Physics of Gravitational Waves

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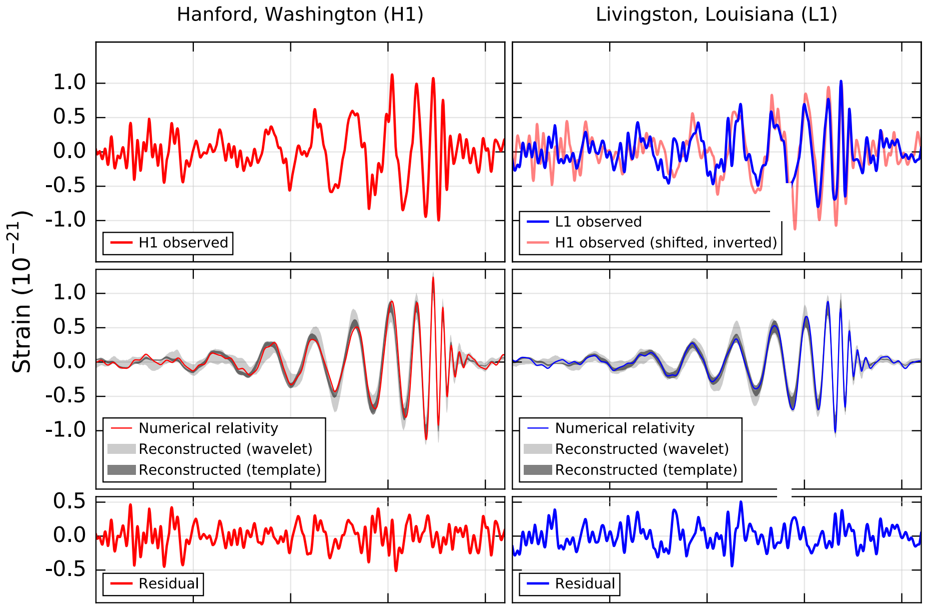
September 15th, 2015: first Gravitational Waves detection



first detection:GW150914

$$m_1 = 29.1^{+3.7}_{-4.4}\ M_\odot$$

$$m_2 = 36.2^{+5.2}_{-3.8}\ M_\odot \qquad a_1, a_2 = -0.06^{+0.14}_{-0.14}$$

$$M = 62.3^{+3.7}_{-3.1}\ M_\odot$$

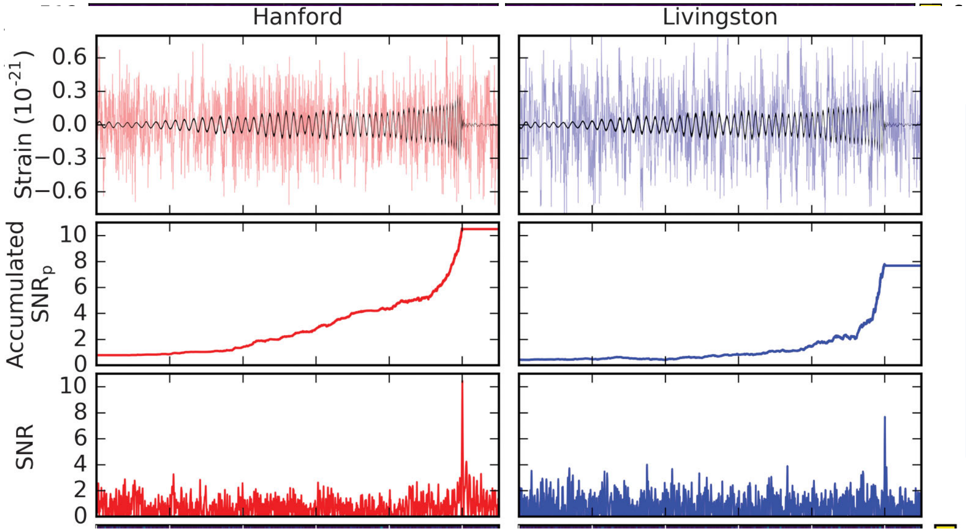
final BH

$$a = J/M = 0.68^{+0.05}_{-0.06}$$

$$D_L = 420^{+150}_{-180} Mpc \qquad z = 0.09^{+0.03}_{-0.04}$$

SNR=23.7

radiated $E_{GW} = 3 M_\odot c^2$



second detection:GW151226

$$m_1 = 7.5^{+2.3}_{-2.3}\ M_\odot$$

$$m_2 = 14.2^{+8.3}_{-3.7}\ M_\odot \qquad a_1, a_2 = 0.21^{+0.20}_{-0.10}$$

$$M = 20.8^{+6.1}_{-1.7}\ M_\odot$$

final BH

$$a = J/M = 0.74^{+0.06}_{-0.06}$$

$$D_L = 440^{+180}_{-190} Mpc \qquad z = 0.09^{+0.03}_{-0.04}$$

SNR=13.0

radiated $E_{GW} = 1 M_\odot c^2$

for both detections the significance is > 5 σ

candidate: LVT151012

$m_1 = 13^{+4}_{-5}\ M_\odot$

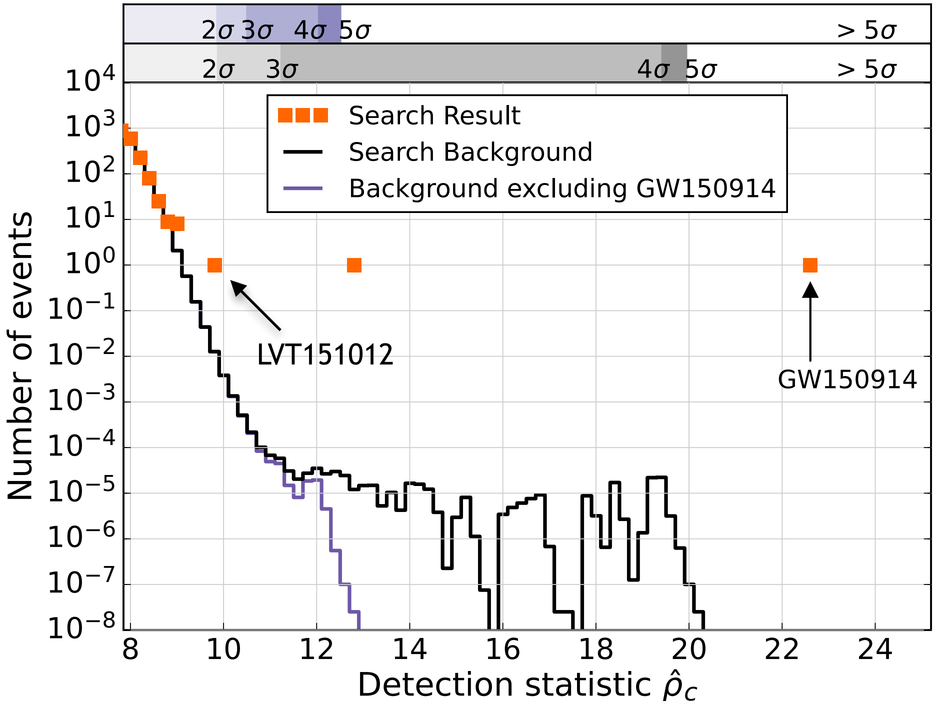
$m_2 = 23^{+18}_{-6}\ M_\odot$ $a_1, a_2 = 0.0^{+0.3}_{-0.2}$

$M = 35^{+14}_{-4}\ M_\odot$

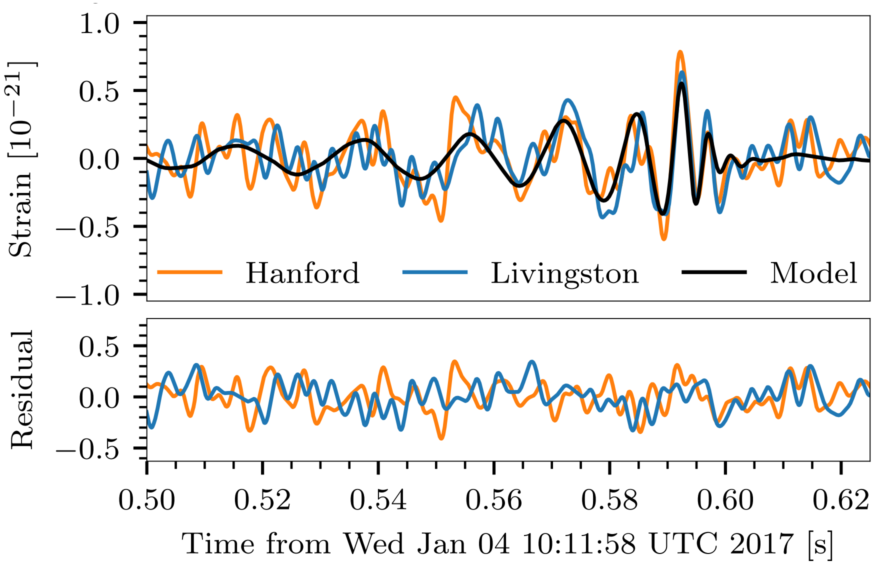
$a = J/M = 0.66^{+0.09}_{-0.10}$

$D_L = 1^{+0.5}_{-0.5} Gpc$ $z = 0.20^{+0.09}_{-0.09}$

the significance is ≤ 2 σ



Third detection :GW170104



SNR= 13

$m_1 = 19.4^{+5.3}_{-5.9}\ M_\odot$

$m_2 = 31.2^{+8.4}_{-6.0}\ M_\odot$

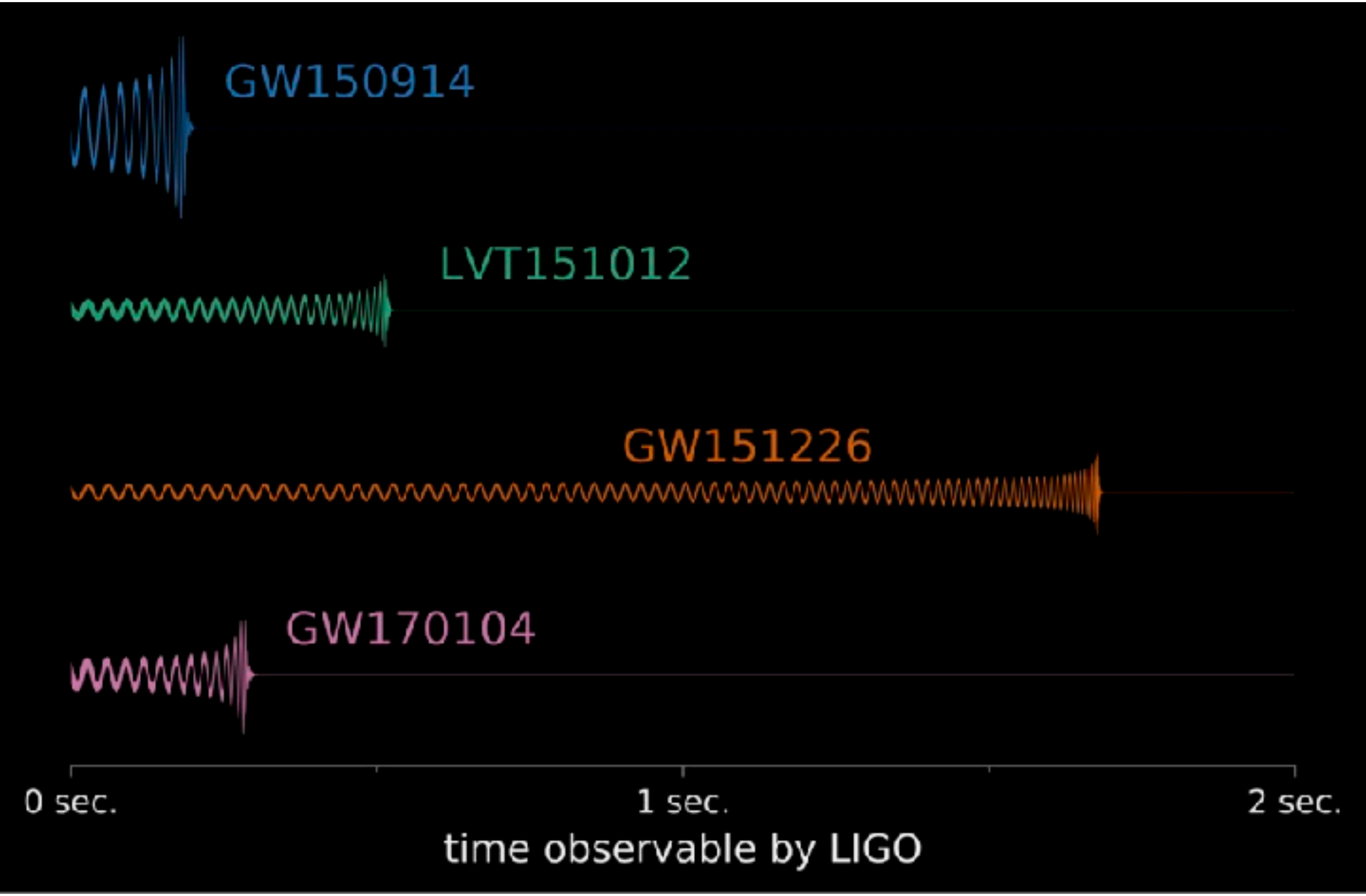
radiated E_{GW} = 2 M⊙ c^2

$M = 48.7^{+5.7}_{-4.6}\ M_\odot$

$a = J/M = 0.64^{+0.09}_{-0.20}$

$D_L = 880^{+450}_{-390} Mpc$ $z = 0.18^{+0.08}_{-0.07}$

None of the signals detected so far have an electromagnetic counterpart


GW150914
LVT151012
GW151226
GW170104
0 sec.
1 sec.
2 sec.
time observable by LIGO

How did the LIGO-Virgo collaboration reach the conclusion that the observed gravitational signal is due to the coalescence of two black holes?

The inspiralling part of the signal is computed by a post-Newtonian expansion of the equations of motion in GR, assuming two point masses in circular orbit

during the inspiralling the orbit shrinks due to GW emission:

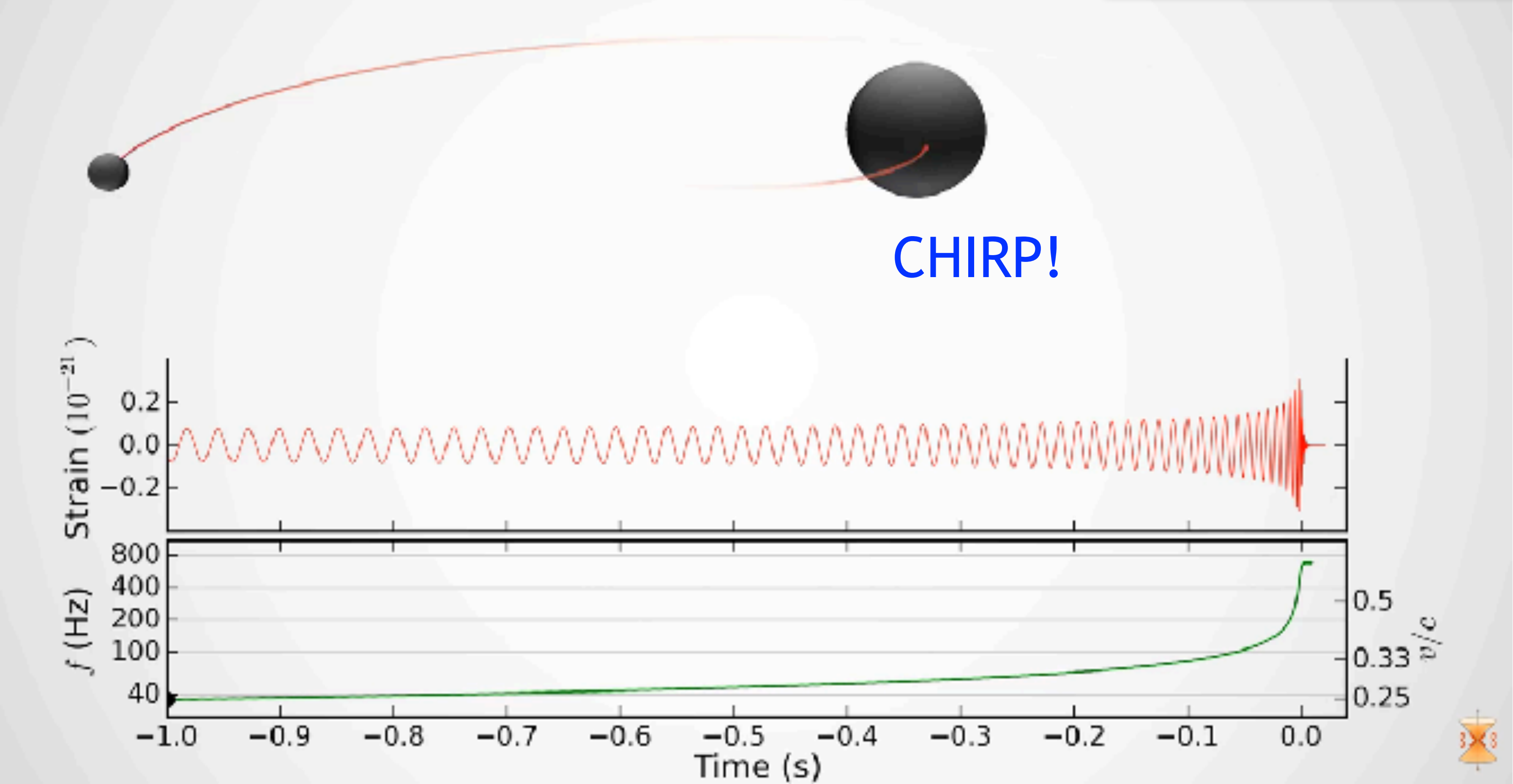
the orbital frequency increases

$\nu_{GW} = 2\,\nu_{orb}$

the frequency increases

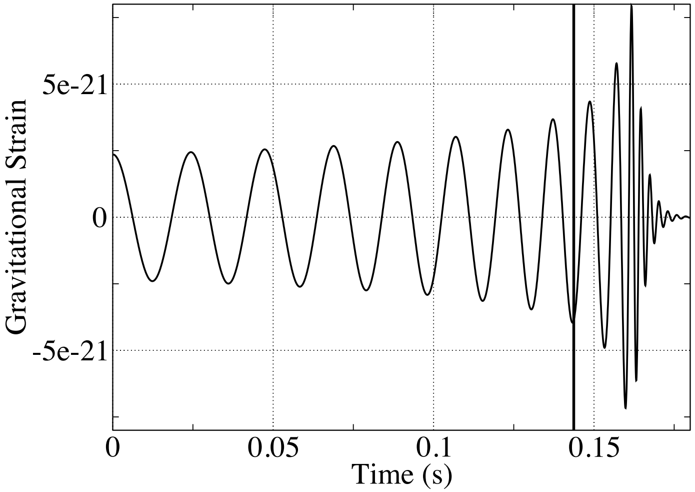
$h_0 \propto \nu_{GW}^{2/3}$

the amplitude increases



CHIRP!

$$\boxed{h(\nu) = h_0 e^{i\Psi(\nu)}}$$



Information from the wave phase

$$\boxed{\Psi = -2\left[\frac{c^3(t_{coal}-t)}{5G\,\mathcal{M}}\right]^{5/8} + \Psi^{in}}$$

"Chirp mass"

$$\boxed{\mathcal{M} = \frac{(m_1 m_2)^{3/5}}{(m_1+m_2)^{1/5}} = \frac{c^3}{G}\left[\frac{5}{96}\pi^{-8/3}\,\nu^{-11/3}\,\dot{\nu}\right]^{3/5}}$$

measuring the wave frequency and its time derivative, we measure the chirp mass

PROBLEM: in the chirp mass formula, frequency and its time derivative are evaluated in the source frame, but the wave frequency is measured in the detector frame:

$$\nu_{obs} = \frac{\nu}{1+z} \quad \Longrightarrow \quad [\nu_{obs}^{-11/3}\dot{\nu}_{obs}]^{3/5} = \frac{[\nu^{-11/3}\dot{\nu}]^{3/5}}{(1+z)}$$

This means that since we do not know the source redshift, what we measure is the "redshifted mass", i.e.

$$\mathcal{M}' = \mathcal{M}(1+z)$$

The same scaling remains true even if we include further terms in the Post-Newtonian expansion

FROM THE WAVE AMPLITUDE WE GAIN INFORMATION ON THE SOURCE DISTANCE

In the detector frame the wave amplitude is

$$h_0(t) = \frac{4\pi^{2/3}G^{5/3}}{c^4} \times \frac{\mathcal{M}'}{D} \times [\mathcal{M}'\nu_{obs}]^{2/3}$$

where $\mathcal{M}' = \mathcal{M}(1+z)$

D is the luminosity distance

$$D(z) = \frac{2}{H_0\Omega_0^2}[\Omega_0 z - (2-\Omega_0)(\sqrt{1+\Omega_0 z} - 1)$$

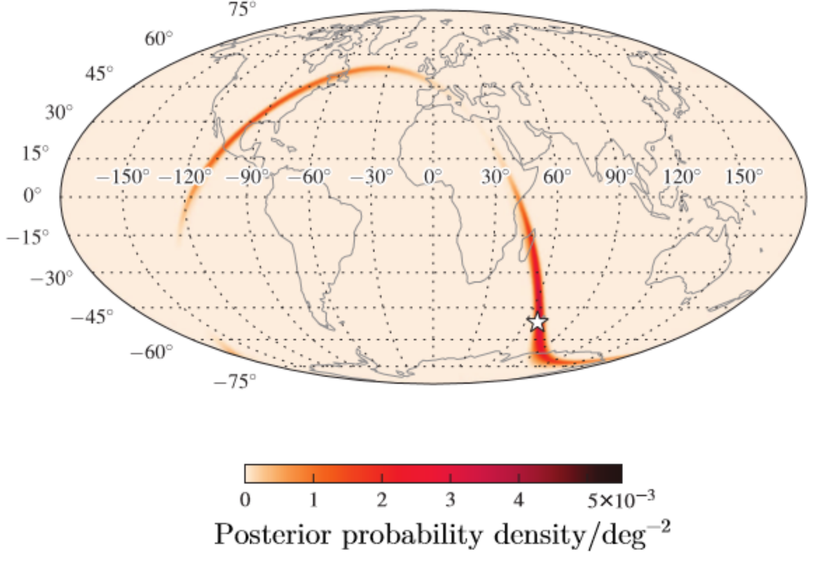
$H_0$is the Hubble constant $\quad \Omega_0 = \frac{8\pi}{3}\frac{\rho_{m_0}}{H_0^2}, \quad \rho_{m_0} =$ present matter density

from the wave amplitude we can infer the source luminosity distance up to a factor (1+z), i.e.

$$d_{eff} = D(z)(1+z)$$

But in the case of the detected signals we do not know the redshift z!

2 detectors



to measure the redshift of the galaxy hosting the source, this must me localized:
more detectors are needed

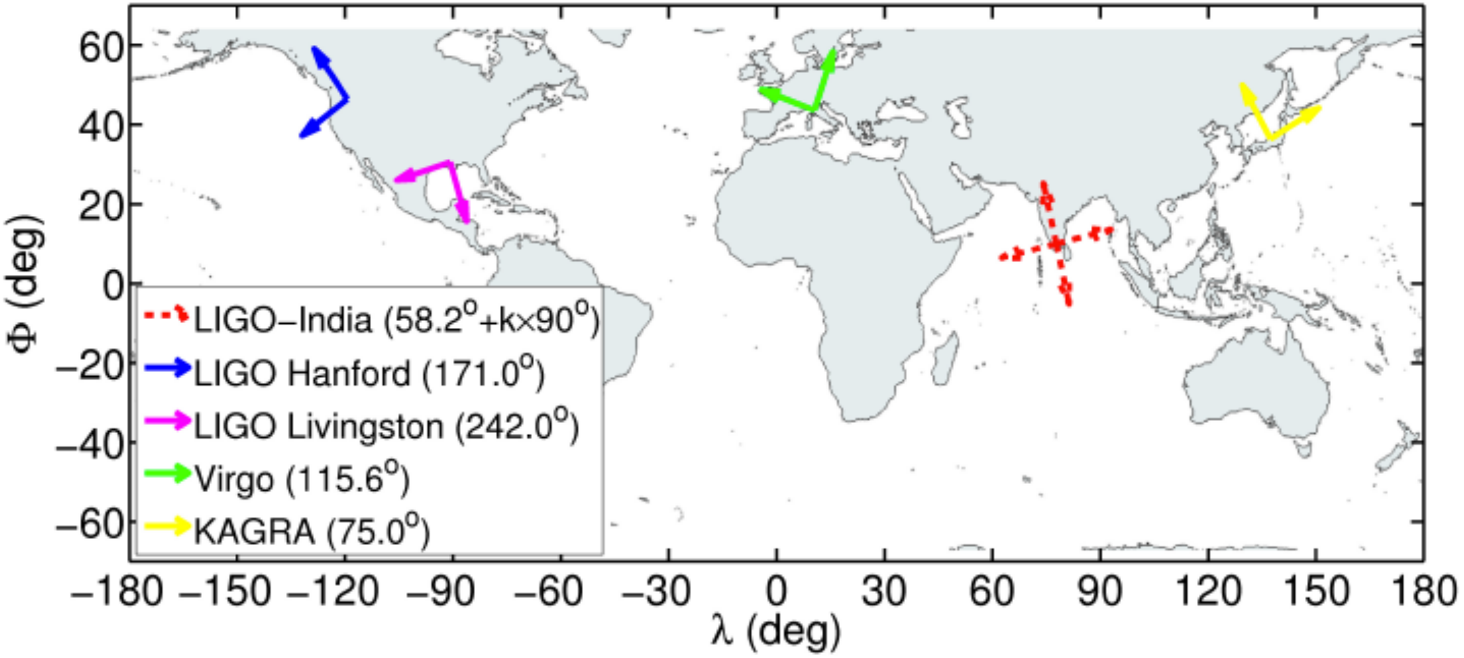
Abbott et al. “*Prospects for Observing and Localizing Gravitational-Wave Transients with Advanced LIGO and Advanced Virgo* “
Living Rev. Relativity, 19, (2016), 1

GW150914 has been localized in a sky area of 230 deg^2

850 deg^2 FOR GW151226

1600 deg^2 FOR LVT151012

DETECTORS WHICH WILL OPERATE IN THE NEXT DECADE



When all detectors will be operating it will be possible to localize the source position within 4-5 deg^2

$$d_{eff} = D(z)(1+z)$$

$$D(z) = \frac{2}{H_0\Omega_0^2}[\Omega_0 z - (2-\Omega_0)(\sqrt{1+\Omega_0 z} - 1)$$

$$d_{eff} = \left[\frac{2}{H_0\Omega_0^2}[\Omega_0 z - (2-\Omega_0)(\sqrt{1+\Omega_0 z} - 1)\right] \cdot (1+z)$$

ASSUMING A COSMOLOGICAL MODEL

ΛCDM cosmology

$H_0 = 67.9\ km\ s^{-1} Mpc^{-1}$

$\Omega_0 = 0.306,\quad$ Planck 2015

from the measured d_{eff}, we can infer the source redshift z

$GW150914: \quad z = 0.09^{+0.03}_{-0.04}$

$GW151226: \quad z = 0.09^{+0.03}_{-0.04}$

$LVT151012: \quad z = 0.2^{+0.09}_{-0.09}$

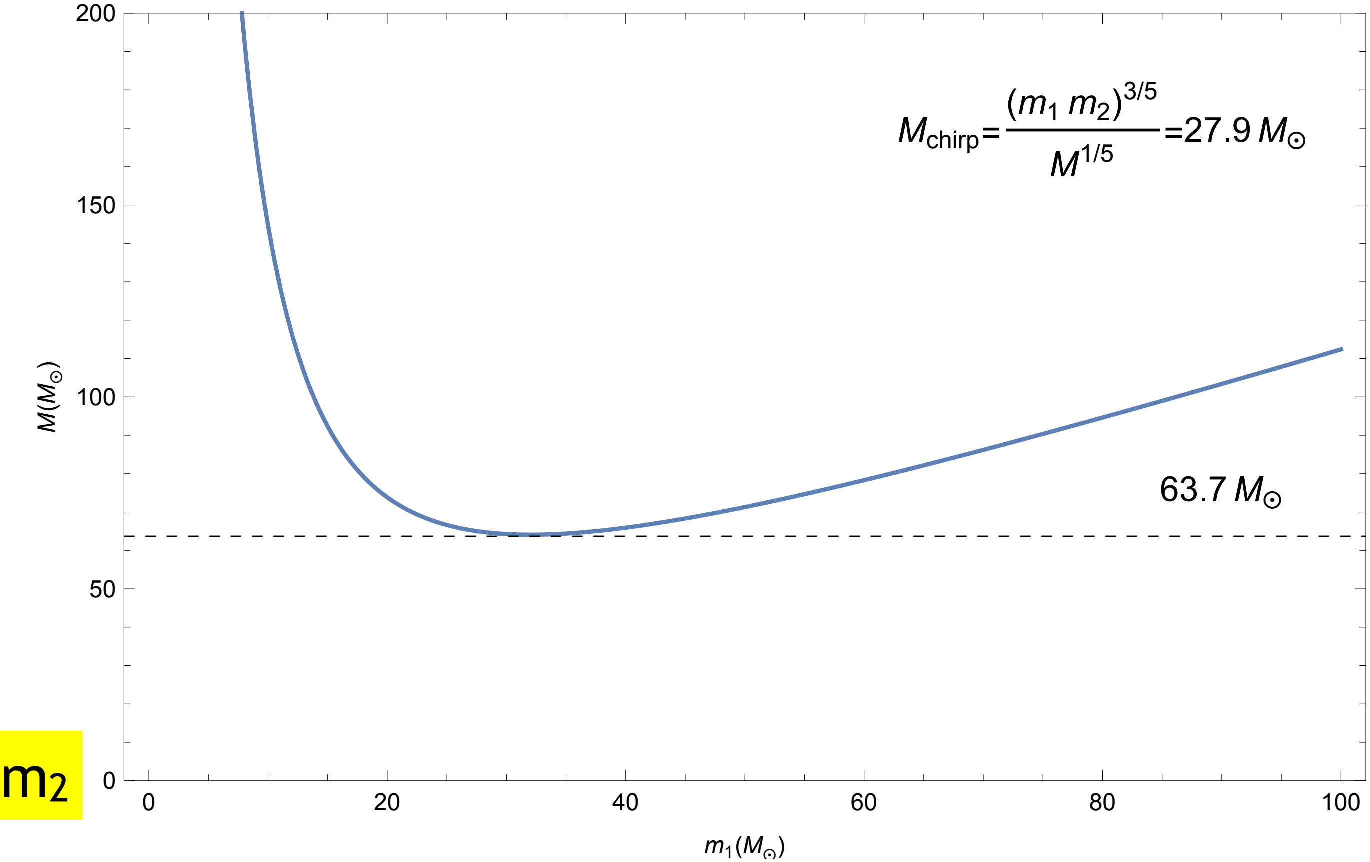
$GW170104: \quad z = 0.18^{+0.08}_{-0.07}$

given the redshift we find the "true" chirp mass

$$\mathcal{M} = \frac{\mathcal{M}'}{1+z}$$

where $\mathcal{M} = \frac{(m_1 m_2)^{3/5}}{(m_1+m_2)^{1/5}}$

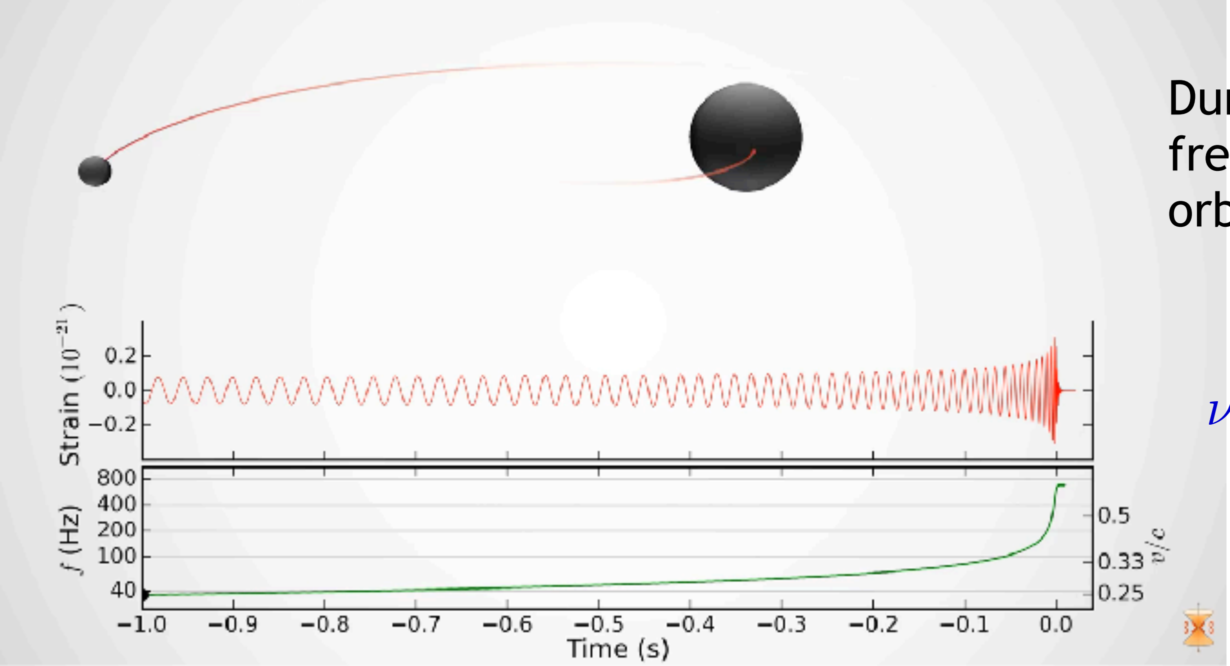
GW150914



$M=m_1+m_2$

$$\mathcal{M} \simeq 28 M_\odot \rightarrow (m_1+m_2) \gtrsim 63.7 M_\odot$$

Too large to be two neutron stars



During the inspiralling the wave frequency is related to the orbital distance by

$$\nu_{GW}(t) = \frac{1}{\pi}\sqrt{\frac{G(m_1+m_2)}{d^3_{orb}(t)}}$$

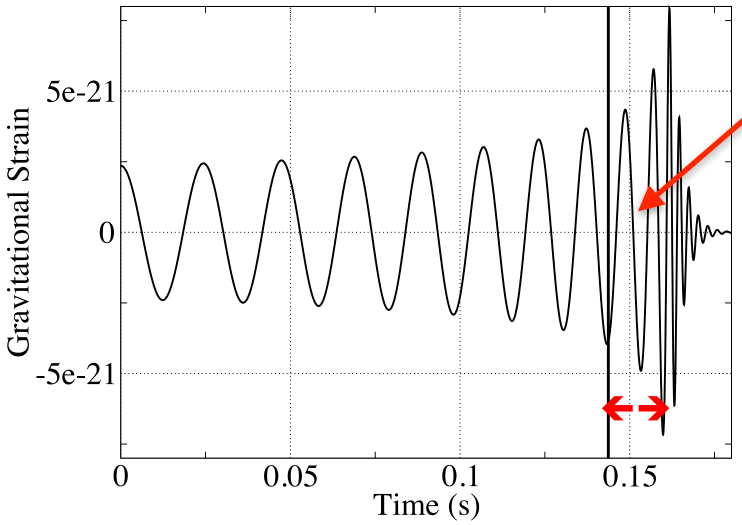
For GW150914 the total mass is ≳ 63.7 M⊙

over 0.2 s the wave frequency increases from 35 to 150 Hz, from which we infer that, just before merging, the distance bewteen the two masses was

$$d_{orb}(150\ Hz) \simeq 339\ km$$

The two objects must be extremely compact!

Are they Black Holes?



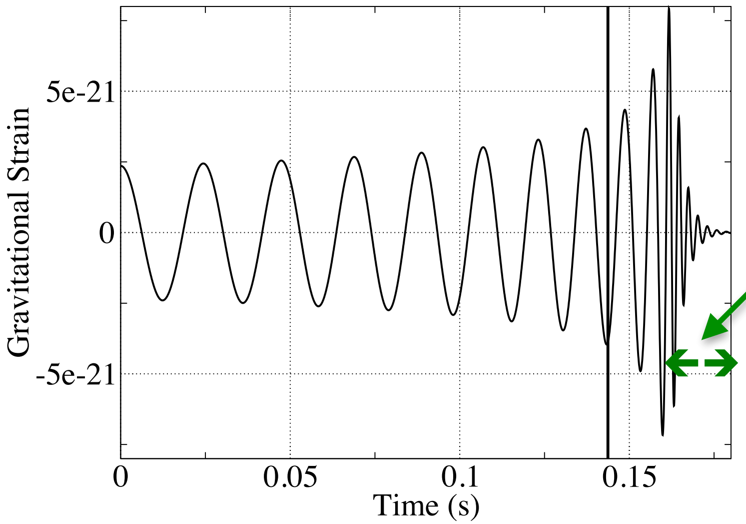
signal emitted during the merging: to be found by solving numerically Einstein’s equations in the non linear regime

These studies started in the late 1990s with the Grand Challenge project to simulate head-on binary black hole collision

After decades of numerical studies on BH coalescence, a bank of templates has been set up

Fitting formulae based on numerical simulations of BH merging have been found, which compared to the merging part of the signal allow to estimate:

individual masses and spins
mass and angular momentum of the final black hole



Ringdown: part of the signal emitted by the final black hole, which oscillates in its proper modes: the Quasi-Normal-Modes (QNM)

the ringdown is a superposition of damped sinusoids at the frequencies and with the damping times of the QNMs

In General Relativity the QNM frequencies depends only on the black hole mass and the angular momentum **(no hair theorem)**

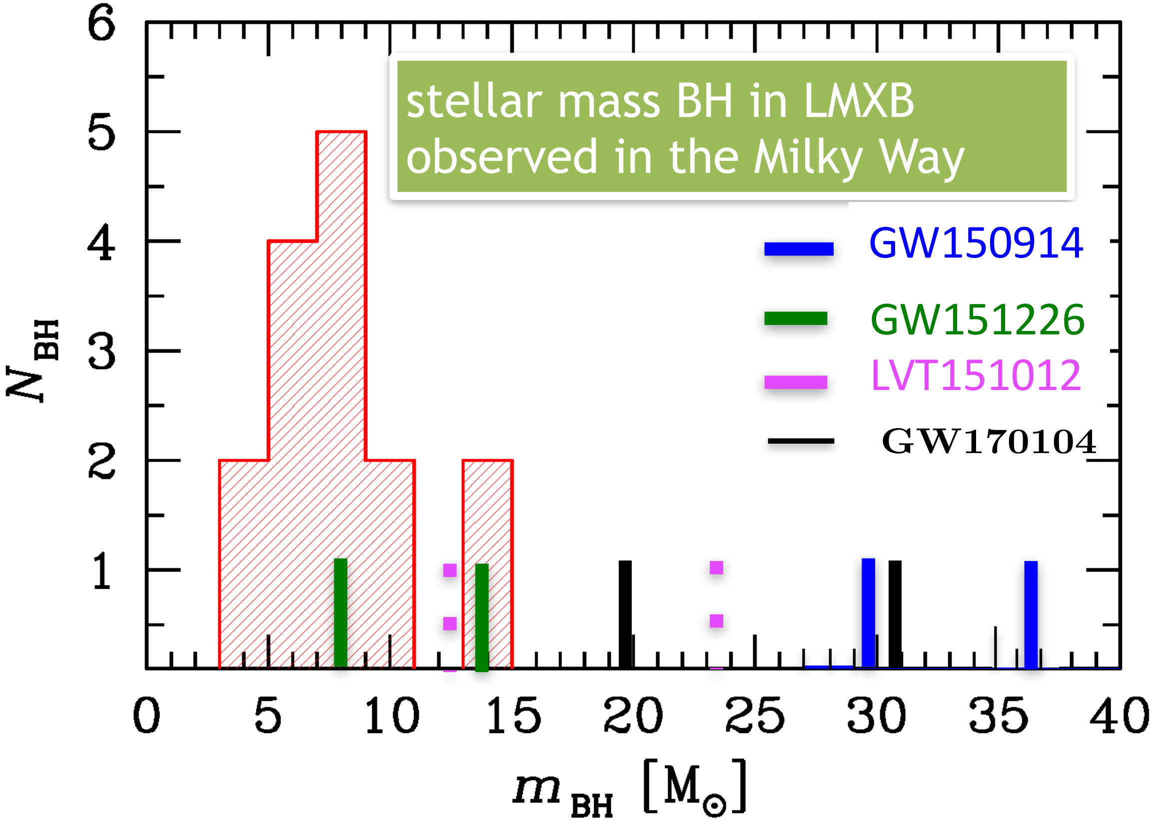
$$\mathbf{M} = \mathbf{n}\mathbf{M}_{\odot} \qquad \nu_0 \sim (\mathbf{12}/\mathbf{n})\mathbf{kHz} \qquad \tau \sim \mathbf{n} \cdot \mathbf{5.5} \times \mathbf{10}^{-5}\ \mathbf{s}$$

frequency increases up to 30% if the BH rotates

The frequency of the lowest quasi-normal mode has been extracted from the detected ringdown of the firts event GW150914. The black hole **mass and angular momentum** agree with the values found from the merging

WHAT DID WE KNOW ABOUT BLACK HOLES BEFORE GW DETECTION

- supermassive black holes $10^6\ M_\odot \lesssim M \lesssim 10^{11}\ M_\odot$
- stellar mass black holes $5\ M_\odot \lesssim M \lesssim 15-20\ M_\odot$



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GW170104

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Orosz etal 2003
Ozel etal 2013

We now know that there is a population of binary black holes with masses ≳ 20 M⊙ and merger rates are large enough to expect more detections.

How did the "heavy" BHs and BH binaries form?

- "heavy" BHs as in GW150914 and in GW170104 ~ 30 M⊙ or larger, are most likely formed in the direct collapse of low metallicity stars
  (below Z ≈ 0.5 Z⊙ , where Z⊙ ≈ 1,6 % of the total mass)

B.P. Abbott et al., Physical Review Letters 116 (2016), 118 (2017)

… but low mass loss may have been possible at higher metallicity if the progenitor stars were strongly magnetized (*Petit etal MNRAS 466, 1052, 2017*).

… or, part of these large mass black holes may be be primordial, i.e. generated by inflation fields fluctuations, which may produce large curvature peaks … (*Carr, Kuhnel, Sandstad, Phys. Rev. D 94 2016*)

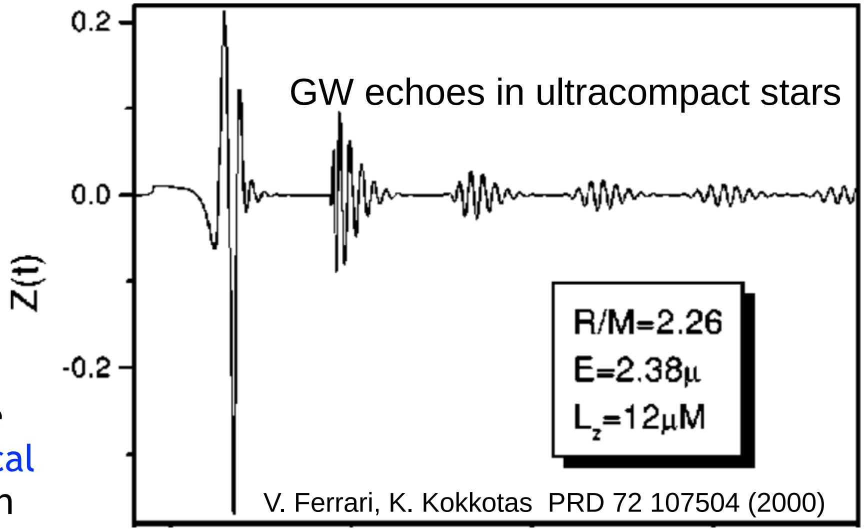
- the observed BH binaries may have been formed:
  - by the evolution of isolated binaries by a BH and a star,
  - or dinamically, by close encounters in three-body systems possible in dense clusters

Ziosi et al MNRAS 441, 2014, Kimpson et al MNRAS 463, 2016

The formation channel depends not only on the mass ratio, but also on the BH spins: these are not measured with sufficient accuracy in the detected signals. More events and larger signal-to-noise ratios will be needed

The coalescing compact objects were two black holes or … something else?

- ★ We are sure that the coalescing objects are extremely compact
- ★ the mass and spin of the final BH estimated from the merging part of the signal agrees with those extracted from the ringing tail, in the frame of General Relativity
- ★ However, the quality of the data is such that some room is left for alternative interpretations that do not involve black holes, but other objects that, either within classical General Relativity, or in modified theories of gravity, can be equally massive and compact, i.e. gravastars, boson stars, whormholes etc



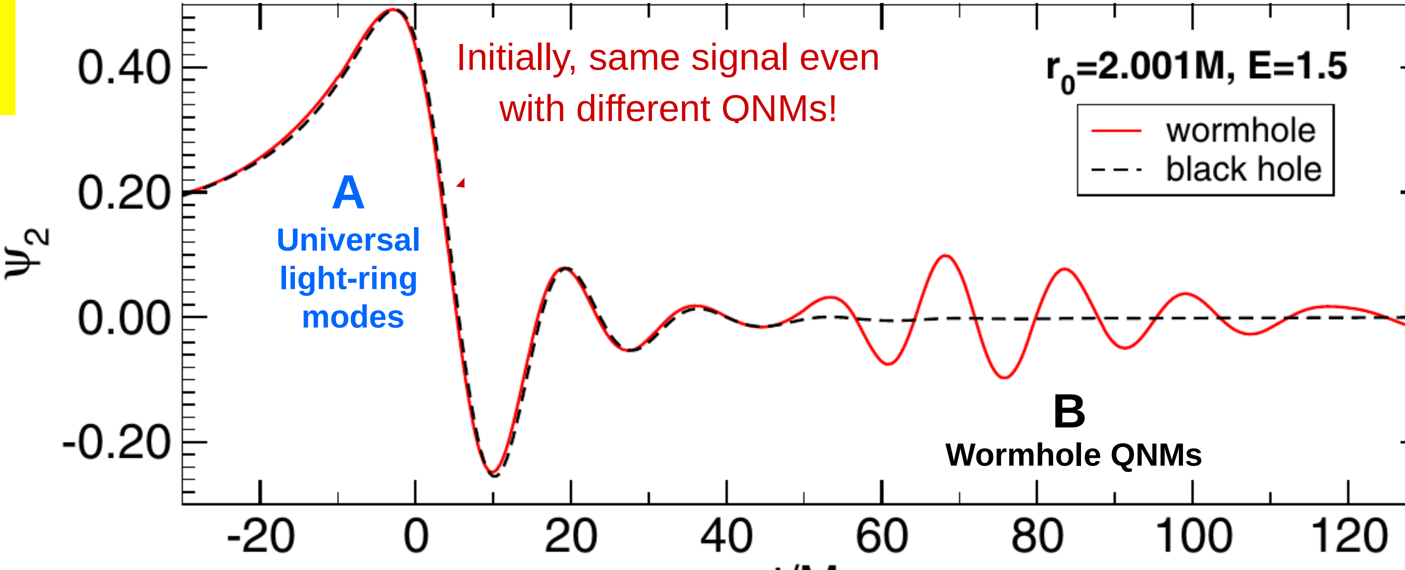
GW echoes in ultracompact stars

V. Ferrari, K. Kokkotas PRD 72 107504 (2000)

More signature to be considered: tidal heating, tidal deformability, etc

Future detections with larger SNR will shed light on this important question

Cardoso, Franzin, Pani; Phys.Rev.Lett. 116 (2016) 17110



Initially, same signal even with different QNMs!

A
Universal light-ring modes

B
Wormhole QNMs

There are other sources we are looking for and unsolved questions we want to answer using gravitational waves

Neutron stars (NS) in different phases of their life:

- Coalescing binaries composed of two neutron stars, or of a neutron star and a black hole
- gravitational collapse to a neutron stars
- spinning neutron stars
- oscillating of neutron stars

Astrophysics:

are coalescing NS-NS or NS-BH sourcing Gamma Ray Bursts?

gravitational collapse: how is it ignited?

what is the shape of a neutron star?

are there sources which we do not know?

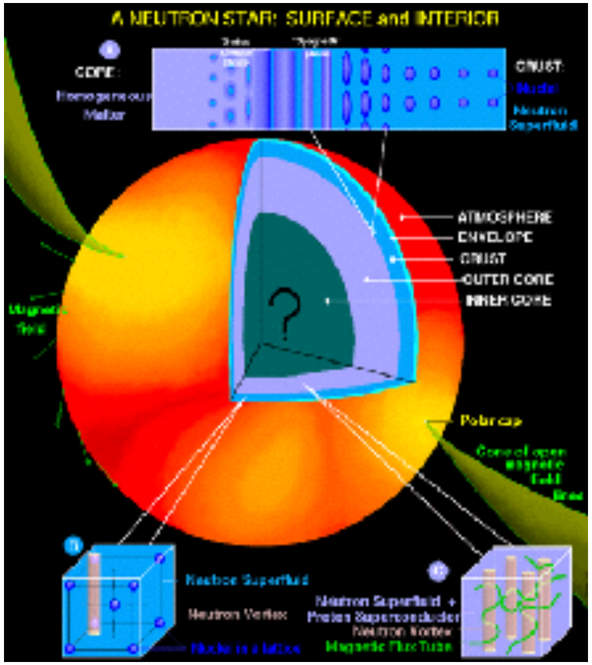
Fundamental physics:

how does matter behave at the extreme densities of a neutron star core?

NEUTRON STARS:

observed mass: [1-2]$M_\odot$

radius: difficult to measure (about 13-15 % accuracy)
[10-15] km (teoretical)



credits D. Page

In the inner part of the core of a neutron star, the density can be larger than the equilibrium density of nuclear matter

$\rho_0 = 2.67 \times 10^{14}$ g /cm3

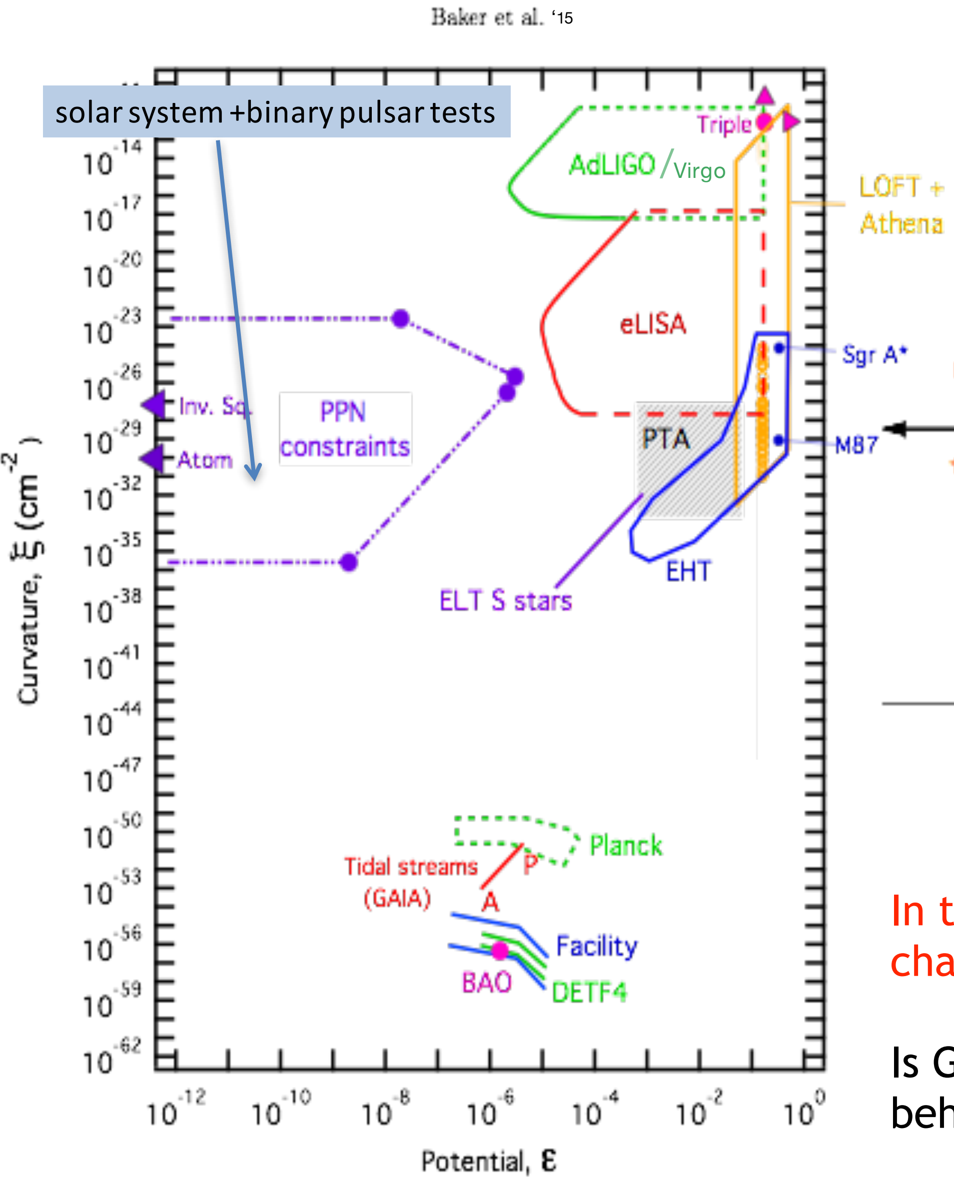
typical densities ≈ 2-5 ρ_0 or more

At these densities (unreachable in a laboratory) hadrons interactions cannot be neglected, and have to be treated in the framework of the theory of Quantum Cromo Dynamics

even the particle content is unknown: Hadrons? Hyperons? Meson condensates?
Deconfined quark matter?

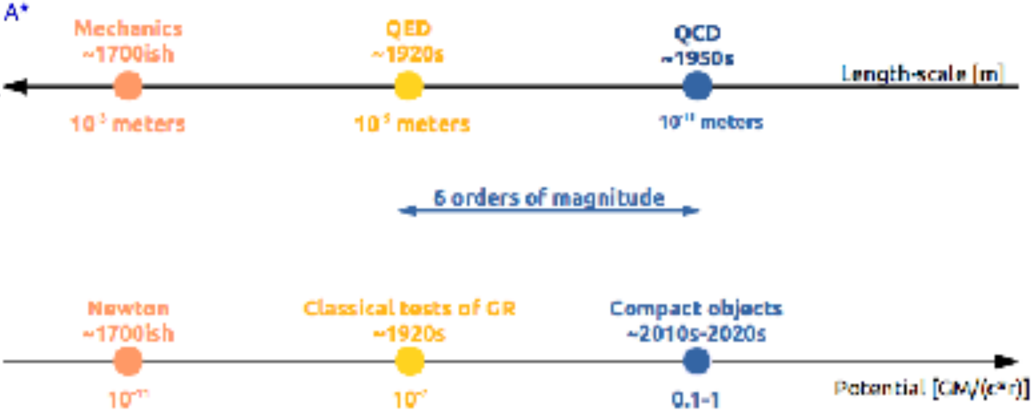
Several different models have been proposed which have to be tested

- **Before GW150914, we had tested only the weak-field regime of gravity (solar system tests, binary pulsars) Now, the realm of strong gravity is open to exploration!**



$$\epsilon = \frac{GM}{r}$$ gravitational potential

$$\zeta = \frac{GM}{r^3}$$ spacetime curvature



In the past, when changing scale interaction also changed!

Is General Relativity appropriate to describe the behaviour of gravity at the horizon scale?

Gravitational waves will be the probe through which we will be able to explore this mysterious and fascinating region of the spacetime