Non-extensive Statistical Analysis of High Energy Particle Collisions

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Zimányi School'16

Dec. 5. - Dec. 9. 2016.

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Content

- Motivation
  - Why de we need high energy collisions?
- Non-extensive statistics
  - Small system, large fluctuations: Tsallis – Pareto distribution
- Fitting hadron spectra
  - CM energy evolution of the parameters
- Summary

Why do we need heavy ion collisions?

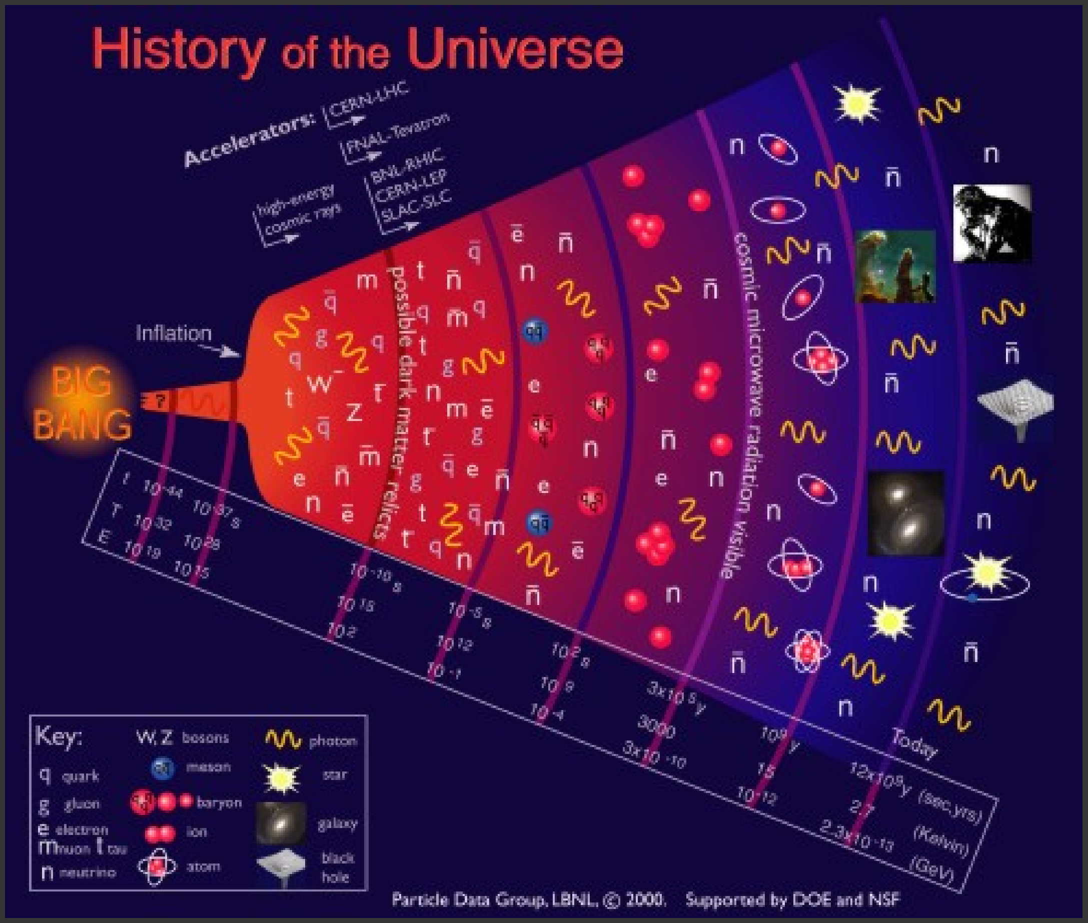


Image source: https://www.phy.duke.edu

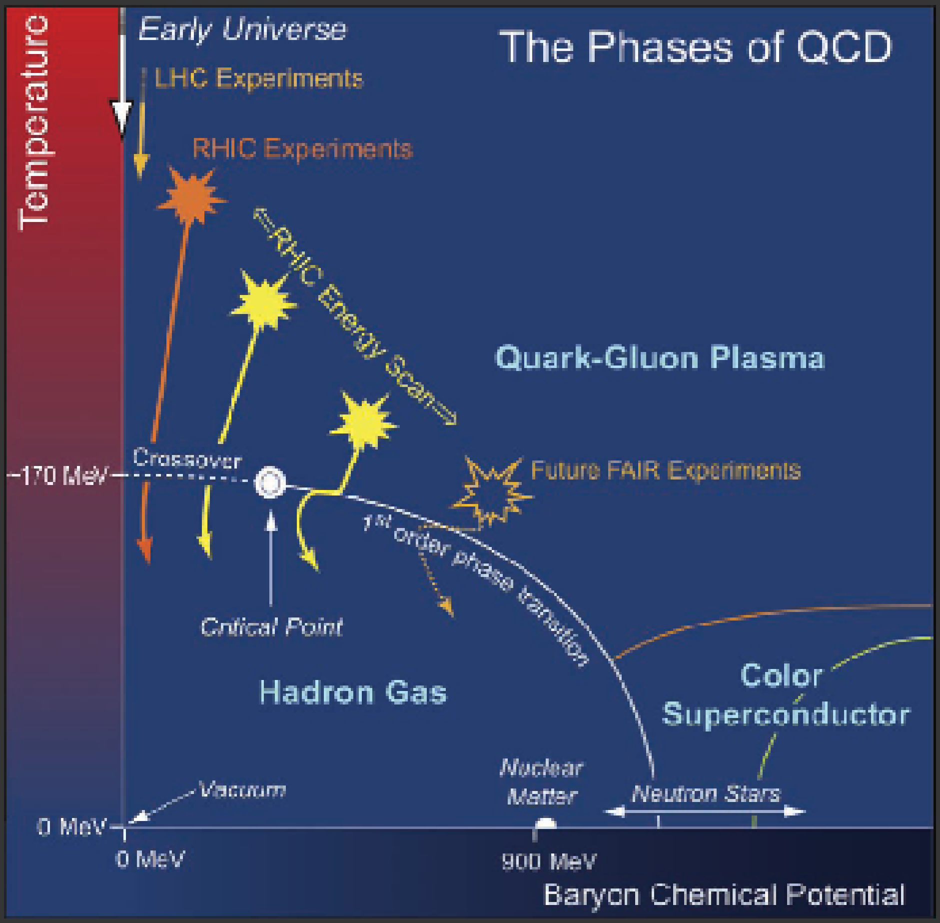


Image source: arXiv:1308.3328

Why do we need heavy ion collisions?

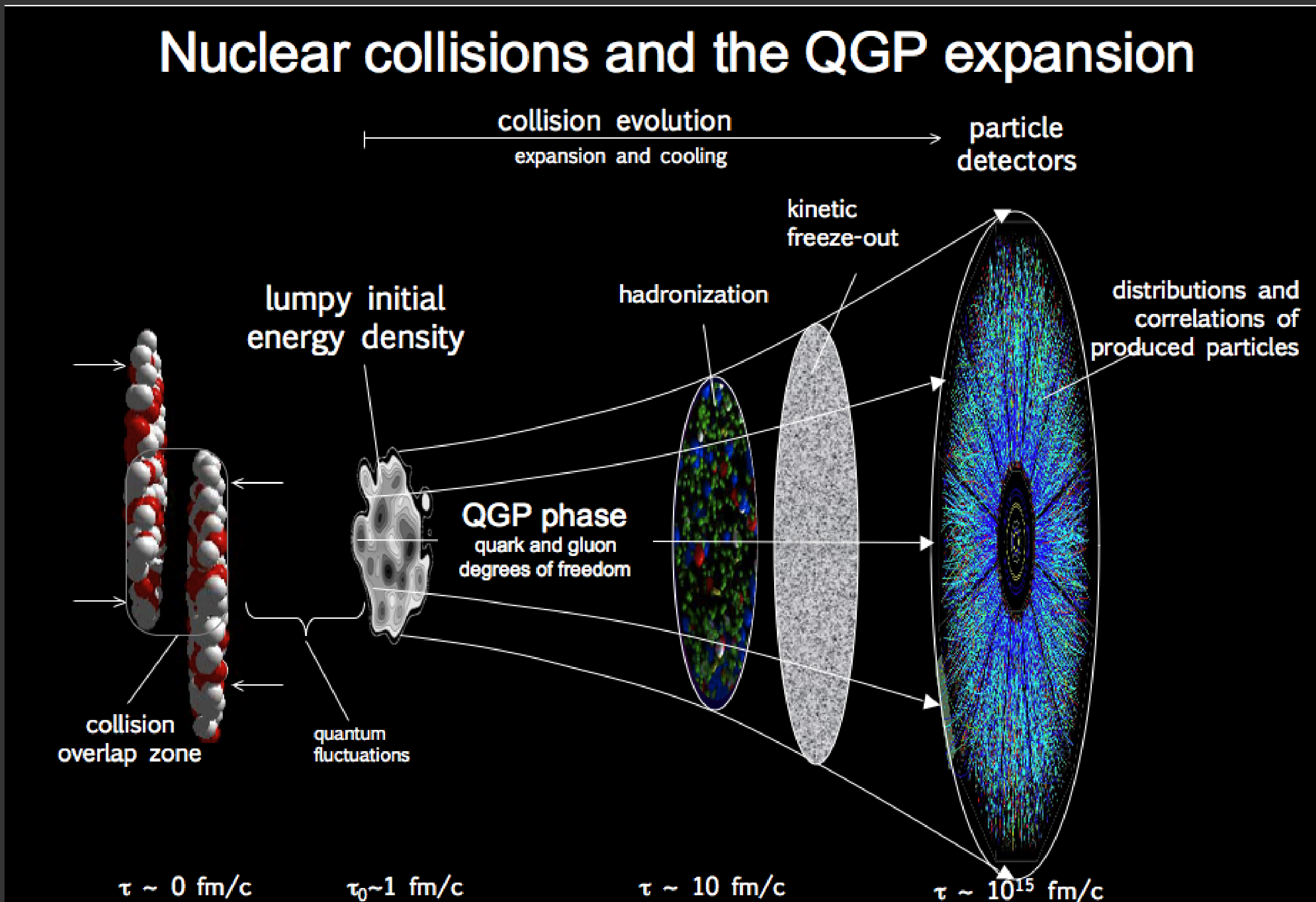


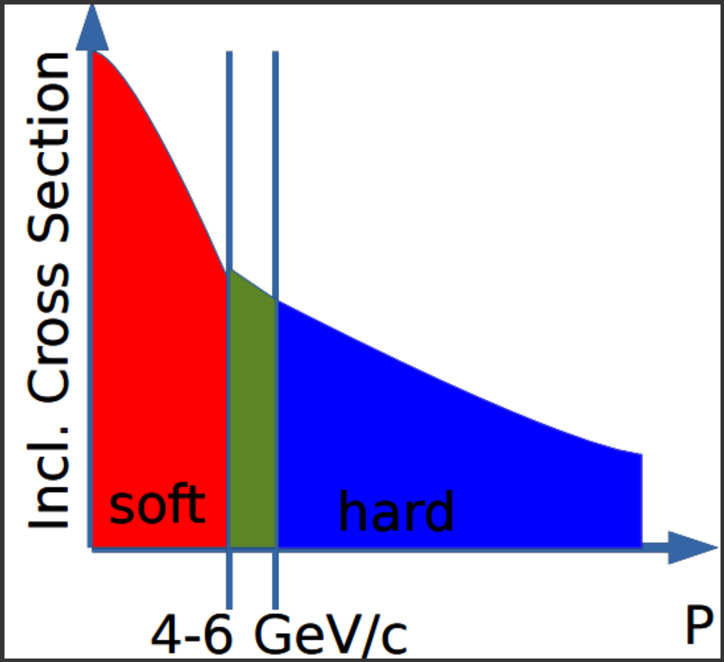
Image source: http://icc.ub.edu/

- Quark-gluon plasma (QGP): strongly interacting hot, dense matter (and perfect fluid)
  - Macro-thermo: Ok
  - Micro-thermo: ???

- Temperature: ~10^{12} K
- Lifetime: *~fm/c*
- Hadronization is still „mystery”

Small system, large fluctuations

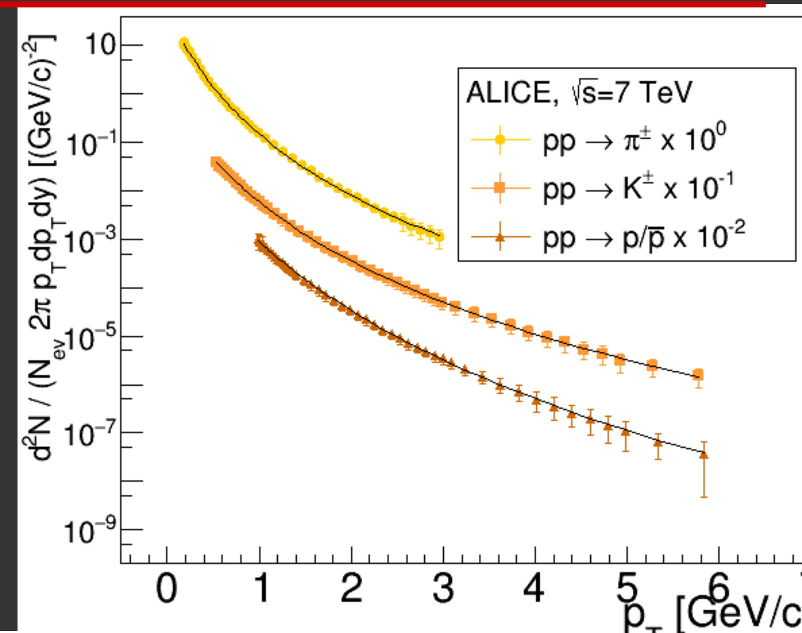
- Spectrum:
  - low p_T (soft): Boltzmann – Gibbs
  - high p_T (hard): power-law tailed (pQCD)
  - the whole range is difficult

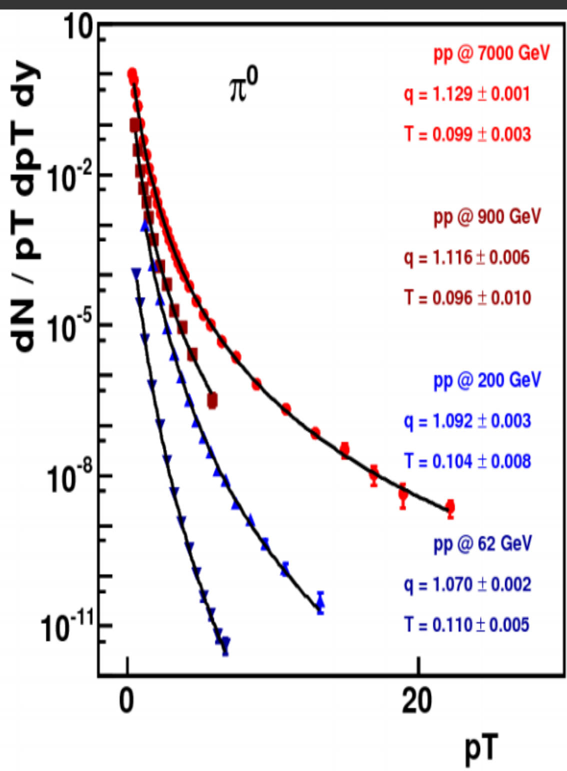


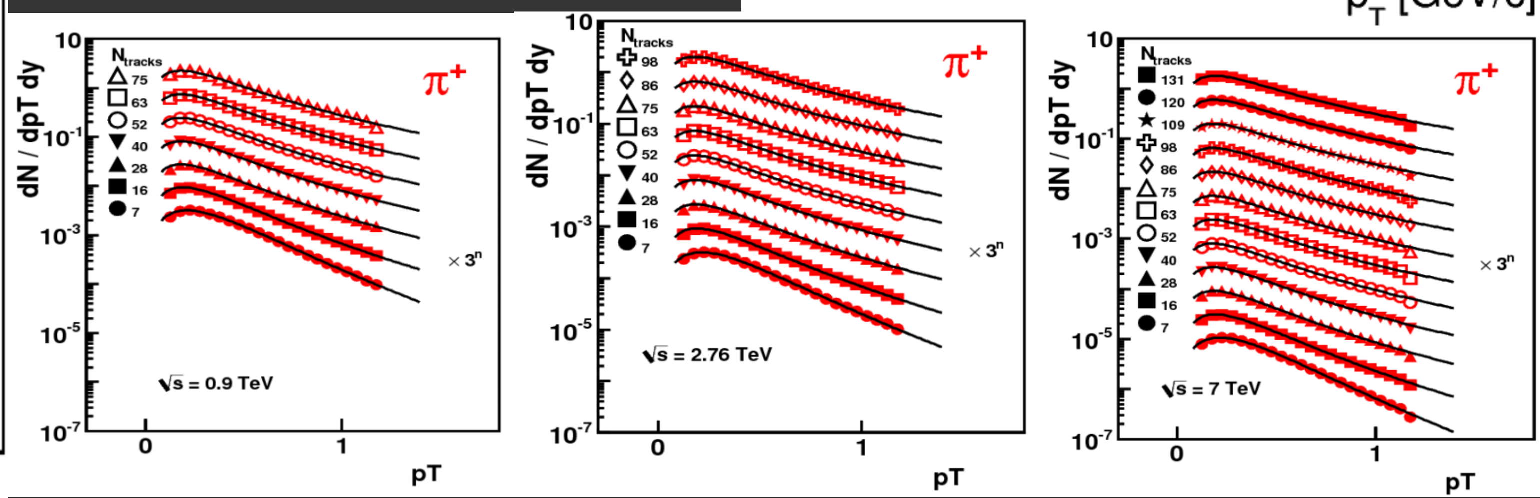
- # of particles in classical atomic matter: $\sim \mathcal{O}(10^{24})$
- # of particles produced in heavy ion collisions: $\sim \mathcal{O}(10^4 - 10^6)$
- # of particles produced in high energy *pp* collisions: $\sim \mathcal{O}(10 - 100)$

Small system, large fluctuations

- High energy physics: new particles from collisions
- Hadron spectra in *pp* collisions can be described by the Tsallis distribution
- π spectra in *pp* collisions depends similarly on $\sqrt{s}$ and on the multiplicity N







Phys.Lett.B701 (2011) 111-116, J.Phys. G36 (2009) 064044 arXiv: 1405.3963, 1501.02352, 1212.0260v2

Under the hood

- **Extensive Boltzmann – Gibbs statistics:**

$$\begin{aligned} S_{12} &= S_1 + S_2 \\ E_{12} &= E_1 + E_2 \end{aligned} \quad \Rightarrow \quad S_B = -\sum_i p_i \ln p_i$$

- **Non-extensivity due to correlations → generalized entropy**

$$\hat{L}_{12} = \hat{L}_1(S_1) + \hat{L}_2(S_2)$$

$$\hat{L}(S) = \frac{e^{(1-q)S} - 1}{1-q} \quad \Rightarrow \quad S_q = \frac{1}{q-1}\left(1 - \sum_i p_i^q\right) = -\sum_i p_i \ln_q p_i$$

$$\ln_q p = \frac{p^{(1-q)} - 1}{1-q} \qquad e_q^p = (1 + (1-q)p)^{1/(1-q)}$$

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Physica A 392 (2013) 3132

Under the hood

- **Boltzmann – Gibbs entropy as *q* → *1* limit:**

$$S_1 = \lim_{q\to 1} S_q = \lim_{q\to 1} \frac{1}{q-1}\left(1 - \sum_i p_i^q\right)$$
$$= \lim_{q\to 1} \frac{1}{q-1}\left(1 - \sum_i p_i e^{(q-1)\ln p_i}\right)$$
$$= \lim_{q\to 1} \frac{1}{q-1}\left(1 - \sum_i p_i \left(1 + (q-1)\ln p_i + O((q-1)^2)\right)\right)$$
$$= -\sum_i p_i \ln p_i$$

- **Maximizing the Tsallis entropy: the Tsallis – Pareto distribution can be obtained**

$$f(\varepsilon) = \left[1 + (q-1)\frac{\varepsilon}{T}\right]^{-\frac{1}{q-1}}$$

Eur. Phys. J. A49 (2013) 110
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Under the hood

- **In high energy collisions:** E **fixed and** $\epsilon \ll E$ **and the particle number fluctuates**

$$q = \frac{\left\langle S'(E)^2 + S''(E) \right\rangle}{\langle S'(E) \rangle^2} = \frac{\langle N(N-1) \rangle}{\langle N \rangle^2} = 1 - \frac{1}{C} + \frac{\Delta T^2}{T^2},$$

$$\frac{1}{T} = \langle S'(E) \rangle , \qquad T = \frac{E}{\langle N \rangle}$$

where $$C = \frac{dE}{dT} = \frac{E}{T}$$

- ***q*: the measure of non-extensivity**
  - **If *q-1* is large, that means that fluctuations due to small size effects are significant**

arXiv:1409.5975

Eur. Phys. J. A49 (2013) 110
Physica A 392 (2013) 3132

Different parametrisations

$$f(p_T) = \frac{N}{y}\frac{(n-1)(n-2)}{2\pi nC(nC+m(n-2))}\left(1+\frac{m_T(p_T)-m}{nC}\right)^{-n}$$

$$f(p_T) = Am_T \cdot \left[1+(q-1)\frac{m_T(p_T)}{T}\right]^{-\frac{1}{q-1}}$$

$$f(p_T) = A \cdot \left[1+(q-1)\frac{m_T(p_T)}{T}\right]^{-\frac{q}{q-1}}$$

$$\boldsymbol{f(p_T) = A \cdot \left[1+(q-1)\frac{(m_T(p_T)-m)}{T}\right]^{-\frac{1}{q-1}}}$$

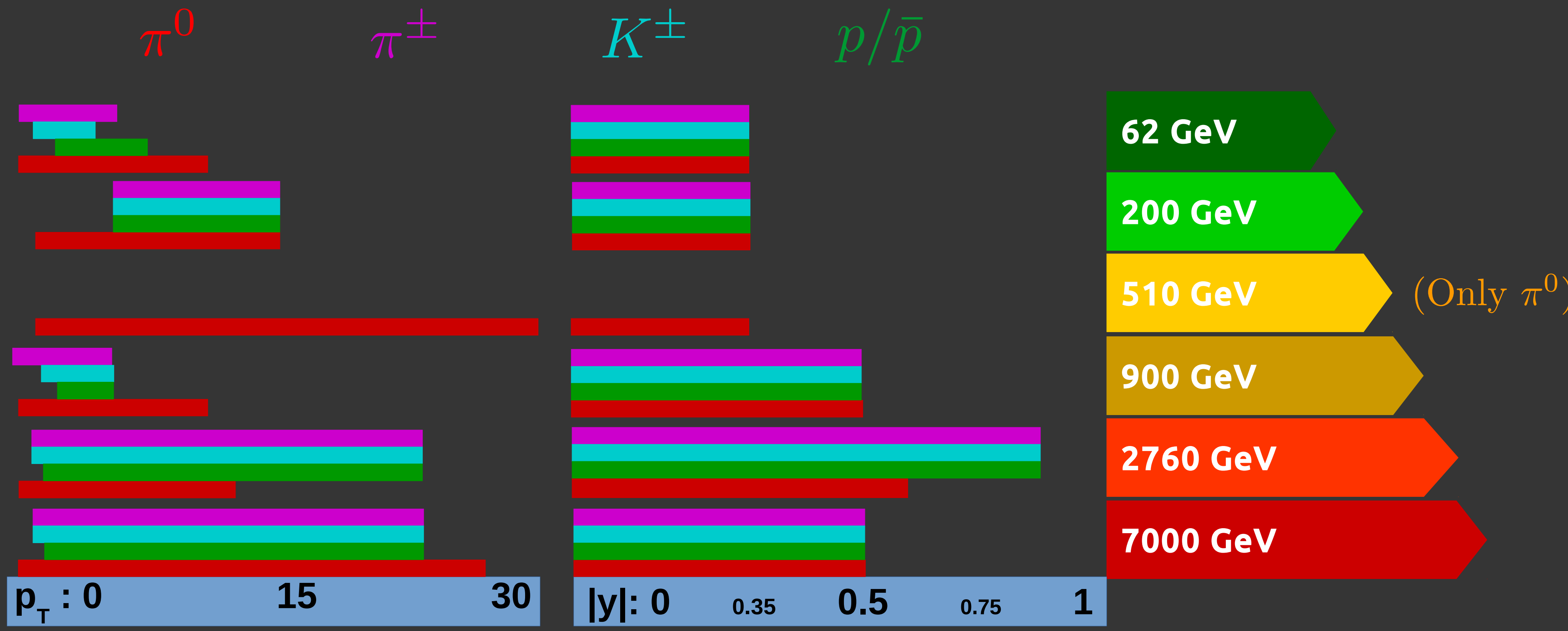
Program:

- **Fit spectra of identified hadrons measured in pp, pA and AA collisions**
- **Investigate the $\sqrt{s}$ dependency of the fitted parameters (and other dependencies: mass, strangeness content, centrality, multiplicity...)**
- **Verification of the scale evolution**
- **Predictions for other collision energies (13-14TeV)**

Phys.Rev. D. 92(7) (2015) 074009.

Results

Data references: identified $\pi^0, \pi^\pm, \mathrm{K}^\pm, \mathrm{p}/\bar{\mathrm{p}}$ spectra



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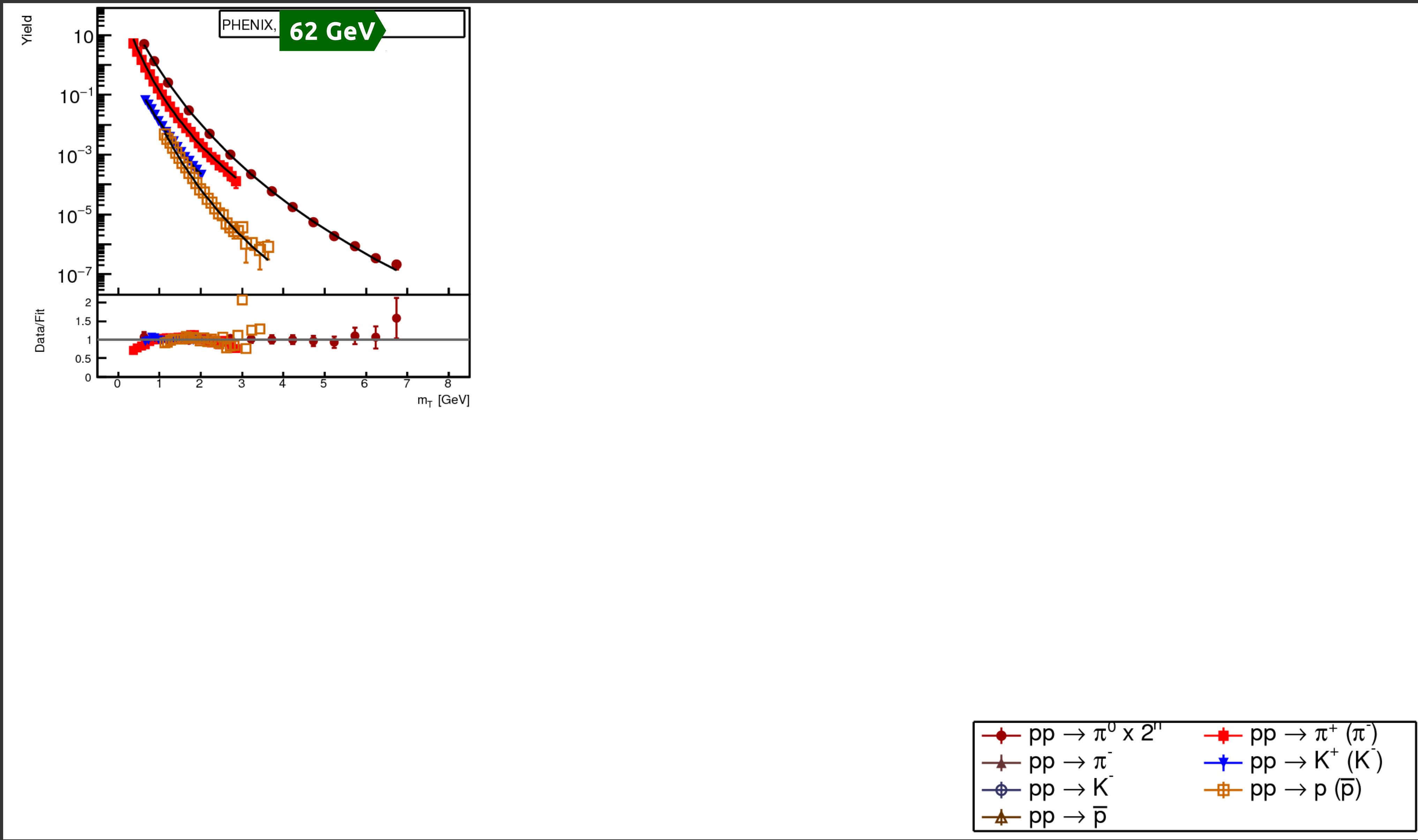
Fitted pp→ PID hadron spectra



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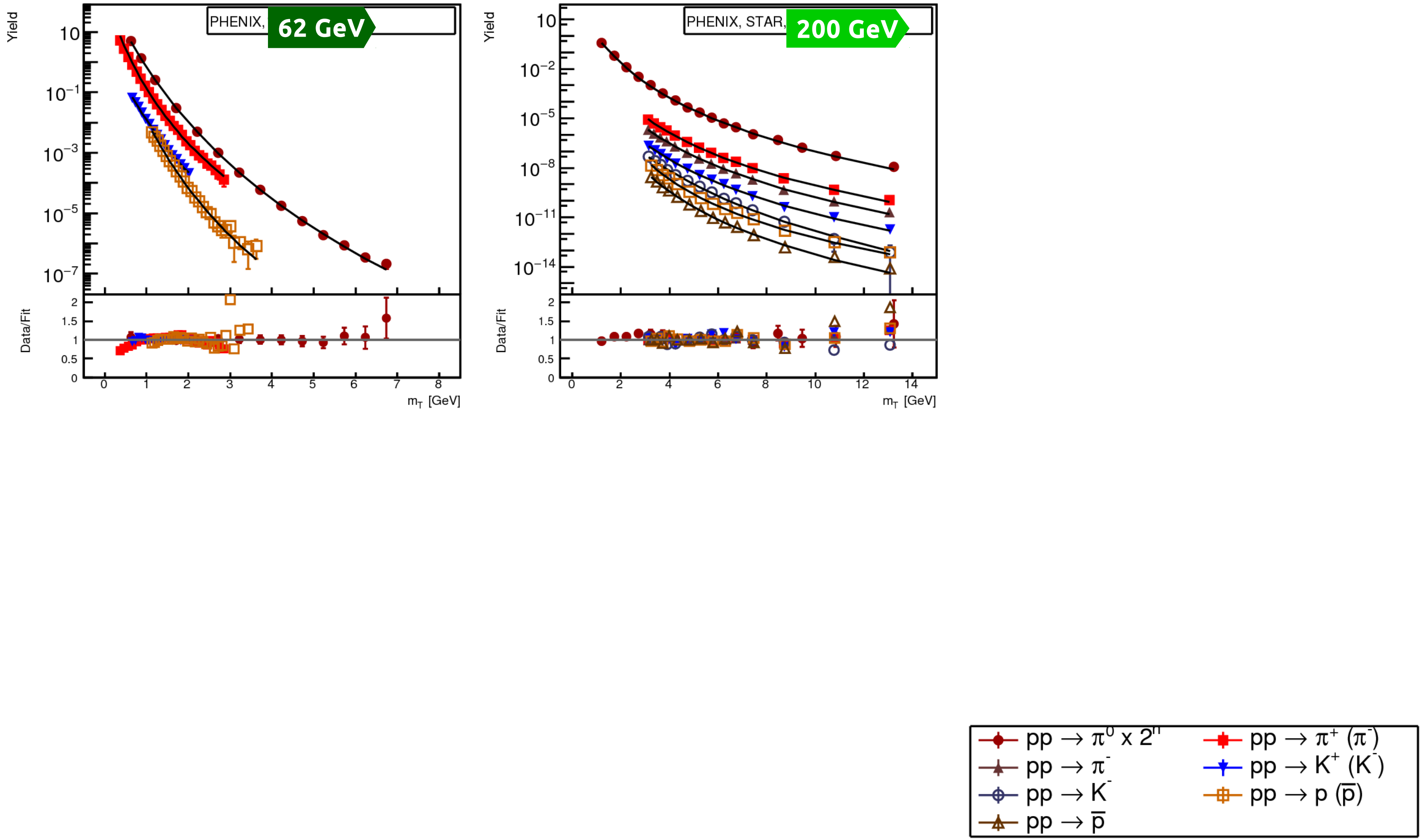

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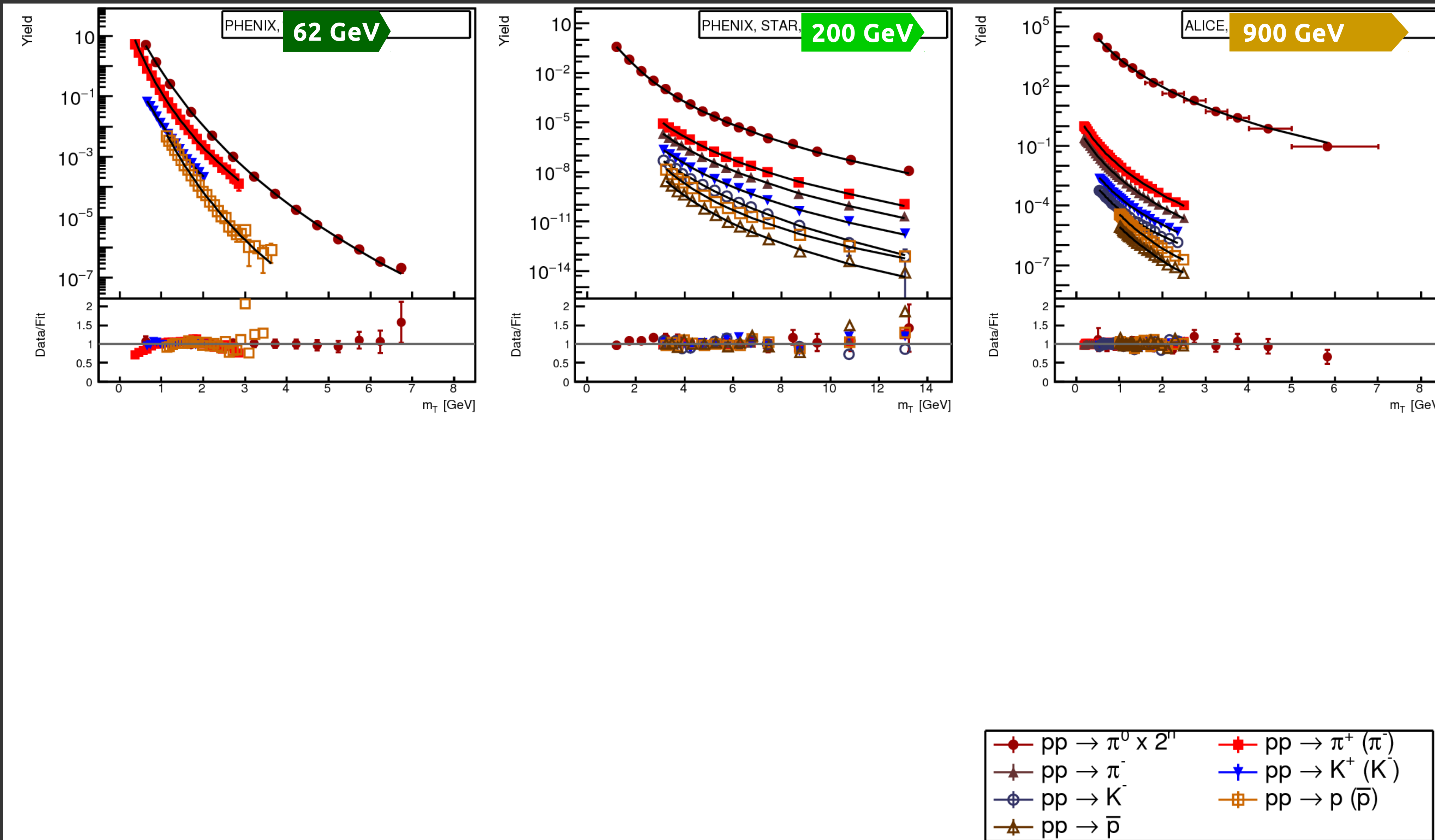

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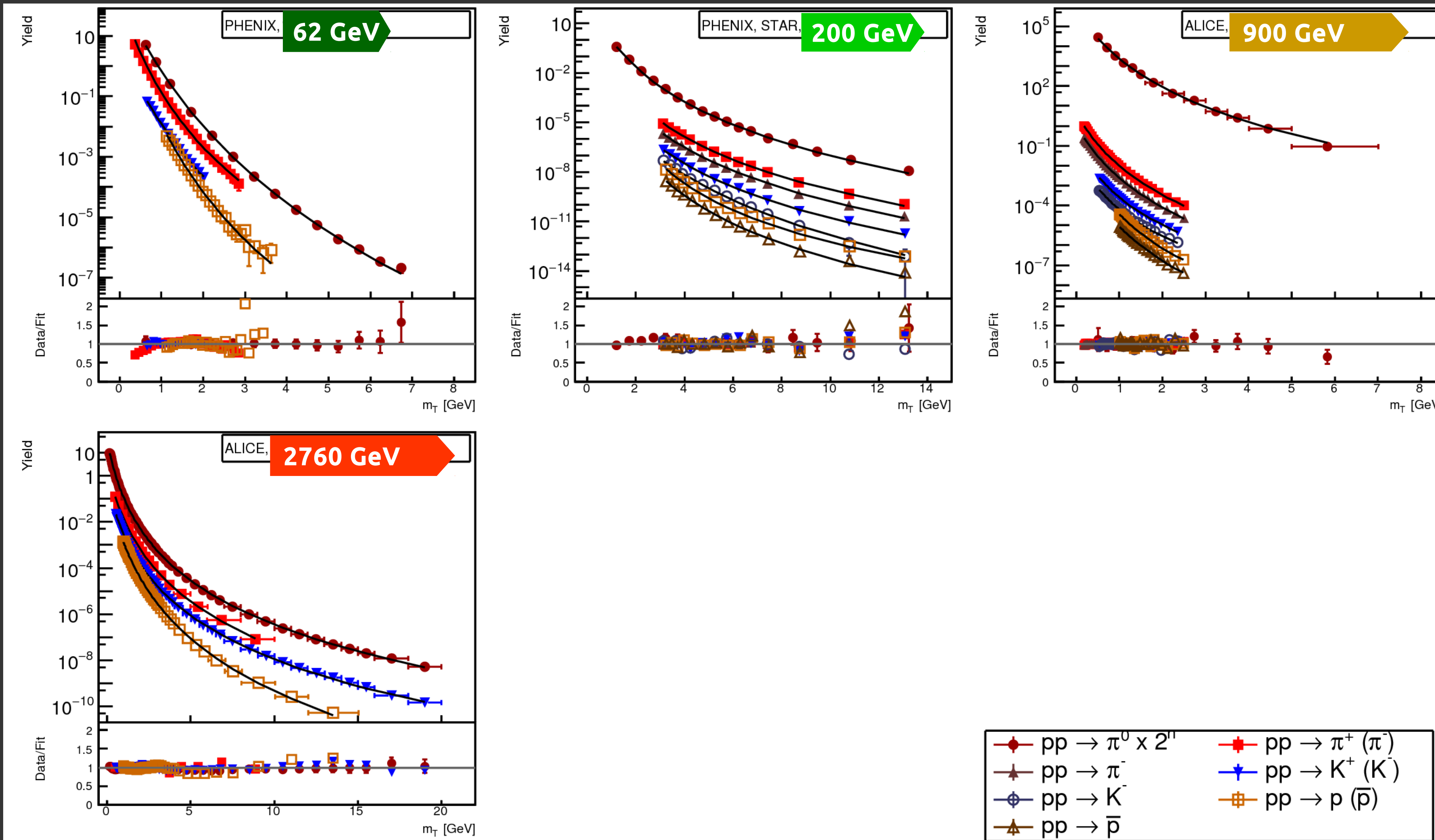

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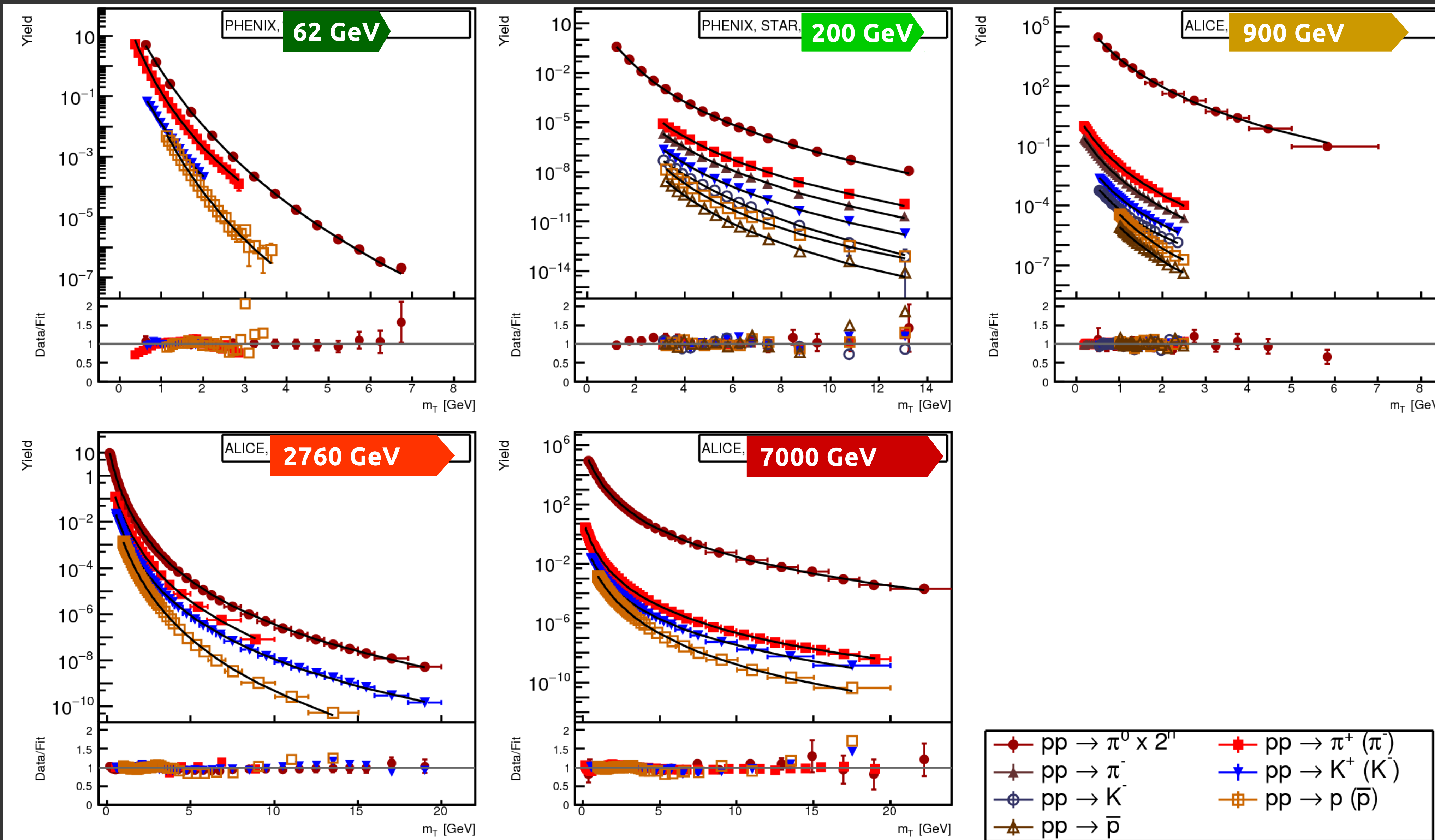
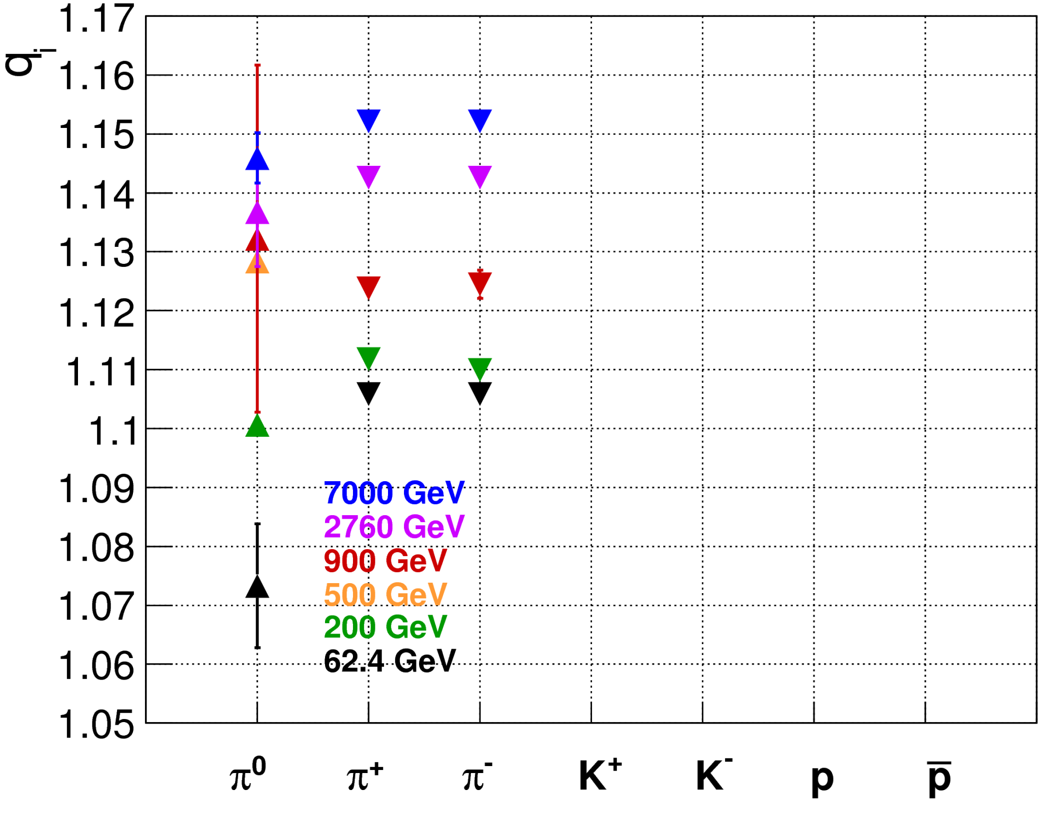
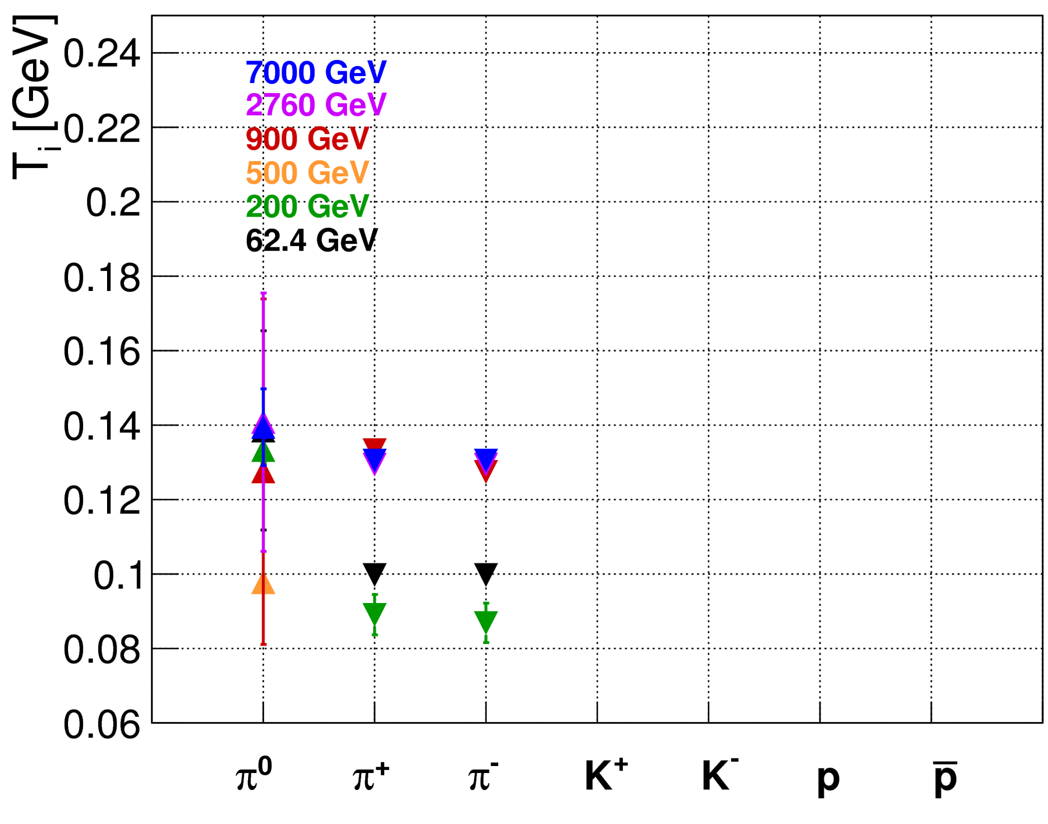


Mass dependency of fitted q and T parameters…?





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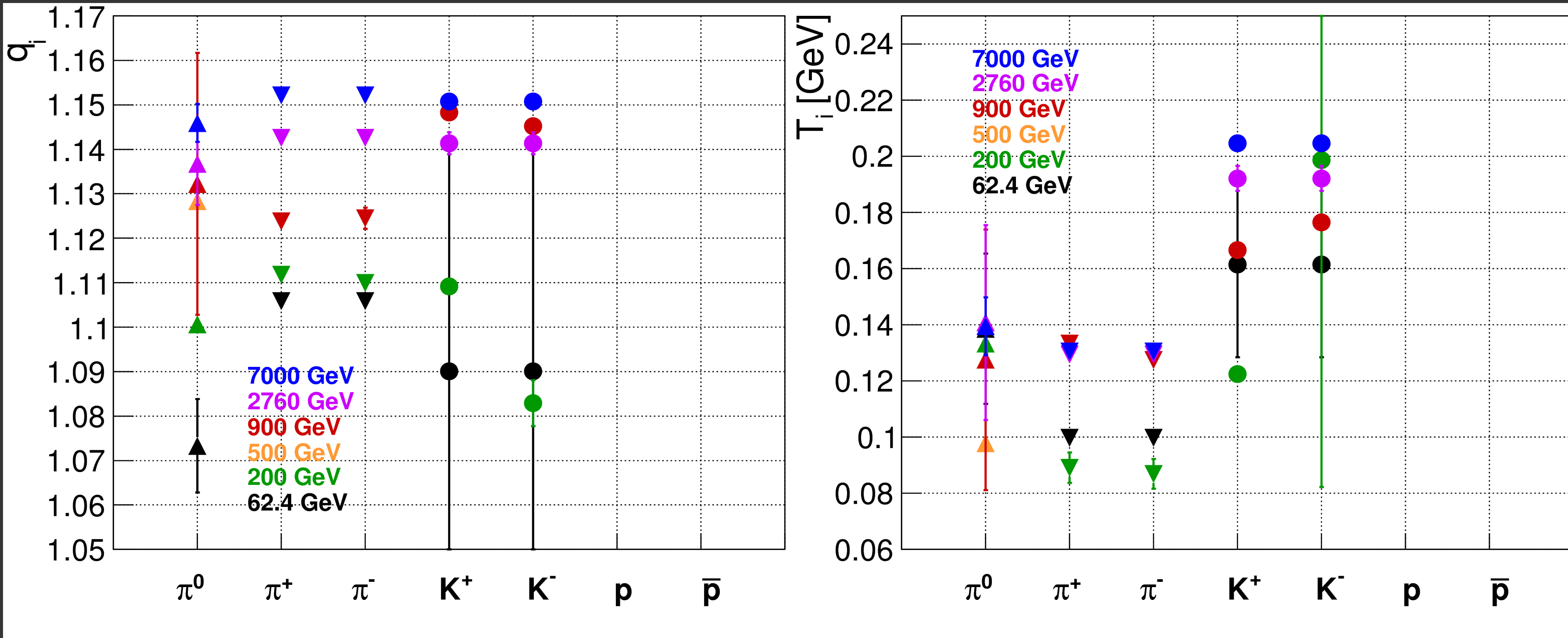

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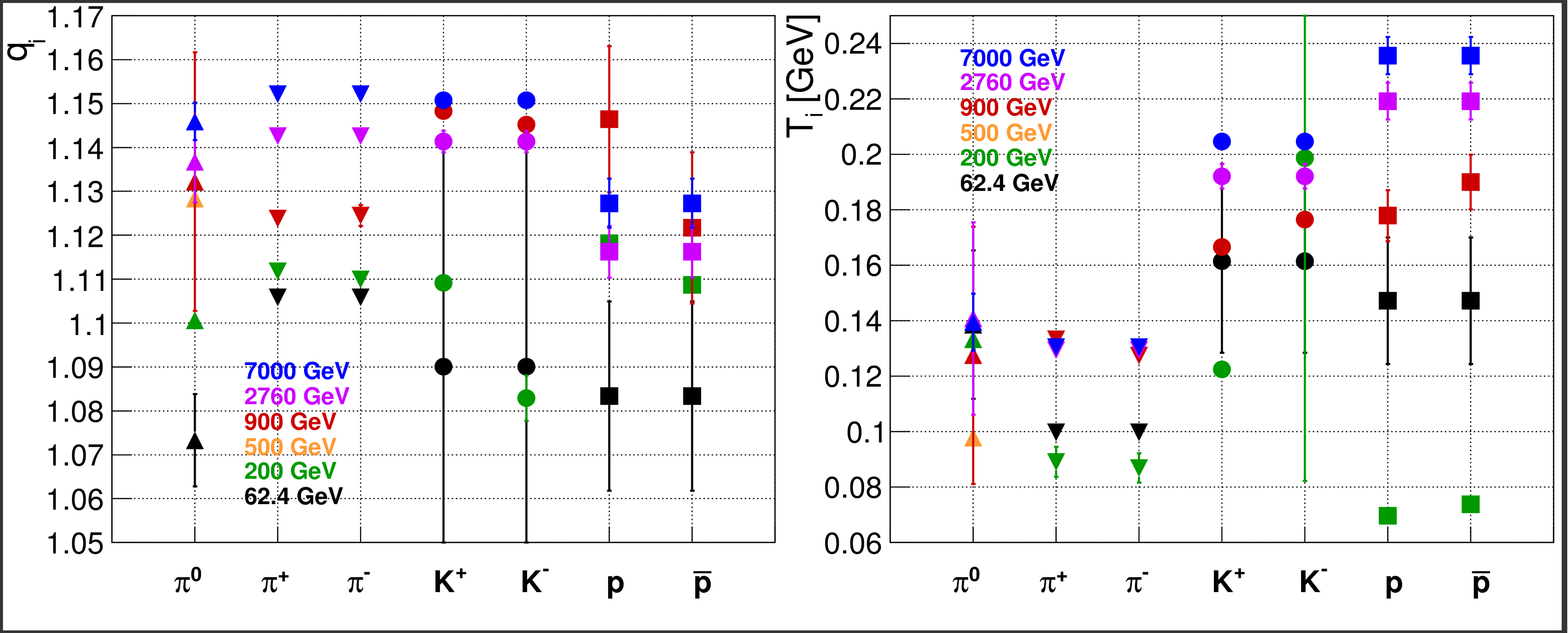

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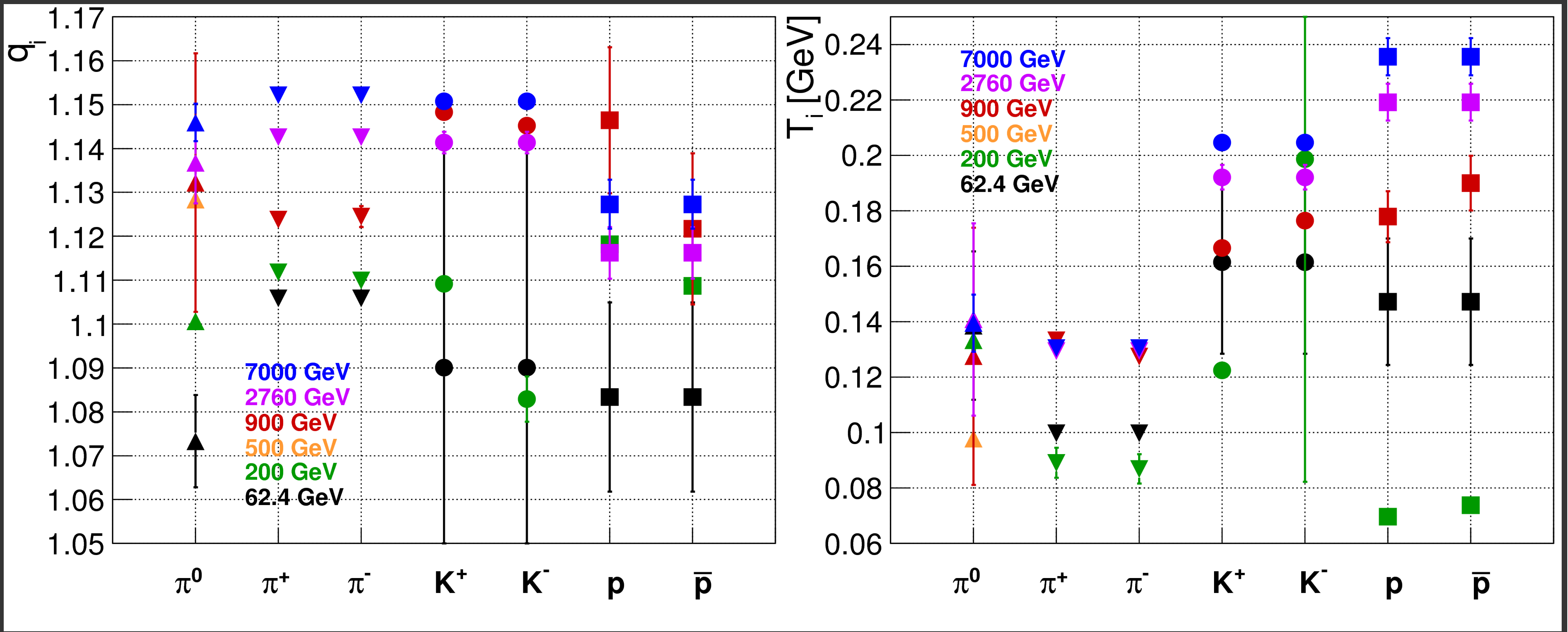
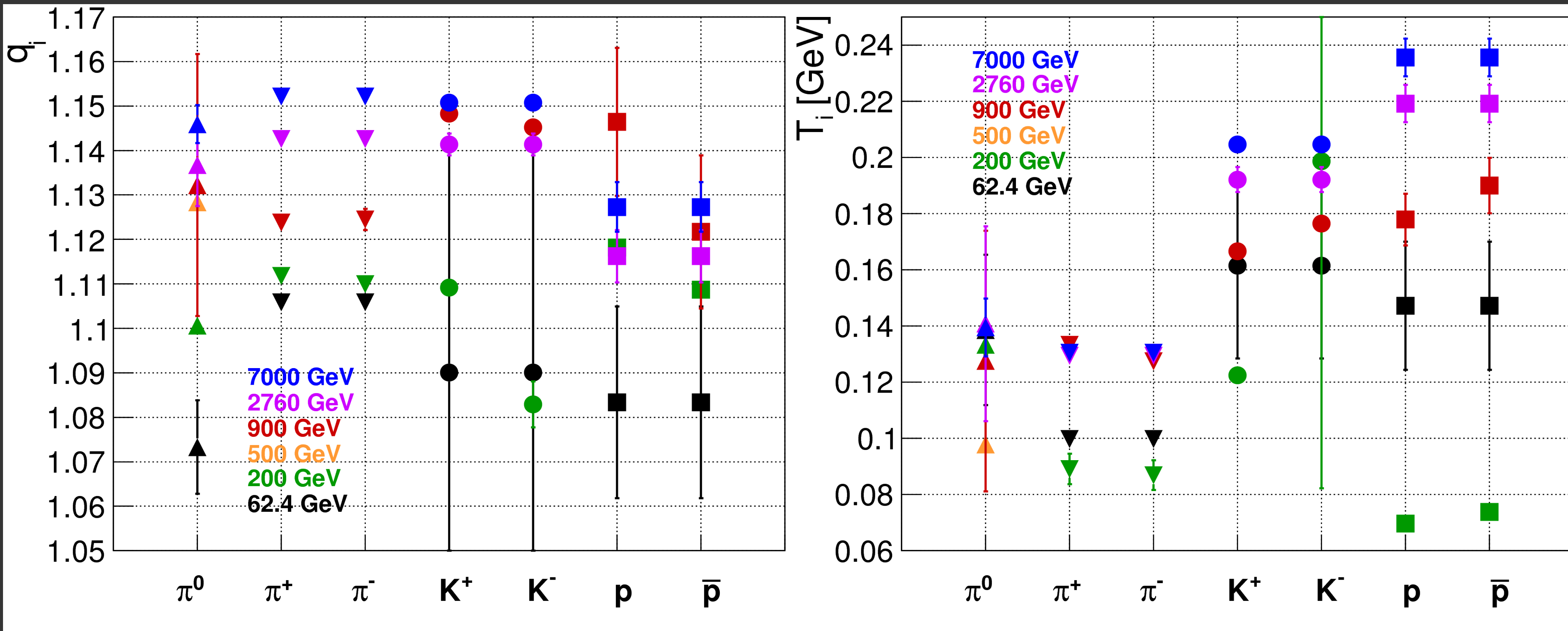


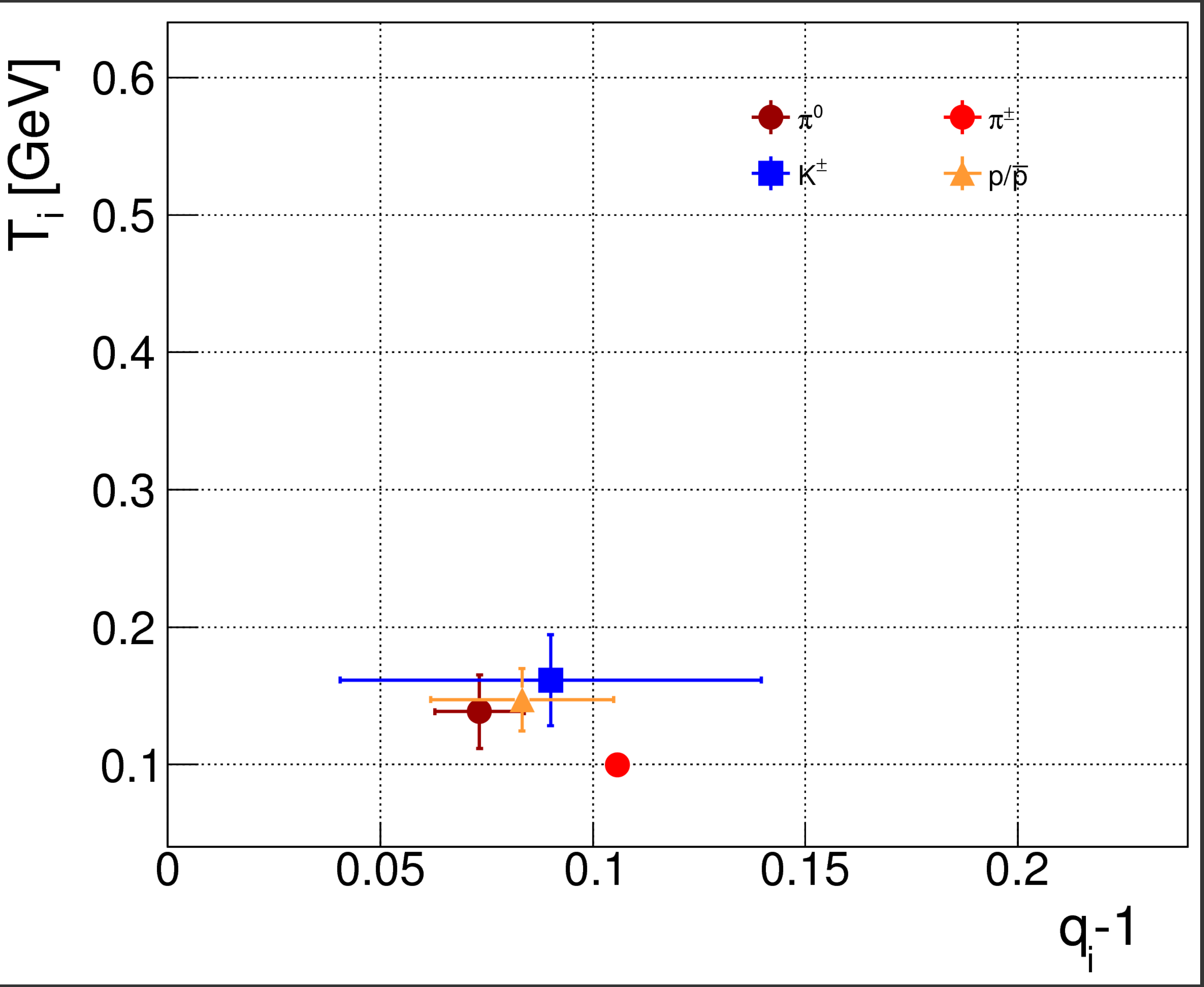
~constant

Mass dependency of fitted q and T parameters…?

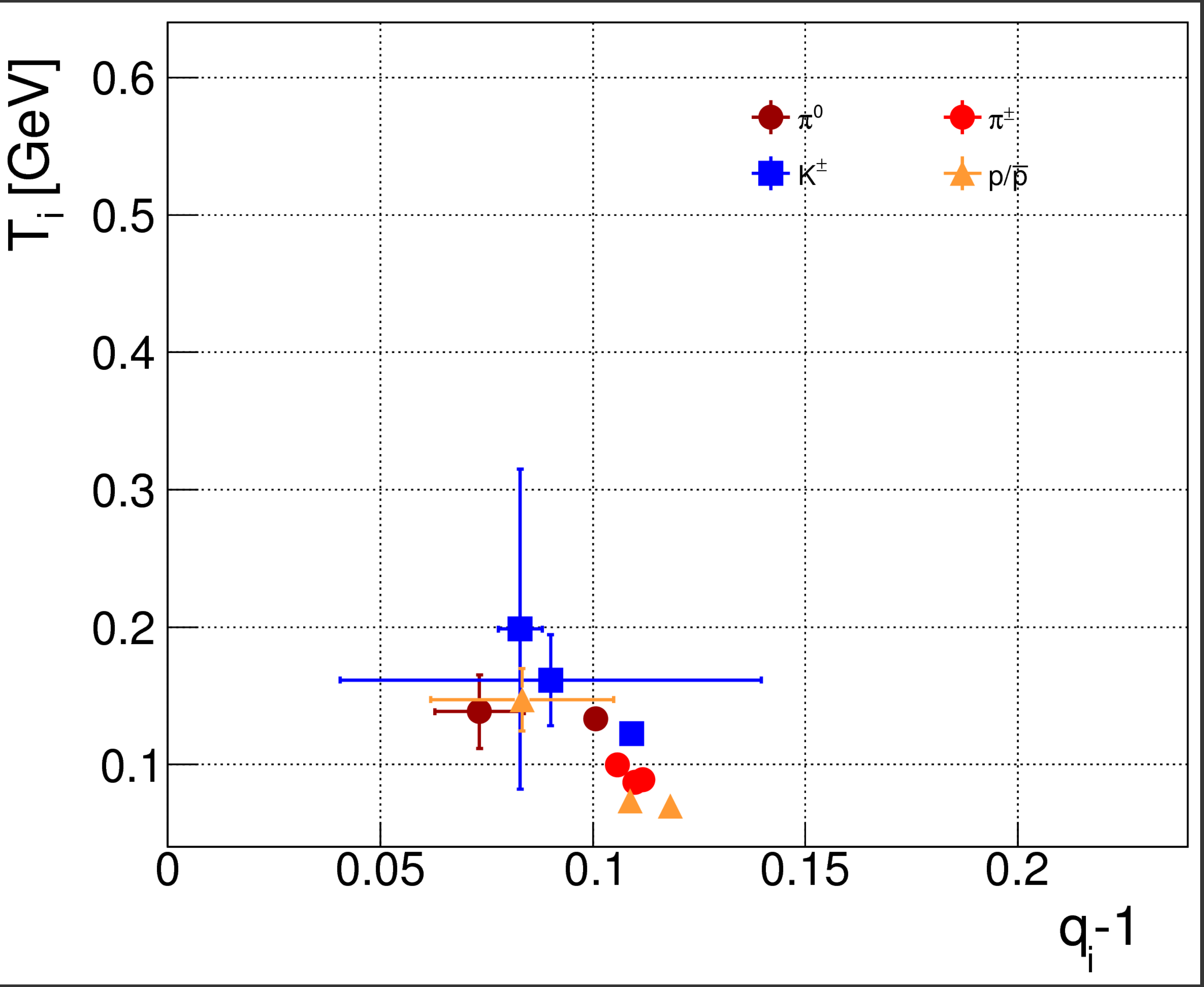


~constant

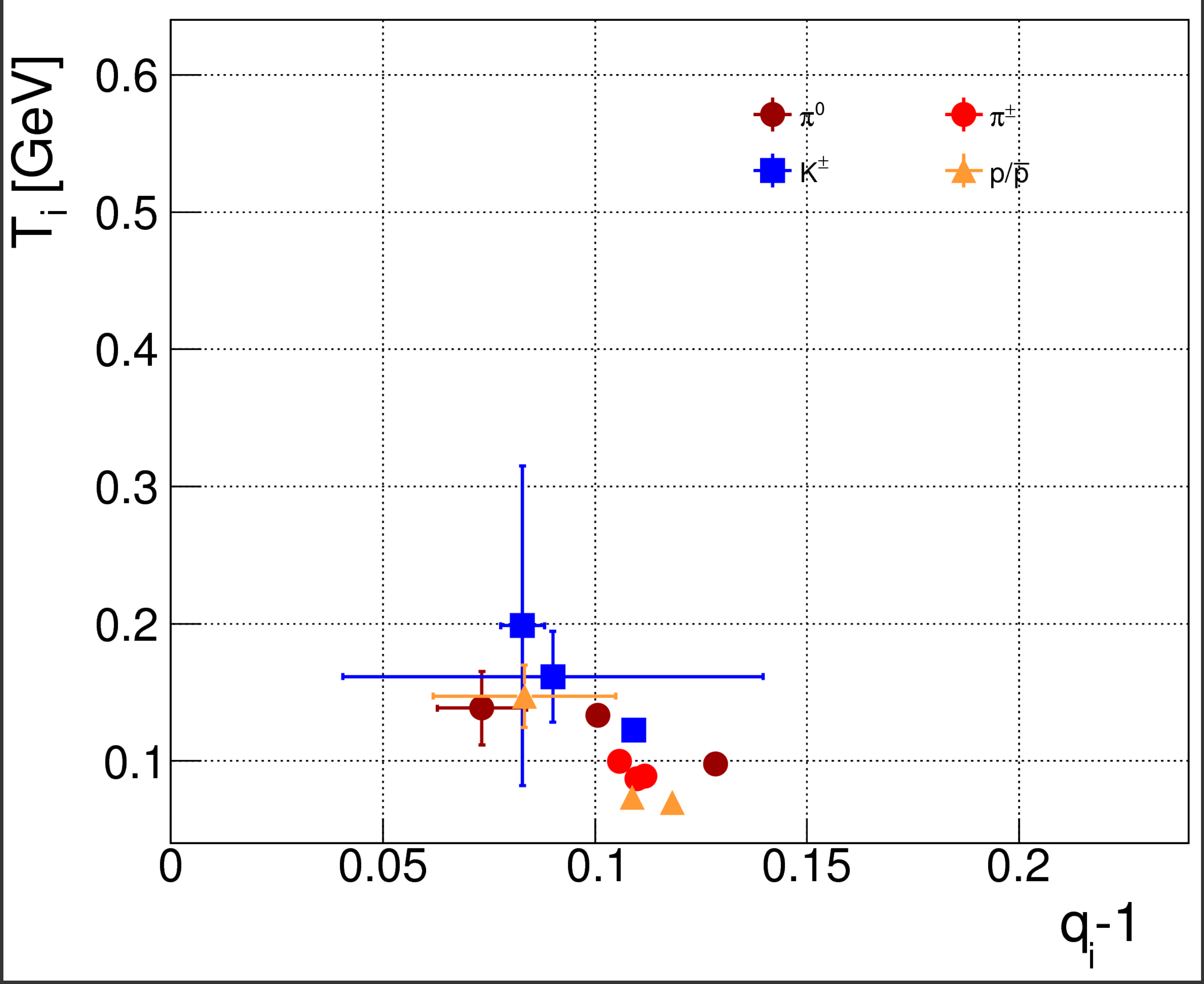
T(q-1), 62 GeV



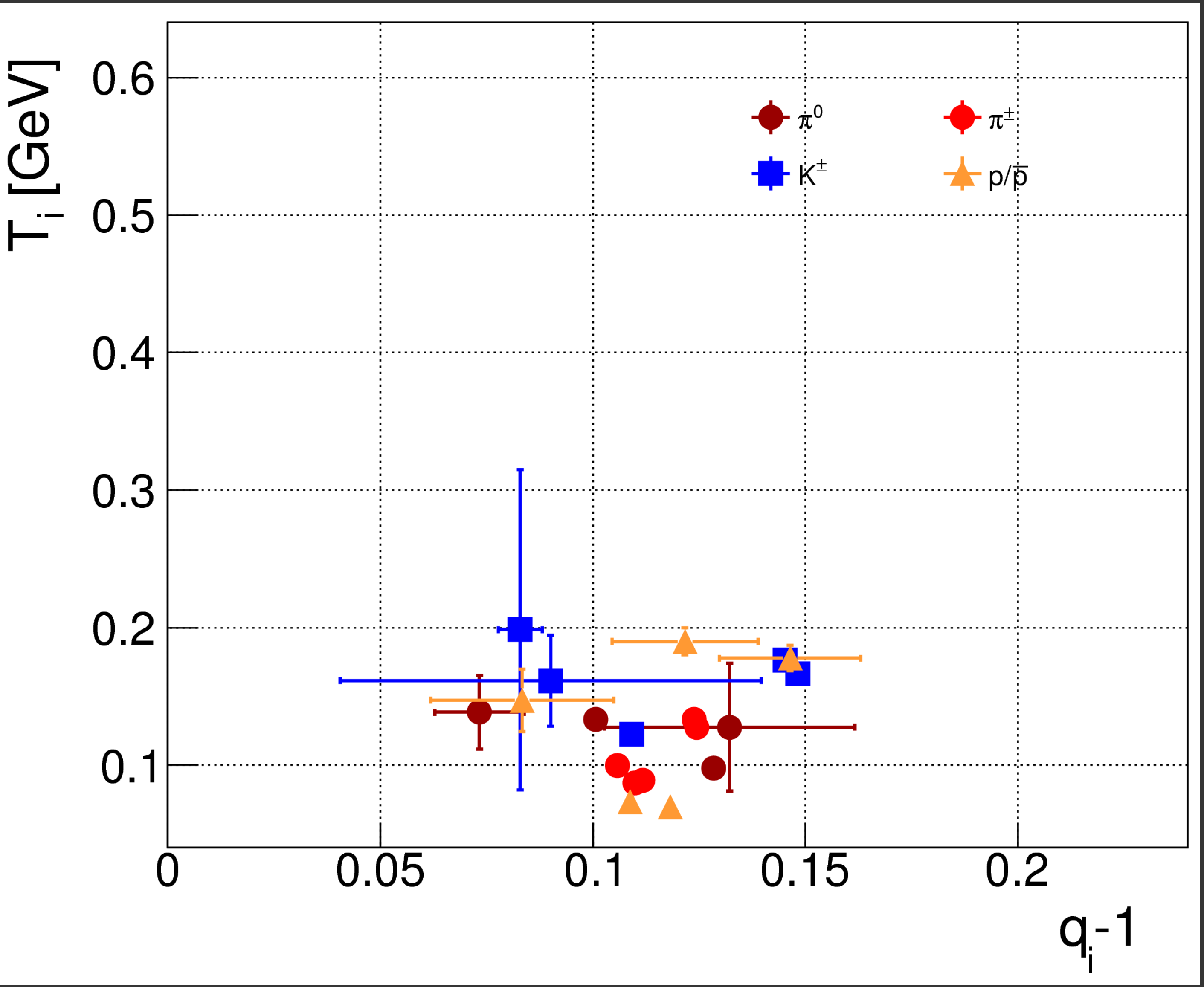
T(q-1), 200 GeV



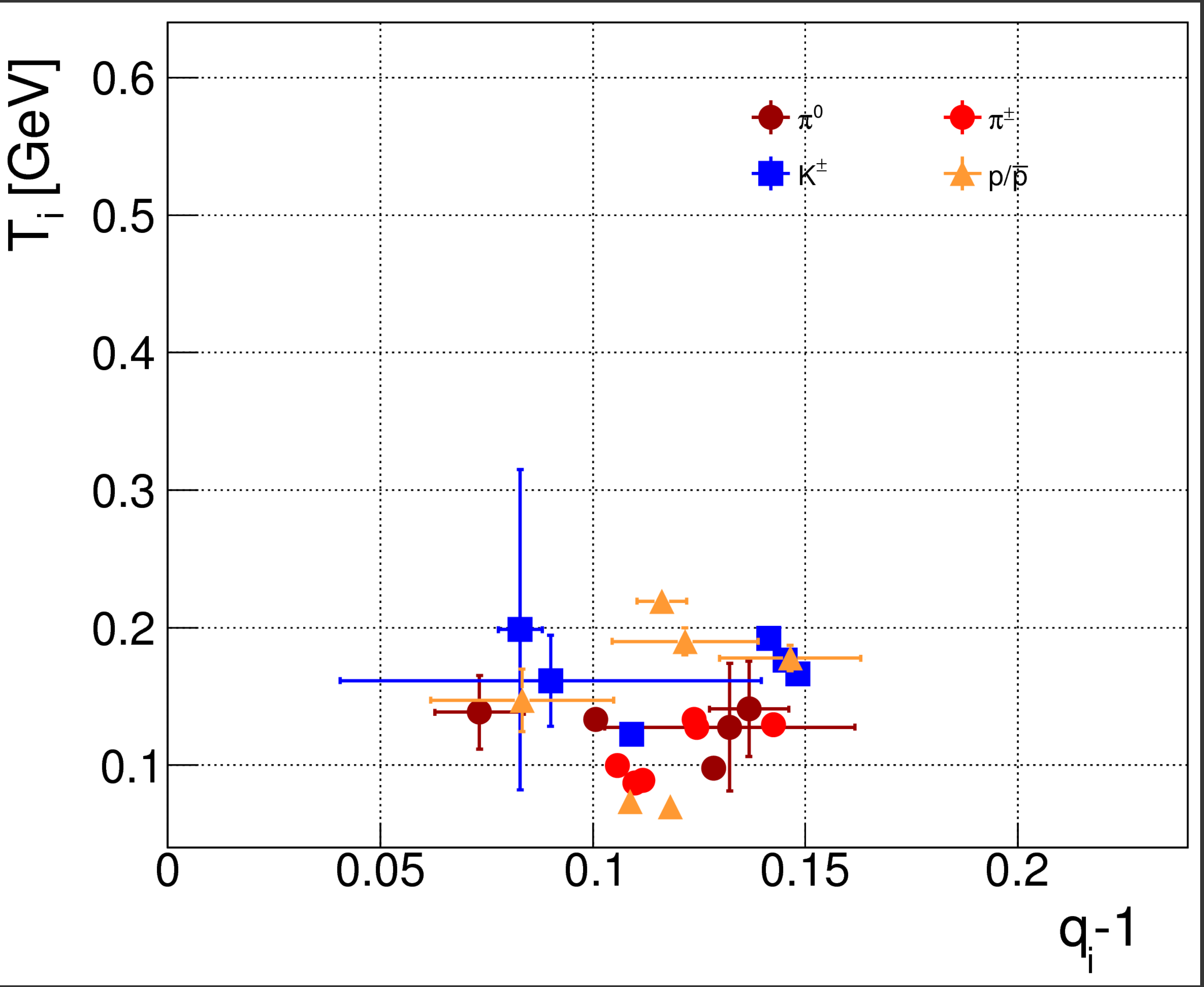
T(q-1), 510 GeV



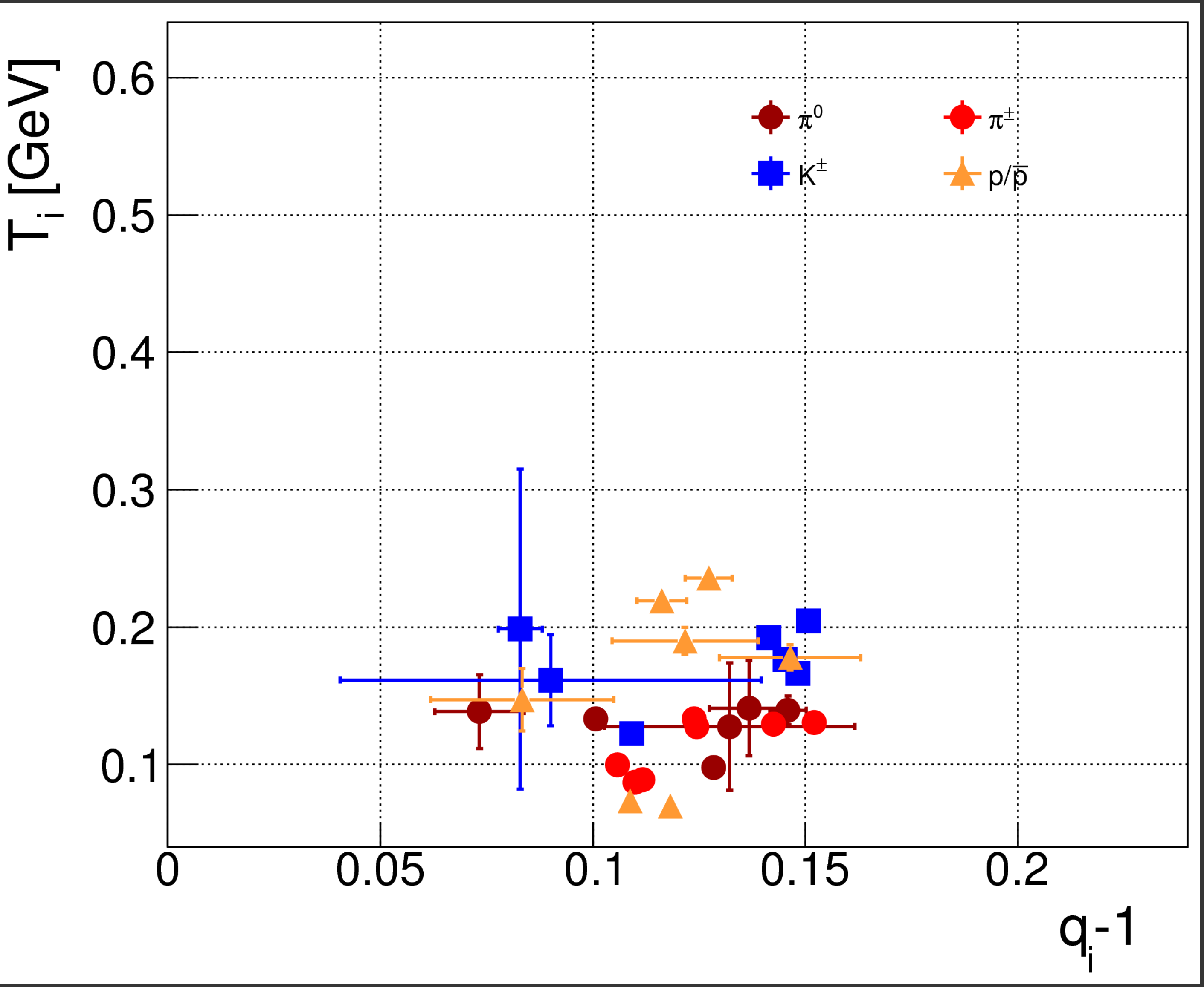
T(q-1), 900 GeV



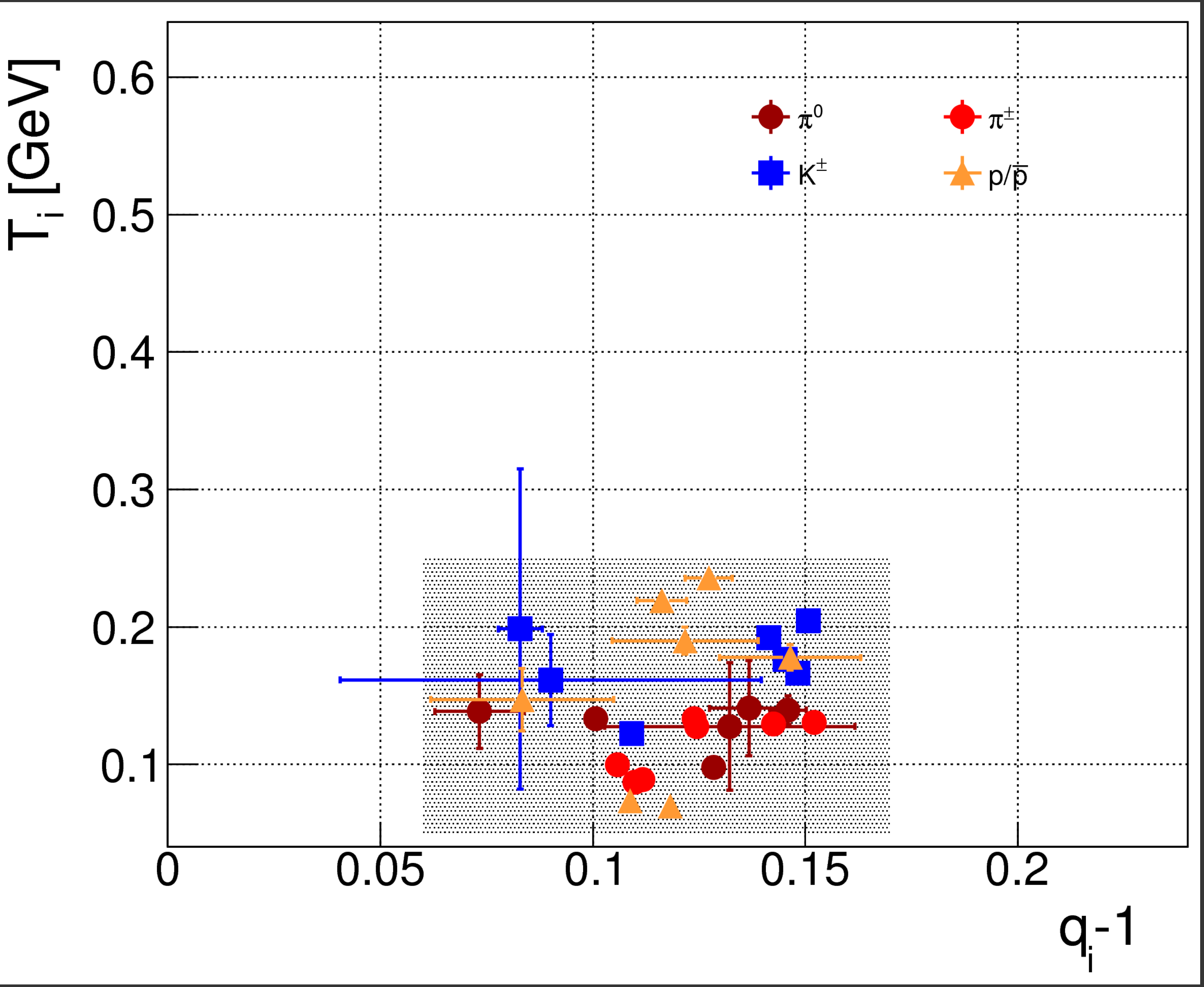
T(q-1), 2760 GeV



T(q-1), 7000 GeV



T(q-1), all energies

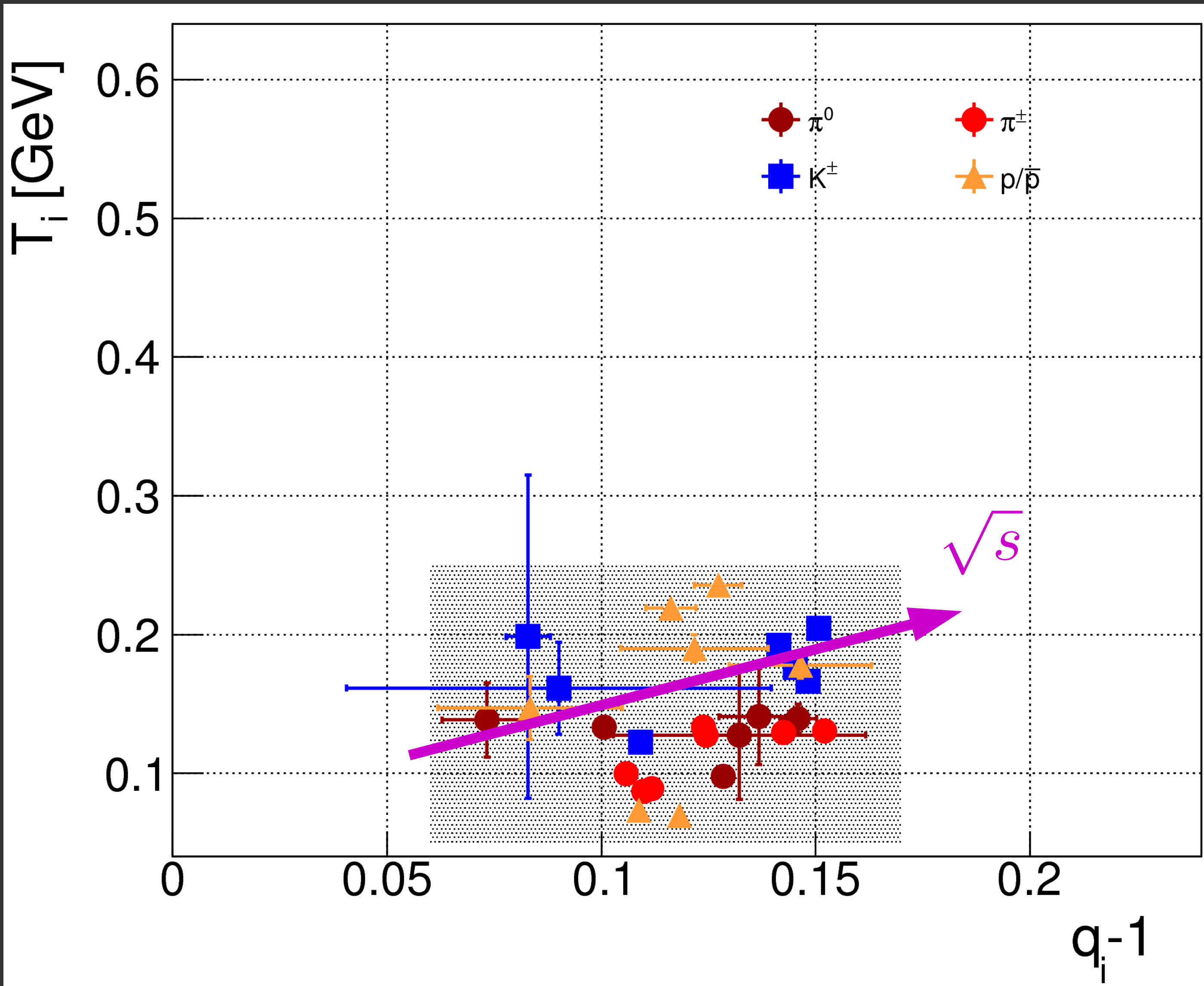


- **The parameter space is very compact**
- **We observed a tendency in CM and in mass**

$$0.075 \lesssim q-1 \lesssim 0.15$$

$$70\text{MeV} \lesssim T \lesssim 230\text{MeV}$$

T(q-1), all energies



- The parameter space is very compact
- We observed a tendency in CM and in mass
- Lets assume the following evolution (see more detailed in GGB's and ÁT's talk):

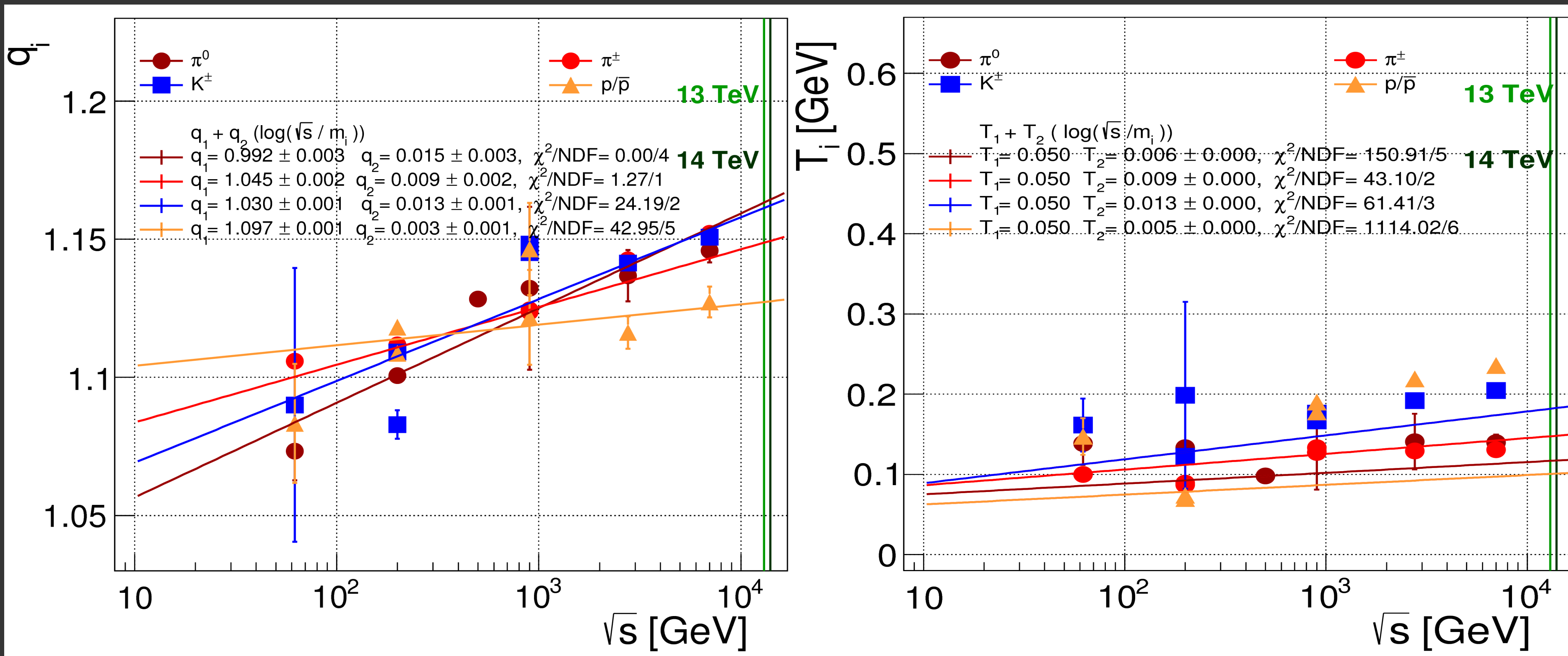
$$q := q_1 + q_2 \log \frac{\sqrt{s}}{m_i}$$

$$T := T_1 + T_2 \log \frac{\sqrt{s}}{m_i}$$

$$0.075 \lesssim q - 1 \lesssim 0.15$$

$$70\text{MeV} \lesssim T \lesssim 230\text{MeV}$$

The evolution of fitted q and T parameters

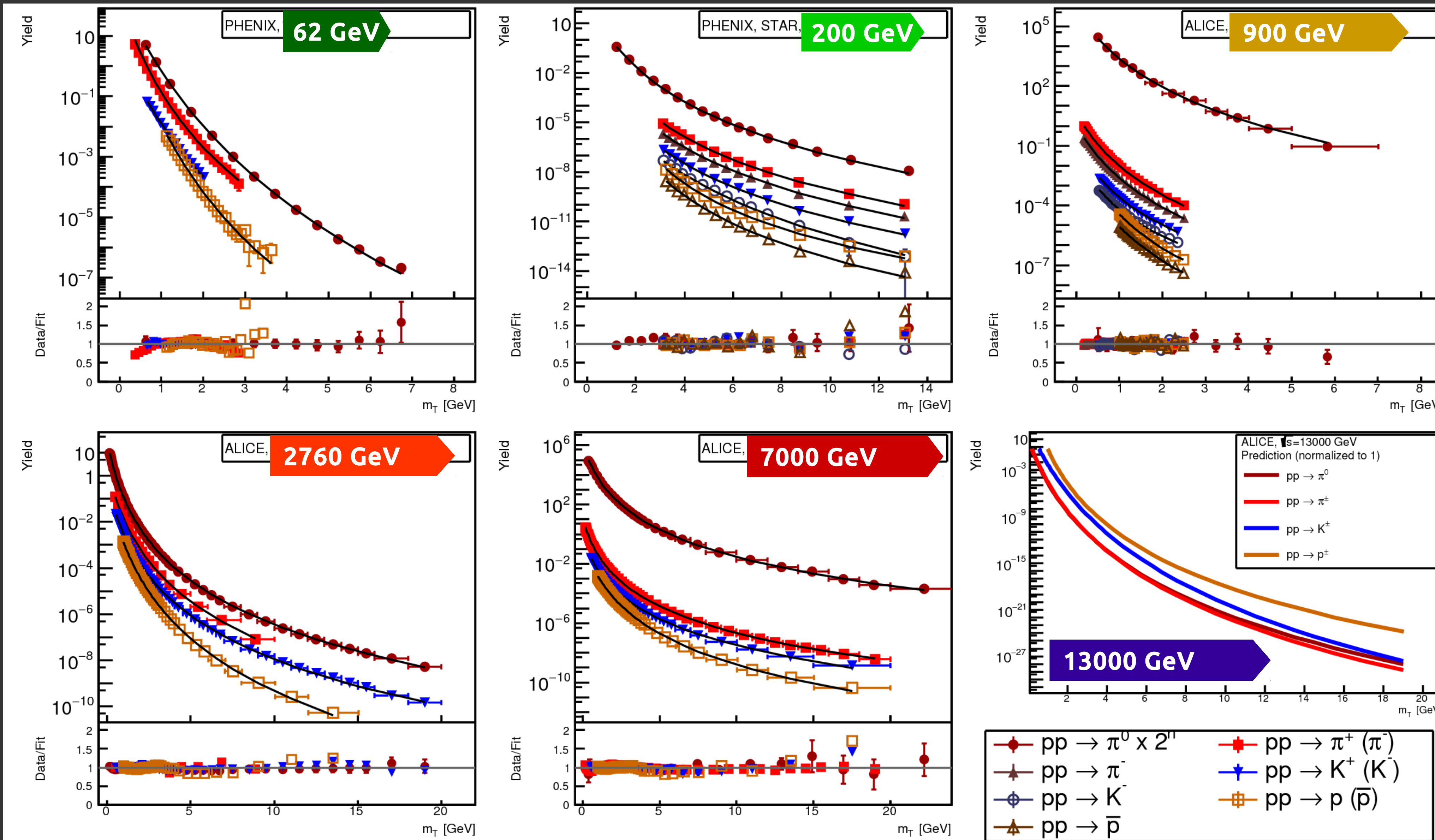


$$q := q_1 + q_2 \log \frac{\sqrt{s}}{m_i}$$

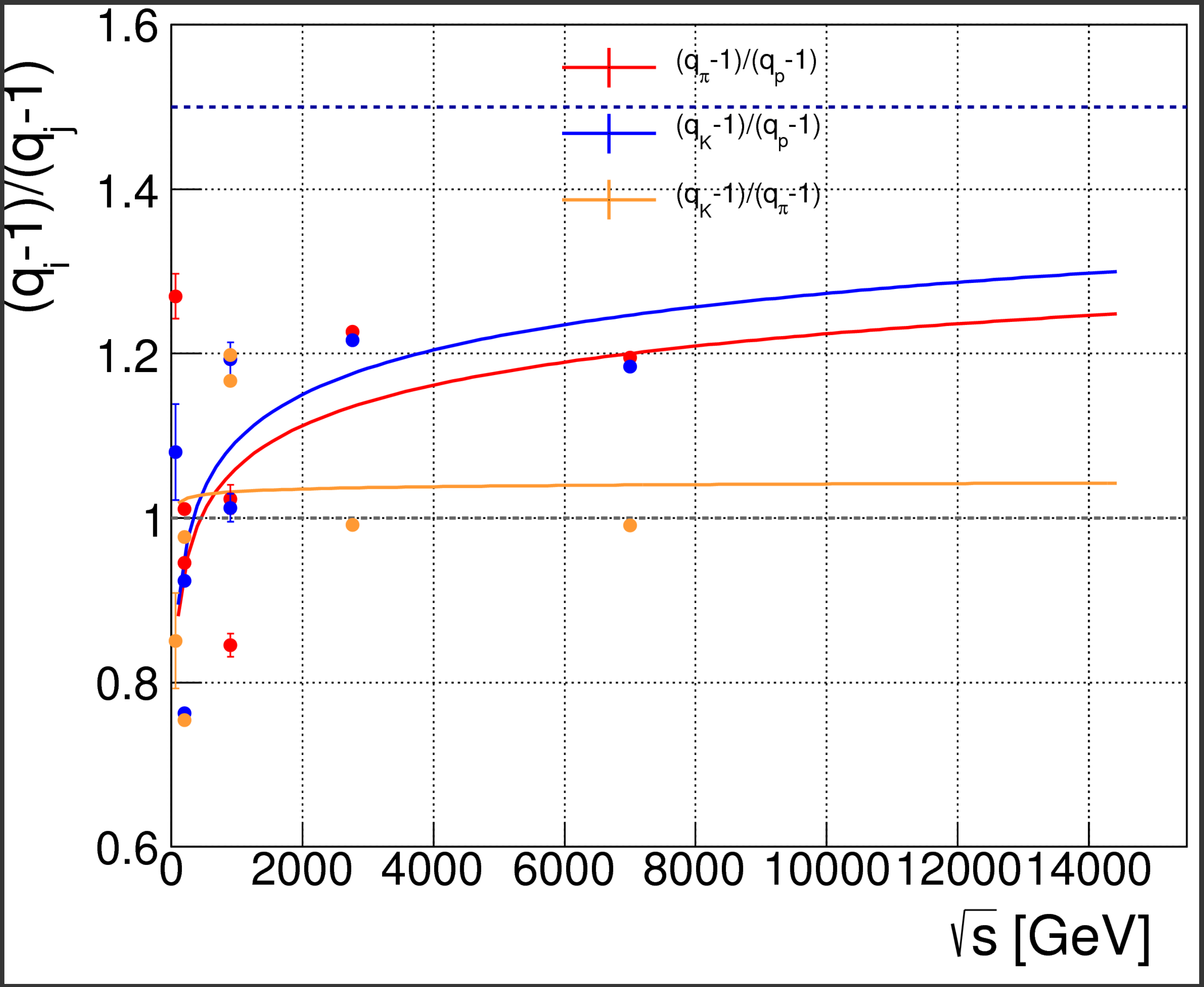
$$T := T_1 + T_2 \log \frac{\sqrt{s}}{m_i}$$

Eur. Phys. J. A40 (2009) 325-340.

Fitted pp→ PID hadron spectra - prediction



Extra feature: coalescence



- **The partons are in equilibrium**
- **The rate of creating a particle consisting *k* partons is a multiplication of *k* T-P distributions:**

$$P_k = f_{TP}^k(p_T)$$

- **Since mesons consist 2 quarks and barions consist 3 quarks, the following ratio (should) be valid:**

$$q' - 1 = \frac{q-1}{k}$$

$$q_q - 1 = 2(q_m - 1)$$

$$= 3(q_b - 1)$$

$$\frac{q_m - 1}{q_b - 1} = \frac{3}{2}$$

J.Phys. G *36* (2009) 064044.

Summary

- Maximizing Tsallis-entropy: thermodinamical expressions for particle spectra obtained in high energy *pp* collisions
  - The fluctuation of the number of particles is large
  - Fitted Tsallis – Pareto distributions describe the spectra very well
  - The q and T parameters show a strong CM energy dependence
  - Physical picture from the T vs $(q-1)$ function
- Similar behaviour in electron-positron, proton-nucleus and nucleus-nucleus collisions
- Prediction for any CM energies

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Fitted pp→ PID hadron spectra