QCD thermodynamics: beyond perturbation theory

A. Jakovác

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Goal describe thermodynamics of strongly interacting matter.

Description of the strongly interacting matter

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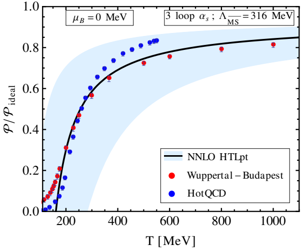

(J.O. Andersen *et.al.* 2014)

- at high energy scales (hight temperature): asymptotic freedom $\Rightarrow$ perturbative QCD; from $T \gtrsim 200 - 250$ MeV

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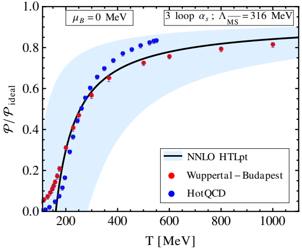

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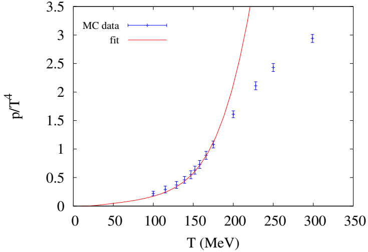


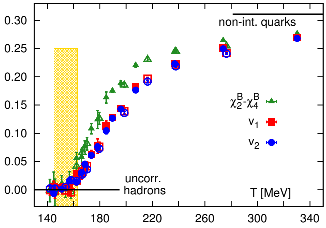
(T.S. Biró, A.J. 2014)

- at high energy scales (hight temperature): asymptotic freedom $\Rightarrow$ perturbative QCD; from $T \gtrsim 200 - 250$ MeV
- at low energy scales (low temperature): bound states are formed (hadrons) which interact "weakly" $\Rightarrow$ perturbative hadron gas (HRG) description; up to $T \lesssim 170$ MeV

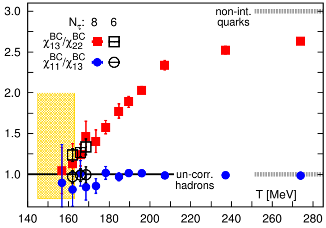
Phase transition regime

at $T \sim 156$ MeV (crossover) phase transition

Width of the phase transition regime



B and BS flututations
(A. Bazazov *et.al.* 2013)



BC fluctutations
(A. Bazazov *et.al.* 2014)

$\mathbf{150 \lesssim T \lesssim 250}$ MeV regime:
non-quasiparticle regime, changing degrees of freedom

nonperturbative, advanced methods are needed

Questions we address

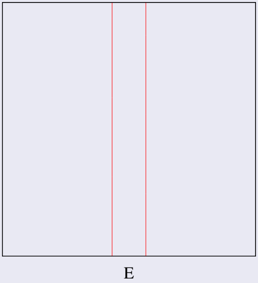
- How can we describe changing degrees of freedom?
- Where is the CEP in the $\mu - T$ plane?

Two particles with the same quantum numbers

same quantum number $\Rightarrow$ only their mass can differ!

What do we observe in a mass spectrometer?

spectrum of two particles



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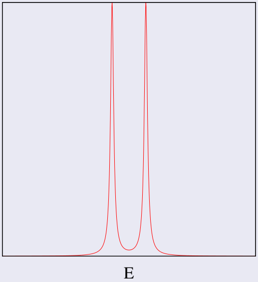

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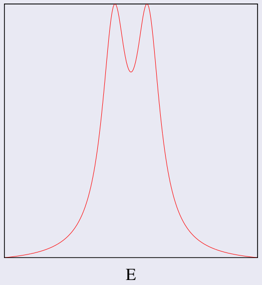

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  no measurements can resolve the peak structure!
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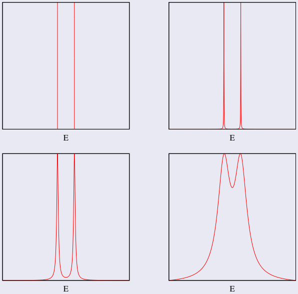

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- realistic: broadened 2 spectral lines
- widht $\sim$ mass difference:
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  the sates become indistinguishable
  $\Rightarrow$ represent 1 dof
- changing width
  $\Rightarrow$ changing degr. of freedom!

thermodynamic definition of # dof: $P = N_{eff} P_0$

P full pressure, P_0 one-particle pressure

Thermodynamics from spectral function

Goal: calculate pressure $P(\varrho)$

(T.S. Biro, A.J. and Zs. Schram 2016; T.S. Biro and A.J. 2014; AJ. 2012,2013)

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Strategy

- **represent** ϱ with a (quadratic) effective model
- calculate thermodynamics from this theory
  energy density $\varepsilon = \frac{1}{Z}\,\mathrm{Tr}\, e^{-\beta H} T_{00}$, use KMS relation

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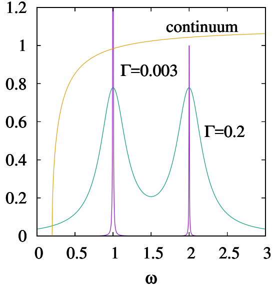
Pressure as a function of the spectral function

$$P = \mp T \int \frac{d^4q}{(2\pi)^4} \frac{\partial \mathcal{K}}{\partial q_0} \ln\left(1 \mp e^{-\beta q_0}\right) \varrho(q), \qquad \varrho = \mathrm{Disc}\, i\mathcal{K}^{-1}$$

- consistency check: for free gas mixture with masses m_i we obtain $P = \sum_i P_0(m_i)$: sum of one-particle partial pressures;
- generally nonlinear ϱ dependence (because $\mathcal{K}$ depends on ϱ)

Changing degrees of freedom for two particles

How thermodynamics changes when peaks are merged?



- spectrum for two particles with different width, and a typical multiparticle continuum (non-quasiparticle system)

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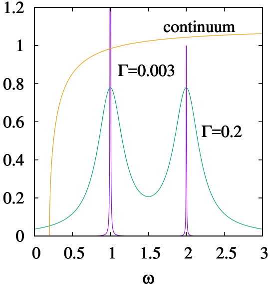

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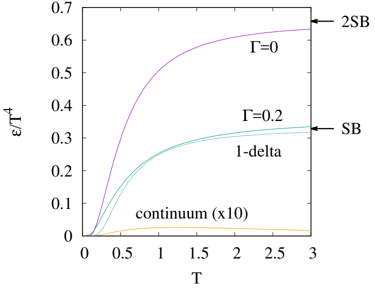
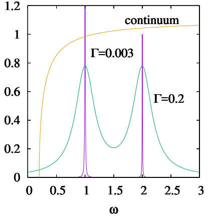


- spectrum for two particles with different width, and a typical multiparticle continuum (non-quasiparticle system)
- at small width ⇒ two-particle energy density
- at large width ⇒ ∼ one-particle energy density
- continuum: practically negligible energy density contibution

Effective number of degrees of freedom N_{eff}

merging peaks, $N_{eff} \to 1$ $\Rightarrow$ **indistinguihability**



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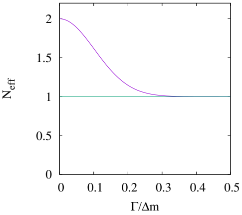

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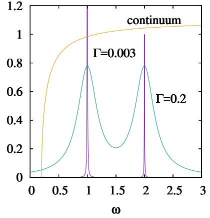

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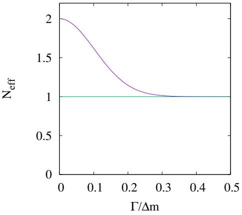
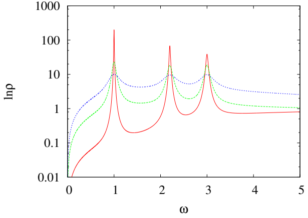


merging with continuum: $N_{eff} \to 0$



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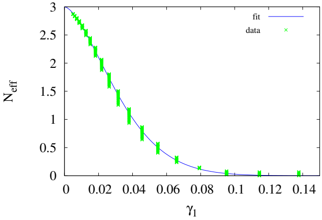

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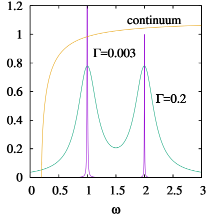

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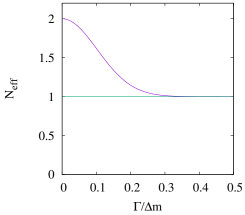
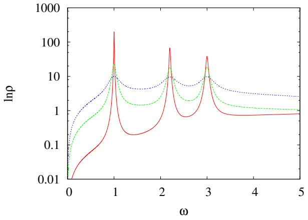


merging with continuum: $N_{eff} \to 0$ $\Rightarrow$ **melting**



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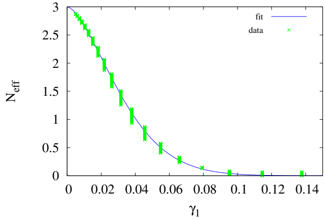

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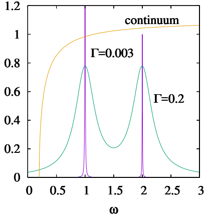

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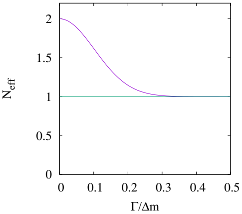

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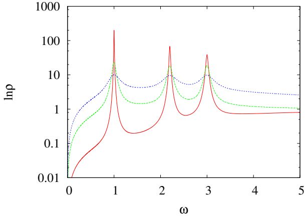

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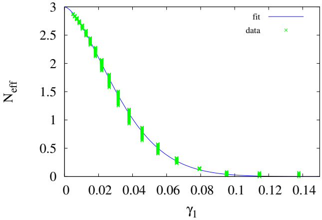

good fitting function: $N_{eff} = N_0 + N_1 e^{-a\gamma^b}$ (typically $b = 1.5 - 2$)

Application to QCD

Simple Ansatz: $P = N_{eff}^{(hadr)} P_{hadr} + N_{eff}^{(QGP)} P_{QGP}$

(T.S. Biro, A.J. and Zs. Schram 2016; T.S. Biro and A.J. 2014)

- hadrons: Hagedorn spectrum; common, T-dependent width
- partons: $N_{eff}^{(QGP)}$ depends on the density of hadrons!

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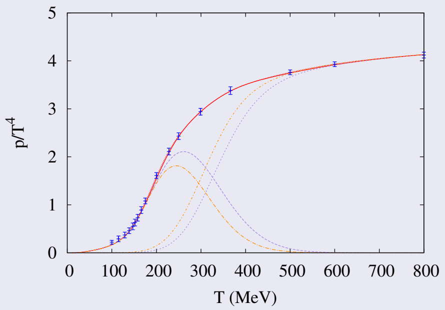
QCD partial pressures



- total pressure is well reproduced
- width of melting interval is tunable
- hadrons do not vanish at T_c: they just start to melt there.
- quarks just start to appear at T_c

MC calculation of phase diagram

Formula to compute on lattice:

$$\mathrm{Tr}\, e^{-\beta(H-\mu N)}\,\hat{O} = \frac{1}{Z}\int \mathcal{D}U\, e^{-S_g}(\det \mathcal{M}(\mu))\,\bar{O}[U]$$

where

- S_g gauge action
- $S_f(\mu) = \int d^4x \bar{\Psi}(D_\mu\gamma_\mu + m - \gamma_0\mu)\Psi = \int d^4x \bar{\Psi}\mathcal{M}(\mu)\Psi$
  fermionic action with chemical potential

Problem **(sign problem)**: $\det \mathcal{M}(\mu)$ is not real!

$\gamma_5\mathcal{M}(-\mu)\gamma_5 = \mathcal{M}^\dagger(\mu) \quad \Rightarrow \quad \boxed{\det \mathcal{M}^*(\mu) = \det \mathcal{M}(-\mu).}$

Consequence: $e^{-S_g[U]} \det \mathcal{M}(\mu)$ is not a probability measure

$\Rightarrow$ **No importance sampling!**

Solutions: Reweighting

Idea: generate configurtions at $T' \neq T$ and $\mu = 0$, and use them to calculate the finite μ case:

$$\mathrm{Tr}\, e^{-\beta(H-\mu N)} = \int \mathcal{D}U\, e^{-S_g(\beta)} (\det \mathcal{M}(\beta,\mu)) =$$

$$= \int \mathcal{D}U \frac{e^{-S_g(\beta)} (\det \mathcal{M}(\beta,\mu))}{e^{-S_g(\beta')} (\det \mathcal{M}(\beta',0))} e^{-S_g(\beta')} (\det \mathcal{M}(\beta',0)) =$$

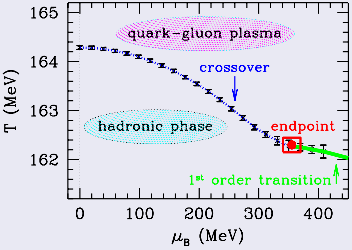
$$= \left\langle \frac{e^{-S_g(\beta)} (\det \mathcal{M}(\beta,\mu))}{e^{-S_g(\beta')} (\det \mathcal{M}(\beta',0))} \right\rangle Z(\beta', \mu = 0).$$

critical endpoint

(Z. Fodor, S. Katz, 2001, 2004)

$$T_E = 162 \pm 2\,\mathrm{MeV},$$
$$\mu_E = 360 \pm 40\,\mathrm{MeV}.$$

The phase diagram



How reliable is this result?

Radius of convergence of rescaling (de Forcrand 2010)

- rescaling: ratio of two partition functions with energy difference
$$\frac{Z(\mu)}{Z(0)} = e^{-\beta V \Delta f}$$
$\Rightarrow$ overlap exponentially vanishes for large volumes
Statistics grows with $\sqrt{N_{step}}$ $\Rightarrow$ exponentially large number of steps are required in the thermodynamic limit
- characterization of sign problem: isospin chemical potential μ
$$\langle e^{2i\Theta} \rangle = \left\langle \frac{\det^2 \mathcal{M}(\mu)}{|\det \mathcal{M}(\mu)|^2} \right\rangle \sim e^{-\#\mu^2} \quad \Rightarrow \quad \# \text{ config.} \sim e^{\#\mu^2}.$$
- At imaginary $\mu/T \approx i\pi/3$ Roberge-Weiss phase transition
$\Rightarrow$ restrict radius of convergence

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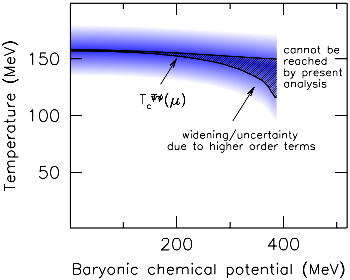
Conclusion

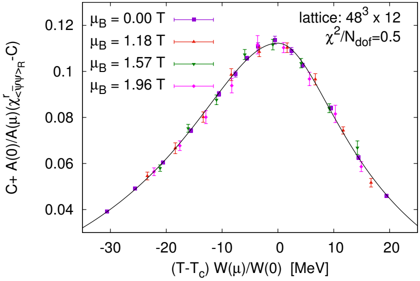
MC methods are reliable for $\frac{\mu_B}{3} = \mu \lesssim T$

Latest MC results

(Bellwied et.al (BMW group) 2015)

- Imaginary μ_B calculation (no sign problem)
- Taylor expand results in μ and continue to real axis





- left panel: radius of convergence
- right panel: for all μ_B the susceptibility curves can be scaled to each other $\Rightarrow$ analytic

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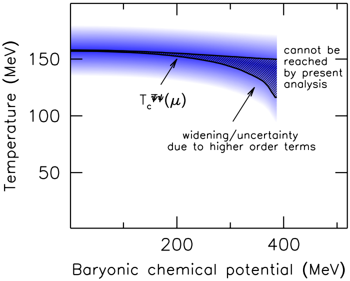
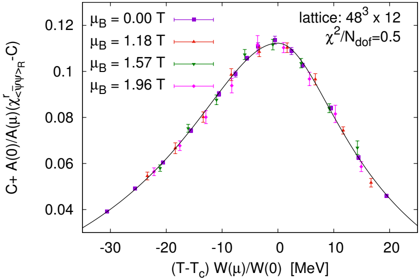




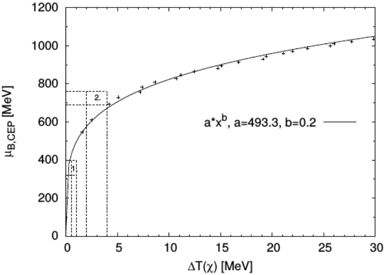
Consequence

no sign of a phase transition until $\mu_B \lesssim 400$ MeV!

Semianalytic method

(cf. P. Kovács dissertation)

- CEP found at nonphysical pion mass, not at cont. limit.
- **one-parameter scaling hypothesis**: assume that one parameter determines the extrapolation to physical point
  choose $\Delta T(\chi)$ width of the susceptibility curve



- μ_{CEP} vs $\Delta T(\chi, \mu = 0)$: model calculations fits to numerical data (Fodor, Katz 2001, 2004).
- Phyiscal point at $\mu = 0$: width of susc. curve is $\Delta T(\chi) \approx 28$ MeV (Aoki, Fodor et.al. 2006)

Prediction: $\mu_{CEP} \sim 1000$ MeV

Resummed perturbation theory

Effective model: chiral sigma model

(A.J, A. Patkós, Zs. Szép, P. Szépfalusy, 2004)

$$\mathcal{L} = \frac{1}{2}\varphi(-d^2 - m^2)\varphi - \frac{\lambda}{24N}(\varphi^2)^2 + \bar{\psi}[i\partial\!\!\!/ - m_q - \frac{g}{\sqrt{N}}\varphi T]\psi$$

$(\varphi = (\sigma, \pi_a),\ T = (1, i\sqrt{2N_f}T_a\gamma_5),\ N = 4, N_f = 2)$

- 1-loop resummed perturbation theory in large N expansion
- effective potential (free energy) $\Rightarrow$ phase transition at

$$\frac{g^2 N_f}{2\pi^2}\mu^2 + \left(\frac{\lambda}{36} + \frac{g^2 N_f}{6}\right) T^2 = m_\sigma^2$$

$\Rightarrow$ an ellipse in the $\mu - T$ plane

- position of the CEP analytically determined

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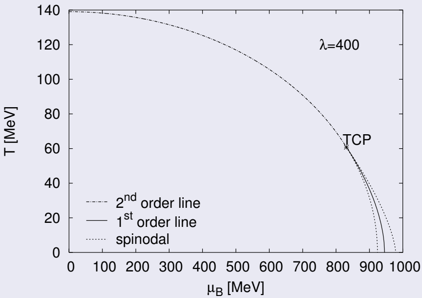
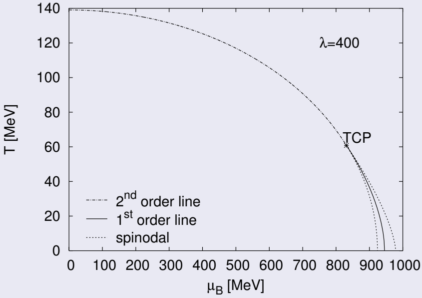
CEP from large N expansion



Prediction: $\mu_{CEP} \sim 850$ MeV

Other analytic models, role of bosonic fluctuations

QM large N with bosonic fluctuations

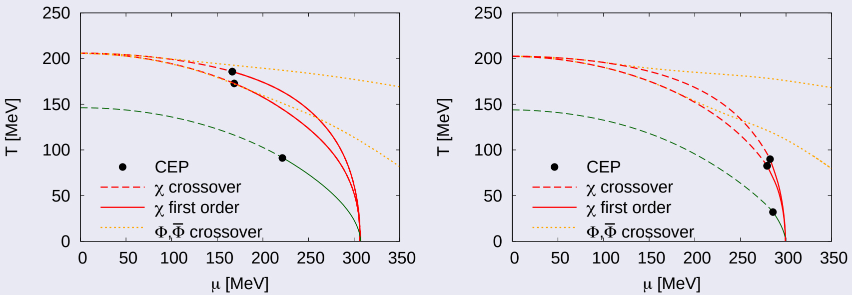


(A.J, A. Patkós, Zs. Szép, P. Szépfalusy, 2004)

QM: quark-meson, P: Polyakov-loop, DSE: Dyson-Schwinger-eq.

Other analytic models, role of bosonic fluctuations

chiral sigma model with/without Polyakov loops



(B.J Schaefer, M. Wagner, 2011)

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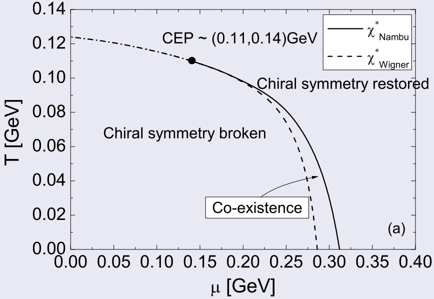
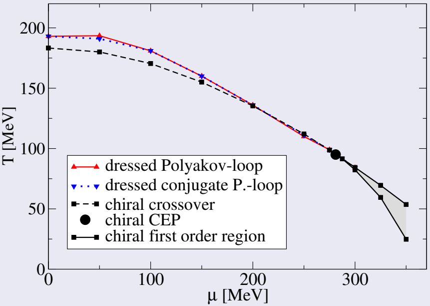
Other analytic models, role of bosonic fluctuations

DSE with/without Polyakov loops





(Craig D. Roberts *et.al.* 2010) (Ch.S. Fischer, J. Luecker, J.A. Mueller, 2011)

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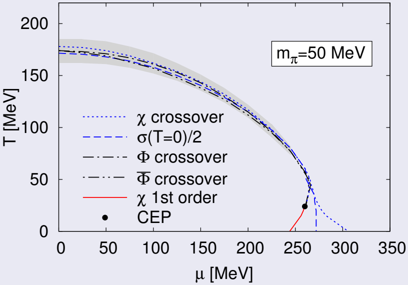
Other analytic models, role of bosonic fluctuations

PQM with FRG (full fluctuations)



(T.K. Herbst, J.M. Pawlowski, B.J Schaefer, 2013)

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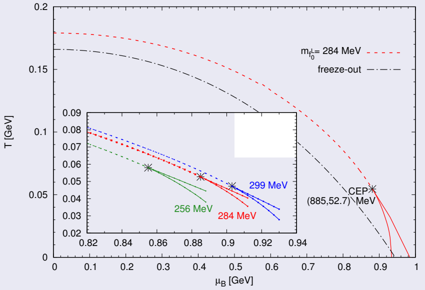
Vector meson extended PQM



(P. Kovács, Zs. Szép, Gy. Wolf, 2016)

QM: quark-meson, P: Polyakov-loop, DSE: Dyson-Schwinger-eq.

Other analytic models, role of bosonic fluctuations

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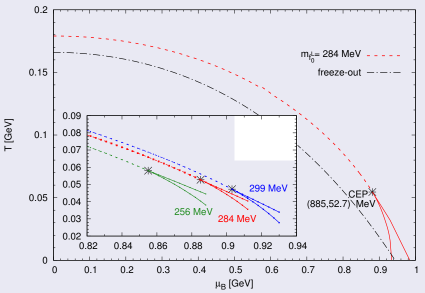

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QM: quark-meson, P: Polyakov-loop, DSE: Dyson-Schwinger-eq.

Lesson

- correct treatment of bosonic fluctuations are important!
- analytic methods with fluctuations: $\mu_{B,CEP} \approx 800 - 1000$ MeV
  DSE pure QCD approach: $\mu_{B,CEP} \approx 500$ MeV

Conclusions

- Thermodynamics of strongly interacting matter is perturbative for $T < 150$ MeV (HRG), and $T > 250$ MeV (QCD) (at $\mu = 0$)
- in the critical domain (analytically) changing dof
  $\Rightarrow$ hadron melting
  crucial: correct treatment of spectral properties

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- Thermodynamics of strongly interacting matter is perturbative for $T < 150$ MeV (HRG), and $T > 250$ MeV (QCD) (at $\mu = 0$)
- in the critical domain (analytically) changing dof
  $\Rightarrow$ hadron melting
  crucial: correct treatment of spectral properties
- location of the CEP: direct MC methods $\mu_{B,CEP} > 450$ MeV
- **crucial:** correct treatement of bosonic fluctuations (direct or Polyakov loops)
- latest results: $\mu_{B,CEP} \approx 800 - 1000$ MeV

Euclidean formalism of QCD

Quantum statistical averages can be computed as

$$Tr e^{-\beta H}\hat{O} = \int \mathcal{D}U\mathcal{D}\bar{\Psi}\mathcal{D}\Psi\, e^{-S}\, O[\Psi, U],$$

the action consists of a fermion and a gauge part $S = S_f + S_g$.

- The fermionic part (with D covariant derivative) :

$$S_f = \int d^4x\bar{\Psi}(D_\mu\gamma_\mu + m)\Psi = \int d^4x\bar{\Psi}\mathcal{M}\Psi, \qquad \{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu}$$

- The fermionic path integral yields

$$\int \mathcal{D}\bar{\Psi}\mathcal{D}\Psi\, e^{-S_f} = \det \mathcal{M}$$

- This contributes to the gauge action as

$$\boxed{Tr e^{-\beta H}\hat{O} = \int \mathcal{D}U\, e^{-S_g}(\det \mathcal{M})\, \bar{O}[U]}$$

Consistency: real expression, since ($\det \gamma_5 = 1$)

$$\gamma_5\mathcal{M}\gamma_5 = \mathcal{M}^\dagger \quad \Rightarrow \quad \det \gamma_5\mathcal{M}\gamma_5 = \det \mathcal{M} = \det \mathcal{M}^\dagger$$

Algorithm: produce configurations with probability $\sim e^{-S_g + \ln \det \mathcal{M}}$

Chemical potential

For a conserved quantity $N_q = \int d^4x \bar{\Psi}_q \gamma_0 \Psi_q$ we can introduce a chemical potential

$$e^{-\beta H} \to e^{-\beta(H-\mu N)}$$

This modifies the fermionic action

$$S_f(\mu) = \int d^4x \bar{\Psi}(D_\mu \gamma_\mu + m - \gamma_0 \mu)\Psi = \int d^4x \bar{\Psi} \mathcal{M}(\mu) \Psi.$$

Problem (sign problem): $\det \mathcal{M}(\mu)$ is not real!

$\gamma_5 \mathcal{M}(-\mu)\gamma_5 = \mathcal{M}^\dagger(\mu) \quad \Rightarrow \quad \boxed{\det \mathcal{M}^*(\mu) = \det \mathcal{M}(-\mu).}$

Consequence: $e^{-S_g[U]} \det \mathcal{M}(\mu)$ is not a probability measure

$\Rightarrow$ **No importance sampling!**

How reliable is this result?

Numerical arguments (de Forcrand 2010)

- rescaling: ratio of two partition functions with energy difference

$$\frac{Z(\mu)}{Z(0)} = e^{-\beta V \Delta f}$$

  $\Rightarrow$ overlap exponentially vanishes for large volumes
  Statistics grows with $\sqrt{N_{step}}$ $\Rightarrow$ exponentially large number of steps are required in the thermodynamic limit
- characterization of sign problem: isospin chemical potential μ

$$\langle e^{2i\Theta} \rangle = \left\langle \frac{\det^2 \mathcal{M}(\mu)}{|\det \mathcal{M}(\mu)|^2} \right\rangle$$

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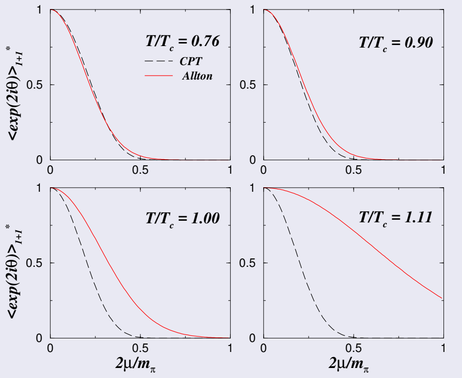

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- At imaginary $\mu/T \approx i\pi/3$ Roberge-Weiss phase transition
$\Rightarrow$ radius of convergence of the overlap to $\mu = 0$ case is of the order $\mu \sim T$ (ie. $\mu_B \lesssim T$).

Conclusion

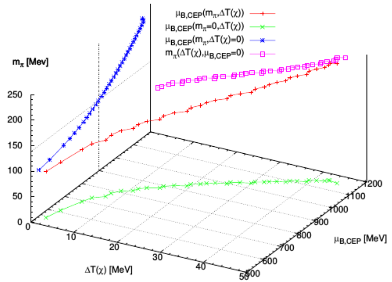
MC methods are reliable for $\frac{\mu_B}{3} = \mu \lesssim T$

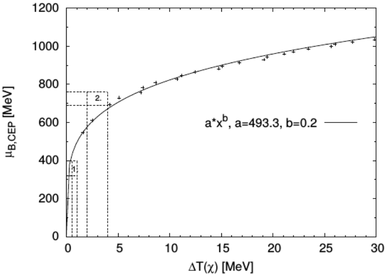
CEP at the physical point

The MC determined CEP is not at continuum limit, not at physical point (large quark masses)...
P. Kovács and Zs. Szép had an elegant line of thought to assess the CEP in the physical point (cf. P. Kovács dissertation)

- Assume that Z. Fodor *et.al.* found the CEP
- one-parameter scaling hypothesis
  experience: most sensitive quantity is pion mass m_π $\Rightarrow$
  Assume that the value of m_π determines the extrapolation to physical point
- in effective model calculation determine m_π-dependence of
  - width of the susceptibility peak $\Delta T(\chi)$
  - position of the CEP (μ_{CEP}, T_{CEP})
- finally determine the $\Delta T(\chi)$ dependence of the CEP!

One parameter scaling to continuum limit





- Model calculation: $\Delta T(\chi)$ vs m_π approx. linear
- μ_{CEP} vs $\Delta T(\chi)$ fits to numerical data (Fodor, Katz 2001, 2004).
- At phyiscal point at $\mu = 0$ the width of susceptibility curve is $\Delta T(\chi) \approx 28$ MeV (Aoki, Fodor et.al. 2006)

  $\Rightarrow$ Prediction $\mu_{CEP} \sim 1000$ MeV

Melting degrees of freedom in QCD

Simplified realization of these ideas to QCD

$P = P_{hadr} + P_{QGP}$ total pressure, P_0 ideal gas pressure

$$P_{hadr}(T) = N_{eff}^{(hadr)} \sum_{n \in \text{hadrons}}^{N} P_0(T, m_n), \qquad \ln N_{eff}^{(hadr)} = -(T/T_0)^b,$$

$$P_{QGP}(T) = N_{eff}^{(part)} \sum_{n \in \text{partons}} P_0(T, m_n), \qquad \ln N_{eff}^{(part)} = G_0 - c(N_{eff}^{(hadr)})^d.$$

- hadrons: Hagedorn-sp. up to a certain mass ($m \lesssim 3\,\mathrm{GeV}$)
- partons quark and gluon quasiparticles
- $N_{hadr}(\gamma)$ common suppression factor for all hadrons: stretched exponential, and $\gamma \sim T$
- $N_{part}(N_{hadr})$ partonic suppression factor grows with the # of available hadronic resonances.

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