Alternative mechanism to SUSY

based on Int.J.Mod.Phys.**A32**(2016)1645041 and `arXiv:1507.08039`

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Zimányi Winter School
6^{th} December 2016

Outline

Symmetries of local field theories around an arbitrary spacetime point will be studied.

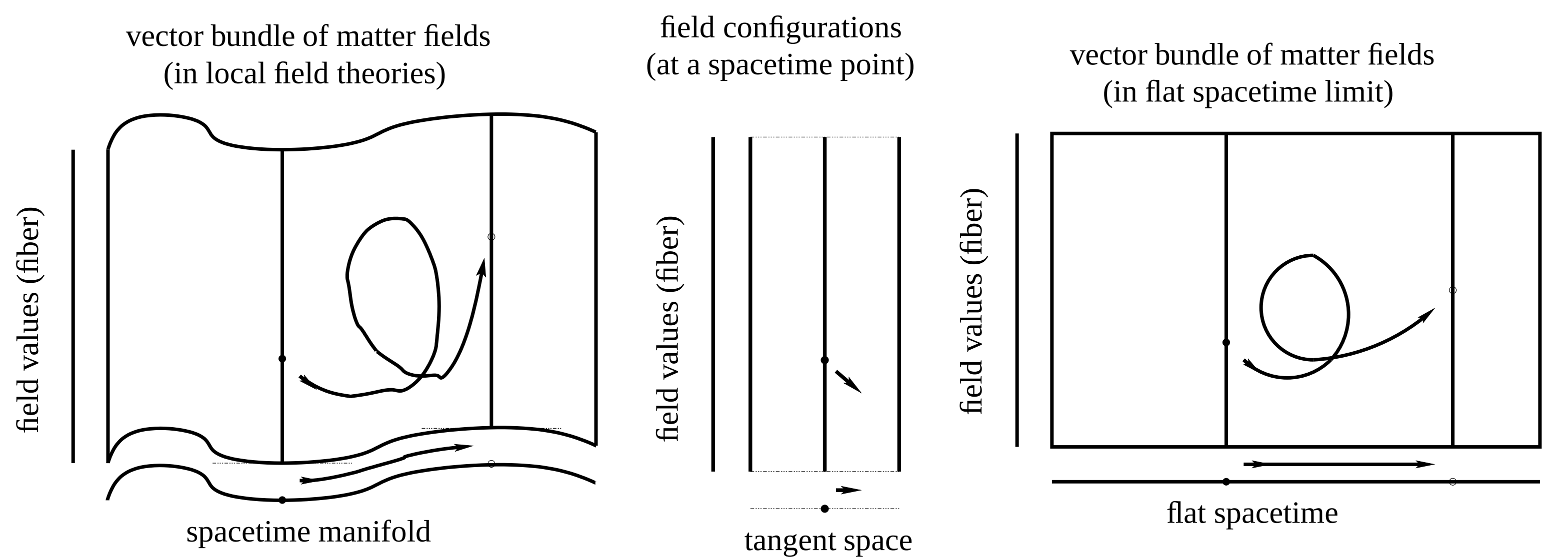
(That is, we study their structure at infinitesimal —Lie algebra— level.)

- Introduction
- General structure of Lie groups and the SUSY
- All possible extensions of the spacetime symmetry group
- Non-SUSY gauge and spacetime symmetry unification
- Summary

Introduction

- **More symmetries simplify a theory.** Larger symmetry requirement reduces the number of variants of a field theoretical Lagrangian, and relates its coupling constants.
- **Grand unification (GUT) strategy.** Models with large, direct-indecomposable symmetry group is looked for.
- **Unification no-go theorems.** Spacetime symmetries (Poincaé group) and compact internal symmetries (compact gauge group) cannot be simply unified (McGlinn1964, Coleman-Mandula1967).
- **Supersymmetry (SUSY).** With this, the no-go theorems are circuimventable (Haag-Lopuszanski-Sohnius1975).
  SUSY seemed justified by its convenient properties.
  But a bit unusual, it is “super-Lie algebra”.
- **SUSY is not seen experimentally.** At present status (ICHEP2016 conference).
- **Alternative exists.** We found a group theoretical mechanism to possibly substitute SUSY.

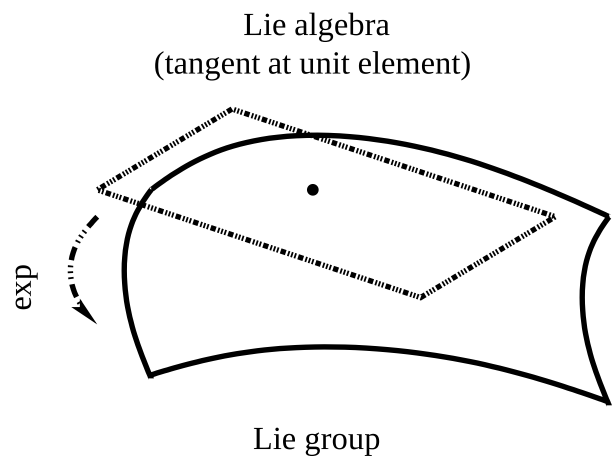
Remark: the no-go theorems are based on global symmetry arguments.
This happens to be enough, since global symmetries infinitesimally act in the same way on the fields at a point of spacetime as local symmetries would do.



For studying symmetries at a point it is enough to study finite dimensional Lie algebras.
Lie algebras are infinitesimal versions of Lie groups, i.e. of parametric groups.

General structure of Lie groups and the SUSY

- **Group.** A collection of transformations, which can be composed, inverted, and there is unit transformation within the collection. (All groups arise in this way.)
- **Lie group.** A parametric group, parametrized by a finite collection of real parameters. E.g.: rotation group, symmetry group of flat plane, Poincaré group, $\mathrm{SU}(N)$ etc.
- **Lie algebra.** Derivatives (or, equivalently, the tangent) of a Lie group at the unit element.



I.e. the Lie algebra is the infinitesimal version of the Lie group.
Exponential map makes a Lie group element from Lie algebra element.

- **Ado's theorem.** Lie algebra completely characterizes Lie group, modulo global topology.
- **Lie bracket.** Lie algebra has the Lie bracket $[\ ,\ ]$ (also called commutator).

- **Subgroup.** A sub-group of a group.
- **Normal, or invariant subgroup.** A subgroup N is normal, whenever $g\,N\,g^{-1} \subset N$, for all group elements g.
  E.g. the translations in the symmetry group of flat plane.
- **Extension, or semi-semi-direct product.** Just synonym to above. If N is normal subgroup, and H is a complementing part, then we can write $N.H$ for the entire group.
- **Semi-direct product.** As above, but complementing part H is subgroup. Notation: $N \rtimes H$.
  E.g. the symmetry group of flat plane is semi-direct product of translations and rotations.
- **Direct product.** As above, but H is normal subgroup. Notation: $N \times H$ or $H \times N$.
  Direct product means that the large group is built of completely independent parts N, H.
  E.g. SM gauge group $\mathrm{U}(1) \times \mathrm{SU}(2) \times \mathrm{SU}(3)$.
  GUT strategy tries to avoid direct product (requirement of direct-indecomposability).

- **Killing-form**: it is an invariant scalar product $x \cdot y = \mathrm{Tr}\,(\mathrm{ad}_x\,\mathrm{ad}_y)$ on any Lie algebra, where $\mathrm{ad}_x(\,) = [x,\,]$. (E.g. it appears in the $F_{ab} \cdot F^{ab}$ Yang-Mills Lagrangian.)

- **Levi decomposition theorem**:

$$\underbrace{E}_{\text{Lie group}} = \underbrace{R}_{\substack{\text{degenerate directions of Killing form} \\ \text{(\textit{radical}, or \textit{solvable part})}}} \rtimes \underbrace{L}_{\substack{\text{non-degenerate directions of Killing form} \\ \text{(\textit{Levi factor}, or \textit{semisimple part})}}}$$

holds for any Lie group, where $\rtimes$ denotes semi-direct product.
The symmetries of flat plane (translations $\rtimes$ rotations) is a typical example.

- **Poincaré group**:

$$\underbrace{\mathcal{P}}_{\text{Poincaré group}} = \underbrace{\mathcal{T}}_{\text{translation group (radical)}} \rtimes \underbrace{\mathcal{L}}_{\text{homogeneous Lorentz group (Levi factor)}}$$

is also a typical demonstration of Levi's decomposition theorem.

super-Poincaré group (SUSY group):

$$\underbrace{\mathcal{P}_s}_{\text{SUSY group}} = \underbrace{\mathcal{S}}_{\text{supertranslation group (radical)}} \rtimes \underbrace{\mathcal{L}}_{\text{homogeneous Lorentz group (Levi factor)}}$$

is a similar example, with a bit larger radical.

Supertranslations: it is a transformation group on the vector bundle of superfields. They act as

$$\begin{pmatrix} \theta^A \\ x^a \end{pmatrix} \mapsto \begin{pmatrix} \theta^A + \epsilon^A \\ x^a + d^a + \sigma^a_{AA'}\mathrm{i}\big(\theta^A\bar{\epsilon}^{A'} - \epsilon^A\bar{\theta}^{A'}\big) \end{pmatrix}$$

on the "supercoordinates" and the affine spacetime coordinates.

One can write:

$$\underbrace{\mathcal{P}_s}_{\text{SUSY group}} = \underbrace{\underbrace{\Big(\underbrace{\mathcal{T}}_{\text{translations}} \cdot \underbrace{\mathcal{Q}}_{\text{supercharges}} \Big)}_{=\mathcal{S}\text{ (group of supertranslations)}} \rtimes \underbrace{\mathcal{L}}_{\text{Lorentz group}}}_{\text{super-Poincer\'e group (SUSY group)}}$$

where arrows indicate which subgroup acts nontrivially on which normal subgroup.

Super-Lie algebra presentation: traditionally they are not presented as Lie algebras, but "super Lie algebras".

$$
\begin{array}{lll}
\checkmark & [\, P_a \;, P_b \,] &= 0, \\
\checkmark & [\, P_a \;, Q_A \,] &= 0, \\
\checkmark & [\, P_a \;, \bar{Q}_{A'} \,] &= 0, \\
!!! \rightarrow & \{\, Q_A \;, Q_B \,\} &= 0, \\
!!! \rightarrow & \{\, \bar{Q}_{A'} \;, \bar{Q}_{B'} \,\} &= 0, \\
!!! \rightarrow & \{\, Q_A \;, \bar{Q}_{A'} \,\} &= 2\,\sigma^a_{\;AA'}\,P_a .
\end{array}
$$

This has also an ordinary Lie algebra presentation [Nucl.Phys.**B76**(1974)477, Phys.Lett.**B51**(1974)239].

For this, instead of the supercharges Q_A, one uses $\delta_{(i)} = \epsilon^A_{(i)}\, Q_A$ generators, where $\epsilon^A_{(i)}$ ($i = 1, 2$) is "supercoordinate" basis (Grassmann valued two-spinor basis).

Via exponentiating this Lie algebra, we get the discussed Lie group, i.e. SUSY is not that exotic. Ordinary Lie group / Lie algebra theory applies.

$\Rightarrow$
With super-Lie algebra, one can exactly generate those ordinary Lie algebras (Lie groups) which have the structure

$$\underbrace{\left(\mathcal{T}\ \cdot\ \mathcal{Q}\right)}_{=\mathcal{S}} \ \rtimes\ \mathcal{L}$$

with

- $\mathcal{S}$ being a normal subgroup,
- $\mathcal{L}$ being a subgroup,
- $\mathcal{T}$ being a normal subgroup in $\mathcal{S}$, and
- $\mathcal{T}$ as well as $\mathcal{Q} \equiv \mathcal{S}/\mathcal{T}$ being abelian. $\leftarrow$!!!
  I.e.: translations are abelian,
  supercharges without considering contribution of translations are abelian.

Last point: makes it possible to switch the sign of “odd” part ($\mathcal{Q}$) with Grassmannian basis.

All possible extensions of the spacetime symmetry group

Any unified gauge - spacetime symmetry group infinitesimally must be Poincaré extension.

These are classified by O'Raifeartaigh theorem (1965), via Levi decomposition:

- **Either:**

$$\begin{array}{ccccc} E & = & R & \rtimes & L \\ \cup & & \cup & & \cup \\ \mathcal{P} & = & \mathcal{T} & \rtimes & \mathcal{L} \end{array}$$

(A) $E = \mathcal{P} \times \{\text{some further Lie group}\}$
(*trivial extension, case of no-go theorems*)

(B) Not (A), $\mathcal{L} \subset L$, R is solvable extension of $\mathcal{T}$.
(*SUSY, and our new example is such*)

- **Or:**

$$\begin{array}{ccccc} E & = & R & \rtimes & L \\ \cup & & & & \cup \\ \mathcal{P} & = & & & (\mathcal{T} \rtimes \mathcal{L}) \end{array}$$

(C) L contains entire $\mathcal{P}$, and L is simple Lie group.
(*conform, and* E_8, $\mathrm{SO}(1,13)$ *theories*)

Consequently: if non-trivial extension is looked for, and we do not want to achieve this via symmetry breaking of a giant symmetry group, then we need to extend the radical (case B).

How (extended) SUSY works?

- **Unification via *extended SUSY group*:**

$$\underbrace{\underbrace{\Big(\big(\underbrace{\mathcal{T}}_{\text{translations}} \times \underbrace{\mathcal{Z}}_{\text{central charges}}\big) \cdot \underbrace{\mathcal{Q}}_{\text{supercharges}}\Big)}_{\text{group of supertranslations}} \rtimes \Big(\underbrace{\mathcal{G}}_{\text{compact internal symmetries}} \times \underbrace{\mathcal{L}}_{\text{Lorentz group}}\Big)}_{\text{group of global extended supersymmetries}}$$

  Arrows: indicate nontrivial subgroup action.

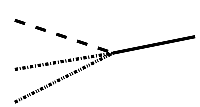
  Parts not connected by arrows: are independent.

- **The extended SUSY group is direct-indecomposable.**

  $\Rightarrow$ Connects spacetime symmetries with compact internal (gauge) symmetries.

  $\Rightarrow$ Connects potentially independent compact internal symmetries with each-other.

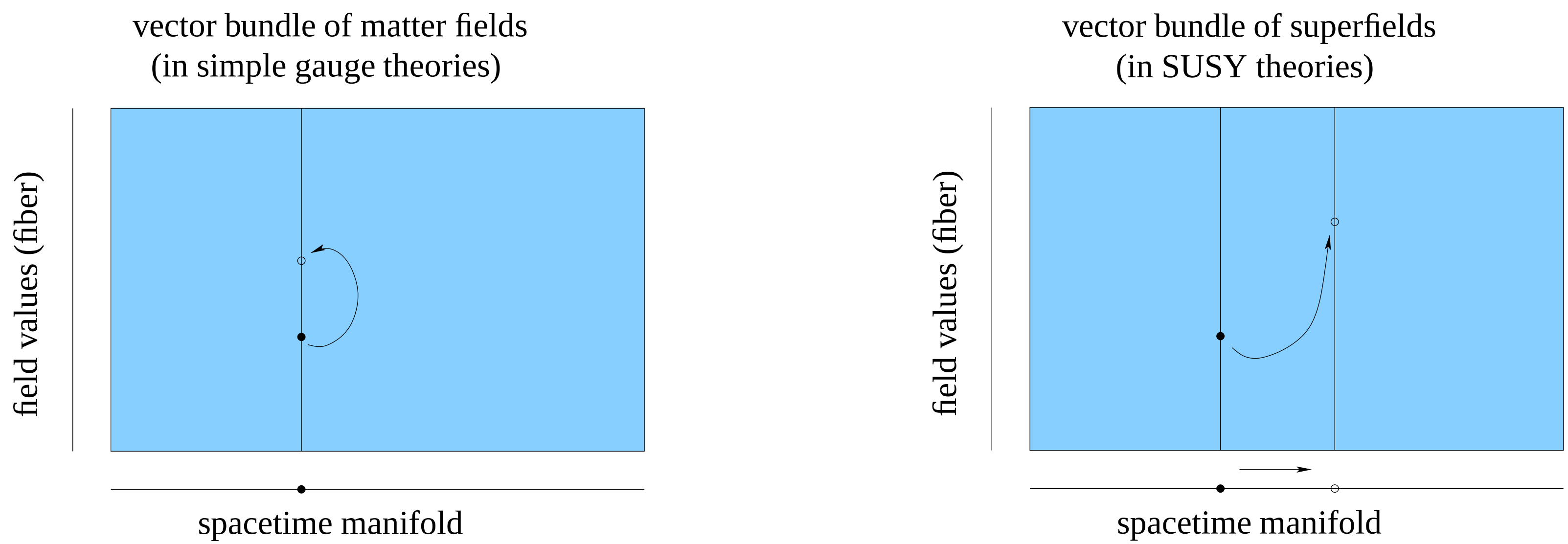
  $\Rightarrow$ Running of coupling factors do unify.



Running of gauge couplings

- **Operated by O'Raifeartaigh theorem case B.** Via the extension of the radical.

- **Symmetry breaking needed.** Because in the (extended) SUSY the complementing symmetries to spacetime symmetries couple too strongly to spacetime symmetries.



Experimental evidence not seen for this, so symmetry breaking needed for an SM-like limit.

Non-SUSY gauge and spacetime symmetry unification

- ***Conservative* extentsions of the Poincaré group.**

  $\Leftrightarrow$ The complementing symmetries are all *inner*, i.e. do not act on spacetime.

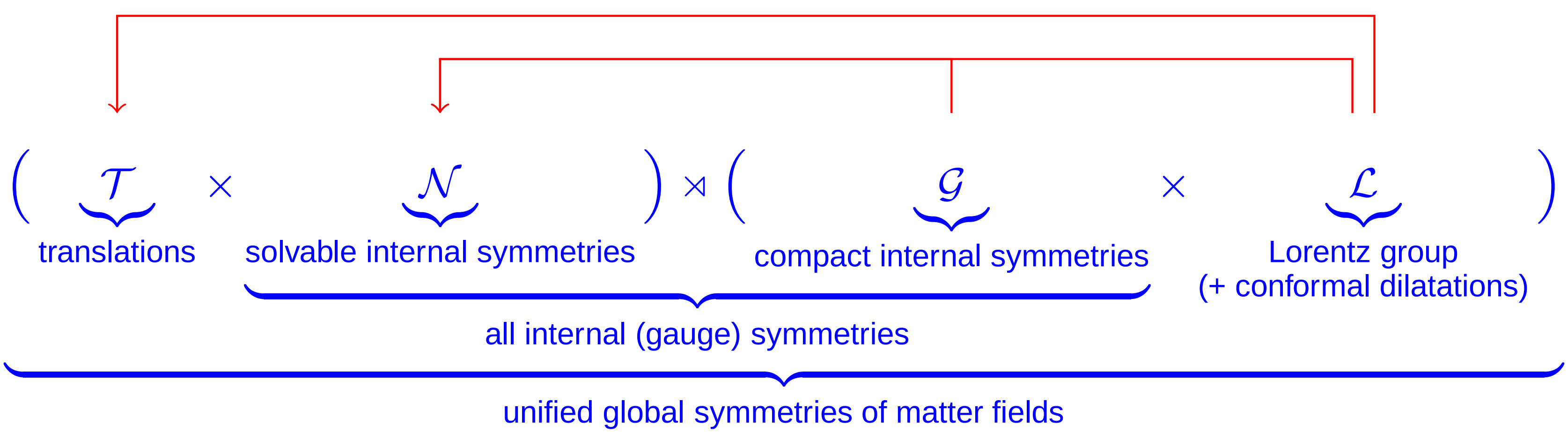
  $\Leftrightarrow$ There exists $\mathcal{P} \stackrel{i}{\longrightarrow} E \stackrel{o}{\longrightarrow} \mathcal{P}$ homomorphisms, such that $o \circ i =$ identity.

  $\Leftrightarrow$ Extension and its invariant restriction both exists.

  $\Leftrightarrow$ The extended symmetries do not need symmetry breaking for an SM-like limit.

- **The (extended) SUSY group is a non-conservative extension of the Poincaré group.**

- **All possible conservative extensions of the Poincaré group:**

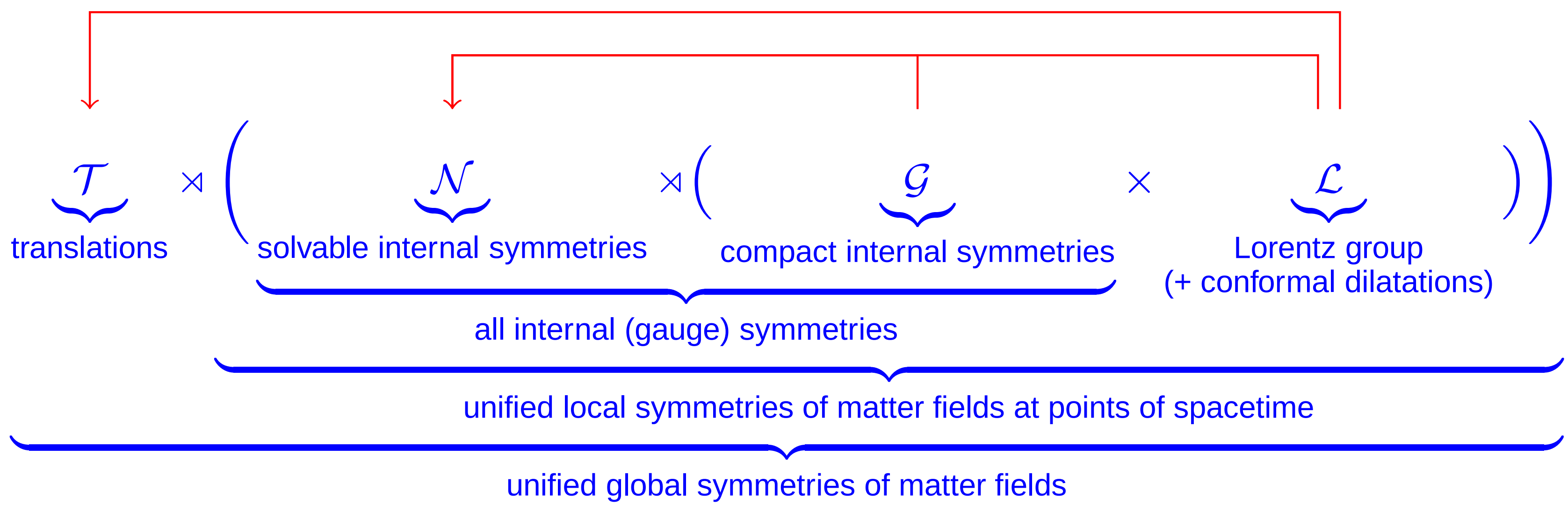


Arrows: indicate nontrivial subgroup action.

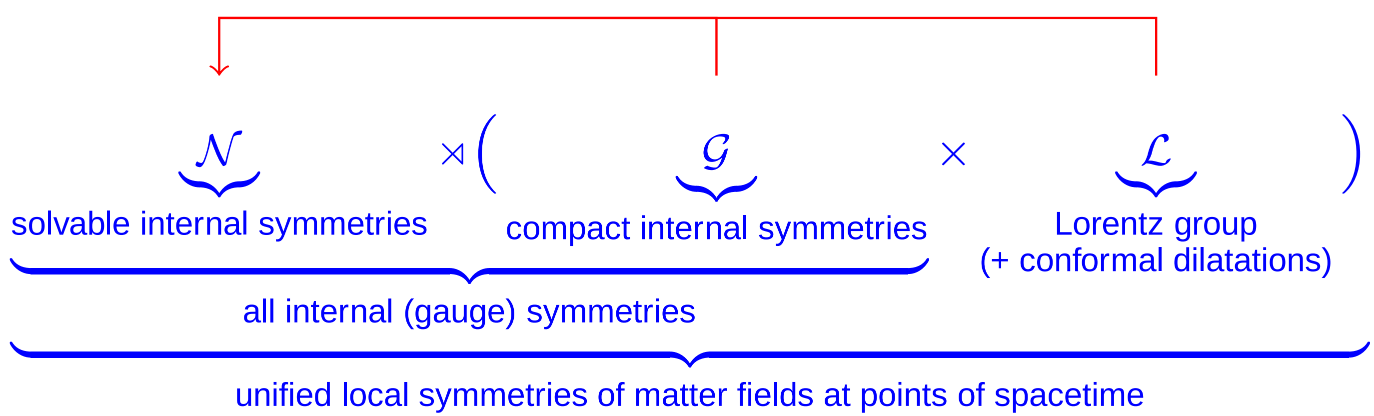
O'Raifeartaigh theorem + energy non-negativity $\Rightarrow$ these are only possible ones.

Similar gauge - spacetime symmetry unification to extended SUSY, via extended radical.

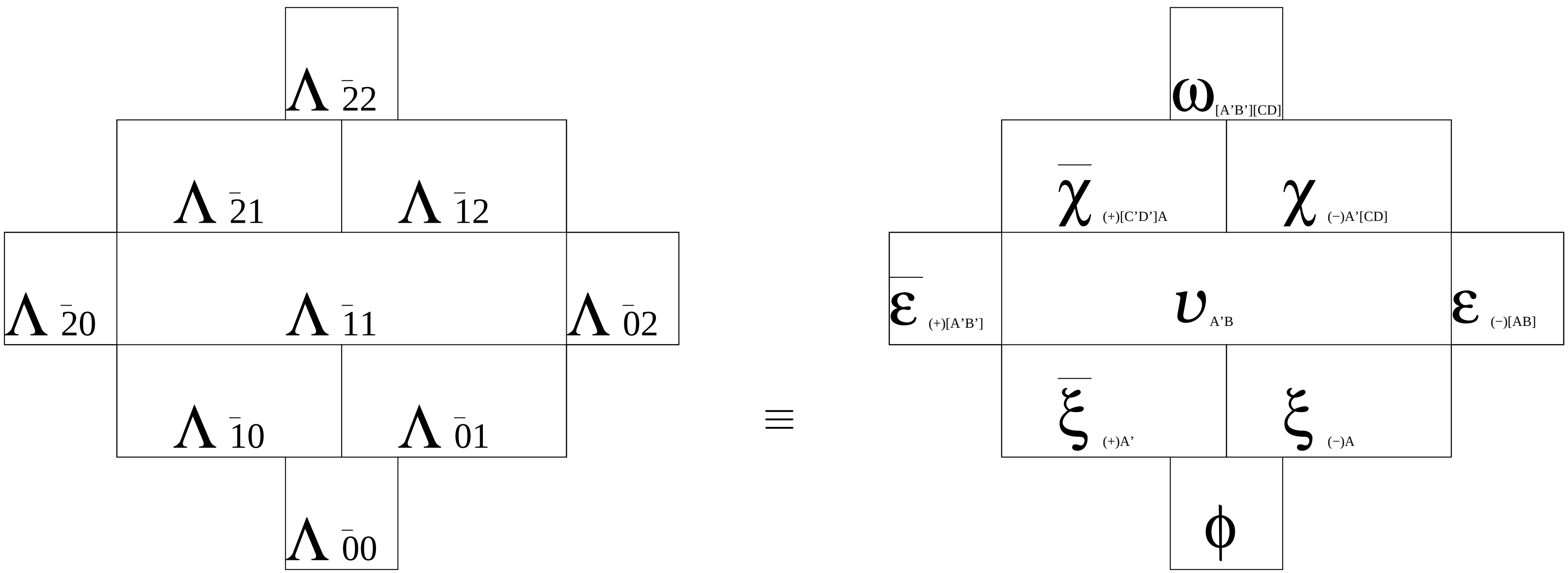
A conservative Poincaré group extension can be made local. The discussed group structure can be equivalently reordered as:



The local, i.e. spacetime pointwise acting part is the key ingredient:



Constructed example for $\mathcal{G} = \mathrm{U}(1)$ in IJMP**A32**(2016)1645041 and `arXiv:1507.08039`.
As a symmetry group of a QFT-related algebra:



$\sim$ algebra of creation operators of particle and antiparticle in the limit if we only had spin. (Encoding 2 fundamental degrees of freedom, Pauli principle, and charge conjugation.)

$\sim$ $\Lambda(\bar{S}^*) \otimes \Lambda(S^*)$, where S^* is lower index two-spinor space.

Price to pay:

- Full gauge group is $\mathcal{N} \rtimes \mathcal{G}$, i.e. “zero-energy” non-propagating gauge field modes ($\mathcal{N}$).
- In QFT analogy: CM is bypassed by relaxing preservation of 1-particle space.

Mechanism singled out group theoretically by “conservativeness” of the Poincaré extension.

Summary

- **SUSY experimentally not visible at present.** See e.g.: ICHEP2016 conference.
- **Mathematical alternatives to SUSY exist.** These are also O'Raifeartaigh B type, as SUSY.
- **The alternative: ”conservative” extensions of the spacetime symmetries.** The complementing symmetries to spacetime symmetries are all inner. Symmetry breaking not needed.
- **Concrente example constructed.** At present, merely with $\mathrm{U}(1)$ as compact gauge group.
- **It connects gauge and spacetime symmetries.** Just like extended SUSY.
- **Harmonizes with present experimental situation.** Extra symmetries are inherently "hidden".