PHENIX results on Levy analysis of Bose–Einstein correlation functions

Zimányi Winter School on Heavy Ion Physics

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The PHENIX Experiment

The PHENIX detector system

- Observing collisions of p, d, Cu, Au, Al, He, U
- Charged pion ID from $\sim 0.2$ to 2 GeV/c
- Beam energy scan is important
# The RHIC Beam Energy Scan

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- **p+p**
- **Au+Au**
- **d+Au**
- **Cu+Cu**
- **U+U**
- **Cu+Au**
- **He+Au**
- **p+Au**
- **p+Al**

## Introduction to Bose-Einstein correlations

\( N_1(p), N_2(p) \) - invariant momentum distributions, the definition of the correlation function:

\[
C_2(p_1, p_2) = \frac{N_2(p_1, p_2)}{N_1(p_1) N_1(p_2)} \tag{1}
\]

**The invariant momentum distributions**

\[
N_1(p) - \text{norm.}, \quad N_2(p_1, p_2) = \int S(x_1, p_1) S(x_2, p_2) |\Psi_2(x_1, x_2)|^2 \, d^4x_2 \, d^4x_1 \tag{2}
\]

- \( S(x, p) \) source func. (usually assumed to be Gaussian - Lévy is more general)
- \( \Psi_2 \) - interaction free case - \( |\Psi_2|^2 = 1 + \cos(qx) \)

If \( k_1 \approx k_2 \):

\[
C_2(q, K) \approx 1 + \left| \frac{\tilde{S}(q, K)}{\tilde{S}(0, K)} \right|^2, \quad \tilde{S}(q, k) = \int S(x, k) e^{iqx} \, d^4x
\]

- Sometimes this simple formula fails (cf. experimentally observed oscillations at L3, CMS)
Final state interactions, resonances

- Final state interactions distort the simple Bose-Einstein picture
  - identical charged pions - Coulomb interaction
    - different methods of handling, an usual practice: Coulomb-correction
    - $C_{B-E}(q) = K(q) \cdot C_{\text{measured}}(q)$
    - An other possibility to fit with the effect incorporated in the fitted func.
- Resonance pions reduce the correlation function
  - $S = S_C + S_H$
- Primordial pions - Core $\lesssim 10$ fm
- Resonance pions - from very far regions - Halo

The out-side-long system, HBT radii

- Corr. func. (with Gaussian source): \( C_2(q) = 1 + \lambda \cdot e^{-R_{\mu\nu}^2 q^\mu q^\nu} \)

- Bertsch-Pratt pair coordinate-system
  - out direction: direction of the average transverse momentum (\( K_t \))
  - long direction: beam direction (z axis)
  - side direction: orthogonal to the latter two

- LCMS system (Lorentz boost in the long direction)

- From the \( R_{\mu\nu}^2 \) matrix, \( R_{out}, R_{side}, R_{long} \) nonzero - HBT radii

- Out-side difference - \( \Delta \tau \) emission duration

- From a simple hydro calculation:
  \[
  R_{out}^2 = \frac{R^2}{1 + \frac{m}{T_0} u_T^2} + \beta_T^2 \Delta \tau^2 \\
  R_{side}^2 = \frac{R^2}{1 + \frac{m}{T_0} u_T^2}
  \]

- RHIC: ratio is near one \( \rightarrow \) no strong 1\textsuperscript{st} order phase trans.

Beam energy & system size dependence of HBT radii


- quantities related to emission duration and expansion velocity
- non-monotonic patterns
- indication of CEP?

- More precise mapping and further detailed studies required
- Is there any other way to find the critical point?
- Maybe Levy exponent $\alpha$!
A possible way of finding the critical point

- Generalized Gaussian - Levy-distribution
  - Anomalous diffusion
  - Generalized central limit th.
  - \( \alpha = 2 \) Gaussian, \( \alpha = 1 \) Cauchy

\[
\mathcal{L}(\alpha, R, r) = \frac{1}{(2\pi)^3} \int d^3 q e^{iqr} e^{-\frac{1}{2} |qR|^\alpha}
\]

- Csörgő, PoS HIGH-pTLHC08:027 (2008), nucl-th/0903.0669

- Shape of the correlation functions with Levy source:

\[
C_2(|k|) = 1 + \lambda \cdot e^{-(2R|k|)^\alpha} \quad \alpha = 2 : \text{Gaussian} \\
\alpha = 1 : \text{Exponential}
\]

- Critical behaviour \( \rightarrow \) described by critical exponents
- Spatial corr. \( \propto r^{-(d-2+\eta)} \) \( \rightarrow \) defines \( \eta \) exponent
- Symmetric stable distributions (Levy) \( \rightarrow \) spatial corr. \( \propto r^{-1-\alpha} \)
- \( \alpha \) identical to critical exponent \( \eta \)
A possible way of finding the critical point

- Generalized Gaussian - Levy-distribution
  - Anomalous diffusion
  - Generalized central limit th.
  - $\alpha = 2$ Gaussian, $\alpha = 1$ Cauchy

$$\mathcal{L}(\alpha, R, r) = \frac{1}{(2\pi)^3} \int d^3 q e^{iqr} e^{-\frac{1}{2}|qR|^\alpha}$$

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- Shape of the correlation functions with Levy source:
  $$C_2(|k|) = 1 + \lambda \cdot e^{-(2R|k|)^\alpha}$$
  $\alpha = 2$ : Gaussian
  $\alpha = 1$ : Exponential

- Critical behaviour $\rightarrow$ described by critical exponents
- Spatial corr. $\propto r^{-(d-2+\eta)}$ $\rightarrow$ defines $\eta$ exponent
- Symmetric stable distributions (Levy) $\rightarrow$ spatial corr. $\propto r^{-1-\alpha}$

- $\alpha$ identical to critical exponent $\eta$
A possible way of finding the critical point

- QCD universality class ↔ 3D Ising

- At the critical point:
  - random field 3D Ising: $\eta = 0.50 \pm 0.05$
  - 3D Ising: $\eta = 0.03631(3)$

- Change in $\alpha_{\text{Levy}}$ ↔ proximity of CEP

- Motivation for precise Levy HBT!
PHENIX Levy HBT analysis

A brief overview

- **Dataset:**
  - $\sqrt{s_{NN}}=200$ GeV Au+Au, min. bias, $\sim 7$ billion events → fine $p_T$ binning

- **Goal:**
  - Detailed shape analysis of 1D two-pion corr. func.
    - Levy source instead of Gaussian → better agreement with data
  - Extraction and analysis of the source parameters
    - Precision measurement of $\lambda(m_T), \alpha_{Levy}(m_T), R_{Levy}(m_T)$
    - Lot of new physics in these results
    - Search for CEP → lower energies
An example correlation function

PHENIX MinBias Au+Au @ $\sqrt{s_{\text{NN}}} = 200$ GeV, $\pi^+\pi^+$, $p_T = 0.2$-0.22 GeV/c

$\lambda = 0.72 \pm 0.02$
$R = 8.74$ fm $\pm 0.24$ fm
$\alpha = 1.16 \pm 0.03$
$\varepsilon = -0.102 \pm 0.005$
$N = 1.0095 \pm 0.0005$

$\chi^2$/NDF = 93/97
conf. level = 59.1%

$C_{\text{Coul}}(\lambda, R, \alpha, k) \times N(1 + \varepsilon k)$
$C_{\text{Levy}}^0(\lambda, R, \alpha, k) \times N(1 + \varepsilon k)$
$N(1 + \varepsilon k)$

$C_{\text{Levy}}^0 = 1 + \lambda \cdot \exp(-2 \cdot R \cdot k^\alpha)$

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Levy scale parameter $R$

- Similar decreasing trend as HBT radii
- Hydro behaviour not invalid
- Hard to say whether the $1/R^2$ scaling is linear or not
Levy scale parameter $R$

- Similar decreasing trend as HBT radii
- Hydro behaviour not invalid
- Hard to say whether the $1/R^2$ scaling is linear or not
Correlation strength $\lambda$

- From the Core-Halo model: \[ \lambda = \left( \frac{N_C}{N_C + N_H} \right)^2 \]
- Observed decrease at small $m_T \rightarrow$ increase of halo fraction
- Different effects can cause change in $\lambda$
  - Resonance effects
  - Partial coherence of the fireball
- Precise measurement is important
The measured value is far from Gaussian ($\alpha = 2$) and expo. ($\alpha = 1$).
Also far from the rfd.3D Ising value at CEP ($\alpha = 0.5$).
More or less constant (at least within systematic errors).
Although the constant fit is statistically not acceptable.
Motivation to do fits with fixed $\alpha = 1.134$. 

$\alpha_0 = 1.134 \pm 0.003$
$\chi^2$/ndf = 159/61
Newly discovered scaling parameter $\hat{R}$

- Empirically found scaling parameter
- Linear in $m_T$
- Physical interpretation $\rightarrow$ open question
PHENIX Levy HBT analysis preliminary results:

- Dataset: Run-10 200 GeV Au+Au, $\sim 7$ billion evts.
- Precise measurement of Levy source parameters ($R, \lambda, \alpha$)
- New empirically found scaling parameter ($\hat{R}$)
- Future plans: lower energies, 3 pion corr., pion-kaon comparison
- Expected physics info: CEP, partial coherence, resonance effects

Thank you for your attention!