A natural $S_4 \times SO(10)$ model of flavour

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in collaboration with:

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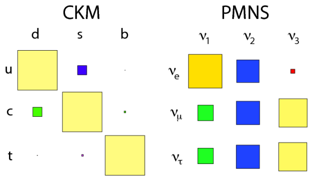
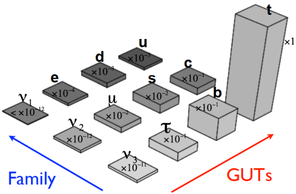
Motivation

- **Flavour problem**:
  The origin of the three families of quarks and leptons with their pattern of masses and mixing.





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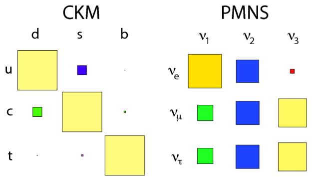

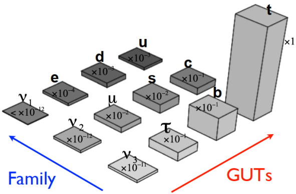

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  "Horizontal" unification of SM fermions.

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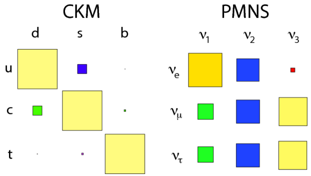

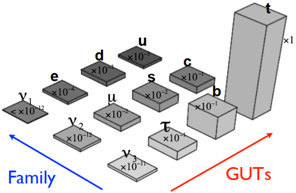

- **Family symmetry**:
  "Horizontal" unification of SM fermions.

- **Grand Unified Theory**:
  Unifies fermions within each family and reproduces an universal mass matrix structure.

The model

We propose a natural $S_4 \times SO(10)$ supersymmetric grand unified theory of flavour.

S_4:

$SO(10)$:

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S_4:

- Enforces CSD(3) vacuum alignments

$SO(10)$:

- Predicts right-handed (RH) neutrinos $\implies$ type-I seesaw mechanism.

The model

We only allow small Higgs representations **10**, **16** and **45**.

Field	Representation			
	S_4	$SO(10)$	$\mathbb{Z}_4^R$	
ψ	$3'$	16	1	Quarks and leptons
$H_{10}^{u,d}$	1	10	0	Break electroweak symmetry
$H_{\overline{16},16}$	1	$\overline{16}$	0	Break $SO(10)$ and give RH neutrino masses
$H_{45}^{X,Y,Z}$	1	45	0	Separate quarks and lepton masses
H_{45}^{B-L}	1	45	2	Gives DT splitting via DW mechanism
ϕ_i	$3'$	1	0	Acquire CSD3 vacuum alignments

$\mathbb{Z}_4^R$ breaks to $\mathbb{Z}_2^R$, the usual R parity in the MSSM.

CSD(3) from S_4

S_4 enforces the flavon vacuum alignments

$$\langle \phi_1 \rangle = v_1 \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix}, \quad \langle \phi_2 \rangle = v_2 \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}, \quad \langle \phi_3 \rangle = v_3 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}.$$

VEVs driven to scales with the hierarchy

$$v_1 \ll v_2 \ll v_3 \sim M_{\mathrm{GUT}}.$$

Yukawa matrices will have an **universal structure** dictated by CSD(3).

Yukawa Matrices

- Up-type quarks and neutrinos couple to one Higgs H_{10}^{u}, leading to Yukawa matrices $Y_{ij} \sim \langle\phi_i\rangle\langle\phi_j\rangle^T$ with an universal structure

$$Y^{u,\nu} = y_1^{u,\nu} \begin{pmatrix} 1 & 1 & 3 \\ 1 & 1 & 3 \\ 3 & 3 & 9 \end{pmatrix} + y_2^{u,\nu} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} + y_3^{u,\nu} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

The right-handed (RH) neutrino mass M^R has the same structure as the Y^ν.

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The right-handed (RH) neutrino mass M^R has the same structure as the Y^ν.

- Each matrix is **rank 1**.
- **Natural** understanding of the hierarchical Yukawa couplings: $y_u \sim v_1^2/M_{\text{GUT}}^2$, $y_c \sim v_2^2/M_{\text{GUT}}^2$, $y_t \sim v_3^2/M_{\text{GUT}}^2$.

Yukawa matrices

- Down-type quarks and charged leptons couple to a second Higgs H_{10}^d, with a new mixed term involving $Y_{12} \sim \langle\phi_1\rangle\langle\phi_2\rangle^T$

$$Y^{d,e} = y_{12}^{d,e} \begin{pmatrix} 0 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 4 & 6 \end{pmatrix} + y_2^{d,e} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} + y_3^{d,e} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} + y^P \begin{pmatrix} 0 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 0 \end{pmatrix}$$

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- This new term enforces a **texture zero** in the (1,1) element of Y^d, giving the GST relation for the Cabbibo angle, i.e. $\vartheta_{12}^q \approx \sqrt{y_d/ys}$.

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- This new term enforces a **texture zero** in the (1,1) element of Y^d, giving the GST relation for the Cabbibo angle, i.e. $\vartheta_{12}^q \approx \sqrt{y_d/ys}$.

- It also leads to a **milder hierarchy** in the down and charged lepton sectors.

Light neutrino mass matrix

- The light neutrino Majorana matrix, after **seesaw**, will also have the CSD(3) structure

$$m^{\nu} = \mu_1^{\nu} \begin{pmatrix} 1 & 1 & 3 \\ 1 & 1 & 3 \\ 3 & 3 & 9 \end{pmatrix} + \mu_2^{\nu} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} + \mu_3^{\nu} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} .$$

where the parameters μ_i are given by

$$\mu_i = v_u^2 \frac{(y_i^{\nu})^2}{M_i^R}$$

- Flavons yield to **normal hierarchy** $m_1 \ll m_2 \ll m_3$.

Numerical fit

- The model accurately fits all available quark and lepton data within 1σ, with a minimum $\chi^2 \approx 3.4$.

- The CP phase δ^l is left as a pure prediction and 2 preferred regions are given by

$$280.7^\circ < \delta^l < 308.3^\circ \qquad \text{and} \qquad 225.1^\circ < \delta^l < 253.2^\circ.$$

- The model predicts significant deviation from both zero and maximal CP violation.

Conclusion

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- Minimal field content.
- Low-dimensional representations.
- CSD(3) from S_4.

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- "Universal Sequential Dominance".
- $\mathcal{O}(1)$ dimensionless parameters.
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- "Universal Sequential Dominance".
- $\mathcal{O}(1)$ dimensionless parameters.
- Explains mass hierarchies.

Complete

- Reproduces all available quark and lepton data.
- Doublet-triplet splitting.
- μ term of $\mathcal{O}(\text{TeV})$.
- Acceptable proton decay.

If you want to know more...

A natural $S_4 \times SO(10)$ model of flavour

(based on arXiv:1705.01555)

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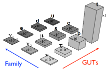


Motivation

Flavour problem
Origin of the three families of quarks and leptons. Very hierarchical charged fermion masses, small and hierarchical quark mixing, small neutrino masses and large lepton mixing.

Family symmetry
A non-Abelian discrete symmetry imposes constraints on the Yukawa couplings and reproduces precise predictions for masses and mixing. S_4 enforces CSD(3).

Grand Unified Theory
Unifies fermions within each family and reproduces an universal mass matrix structure, predicting relationships between quark and lepton Yukawa matrices.

Unified model of flavour

- We present a model with quarks and leptons unified in a single ψ representation of $\mathbf{S_4} \times \mathbf{SO(10)}$.
- The essential superfields are given in the table below. We only allow small Higgs representations **10**, **16** and **45**.

Field	Representation S_4	$SO(10)$	$\mathbb{Z}_4^R$	
ψ	$3'$	16	1	Quarks and leptons
$H_{10}^{u,d}$	1	10	0	Break electroweak symmetry
$H_{\overline{16},16}$	1	$\overline{16}$	0	Break $SO(10)$ and give RH Majorana masses
$H_{45}^{X,Y,Z}$	1	45	0	Separate quarks and lepton masses
H_{45}^{B-L}	1	45	2	Gives DT splitting via DW mechanism
ϕ_i	$3'$	1	0	Acquire CSD3 vacuum alignments

- The discrete symmetry $\mathbb{Z}_4^R$ is broken at the GUT scale by the H_{45}^{B-L} VEV to $\mathbb{Z}_2^R$, the usual R parity in the MSSM.

CSD(3) flavon vacuum alignments

The Yukawa structure of all fermions is determined by the hierarchical vacuum expectation values of three S_4 triplet flavons, with the following CSD(3) vacuum alignments [1]

$$\langle\phi_1\rangle = v_1 \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix}, \quad \langle\phi_2\rangle = v_2 \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}, \quad \langle\phi_3\rangle = v_3 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}.$$

VEVs driven to scales with the hierarchy $v_1 \ll v_2 \ll v_3 \sim M_{\mathrm{GUT}}$.

Yukawa matrices

- Up-type quarks and neutrinos couple to one Higgs H_{10}^u, leading to Yukawa matrices $Y_{ij} \sim \langle\phi_i\rangle\langle\phi_j\rangle^T$ with an universal structure

$$Y^{u,\nu} = y_1^{u,\nu} \begin{pmatrix} 1&1&3 \\ 1&1&3 \\ 3&3&9 \end{pmatrix} + y_2^{u,\nu} \begin{pmatrix} 0&0&0 \\ 0&1&1 \\ 0&1&1 \end{pmatrix} + y_3^{u,\nu} \begin{pmatrix} 0&0&0 \\ 0&0&0 \\ 0&0&1 \end{pmatrix}.$$

Natural understanding of the hierarchical Yukawa couplings $y_u \sim v_1^2/M_{\mathrm{GUT}}^2$, $y_c \sim v_2^2/M_{\mathrm{GUT}}^2$, $y_t \sim v_3^2/M_{\mathrm{GUT}}^2$.

- Down-type quarks and charged leptons couple to a second Higgs H_{10}^d, with a new mixed term involving $Y_{12} \sim \langle\phi_1\rangle\langle\phi_2\rangle^T$

$$Y^{d,e} = y_{12}^{d,e} \begin{pmatrix} 0&1&1 \\ 1&2&4 \\ 1&4&6 \end{pmatrix} + y_2^{d,e} \begin{pmatrix} 0&0&0 \\ 0&1&1 \\ 0&1&1 \end{pmatrix} + y_3^{d,e} \begin{pmatrix} 0&0&0 \\ 0&0&0 \\ 0&0&1 \end{pmatrix} + y^P \begin{pmatrix} 0&0&-1 \\ 0&2&0 \\ -1&0&0 \end{pmatrix}.$$

This new term enforces a zero in the (1,1) element of Y^d, giving the GST relation [2] for the Cabbibo angle, i.e. $\vartheta_{12}^q \approx \sqrt{y_d/ys}$. It also leads to a milder hierarchy in the down and charged lepton sectors.

Seesaw mechanism

The right-handed (RH) neutrino mass M^R has the same structure as Y^ν. The light neutrino mass matrix is obtained by the **type-I seesaw mechanism** [3,4] and will also have the CSD(3) structure

$$m^\nu = \mu_1^\nu \begin{pmatrix} 1&1&3 \\ 1&1&3 \\ 3&3&9 \end{pmatrix} + \mu_2^\nu \begin{pmatrix} 0&0&0 \\ 0&1&1 \\ 0&1&1 \end{pmatrix} + \mu_3^\nu \begin{pmatrix} 0&0&0 \\ 0&0&0 \\ 0&0&1 \end{pmatrix}.$$

The parameters μ_i are given in terms of the parameters y_i^ν and M_i^R simply by

$$\mu_i = v_u^2 \frac{(y_i^\nu)^2}{M_i^R}$$

The flavons yield a light neutrino mass matrix m^ν, where the normal hierarchy $m_1 \ll m_2 \ll m_3$ after seesaw is due to the very hierarchical RH neutrino masses.

Doublet-triplet splitting and proton decay

$$H_{10}^{u,d}: \quad \mathbf{10} \to \mathbf{2} + \mathbf{2} + \mathbf{3} + \bar{\mathbf{3}}$$

- Light MSSM doublets at electroweak scale.
- Heavy doublets **to preserve gauge unification**.
- Colour triplets of $\mathcal{O}(M_{GUT})$ leading to **acceptable proton decay** [5].
- DT splitting: Dimopoulos-Wilczek (DW) mechanism [6].
- Construct the doublet and triplet mass matrices from $H_{10}^{u,d}$ and $H_{16,\overline{16}}$. The triplets mass matrix has three eigenvalues of $\mathcal{O}(M_{\mathrm{GUT}})$. The doublets mass matrix has two eigenvalues at $\mathcal{O}(M_{\mathrm{GUT}})$ and one at $\mathcal{O}(TeV)$, which we identify with the $\boldsymbol{\mu}$ **term**.

Numerical fit

The model accurately fits all available quark and lepton data, with a minimum $\chi^2 \approx 3.4$. It predicts **normal neutrino hierarchy**. The *CP* phase δ^l is left as a pure prediction and 2 preferred regions are given by

$$280.7° < \delta^l < 308.3° \quad \text{and} \quad 225.1° < \delta^l < 253.2°.$$

The neutrino masses are also predicted

$$m_1 \approx 0.44 \text{ meV}, \quad m_2 \approx 8.68 \text{ meV}, \quad m_3 \approx 50.24 \text{ meV}.$$

The model predicts significant deviation from both zero and maximal *CP* violation.

Conclusion

Simple	Natural	Complete
Minimal field content	"Universal sequential dominance"	Renormalisable
Low-dimensional representations	No tuning of	Reduces to MSSM
CSD(3) from S_4	$\mathcal{O}(1)$ parameters	μ term of $\mathcal{O}(TeV)$
		DT splitting
		Proton decay suppressed

Field Content

Field	Representation				
	S_4	$SO(10)$	$\mathbb{Z}_4$	$\mathbb{Z}_4$	$\mathbb{Z}_4^R$
ψ	$3'$	16	1	1	1
H_{10}^u	1	10	0	2	0
H_{10}^d	1	10	2	0	0
$H_{\overline{16}}$	1	$\overline{16}$	2	1	0
H_{16}	1	16	1	2	0
$H_{45}^{X,Y}$	1	45	2	1	0
H_{45}^{Z}	1	45	1	2	0
H_{45}^{B-L}	1	45	2	2	2
ξ	1	1	2	2	0
ϕ_1	$3'$	1	0	0	0
ϕ_2	$3'$	1	2	0	0
ϕ_3	$3'$	1	2	2	0

Field	Representation				
	S_4	$SO(10)$	$\mathbb{Z}_4$	$\mathbb{Z}_4$	$\mathbb{Z}_4^R$
$\bar{\chi}_1$	1	$\overline{16}$	3	3	1
χ_1	1	16	0	3	1
$\bar{\chi}_2$	1	$\overline{16}$	1	3	1
χ_2	1	16	2	3	1
$\bar{\chi}_3$	1	$\overline{16}$	1	1	1
χ_3	1	16	2	1	1
χ_3'	1	16	1	2	1
χ_2'	1	16	1	0	1
ρ	1	1	0	2	1

Superpotential

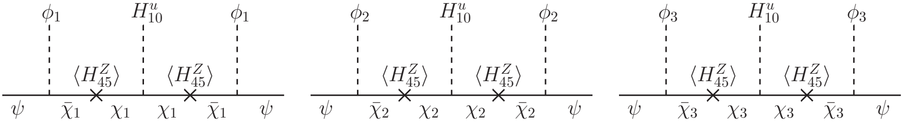
At the GUT scale, the renormalisable Yukawa superpotential is given by

$$W_Y^{(\mathrm{GUT})} = \psi\phi_a\bar{\chi}_a + \bar{\chi}_a\chi_a H_{45}^Z + \chi_a\chi_a H_{10}^u + \rho\chi_3 H_{\overline{16}} + M_\rho\rho\rho \\ + \bar{\chi}_b\chi_b' \left(H_{45}^X + H_{45}^Y\right) + \chi_b'\chi_b' H_{10}^d + \chi_1\chi_2 H_{10}^d,$$

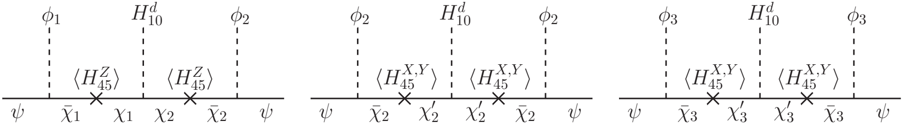
There are also Planck-suppressed terms

$$W_Y^{(\mathrm{Planck})} = \frac{\chi_a\chi_a H_{\overline{16}} H_{\overline{16}}}{M_P} + \frac{\psi\psi\phi_3 H_{10}^d}{M_P},$$

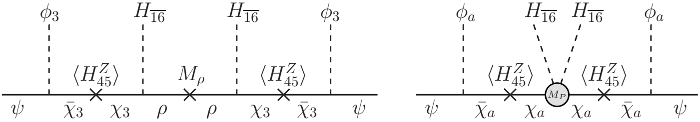
Up-type quarks and Dirac neutrinos



Down-type quarks and charged leptons



Right-handed neutrinos



Analytical estimates

Flavon VEV scales:

$$v_1 \approx 0.002 M_{\text{GUT}}, \qquad v_2 \approx 0.05 M_{\text{GUT}}, \qquad v_3 \approx 0.5 M_{\text{GUT}},$$

Estimated Yukawa couplings:

$$\begin{aligned}
y_1^u \sim y_1^\nu \sim \; & v_1^2/M_{\text{GUT}}^2 \approx 4 \times 10^{-6}, \\
y_2^u \sim y_2^\nu \sim y_2^d \sim y_2^e \sim \; & v_2^2/M_{\text{GUT}}^2 \approx 2.5 \times 10^{-3}, \\
y_3^u \sim y_3^\nu \sim y_3^d \sim y_3^e \sim \; & v_3^2/M_{\text{GUT}}^2 \approx 0.25, \\
y_{12}^d \sim y_{12}^e \sim & \, v_1 v_2/M_{\text{GUT}}^2 \approx 1 \times 10^{-4}, \\
y^P \sim \; & v_3/M_P \approx 5 \times 10^{-4}.
\end{aligned}$$

Estimated RH neutrino mass parameteres:

$$M_1^{\text{R}} \sim 4 \times 10^7 \text{ GeV}, \quad M_2^{\text{R}} \sim 2.5 \times 10^{10} \text{ GeV}, \quad M_3^{\text{R}} \sim 10^{16} \text{ GeV}.$$

Analytical estimates for quark mixing

Strong hierarchy in $Y^u \implies$ almost all mixing in Y^d.

Leading terms in Y^d (and ignoring phases):

$$Y^d \approx \begin{pmatrix} 0 & y^d_{12} & y^d_{12} - y^P \\ y^d_{12} & y'_2 & y'_2 + 2(y^d_{12} - y^P) \\ y^d_{12} - y^P & y'_2 + 2(y^d_{12} - y^P) & y^d_3 \end{pmatrix}.$$

Mixing angles estimated by

$$\theta^q_{12} \approx \frac{Y^d_{12}}{Y^d_{22}} = \frac{y^d_{12}}{y'_2}, \quad \theta^q_{13} \approx \frac{Y^d_{13}}{Y^d_{33}} = \frac{y^d_{12} - y^P}{y^d_3}, \quad \theta^q_{23} \approx \frac{Y^d_{23}}{Y^d_{33}} = \frac{y'_2 + 2(y^d_{12} - y^P)}{y^d_3}.$$

Down-type quark Yukawa eigenvalues

$$y_d \approx (y^d_{12})^2 / y'_2, \quad y_s \approx y'_2, \quad y_b \approx y^d_3.$$

Analytical estimates for quark mixing

Solving for y^d_{12}, y'_2 and y^d_3, we have

$$\theta^q_{12} \approx \sqrt{\frac{y_d}{y_s}}, \quad \theta^q_{13} \approx \frac{\sqrt{y_d y_s} - y^P}{y_b}, \quad \theta^q_{23} \approx \frac{y_s + 2(\sqrt{y_s y_d} - y^P)}{y_b}.$$

The first equality is exactly the GST relation.

The GUT-scale values* from observation (with $y^P = 0$) are

$$\theta^q_{12} \approx 12.85^\circ, \quad \theta^q_{13} \approx 0.23^\circ, \quad \theta^q_{23} \approx 1.48^\circ.$$

*assuming no SUSY threshold corrections

Quarks

Observable	Data		Model	
	Central value	1σ range	Best fit	Interval
θ_{12}^q /°	13.03	$12.99 \to 13.07$	13.02	$12.94 \to 13.10$
θ_{13}^q /°	0.039	$0.037 \to 0.040$	0.039	$0.036 \to 0.041$
θ_{23}^q /°	0.445	$0.438 \to 0.452$	0.439	$0.426 \to 0.450$
δ^q /°	69.22	$66.12 \to 72.31$	69.21	$63.22 \to 73.94$
y_u $/10^{-6}$	2.988	$2.062 \to 3.915$	3.012	$1.039 \to 4.771$
y_c $/10^{-3}$	1.462	$1.411 \to 1.512$	1.493	$1.445 \to 1.596$
y_t	0.549	$0.542 \to 0.556$	0.547	$0.532 \to 0.562$
y_d $/10^{-5}$	2.485	$2.212 \to 2.758$	2.710	$2.501 \to 2.937$
y_s $/10^{-4}$	4.922	$4.656 \to 5.188$	5.168	$4.760 \to 5.472$
y_b	0.141	$0.136 \to 0.146$	0.137	$1.263 \to 1.429$

Leptons

Observable	Data		Model	
	Central value	1σ range	Best fit	Interval
θ^{ℓ}_{12} $/^\circ$	33.57	$32.81 \to 34.32$	33.62	$31.69 \to 34.46$
θ^{ℓ}_{13} $/^\circ$	8.460	$8.310 \to 8.610$	8.455	$8.167 \to 8.804$
θ^{ℓ}_{23} $/^\circ$	41.75	$40.40 \to 43.10$	41.96	$39.47 \to 43.15$
δ^{ℓ} $/^\circ$	261.0	$202.0 \to 312.0$	300.9	$280.7 \to 308.4$
y_e $/10^{-5}$	1.017	$1.011 \to 1.023$	1.017	$1.005 \to 1.029$
y_μ $/10^{-3}$	2.147	$2.134 \to 2.160$	2.147	$2.121 \to 2.173$
y_τ $/10^{-2}$	3.654	$3.635 \to 3.673$	3.654	$3.616 \to 3.692$
$\Delta m^2_{21}/(10^{-5}\,\mathrm{eV}^2)$	7.510	$7.330 \to 7.690$	7.515	$7.108 \to 7.864$
$\Delta m^2_{31}/(10^{-3}\,\mathrm{eV}^2)$	2.524	$2.484 \to 2.564$	2.523	$2.443 \to 2.605$
m_1 /meV			0.441	$0.260 \to 0.550$
m_2 /meV			8.680	$8.435 \to 8.888$
m_3 /meV			50.24	$49.44 \to 51.05$
$\sum m_i$ /meV	< 230		59.36	$58.49 \to 60.19$

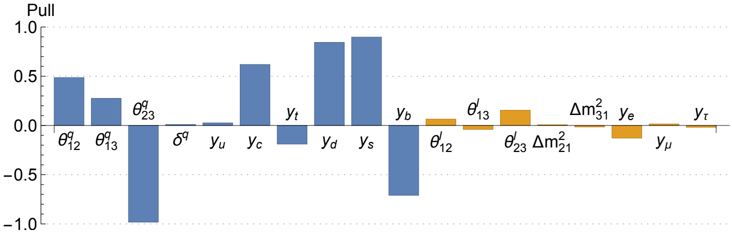
Input parameters

Parameter	Value
$y_1^u\ /10^{-6}$	3.009
$y_2^u\ /10^{-3}$	1.491
y_3^u	0.549
$y_{12}^d\ /10^{-4}$	-1.186
$y_2^d\ /10^{-4}$	6.980
y_3^d	0.137
$y^P\ /10^{-4}$	1.243

Parameter	Value
$y_{12}^e\ /10^{-4}$	1.558
$y_2^e\ /10^{-3}$	2.248
$y_3^e\ /10^{-2}$	3.318
μ_1 /meV	2.413
μ_2 /meV	27.50
μ_3 /meV	2.900

Parameter	Value
α_d	0.043π
β_d	0.295π
α_e	1.692π
β_e	1.755π
γ	0.918π
η'	1.053π

Pulls



Pull
1.0
0.5
0.0
−0.5
−1.0
θ^q_{12}
θ^q_{13}
θ^q_{23}
δ^q
y_u
y_c
y_t
y_d
y_s
y_b
θ^l_{12}
θ^l_{13}
θ^l_{23}
Δm^2_{21}
Δm^2_{31}
y_e
y_μ
y_τ