

Zimányi Winter School

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ELASTIC & INELASTIC DIFFRACTION at the LHC

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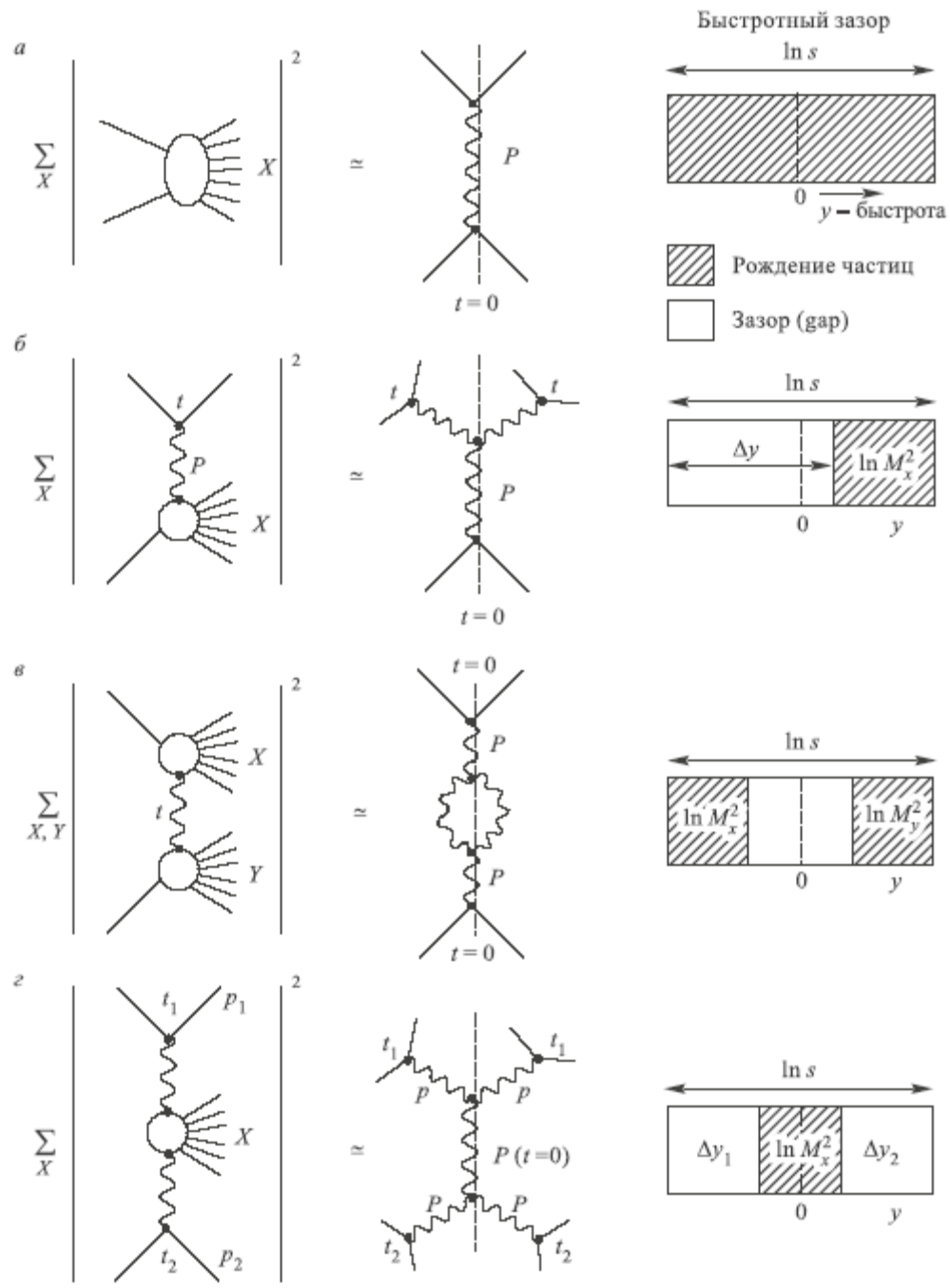
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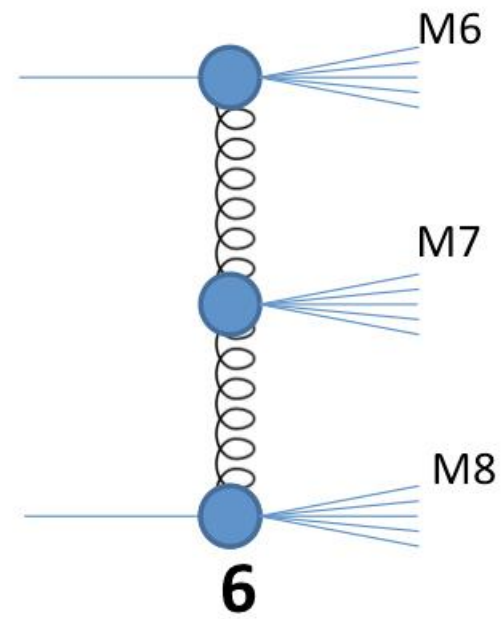
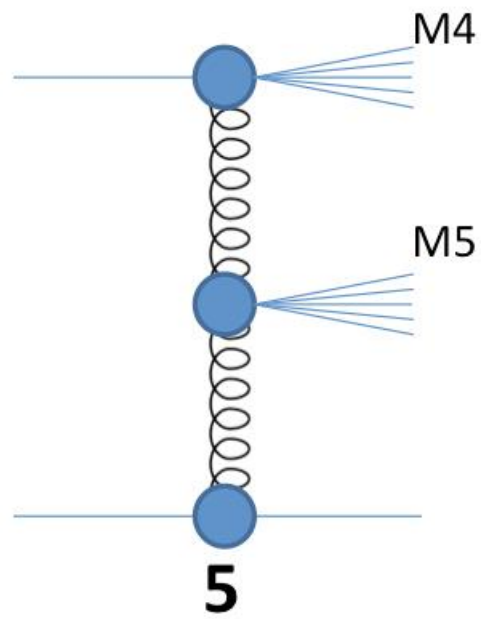
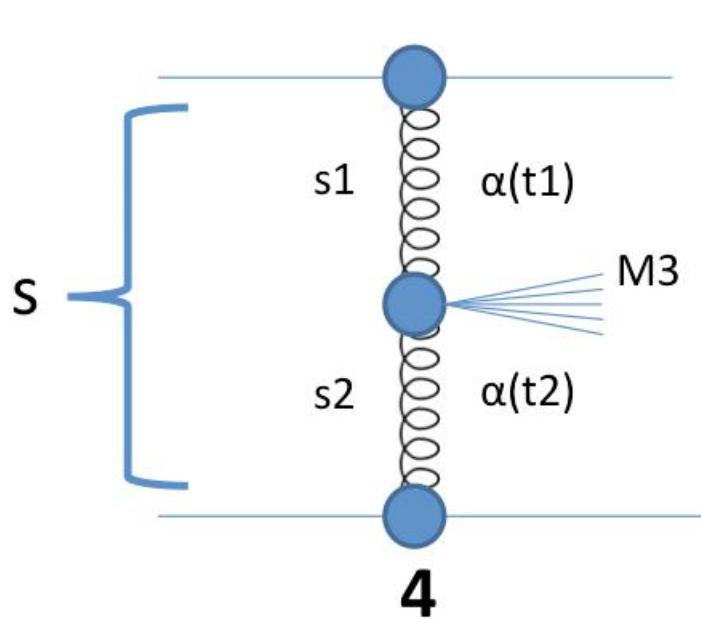
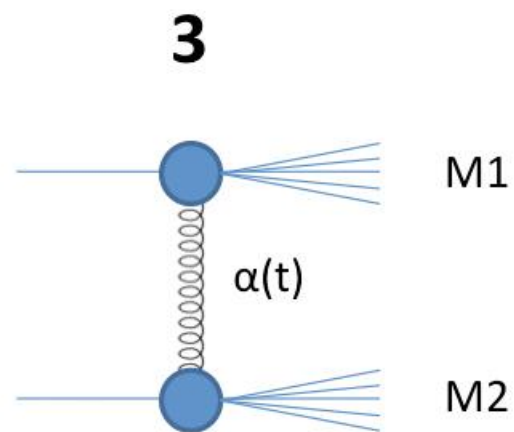
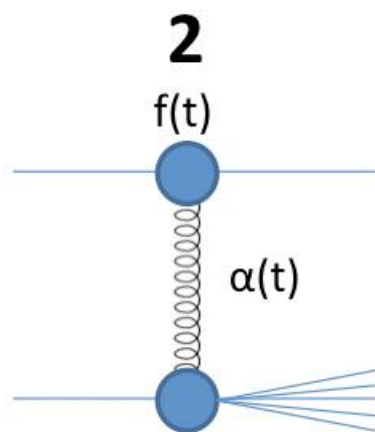
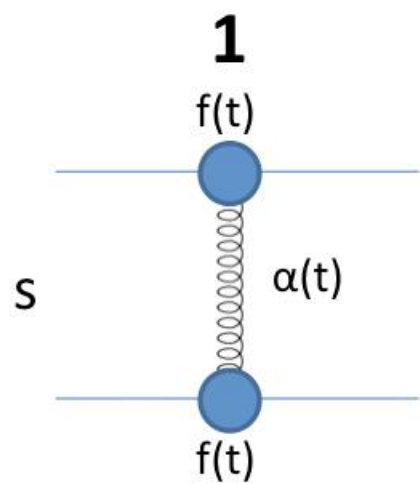
Roberto Fiore (Calabria), Risto Orava (Helsinki),
Mikael Mieskolainen (Helsinki) and Rainer Schicker (Heidelberg)

Abstract:

Salient feature of elastic and inelastic diffraction in proton-proton collisions with emphases on the recent LHC experimental data and future expectations are highlighted. These are:

1. Rise with energy, unitarity, "flux renormalization";
2. Physics behind the non-exponential behavior of the elastic cone at low- $|t|$, observed both at the ISR and the LHC; will it be seen in diffraction dissociation? (**Follow elastic!**)
3. The dip-bump structure in elastic scattering and its possible appearance in diffraction dissociation (**following elastic?**)
4. Pomeron dominance at the LHC, Regge factorization and its breakdown;
5. Importance and open problems in the description of **low missing mass, resonance structure** in single- (SD) and double (DD) diffraction dissociation; misuse of the triple Regge limit; duality in the missing mass: finite mass sum rules (FMSR);
6. From differential to integrated cross sections: incompatibility caused by different integration limits used at the LHC;
7. Central exclusive production of glueballs and other meson resonances





Factorization (nearly perfect at the LHC!)

$$(g_1 g_2)^2 = \frac{(g_1 f_1)^2 (f_1 g_2)^2}{(f_1 f_2)^2}.$$

Hence

$$\frac{d^3 \sigma}{dt dM_1^2 dM_2^2} = \frac{d^2 \sigma_1}{dt dM_1^2} \frac{d^2 \sigma_2}{dt dM_2^2} \frac{d\sigma_{el}}{dt}.$$

Assuming exponential cone, t^{bt} and integrating in t , one gets

$$\frac{d^2 \sigma_{DD}}{dM_1^2 dM_2^2} = k \frac{1}{\sigma_{el}} \frac{d\sigma_1}{dM_1^2} \frac{d\sigma_2}{dM_2^2},$$

where $k = r^2 / (2r - 1)$, $r = b_{SD} / b_{el}$.

Further integration in M^2 yields $\sigma_{DD} = k \frac{\sigma_{SD}^2}{\sigma_{el}}$.

Pomerons (diffraction's) fraction

Energy variation of the relative importance of the Pomeron with respect to contributions from the secondary trajectories and the Odderon:

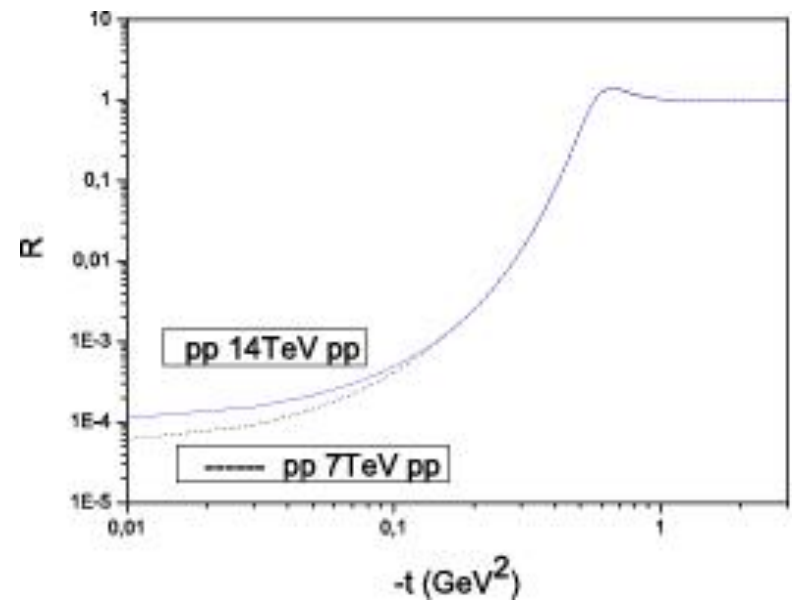
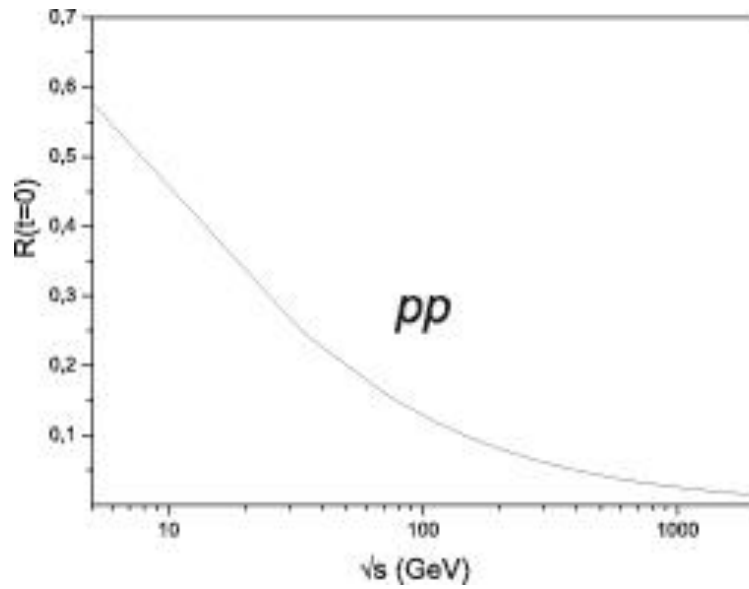
$$R(s, t = 0) = \frac{\Im m(A(s, t) - A_P(s, t))}{\Im A(s, t)}, \quad (1)$$

where the total scattering amplitude A includes the Pomeron contribution A_P plus the contribution from the secondary Reggeons and the Odderon.

Starting from the Tevatron energy region, the relative contribution of the non-Pomeron terms to the total cross-section becomes smaller than the experimental uncertainty and hence at higher energies they may be completely neglected, irrespective of the model used.

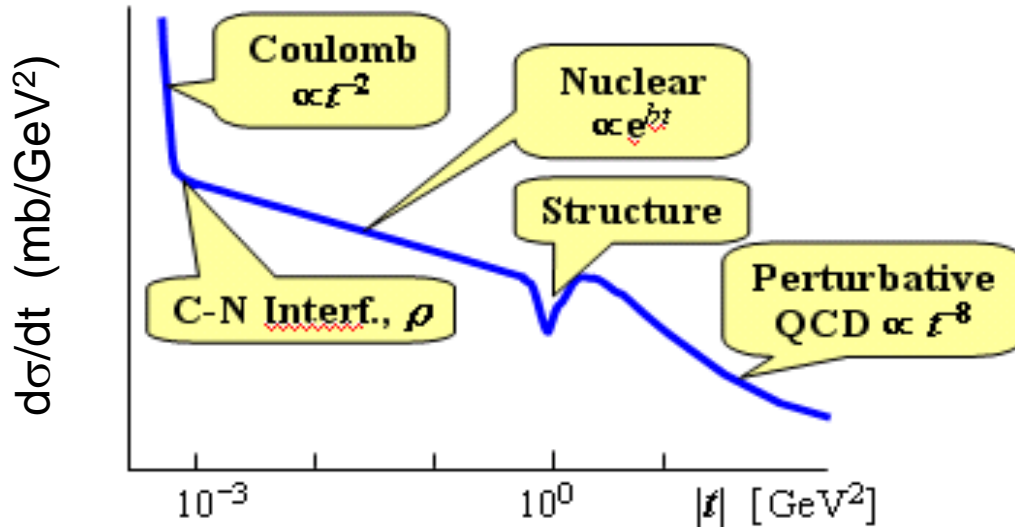
$$R(s, t) = \frac{|(A(s, t) - A_P(s, t))|^2}{|A(s, t)|^2}. \quad (2)$$

Pomeron dominance at the LHC



Elastic Scattering

$\sqrt{s} = 14$ TeV prediction of BSW model



momentum transfer $-t \sim (p\theta)^2$

θ = beam scattering angle

p = beam momentum

$$\rho = \frac{\text{Re}(f_{el}(t))}{\text{Im}(f_{el}(t))} \Big|_{t \rightarrow 0}$$

$$\left. \frac{dN}{dt} \right|_{t=CNI} = L\pi |f_C + f_N|^2 \approx L\pi \left| -\frac{2\alpha_{EM}}{|t|} + \frac{\sigma_{tot}}{4\pi} (i + \rho) e^{-\frac{b|t|}{2}} \right|^2$$

L , σ_{tot} , b , and ρ
from FIT in CNI
region (UA4)

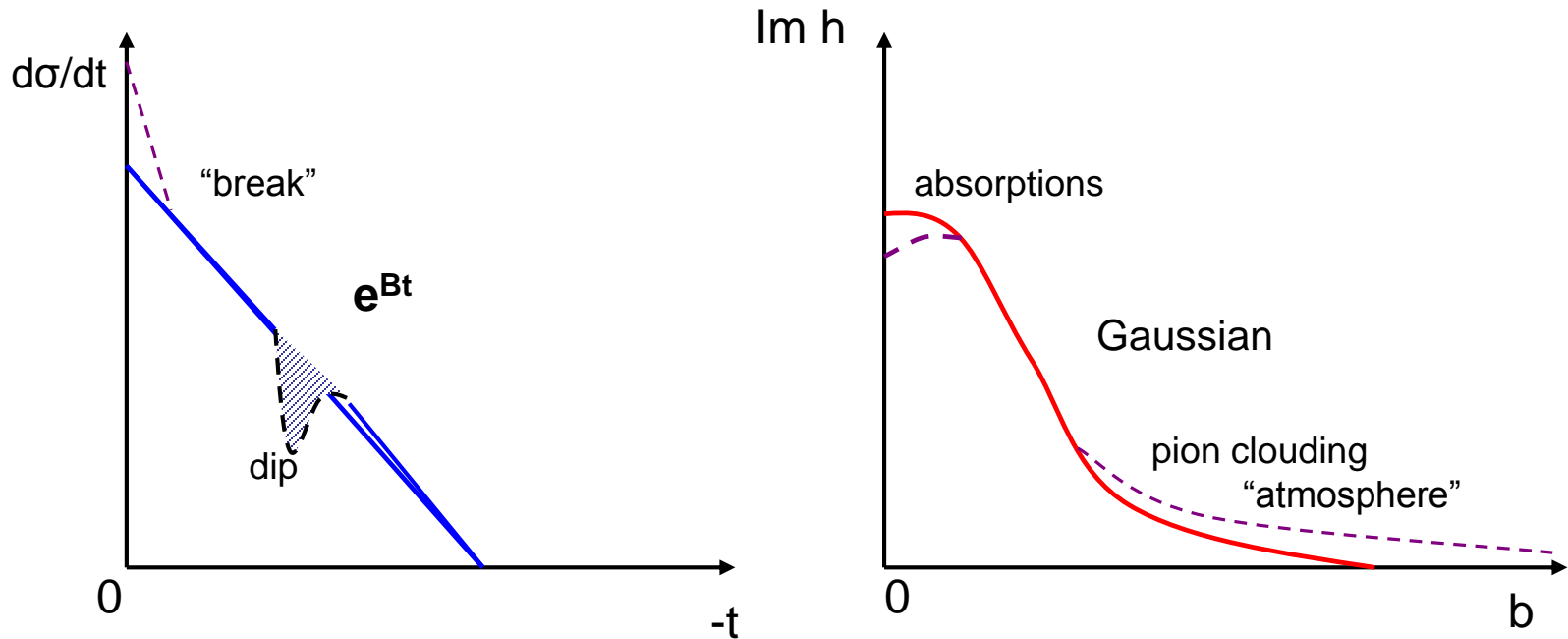
CNI region: $|f_C| \sim |f_N| \rightarrow$ @ LHC: $-t \sim 6.5 \cdot 10^{-4} \text{ GeV}^2$; $\theta_{min} \sim 3.4 \text{ } \mu\text{rad}$

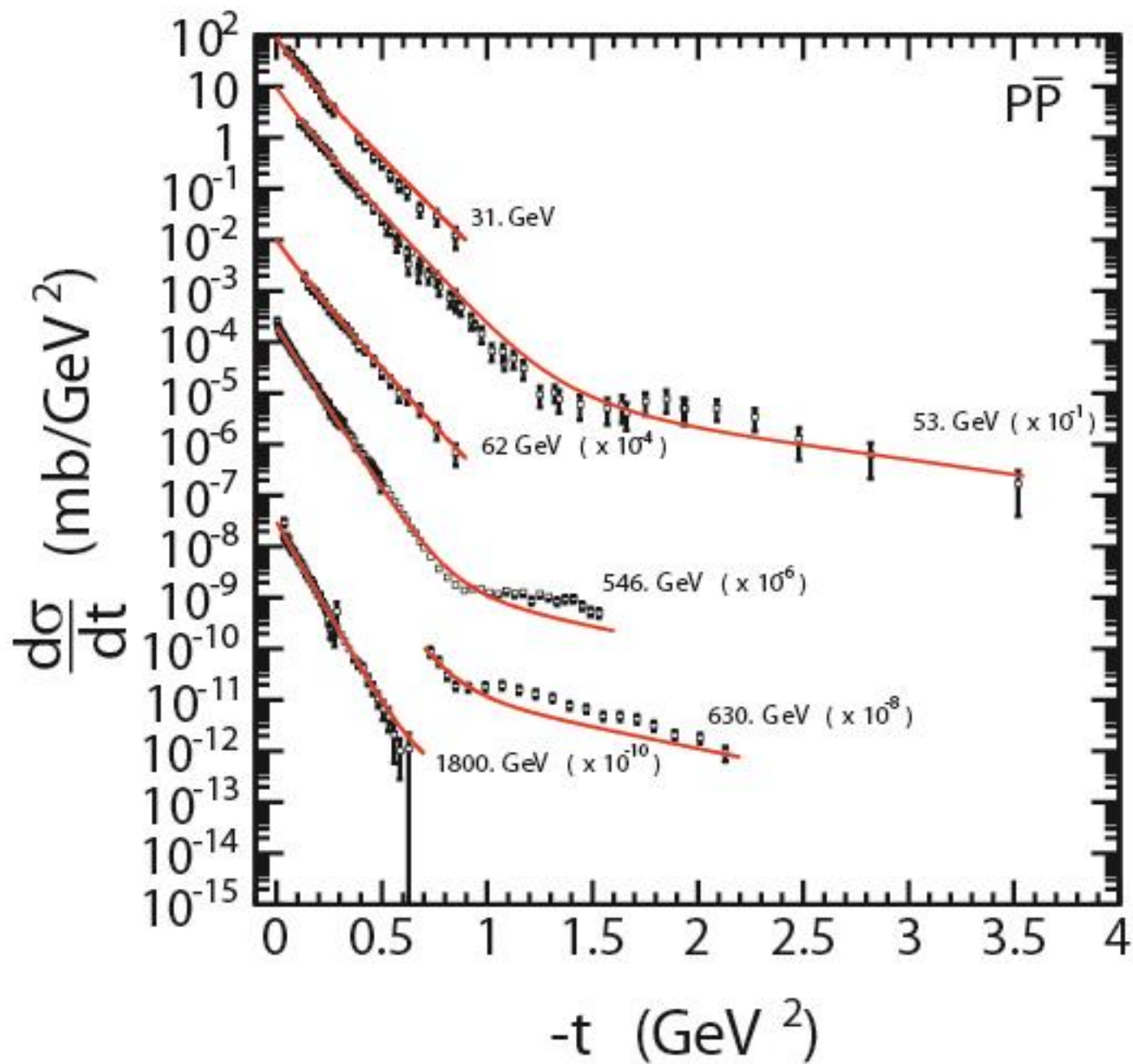
($\theta_{min} \sim 120 \text{ } \mu\text{rad}$ @ SPS)

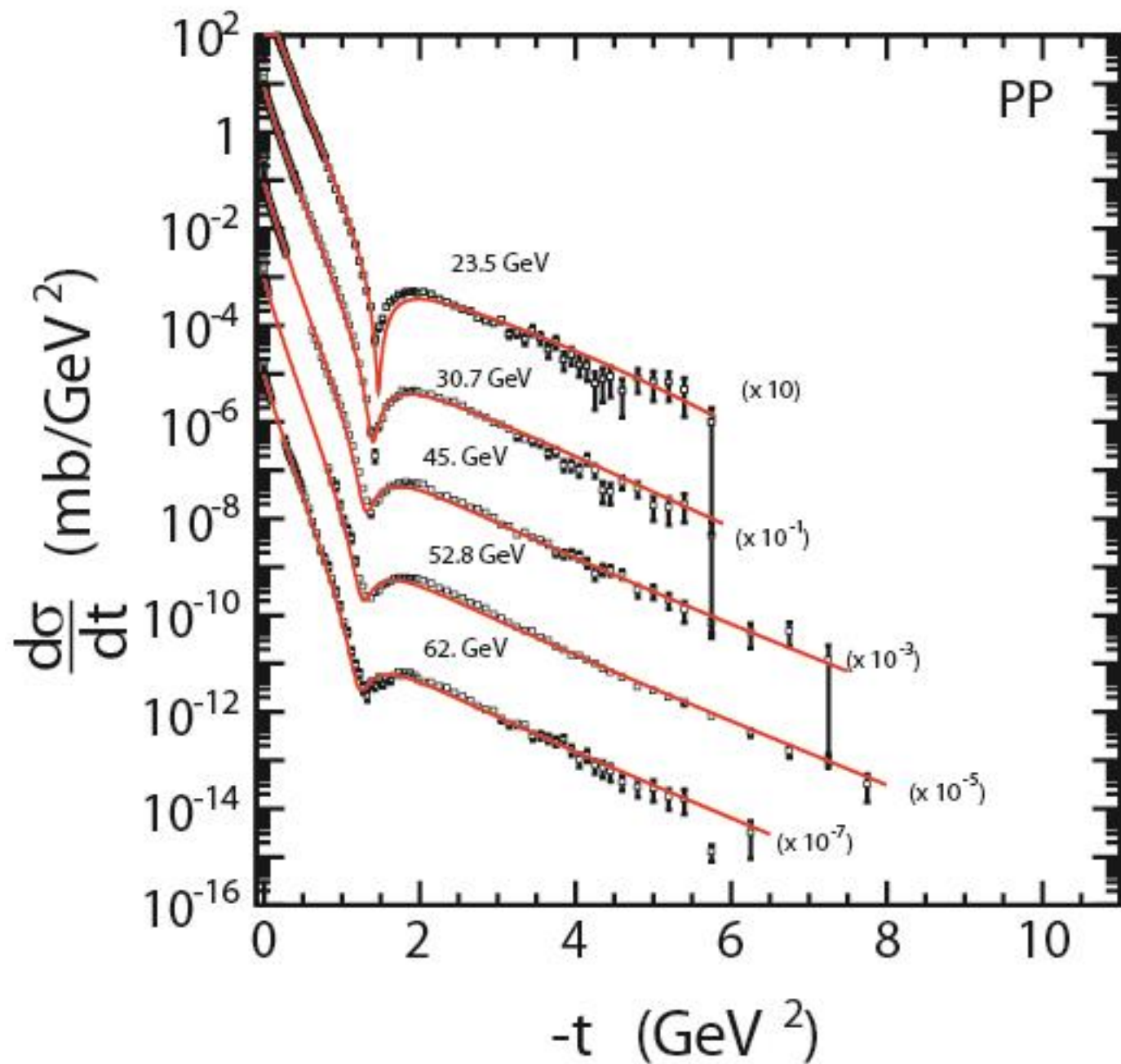
Geometrical scaling (GS), saturation and unitarity

1. On-shell (hadronic) reactions ($s, t, Q^2=m^2$);

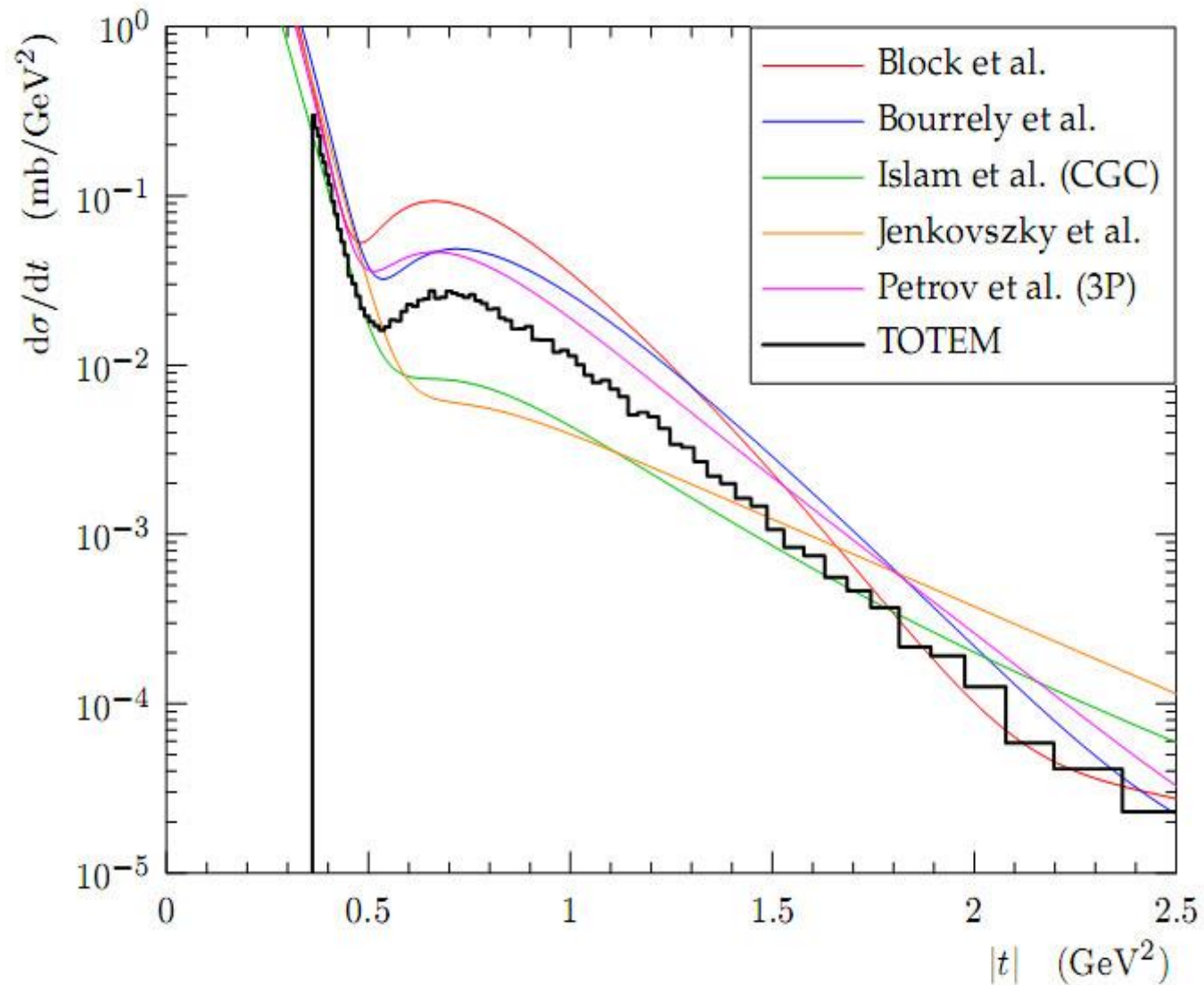
$t \leftrightarrow b$ transformation: $h(s, b) = \int_0^\infty d\sqrt{-t} \sqrt{-t} A(s, t)$
and dictionary:

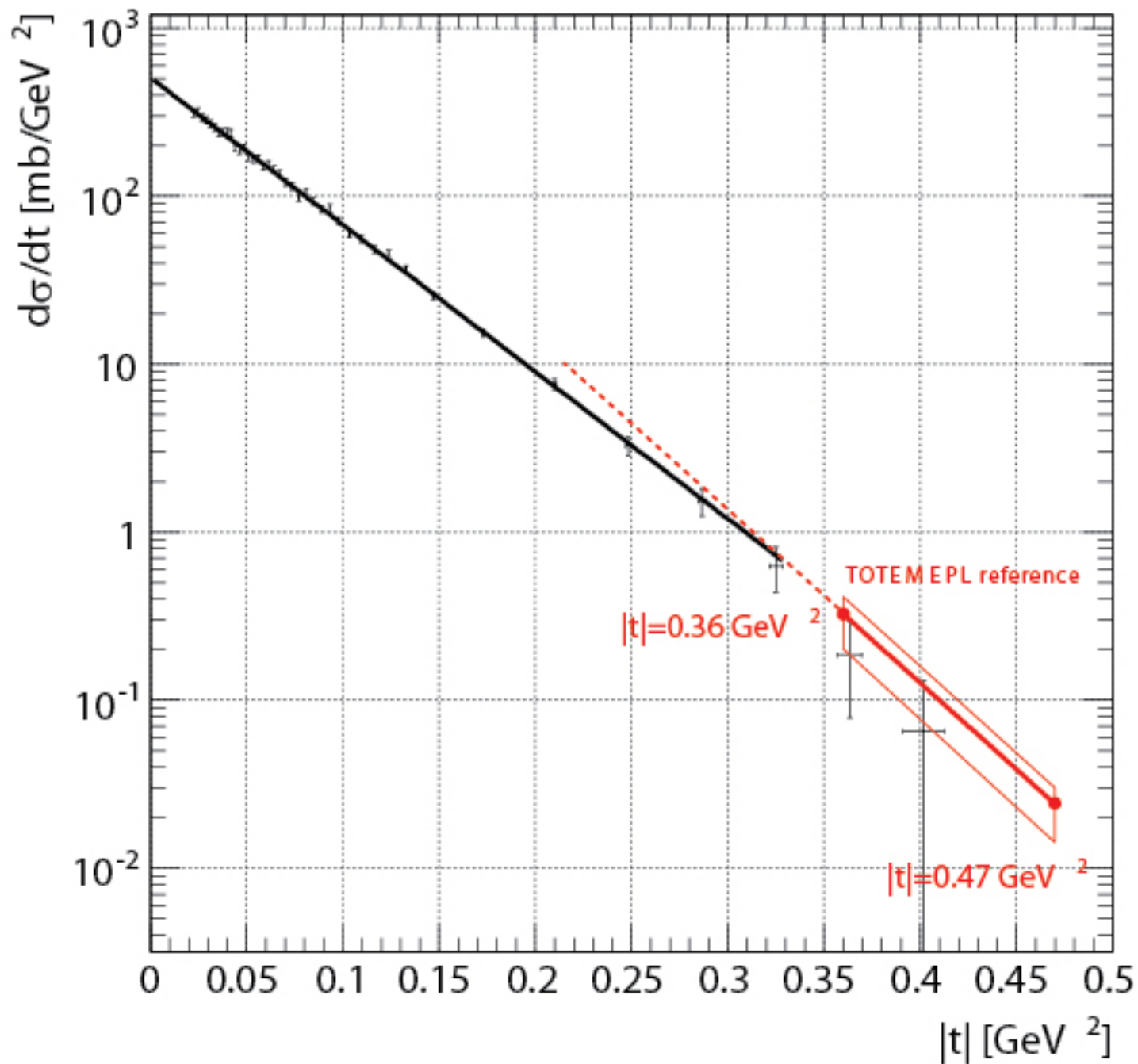




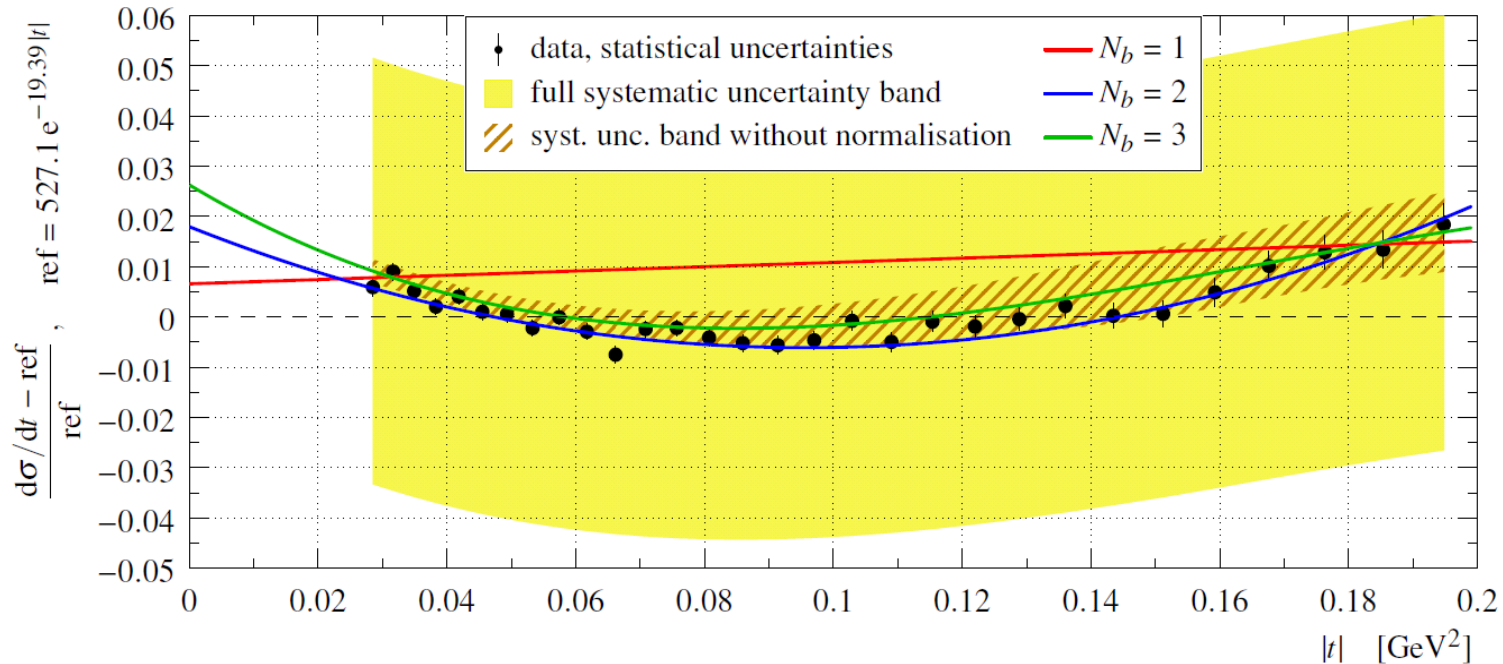


CERN LHC, TOTEM Collab., June 26, 2011:





Elastic scattering: non-exponentiality at low $|t|$

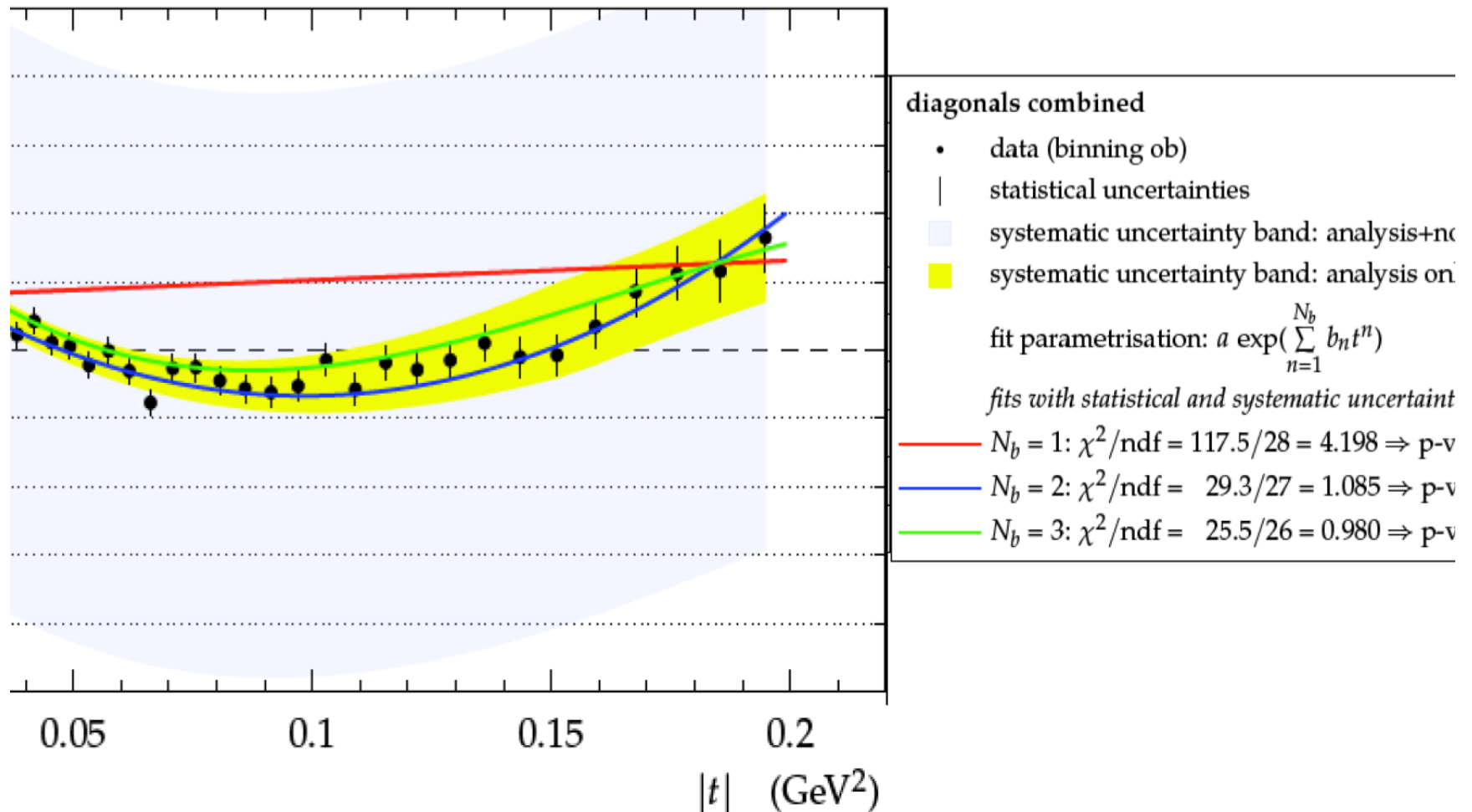


$$\frac{d\sigma}{dt}(t) = \frac{d\sigma}{dt} \Big|_{t=0} \exp \left(\sum_{i=1}^{N_b} b_i t^i \right)$$

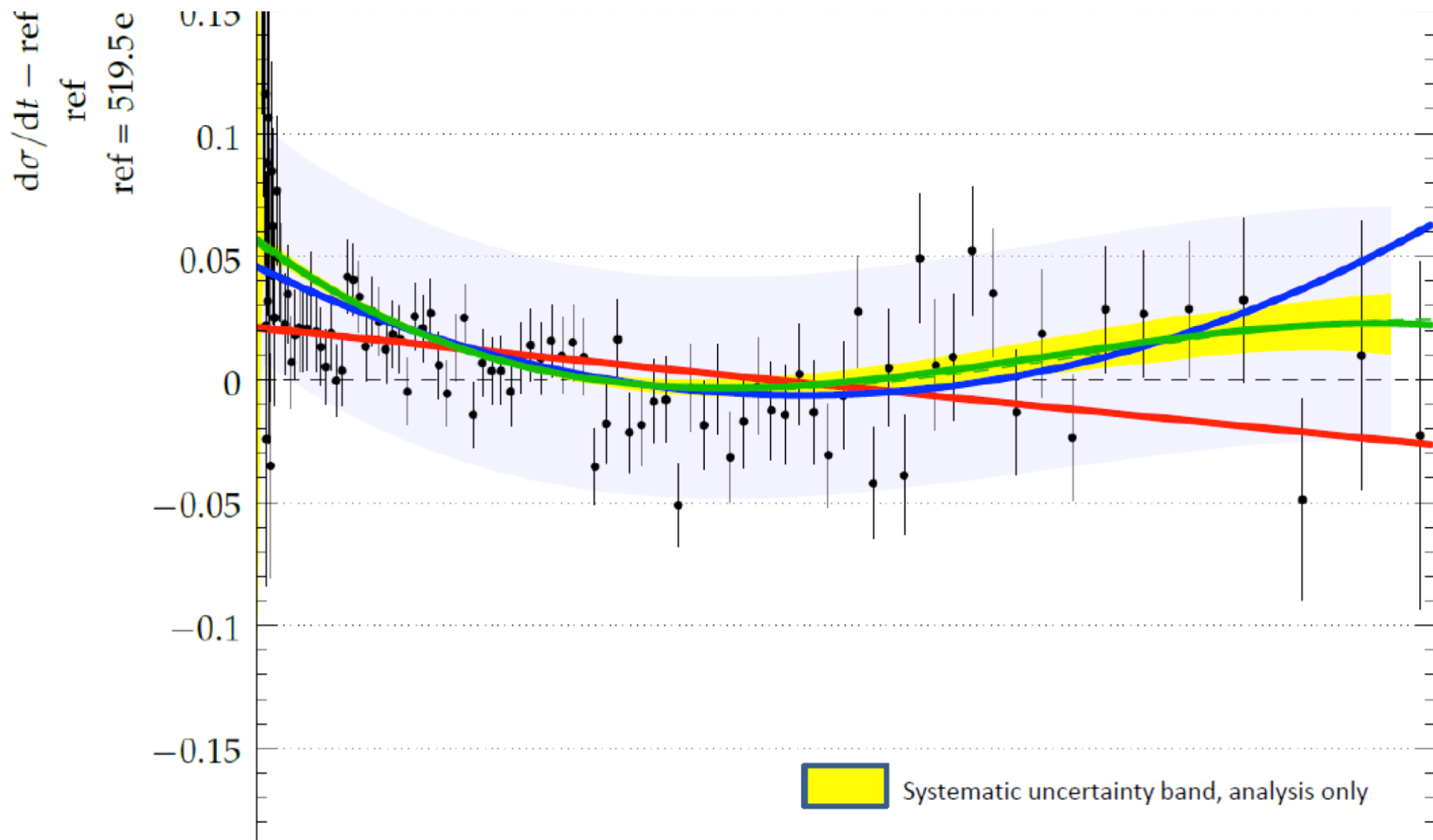
$N_b = 1$ excluded with 7σ significance!

$N_b = 2 : \sigma_{\text{tot}} = (101.5 \pm 2.1) \text{ mb}$
 $N_b = 3 : \sigma_{\text{tot}} = (101.9 \pm 2.1) \text{ mb}$

Fine structure of the Pomeron (at the LHC)



Fine structure of the Pomeron (TOTEM)



The Pomeron trajectory

The Pomeron trajectory has threshold singularities, the lowest one being due to the two-pion exchange, required by the t -channel unitarity. There is a constrain (Barut, Zwanziger; Gribov) from the t -channel unitarity, by which

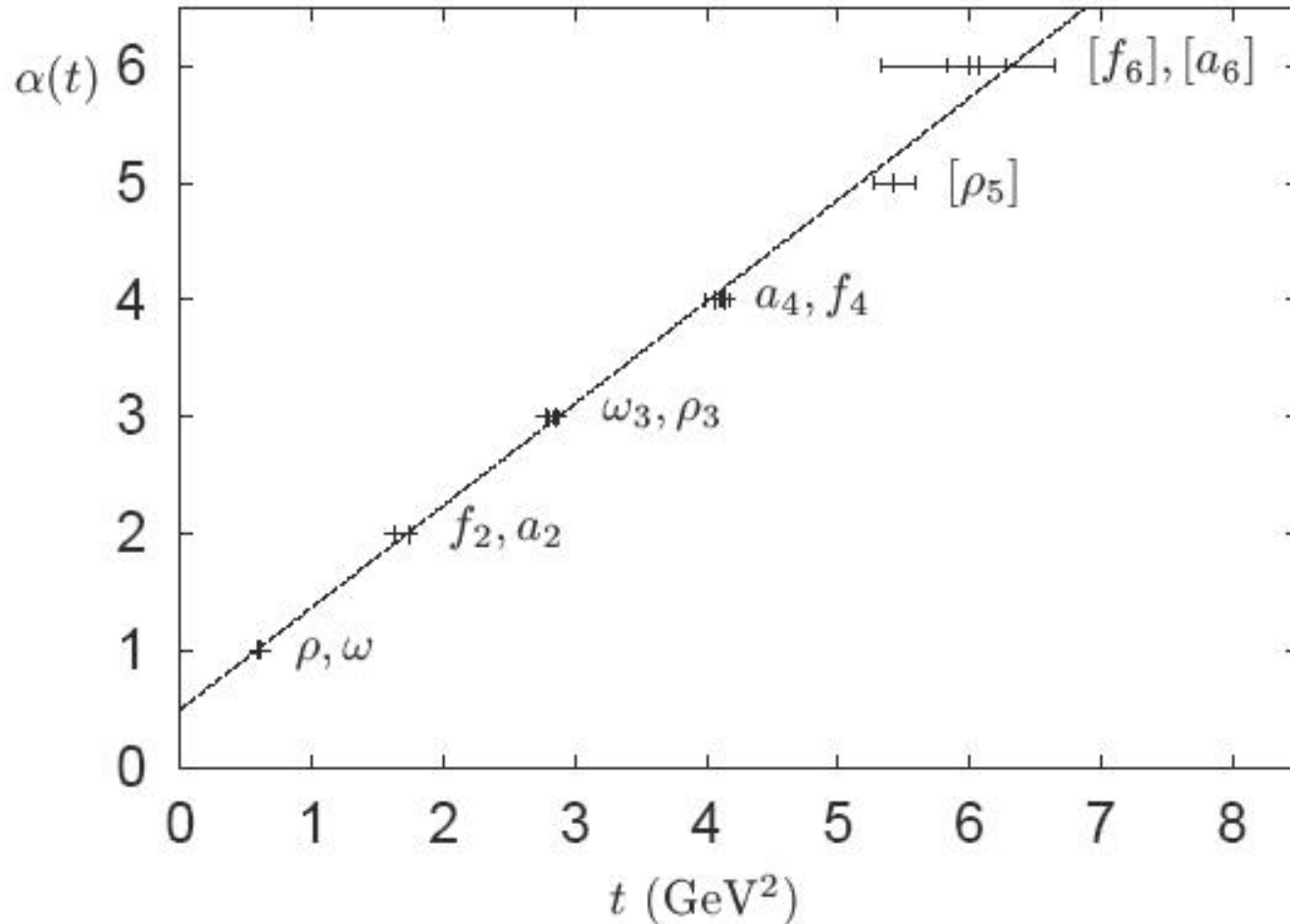
$$\Im\alpha(t) \sim (t - t_0)^{\Re\alpha(t_0)+1/2}, \quad t \rightarrow t_0,$$

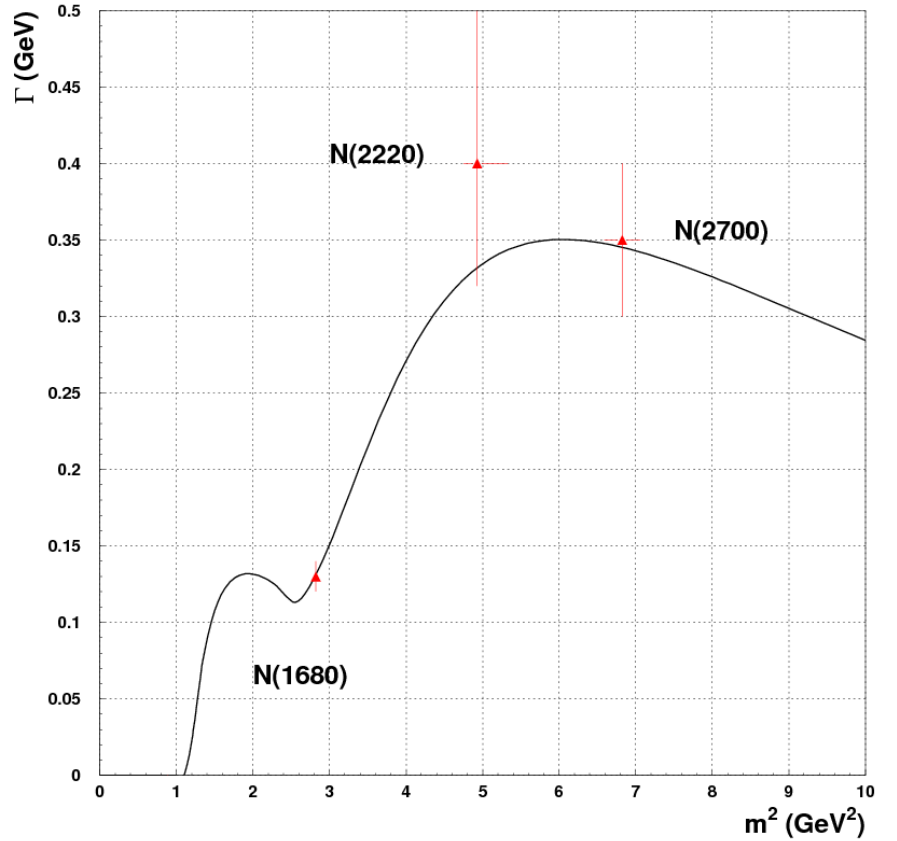
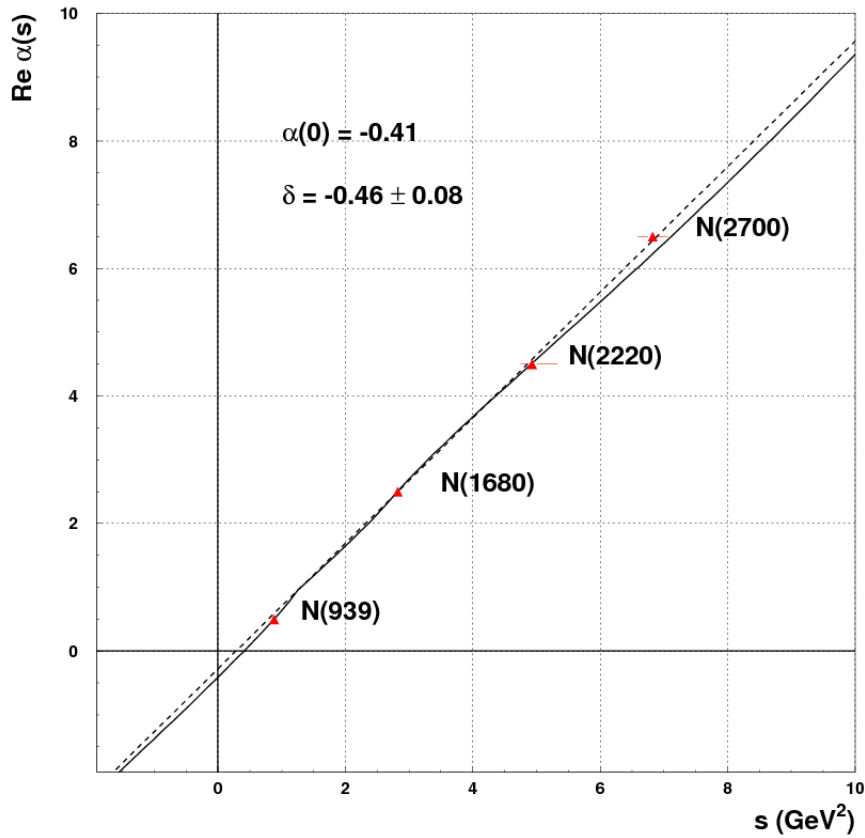
where t_0 is the lightest threshold. For the Pomeron trajectory it is $t_0 = 4m_\pi^2$, and near the threshold:

$$\alpha(t) \sim \sqrt{4m_\pi^2 - t}. \quad (1)$$

Linear particle trajectories

Plot of spins of families of particles against their squared masses:



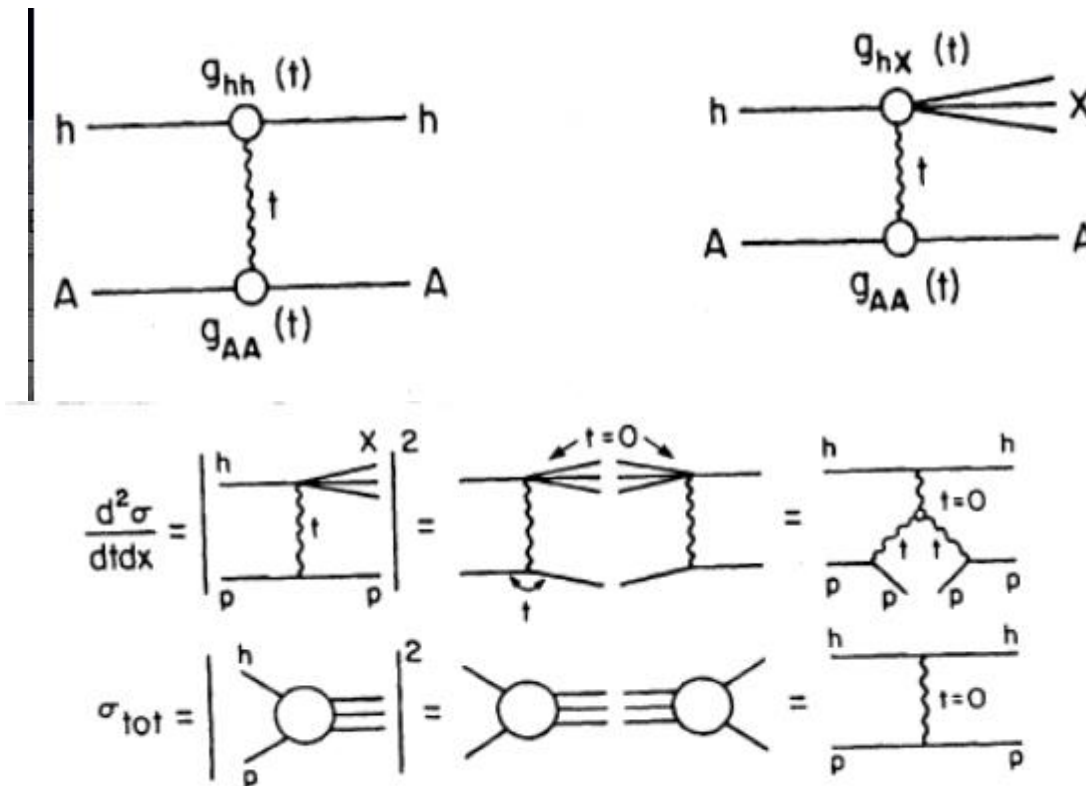


The imaginary part of the trajectory can be written in the following way:

$$\text{Im } \alpha(s) = s^\delta \sum_n c_n \left(\frac{s - s_n}{s} \right)^{\lambda_n} \cdot \theta(s - s_n), \quad (1)$$

where $\lambda_n = \text{Re } \alpha(s_n)$.

The optical (generalised optical (Müller) theorem and triple-Regge limit (for high M only!)



The differential cross section for $1 + 2 \rightarrow X$ is

$$\frac{d^2\sigma}{dt dM^2} = \frac{G(t)}{16\pi^2 s_0^2} \left(\frac{s}{s_0}\right)^{2\alpha(t)-2} \left(\frac{M^2}{s_0}\right)^{\alpha(0)-2\alpha(t)}, \quad (1)$$

where $G(t)$ is the triple Pomeron vertex, $G(t) = Ge^{at}$ for simplicity, and $\alpha(t) = \alpha^0 + \alpha'(t)$ is the (linear for the moment) Pomeron trajectory.

For a critical Pomeron, $\alpha^0 = 1$, one can use the formula

$$\int \frac{dx}{x \ln x} = \ln(\ln x) \quad (2)$$

to get

$$\sigma^{SD}(s) \sim (2\alpha')^{-1} \ln\left(1 + \frac{2\alpha'}{a} \ln s\right) \sim \ln(\ln s), \quad (3)$$

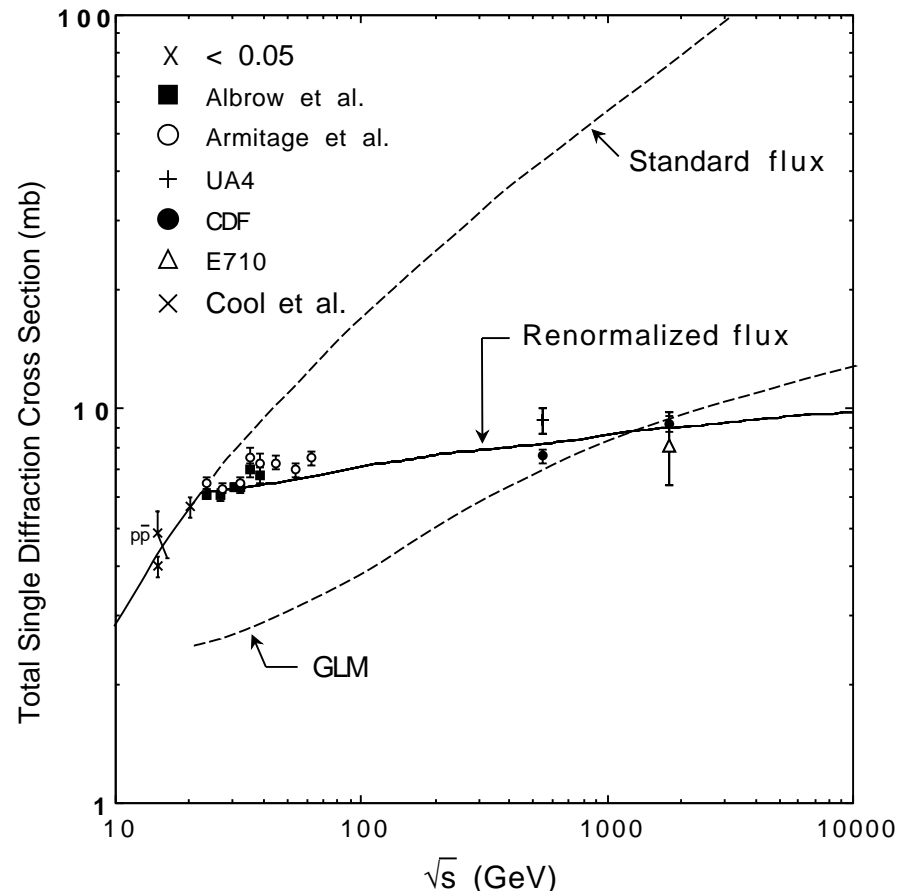
while the total cross section

$$\sigma^{tot}(s) \rightarrow const. \quad (4)$$

It contradicts unitarity since e.g. for critical Pomeron, $\alpha^0 = 1$, the partial (SD) cross section overshoots the total cross section $\sigma^{SD} > \sigma^{tot}$.

A trivial trick to avoid violation of unitarity is to assume the triple Pomeron vertex $G(t)$ vanishing at $t = 0$. Huge literature (Kaidalov, Brower, Ganguli, Kopeliovich,...) exists reflecting the efforts along this direction. The main conclusion is that decoupling (vanishing of the triple Pomeron vertex at $t = 0$) is incompatible with the data.

To remedy this difficulty, Dino Goulianos suggested a renormalization procedure, by which the Pomeron flux is multiplied by a factor $N(s)$ moderating the rise of inelastic diffraction starting from a certain threshold. The appearance of a threshold, however may violate analyticity.

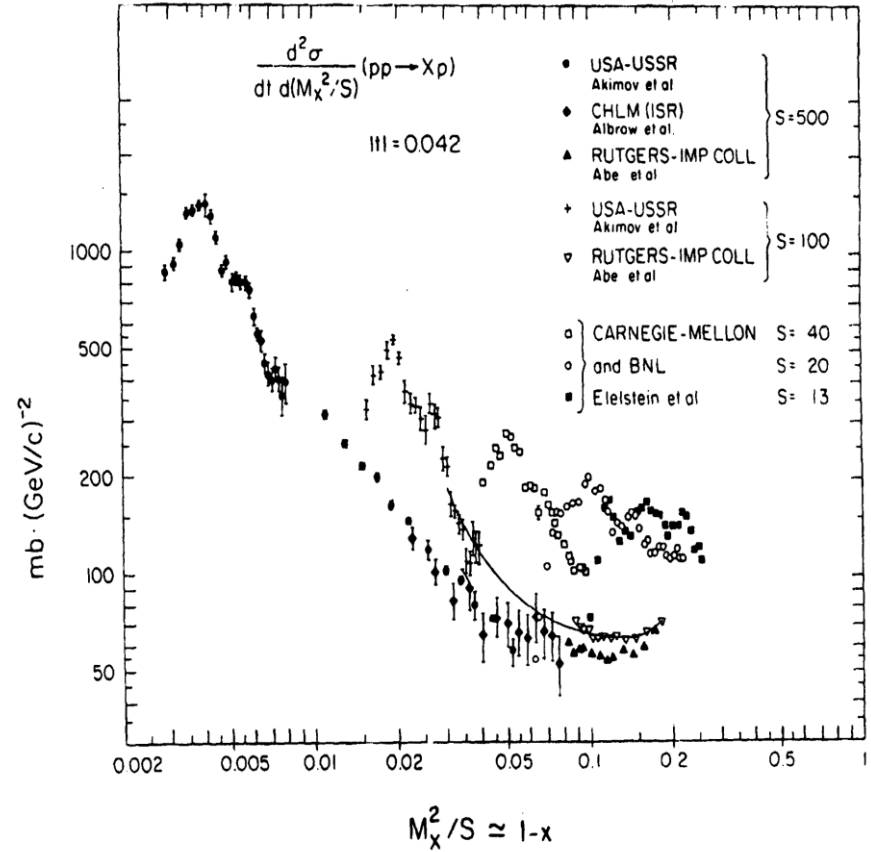
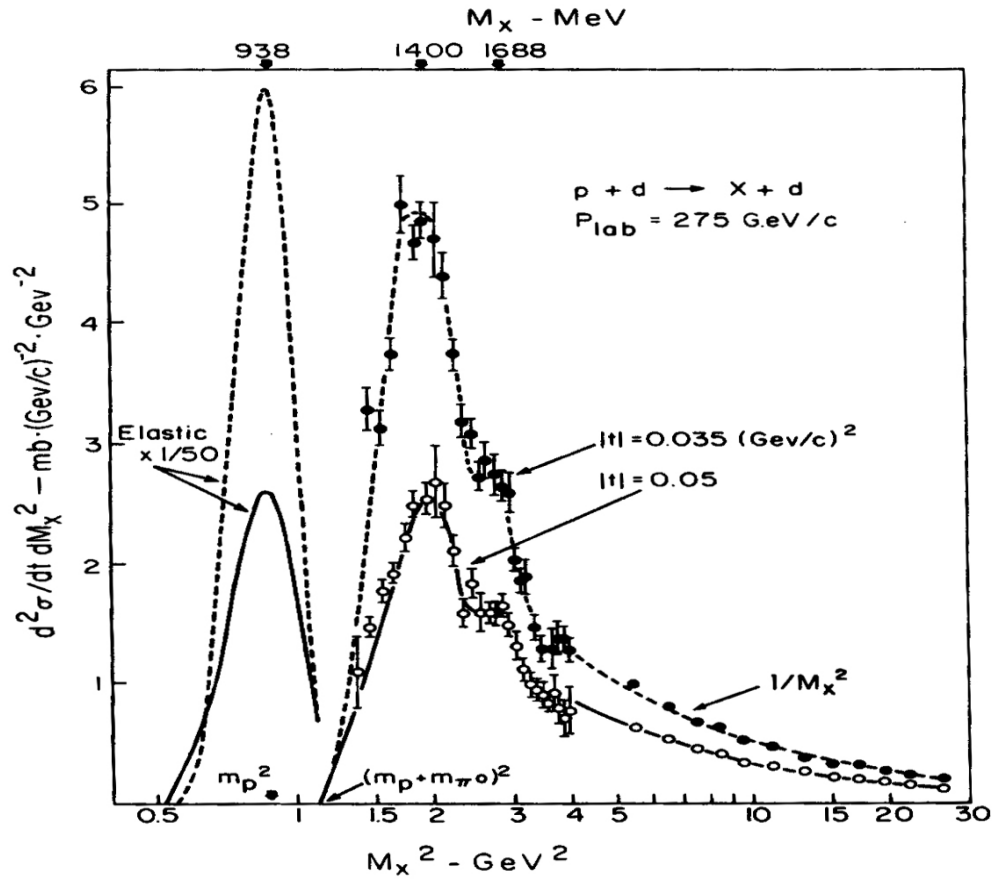


The function $N(s)$ is the so-called renormalization factor, introduced by K. Goulianos,

$$N_s \equiv \int_{\xi(\min)}^{\xi(\max)} \int_{t=0}^{-\infty} dt f_{P/p}(\xi, t) \sim s^{2\epsilon},$$

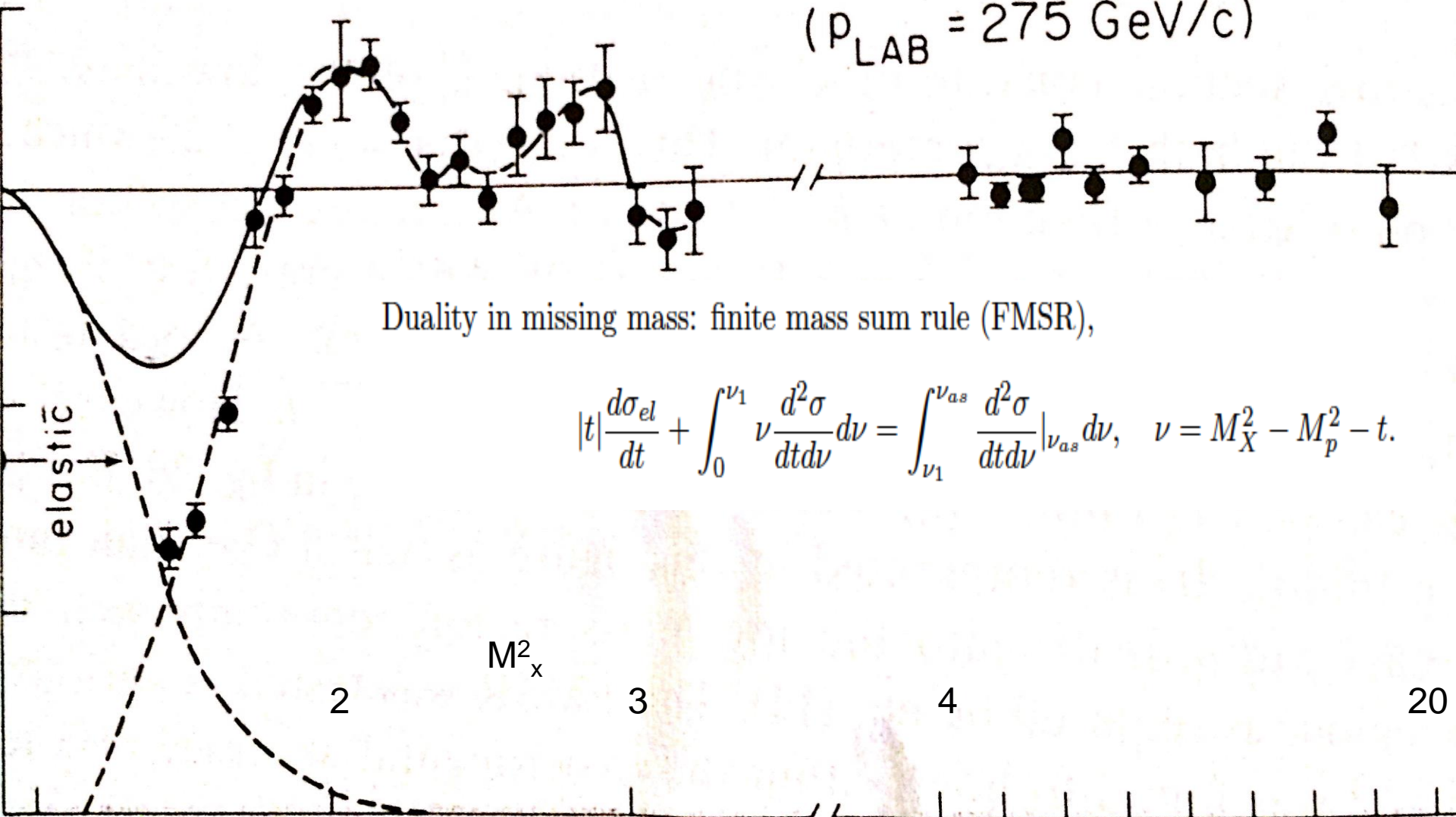
where $\xi(\min) = 1.4/s$ and $\xi(\max) = 0.1$. This factor secures unitarity.

FNAL



$$\nu \frac{d^2\sigma}{dt dM_X^2} \Big|_{|t|=0.035} \quad (p+d \rightarrow X+d) / F_d$$

$(p_{LAB} = 275 \text{ GeV}/c)$



Low-mass diffraction dissociation at the LHC

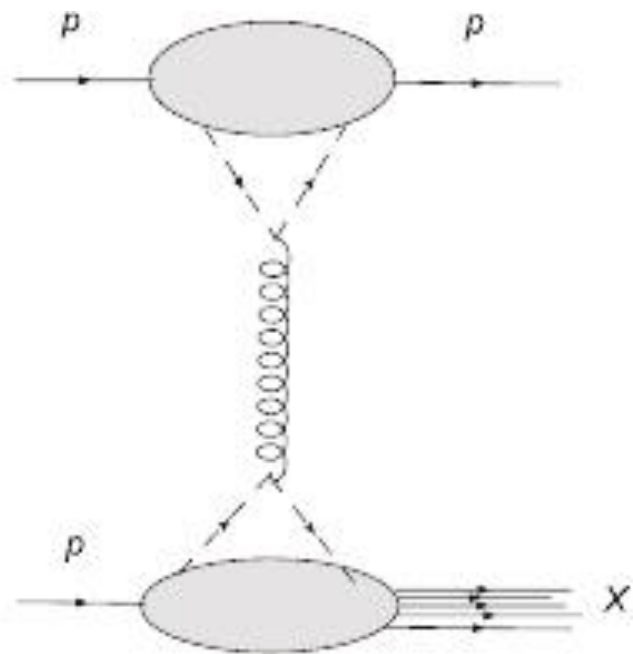
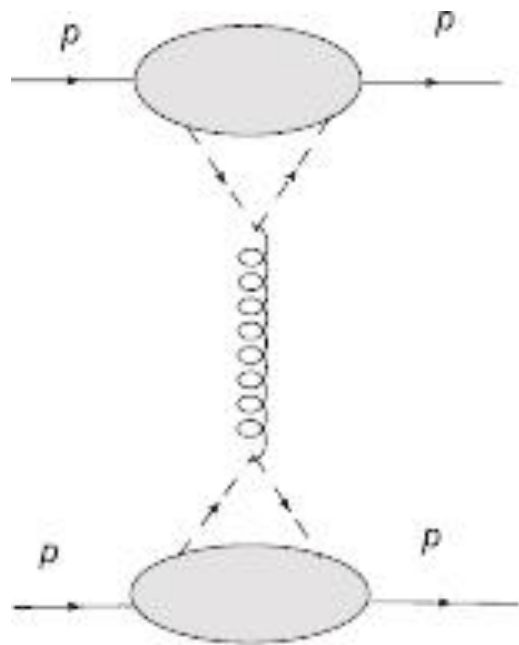
L. Jenkovszky, O. Kuprash, J. Lamsa, V. Magas, and R. Orava:
Dual-Regge approach to high-energy, low-mass DD at the LHC,
Phys. Rev. D83(2011)0566014; hep-ph/1-11.0664.

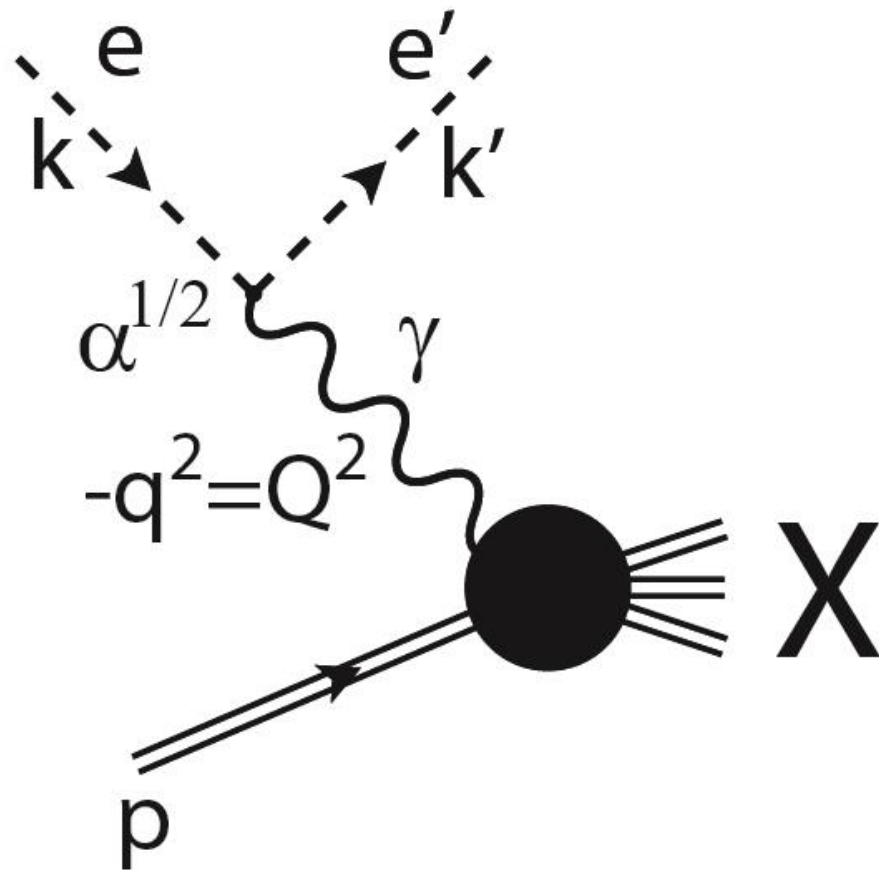
L. Jenkovszky, O. Kuprash, J. Lamsa and R. Orava: hep-ph/11063299,
Mod. Phys. Letters A. **26**(2011) 1-9, August 2011;

L. Jenkovszky, O. Kuprash, Risto Orava, A. Sali, arXiv:**1211.584**,
Low missing mass, single- and double diffraction dissociation at the LHC

Experimentally, diffraction dissociation in proton-proton scattering was intensively studied in the '70-ies at the Fermilab and the CERN ISR. In particular, double differential cross section $\frac{d\sigma}{dt dM_X^2}$ was measured in the region $0.024 < -t < 0.234$ (GeV/c)², $0 < M^2 < 0.12s$, and $(105 < s < 752)$ GeV², and a single peak in M_X^2 was identified.

Low-mass single diffraction dissociation (SDD) of protons, $pp \rightarrow pX$ as well as their double diffraction dissociation (DDD) are among the priorities at the LHC. For the CMS Collaboration, the SDD mass coverage is presently limited to some 10 GeV. With the Zero Degree Calorimeter (ZDC), this could be reduced to smaller masses, in case the SDD system produces very forward neutrals, i.e. like a N^* decaying into a fast leading neutron. Together with the T2 detectors of TOTEM, SDD masses down to 4 GeV could be covered.





$$\left| \begin{array}{c} \text{wavy } q \\ \bullet \\ \text{double } p \\ \text{X} \end{array} \right|^2 = \sum_X \begin{array}{c} \text{wavy } q \\ \bullet \\ \text{double } p \\ \text{X} \end{array} = \begin{array}{c} \text{wavy } q \\ \bullet \\ \text{double } p \\ \text{X} \end{array} \begin{array}{c} \text{wavy } q \\ \bullet \\ \text{double } p \\ \text{X} \end{array} = \sum_R \begin{array}{c} \text{wavy } q \\ \bullet \\ \text{double } p \\ \text{R} \end{array} = \sum_{\text{Res}} \begin{array}{c} \text{wavy } q \\ \bullet \\ \text{double } p \\ \text{Res} \end{array}$$

Unitarity $t=0$

Veneziano duality

Similar to the case of elastic scattering, the double differential cross section for the SDD reaction, by Regge factorization, can be written as

$$\frac{d^2\sigma}{dt dM_X^2} = \frac{9\beta^4 [F^p(t)]^2}{4\pi \sin^2[\pi\alpha_P(t)/2]} (s/M_X^2)^{2\alpha_P(t)-2} \times \quad (1)$$

$$\left[\frac{W_2}{2m} \left(1 - M_X^2/s\right) - mW_1(t + 2m^2)/s^2 \right],$$

where W_i , $i = 1, 2$ are related to the structure functions of the nucleon and $W_2 \gg W_1$. For high M_X^2 , the $W_{1,2}$ are Regge-behaved, while for small M_X^2 their behavior is dominated by nucleon resonances. The knowledge of the inelastic form factors (or transition amplitudes) is crucial for the calculation of low-mass diffraction dissociation.

The pp scattering amplitude

$$A(s, t)_P = -\beta^2 [f^u(t) + f^d(t)]^2 \left(\frac{s}{s_0}\right)^{\alpha_P(t)-1} \frac{1 + e^{-i\pi\alpha_P(t)}}{\sin \pi\alpha_P(t)}, \quad (1)$$

where $f^u(t)$ and $f^d(t)$ are the amplitudes for the emission of u and d valence quarks by the nucleon, β is the quark-Pomeron coupling, to be determined below; $\alpha_P(t)$ is a vacuum Regge trajectory. It is assumed that the Pomeron couples to the proton via quarks like a scalar photon.

A single-Pomeron exchange is valid at the LHC energies, however at lower energies (e.g. those of the ISR or the SPS) the contribution of non-leading Regge exchanges should be accounted for as well.

Thus, the unpolarized elastic pp differential cross section is

$$\frac{d\sigma}{dt} = \frac{[3\beta F^p(t)]^4}{4\pi \sin^2[\pi\alpha_P(t)/2]} (s/s_0)^{2\alpha_P(t)-2}. \quad (2)$$

The final expression for the double differential cross section reads:

$$\begin{aligned}
 \frac{d^2\sigma}{dt dM_X^2} = & \\
 A_0 \left(\frac{s}{M_X^2} \right)^{2\alpha_P(t)-2} & \frac{x(1-x)^2 [F^p(t)]^2}{(M_x^2 - m^2) \left(1 + \frac{4m^2 x^2}{-t} \right)^{3/2}} \times \\
 \sum_{n=1,3} & \frac{[f(t)]^{2(n+1)} \text{Im } \alpha(M_X^2)}{(2n + 0.5 - \text{Re } \alpha(M_X^2))^2 + (\text{Im } \alpha(M_X^2))^2} .
 \end{aligned} \tag{1}$$

SD and DD cross sections

$$\frac{d^2 \sigma_{SD}}{dt dM_x^2} = F_p^2(t) F(x_B, t) \frac{\sigma_T^{Pp}(M_x^2, t)}{2m_p} \left(\frac{s}{M_x^2} \right)^{2(\alpha(t)-1)} \ln \left(\frac{s}{M_x^2} \right)$$

$$\begin{aligned} \frac{d^3 \sigma_{DD}}{dt dM_1^2 dM_2^2} &= C_n F^2(x_B, t) \frac{\sigma_T^{Pp}(M_1^2, t)}{2m_p} \frac{\sigma_T^{Pp}(M_2^2, t)}{2m_p} \\ &\times \left(\frac{s}{(M_1 + M_2)^2} \right)^{2(\alpha(t)-1)} \ln \left(\frac{s}{(M_1 + M_2)^2} \right) \end{aligned}$$

“Reggeized (dual) Breit-Wigner” formula:

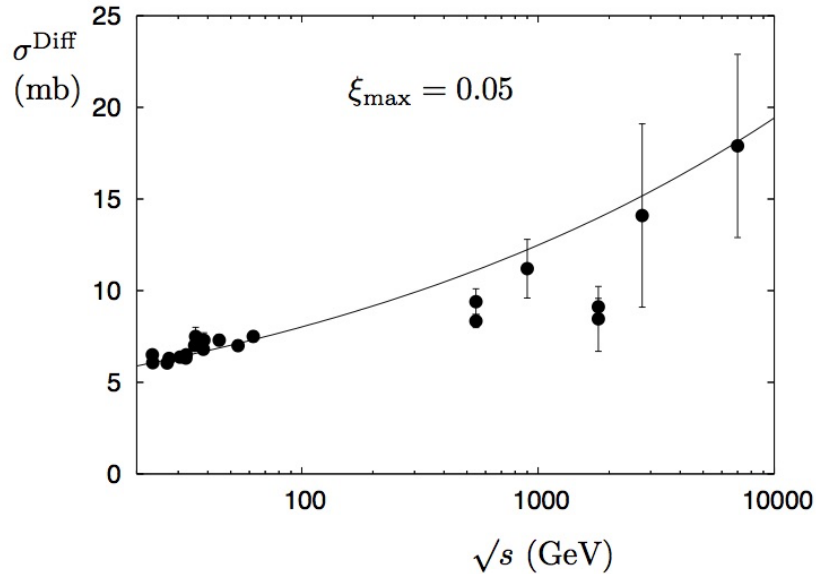
$$\begin{aligned} \sigma_T^{Pp}(M_x^2, t) &= \text{Im} A(M_x^2, t) = \frac{A_{N^*}}{\sum_n n - \alpha_{N^*}(M_x^2)} + Bg(t, M_x^2) = \\ &= A_n \sum_{n=0,1,\dots} \frac{[f(t)]^{2(n+1)} \text{Im} \alpha(M_x^2)}{(2n + 0.5 - \text{Re} \alpha(M_x^2))^2 + (\text{Im} \alpha(M_x^2))^2} + B_n e^{b_{in}^{bg} t} (M_x^2 - M_{p+\pi}^2)^\epsilon \end{aligned}$$

$$F(x_B, t) = \frac{x_B(1 - x_B)}{(M_x^2 - m_p^2) (1 + 4m_p^2 x_B^2 / (-t))^{3/2}}, \quad x_B = \frac{-t}{M_x^2 - m_p^2 - t}$$

$$F_p(t) = \frac{1}{1 - \frac{t}{0.71}}, \quad f(t) = e^{b_{in} t}$$

$$\alpha(t) = \alpha(0) + \alpha' t = 1.04 + 0.25t$$

Diffraction dissociation data problems



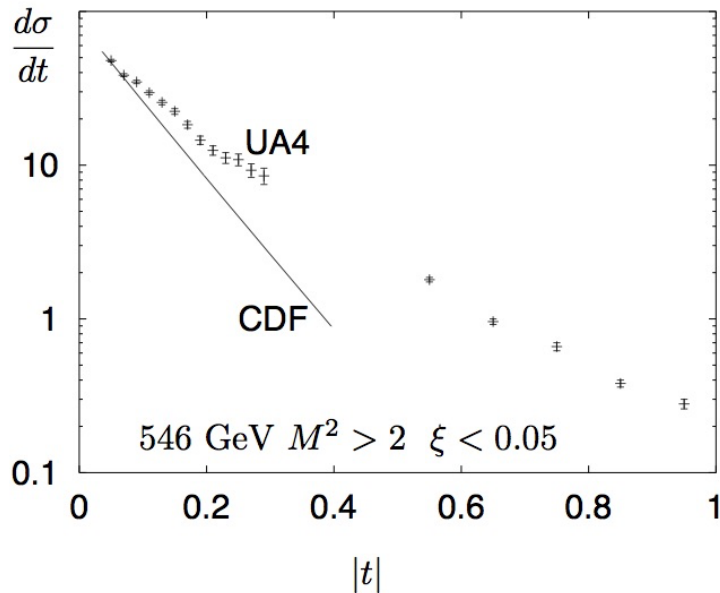
$$\sigma^{\text{Diff}}(s) = \int dt \int_{\xi_{\text{min}}}^{\xi_{\text{max}}} d\xi d^2\sigma/dtd\xi$$

Sensitive to ξ_{max} , and more so to ξ_{min}

Uppermost three points: ALICE at LHC

The curve rises as $s^{0.08}$

Issues with the data – often unclear experiments are measuring the same thing

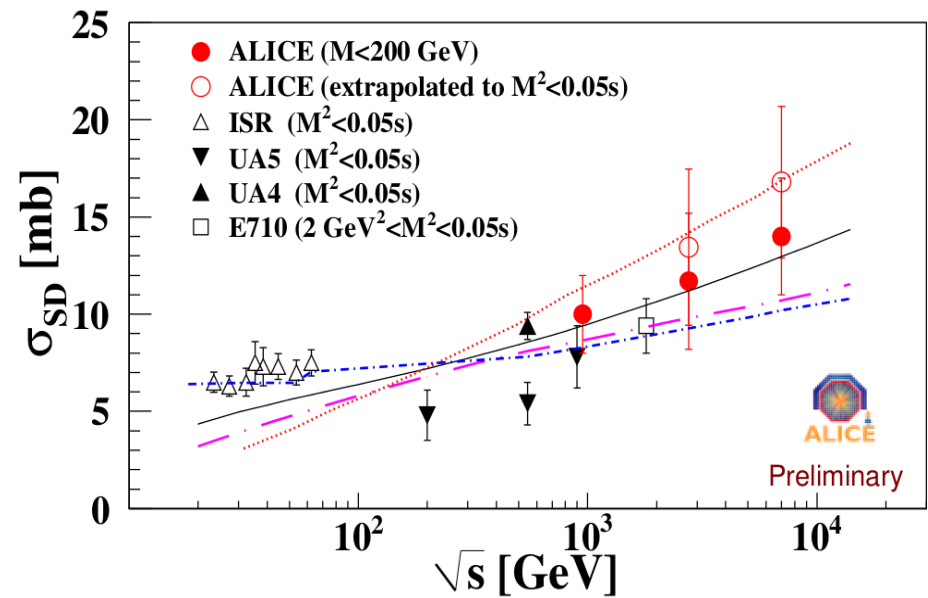
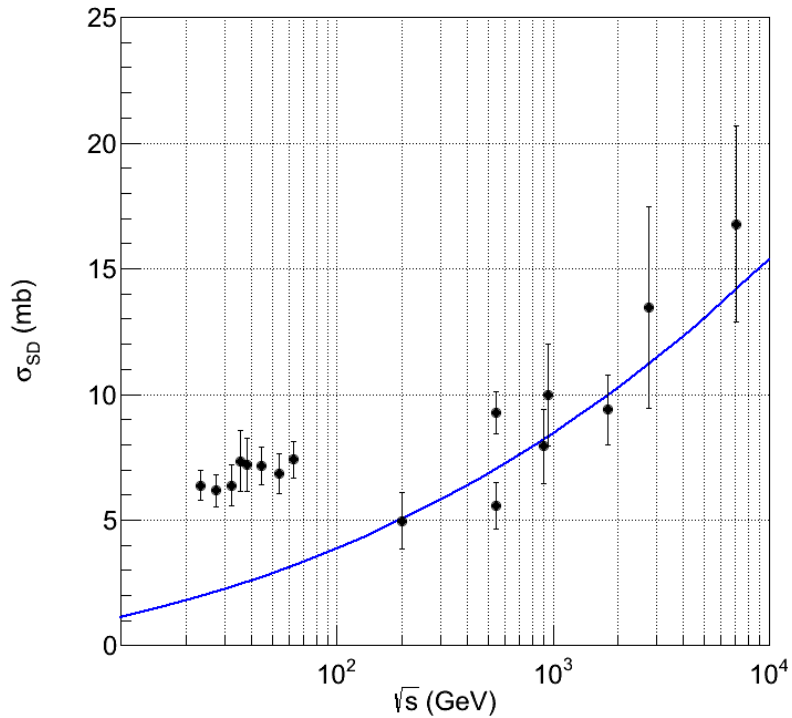


Data at $\sqrt{s} = 546$ GeV

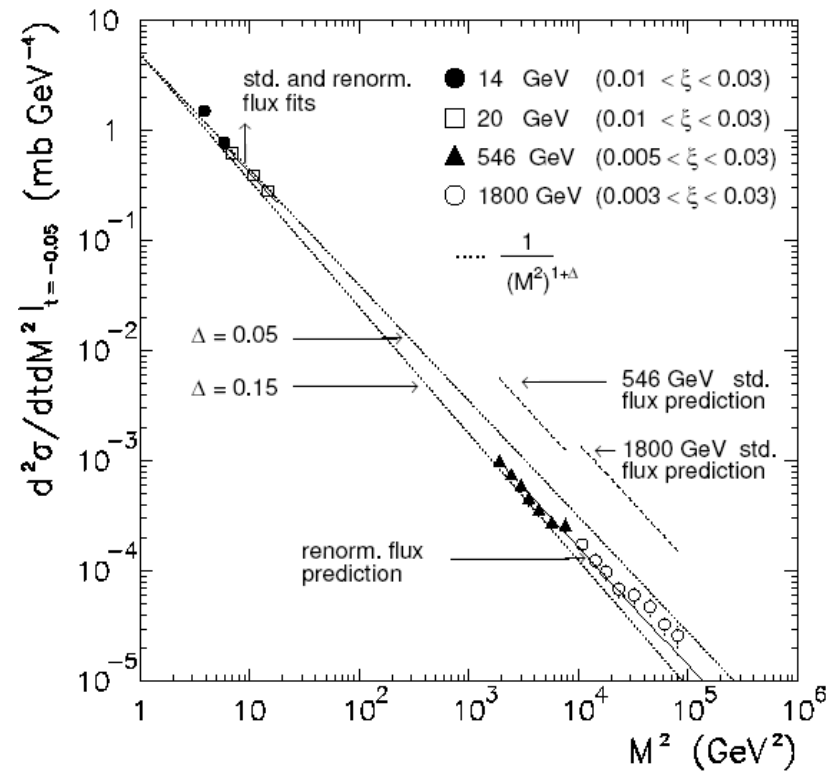
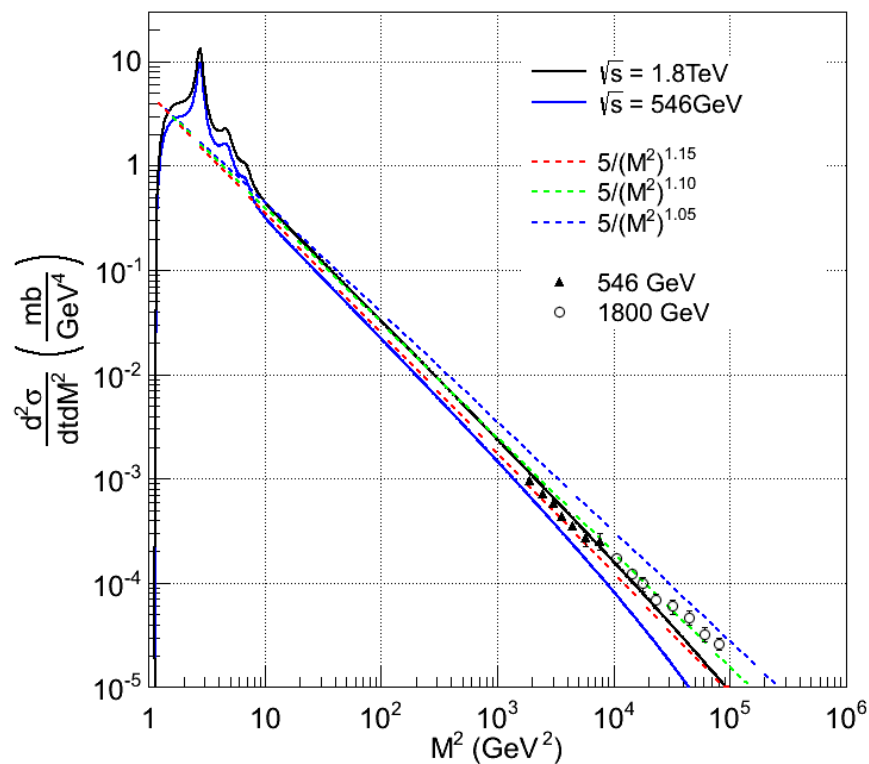
The line is CDF's parametrisation of its data for $d^2\sigma/dtd\xi$ integrated over ξ

Courtesy: Peter Landshoff

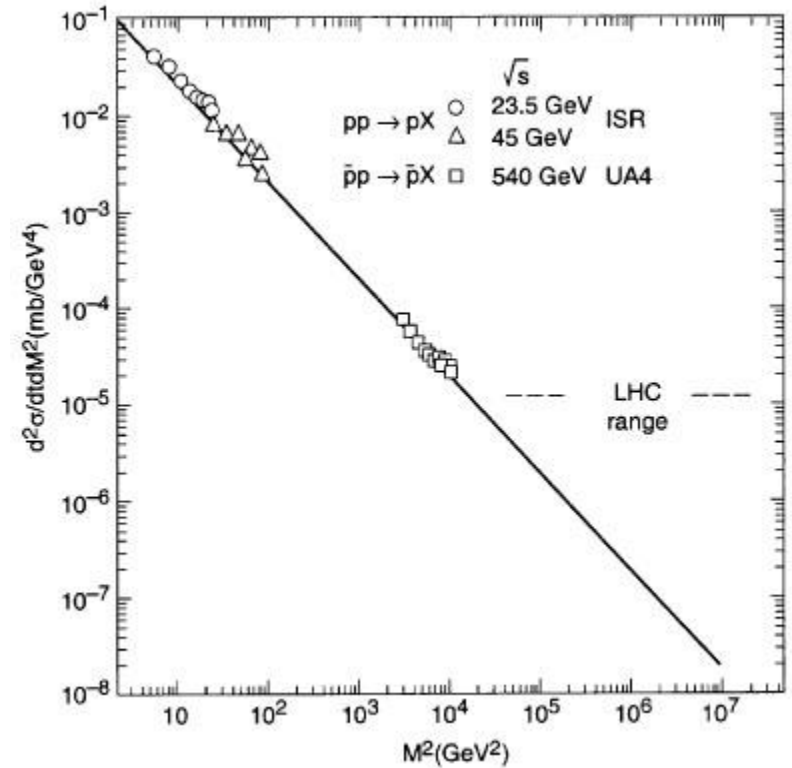
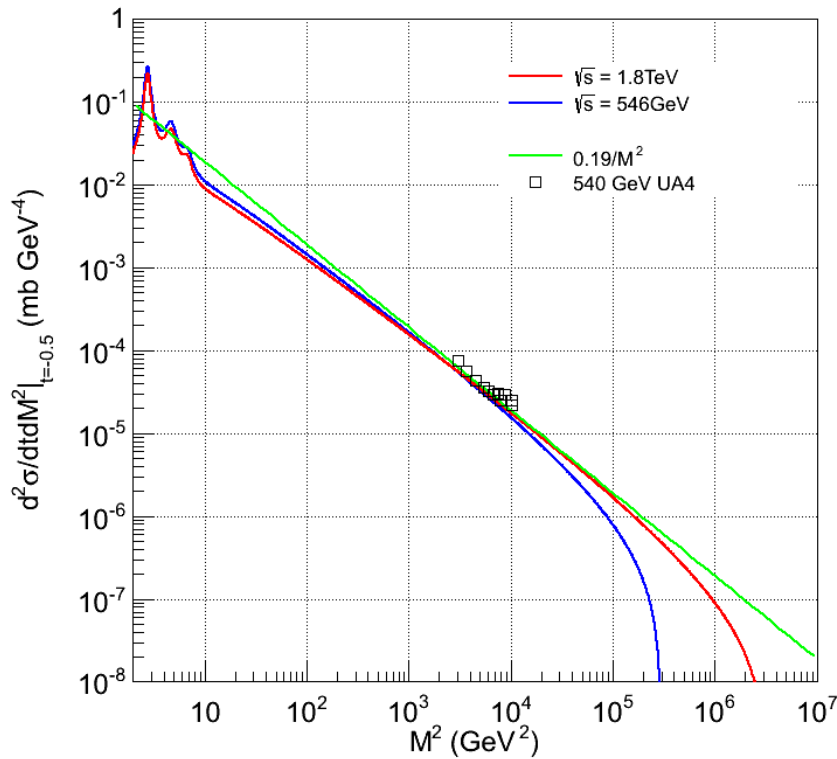
SDD cross sections vs. energy.



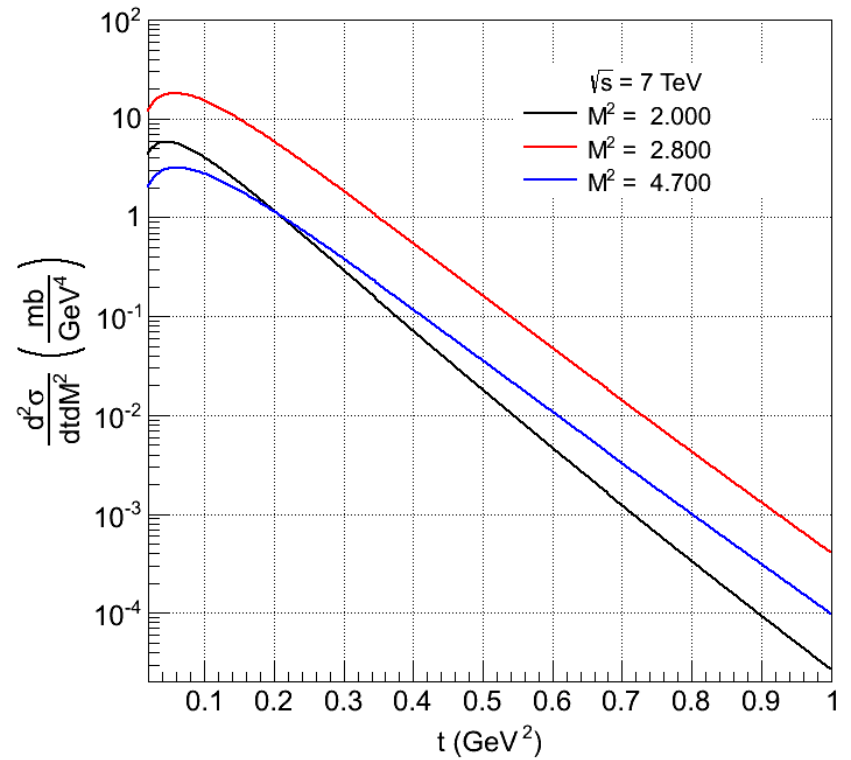
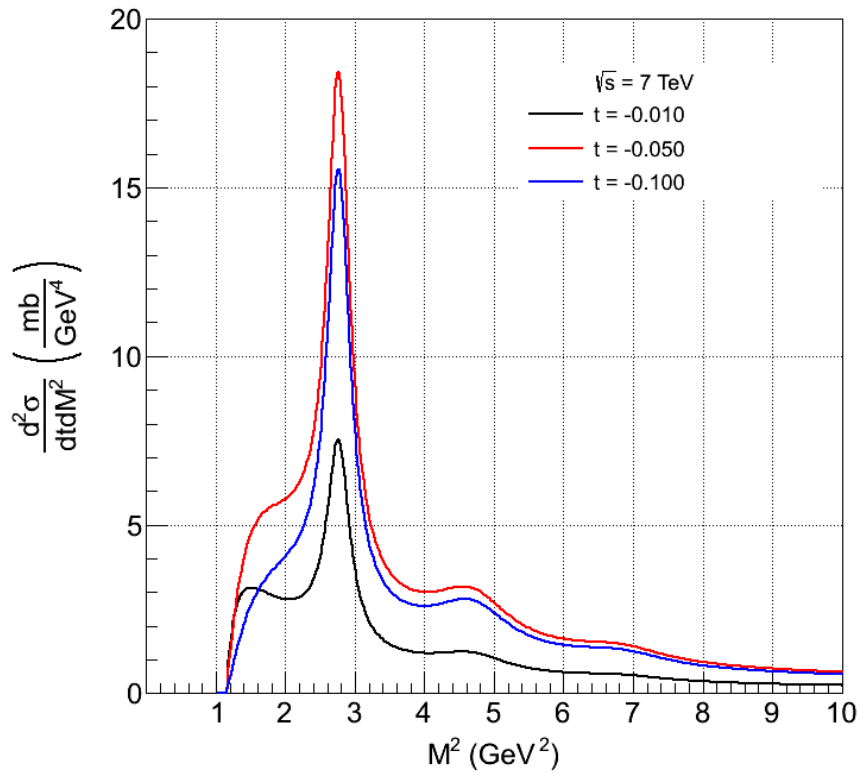
Approximation of background to reference points (t=-0.05)



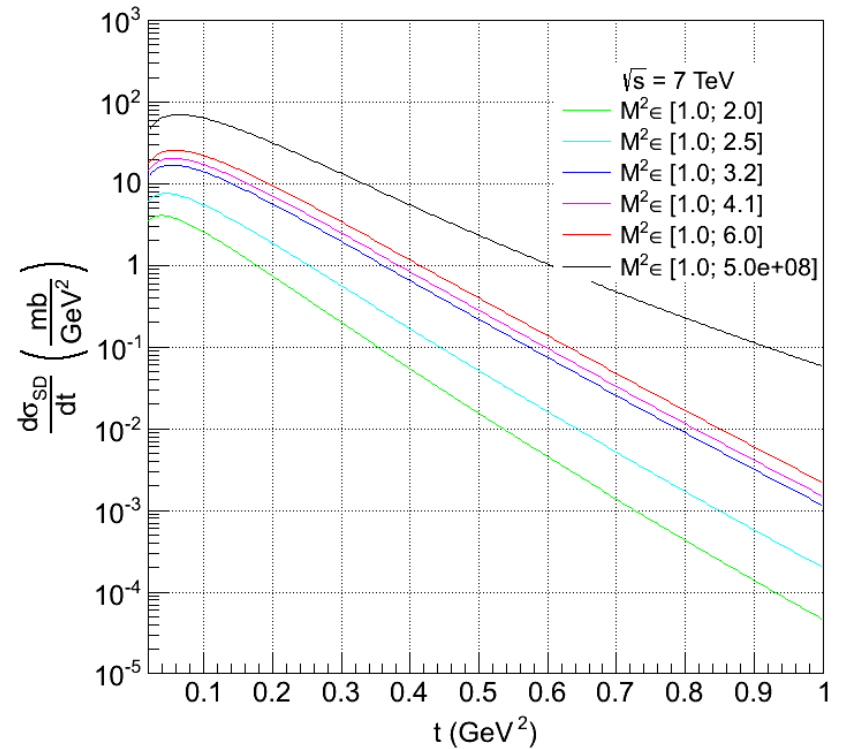
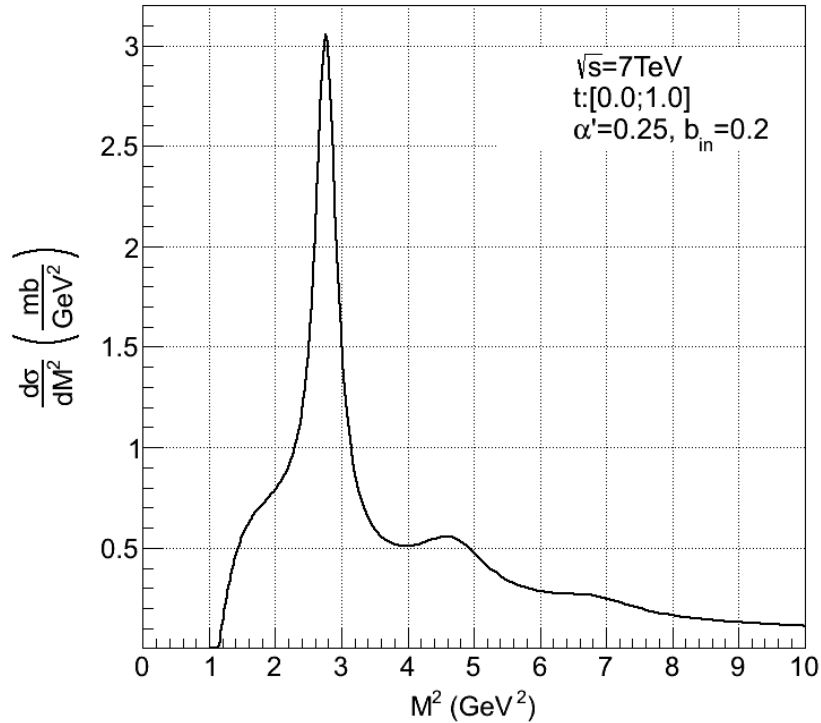
Approximation of background to reference points ($t=-0.5$)



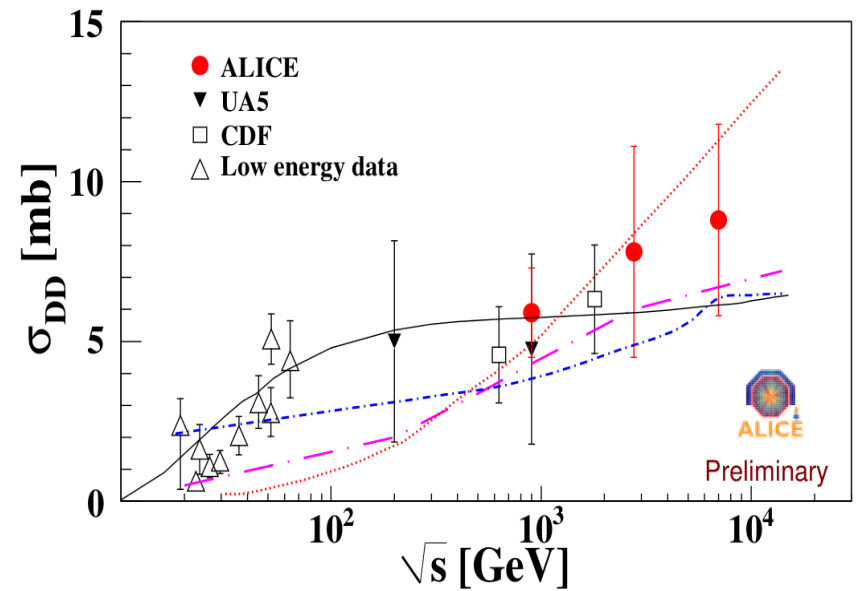
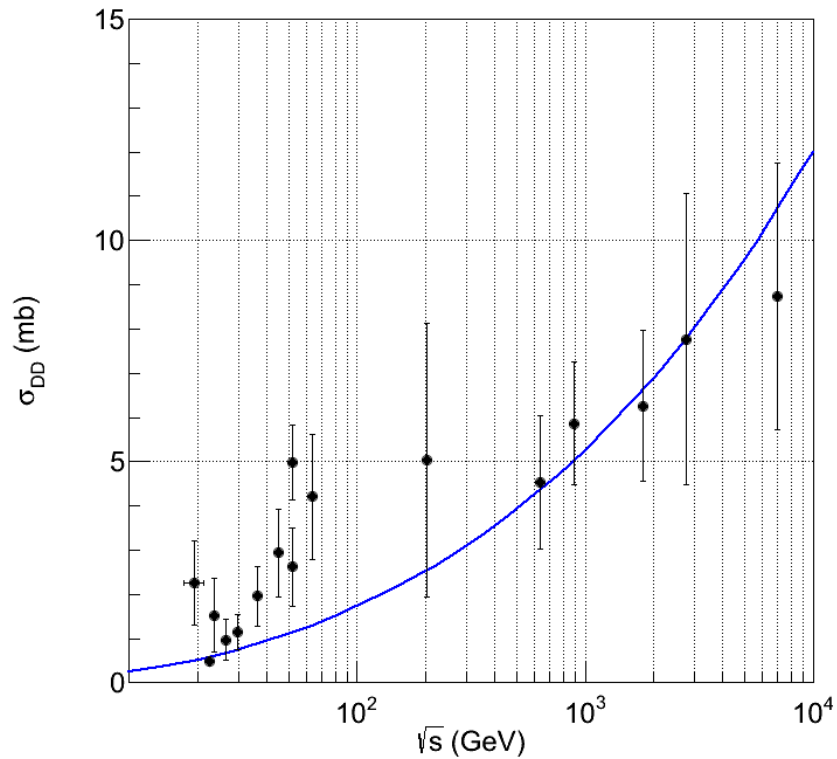
Double differential SD cross sections



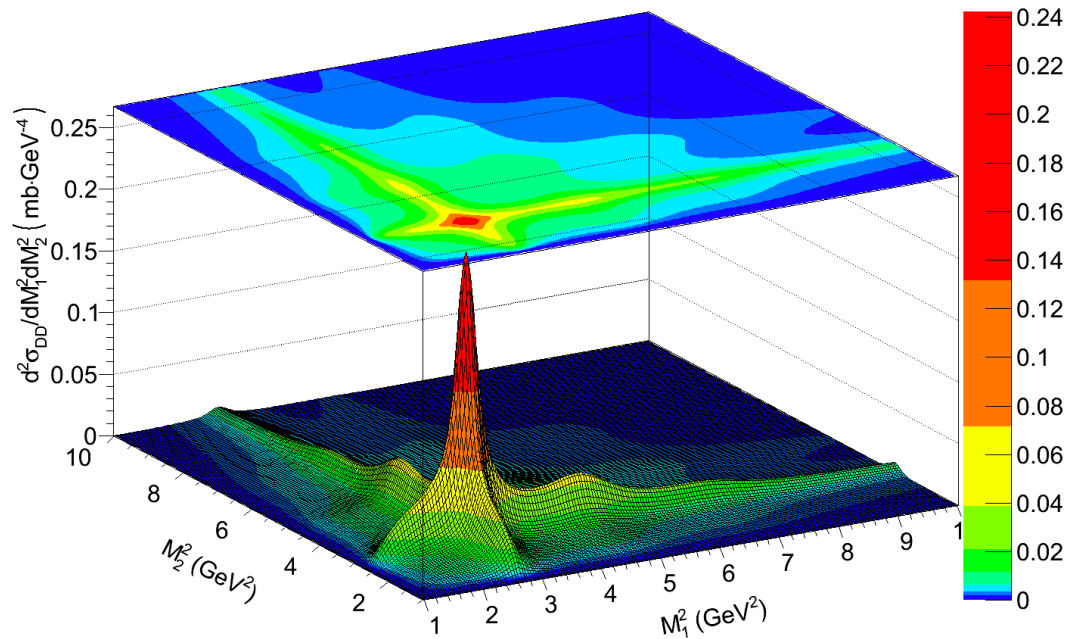
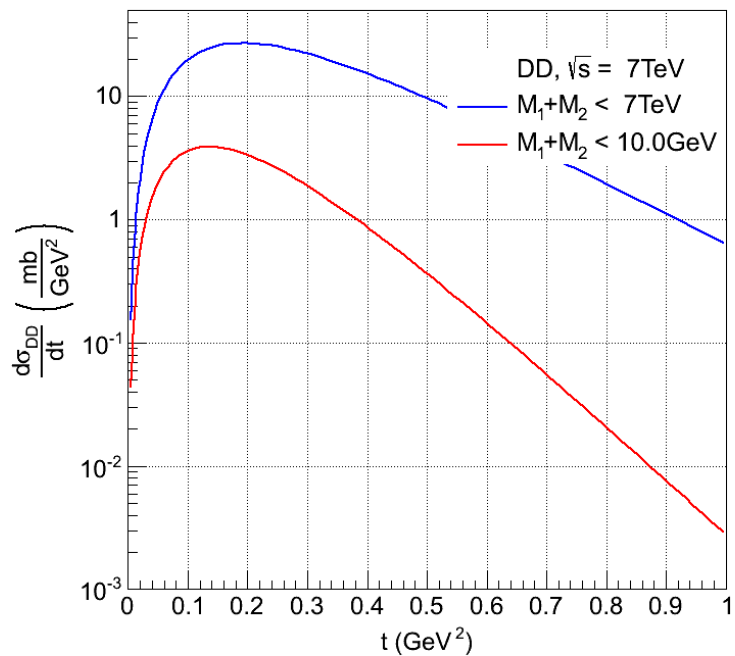
Single differential integrated SD cross sections



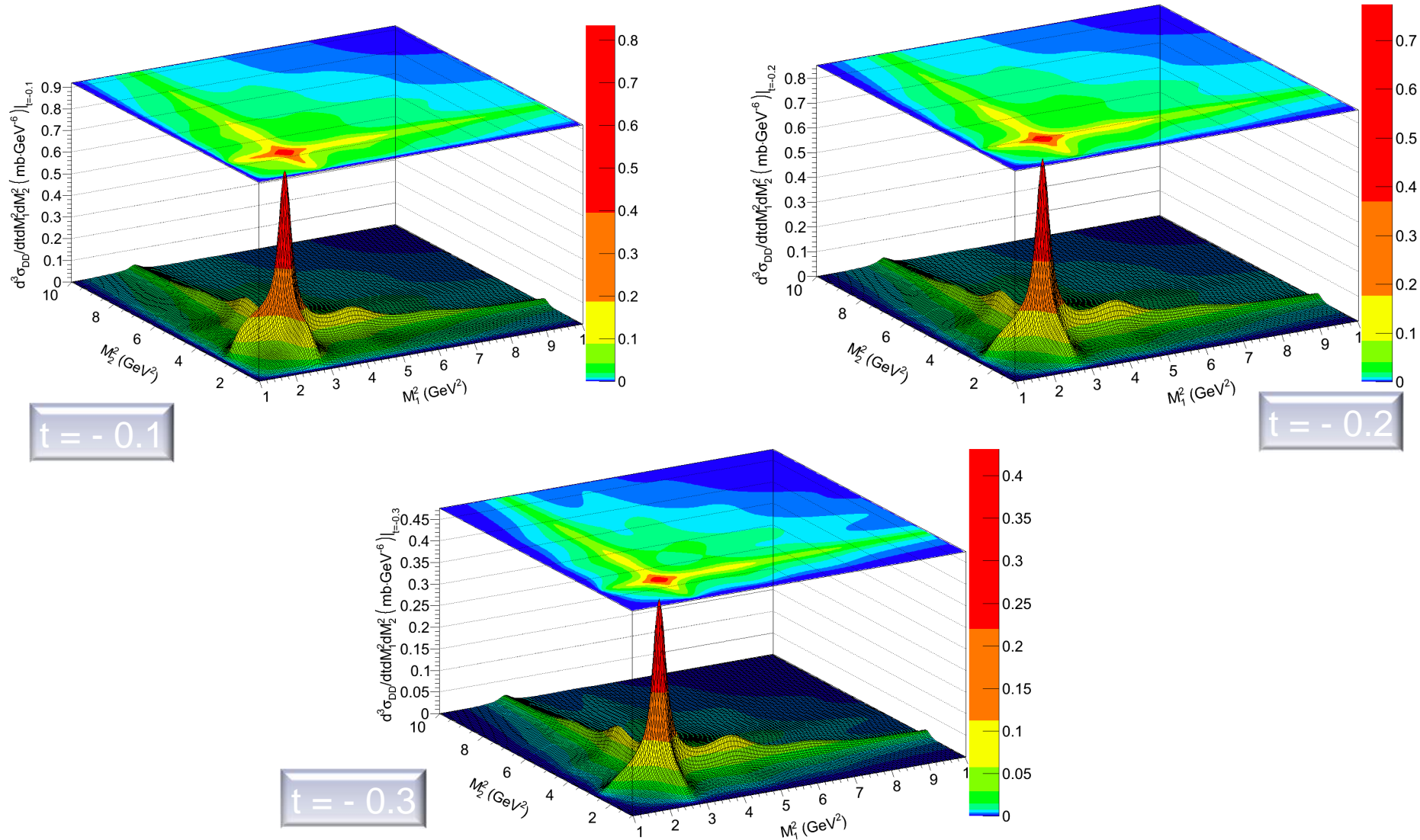
DDD cross sections vs. energy.



Integrated DD cross sections



Triple differential DD cross sections



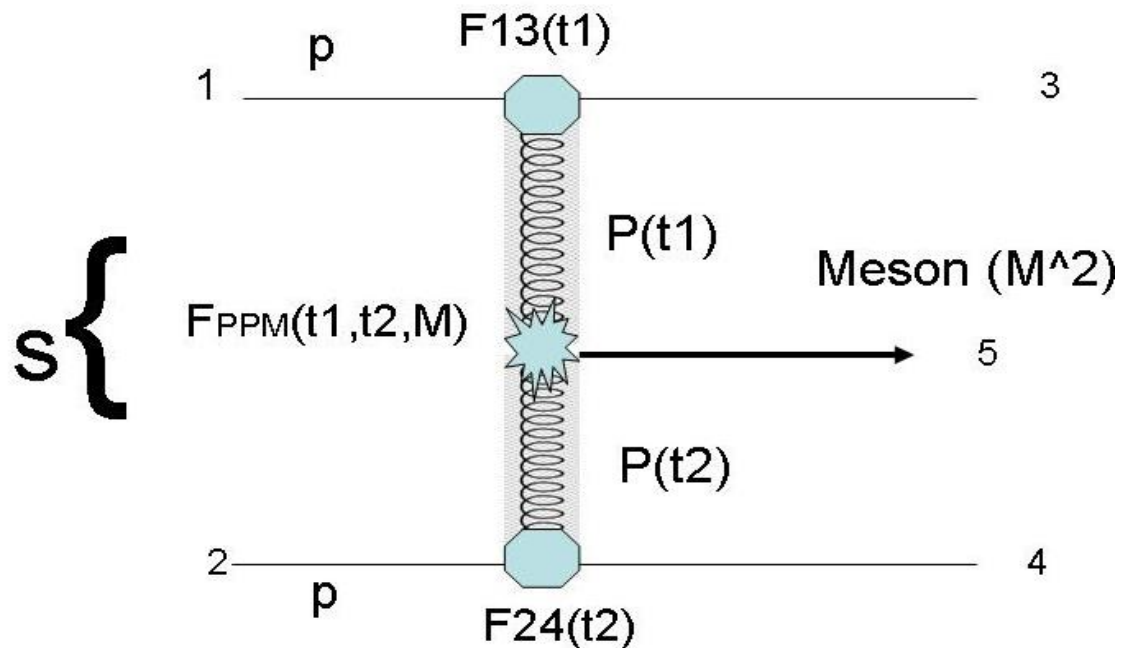
The parameters and results

$b_{in} \text{ (GeV}^{-2}\text{)}$	0.2
$b_{in}^{bg} \text{ (GeV}^{-2}\text{)}$	3
$\alpha' \text{ (GeV}^{-2}\text{)}$	0.25
$\alpha(0)$	1.04
ϵ	1.03
A_n	18.7
B_n	8.8
C_n	3.79e-2

$\sigma_{SD} \text{ (mb)}$	14.13
$\sigma_{SD}(M < 3.5\text{GeV}) \text{ (mb)}$	4.68
$\sigma_{SD}(M > 3.5\text{GeV}) \text{ (mb)}$	9.45
$\sigma_{Res}^{SD} \text{ (mb)}$	2.48
$\sigma_{Bg}^{SD} \text{ (mb)}$	9.45
$\sigma_{DD} \text{ (mb)}$	10.68
$\sigma_{DD}(M < 10\text{GeV}) \text{ (mb)}$	1.05
$\sigma_{DD}(M > 10\text{GeV}) \text{ (mb)}$	9.63

Prospects (future plans):

1. Central diffractive meson production (double Pomeron exchange);



2. Charge exchange reactions at the LHC (single Reggeon exchange), e.g. $pp \rightarrow n\Delta$ (in collaboration with Oleg Kuprash and Rainer Schicker)

Open problems:

- Need for data (LHC) on differential (nonintegrated!) data;
- Interpolation in energy: from the Fermilab and ISR to the LHC;
(Inclusion of non-leading contributions);
- Deviation from a simple Pomeron pole model and breakdown of
Regge-factorization;
- Need for a bank of models. Open a relevant international
PROJECT?!

**Elastic and total pp and p-\bar{p} scattering,
diffraction and the Pomeron;
nucleon's “shape”, the “black disc limit”**

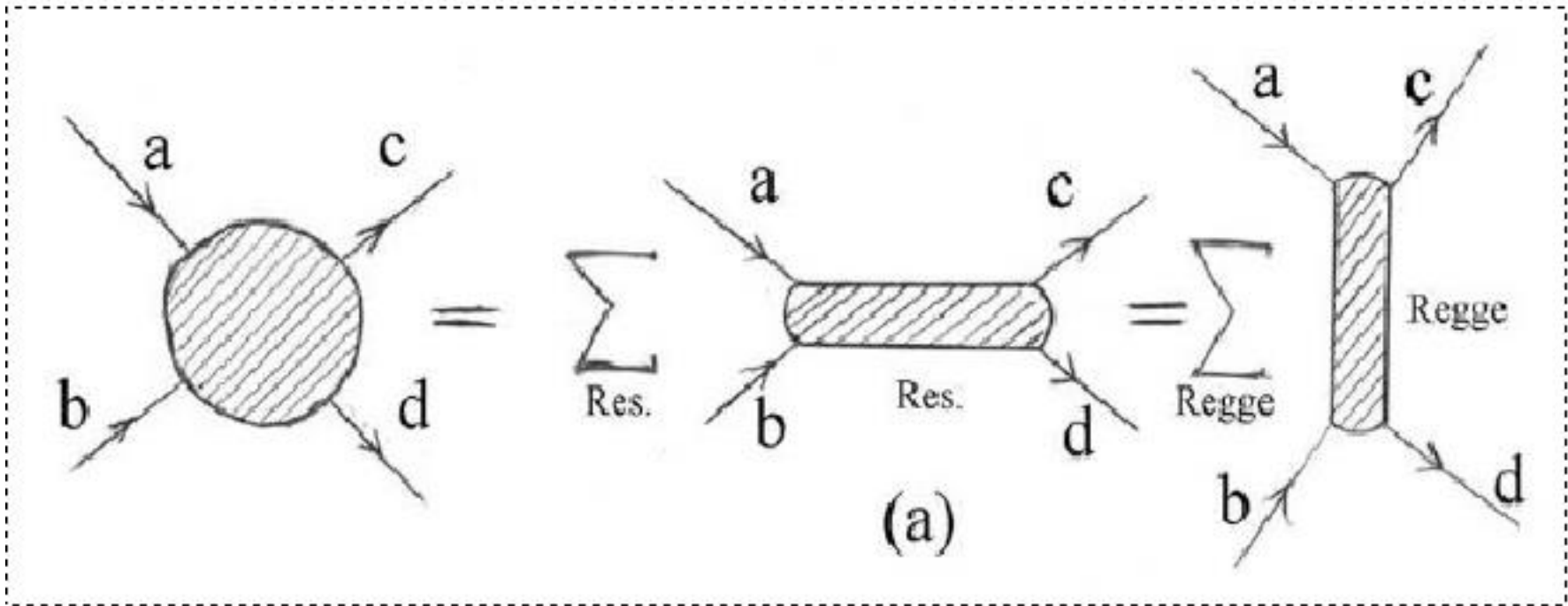
R. Fiore, L. Jenkovszky, R. Orava, E. Predazzi,
A. Prokudin, O. Selyugin, *Forward Physics at the LHC;
Elastic Scattering*, Int. J.Mod.Phys., **A24**: 2551-2559
(2009).

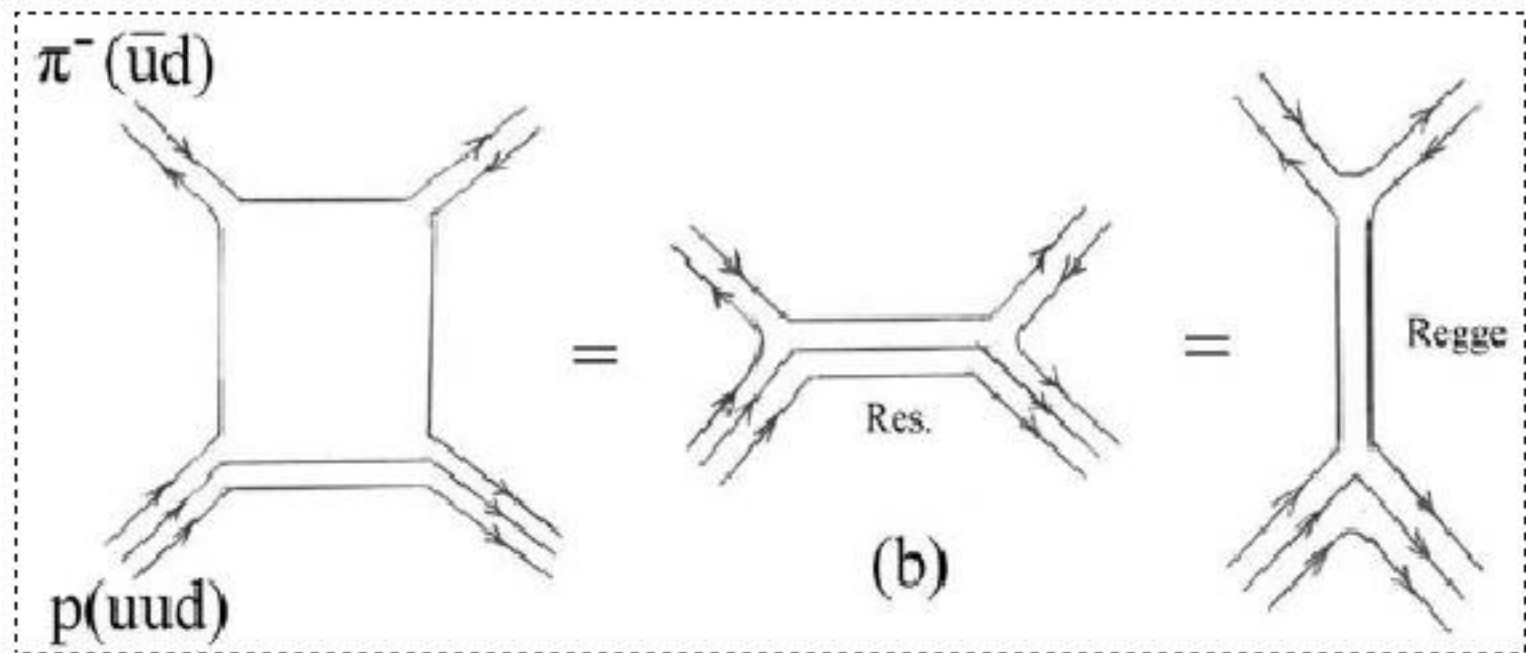
Thank you !

Does GS imply saturation? Not necessarily!

$ImH(s, b) = |h(s, b)|^2 + G_{in}(s, b)$, (h is associated with the "opacity"), Here from: $0 \leq |h(s, b)|^2 \leq \Im h(s, b) \leq 1$. The Black Disc Limit (BDL) corresponds to $\Im h(s, b) = 1/2$, provided $h(s, b) = i(1 - \exp[i\omega(s, b)])/2$, with an imaginary eikonal $\omega(s, b) = i\Omega(s, b)$.

There is an alternative solution, that with the "minus" sign in $h(s, b) = [1 \pm \sqrt{1 - 4G_{in}(s, b)}]/2$, giving (S. Troshin and N. Tyurin (Protvino)): $h(s, b) = \Im u(s, b) / [1 - iu(s, b)]$,





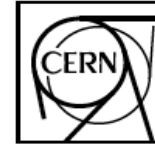
$$\sigma_t(s) = \frac{4\pi}{s} \text{Im}A(s, t=0); \quad \frac{d\sigma}{dt} = \frac{\pi}{s^2} |A(s, t)|^2; \quad \mathbf{n(s)};$$

$$\sigma_{el} = \int_{t_{min} \approx -s/2 \approx \infty}^{t_{thr.} \approx 0} \frac{d\sigma}{dt}; \quad \sigma_{in} = \sigma_t - \sigma_{el}; \quad B(s, t) = \frac{d}{dt} \ln\left(\frac{d\sigma}{dt}\right);$$

$$A_{pp}^{\bar{p}}(s, t) = P(s, t) \pm O(s, t) + f(s, t) \pm \omega(s, t) \rightarrow_{LHC} \approx P(s, t) \pm O(s, t),$$

where P , O , f , ω are the Pomeron, odderon and non-leading Reggeon contributions.

$\alpha(\mathbf{0}) \setminus \mathbf{C}$	+	-
1	P	O
1/2	f	ω



TOTEM 2011-01
22 June 2011

CERN-PH-EP-2011-101
26 June 2011

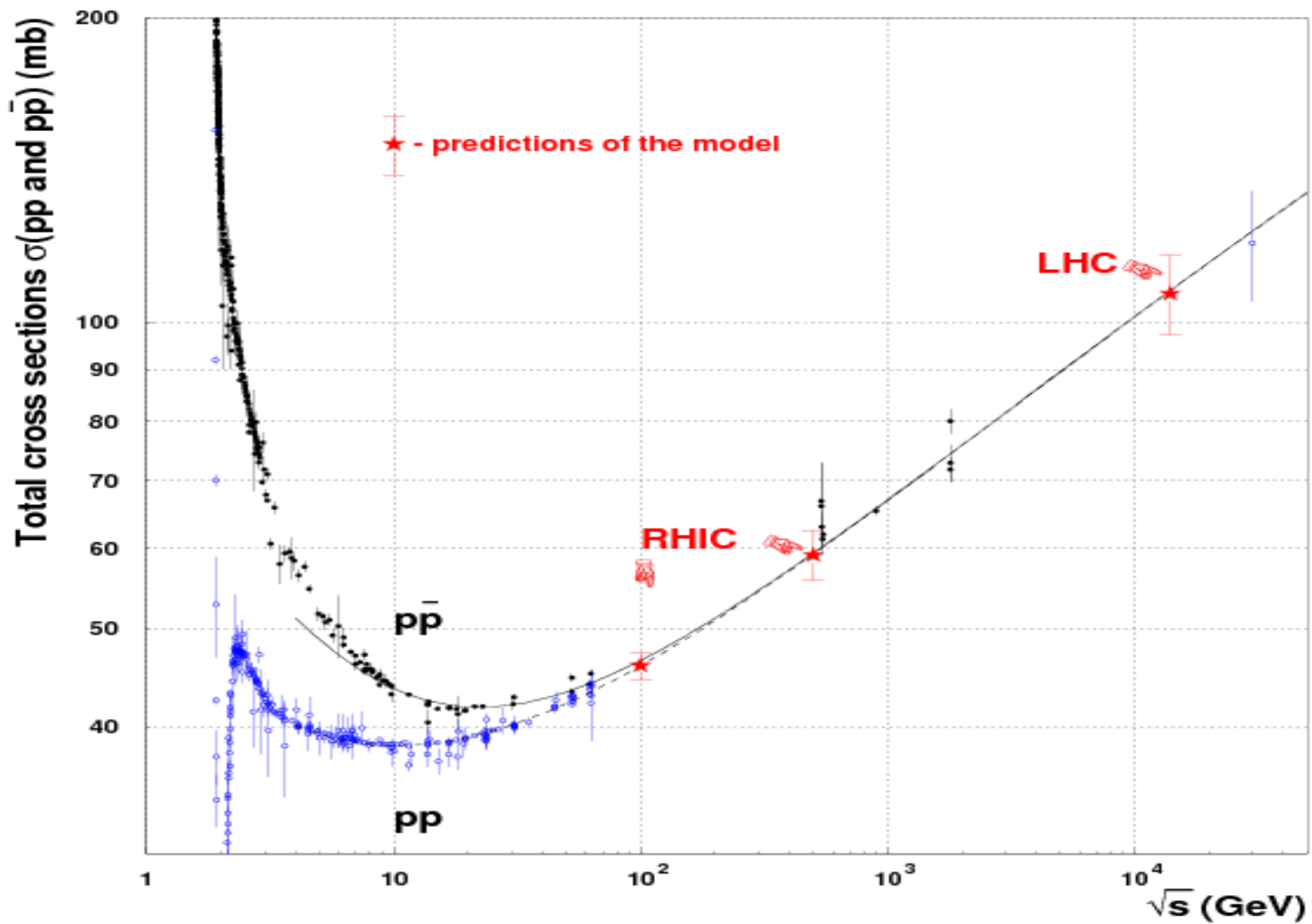
Elastic pp Scattering at the LHC at $\sqrt{s} = 7$ TeV.

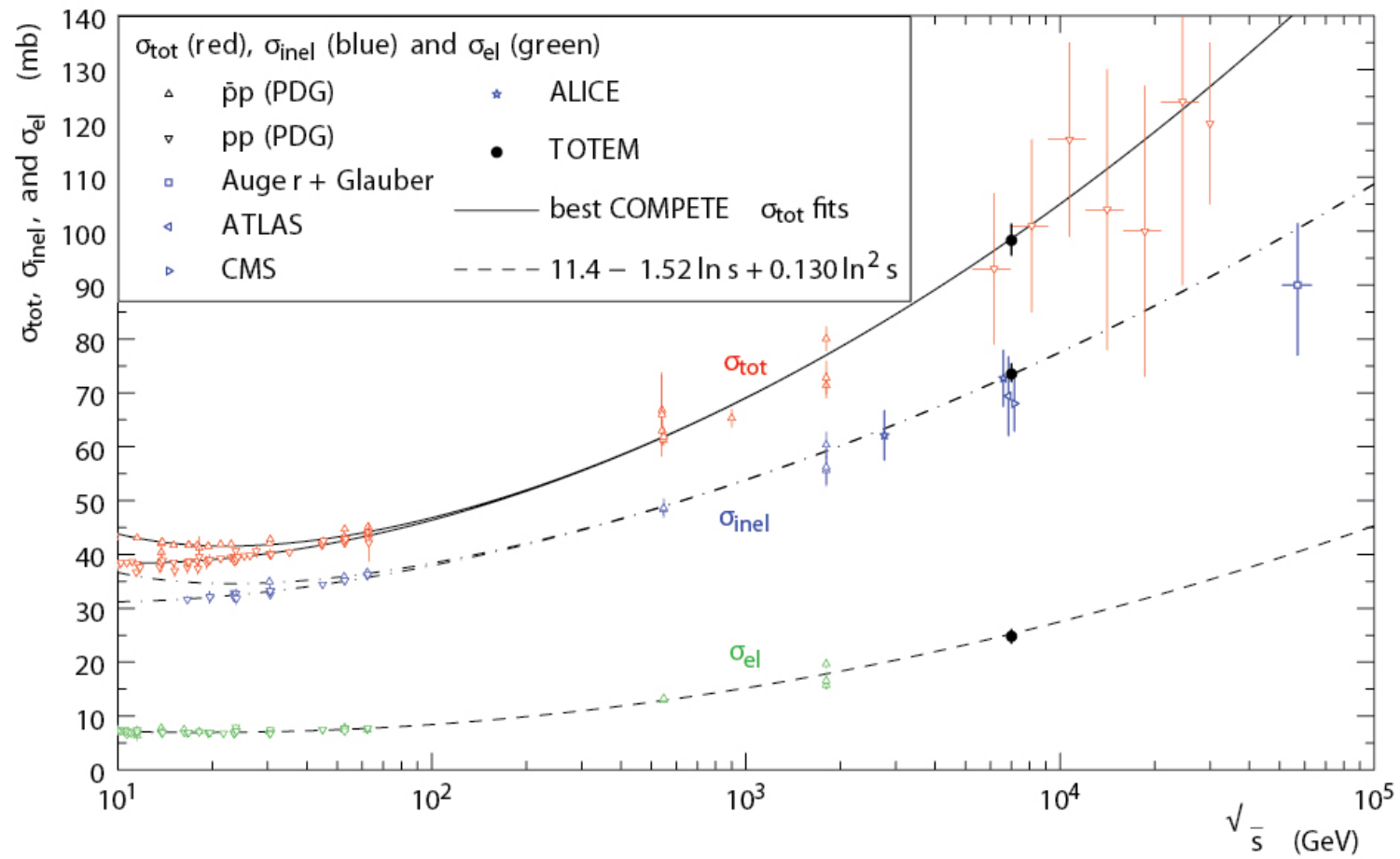
The TOTEM Collaboration

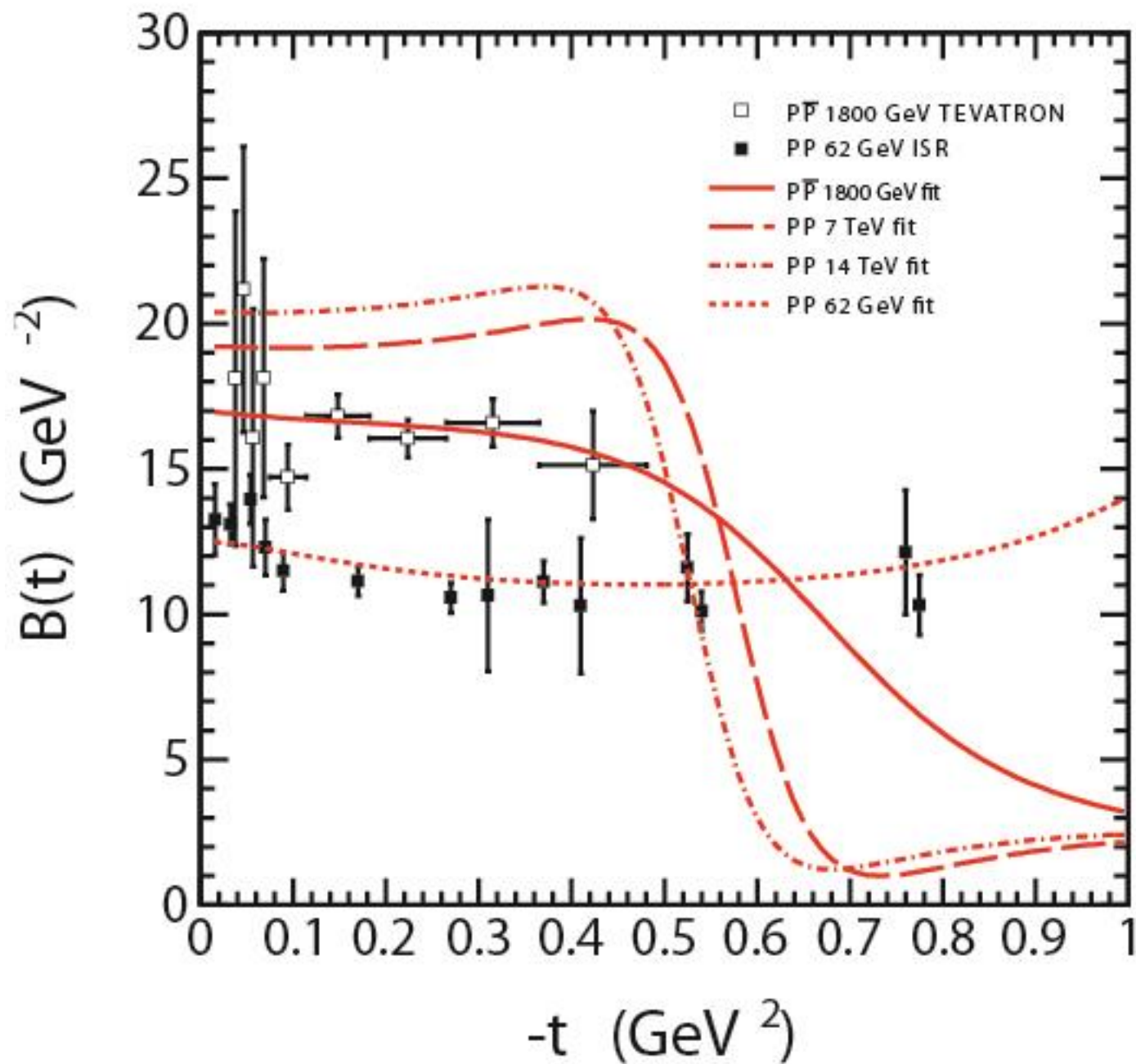
G. Antchev^{*}, P. Aspell⁸, I. Atanassov^{8,*}, V. Avati⁸, J. Baechler⁸, V. Berardi^{5b,5a}, M. Berretti^{7b},
M. Bozzo^{6b,6a}, E. Brücken^{3a,3b}, A. Buzzo^{6a}, F. Cafagna^{5a}, M. Calicchio^{5b,5a}, M. G. Catanesi^{5a},
C. Covault⁹, M. Csanád^{4†}, T. Csörgö⁴, M. Deile⁸, E. Dimovasili⁸, M. Doubek^{1b}, K. Eggert⁹,
V. Eremin[‡], F. Ferro^{6a}, A. Fiergolski[§], F. Garcia^{3a}, S. Giani⁸, V. Greco^{7b,8}, L. Grzanka^{8,¶}, J. Heino^{3a},
T. Hilden^{3a,3b}, M. Janda^{1b}, J. Kašpar^{1a,8}, J. Kopal^{1a,8}, V. Kundrať^{1a}, K. Kurvinen^{3a}, S. Lami^{7a},
G. Latino^{7b}, R. Lauhakangas^{3a}, T. Leszko[§], E. Lippmaa², M. Lokajíček^{1a}, M. Lo Vetere^{6b,6a},
F. Lucas Rodríguez⁸, M. Macrí^{6a}, L. Magaletti^{5b,5a}, G. Magazzù^{7a}, A. Mercadante^{5b,5a}, M. Meucci^{7b},
S. Minutoli^{6a}, F. Nemes^{4,†}, H. Niewiadomski⁸, E. Noschis⁸, T. Novak^{4,||}, E. Oliveri^{7b}, F. Oljemark^{3a,3b},
R. Orava^{3a,3b}, M. Oriunno^{8**}, K. Österberg^{3a,3b}, A.-L. Perrot⁸, P. Palazzi⁸, E. Pedreschi^{7a},
J. Petäjäjärvi^{3a}, J. Procházka^{1a}, M. Quinto^{5a}, E. Radermacher⁸, E. Radicioni^{5a}, F. Ravotti⁸,
E. Robutti^{6a}, L. Ropelewski⁸, G. Ruggiero⁸, H. Saarikko^{3a,3b}, A. Santroni^{6b,6a}, A. Scribano^{7b},
G. Sette^{6b,6a}, W. Snoeys⁸, F. Spinella^{7a}, J. Sziklai⁴, C. Taylor⁹, N. Turini^{7b}, V. Vacek^{1b}, J. Welti^{3a,b},
M. Vítek^{1b}, J. Whitmore¹⁰.

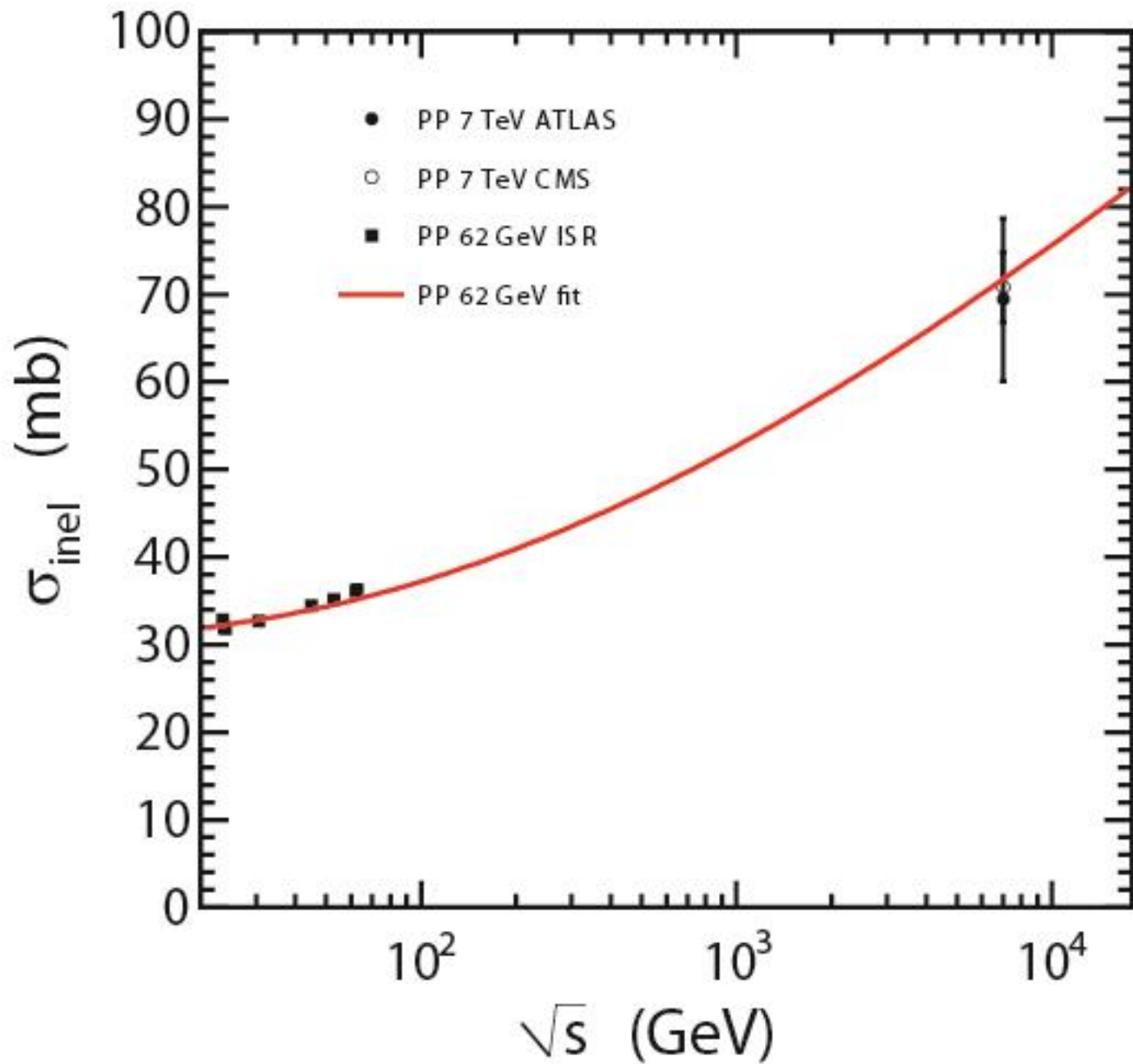
P. Aspell et. al. (TOTEM Collaboration), *Proton-proton elastic scattering at the LHC energy 7 TeV*, Europhys. Lett. **95** (2011) 41001; arXiv:1110.1385.

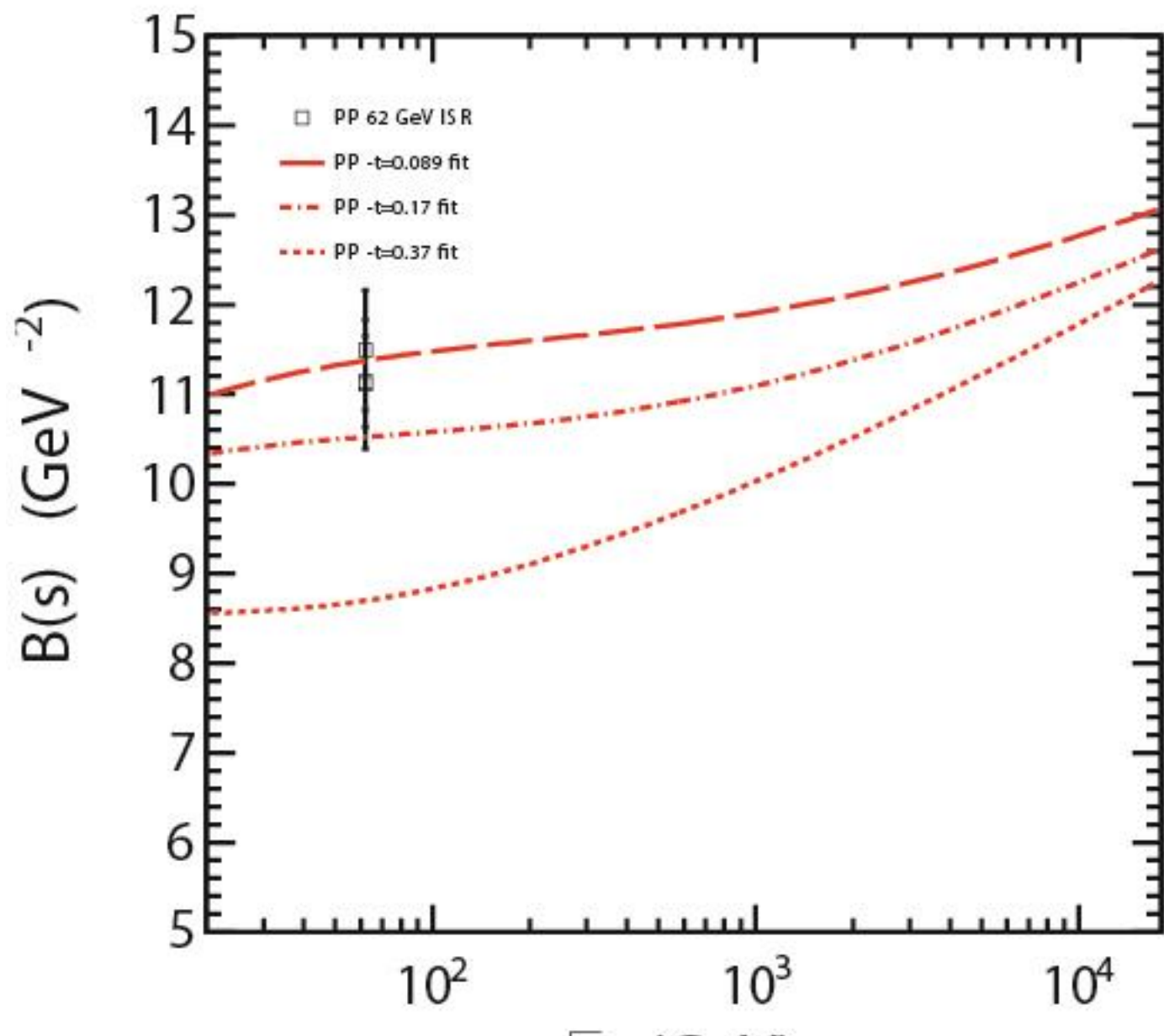
G. Antchev et al. (TOTEM Collab.), *First measurement of the total proton-proton cross section at the LHC energy of 7 TeV*, EPL; arXiv:1110.1395.











Phillips and Barger in 1973 [], right after its first observation at the ISR. Their formula reads

$$\frac{d\sigma}{dt} = |\sqrt{A} \exp(Bt/2) + \sqrt{C} \exp(Dt/2 + i\phi)|^2, \quad (1)$$

where A , B , C , D and ϕ are determined independently at each energy.

**L.Jenkovszky, A. Lengyel, D. Lontkovskyi:
The Pomeron and Odderon in elastic, inelastic and total cross-sections,
hep-ph/056014.**

The Pomeron is a dipole in the j -plane

$$A_P(s, t) = \frac{d}{d\alpha_P} \left[e^{-i\pi\alpha_P/2} G(\alpha_P) \left(s/s_0 \right)^{\alpha_P} \right] = \quad (1)$$

$$e^{-i\pi\alpha_P(t)/2} \left(s/s_0 \right)^{\alpha_P(t)} \left[G'(\alpha_P) + \left(L - i\pi/2 \right) G(\alpha_P) \right].$$

Since the first term in squared brackets determines the shape of the cone, one fixes

$$G'(\alpha_P) = -a_P e^{b_P[\alpha_P-1]}, \quad (2)$$

where $G(\alpha_P)$ is recovered by integration, and, as a consequence, the Pomeron amplitude can be rewritten in the following “geometrical” form

$$A_P(s, t) = i \frac{a_P s}{b_P s_0} \left[r_1^2(s) e^{r_1(s)[\alpha_P-1]} - \varepsilon_P r_2^2(s) e^{r_2(s)[\alpha_P-1]} \right], \quad (3)$$

where $r_1^2(s) = b_P + L - i\pi/2$, $r_2^2(s) = L - i\pi/2$, $L \equiv \ln(s/s_0)$.