Induced surface tension within novel equation of state of nuclear and hadron matter

O. Ivanytskyi

collaborators: K. Bugaev and V. Sagun

Refs: Nucl. Phys. A 924, 24 (2014)
arXiv:1611.07569 [nucl-th]

Budapest, 7 December 2016

Zimaniy School

Models of hadronic/nuclear matter EoS

- **Relativistic Mean Field models** - Walecka Model, Chiral PerturbationTheory, etc.
  Advantage: QCD symmetries are preserved
  Problems: 1. NO fluctuations $\Rightarrow$ unrealistic critical point
  2. Only few particle species
- **Statistical Cluster models** - Fisher Droplet Model, Gas of Bags Model, Statistical Multifragmentation Model, Hadron Resonance Gas Model
  Advantage: fluctuations $\Rightarrow$ physical critical point
  Problems: Hard core repulsion violates causality
- **Hybrid approach** - Walecka model with nonrelativistic proper volume of nucleons

$$p(T,\mu) = p_{Walecka}(T, \mu - pV_{eigen})$$

D.H. Rischke, M.I. Gorenstein, H.Stoecker and W. Greiner, Z. Phys. C 51(1991) 485

Why hard core repulsion?

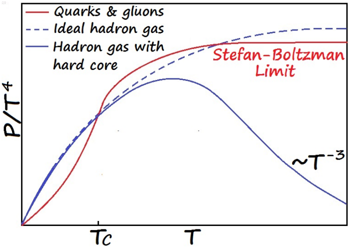
Hadronic hard core

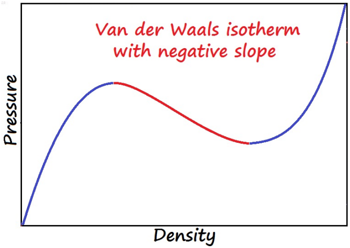
- Prevents phenomenological EoS of QCD from quark confinement at high temperatures

$$ideal\ gas:\ \ p \sim T^4$$

$$hard\ core:\ \ p \sim T$$

- Accounts for short range repulsion between the constituents (hadrons, nuclear fragments, etc.)
- Is necessary for statistical (not Van der Waals) liquid-gas phase transition in cluster models
- Important element in description of particle yields (ideal hadron gas is proven to be inadequate at high A+A collision energies)





J. Cleymans and H. Satz, Z. Phys. C 57, 135 (1993)
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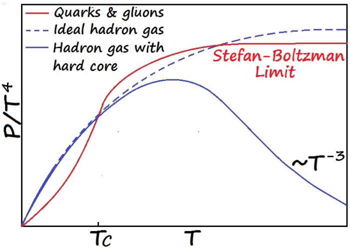

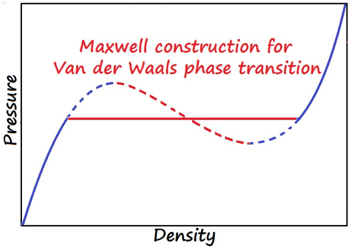

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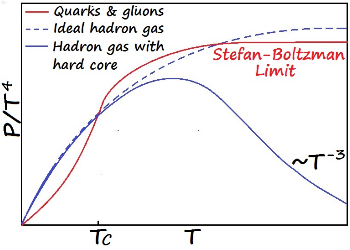

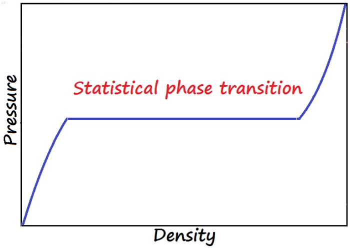

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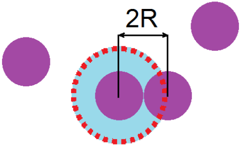
Constraints on hadronic/nuclear matter EoS

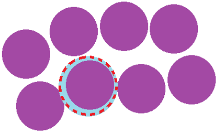
- **Multicomponent EoS** $\Rightarrow$ Grand Canonical Ensemble (GCE) is natural choice
- **Thermodynamic consistency** (in GCE pressure is function of T and μ only)
  $p = p(T, \mu, n)$ $\Rightarrow$ contradiction with thermodynamic relation $n = \frac{\partial p}{\partial \mu}$
  L. van Hove, Physica 15, 951 (1949) and Physica 16, 137 (1950)
- **Switching between excluded and eigen volumes (per particle)**

Low densities



$V_{excl} \simeq 4 V_{eigen}$

High densities



$V_{excl} \simeq V_{eigen}$

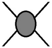
high order virial coefficients are needed

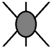
- **Causality:** $c_{sound} \leq c_{light} = 1$, where $c^2_{sound} = \frac{dp}{d\epsilon}\Big|_{s/n=const}$
  Existing approaches to restore causality violated by hard core are rather complicated
  K. Bugaev, Nucl. Phys. A 807 (2008)

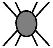
Something more convenient for practical applications is needed

Application to compact astrophysical objects

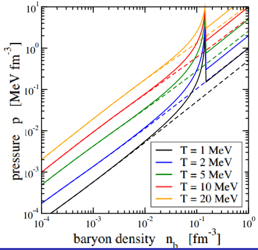
- Hadronic EoS used for modeling of the neutron star interiors **violates causality**
  H. Grigorian, COST Action @ CPOD 2016, Wroclaw
- Three, four, ... particle forces are needed - **high order virial coefficients**
  G. Baym, COST Action @ CPOD 2016, Wroclaw

$2 \rightarrow 2$ reactions  $\Rightarrow$ **2^{nd} virial coefficient**

$3 \rightarrow 3$ reactions  $\Rightarrow$ **3^{rd} virial coefficient**

$4 \rightarrow 4$ reactions  $\Rightarrow$ **4^{th} virial coefficient**

- Observational constrains require soft/stiff EoS at low/high densities
  T. Kojo, COST Action @ CPOD 2016, Wroclaw
  hyperons $\Rightarrow$ softening of the hadronic EoS
  multicomponent EoS in GCE is needed
- Thermodynamically inconsistent suppression of deutrons in the neutron star interiors $\Rightarrow$
  pressure discontinuity = zero order PT
  S. Typel, Eur. Phys. J. A (2016) 52, 16



Objectives

- Simple multicomponent EoS with hard core repulsion
- Thermodynamic consistency
- Correct asymptotics of excluded volume EoS at low and high densities
- Higher virial coefficients
- Causality up to densities where QGP is expected

Multicomponent mixture of Boltzmann hard spheres

- Virial expansion in powers of one particle thermal densities $\{\phi_i\}$

$$\frac{p}{T} = \sum_i \phi_i e^{\frac{\mu_i}{T}} - \sum_{i,j} \overbrace{\frac{2\pi}{3}(R_i + R_j)^3}^{2^{nd}\ \ virial\ \ coefficient} \phi_i \phi_j e^{\frac{\mu_i+\mu_j}{T}} + \mathcal{O}(\phi^3)$$

$$= \sum_i \phi_i e^{\frac{\mu_i}{T}} \left(1 - \underbrace{V_i \sum_j \phi_j e^{\frac{\mu_j}{T}}}_{bulk\ \ term} - \underbrace{S_i \sum_j R_j \phi_j e^{\frac{\mu_j}{T}}}_{surface\ \ term}\right) + \mathcal{O}(\phi^3)$$

- VdW like extrapolation (gives exponentials)

$$\begin{matrix} \sum_j \phi_j e^{\frac{\mu_j}{T}} \simeq \frac{p}{T} \\ \sum_j R_j \phi_j e^{\frac{\mu_j}{T}} \simeq \frac{\Sigma}{T} \end{matrix} \quad \Rightarrow \quad \left\{ \begin{matrix} \frac{p}{T} = \sum_i \phi_i \exp\left(\frac{\mu_i - pV_i - \Sigma S_i}{T}\right) - \textbf{pressure} \\ \frac{\Sigma}{T} = \sum_i \phi_i \exp\left(\frac{\mu_i - pV_i - \Sigma S_i}{T}\right) R_i - \textbf{surface tension} \end{matrix} \right.$$

V.Sagun, A.Ivanytskyi, K. Bugaev, I. Mishustin, Nucl. Phys. A 924, 24 (2014)

- Hard core repulssion only in part is accounted by eigen volume. The rest corresponds to **induced surface tension**

Extrapolation to high densities

- Extrapolation to high densities is not unique $\Rightarrow$ equations for p and Σ can differ

$$\frac{p}{T} = \sum_i \phi_i \exp\left(\frac{\mu_i - pV_i - \Sigma S_i}{T}\right)$$

$$\frac{\Sigma}{T} = \sum_i R_i \phi_i \exp\left(\frac{\mu_i - pV_i - \Sigma S_i}{T}\right) \cdot \underbrace{\exp\left(\frac{(1-\alpha)S_i\Sigma}{T}\right)}_{\text{not uniqueness of extrapolation}}, \quad \alpha = const$$

- Meaning of $\alpha > 1$: one component case

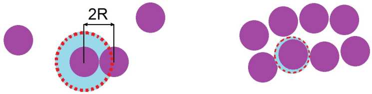
$$\Sigma = pR \exp\left(\frac{(1-\alpha)S\Sigma}{T}\right)$$

$$p = T\phi \exp\left(\frac{\mu - pV_{eff}}{T}\right) \quad \Rightarrow \quad \begin{array}{l} low\ densities\ (\Sigma \to 0) : V_{eff} = 4V \\ high\ densities\ (\Sigma \to \infty) : V_{eff} = V \end{array}$$

$$V_{eff} = V\left[1 + 3\exp\left(\frac{(1-\alpha)S\Sigma}{T}\right)\right]$$

α switches excluded and eigen volume regimes

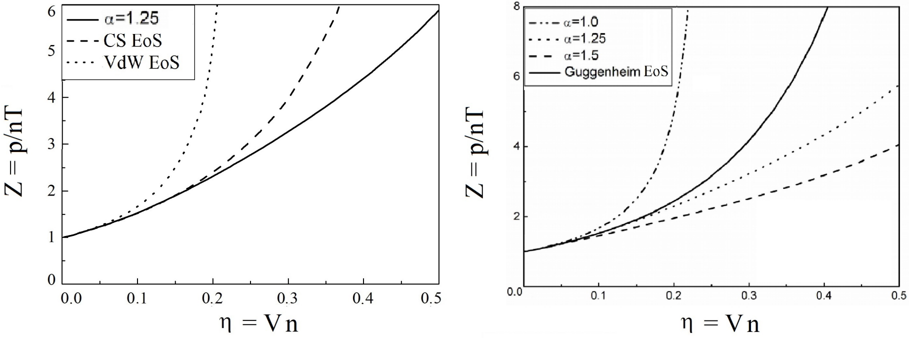
high order virial coefficients?



- Higher virial coefficients of hard spheres
  - **Second virial coefficient** – reproduced for any α
  - **Third virial coefficient** – reproduced for $\alpha = 1.245$ whithin 16%
  - **Fourth virial coefficient** – reproduced for $\alpha = 1.245$

Comparison with other one component EoS

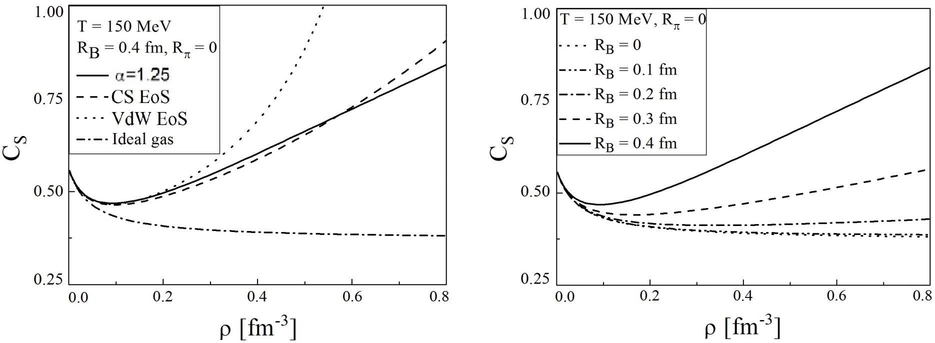
- One component Boltzmann gas of hard spheres



- VdW EoS: $Z = (1-4\eta)^{-1}$ - **is rather stiff**
- Guggenheim EoS: $Z = (1-\eta)^{-4}$ - **reproduced up to** $\eta \simeq 0.2$
- Carnahan-Starling EoS (reproduces 7 virial coefficients): $Z = \frac{1+\eta+\eta^2-\eta^3}{(1-\eta)^3}$
  - **reproduced up to** $\eta \simeq 0.22$

Causality of IST EoS at very extreme cases

- Boltzmann mixture of baryons (N and Δ) and pions



At $\alpha = \mathbf{1.25}$ multicomponent EoS is causal
up to $\simeq 7$ normal nuclear densities where quark matter is expected

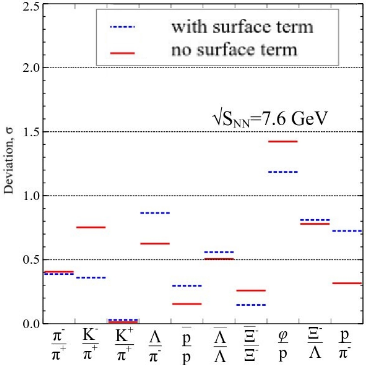
EoS with Induced Surface Tension

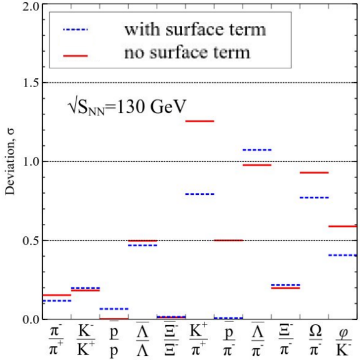
$$p = \sum_i p_0(T, \mu_i) \;\Rightarrow\; \begin{cases} p = \sum_i p_0(T, \mu_i - pV_i - \Sigma S_i) \\ \Sigma = \sum_i p_0(T, \mu_i - pV_i - \alpha \Sigma S_i) R_i \end{cases}$$

- Advantages compared to other EoS with hard core repulsion:
  - Multicomponent character and thermodynamic consistency
  - Correct asymptotic of excluded volume at high and low densities, higher virial coefficients
  - Wide range of causality
  - Straightforward generalization to quantum statistics and mean field models
- Questions
  - Value of α in case of quantum statistics? Medium dependent α?
  - ...

Hadron Resonance Gas

- Hadrons with masses ≤ 2.5 GeV (widths, strong decays, zero strangeness)
- 111 independent particle ratios measured at 14 energies
- 14×4 local parameters $(T, \mu_B, \mu_{I3}, \gamma_s)$ + 5 global parameters (hard core radii)

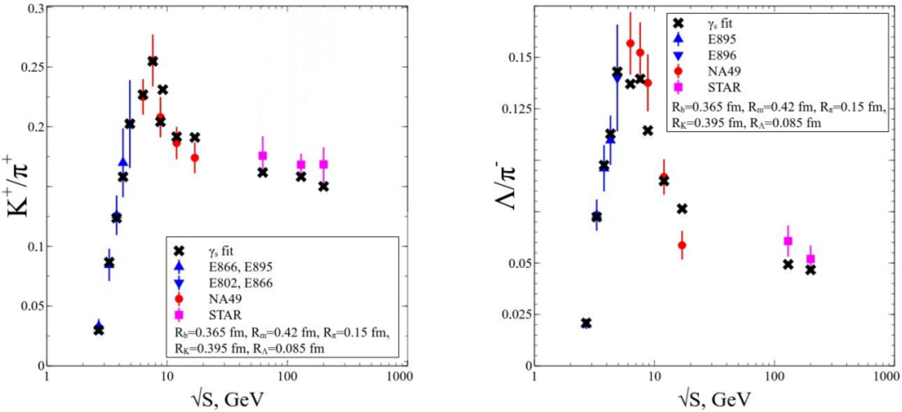




$$R_b = 0.365\ fm,\ R_m = 0.42\ fm,\ R_\pi = 0.15\ fm,\ R_K = 0.395\ fm,\ R_\Lambda = 0.085\ fm$$

$$Overall\ \chi^2/dof \simeq 1.038$$

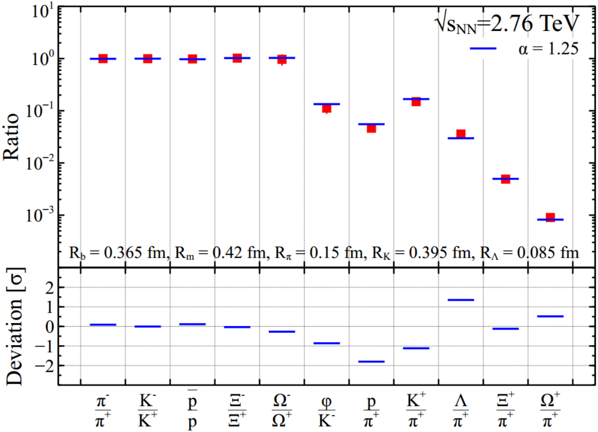
K^+/π^+ and Λ/π^- ratios



- Induced surface tension correction improves quality of the data description
- Advantage in multicomponent case - **EoS includes just two equations EoS of earlier HRG model versions - N equations for N components**

Hadron Resonance Gas at ALICE energies

- 11 independent particle yields, 6 parameters (temperature + 5 hard core radii)
- Overal $\chi^2/dof \simeq 1.038$
- Freeze out temperature $T_{FO} = 154 \pm 7 \ MeV$



Applicability test with multicomponent Carnahan-Starling EOS

5 % deviation from MCSL EOS

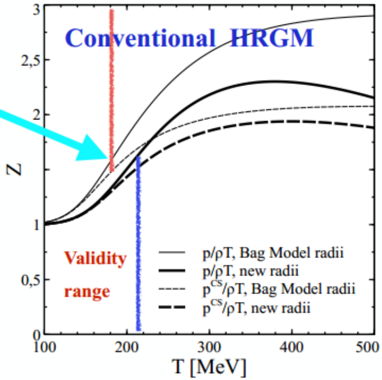
compressibility $Z = p/(\rho T)$

total pressure of the system is p,

total particle density is $\rho = \sum_{k=1}^{N} \rho_k$

$$p^{CS} = \frac{6T}{\pi}\left[\frac{\xi_0}{1-\xi_3} + \frac{3\,\xi_1\xi_2}{(1-\xi_3)^2} + \frac{3\,\xi_2^3}{(1-\xi_3)^3} - \frac{\xi_3\xi_2^3}{(1-\xi_3)^3}\right],$$

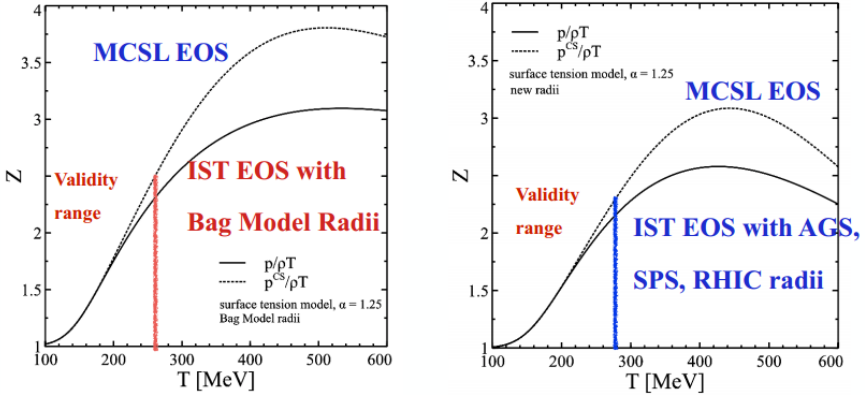
$$\xi_n = \frac{\pi}{6}\sum_{k=1}^{N} \rho_k\,[2\,R_k]^n\,.$$



Using densities, T and hard-core radii we calculated compressibility for each EOS and compared them with Z of MCSL EOS

MCSL EOS: G. A. Mansoori, N. F. Carnahan, K. E. Starling and T. W. Leland, Jr., J. Chem. Phys. 54, 1523 (1971)

Same Applicability Test for IST EOS



IST EOS with Bag Model radii is valid for T < 260 MeV

IST EOS with new radii is valid for T < 280 MeV

IMPORTANT: IST EOS is much softer than MCSL EOS at high densities!

Summary

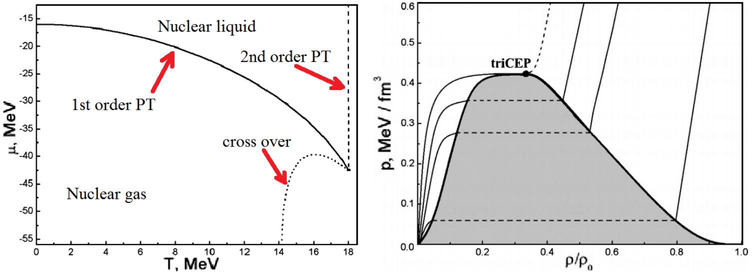
- Thermodynamically consistent approach to account for the excluded volume effects is proposed.
- Developed EoS reproduces 3rd and 4th virial coefficients and provides causality up to 7 normal nuclear densities.
- Approach to study induced surface tension. is developed.
  Tolman length caused by curvature effects is evaluated. **Not in this talk :(**
- Novel EoS is applied to study properties of hadronic matter.
  Phase diagram of nuclear matter is analyzed. **Not in this talk :(**
- Generalization of EoS with induced surface tension is discussed.

Thank you for attention

Nuclear matter in Statistical Multifragmentation Model

- Degrees of freedom: nucleons and composite nuclear fragments
- Interaction: hard-core repulsion, effective attraction due to large number of constituents like in the original Statistical Bootstrap Model

  J. P. Bondorf et al., Phys. Rep. 257, 131 (1995) and references therein
- Phase diagram with statistical liquid-gas phase transition and critical point



  - Properties of normal nuclear matter
  - Compressible nuclear liquid ⇒ critical density $\simeq \frac{\rho_0}{3}$ which is typical for liquid-gas PT (standard SMM with incompressible nuclear liquid predicts $\rho_{cep} = \rho_0$)

V. V. Sagun, A. I. Ivanytskyi, K. A. Bugaev, I. N. Mishustin, Nucl. Phys. A 924, 24 (2014)

Thermodynamic consistency

- Grand Canonical Ensemple $\Rightarrow$ pressure is function of T, μ and V only, $p = p(T, \mu, n)$ $\Rightarrow$ contradiction with thermodynamic relation $n = \frac{\partial p}{\partial \mu}$
  L. van Hove, Physica 15, 951 (1949) and Physica 16, 137 (1950)
- van der Waals EoS is thermodynamically consistent

$$p = \underbrace{\frac{Tn}{1 - nb}}_{CE} = \underbrace{T \phi \exp\left(\frac{\mu - pb}{T}\right)}_{GCE} \quad if \;\; n = \frac{\partial p}{\partial \mu}, \;\; \phi = \underbrace{g \int \frac{d\vec{k}}{(2\pi)^3} e^{-\frac{\omega(\vec{k})}{T}}}_{one\;\;particle\;\;thermal\;\;density}$$

- Available volume fraction commonly used to account for hard core repulsion

$$motivated\;\;by\;\;VdW\;\;EoS\;\;in\;\;CE: \; p = \frac{Tn}{V\Phi_{VdW}(n)}, \quad \Phi_{VdW}(n) = 1 - bn$$

$$GCE\;\;generalizations: \; p = \frac{p_{id}(T, \mu)}{\Phi(n)} \quad - \quad \textbf{not thermodynamically consistent}$$

S. Typel, Eur. Phys. J. A (2016)52:16

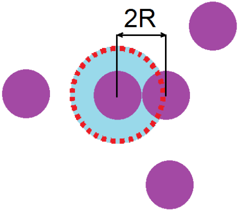
- GCE is much more suitable for multicomponent mixtures than CE

Thermodynamic consistency should be provided

Excluded or eigen volume?

hard core repulsion blocks the part of space for particle motion $\Rightarrow$ **excluded volume**

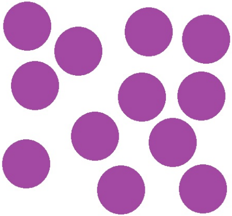
Low densities



translation of one particle around another

$$V_{excl} = \frac{1}{2} \cdot \frac{4\pi}{3}(2R)^3 = 4V_{eigen}$$

High densities



motion of particles is restricted

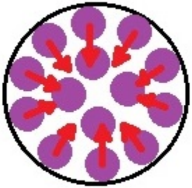
$$V_{excl} \simeq V_{eigen}$$

- replacement $\mu \to \mu - pV_{eigen}$ is based on the **high density approximation**
- VdW EoS is extrapolation of the **low density expastion**

Contradiction?

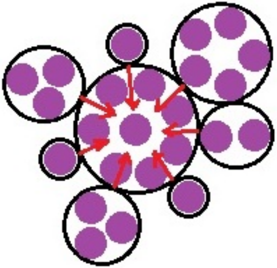
Physical origin of the induced surface tension

Vacuum



attraction of constituents
⇒ **eigen surface tension**

Medium



repulsion of clusters
⇒ **induced surface tension**

- Hard core repulsion only in part is accounted by eigen volume
- The rest corresponds to surface tension and curvature tension
  Curvature tension can be accounted explicitly or implicitly
- Physical clusters tend to have spherical (in average) shape

Causality

- Causality condition: $c_{sound} \leq c_{light} = 1$, where $c^2_{sound} = \left.\frac{dp}{d\epsilon}\right|_{\frac{s}{n}=const}$
- VdW EoS violates causality:

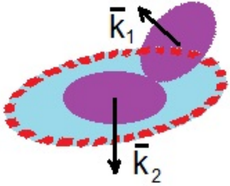
$$p = T\phi(T)\exp\left(\frac{\mu - pb}{T}\right) \quad \Rightarrow \begin{cases} p \to \mu/b \\ n \to 1/b \\ s \to T\phi'/b\phi \\ \epsilon = Ts + \mu n - p \to T^2\phi'/b\phi \end{cases} \Rightarrow c_{sound} \to \infty$$

dense packing $(\mu \to \infty)$

absolutely rigid objects are absent in nature due to Lorentz contraction

- Lorentz contraction of hard spheres is accounted for non-quantum VdW EoS
  K. Bugaev, Nucl. Phys. A 807 (2008)

relativistic hard core potential
is momentum dependent
$\Downarrow$
prescription is rather complicated



One should look for something
more convenient for practical applications

High order virial coefficients

- Virial expansion of one component EoS with induced surface tension

$$p = nT\Big[1 + \overbrace{4V}^{a_2} n + \overbrace{\Big(16 - 18(\alpha - 1)\Big) V^2}^{a_3} n^2 + \underbrace{\Big(64 - 216(\alpha - 1) + \frac{243}{2}(\alpha - 1)^2\Big) V^3}_{a_4} n^3\Big] + \mathcal{O}(n^5)$$

- Second virial coefficient of hard spheres $a_2 = 4V$ is reproduced always
- Third virial coefficient of hard spheres

$a_3 = 10V^2 \;\Rightarrow\; \alpha = \frac{4}{3},\; a_4 = \frac{11V^3}{2}$ - not reproduced

- Fourth virial coefficient of hard spheres

$a_4 \simeq 18.365V^3 \;\Rightarrow\; \alpha \simeq 2.537,\; a_3 \simeq -11.666V^2$ - not reproduced
$\alpha \simeq 1.245,\; a_3 \simeq 11.59V^2$ - reproduced with 16 % accuracy

One parameter reproduces two (3rd and 4th) virial coefficients and allows generalization for multicomponent case

Tolman correction

- Extrapolation to high densities is not unique $\Rightarrow$ surface and curvature terms can be treated separately

$$\frac{P}{T} = \sum_i \phi_i e^{\frac{\mu_i}{T}} \left(1 - V_i p - \overbrace{\frac{S_i}{2} \sum_j R_j \phi_j e^{\frac{\mu_j}{T}}}^{surface\ \ term} - \overbrace{2\pi R_i \sum_j R_j^2 \phi_j e^{\frac{\mu_j}{T}}}^{curvature\ \ term}\right) + \mathcal{O}(\phi^3)$$

- Term proportional to circumference $L_i = 2\pi R_i^2$ is accounted explicitly

$$\frac{p}{T} = \sum_i \phi_i \exp\left(\frac{\mu_i - pV_i - \Sigma S_i - CL_i}{T}\right) - \textbf{pressure}$$

$$\frac{\Sigma}{T} = \sum_i \phi_i \exp\left(\frac{\mu_i - pV_i - \Sigma S_i - CL_i}{T}\right) \frac{R_i}{2} - \textbf{surface tension}$$

$$\frac{C}{T} = \sum_i \phi_i \exp\left(\frac{\mu_i - pV_i - \Sigma S_i - CL_i}{T}\right) \frac{R_i^2}{2} - \textbf{curvature tension}$$

- Tolman correction to surface tension

$$\Sigma_i^{tot} = \Sigma \left(1 - \frac{2\delta}{R_i}\right), \qquad \delta = \frac{C}{\Sigma} - \textbf{Tolman length}$$

R.C. Tolman, J. Chem. Phys. 17, 333 (1949)

Negative values of surface tension

Surface Free Energy: F = E -TS

To find surface F one has to count for ALL surface deformations together with energy costs
Can be exactly done within Hills and Dales Model for v-volume cluster:

K.A. Bugaev et al, PRE 72 (2005)

$$\underbrace{\exp\left[-\frac{\sigma_o v^{2/3}}{T}\right]}_{Energy\ part}\ \underbrace{\exp(S)}_{Entropy\ part} = \underbrace{\exp\left[-\frac{\sigma_o v^{2/3}}{T}\right]}_{Sphere's\ Energy} \times \left\{1 + \Big(\underbrace{w_H N_H}_{1\ Hill} + \underbrace{w_D N_D}_{1\ Dale}\Big) \exp\left[-\frac{\sigma_o \Delta S_1}{T}\right] + 2,\ 3,\ \text{etc deformations}\right\}$$

$$= \underbrace{\exp\left[-\frac{\sigma_o v^{2/3}}{T}\right]}_{Energy\ part}\ \underbrace{\exp\left[+\frac{\sigma_o v^{2/3}}{T_c}\right]}_{Entropy\ part}$$

Simplest case (M. Fisher)

Also one can find supremum and infimum for surface F and surface partition

$$\sigma_0(1-\lambda_L T)\, v^{\frac{2}{3}} \;\geq\; F \;\geq\; \sigma_0(1-\lambda_U T)\, v^{\frac{2}{3}}, \quad \lambda_L \approx 0.28\, T_c^{-1}, \quad \lambda_U \approx 1.06\, T_c^{-1}$$

K.A. Bugaev & J. Elliott, UJP 52 (2007)

Thus, there is NOTHING wrong, if surface free energy $F < 0$ for high T!

This means only that entropy contribution dominates!