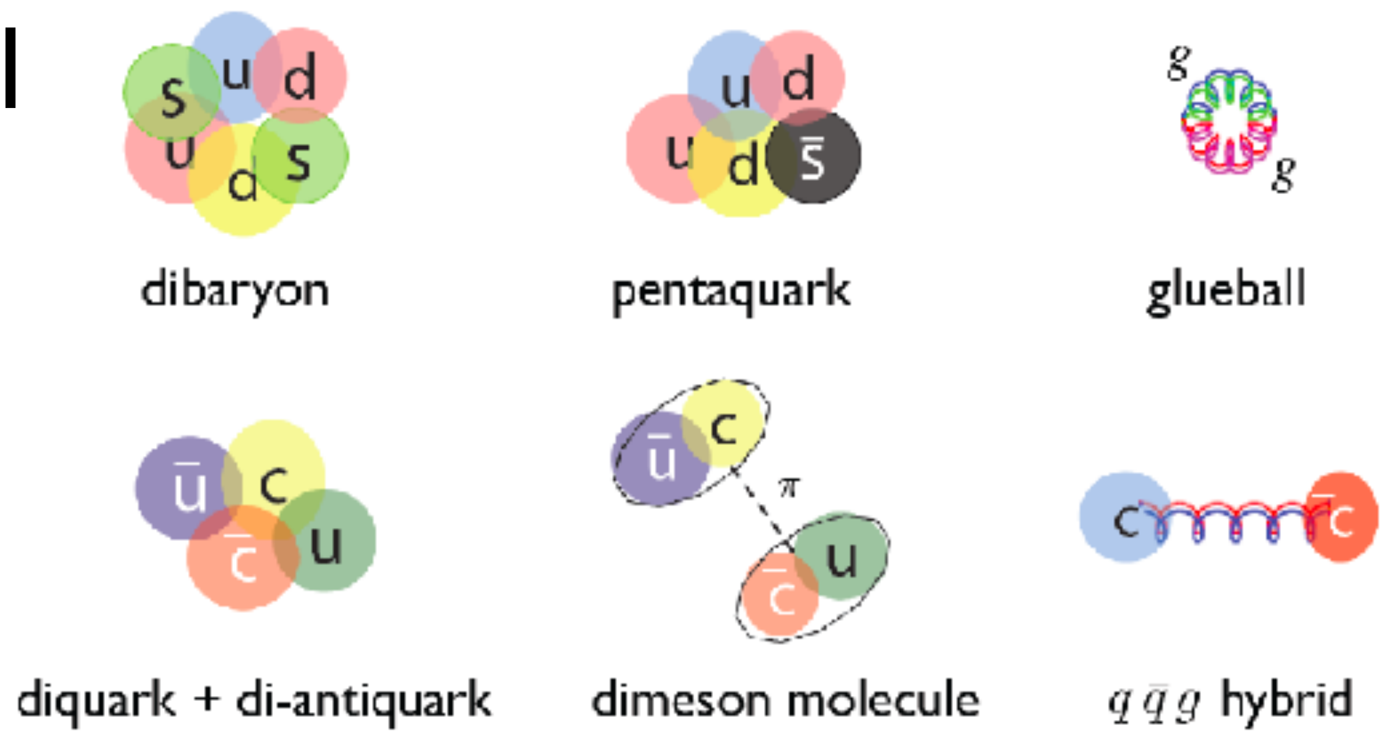
Meson Spectroscopy (in the context of) Amplitude Analysis

Adam Szczepaniak IU/JLab

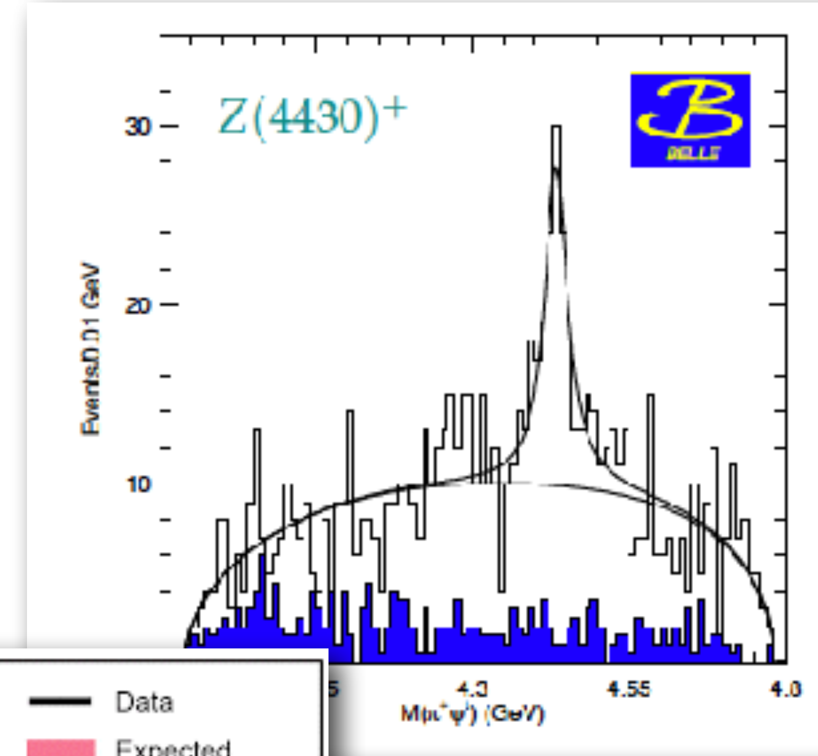
QCD predicts matter is made from confined (non-existing) quarks and gluons with >95% mass coming from interactions!

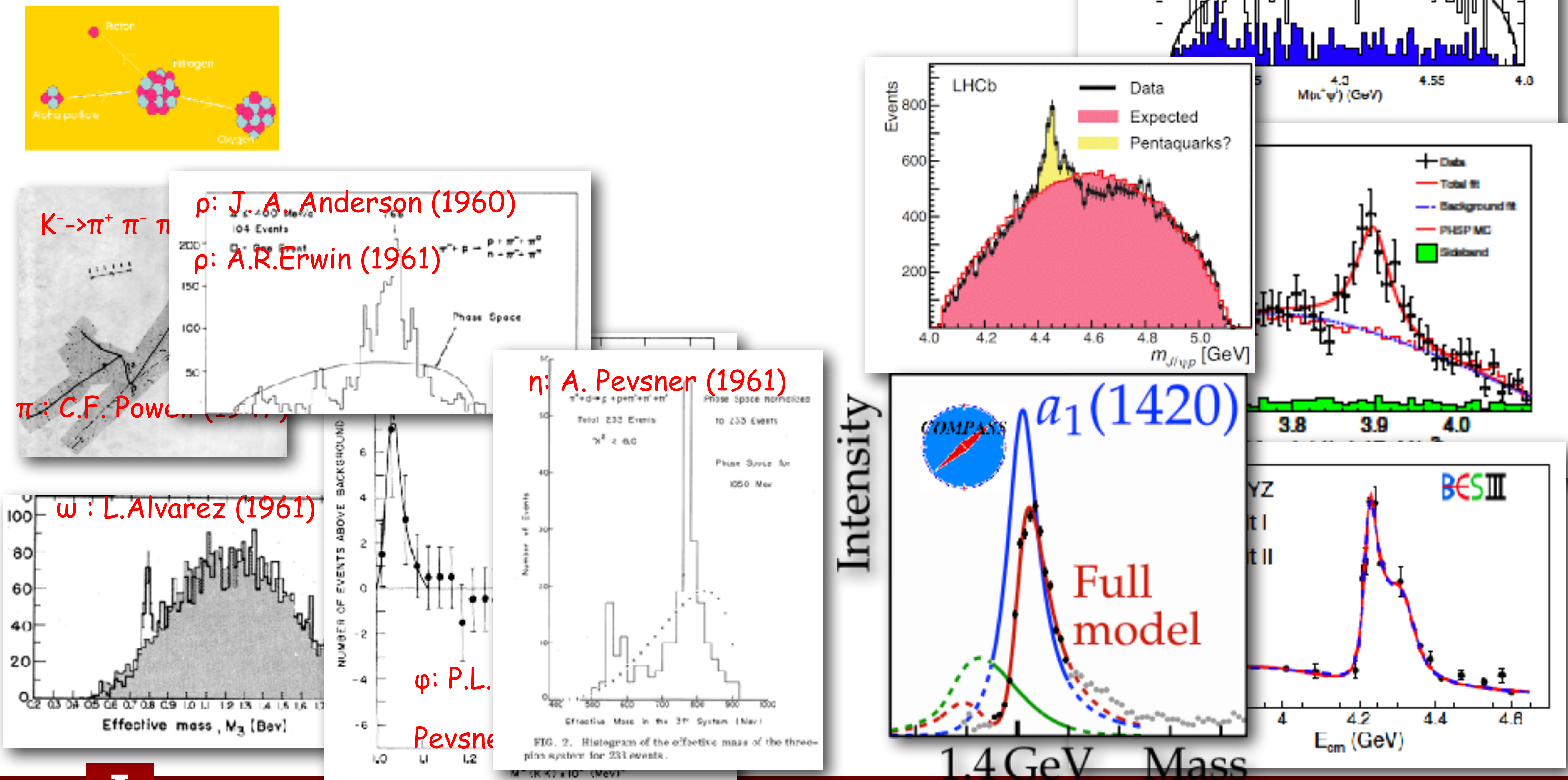
this is a highly nontrivial environment :
small world (10^{-15}m)
of fast (v~c) particles
exerting ~1T forces !!!



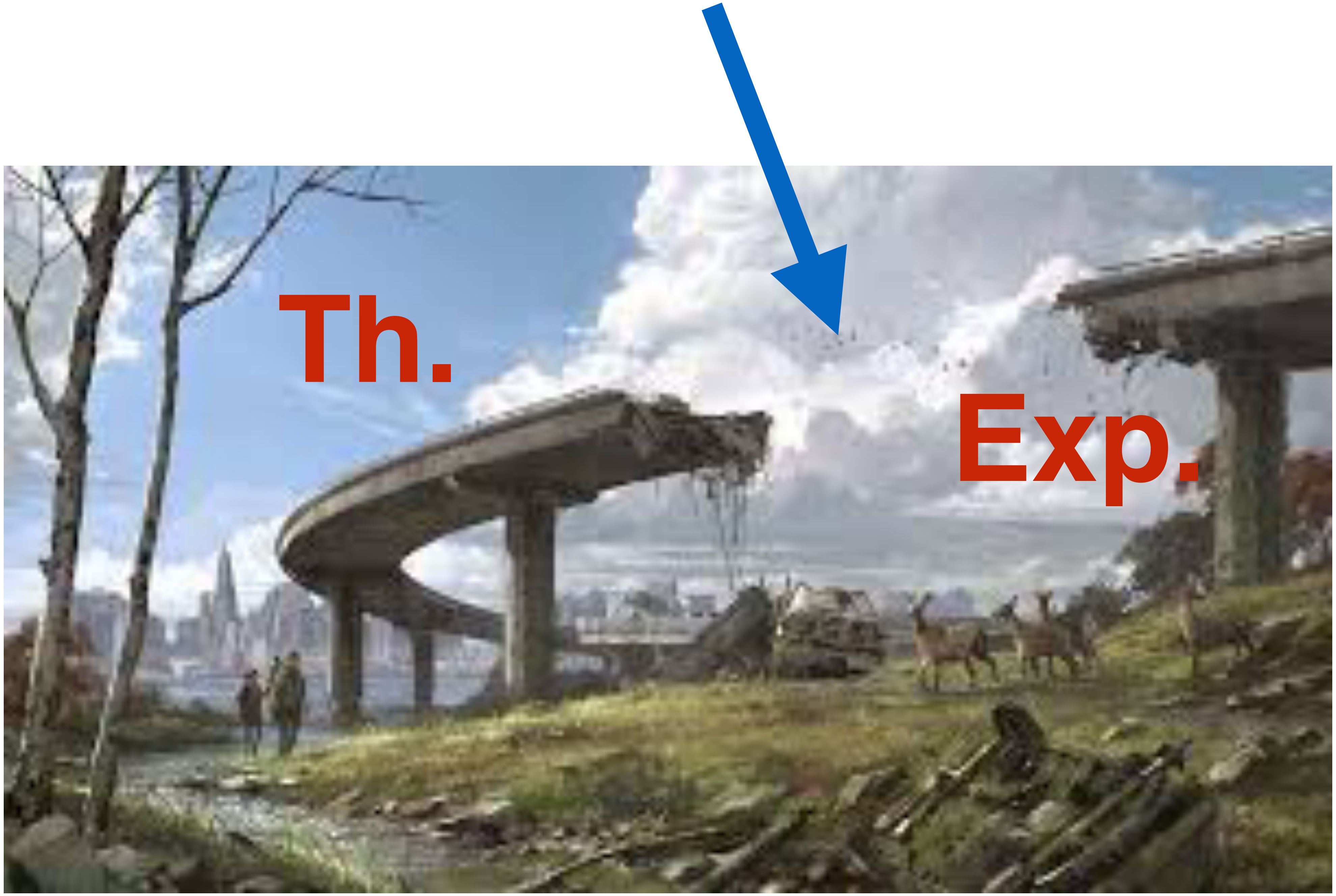
What are the constituents of hadrons ?

How do resonances appear in the data (experiment/lattice) ?



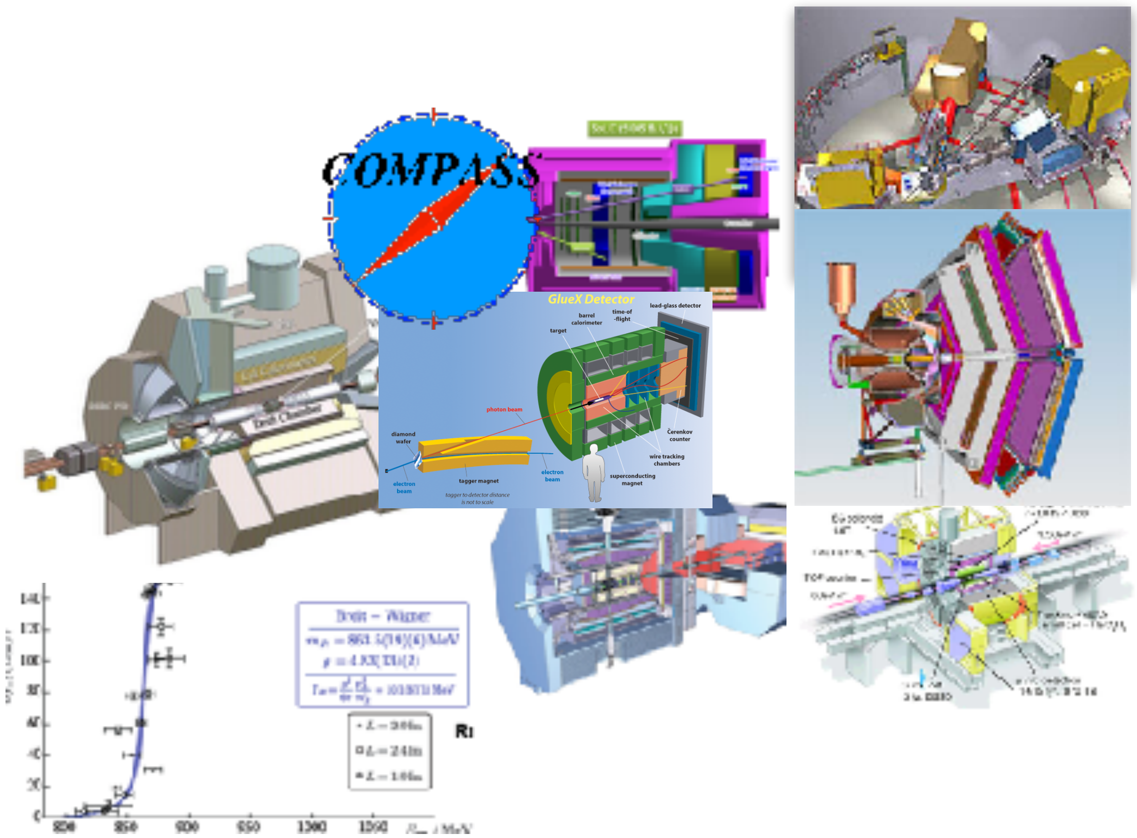


Amplitude Analysis



JPAC

- Started in the Fall of 2013 to support the extraction of physics results from analysis of experimental data from JLab12 and other accelerator laboratories.
- Work is on theoretical, phenomenological and data analysis tools in close collaboration with theorists and experimentalists worldwide.
- Contribute to education of new generation of practitioners in physics of strong interactions.

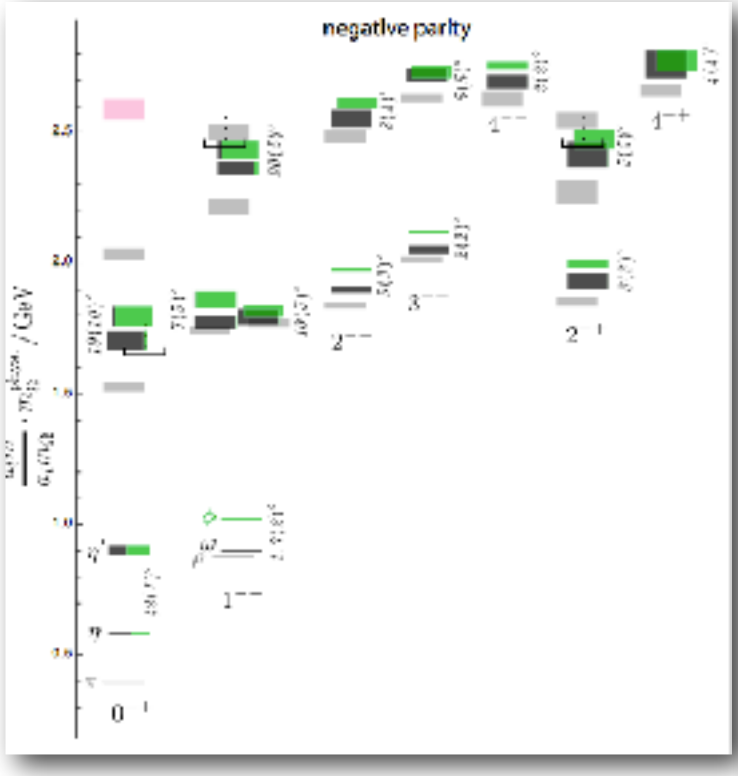


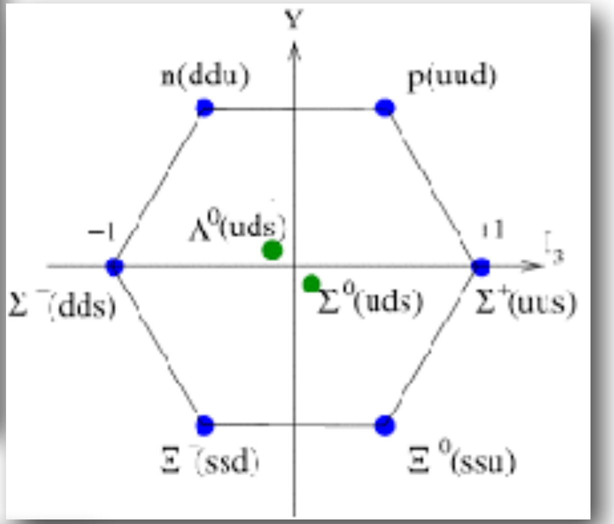
Amplitude analysis in hadron spectroscopy

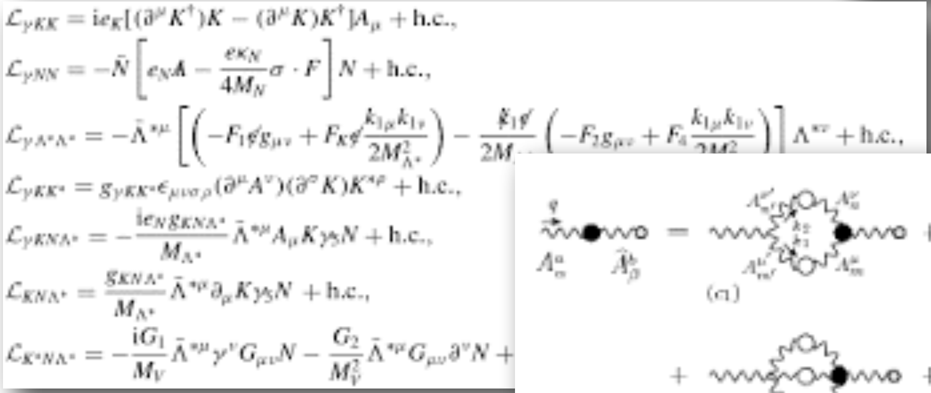
- Resonances appear as poles in partial wave amplitudes.
- Amplitudes are agnostic as to the nature of these poles (other "dials" needed to discriminate)
- We do not know (aka. from "exact calculations") where these poles are.
- There can be other singularities producing "bumps" in the physical region, or resonances can produce "dips"
- We put these "by hand" and have data (and lattice) to discriminate between hypotheses.

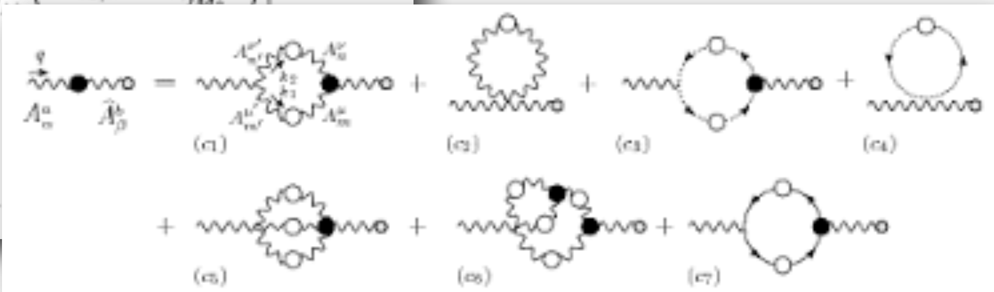
Resonance input

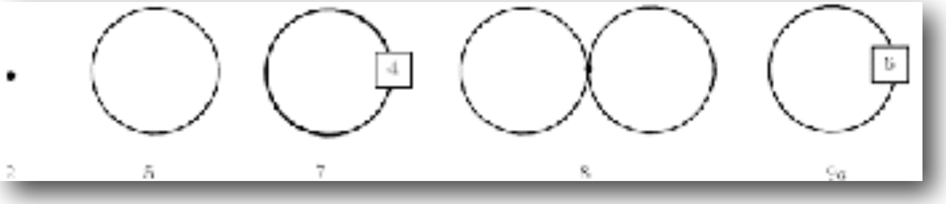
- PDG
- Quark Models
- Lattice
- Regge theory
- Other

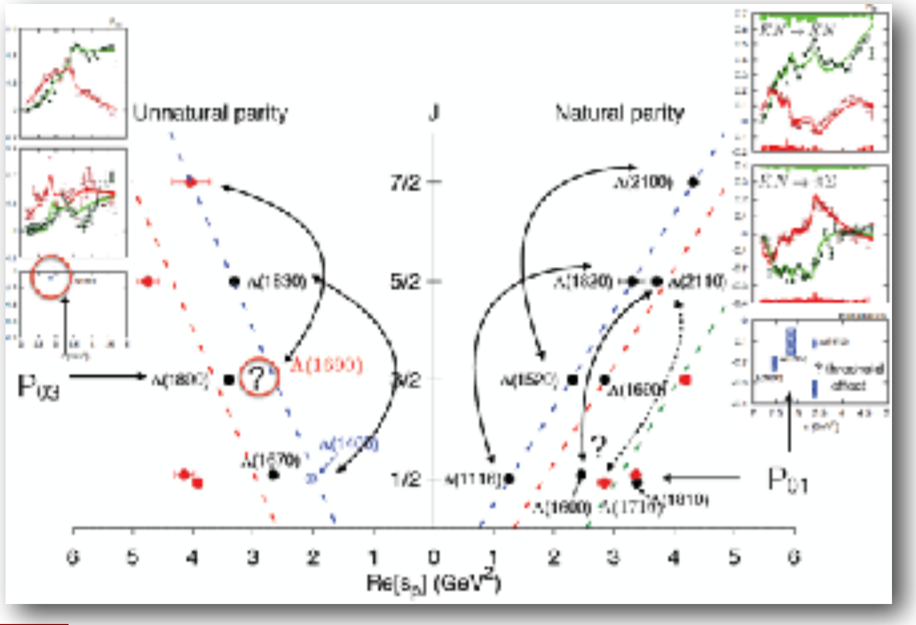












S -matrix constraints

"All constraints are equal but some are more equal the another"

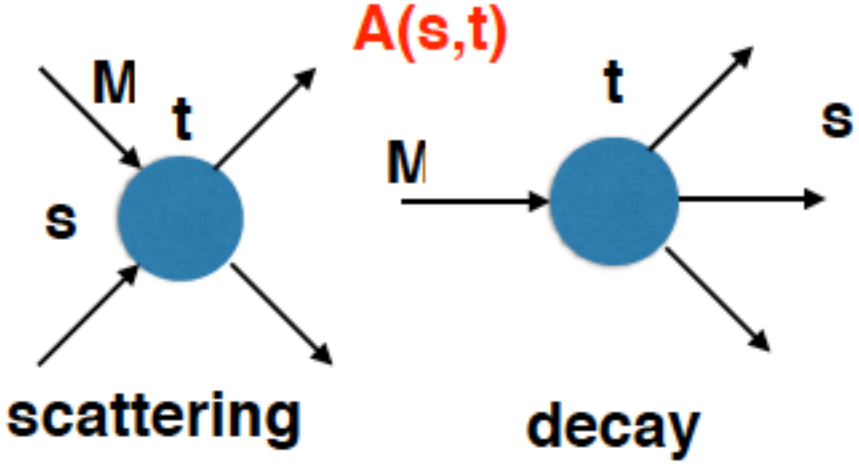
In constructing amplitudes, one must make a judgment which ones are the most important.

- Conservation of probability: appearances of real axis singularities, bound state vs resonance, Regge phenomena, absence of overlapping singularities,…
- Causality/Lorentz symmetry: crossing, analyticity, dualities, barrier factors, kinematical singularities, polynomial bounds,…
- QCD: appearance of bound states/ resonances, low energy limits, exchange degeneracy, …

$$A(s,t) = \sum_l A_l(s) P_l(z_s)$$

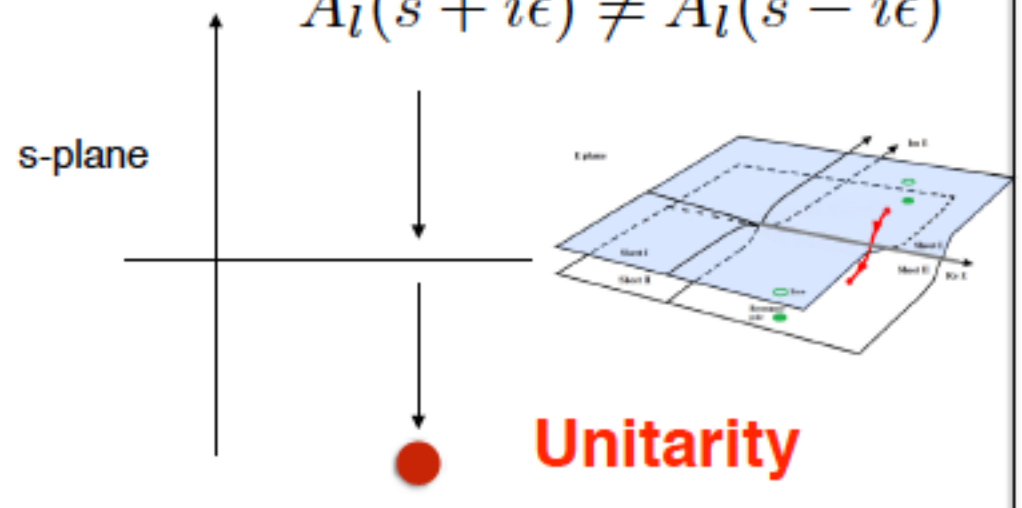
Analyticity

$$A_l(s) = \lim_{\epsilon \to 0} A_l(s + i\epsilon)$$



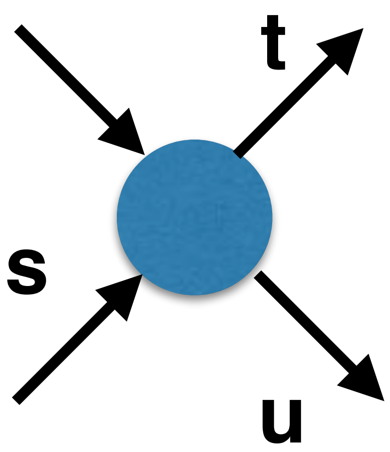
Crossing

$$A_l(s + i\epsilon) \neq A_l(s - i\epsilon)$$



Unitarity

Anatomy of Amplitudes (2-to-2)



$A(s,t)$

There are various representations

- Mandelstam
- Khuri
- Sommerfeld-Watson
- DAMA
- PWA
- …

… but PWA used the most (?)

$$A(s,t) = \sum_l^{\infty} A_l(s) P_l(z_s)$$

- Partial waves have complicated analytical properties; truncation violates crossing symmetry, Mandelstam analyticity, asymptotic behavior ….

Isobar Model $a_l(s) \neq A_l(s)$

$$A(s,t) = \sum_l^{L_{max}<\infty} a_l(s) P_l(z_s) + (s \to t) + (s \to u)$$

Isobar Model

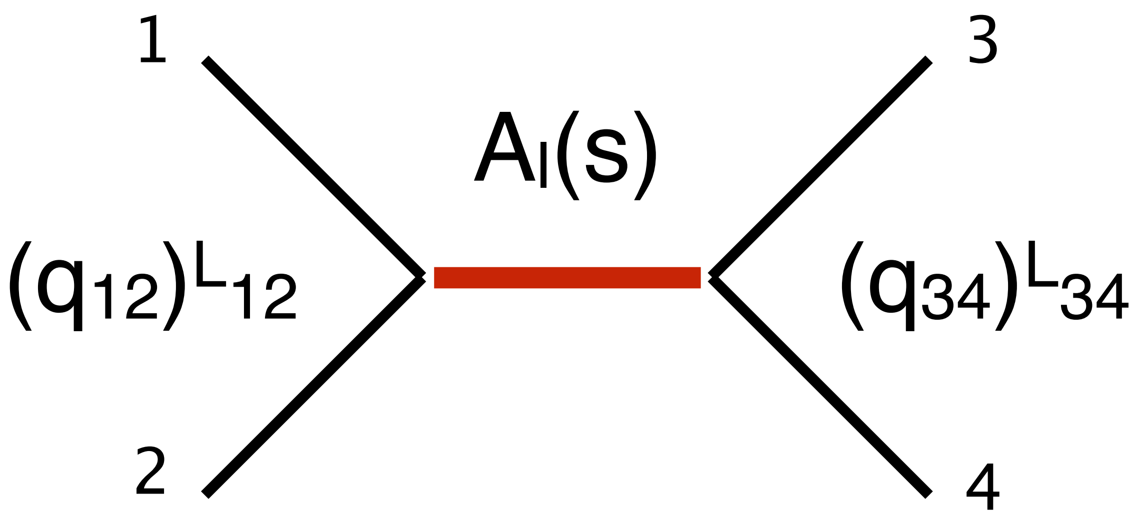
- Focuses on individual partial waves
- Dynamical consequences of crossing are missing

Other representations of $A(s,t)$

- Emphasize duality between resonances and reggeons
- Individual partial waves are “averaged”

What happens when we try to contract an analytical amplitude from a partial wave (this is important !)

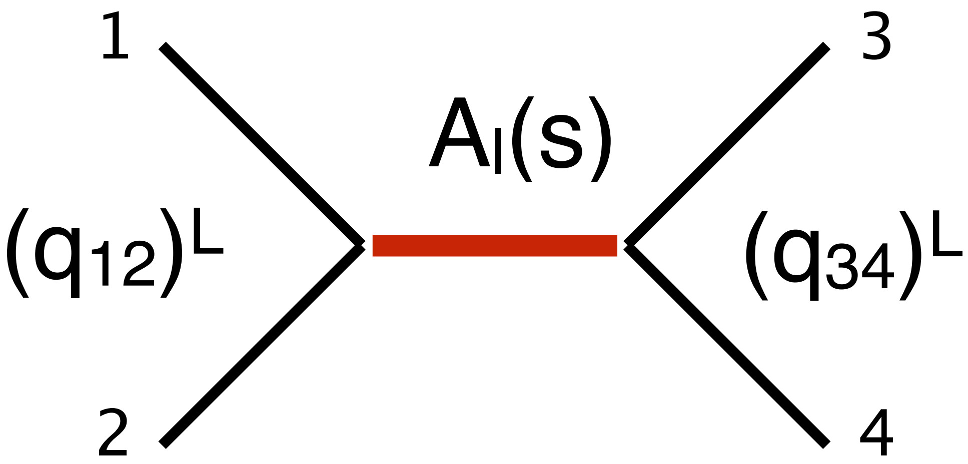
$$A(s,t,u) \leftarrow A_L(s) P_L(z_s)$$



$$q_{ij} = \frac{\lambda^{1/2}(s, m_i^2, m_j^2)}{2\sqrt{s}}$$

threshold factors (λ) => from “forces” e.g. cross channel process

(One needs to be careful and not associate divergencies ~q^{2L} with subtractions e.g. in N/D equations)



$$A(s,t,u) = \frac{g_{12}g_{34}}{m_R^2 - s}(q_{12}q_{34})^L P_L(z_s)$$

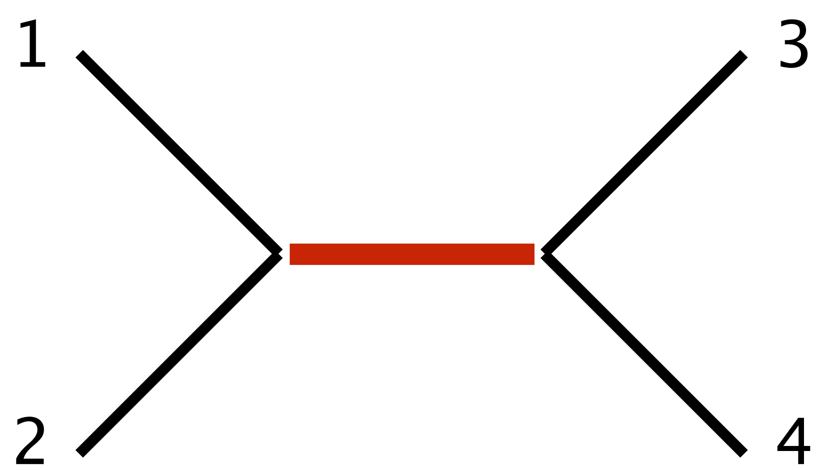
$$\Delta = (m_1^2 - m_2^2)(m_3^2 - m_4^2) \neq 0$$

for UU amplitudes there is a singularity at s = 0!

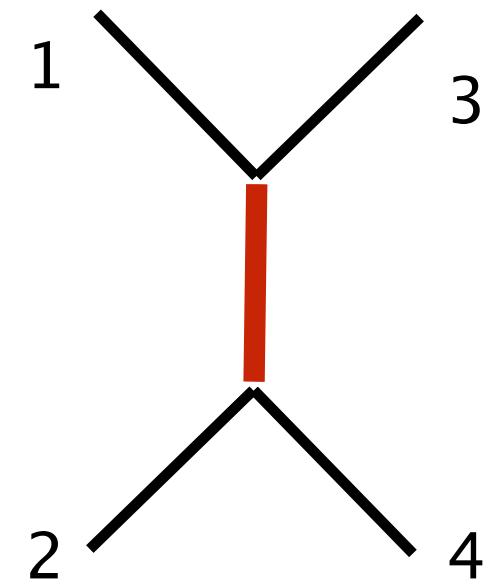
$$(q_{12}q_{34})^L P_L(z_s) = (q_{12}q_{34}z_s)^L + \cdots = f(s,t,u)[1 + O(\frac{\Delta}{s}) + O(\frac{\Delta}{s})^2 \cdots]$$

common solution $g_{12}g_{34} \to s^L g_{12}g_{34}$

but this introduces polynomials (Regge solution based on daughter trajectories)



$$A^{s}_{\lambda_i}(s,t) \text{ vs } A^{t}_{\mu_i}(s,t)$$



for external particles with spin crossing (Lorentz transformations) leads to Wigner rotations

There is an “open market” for amplitude framework

L-S amplitudes, Helicity amplitudes, Spinorial amplitudes

Examples:

$$R_X(m) = B'_{L^X_{\Lambda_b^0}}(p, p_0, d) \left(\frac{p}{M_{\Lambda_b^0}}\right)^{L^X_{\Lambda_b^0}} \mathrm{BW}(m|M_{0X}, \Gamma_{0X})\, B'_{L_X}(q, q_0, d) \left(\frac{q}{M_{0X}}\right)^{L_X}$$

It would be useful to examine such parametrization and determine systematic uncertainties

$q_{12}^L q_{34}^L P_L(\cos\theta)$ analytical function of (t,u) [1 + O(1/s) + ..]

How to handle the 1/s singularities

$$\mathcal{M}^{P_c}_{\lambda_{\Lambda_b^0}, \lambda_p^{P_c}, \Delta\lambda_\mu^{P_c}} \equiv \sum_j \sum_{\lambda_{P_c}} \sum_{\lambda_\psi^{P_c}} \mathcal{H}^{\Lambda_b^0 \to P_{cj} K}_{\lambda_{P_c}, 0} D^{\frac{1}{2}}_{\lambda_{\Lambda_b^0}, \lambda_{P_c}}(\phi_{P_c}, \theta^{P_c}_{\Lambda_b^0}, 0)^*$$

$$\mathcal{H}^{P_{cj} \to \psi p}_{\lambda_\psi^{P_c}, \lambda_p^{P_c}} D^{J_{P_{cj}}}_{\lambda_{P_c}, \lambda_\psi^{P_c} - \lambda_p^{P_c}}(\phi_\psi, \theta_{P_c}, 0)^* R_{P_{cj}}(m_{\psi p})\;\; D^1_{\lambda_\psi^{P_c}, \Delta\lambda_\mu^{P_c}}(\phi_\mu^{P_c}, \theta_\psi^{P_c}, 0)^*$$

Make sure complexities come from physics

Perfecting Isobar Model (light meson decays)

- $\eta \to 3\pi$: Isospin violating decay sensitive to the quark mass difference.

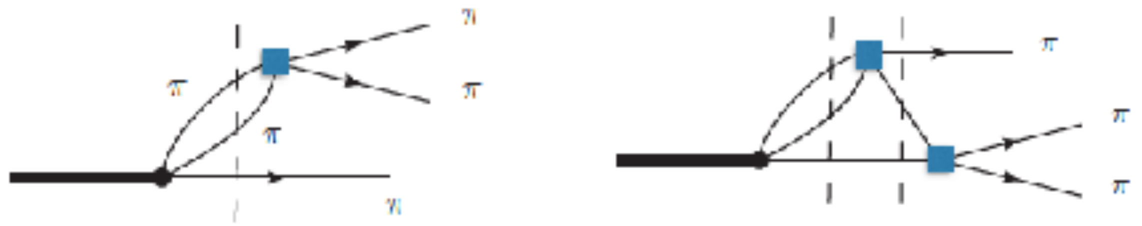
- Slow convergence of ChPT (importance of singularities)

$$\Gamma_{\eta\to\pi^+\pi^-\pi^0} = 66_{[LO]} + 94_{[NLO]} + \ldots = 296 \pm 16\,\mathrm{eV}_{[Exp]}$$

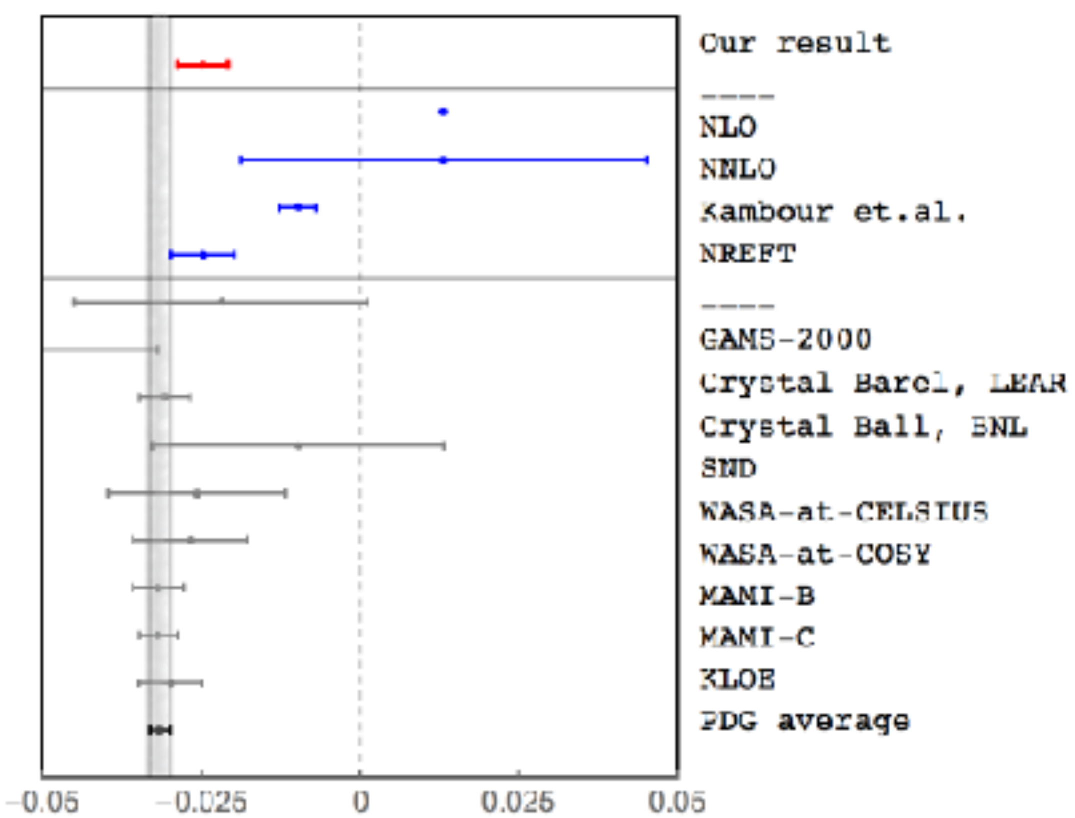
- Slope parameter in neutral decay, a puzzle for ChPT.

$$|A_{\eta\to 3\pi^0}|^2 \propto 1 + 2\alpha z + \ldots$$

Niecknig, Kubis, Schneider'12,
Danilkin et al. JPAC'15,'16
Escribano, Masjuan,Sanz-Cillero'11, Kubis & Schneider'12,
Perotti, Niblaeus, Leupold'15
G. Colangelo, et al'16

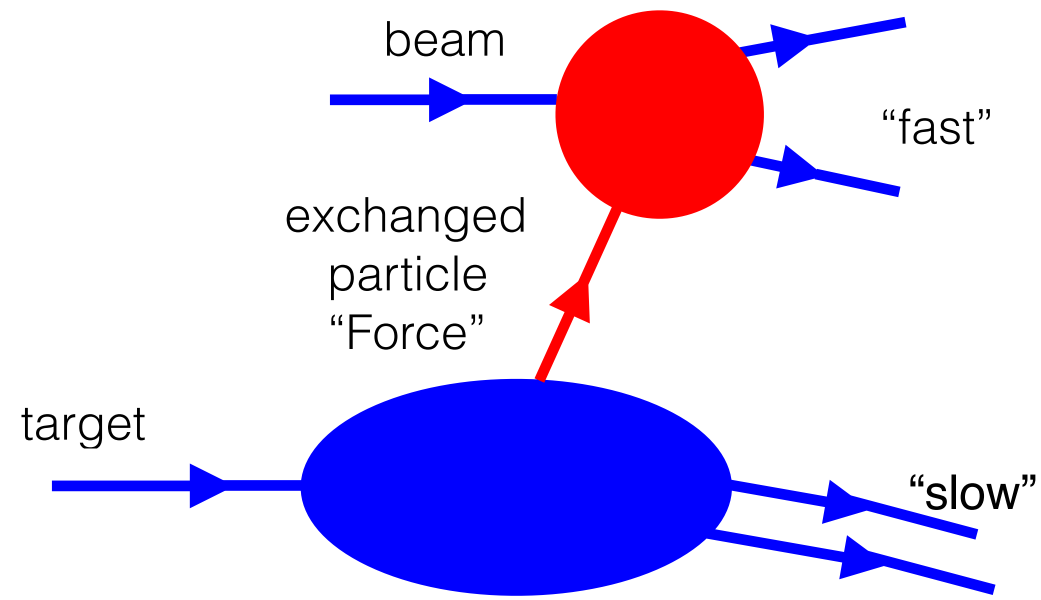


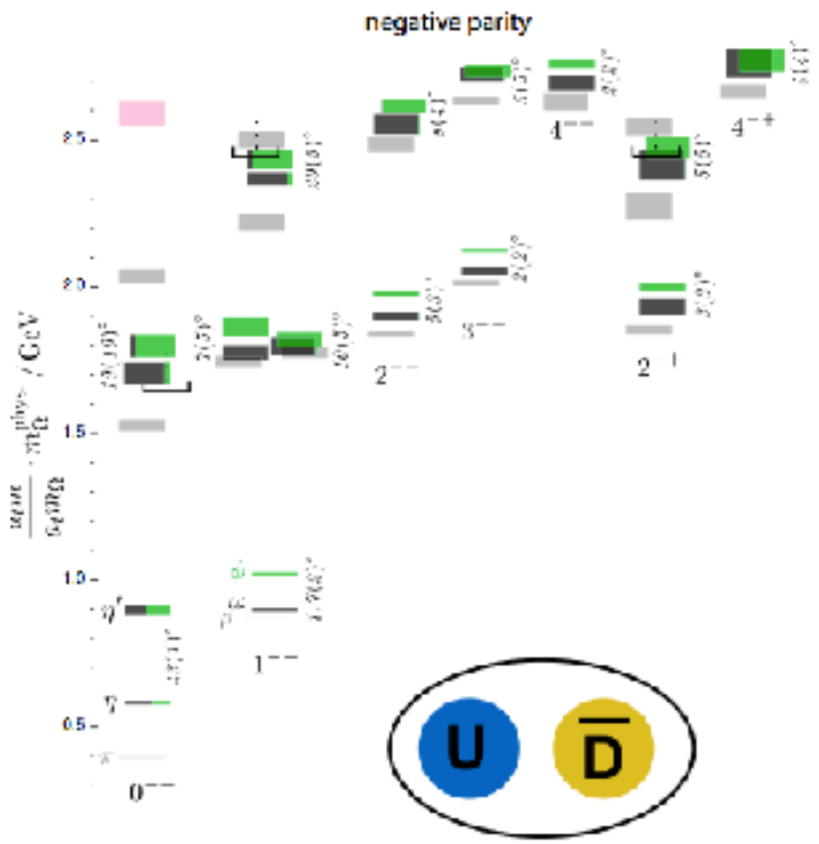
The Good: requires two body amplitudes only, connection to energy elastic scattering, partial wave expansion
The Bad : Difficult to make systematic improvements (e.g. inelastic channels)
The Ugly : High energy is parametric and "very wrong".

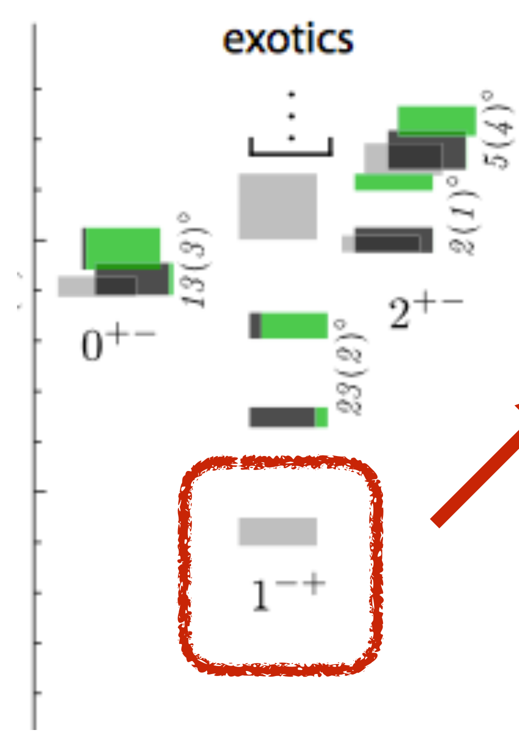


Resonances in peripheral production

- Assess factorization, develop of 2-to-2 reactions, including reggeon - particle scattering.







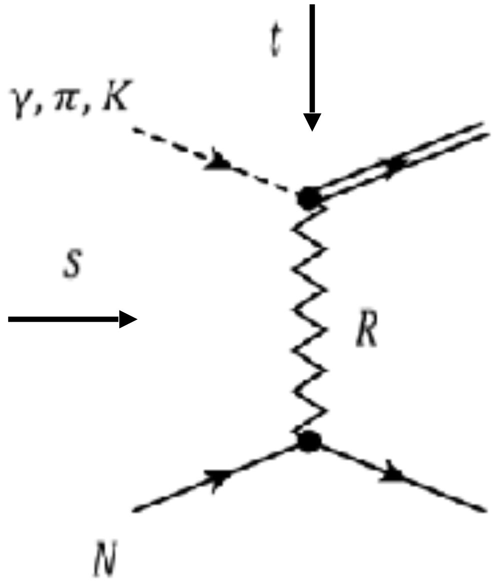
lightest hybrid couples to

$\eta\pi$

$\eta'\pi$

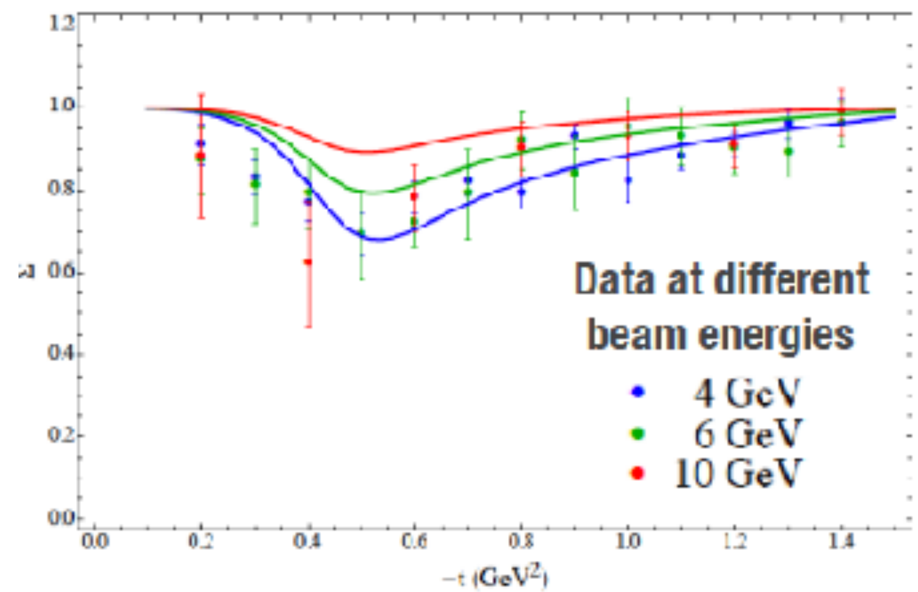
3π

Regge analysis of meson resonance production



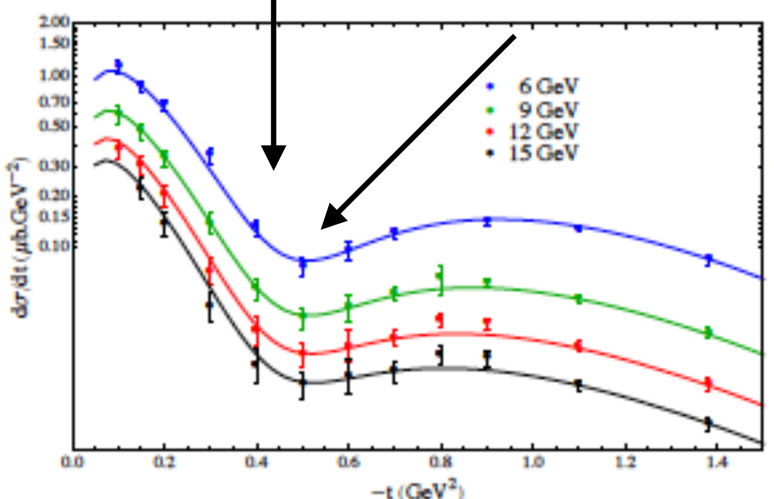
- Key to determine separation meson from baryon resonance production

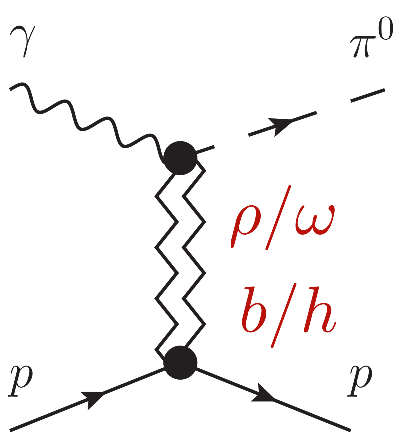
Mathieu et al. PRD 92, 074013



$$H^{(s)}_{\mu_3\mu_4,\mu_2\mu_1} = \frac{V(t)}{\sin\pi\alpha(t)} g_{\mu_3\mu_1}(t) g_{\mu_4\mu_2}(t) \left(\frac{\nu}{-t}\frac{1-\cos\vartheta_s}{2}\right)^{\frac{|\mu_i-\mu_f|}{2}} \left(\frac{1+\cos\vartheta_s}{2}\right)^{\frac{|\mu_i+\mu_f|}{2}}$$

- correction to NWSZ (cut)



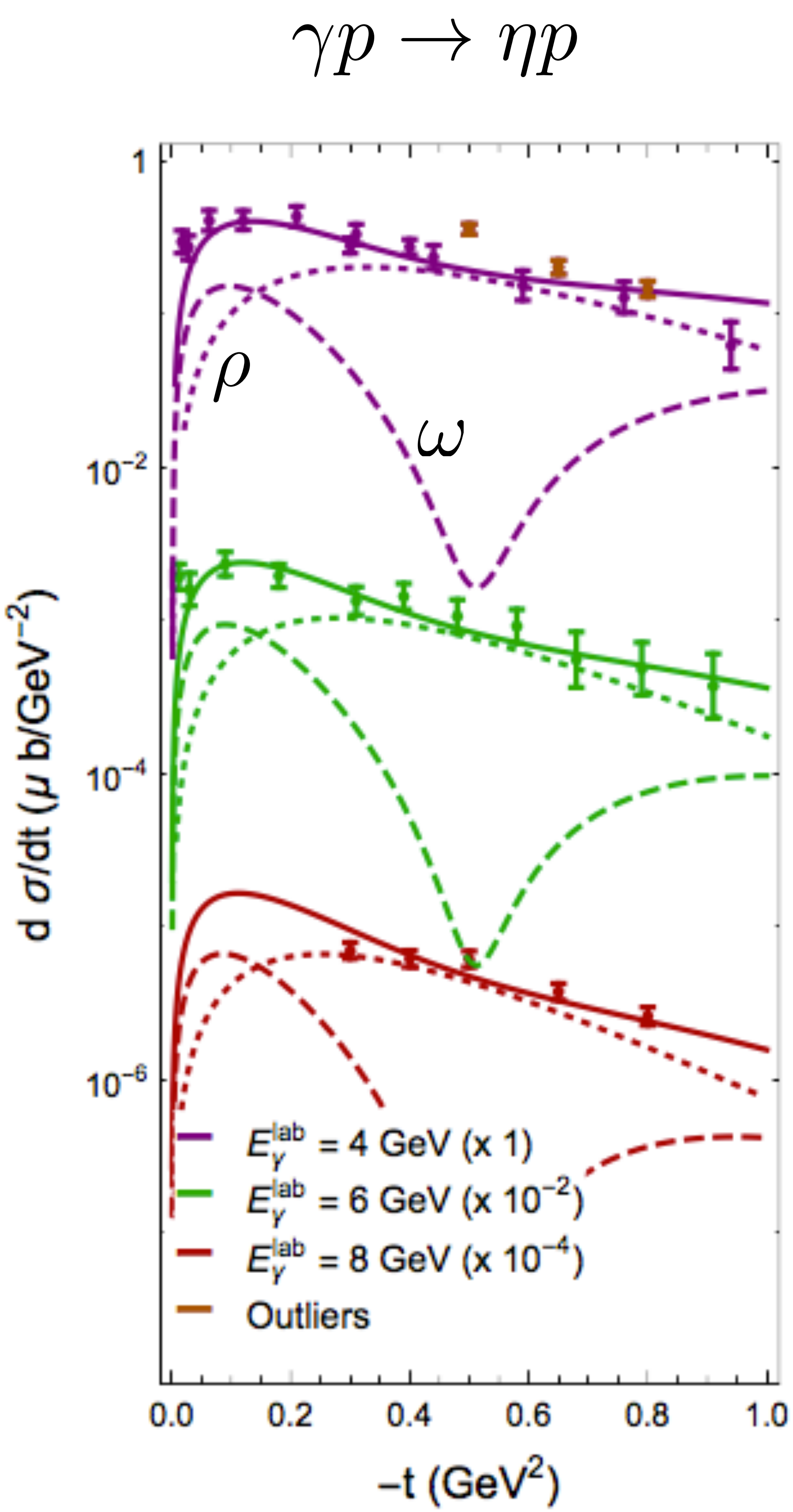


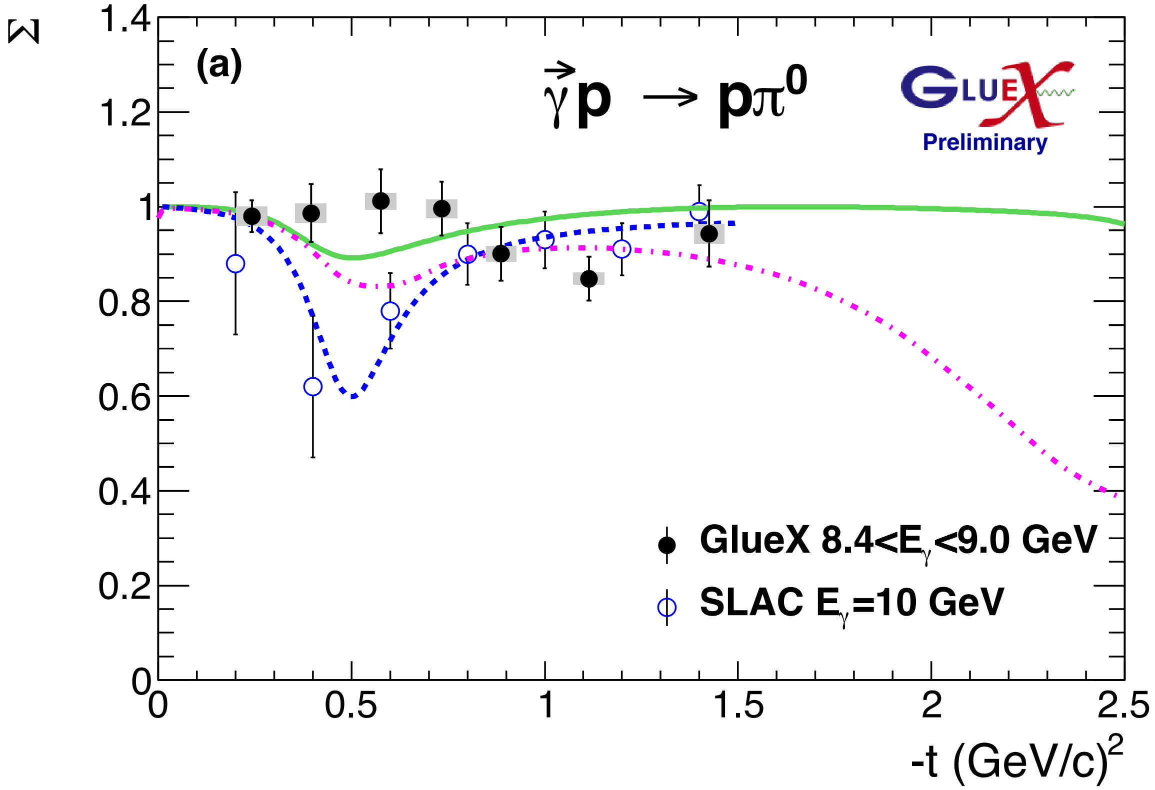
$$\Sigma = \frac{|\omega+\rho|^2 - |h+b|^2}{|\omega+\rho|^2 + |h+b|^2}$$

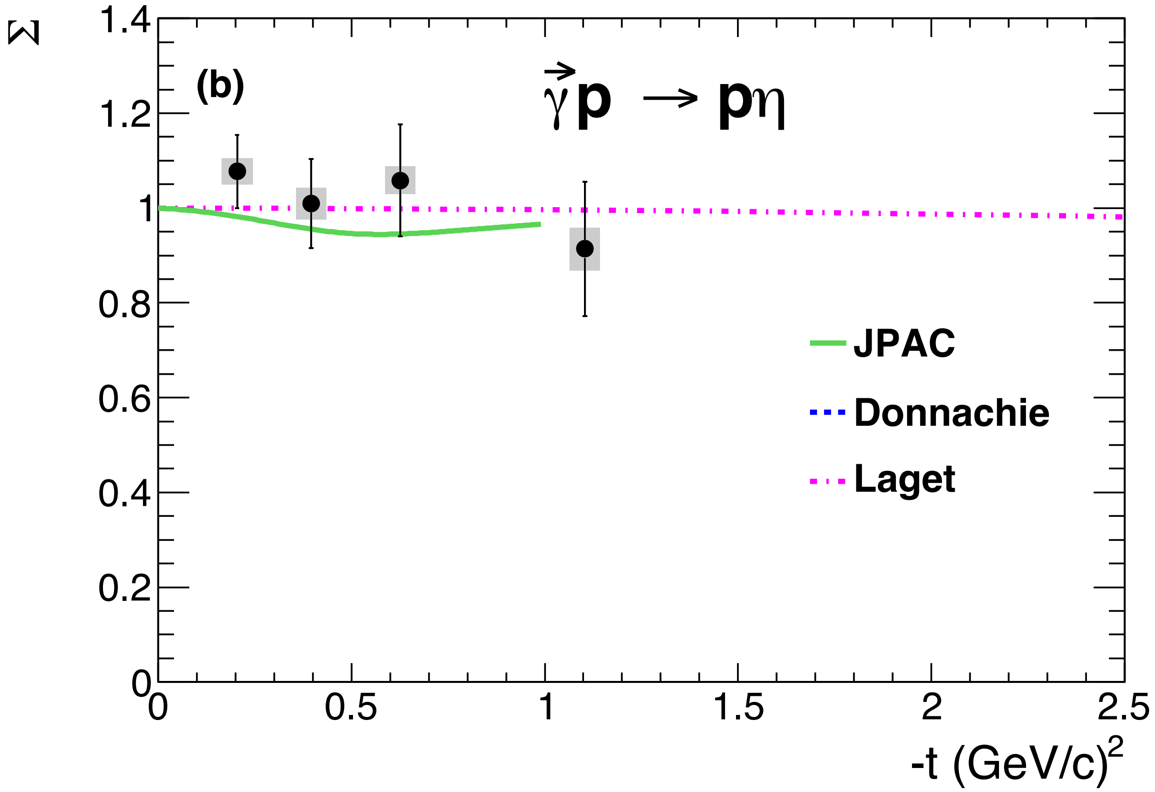
Data collection:

- γ,π,K beams
- 0^-, 1^-, 2^+ peripheral meson production
- Universal data format

axial-vector exchanges strength decreases with energy



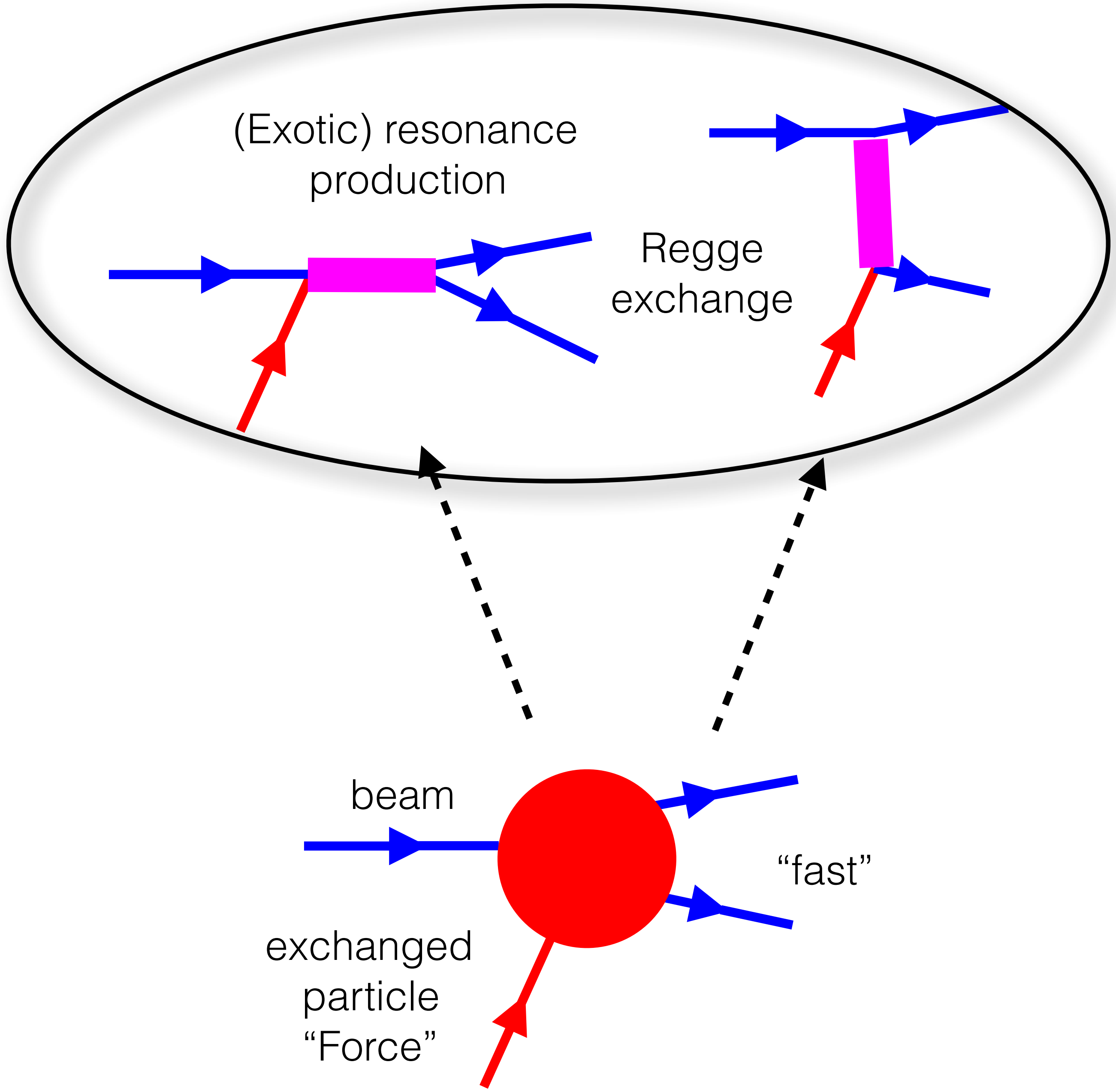




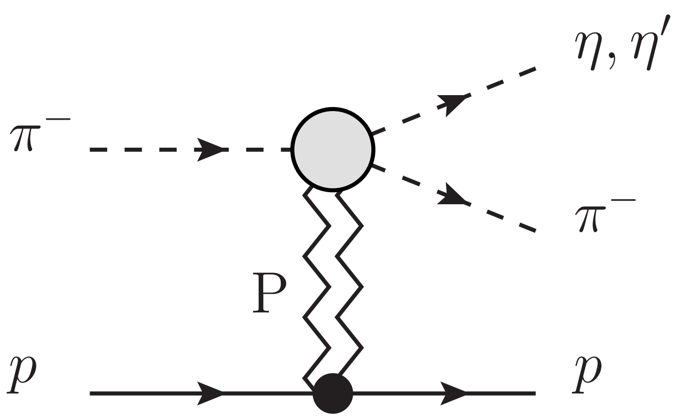
J. Nys, V. Mathieu, at this meeting

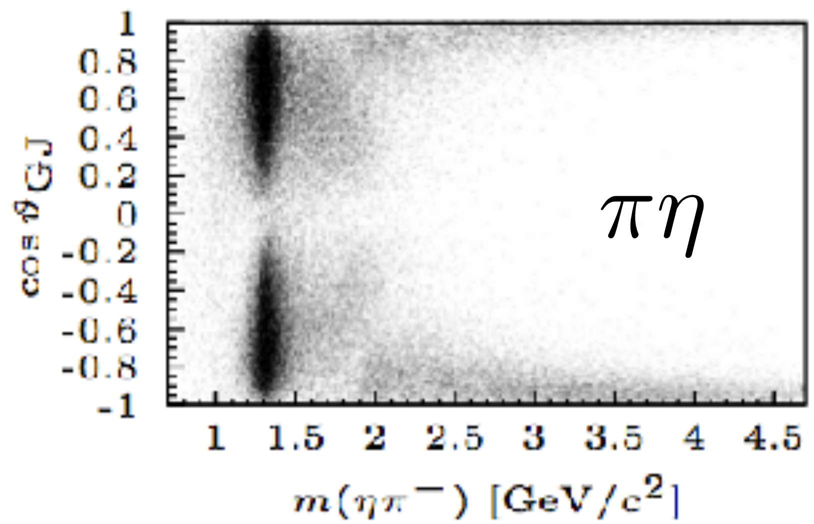
Resonances in peripheral production (cont.)

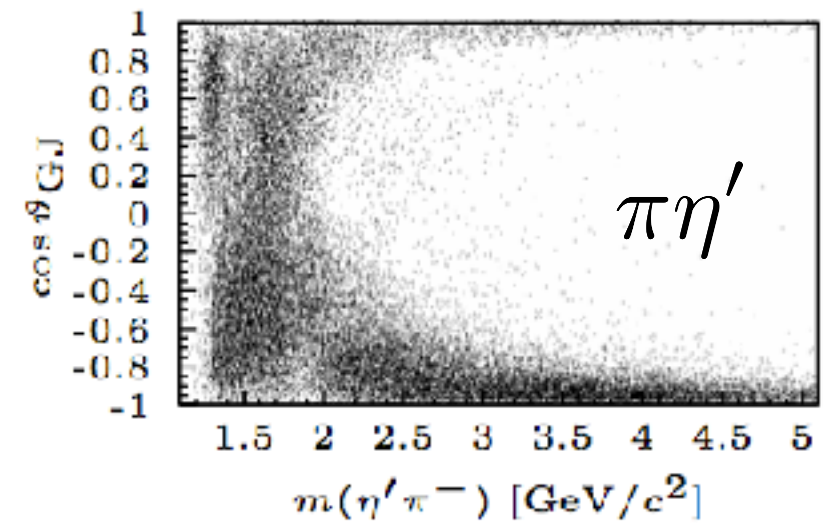
- Develop analytical constraints to relate resonance production with high energy (Regge) dynamics (e.g. FESR's)
- Understand how parametrize the "thick lines" : Dynamics (isobars, K-matrices, left hand cuts, right hand cuts, resonance "seeds", ….
- Understand how to parametrize the "thin lines" : Kinematics (kinematical singularities, helicity, L-S, covariant amplitudes, …)



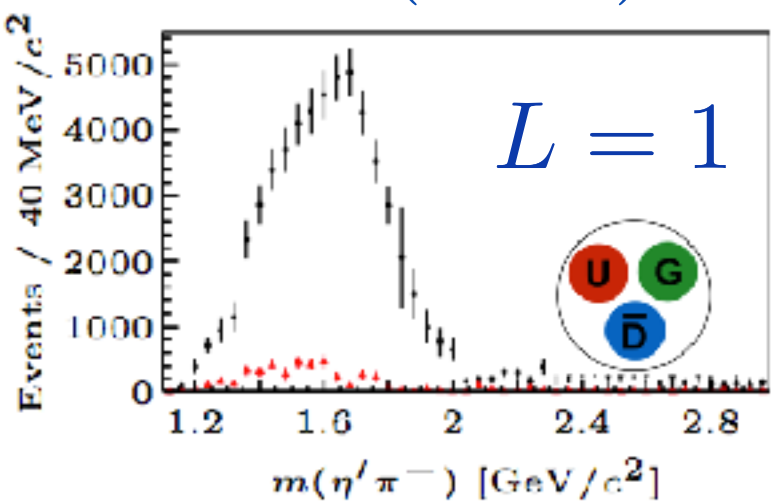
Eta-Pi @COMPASS



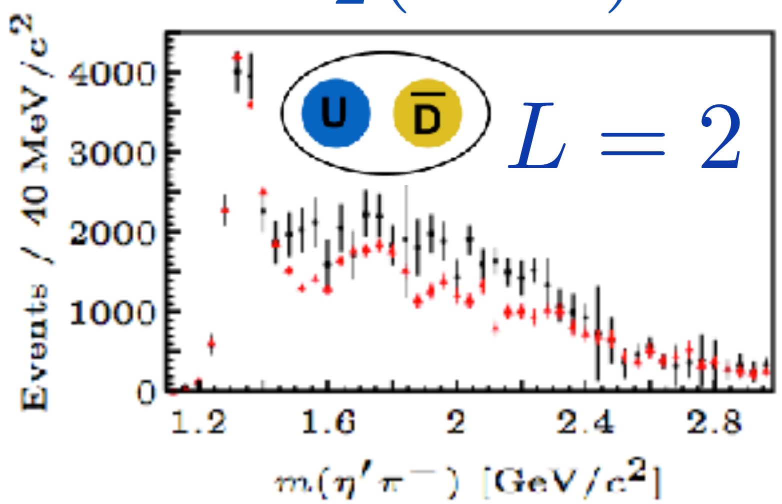




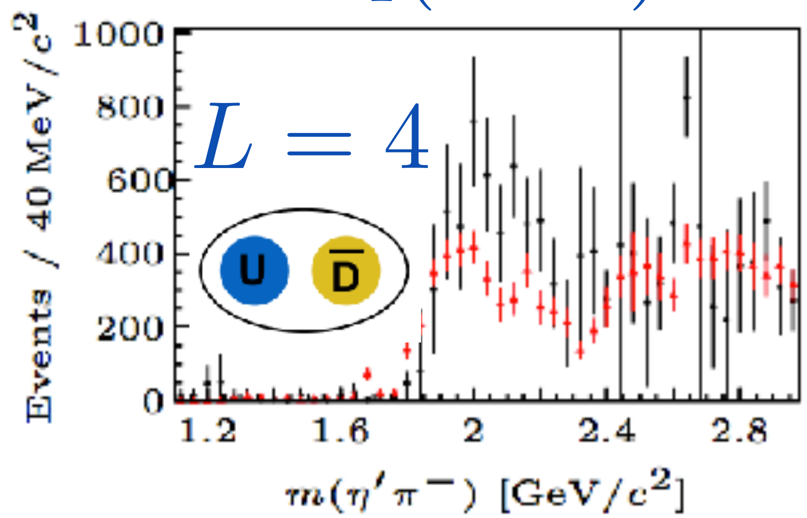
$\pi_1(1600)$?

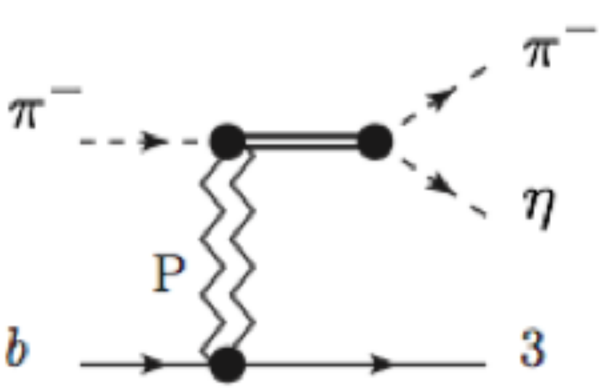


$a_2(1320)$



$a_4(2040)$



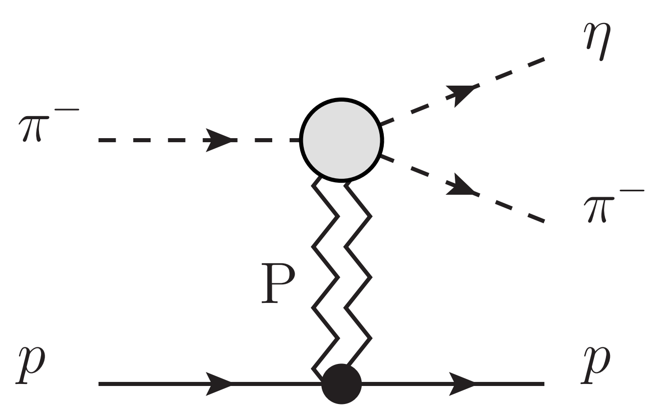


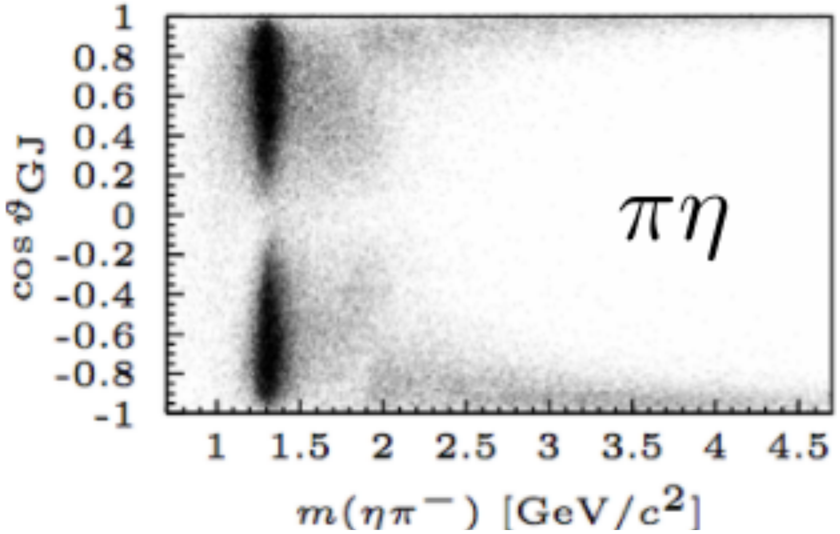
black: $\pi\eta'$

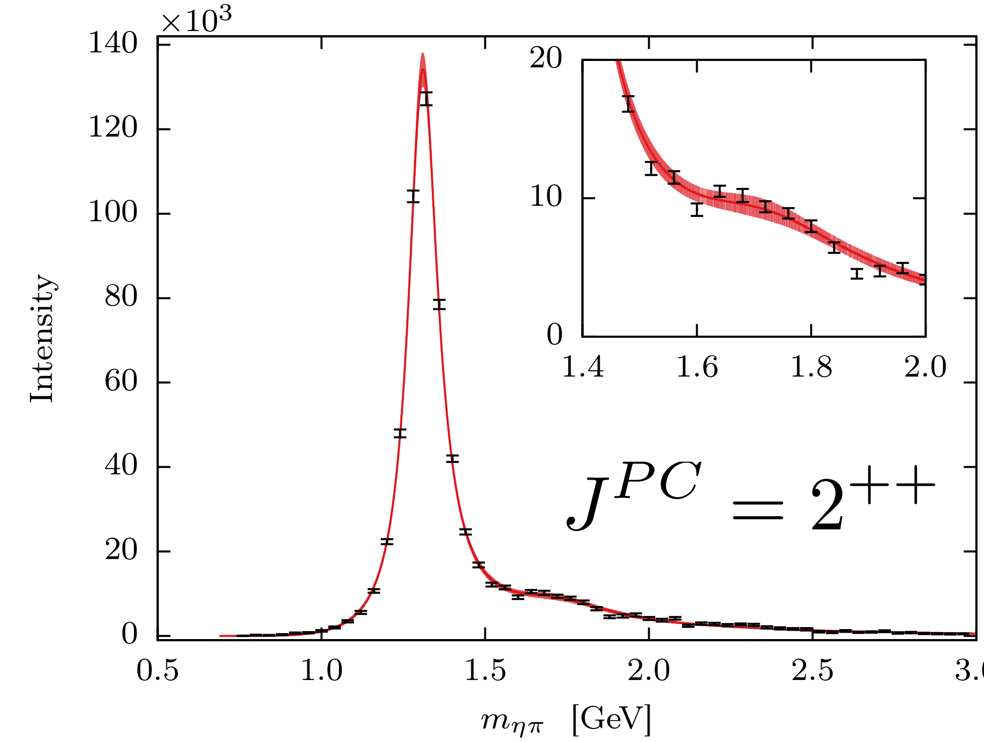
red: (scaled) $\pi\eta$

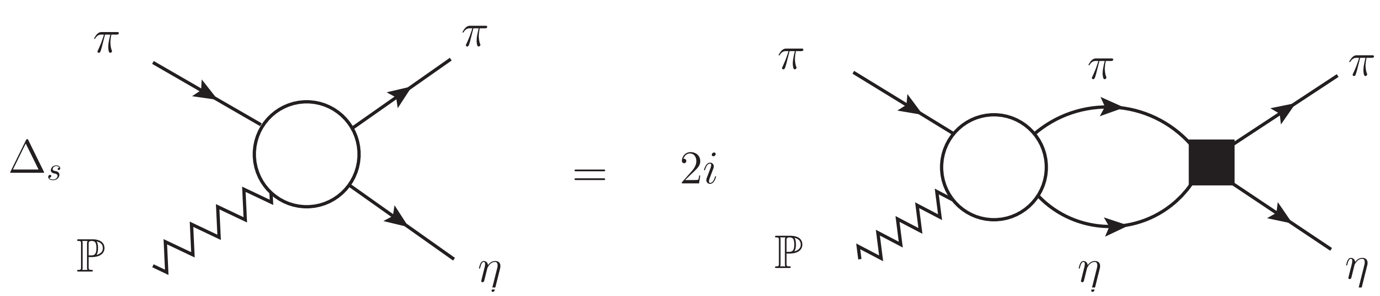
COMPASS Phys. Lett. B740 (2015)

Eta-Pi @COMPASS

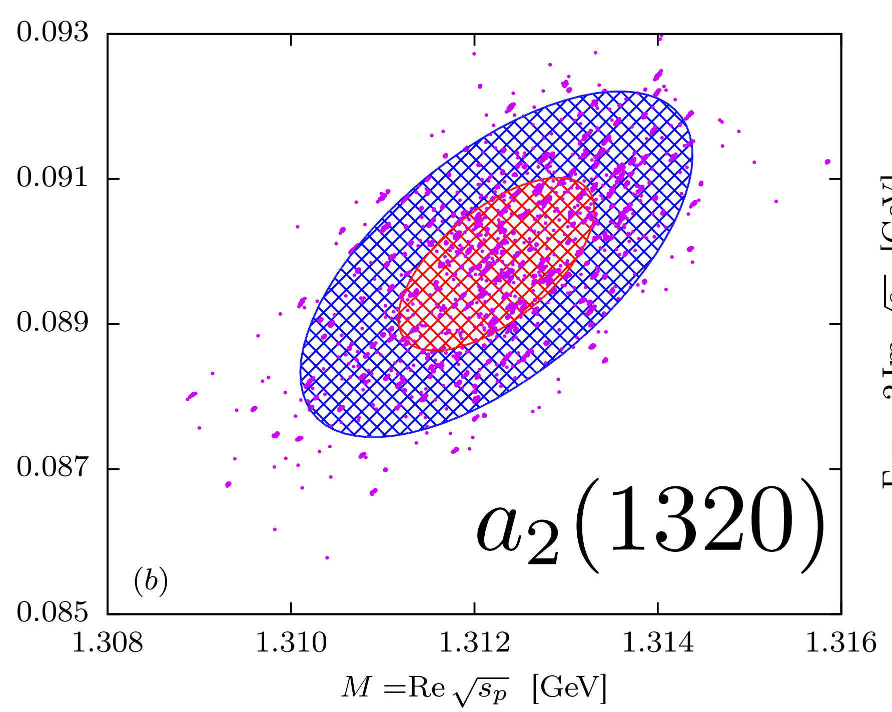


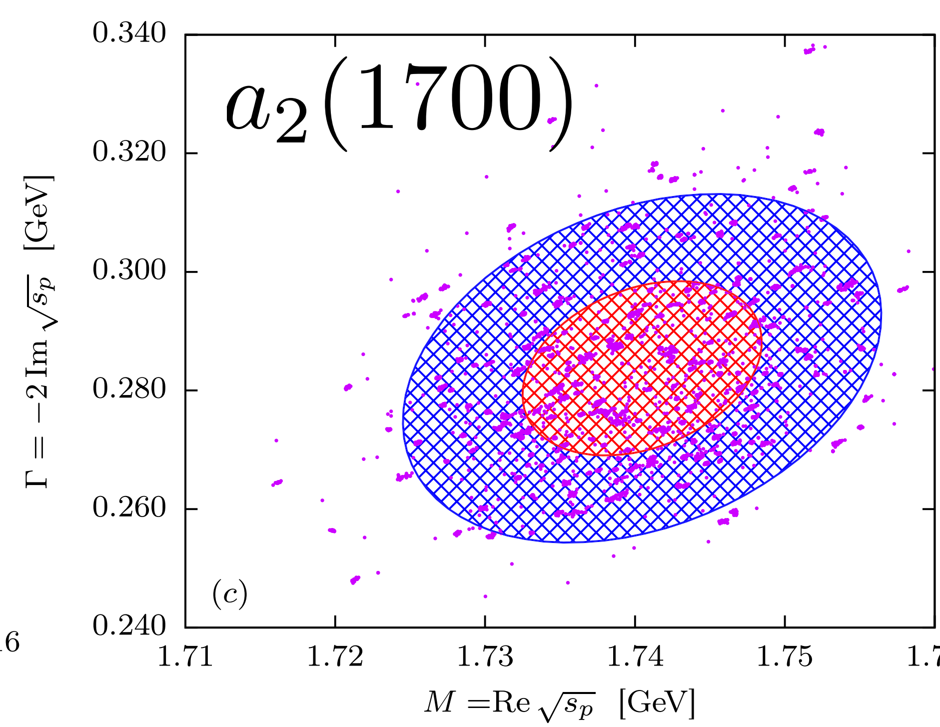






Precise determination of resonance content: (complex plane structure)

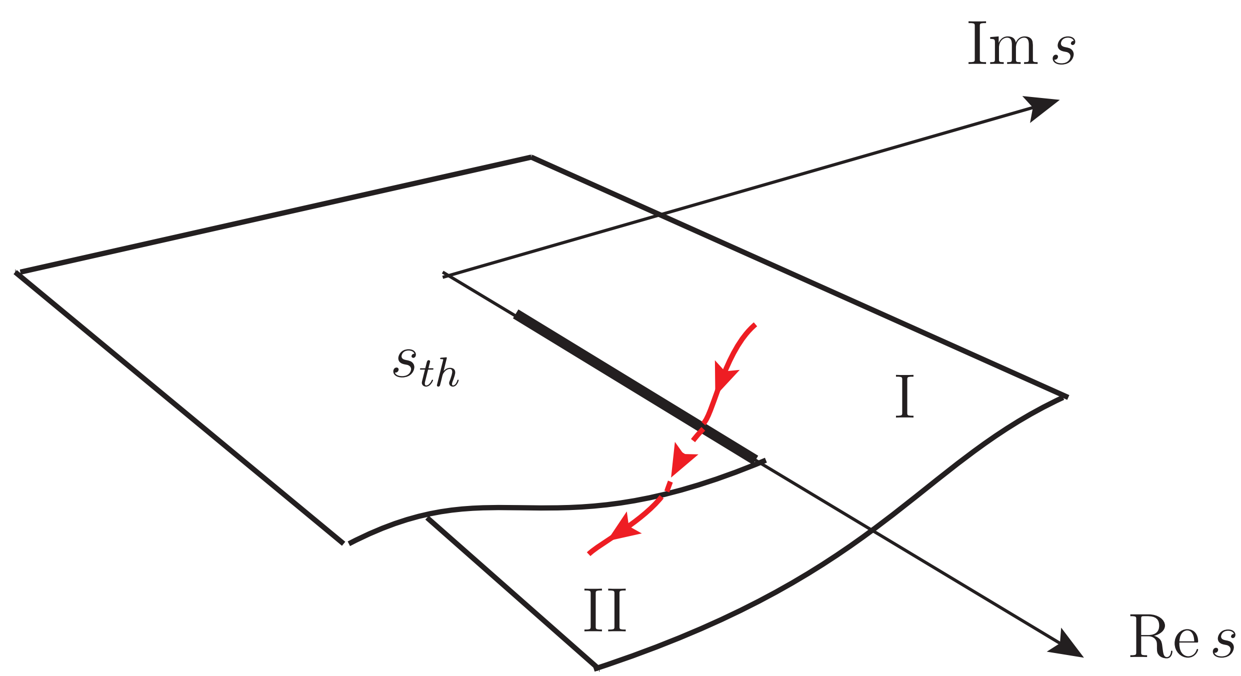




A. Jackura, at this meeting

Pole Extraction - K-matrix vs. CDD poles

- Need reliable parameterizations to extract resonance parameters
- Models must satisfy unitarity conditions
- In addition, resonance pole positions must be on unphysical sheets



$$\mathrm{Im}\, t(s) = \rho(s)|t(s)|^2 \implies t^{-1}(s) = K^{-1}(s) - \frac{s}{\pi}\int_{s_{th}}^{\infty} ds' \, \frac{\rho(s')}{s'(s'-s)}$$

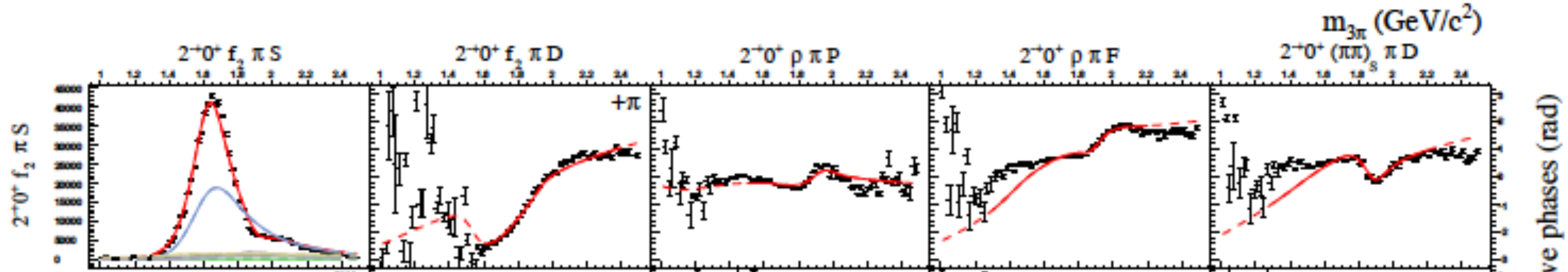
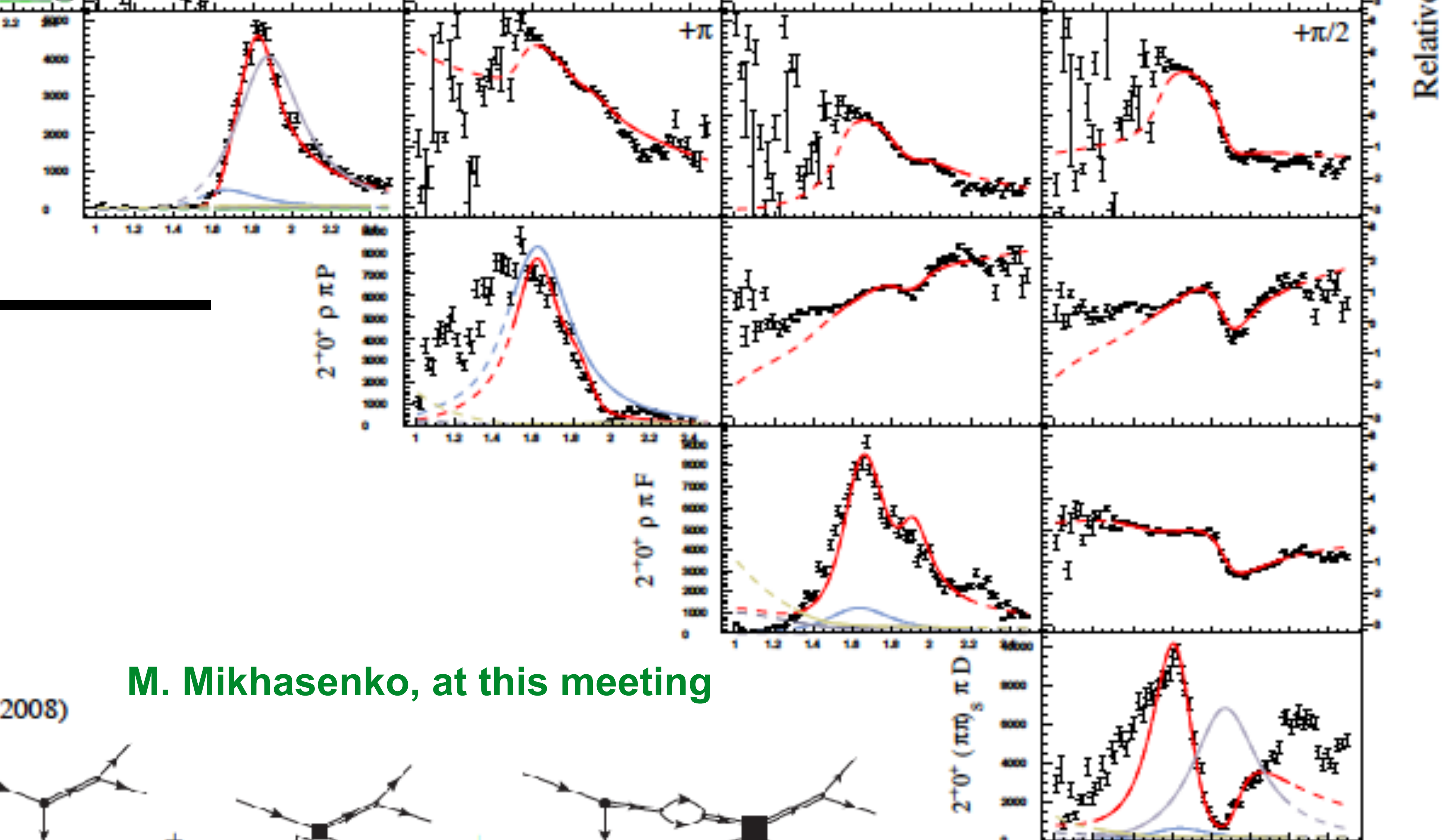
$$K(s) = \sum_r \frac{g_r^2}{m_r^2 - s} + \sum_j \gamma_j \, s^j$$

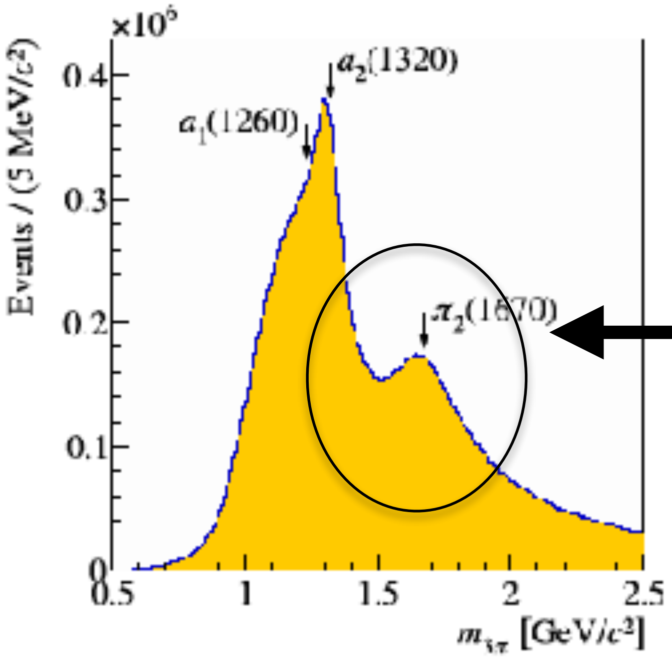
No obvious constraints on parameters

$$K^{-1}(s) = C_0 - C_1 s - \sum_{r=1}^{N} \frac{C_2^r}{C_3^r - s}$$

If $C_1, C_2 > 0$ then NO poles on first sheet! (Herglotz function)

Fit over all t' slices

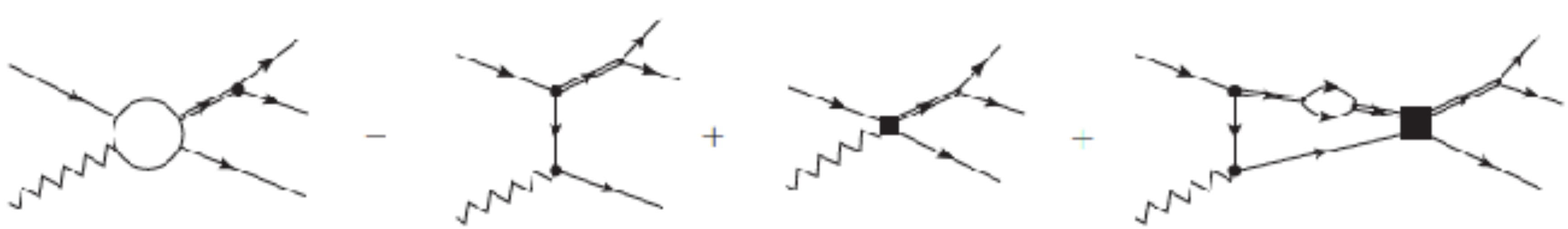





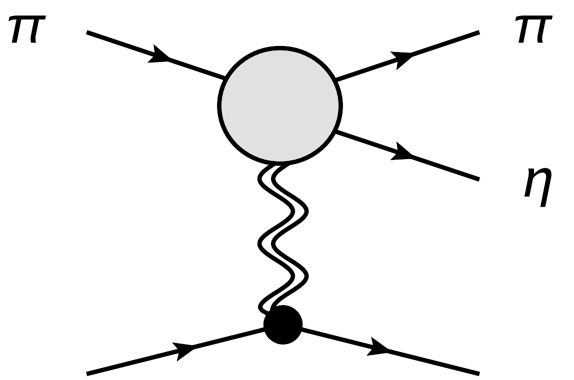
$0.100 < t' < 0.113\ (\mathrm{GeV}/c)^2$

$\pi^- p \to \pi^-\pi^+\pi^- p$ (COMPASS 2008)

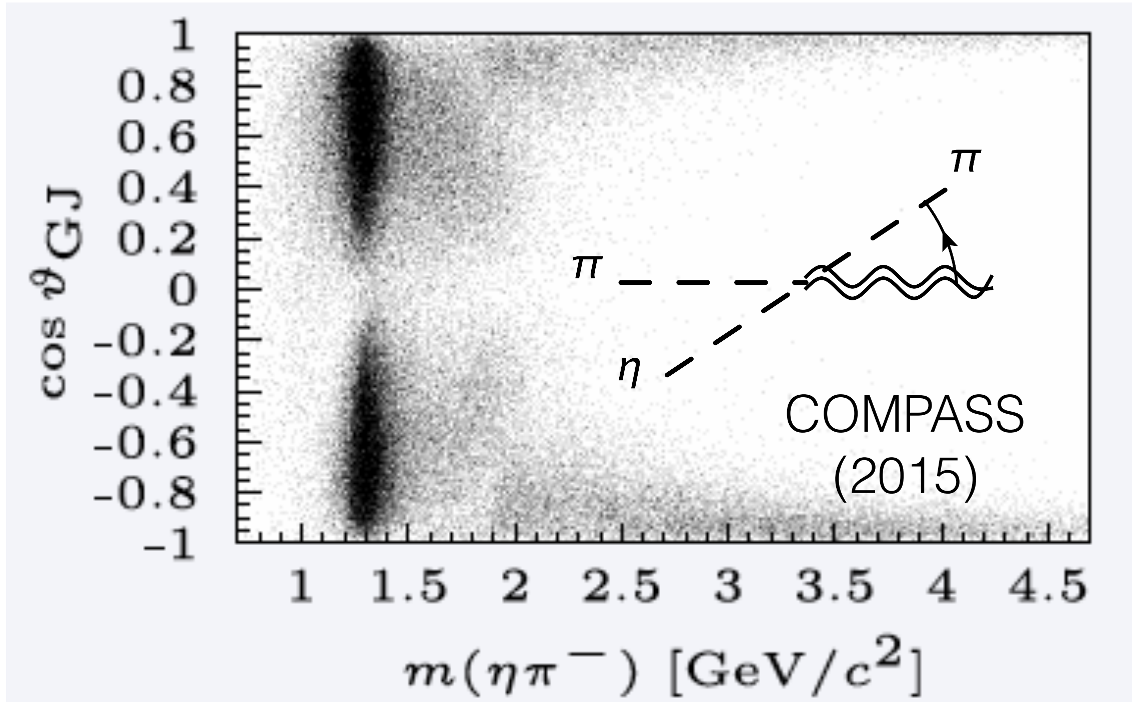
M. Mikhasenko, at this meeting



Finite Energy Sum Rules (FESR's) first time to be applied in analysis of 2-to-3 reactions!

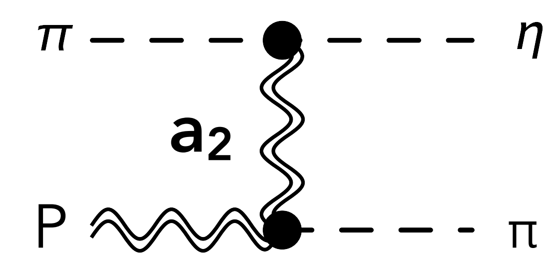


forward-backward asymmetry due to P wave !



fwd η

θ~π

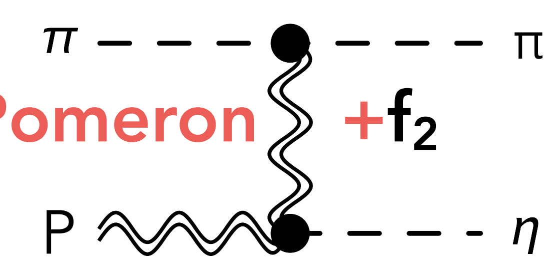


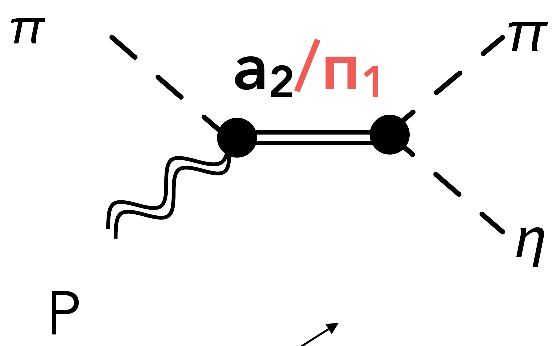
fwd π

θ~0

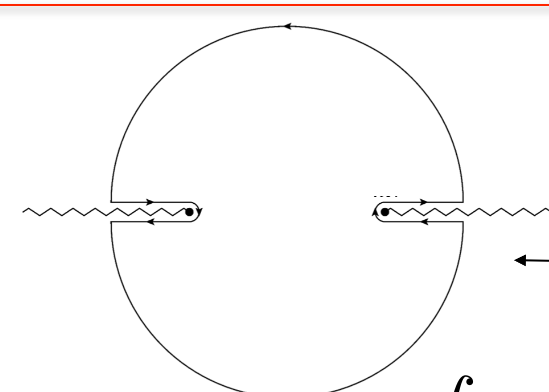
+

Pomeron +f_2





Magnitude of the P wave (exotic resonance) at low η π mass is related by FESR to Regge exchanges at high η π invariant mass

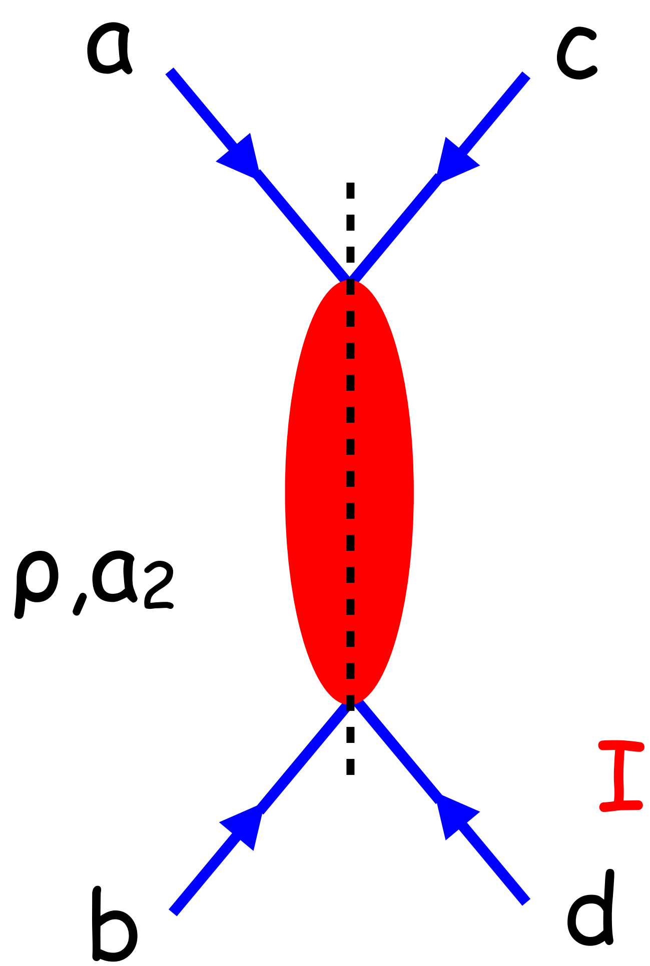


work in progress

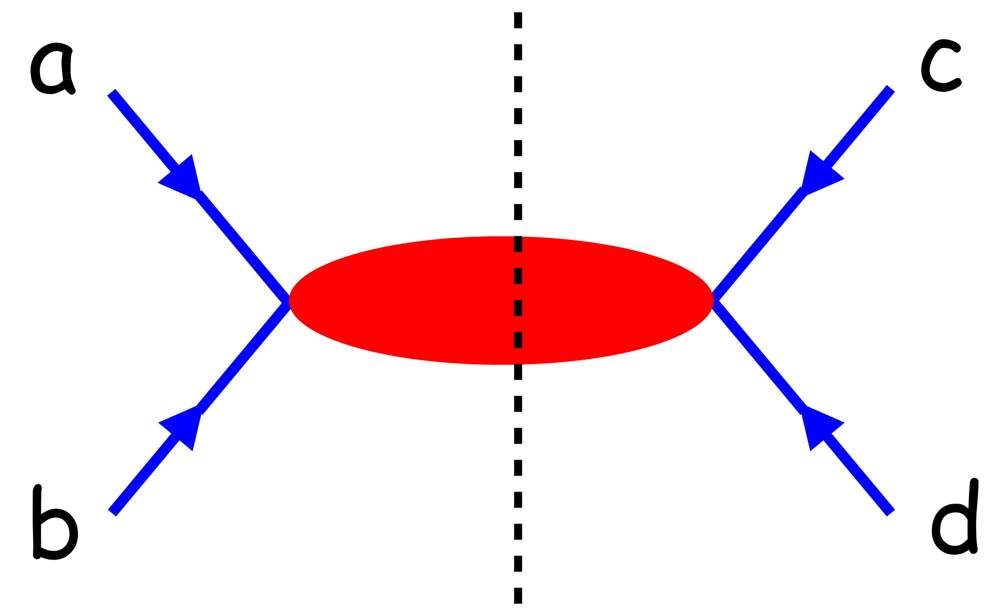
$$\int^{\Lambda} ImA_i(s_1, t_1, t_2) = \int_{\Lambda} ImA_i(s_1, t_1, t_2)$$

More on crossing and duality

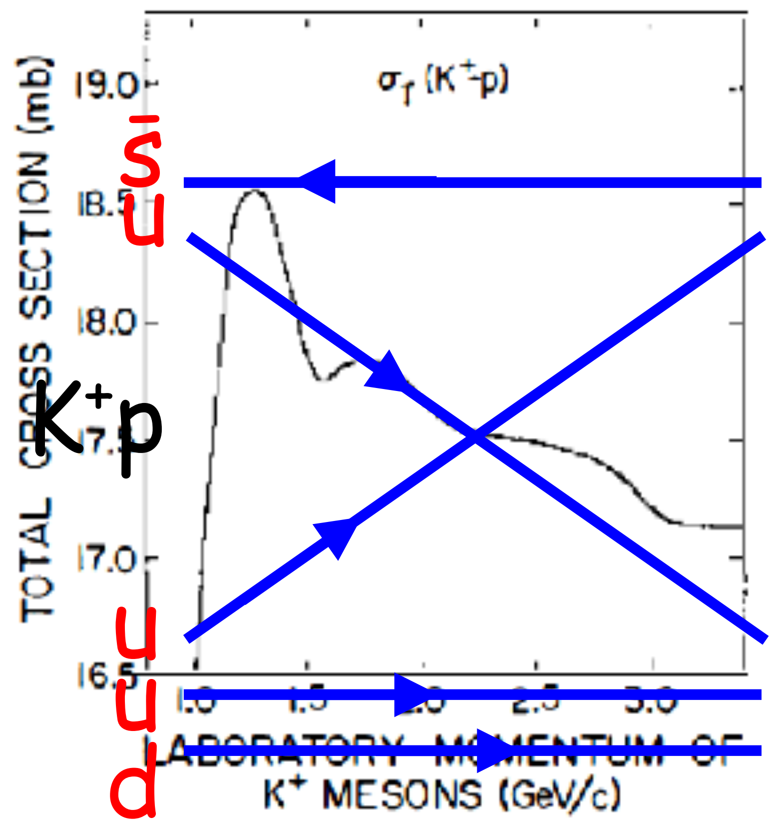
$\mathrm{Im}\, A_{Regge}(N,t)$



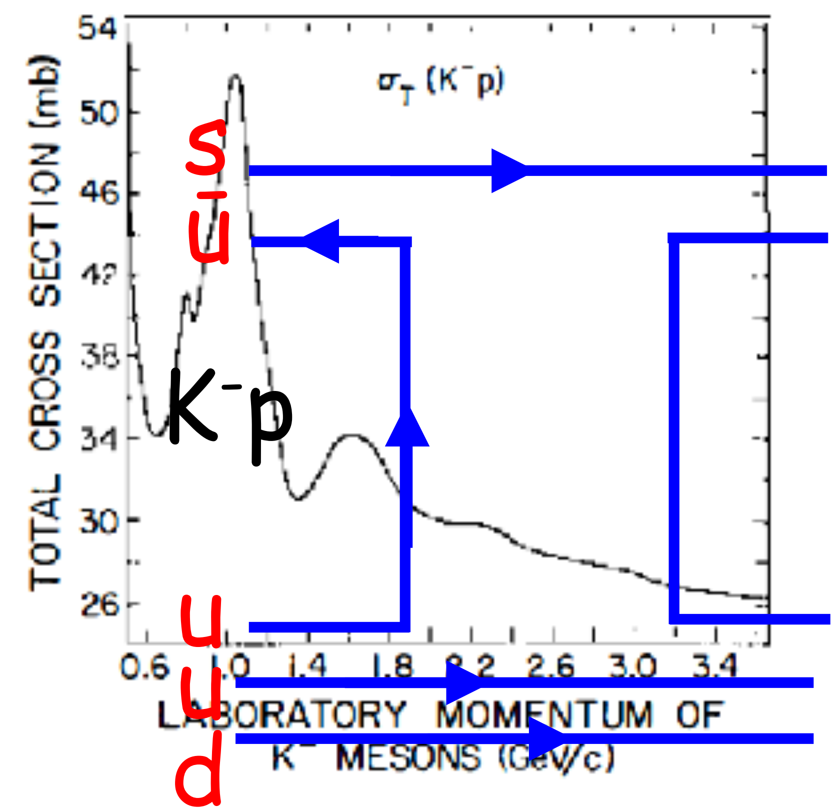
$\int^N ds\, \mathrm{Im}\, A(s,t)$



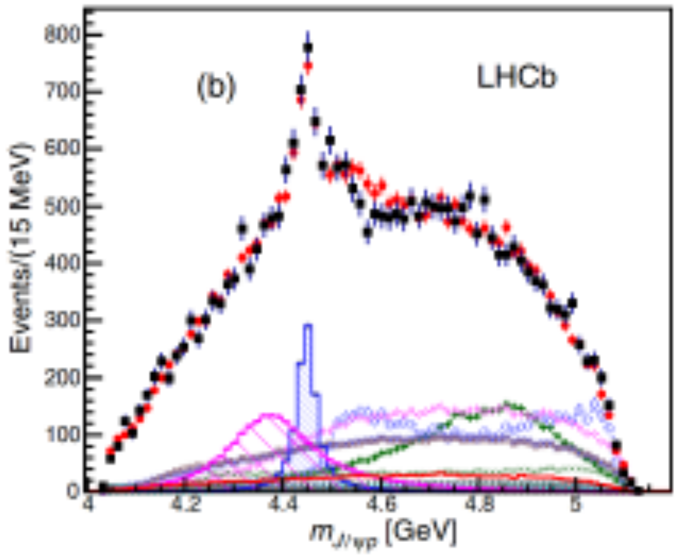
$\mathrm{Im}\, A(s,t) = 0$

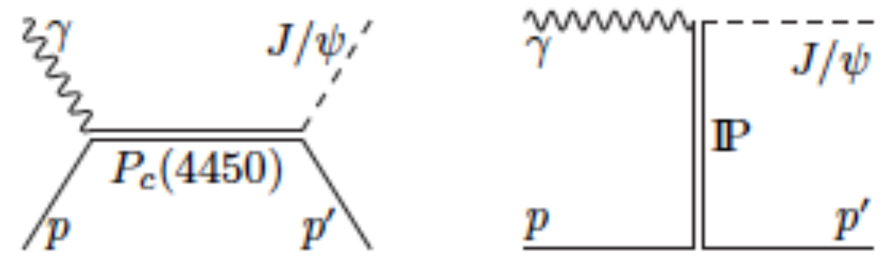


$\mathrm{Im}\, A(s,t) \neq 0$



$P_c(4450)$ in J/ψ photo production





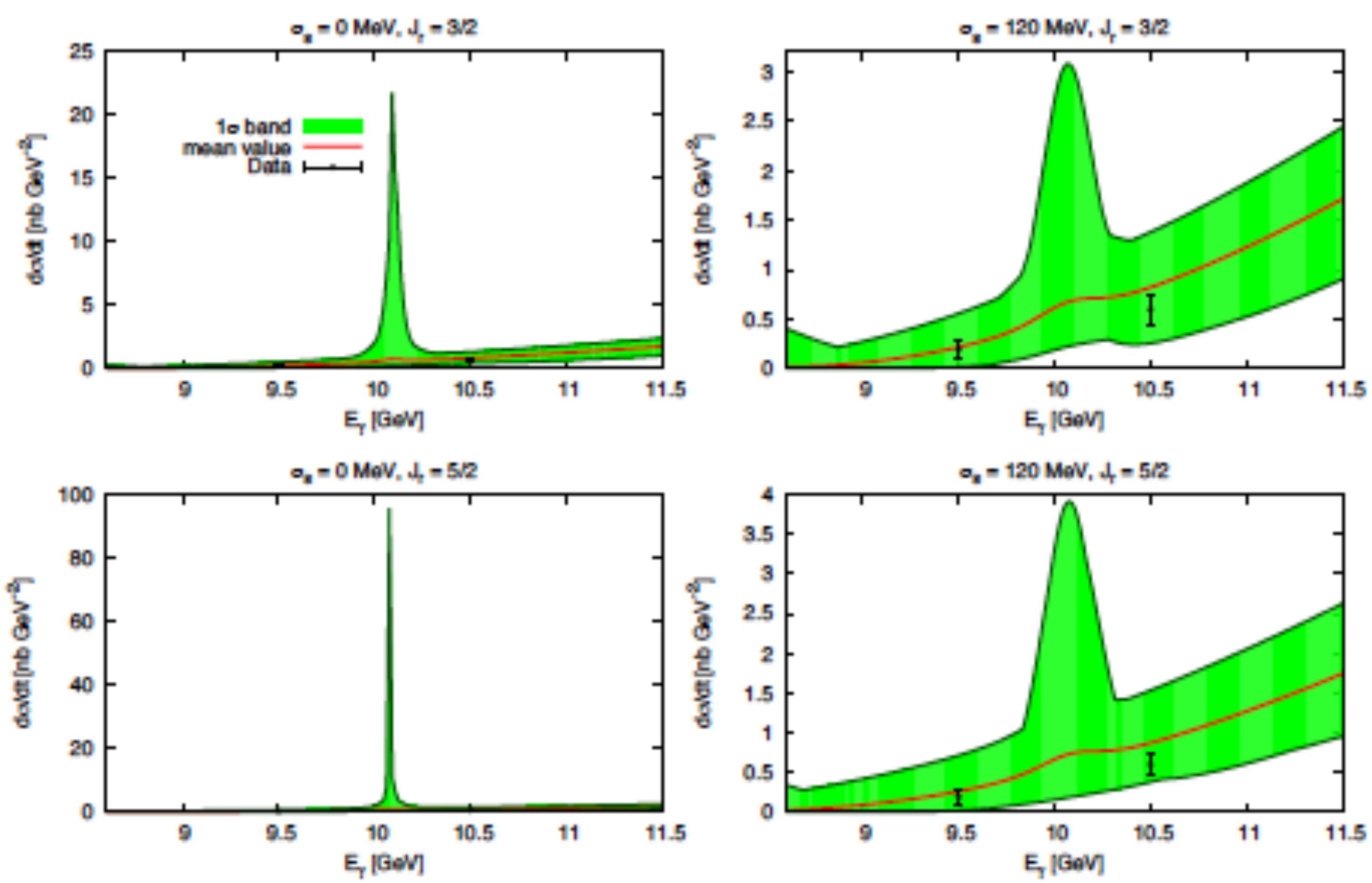
LHCb Collaboration, PRL 115, 072001 (2015)

Fit to data! W from threshold to ~ 300 GeV.

Upper bound for partial decay width!

$$\begin{cases} J_r = 3/2 \Rightarrow 23-30\% \\ J_r = 5/2 \Rightarrow 8-17\% \end{cases}$$

Also angular distributions and photocouplings studied.

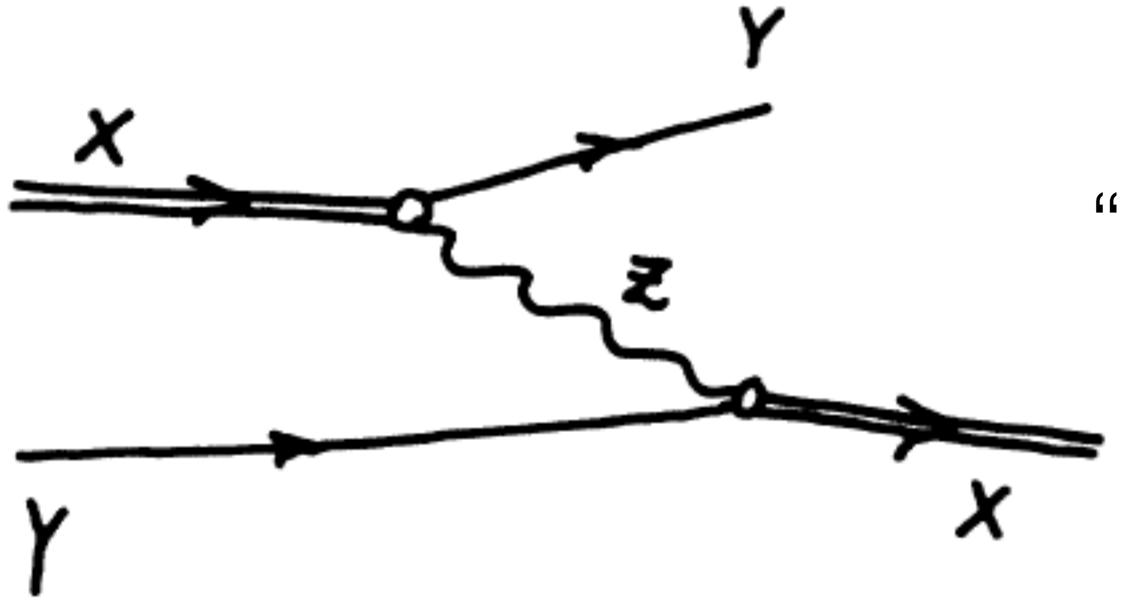


A.Blin, at this meeting

REMARK ON ENERGY PEAKS IN MESON SYSTEMS

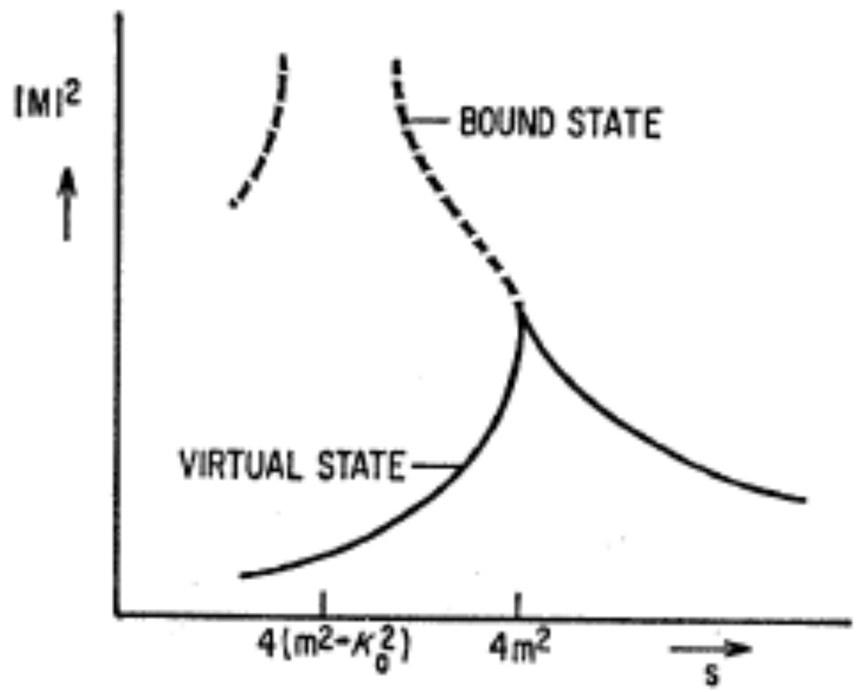
M. Nauenberg A. Pais

If the width of particle X is not very large we will stay close to the physical region. This almost singular behavior of $A(s)$ for certain physical s causes the peaking effect to which we refer as an (X, Y, Z) peak.



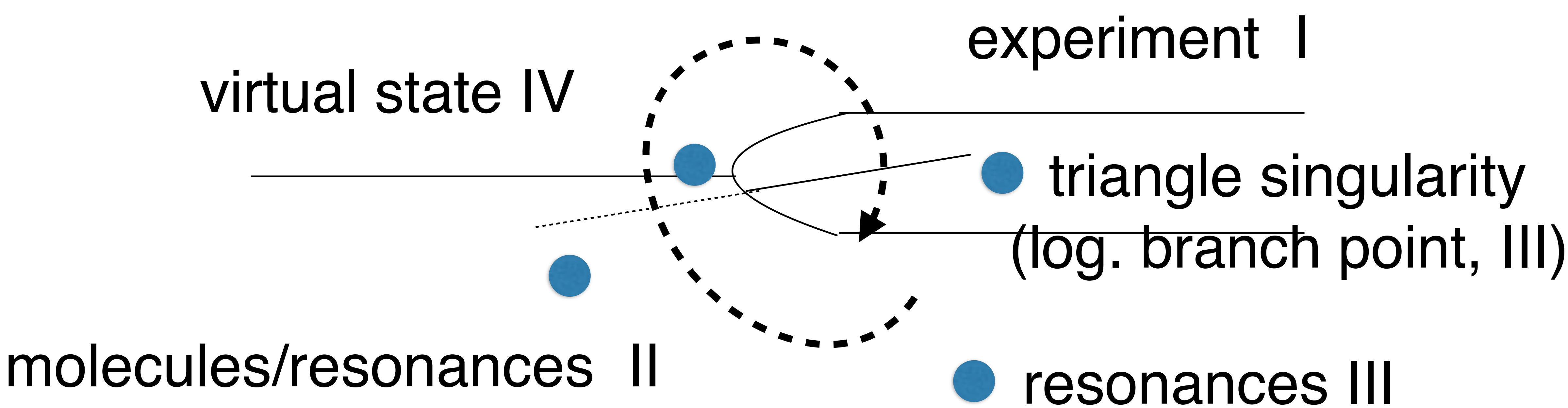
"Peierls mechanism"

Singularities, is all that matters: poles and cuts

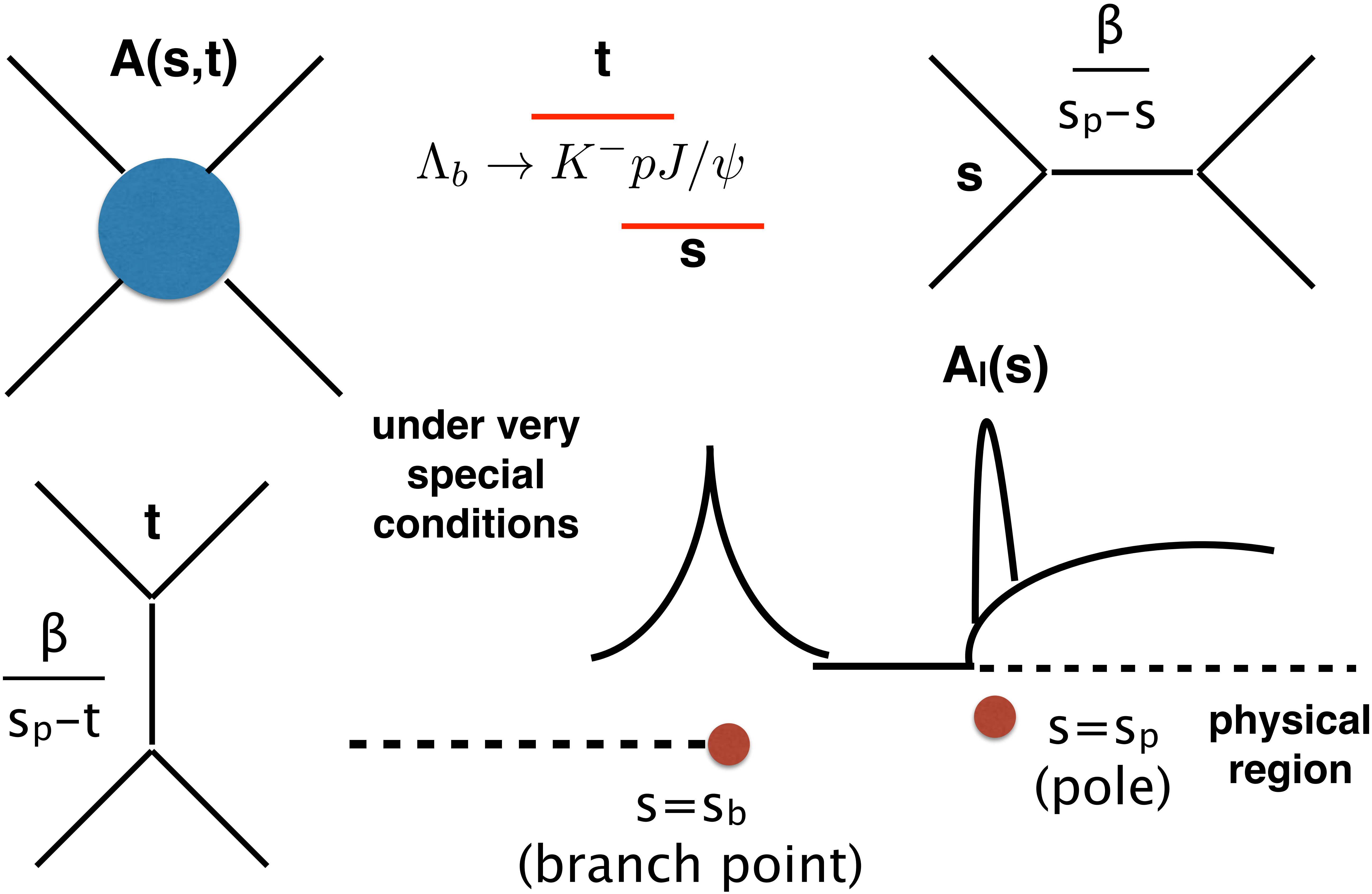


Cusps from singularities below threshold
Bumps from singularities above threshold

The view from above the heavier threshold

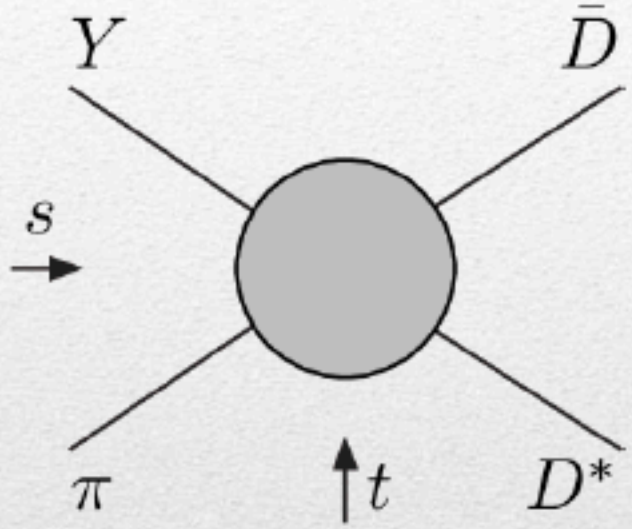


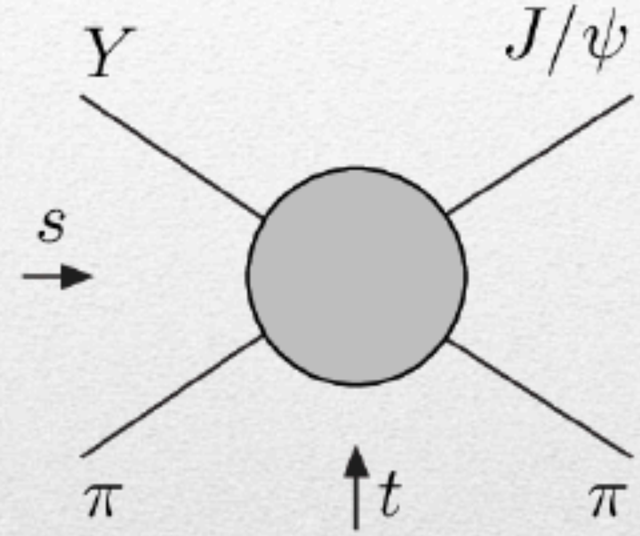
Origin of singularities (exchanges constrained by unitarity)



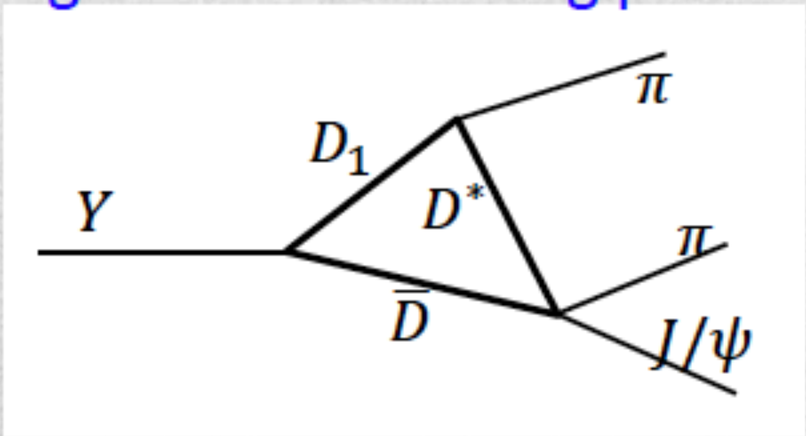
Case study, $Z_c(3900)$

One can test different parametrizations of the amplitude, which correspond to different singularities → different natures



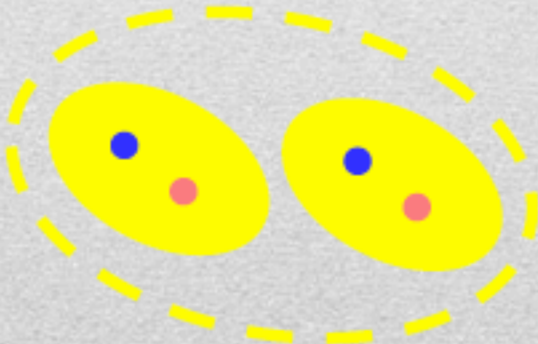


Triangle rescattering,
logarithmic branching point



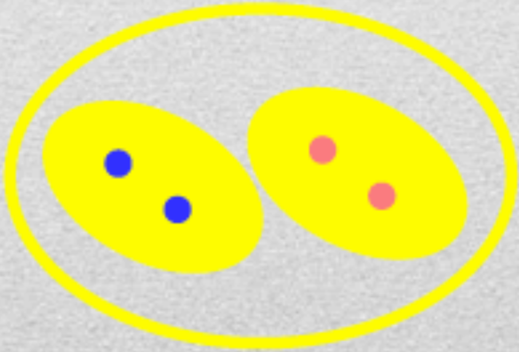
Szczepaniak, PLB747, 410-416
Szczepaniak, PLB757, 61-64
Guo *et al.* PRD92, 071502

(anti)bound state,
II/IV sheet pole



Tornqvist, Z.Phys. C61, 525
Swanson, Phys.Rept. 429
Hanhart *et al.* PRL111, 132003

Compact QCD state,
III sheet pole



Maiani *et al.*, PRD71, 014028
Maiani *et al.*, PRD87, 111102
AP *et al.*, Phys.Rept. 668

A.Pilloni at this meeting

Properties:

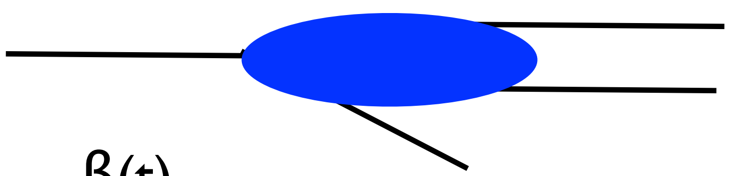
- **Duality:** resonances in direct channel dual to reggeons in cross channels and backgrounds are dual to the pomeron
- **All resonances are "connected":** resonances belong to Regge trajectories (reggeons)
- **Asymptotics:** determined by Regge poles
- **Unitarity:** imaginary parts determined by decay thresholds

Veneziano amplitude satisfies all of the above except unitarity

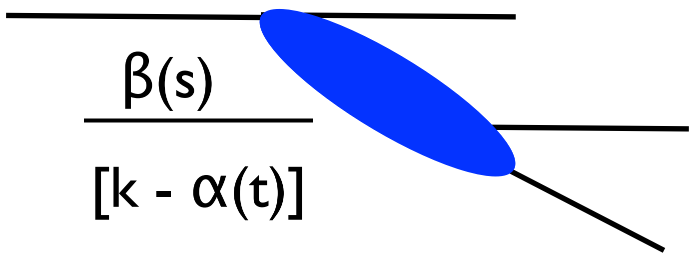
Veneziano amplitude: “compact” expression for the full amplitude

$$A(s,t) = \frac{\Gamma(-\alpha(s))\Gamma(-\alpha(t))}{\Gamma(-\alpha(s)-\alpha(t))} \qquad \alpha(s) = a + bs$$

resonance/reggeon in $s=m_{12}^2$



$\frac{\beta(t)}{[k - \alpha(s)]}$ ~ B.W. propagator



$\frac{\beta(s)}{[k - \alpha(t)]}$

resonance/reggeon in $t=m_{23}^2$

A(s,t) can be written as sum over resonances in ether channel.

$$A(s,t) = \sum_k \frac{\beta_k(t)}{k-\alpha(s)} = \sum_k \frac{\beta_k(s)}{k-\alpha(t)}$$

Note: in V-model resonance couplings, β, are fixed!

$$\beta_k(t) \propto (1+\alpha(t))(2+\alpha(t))\cdots(k+\alpha(t))$$

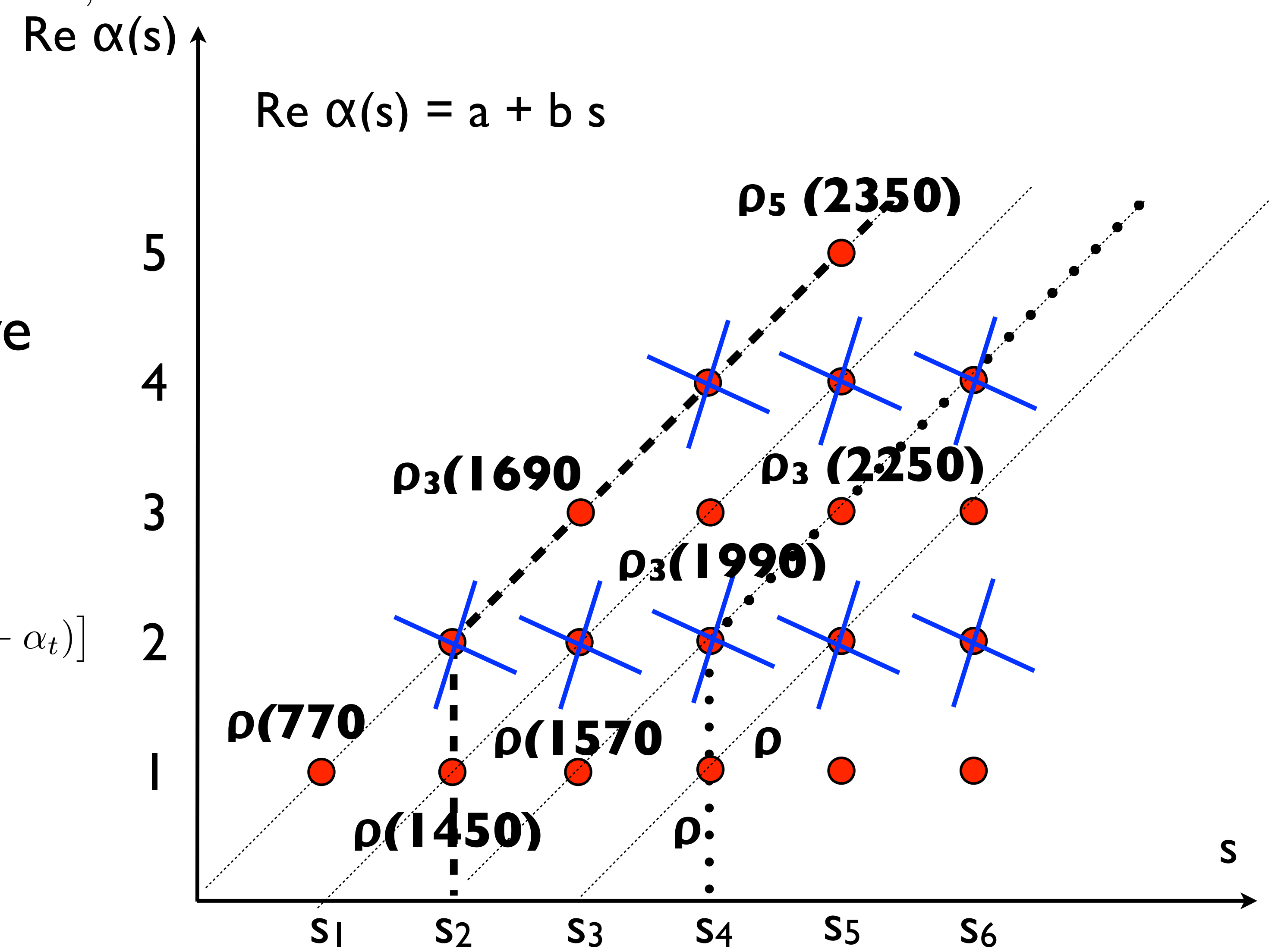
Resonances couplings, β, should depend on final state particles: a linear superposition of Veneziano amplitudes can be used to suppress or enhance individual resonances or trajectories

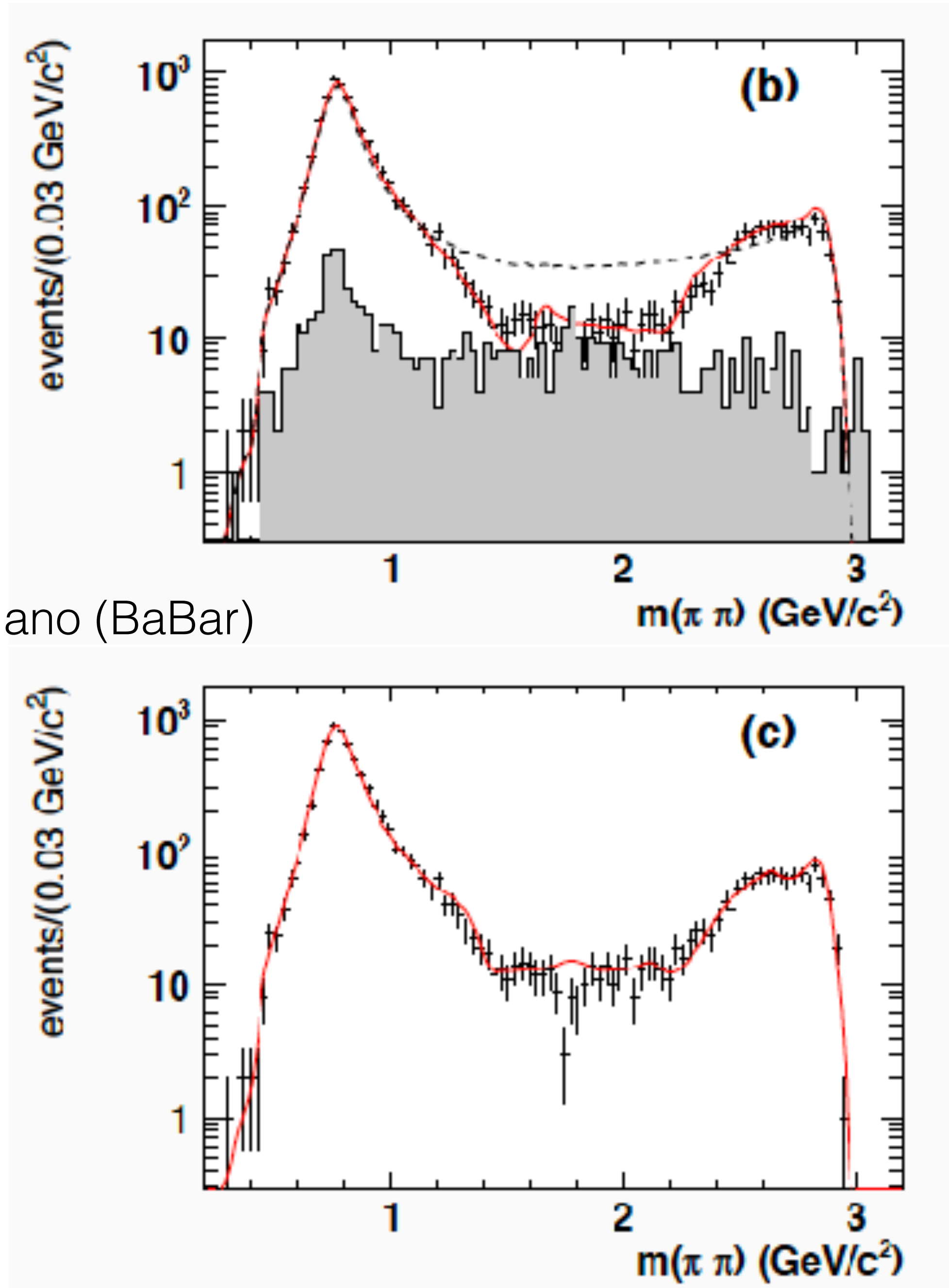
$$M = \epsilon_{\mu\nu\alpha\beta} p_1^\mu p_2^\nu p_3^\alpha \epsilon^\beta A(s,t,u)$$

$$A = \sum_{n,m} c_{n,m} \left[\frac{\Gamma(n-\alpha(s))\Gamma(n-\alpha(t))}{\Gamma(n+m-\alpha(s)-\alpha(t))} + (s,u) + (t,u) \right]$$

Need flexibility in low partial wave the get resonance widths right

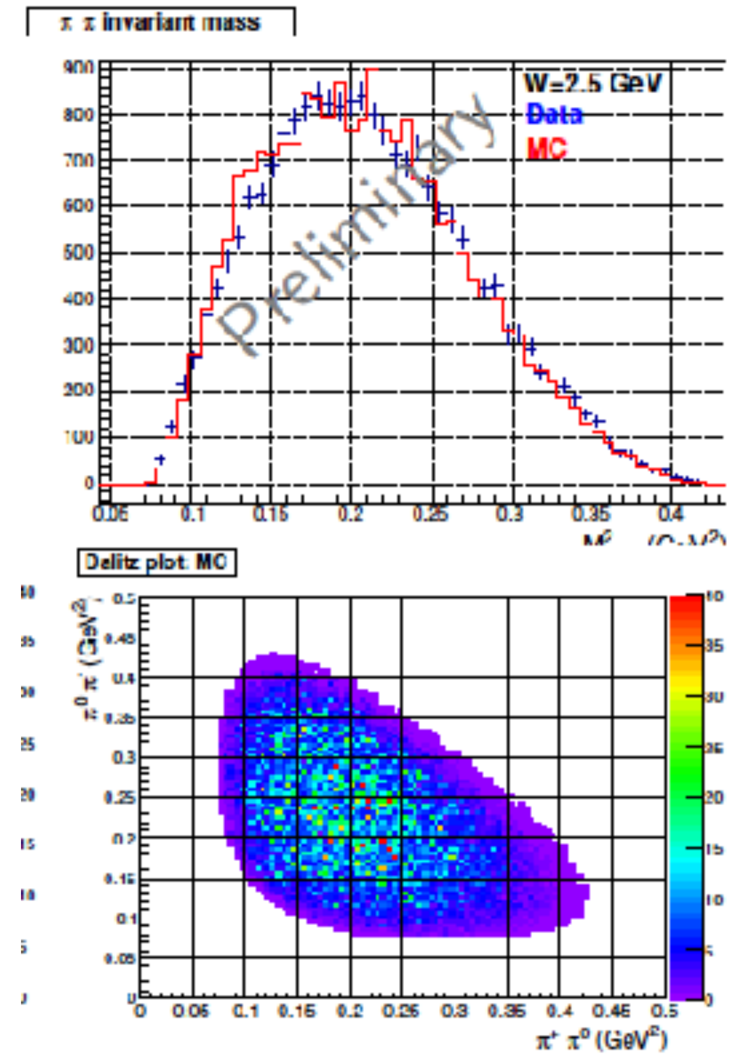
$$\mathcal{A}_n(s,t;N) = a_{n,0} \frac{2n-\alpha_s-\alpha_t}{(n-\alpha_s)(n-\alpha_t)} \left[\Pi_{i=1}^{n-1} (a_{n,i} - \alpha_s - \alpha_t) \right] \times \frac{\Gamma(N+1-\alpha_s)\Gamma(N+1-\alpha_t)}{\Gamma(N+1-n)\Gamma(N+n+1-\alpha_s-\alpha_t)}$$





A. Palano (BaBar)

FIG. 7: (a) Binned scatter diagram of $\cos\theta_{\pi_3}$ *vs* $m(\pi_1\pi_2)$. (b), (c) $\pi\pi$ mass projection in the $|\cos\theta_\pi| < 0.2$ region for all the three $\pi\pi$ charge combinations. The horizontal lines in (a) indicate the $\cos\theta_\pi$ selection. The dashed line in (b) is the result from the fit with only the $\rho(770)\pi$ amplitude. The fit in (b) uses the isobar model and the shaded histogram shows the background distribution estimated from the J/ψ sidebands. The fit in (c) uses the Veneziano model.



A. Celentano (CLAS, g11 data analysis) of ω→3π Dalitz plot distribution and projection

A.Palano (BaBar) **(also S.Fegan (BESIII))**

http://www.indiana.edu/~ssrt/

2017 International Summer Workshop on Reaction Theory

June 12-22, 2017, Bloomington, Indiana, USA

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The 2017 International Summer Workshop on Reaction Theory is dedicated to theory and phenomenology of scattering theory and its application to data analysis of modern experiments in strong interactions physics. As a new frontier in particle and nuclear physics has opened up with advances in experimental, theoretical and computational techniques there is new demand for a qualitatively new level of sophistication in data analysis never before achieved. These require deep knowledge of the methods in relativistic scattering theory. For at least two decades scattering theory has essentially disappeared from the physics curriculum and generations of physicists have been educated without this basic knowledge. Few have working experience with topics related to the analysis of relativistic reactions that involve aspects of Regge phenomenology, crossing relations and duality, analytic continuations, dispersion relations, etc., and the phenomenological application of all these concepts.

Future Directions in Spectroscopy Analysis II
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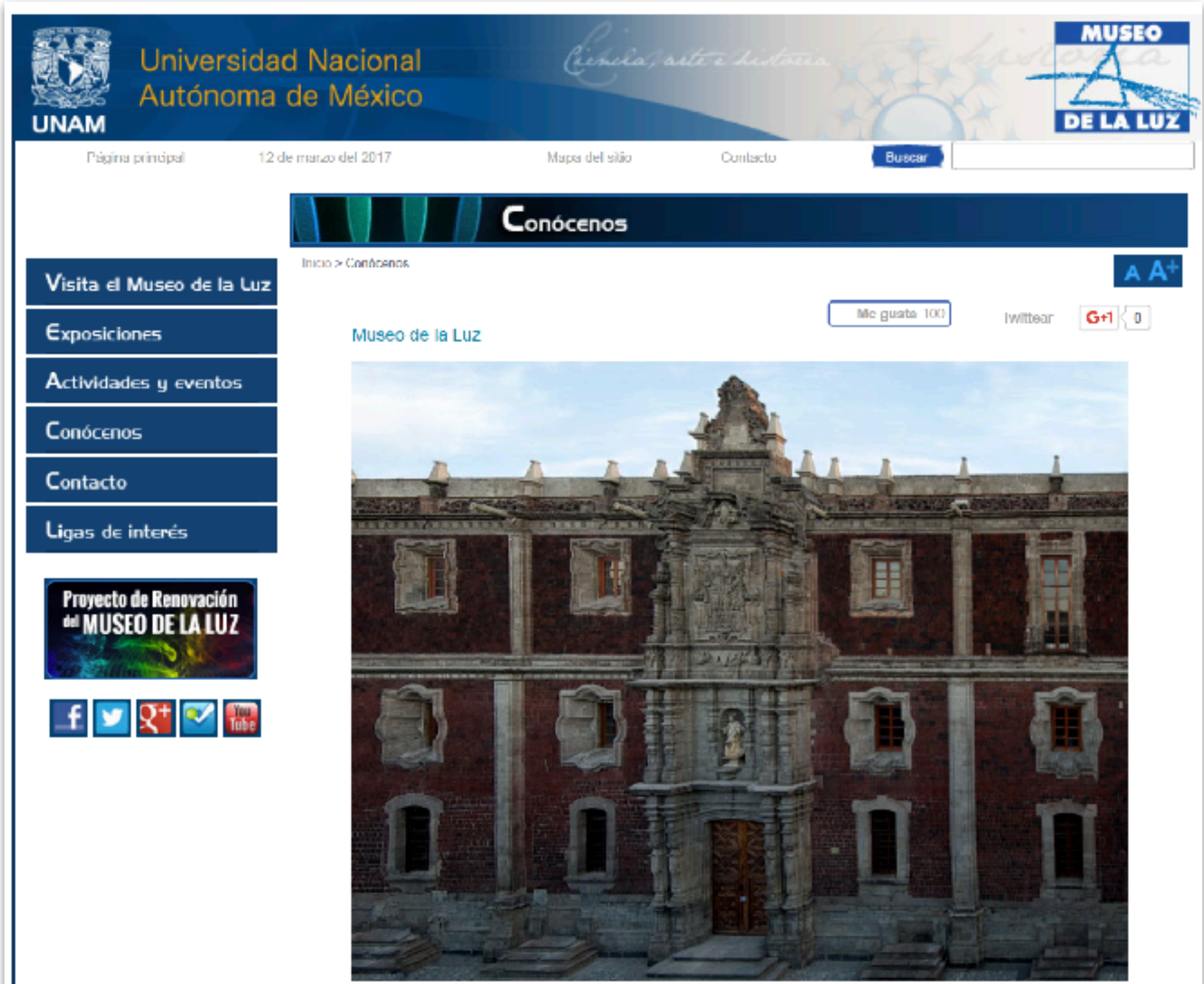

http://epistemia.nucleares.unam.mx/web?name=FDSA2017