Interplay of $U_A(1)$ and chiral symmetry breakings and restorations

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Overview

Some signatures of dynamical chiral symmetry breaking (DChSB) and restoration

$U_A(1)$ symmetry breaking is why $\eta_0 \approx \eta'$ has an anomalous piece of mass

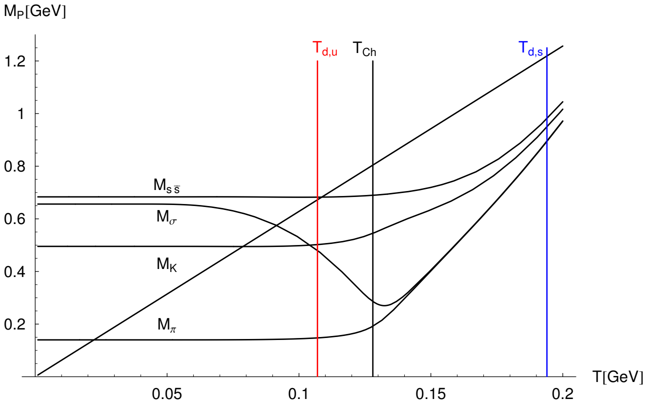
$T = 0$ results on η and η' using WVR

Shore's generalization of WVR - full, and with the chiral condensate approximation

Summary

Some signatures of dynamical chiral symmetry breaking (DChSB) and restoration

- DChSB dresses light ($q = u, d, s$) current quarks and so creates much more massive constituent quarks, and QCD vacuum condensates $\langle q\bar{q}\rangle$, and (very light) pseudoscalar mesons as (almost-) Goldstone bosons



- 'Deconfinement' $T_{d,q}$ from S_q pole - some models predict very different $T_{d,u}$, $T_{d,s}$ ... can be synchronized with $T_{\rm Ch}(= T_{cri})$ by Polyakov loop
- But what about η and η', both at $T = 0$ and $T > 0$?

$U_A(1)$ symmetry breaking is why $\eta_0 \approx \eta'$ has an anomalous piece of mass

$U_A(1)$ symmetry is broken by nonabelian ("gluon") axial anomaly: even in the chiral limit (ChLim, where $m_q \to 0$),

$$\partial_\alpha \bar{\psi}(x)\gamma^\alpha \gamma_5 \frac{\lambda^0}{2}\psi(x) \propto F^a(x)\cdot \widetilde{F}^a(x) \equiv \epsilon^{\mu\nu\rho\sigma} F^a_{\mu\nu}(x) F^a_{\rho\sigma}(x) \neq 0\,.$$

This breaks the $U_A(1)$ symmetry of QCD and precludes the 9^{th} Goldstone pseudoscalar meson $\Rightarrow$ very massive η': even in ChLim, where $m_\pi, m_K, m_\eta \to 0$, still ('ChLim WVR')

$$0 \neq \Delta M^2_{\eta_0} = \Delta M^2_{\eta'} = \frac{(A = \text{qty.dim.mass})^4}{(\text{“}f_{\eta'}\text{''})^2} = \frac{6\,\chi_{\text{YM}}}{f_\pi^2} + O(\frac{1}{N_c})$$

$$\text{Out of ChLim}: \quad {M_{\eta'}}^2 + {M_\eta}^2 - 2\,{M_K}^2 = \frac{2N_f}{f_\pi^2}\,\chi_{\text{YM}} \quad \left(+O(\frac{1}{N_c})\right)$$

Anomalous part of η_0 mass $\Delta M_{\eta_0}^2 = \chi_{\rm YM} \frac{2N_f}{f_\pi^2} + O(\frac{1}{N_c})$

QCD chiral behavior (reproduced by (e.g.) DS approach) of the non-anomalous parts of masses of light $q\bar{q}'$ pseudoscalars (i.e., all parts except ΔM_{η_0}) : $M_{q\bar{q}'}^2 = \mathrm{const}\,(m_q + m_{q'}),\ \ (q, q' = u, d, s)$.

$\Rightarrow$ non-anomalous parts of the masses in WVR cancel:
$M_{\eta'}{}^2 + M_\eta{}^2 - 2\,M_K{}^2 \approx \Delta M_{\eta_0}{}^2$, approx. as in ChLim WVR

$$\chi = \int d^4x\, \langle 0|Q(x)Q(0)|0\rangle \,, \qquad Q(x) = \frac{g^2}{64\pi^2}\epsilon_{\mu\nu\rho\sigma} F^a_{\mu\nu}(x) F^a_{\rho\sigma}(x)$$

- $Q(x)$ = topological charge density operator
- In WV rel., χ is the pure-glue, YM one, $\chi_{\rm YM} \leftrightarrow \chi_{quench}$, reproduced reliably by lattice, but for χ of light-flavor QCD, use Di Vecchia-Veneziano

relation:
$$\chi = -\frac{\langle \bar{q}q\rangle_0}{\sum\limits_{q=u,d,s} \frac{1}{m_q}} + \mathcal{C}(\text{unknown corrections, higher } \mathcal{O} \text{ in small } m_q)$$

.

Results on η and η' (at $T=0$) with $\Delta M_{\eta_0} = 6\chi_{\rm YM}/f_\pi^2$ from WVR

	$\beta_{\rm fit}$	$\beta_{\rm latt.}$	Exp.
θ	-12.22deg	-13.92deg	
M_η [MeV]	548.9	543.1	547.75
$M_{\eta'}$ [MeV]	958.5	932.5	957.78
X	0.772	0.772	
3β [GeV^2]	0.845	0.781	

- $X = f_\pi/f_{s\bar{s}}$ as well as the whole $\hat{M}^2_{NA}$ (consisting of M_π and $M_{s\bar{s}}$) are calculated model quantities (in SD approach).
- $\beta_{\rm latt.} = \Delta M_{\eta_0}/(2+X^2)$ was obtained from $\chi_{\rm YM}(T=0) = (175.7\ {\rm MeV})^4$
- But is an extension to high T possible, as there is a large mismatch of characteristic temperature scales of the pure-gauge YM ($T_c \sim 270$ MeV) vs. full QCD ($T_c \sim 160$ MeV) with quarks?
- $\Rightarrow$ in WVR, $\chi_{\rm YM}$ is more T-resistant than QCD quantities $M_{\eta,\eta',K}$ and f_π.
- $\Rightarrow$ Conflict with experiment [Horvatić&al,PRD76(2011)] ... Does WVR become unusable as T approaches $T_{\rm Ch}$ of full QCD ?
- But Shore's generalization of WVR does **NOT** have this mismatch of the full QCD and pure-gauge YM temperature scales! Try this?

Shore's generalization of WV valid to all orders in $1/N_c$

$$(f^0_{\eta'})^2 M^2_{\eta'} + (f^0_\eta)^2 M^2_\eta = \frac{1}{3}\left(f^2_\pi M^2_\pi + 2 f^2_K M^2_K\right) + 6A \quad (1)$$

$$f^0_{\eta'} f^8_{\eta'} M^2_{\eta'} + f^0_\eta f^8_\eta M^2_\eta = \frac{2\sqrt{2}}{3}\left(f^2_\pi M^2_\pi - f^2_K M^2_K\right) \quad (2)$$

$$(f^8_{\eta'})^2 M^2_{\eta'} + (f^8_\eta)^2 M^2_\eta = -\frac{1}{3}\left(f^2_\pi M^2_\pi - 4 f^2_K M^2_K\right) \quad (3)$$

The role of $\chi_{\rm YM}$ taken over by the full QCD topological charge parameter A ,

$$A = \frac{\chi}{1 + \chi\left(\frac{1}{\langle \bar{u}u\rangle\, m_u} + \frac{1}{\langle \bar{d}d\rangle\, m_d} + \frac{1}{\langle \bar{s}s\rangle\, m_s}\right)} \quad (4)$$

- A should behave with T as a full QCD quantity
- ... **but**, at $T = 0$ it is known that $A = \chi_{\rm YM} + \mathcal{O}(\frac{1}{N_c})$

Note (1)+(3) $\Rightarrow (f^0_{\eta'})^2 M^2_{\eta'} + (f^0_\eta)^2 M^2_\eta + (f^8_\eta)^2 M^2_\eta + (f^8_{\eta'})^2 M^2_{\eta'} - 2 f^2_K M^2_K = 6A$

- Then, large N_c limit and 'off-diagonal' $f^0_\eta, f^8_{\eta'} \to 0$, as well as $f^0_{\eta'}, f^8_\eta, f_K \to f_\pi$, recovers the standard WV.

Approximate all 3 light condensates by $\langle \bar{q} q \rangle_0$, the chiral-limit one!

This reduces the full QCD topological charge A, Eq. (4), to the remarkable Leutwyler-Smilga relation (LS), which is still valid for both large and small values of m_q:

$$\chi_{\rm YM} = \frac{\chi}{1 + \frac{\chi}{\langle \bar{q} q \rangle_0} \sum_{q=u,d,s} \frac{1}{m_q}} \equiv \widetilde{\chi} \to \widetilde{\chi}(T) \approx A(T)$$

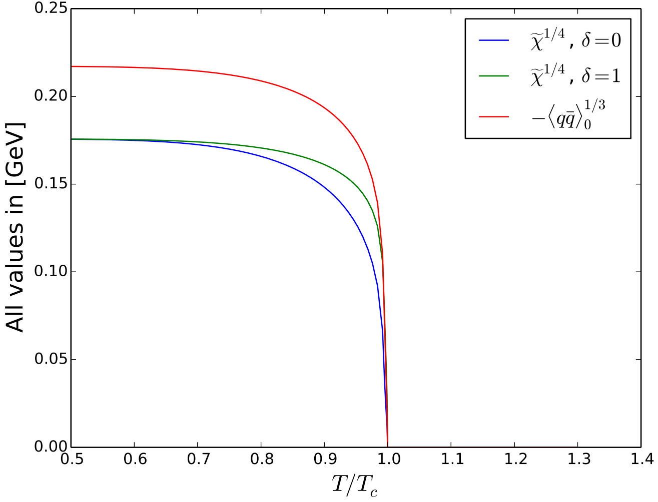
where for the light quarks $\quad \chi = - \frac{1}{\sum_{q=u,d,s} \frac{1}{m_q \, \langle \bar{q} q \rangle_0}} + \mathcal{C}(m)$

.

- $\mathcal{C}(m)$ = small corrections of higher orders in small m_q, ... but $\mathcal{C}(m)$ should not be neglected, since $\mathcal{C}(m) = 0$ would imply that $\chi_{\rm YM} = \infty$.
- LS relation fixes the value of the correction at $T = 0$:

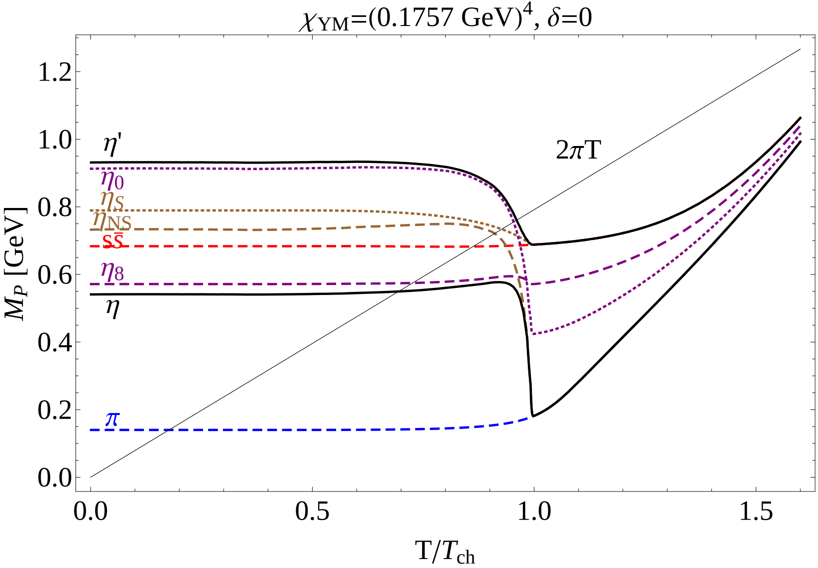
$$\frac{1}{\mathcal{C}(m)} = \sum_{q=u,d,s} \frac{1}{m_q \langle \bar{q} q \rangle_0} - \chi_{\rm YM}(0) \left(\sum_{q=u,d,s} \frac{1}{m_q \langle \bar{q} q \rangle_0} \right)^2 .$$

Chiral condensate $\langle q\bar{q}\rangle_0(T)$ and resulting $\widetilde{\chi}(T)$

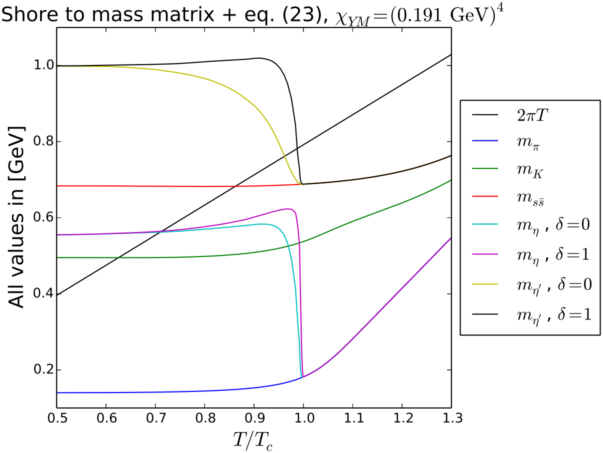


Prediction good for η', but for η not supported by any experiment

[Benić, Horvatić, Kekez and Klabučar, Phys. Rev. D **84** (2011) 016006.]

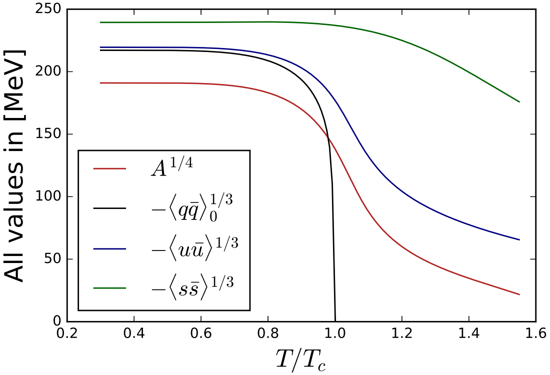


Variations of model, or input or model parameters, do not change much ...



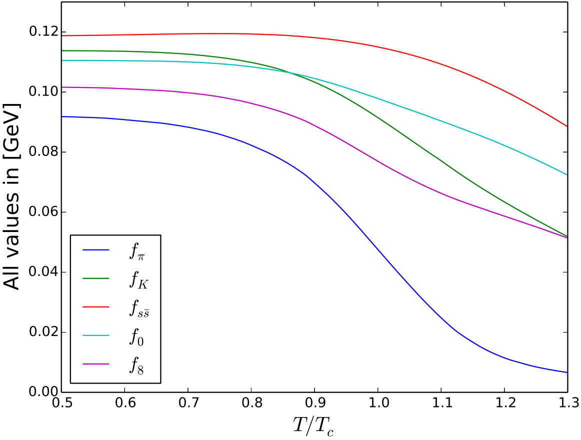
... mass drop prediction still good for η' (where Csörgö and collaborators had found this in RHIC data), but again an even larger mass drop for η, which is not supported by any experiment.

A solution: $U_A(1)$ breaking from realistic condensates



Instead of the fast-falling chiral-limit condensate $\langle \bar{q}q \rangle_0$, try $\langle \bar{q}q \rangle$ condensates with realistic explicit chiral symmetry breaking: replace $m_q \langle \bar{q}q \rangle_0 \to m_q \langle \bar{q}q \rangle$, $(q = u, d, s)$ in χ, like in the original A.

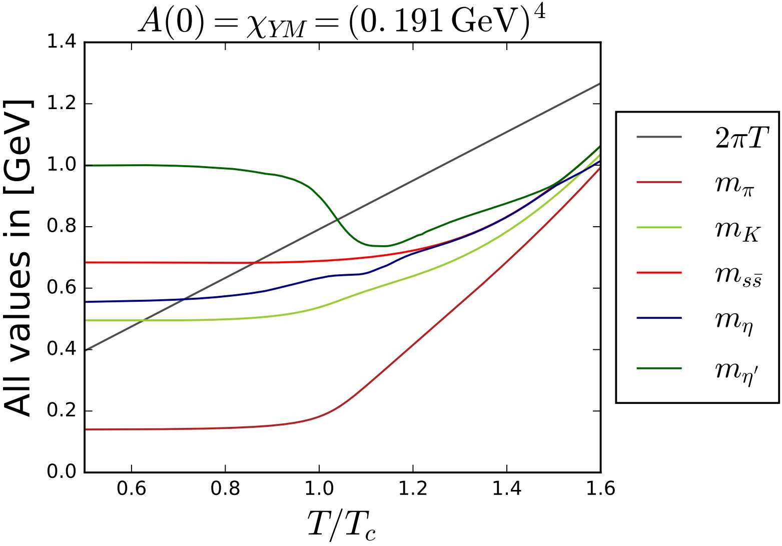
T-dependence of pseudoscalar decay constants



How they influence the elements of the η-η' mass matrix:

$$M_{\rm NS}^2 = M_\pi^2 + \frac{4A}{f_\pi^2}\,, \qquad M_{\rm NSS}^2 = \frac{2\sqrt{2}A}{f_\pi\, f_{s\bar{s}}}\,, \qquad M_{\rm S}^2 = M_{s\bar{s}}^2 + \frac{2A}{f_{s\bar{s}}^2}$$

⇒ Acceptable T dependence of light pseudoscalars including η and η'



Summary

- Our approach tied the $U_A(1)$ SB to the DChSB so closely, that the restoration of the chiral symmetry must lead to the restoration of the $U_A(1)$ symmetry at least partially, on the level of the η' & η masses.
- **We again got the η' mass drop** $\approx$ 300 MeV. But, the lighter isoscalar η suffers a qualitatively different fate due to a quantitative difference in the description of the two "light" $\langle q\bar{q}\rangle$ condensates, which influence results strongest, being associated with the lightest masses m_u & m_d.
- The condensate $\langle q\bar{q}\rangle_0$, evaluated in i the chiral limit $m_q \to 0$, falls to zero abruptly as $T \to T_{Ch}$. This had in the past given us the abrupt η mass drop of $\approx$ 400 MeV at T_{Ch} and abrupt degeneracy with the pion – but heavy ion collisions could not find any increase of η multiplicity.
- The cond's $\langle q\bar{q}\rangle$ $(q = u, d, s)$ calculated with realistic **explicit** chiral symmetry breakings fall with T much more slowly and smoothly than $\langle q\bar{q}\rangle_0$. $\Rightarrow$ similar T-behavior of the topological charge parameter $A(T)$. Its ratios with similarly varying decay constants in the elements of the mass matrix then lead to η not exhibiting any mass drop at all, but just the rise similar to other pseudoscalar octet members, at least till the anticrossing with η'. But, the behavior of η' is not changed much. Compared with the calculation with $\langle q\bar{q}\rangle_0$, $M_{\eta'}(T)$ falls again around T_{Ch} by almost 300 MeV, but much more slowly. After the anticrossing with η, it (the η'), becomes a pure $s\bar{s}$ pseudoscalar resonance without anomalous contributions to its mass, signaling the partial restoration of $U_A(1)$ symmetry.