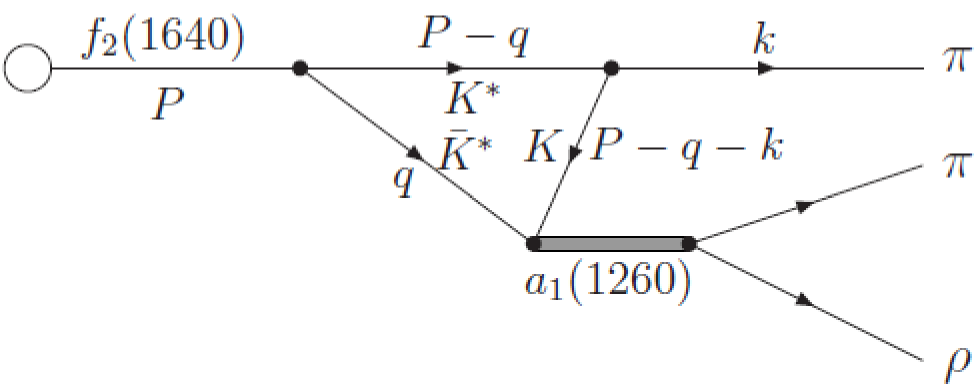
B)

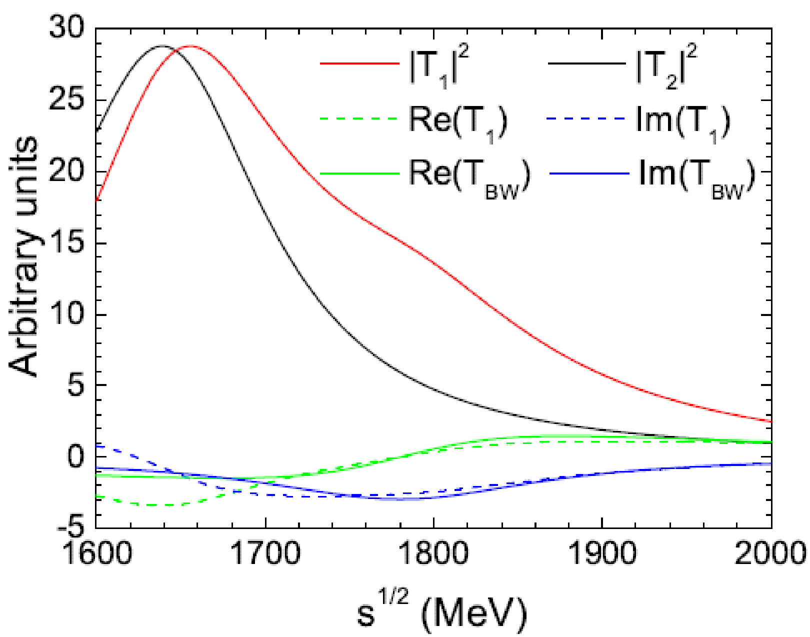


Cross section for the decay $a_1^+(1260) \to \pi^+ f_0(980)$

$f_2(1810)$ as a triangle singularity

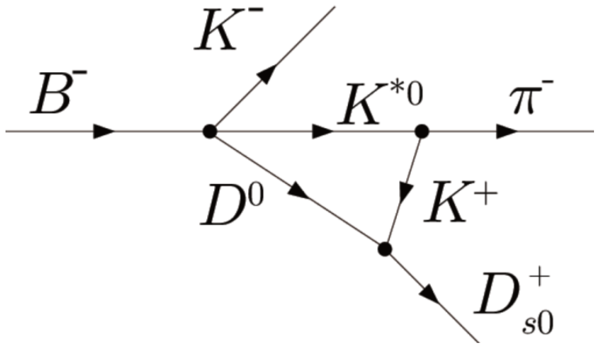
J.J. Xie, L.S. Geng, E. O, PRD 2017





Bbar-> Kbar K* D -> Kbar pi D_{s0}(2317)

Suggested in X.H. Liu, M. Oka, Q. Zhao, PLB 2016



A. Ramos, S. Sakai, E. O

BR($B^- \to K^- K^{*0} D^0$)=7.5 10^{-4}

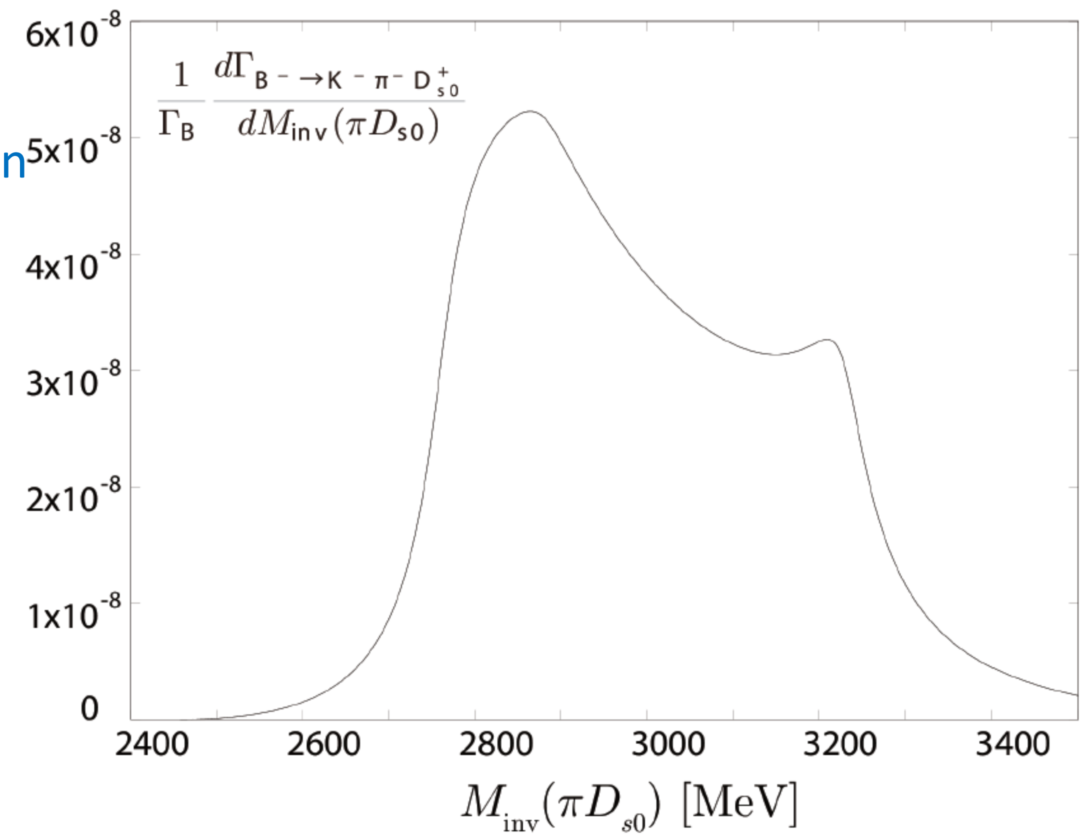
T=C ε . p_K to match angular momentum.
Determine C and evaluate the T diagram

The coupling D_{s0}*(2317) to K D is , g_{KD}=10.21 from the picture where D_{s0}*(2317) is dynamically generated, Kolomeitsev, Gamermann
Corroborated from lattice (Martinez, Oset, Prelovsek, Ramos, JHEP, 2015)

Integrated rate for

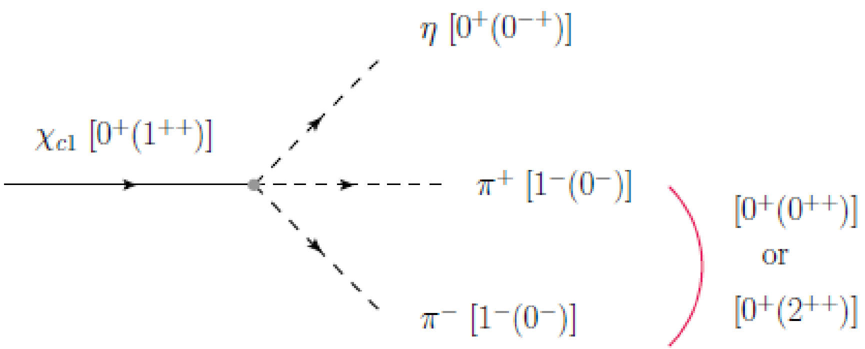
$B^- \to K^- \pi^- D_{s0}$*(2317)

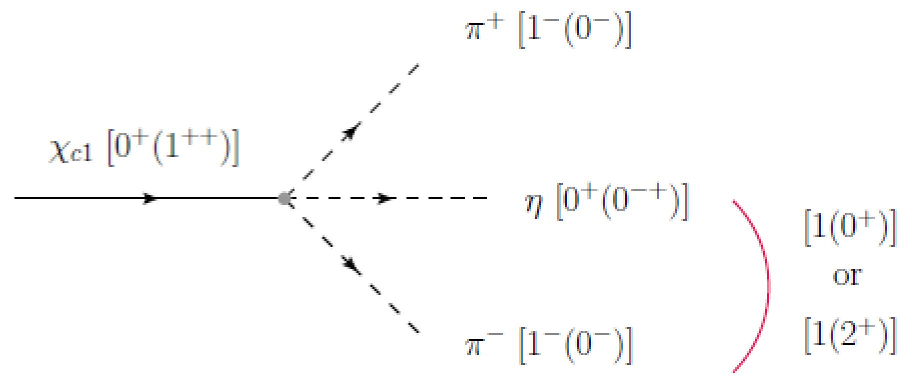
BR=~ 2 10^{-5}



$f_0(500)$, $f_0(980)$, and $a_0(980)$ production in the $\chi_{c1} \to \eta\pi^+\pi^-$ reaction

W.H. Liang, J.J. Xie and E. O, EPJ C , 2016





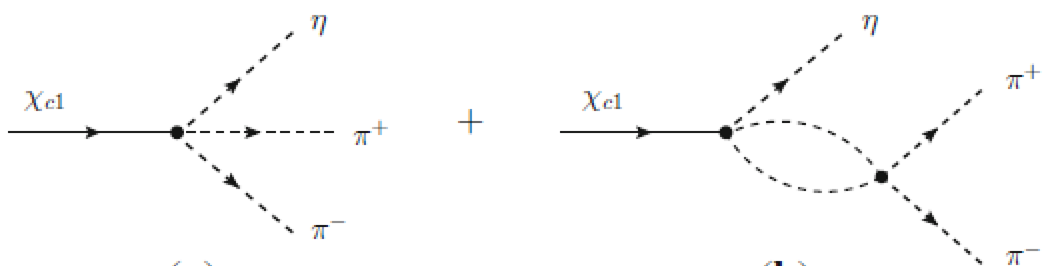
$$
M \to \phi
\equiv \begin{pmatrix}
\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{3}}\eta + \frac{1}{\sqrt{6}}\eta' & \pi^+ & K^+ \\
\pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{3}}\eta + \frac{1}{\sqrt{6}}\eta' & K^0 \\
K^- & \bar{K}^0 & -\frac{1}{\sqrt{3}}\eta + \sqrt{\frac{2}{3}}\eta'
\end{pmatrix}.
$$

$$
\mathrm{SU(3)[scalar]} \equiv \mathrm{Trace}(\phi\phi\phi)
$$

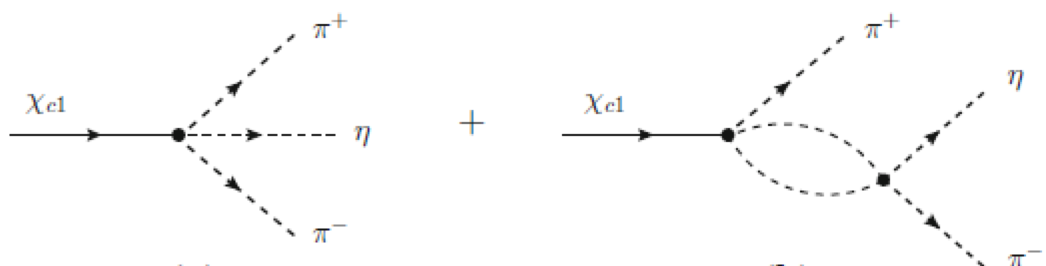
$$C_1: \ \eta\left(\frac{6}{\sqrt{3}}\pi^+\pi^- + \frac{3}{\sqrt{3}}\pi^0\pi^0 + \frac{1}{3\sqrt{3}}\eta\eta\right)$$

$$C_2: \ \pi^+\left(\frac{6}{\sqrt{3}}\pi^-\eta + 3K^0K^-\right)$$

$$C_3: \ \pi^-\left(\frac{6}{\sqrt{3}}\pi^+\eta + 3K^+\bar{K}^0\right)$$



(a) **(b)**



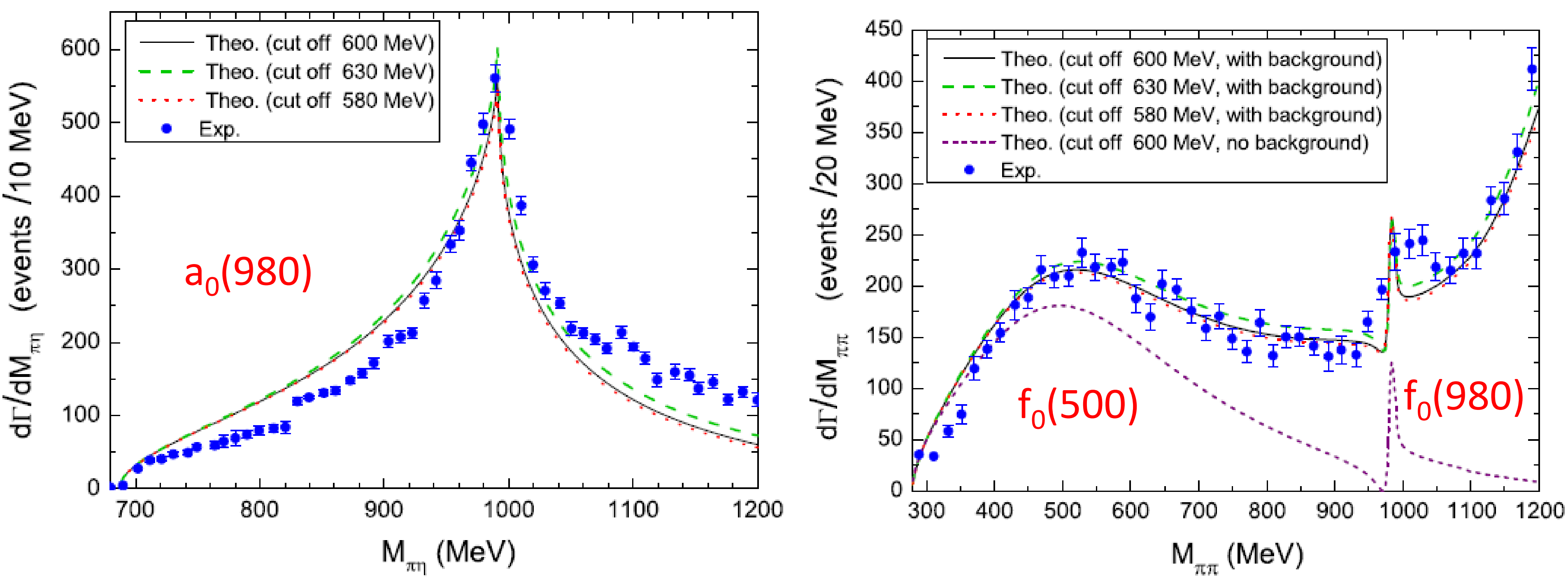
(a) **(b)**

$$\tilde{t}_\eta = V_P \left(h_{\pi^+\pi^-} + \sum_i h_i S_i G_i(M_{\rm inv}) t_{i,\pi^+\pi^-} \right)$$

$$h_{\pi^+\pi^-} = \frac{6}{\sqrt{3}}, \quad h_{\pi^0\pi^0} = \frac{3}{\sqrt{3}}, \quad h_{\eta\eta} = \frac{1}{3\sqrt{3}}$$

$$S_{\pi^0\pi^0} = 2 \times \frac{1}{2} \text{ (for two } \pi^0\text{)}; \quad S_{\eta\eta} = 3!\frac{1}{2} \text{ (for three } \eta\text{)}$$

Data from BESIII, PRD 2017

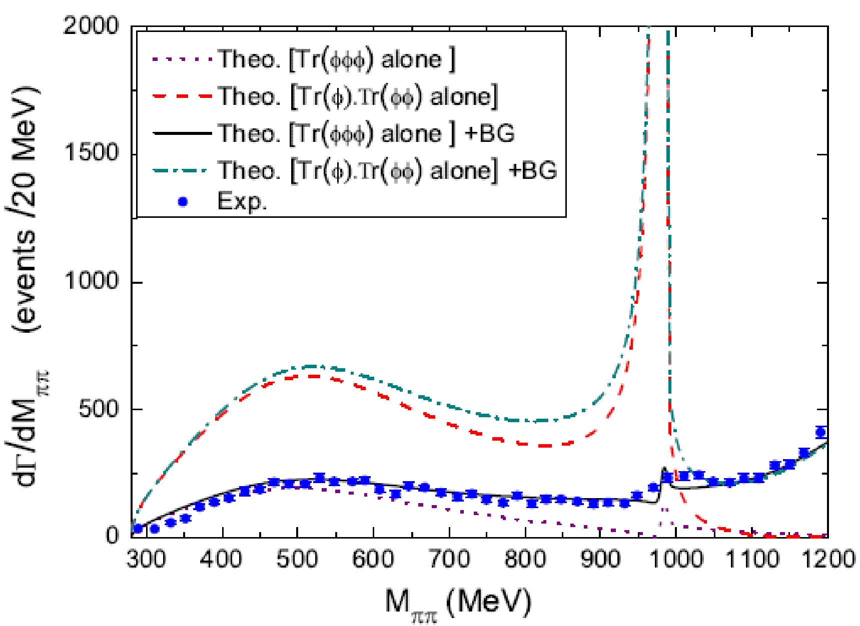


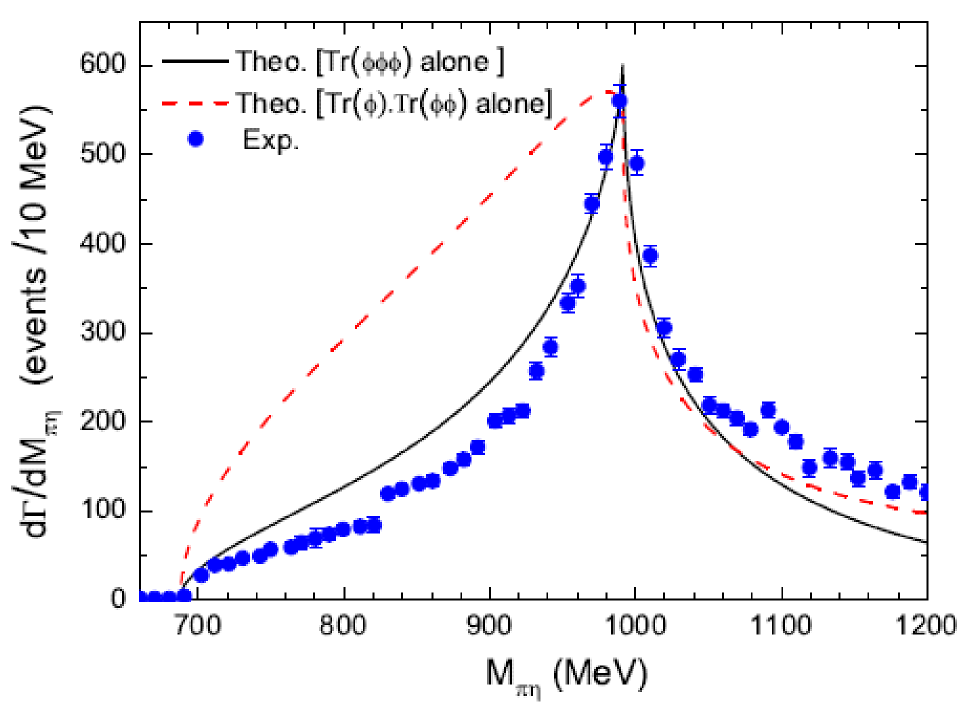
$$\mathrm{Trace}(\phi\phi\phi) = 2\sqrt{3}\eta\pi^+\pi^- + \sqrt{3}\eta\pi^0\pi^0 + \frac{\sqrt{3}}{9}\eta\eta\eta + 3\pi^+K^0K^- + 3\pi^-K^+\bar{K}^0, \quad (3)$$

$$\mathrm{Trace}(\phi)\mathrm{Trace}(\phi\phi) = \frac{\sqrt{3}}{3}\eta\left(2\pi^+\pi^- + \pi^0\pi^0 + 2K^+K^- + 2K^0\bar{K}^0 + \eta\eta\right), \quad (4)$$

$$[\mathrm{Trace}(\phi)]^3 = \frac{\sqrt{3}}{9}\eta\eta\eta. \quad (5)$$

The last two forms are clearly rejected by the data.





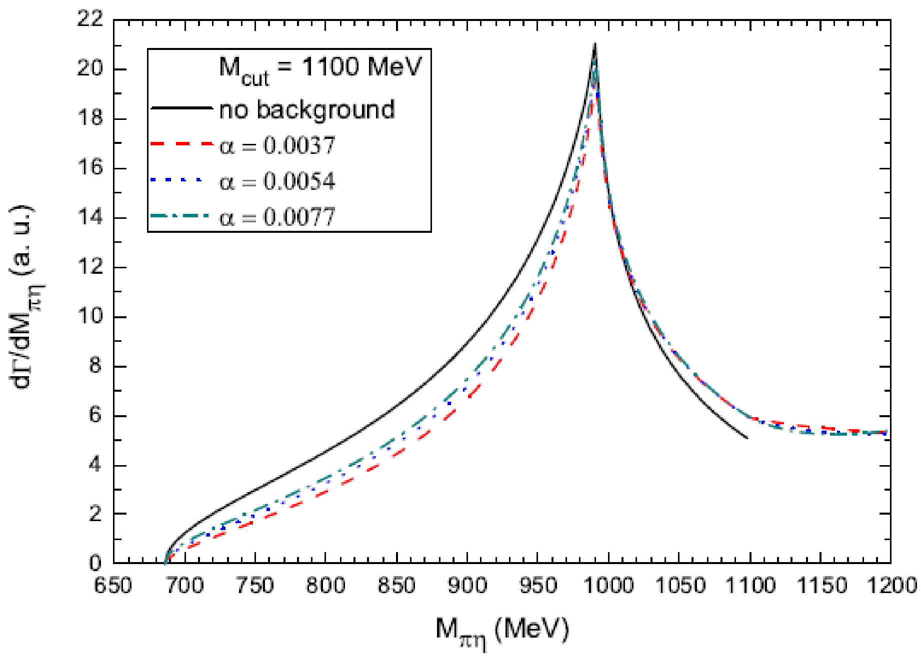
Predictions for $\eta_c \to \eta\pi^+\pi^-$ producing $f_0(500)$, $f_0(980)$ and $a_0(980)$

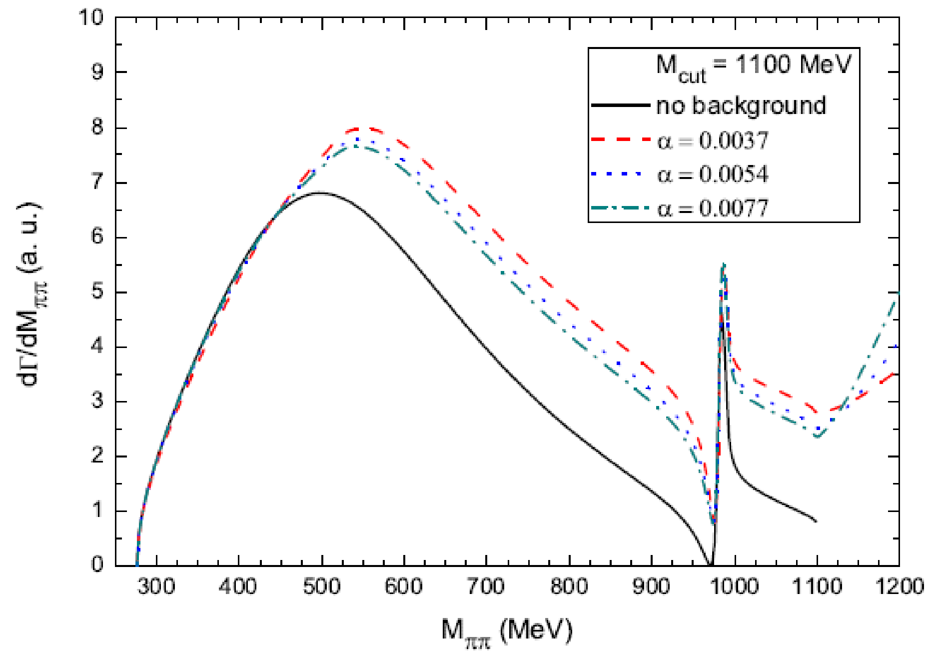
Debastiani, Liang, Xie, E. O. PLB 2017

$$\mathrm{SU(3)[scalar]} \equiv \mathrm{Trace}(\phi\phi\phi)$$

Same flavor combinations as in χ_{c1} decay, but the vertex now is s-wave

The background is estimated extrapolating the πη mass distribution beyond the $a_0(980)$ peak





Conclusions

-- Triangular singularities are seen to play an important role in some reactions:

They can give resonant like amplitudes, leading to claims of new resonances

Other times they reinforce genuine resonances, providing new decay channels at the same resonance energy

Sometimes they simply provide special decay channels with no obvious resonant shape

-- In cases where one knows that the TS is unavoidable, the partial wave analysis should be modified to incorporate them in the formalism.

-- Heavy mesons decays into lighter ones continue providing a very rich source of information on meson resonances and their nature