Correspondence of Many-flavor Limit and Kaluza-Klein Degrees of Freedom in the Description of Compact Stars

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Motivation

- neutron stars are celestial laboratories for the study of dense matter
- recent observations have uncovered both massive and low-mass neutron stars and have also set constraints on neutron star radii

Circinus X-1: X-ray light rings from a binary neutron star
(24 June 2015; Chandra X-ray Observatory)

Motivation

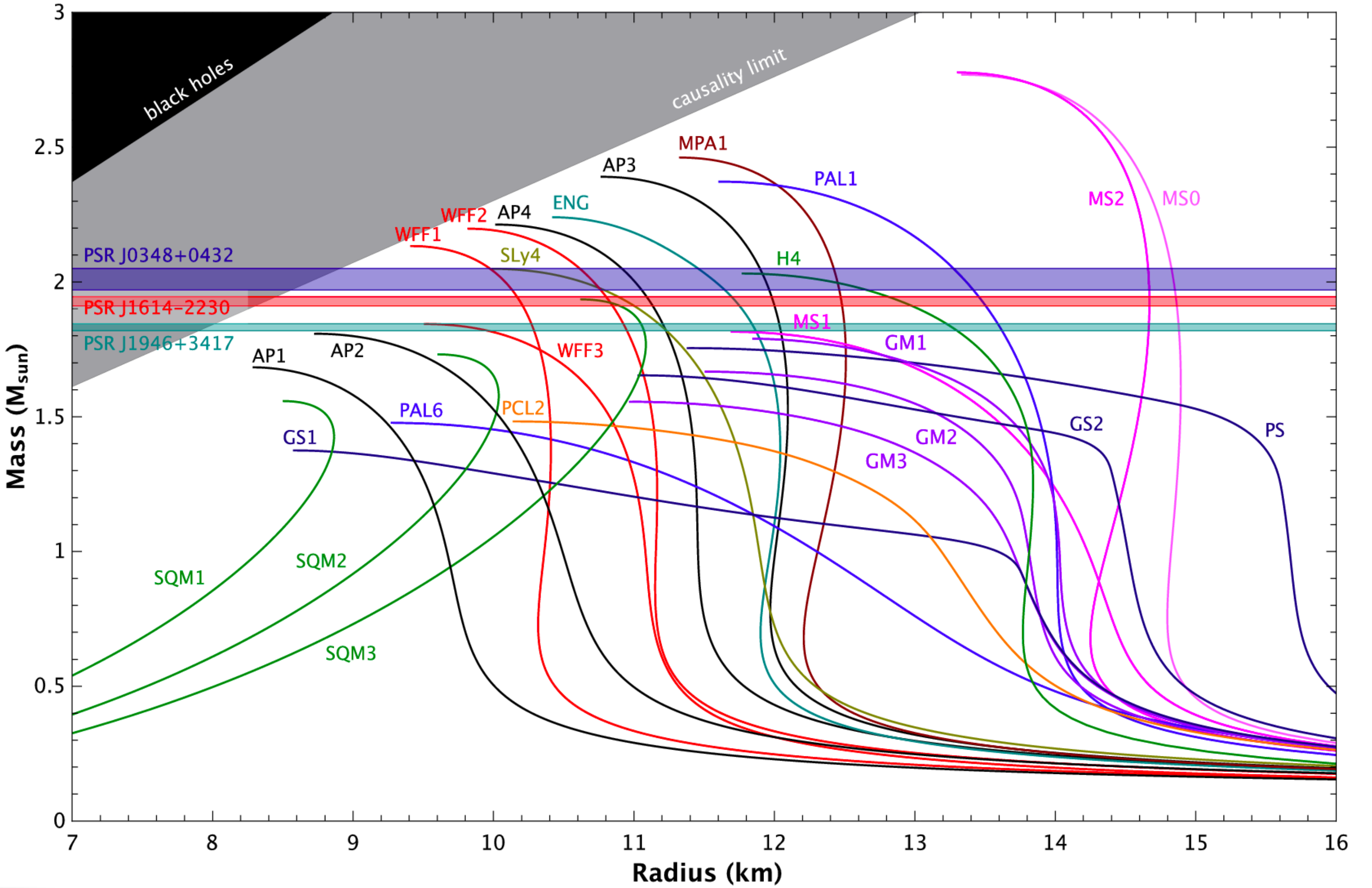
- the largest mass measurements are powerfully influencing the high-density equation of state because of the existence of the neutron star maximum mass
- the smallest mass measurements, and the distributions of masses, have implications for the progenitors and formation mechanisms of neutron stars
- the ensemble of mass and radius observations can realistically restrict the properties of dense matter, and, in particular, the behavior of the nuclear symmetry energy near the nuclear saturation density
- simultaneously, various nuclear experiments are progressively restricting the ranges of parameters describing the symmetry properties of the nuclear equation of state

Neutron stars

- the concept of the neutron star dates back to the 1930s, when it was discussed in the context of general relativity by Oppenheimer ➔ a dense neutron gas could support itself under gravitational attraction provided the total mass was not greater than ~$2M_{\odot}$ (Baade & Zwicky, 1933; Kalogera & Baym, 1996)

- this idea remained largely academic until the discovery of pulsars in 1967 by J. Bell et al.

Jocelyn Bell

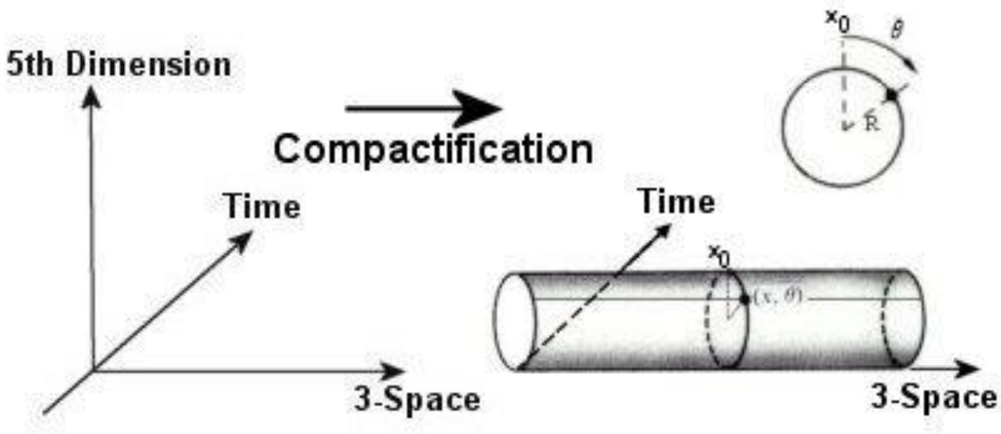


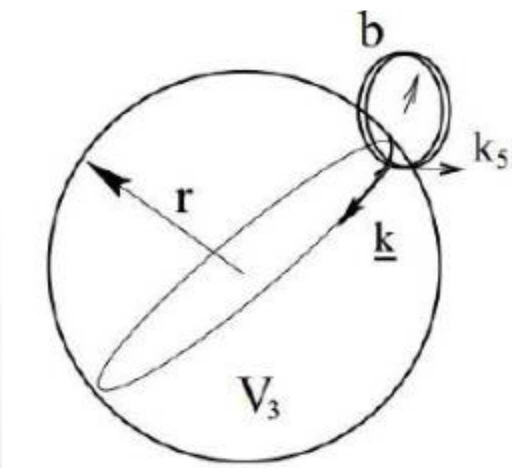
Precise neutron star mass measurements are useful for a variety of purposes. One of them is to constrain the macroscopic behaviour (in particular the relation between density and pressure, known as the equation of state, or EOS) of the cold, super-dense matter at the center of a neutron star. For each EOS (named in the figure) the relation between mass and radius for all neutron stars is indicated by its related curve. If a particular EOS predicts a maximum mass smaller than the largest measured NS mass (horizontal bars are for the three most massive pulsars tabulated below) then it is excluded. Figure created by Norbert Wex.
EOSs tabulated in Lattimer & Prakash (2001) and provided by the authors.

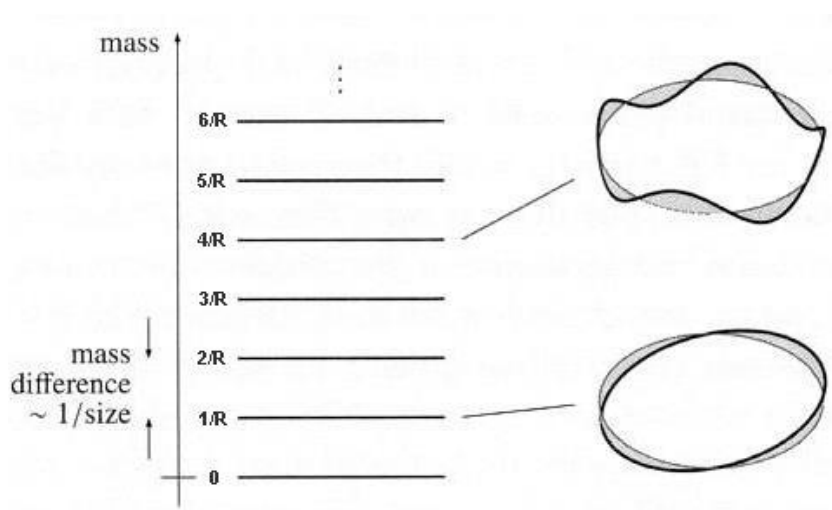
Our model

- in the Kaluza-Klein model the gravity and quantum field theory can be unified at energy scale lower than the Planck's scale (Kaluza,1921; Klein,1926; Antoniadis,1990)
- in the simplest case a $3+1_C+1$ dimensional space-time can be introduced, where excited particles can move freely along the extra x^5 spatial direction as well
- in this manner we 'geometrize' quantum fields, where charges are associated with compactified spatial extra dimensions, induced by the topological structure of the space-time

Our model – Klauza-Klein model







Mass of the excited states

- the compactness of the extra dimension generates a periodic boundary condition, which results a Bohr-type quantization condition for the k_5 momentum component ➔ the relation induces an uncertainty in the position with the size (volume) of $2\pi R_c$
- an interesting feature of this space-time structure, that motion into the 5^{th} dimension generates an extra mass term by k_5, what appears as 'excited mass', *m* in the standard 3+1-dimensional space-time

$$k_5 = n\ \hbar c / R_\mathrm{c} \Rightarrow\ m = \sqrt{\widehat{m}^2 + (n\ \hbar c / R_\mathrm{c})^2}$$

Mass of the excited states

- $\widehat{m}$ is the particle mass in the 5-dimensional description
- *n* is the excitation number
- R_c is the compactified radius ➔ for example

$R_c \approx 10^{-13}$ cm ➔ this 'extra mass' gap $\Delta m \approx 100$ MeV

- an available value in hadron spectroscopy by the TeV energy accelerators, such as the Large Hadron Collider (Arkhipov,2004) or even in superdense compact stars
- in this work the extra compactified 5^{th} dimension represents the hypercharge or the similar quantum number (strangeness, charm, or bottomness) connected to even several number of flavors

Mass of the excited states

- in our first approach:

$$m = \sqrt{\widehat{m}^2 + (n\ \hbar c/R_\mathrm{c})^2} \Rightarrow m = m_{n^0} + n\Delta m$$

- based on the above model on the degrees of freedom, we present our results for the EoS in the case of non-interacting fermion gas in $3+1_C+1$ dimensional space-time with excitations (n) and 'extra mass' gap (Δm) values as highlighted above

Method

- calculated from Tolman-Oppenheimer-Volkov equation in static, spherical symmetric, 5-dimensional spacetime applying several many-component, non-interacting fermion EoS-s for given n and Δm values

$$\frac{\mathrm{d}p(r)}{\mathrm{d}r} = -\frac{[p(r)+\epsilon(r)]\left[M(r)+4\pi r^3 p(r)\right]}{r[r-2M(r)]}$$

- M(r) gives the mass included in a sphere with the same center as the neutron star and the radius r

$$M(r) = 4\pi \int_0^r \epsilon(r) r^2 \mathrm{d}r$$

Method

- the pressure and energy per state:

$$p_i = \frac{g_i}{24\pi^2}\left[\mu_i\sqrt{\mu_i^2 - m_i^2}\left(\mu_i^2 - \frac{5}{2}m_i^2\right) - \frac{3}{2}m_i^4 \ln\frac{m_i}{\mu_i + \sqrt{\mu_i^2 - m_i^2}}\right]$$

$$\epsilon_i = \frac{g_i}{8\pi^2}\left[\mu_i\sqrt{\mu_i^2 - m_i^2}\left(\mu_i^2 - \frac{1}{2}m_i^2\right) + \frac{m_i^4}{2} \ln\frac{m_i}{\mu_i + \sqrt{\mu_i^2 - m_i^2}}\right]$$

- the number density and chemical potential per state:

$$n_i = \frac{g_i}{6\pi^2}\left(\mu_i^2 - m_i^2\right)^{\frac{3}{2}}$$

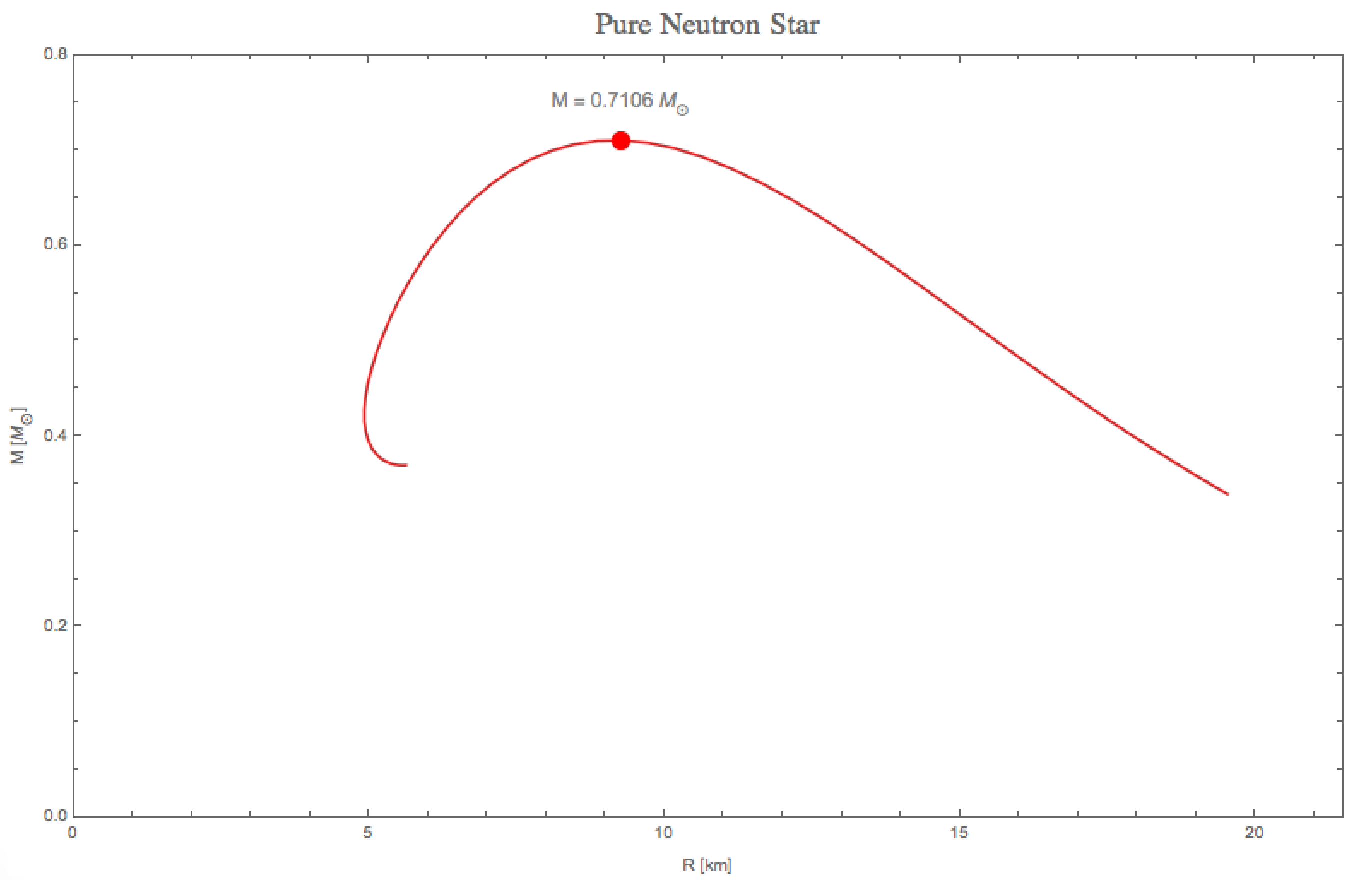
$$\mu_i = \frac{\partial \epsilon}{\partial n_i}$$

Method

- the simplest model for a neutron star is one consisting of a Fermi gas of non-interacting neutrons ➔ pure neutron star
- the complex model ➔ non-interacting fermion gas in $3+1_C+1$ dimensional space-time with excitations (n) and 'extra mass' gap (Δm) values

Results

0. Pure neutron star



The mass-radius relation, M(R) of compact star models.
This plot depicts the mass over the radius for different values of the initial central energy.

Results

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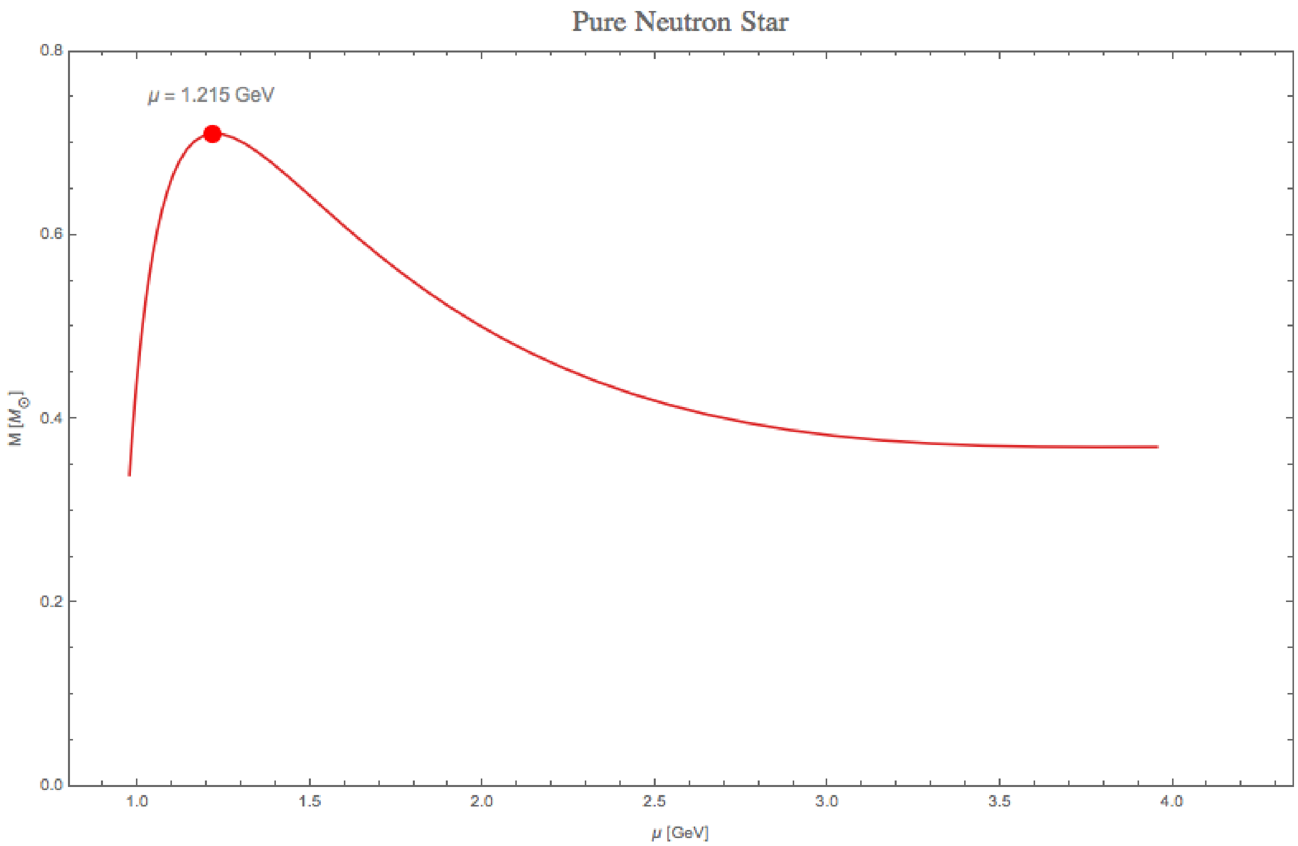
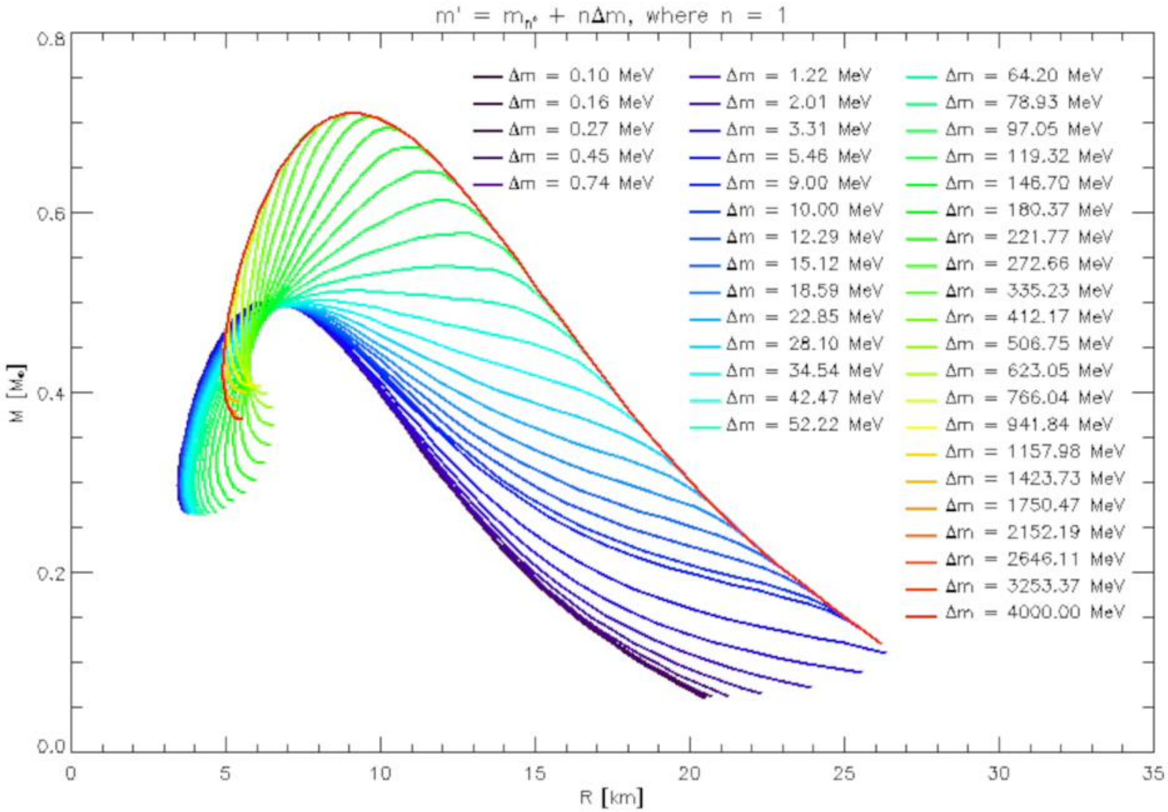


The mass-chemical potential relation, M(μ) of compact star models.
This plot depicts the mass over the radius for different values of the initial central energy.

Results

1. Excitation number (n) increases



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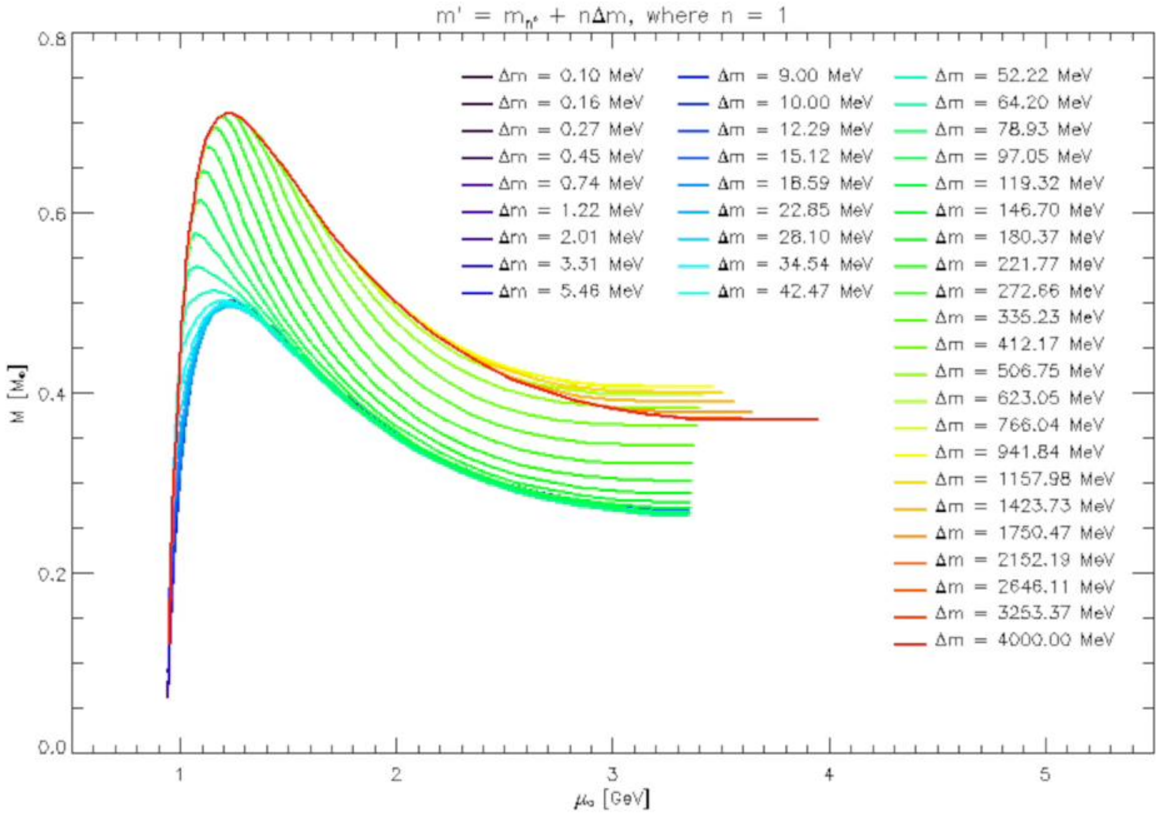
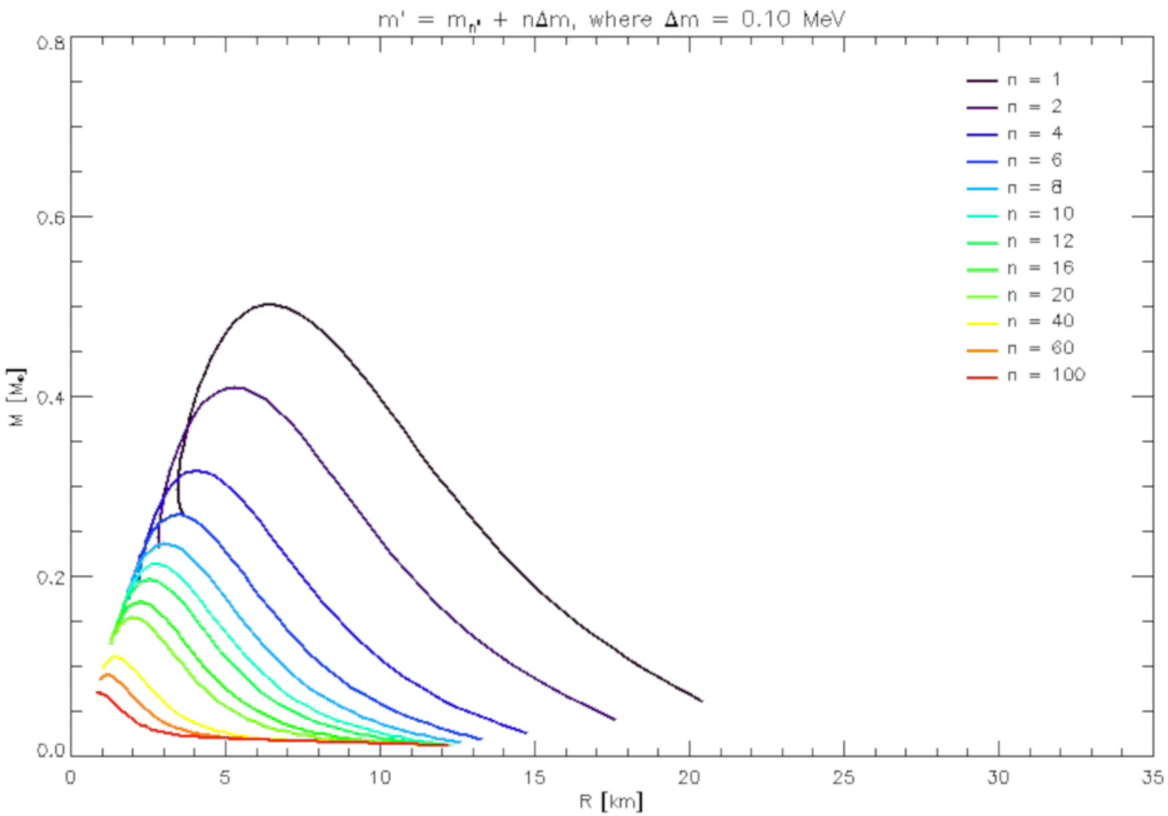


The mass-chemical potential relation, M(μ) of compact star models.

Results

2. The mass gap (Δm) increases



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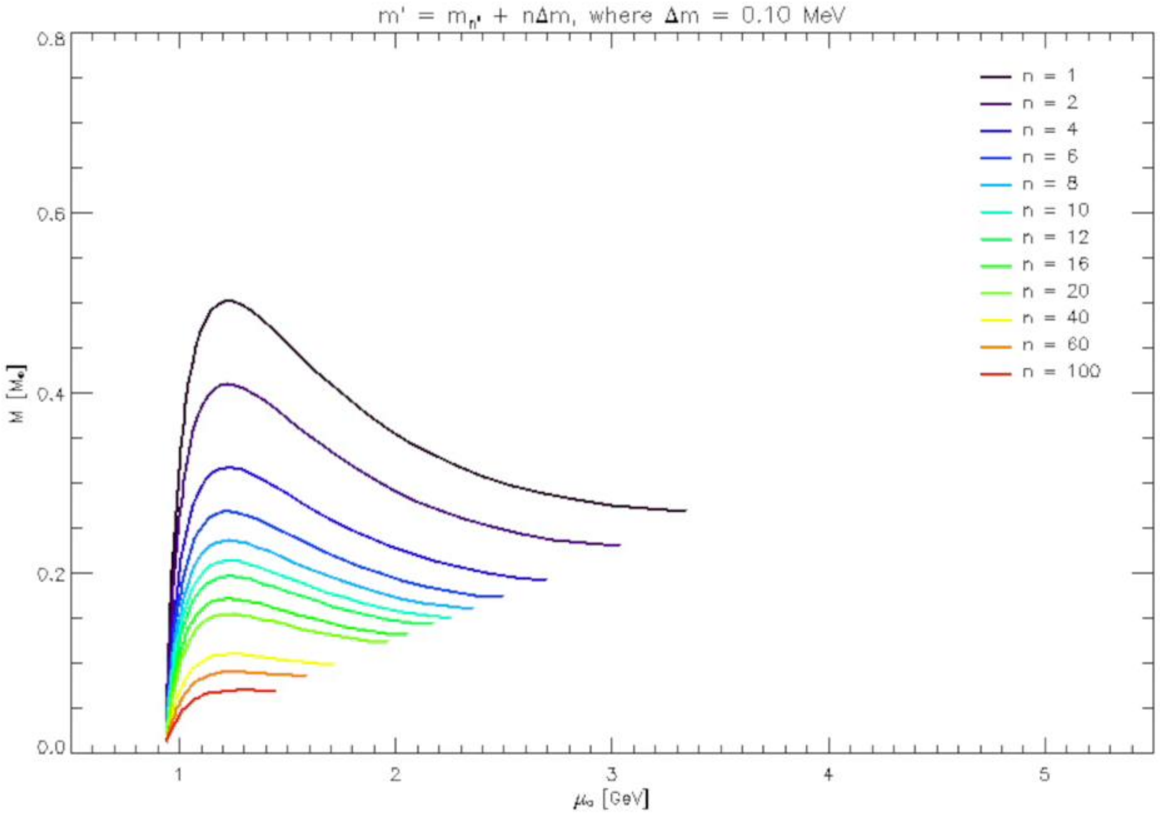
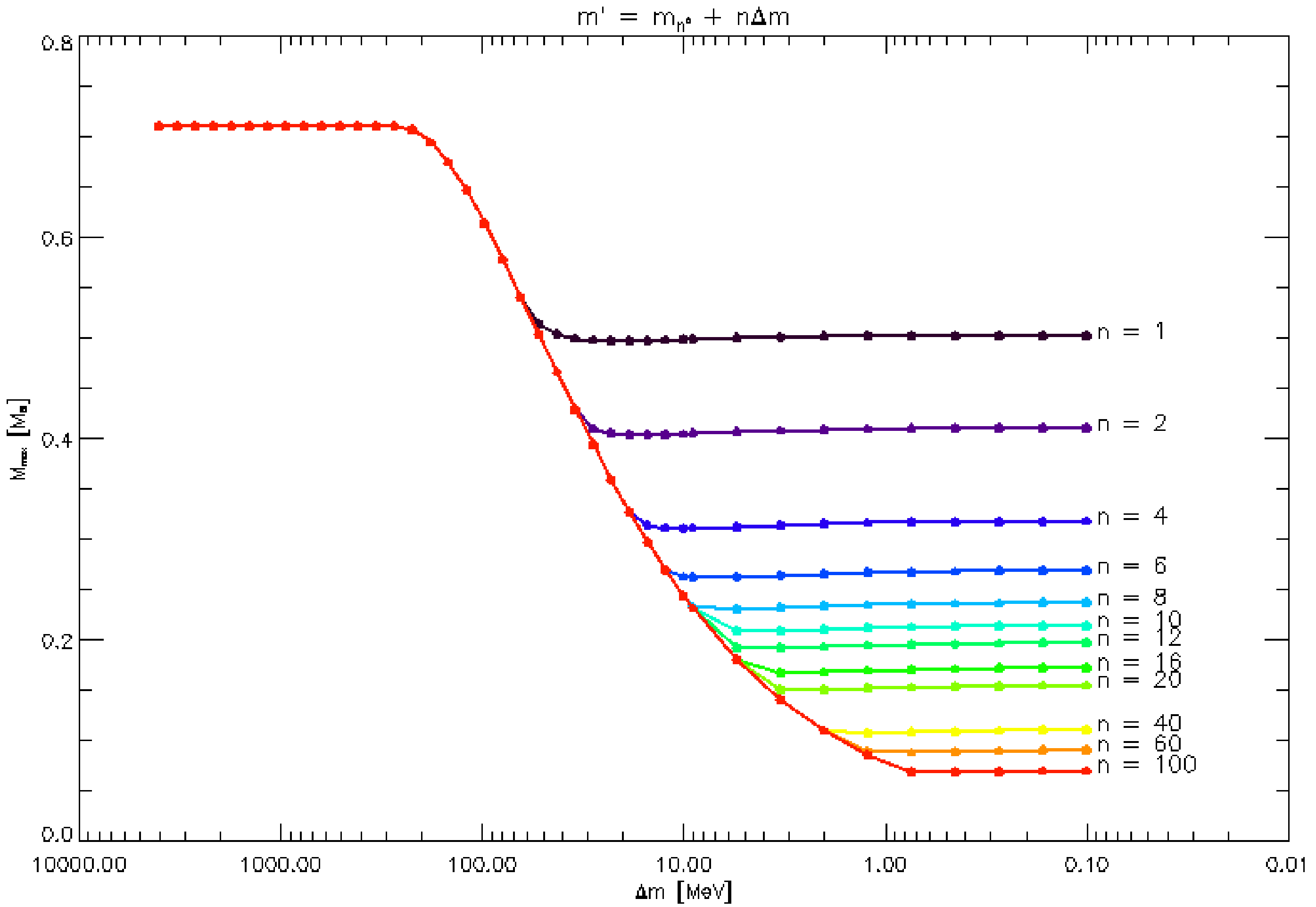


The mass-chemical potential relation, M(μ) of compact star models.

Results

3. The maximum mass of a compact star



M_{max} as a function of Δm and n

Conclusion

- The mass-radius diagram clearly presents
  - that increasing the Δm, the maximum mass of the star is getting larger, since the EoS of the star becomes stiffer
  - that increasing the n, the maximum mass of the star is getting smaller, since the EoS of the star becomes softer
- As increasing Δm and getting $\Delta m \approx 110$ MeV, the M_{max} increases and saturates to a maximum 0.7 $M_{\odot}$ independently of the excitation number
- As decreasing Δm (softens the EoS), M_{max} also saturates, but results smaller-and-smaller M_{max} depending on the possible degrees of freedom, the excitation number

Thank you for your attention!

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