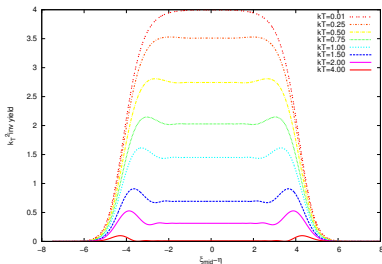


Classical fields, Unruh radiation and heavy ions

Zsuzsanna Szendi, Tamás Sándor Biró

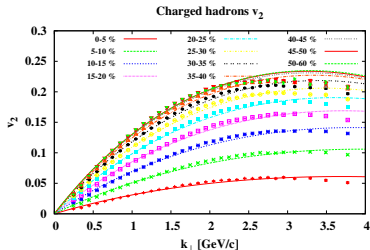
MTA Wigner RCP, Eotvos Roland University, Budapest

Dec. 05-09, 2016, Zimanyi Winter School, Budapest



- Photon rapidity spectra of a single point charge

- Elliptic flow: fits for experimental data



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- Fermi: thermodynamics for high-energy collisions
- Landau: particle production after expansion
- Several new models since then

Bjorken model

- fast thermalization
- expansion: 1D, $y = \eta$
- plateau in rapidity spectrum



Landau model

- fast thermalization
- longitudinal + transversal expansion
- bell shapes in rapidity spectrum



Unruh effect

A relativistic observer with constant acceleration sees blackbody radiation, non accelerating: monochromatic plane wave. Interpretation: Relativistic Doppler-effect.

- No heat bath
- No Brownian-motion
- Planck spectrum

Unruh temperature

proportional to the acceleration (g) :

$$kT = \frac{\hbar g}{2\pi c}$$

Radiation and Doppler effect

Moving observer sees different plane wave: Doppler effect, if there is acceleration radiation can be interpreted due to the Doppler shift.

$$I(\Omega) \propto \left| \int e^{i \int \omega \sqrt{\frac{1-v(\tau)}{1+v(\tau)}} d\tau - \Omega \tau} d\tau \right|^2 \propto \left| \int e^{i c \omega z / g} z^{-i \Omega c / g - 1} dz \right|^2$$

- $z = e^{-\xi}$
- $a_i = (\sinh \xi, \cosh \xi) \frac{d\xi}{d\tau}$
- $a_i a^i = -g^2$

Planck spectra

$$I(\Omega) = \frac{1}{e^{2\pi c \Omega / g} - 1}$$



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Linear acceleration

An accelerating point charge radiates, it can be interpreted as emission of photons.

T.S Biro Z. Szendi, Zs. Schram (1401.1987 → EPJ A 2014)

Number of photons:
$$d^3N = \frac{d^3k}{2k_0(2\pi)^3} \sum |\epsilon \cdot J(k)|^2$$

- $J(k)$: source term $J^i(k) = q \int e^{ik \cdot x(\tau)} u^i(\tau) d\tau$,
- modified source: $\epsilon \cdot J(k) = q \int_{\tau_1}^{\tau_2} e^{ik \cdot x(\tau)} \frac{d}{d\tau} \left(\frac{\epsilon \cdot u}{k \cdot u} \right) d\tau$
- $k_i = k_{\perp} (\cosh \eta, \sinh \eta, \cos \psi, \sin \psi)$

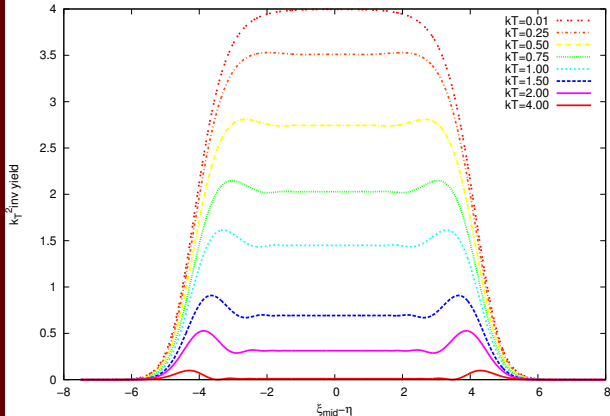
Photon distribution

$$k_{\perp}^2 \frac{dN}{k_{\perp} dk_{\perp} d\eta} = 2\alpha_e \left| \int_{v_1}^{v_2} e^{ik_{\perp} \gamma v} dv \right|^2$$

Photon distr. for large k :

$$\frac{d^2N}{k_{\perp} dk_{\perp} d\eta} = N_0 e^{-\hbar c k_{\perp} / k_B T}$$

$$T = \frac{\hbar c}{2k_B \ell} = \pi T_{Unruh}$$



- longer deceleration time
- plateau
- $lk_{\perp} > 0.5?$

T.S Biro Z. Szendi, Zs. Schram (1401.1987 → EPJ A 2014)

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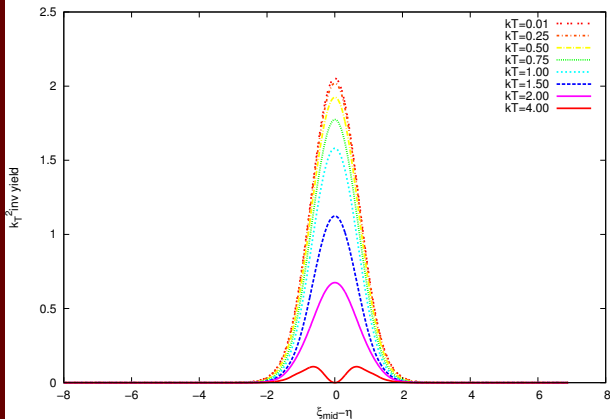
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- short deceleration time
- $g = 1$
- bell shape

T.S Biro Z. Szendi, Zs. Schram (1401.1987 → EPJ A 2014)

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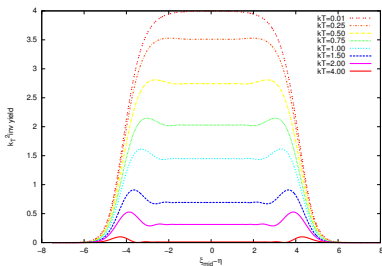
Radiation due to a single point charge

Radiation due to two point charges

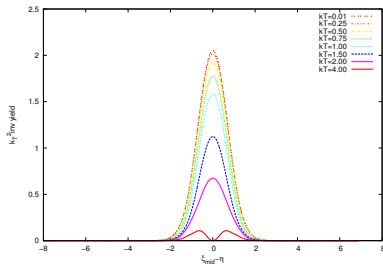
Elliptic flow due to radiation

Summary

Bjorken



Landau



short time acc.
plateau



long time acc.
bell shape

shift: $gT = \pi$

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Two point like sources

Two point charges moving in opposite directions on parallel paths. The yield is: $Y \propto \left| A_1 e^{ik_{\perp} \frac{d}{2} \cos(\alpha - \psi)} + A_2 e^{-ik_{\perp} \frac{d}{2} \cos(\alpha - \psi)} \right|^2$

- A_i : amplitudes
- ψ : angle of the event
- α : detector angle
- d : distance

After expanding the square:

$$Y \propto |A_1|^2 + |A_2|^2 + A_1 A_2^* e^{ik_{\perp} \frac{d}{2} \cos(\alpha - \psi)} + A_1^* A_2 e^{-ik_{\perp} \frac{d}{2} \cos(\alpha - \psi)}$$

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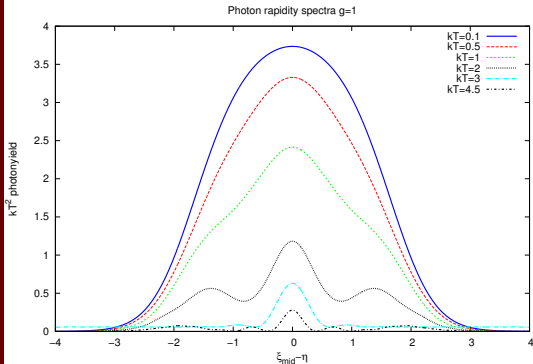
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- $g = 1$
- phase: 0
- bell shape
- interference patterns

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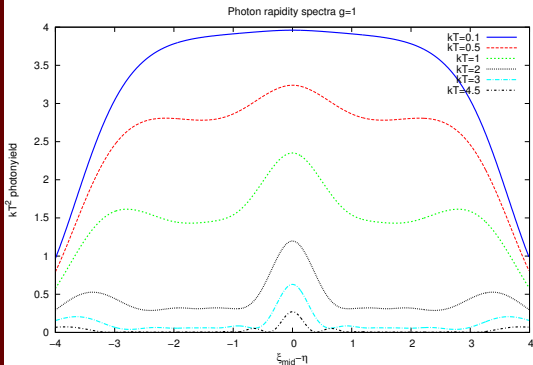
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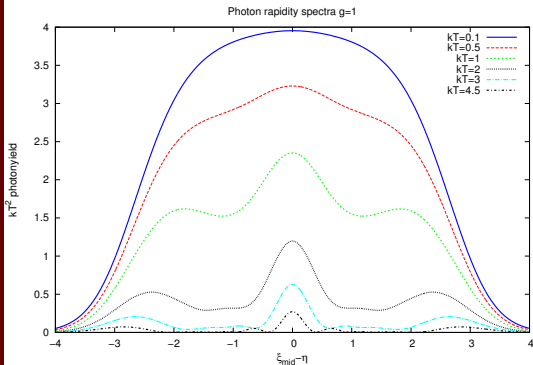
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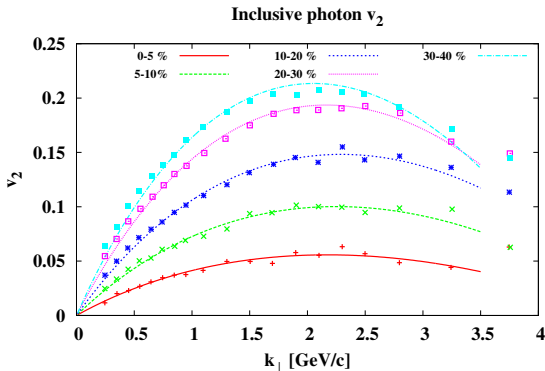
Flow coefficients

Flow coefficients are the relative amplitudes of $\cos(n\theta)$ terms to the zeroth order term.

 v_n

$$v_n = \frac{2R_n J_n(k_{\perp} d)}{|A_1|^2 + |A_2|^2 + R_0 J_0(k_{\perp} d)}$$

- J_n : Bessel functions (first kind)
- $R_n := 2\Re e (i^n A_1 A_2^*) = 2 |A_1| |A_2| \cos(\delta + n\frac{\pi}{2})$
- δ : phase factor



- Pb-Pb collisions
- several centrality classes
- $\sqrt{s_{NN}} = 2.76\text{TeV}$

Figure: v_2 for inclusive photons

J. Phys. Conf. Ser. 446, 012028 (2013), M.Horvath, Z.Schram, T.S. Biro EPJA 51 7 (2015) 75

Summarizing the results from the fits:

- d is stable



- $d \approx 0.07 - 0.1$ [fm]

- γ is stable



- $\gamma \approx 1$

- fits for different centrality



- $c : 0 - 60\%$

- comes close to data



- hadron, photon

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Radiation-single point charge

- no initial conditions
- no fluctuations
- charge: linear trajectory
- Landau, Bjorken like behaviour



Hydro, thermo

- initial conditions
- fluctuations
- p-p elliptic flow?
- heath bath?



Radiation-Two charges

- Dipole like sturcture
- v_2 calculations
- Fits to experimental data
- (Illusory) flow without hydro



Flow coefficients

Flow coefficients are the relative amplitudes of $\cos(n\theta)$ terms to the zeroth order term.

Using the Jacobi-Anger formula:

$$e^{ix \cos \Theta} = J_0(x) + 2 \sum_{n=1}^{\infty} i^n J_n(x) \cos(n\Theta).$$

 v_n

$$v_n = \frac{2R_n J_n(k_{\perp} d)}{|A_1|^2 + |A_2|^2 + R_0 J_0(k_{\perp} d)}$$

- J_n : Bessel functions (first kind)
- $R_n := 2 \Re e (i^n A_1 A_2^*) = 2 |A_1| |A_2| \cos(\delta + n\frac{\pi}{2})$
- δ : phase factor

For $n = 2$ the flow coefficient becomes:

$$v_2 = \frac{-2\varepsilon J_2(k_\perp d) \cos(\delta)}{1 + \varepsilon J_0(k_\perp d) \cos(\delta)} \quad \text{with } \varepsilon = \frac{2|A_1||A_2|}{|A_1|^2 + |A_2|^2} \leq 1$$

If the following parametrization is made:

- $A_1 = e^{i\delta_0}$
- γ : ratio of the amplitudes
- $A_2 = e^{i(\delta_0 + \delta)}$
- d : dipole size

The flow coefficient, v_2
becomes:

Final v_2

$$v_2 = \frac{-J_2(k_\perp d) \cos \delta}{\frac{1 + \gamma^2}{2\gamma} + J_0(k_\perp d) \cos \delta}$$

Two sources in the opposite direction:

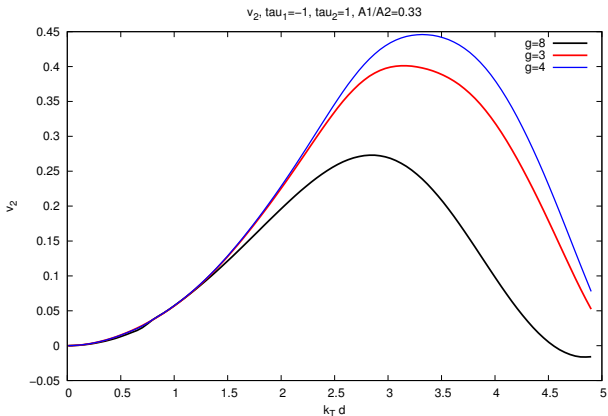
- velocity from c to 0 - first charge
- second: velocity: from c to above 0 - second charge

$$A_1 = \int_{-\infty}^0 dv \frac{e^{iv\Delta}}{(1+v^2)^{\frac{3}{2}}} = \frac{\Delta}{2} (2K_1(\Delta) + i\pi (K_1(\Delta) - L_{-1}(\Delta)))$$

$$A_2 = \int_{\infty}^{v_2} dv \frac{e^{iv\Delta}}{(1+v^2)^{\frac{3}{2}}} = -A_1^* + \int_0^{v_2} dv \frac{e^{iv\Delta}}{(1+v^2)^{\frac{3}{2}}},$$

where the parameters are:

- K_n Bessel functions
- $\Delta = \frac{k_{\perp}}{g}$



- $g = 4$
- $g = 3$
- $g = 8$

v_2 from two radiating point charges with:

- different accelerating times
 - no averaging
- asymmetric tau
given A_1/A_2

Analytically averaging over the phase factor results in:

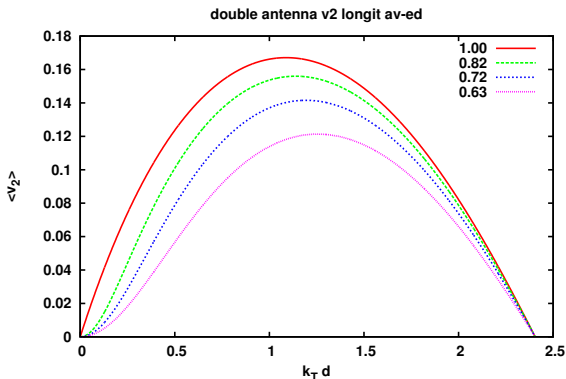
Averaged v_2

$$\langle v_2 \rangle = \frac{J_2(dk_{\perp})}{J_0(dk_{\perp})} \left(\frac{1}{\sqrt{1 - \frac{4\gamma^2}{(1+\gamma^2)^2} J_0^2(dk_{\perp})}} - 1 \right)$$

- leading term: from dipole
- geometric factor: F

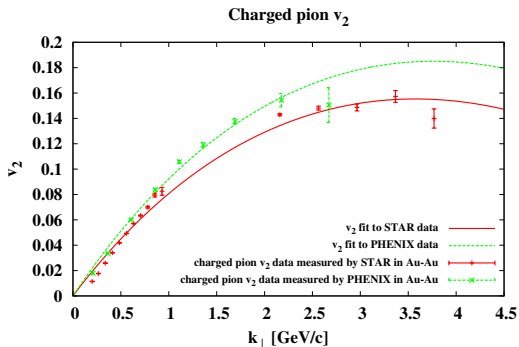
 v_2

$$\langle v_2 \rangle_{fit} = F \cdot \langle v_2 \rangle(dk_{\perp}, \gamma)$$



- $F = 1$
- Only the dipole term exists
- lines: different amplitude ratios

Figure: Analytical results for averaged v_2



- Au-Au collisions
- $\sqrt{s_{NN}} = 62 \text{ GeV} / 200 \text{ GeV}$
- $\gamma \approx 1$
- $d = 0.06 [fm]$

Figure: v_2 of charged pions, data is from STAR (red) and PHENIX (green)

Phys. Rev. C 75, 054906 (2007)
 Phys. Rev. Lett. 91, 182301 (2003)

The model

After Fourier expansion the yield can be written:

$$Y = Y_0 + Y_2 F \cos(2\varphi) \quad \text{with } F: \frac{A^2 - B^2}{A^2 + B^2}$$

How to attach this ellipse to the collisions?

- nuclei disks with radius R
- b : impact parameter
- intersection: ellipse
- subhadronic dipoles
- $F = \frac{b}{2R}$

Dipoles can be ordered parallel or perpendicular to the reaction plane, but F remains the same!