Time evolution of the spatial anisotropies in heavy ion collisions

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Motivation

How fluidity of sQGP determine:

- how simple effects influence time evolution of asymmetries
- effects which can't be discussed analytically

$\Longrightarrow$ Numerical hydrodynamics: realistic models, but effects get mixed

$\Longrightarrow$ Initial condition: close to exact solution, but more realistic

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Table of contents

1 Equations of hydrodynamics

2 Numerical method

3 Code testing

4 Initial conditions

5 Reaction planes

6 Nonrelativistic results

7 Relativistic results

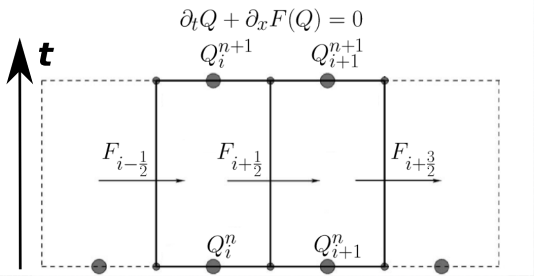
8 Hadronization

Equations of hydrodynamics

- Nonrelativistic :
  - Barion number conservation: $\frac{\partial \rho}{\partial t} + \nabla \rho \boldsymbol{v} = 0$
  - Impulse conservation:
    $$\rho\left(\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v}\nabla)\boldsymbol{v}\right) = -\nabla p + \mu \Delta \boldsymbol{v} + \left(\zeta + \frac{\mu}{3}\right)\nabla(\nabla \boldsymbol{v}) + \boldsymbol{f}$$
  - Energy conservation: $\frac{\partial \varepsilon}{\partial t} + \nabla \varepsilon \boldsymbol{v} = -p\nabla \boldsymbol{v} + \nabla(\boldsymbol{\sigma} \boldsymbol{v})$
  - ρ barion number density, $\boldsymbol{v}$ velocity field, ε energy density, p pressure distribution
- Relativistic hydrodynamics:
  $$T^{\mu\nu} = (\varepsilon + p)u^{\mu}u^{\nu} - pg^{\mu\nu}, \quad \partial_{\mu} T^{\mu\nu} = 0$$
  - $T^{\mu\nu}$ energy-impulse tensor, u^{μ} four-velocity, $g^{\mu\nu}$ metric tensor
- Equation of state: $\varepsilon = \kappa(T)p \quad (\kappa = 1/c_s^2, \kappa = 3/2$ ideal gas)
- Advection form: $\partial_t Q(\rho, \varepsilon, \boldsymbol{v}) + \partial_x F(Q) = 0 \quad$ (F flux)

Numerical scheme

- Mid rapidity: distributions have maximum: $2+1$ dimension
- Numerical solution: discretization $\leftarrow$ finite volume method
- Problem: we need fluxes between grid points $\rightarrow$ approximations
- Instability: perturbation, vanish in grid points $\rightarrow$ CFL condition
- 2 spatial dimension complicated $\rightarrow$ operator splitting
- Viscosity: ideal substep + viscous substep (operator splitting)



MUSTA method

- $\mathrm{n^{th}}$ time step: $Q_i^{(0)} \equiv Q_i^n$, $Q_{i+1}^{(0)} \equiv Q_{i+1}^n$
- ℓ^{th} predicted values: $Q_i^{(\ell)}$, $F_i^{(\ell)} \equiv F\left(Q_i^{(\ell)}\right)$
- Intermediate value and flux:

$$Q_{i+\frac{1}{2}}^{(\ell)} = \frac{1}{2}\left[Q_i^{(\ell)} + Q_{i+1}^{(\ell)}\right] - \frac{1}{2}\frac{\Delta t}{\Delta x}\left[F_{i+1}^{(\ell)} - F_i^{(\ell)}\right], \quad F_M^{(\ell)} \equiv F\left(Q_{i+\frac{1}{2}}^{(\ell)}\right)$$

- Corrected inner flux:

$$F_{i+\frac{1}{2}}^{(\ell)} = \frac{1}{4}\left[F_{i+1}^{(\ell)} + 2F_M^{(\ell)} + F_i^{(\ell)} - \frac{\Delta x}{\Delta t}\left(Q_{i+1}^{(\ell)} - Q_i^{(\ell)}\right)\right]$$

- Next prediction to Q values to better approximation of flux:

$$Q_i^{(\ell+1)} = Q_i^{(\ell)} - \frac{\Delta t}{\Delta x}\left[F_{i+\frac{1}{2}}^{(\ell)} - F_i^{(\ell)}\right]$$

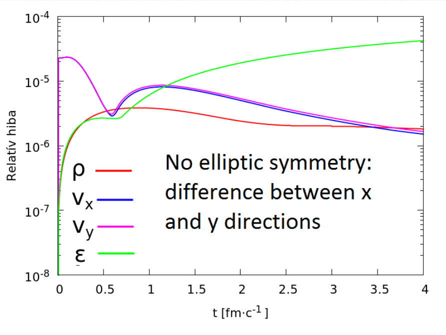
- k step $\rightarrow F_{i+\frac{1}{2}} = F_{i+\frac{1}{2}}^{(k)} \implies Q_i^{n+1} = Q_i^n - \frac{\Delta t}{\Delta x}(F_{i+\frac{1}{2}} - F_{i-\frac{1}{2}})$
- Method published by E. F. Toro et al, 2006, J. Comp. Phys

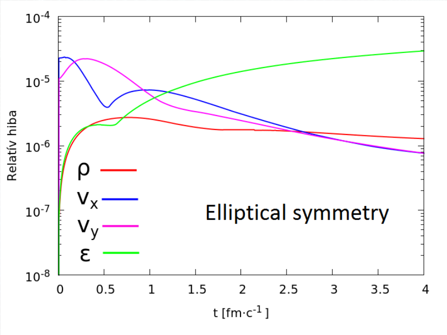
Code testing

- Exact solutions (Csörgő et al, PhysRevC67)
- Relative difference between numerical and exact solution:

$$\int \left|\rho_{\text{analytical}}(t, \underline{x}) - \rho_{\text{numerical}}(t, \underline{x})\right| d^2x \Big/ \int \rho_{\text{analytical}}(t, \underline{x}) d^2x$$

- Relativistic code: tested with Karpenko's hydro code

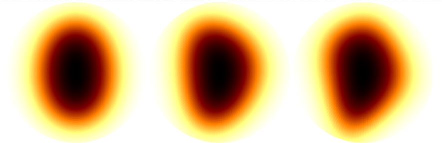




Initial condition

- Variables: space dependency only in scale variable, asymmetry in this variable
- Number density, pressure $\propto \exp\left(-s\right)$
- Scale variable:

$$s = \frac{r^2}{R^2}\Big(1 + \epsilon_2 \cos(2\phi) + \epsilon_3 \cos(3\phi) + \epsilon_4 \cos(4\phi)\Big)$$



- Velocity: Hubble-velocity field or 0
- Effect of pressure gradient: $p \propto \exp\left(-\mathrm{c_p} \cdot \mathrm{s}\right)$
- Constant pressure, multipole exact solution: Csanád és Szabó,

More realistic initial condition

- Gaussian decay $\rightarrow$ values in full space (Ω)
- Better model: Gaussian decay in $\mathcal{A} \subset \Omega$, 0 in $\Omega \setminus \mathcal{A}$
- Problem: numerical instability at $\partial\mathcal{A}$ (non existing derivates)
- Idea: $exp(-s) \rightarrow$ smooth function, example:

$$f(x) = \begin{cases} e^{-\frac{1}{1-x^2}} & \text{if } |x| < 1, \\ 0 & \text{otherwise} \end{cases}$$

- Member of C^∞ but not good for analyzing asymmetries
- Other idea: keep $\exp -s$ distribution for $\mathcal{A}$, 0 for $\Omega \setminus \mathcal{A}$
- But: convolved with $f(x) \rightarrow$ smooth function, derivates will be OK at boundary

Description of asymmetries

- Scale variable: $s = \frac{r^2}{R^2}\left(1 + \epsilon_2\cos(2\phi) + \epsilon_3\cos(3\phi) + \epsilon_4\cos(4\phi)\right)$
- Definition of asymmetry parameters: $\varepsilon_n = \langle\cos(n\phi)\rangle_{\rho/\mathbf{v}/p}$
- ε_n (newly introduced) $\neq \epsilon_m$ (in scale variable)
- Initially ($t = 0$) connection between ε_n and ϵ_m can be derived (Taylor expansion):
  - $\varepsilon_1 = 0 + \epsilon_3(\epsilon_2 + \epsilon_4) + \mathcal{O}(\epsilon^4)$
  - $\varepsilon_2 = -\epsilon_2 + \epsilon_2\epsilon_4 + \epsilon_2\sum_n \epsilon_n^2 + \mathcal{O}(\epsilon^4)$
  - $\varepsilon_3 = -\epsilon_3 + \epsilon_3\sum_n \epsilon_n^2 + \mathcal{O}(\epsilon^4)$
  - $\varepsilon_4 = -\epsilon_4 + \frac{1}{2}\epsilon_2^2 - \epsilon_4\sum_n \epsilon_n^2 + \mathcal{O}(\epsilon^4)$

Generalize the asymmetry parameters

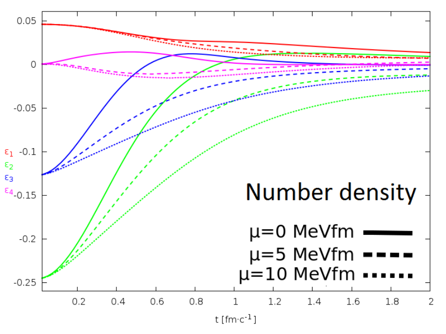
- ε_i evolve in time, it freezes out with phase transition $\rightarrow$ v_n parameters
- Measuring momentum space asymmetry: average for reaction planes
- Reaction plane can be introduced in scale variable:

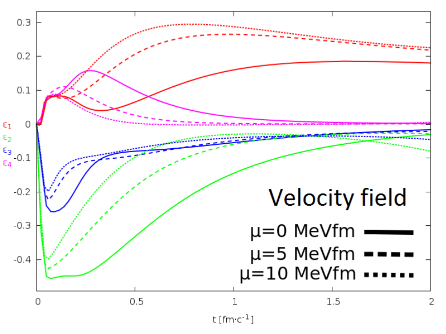
$$s = \frac{r^2}{R^2}\Big(1 + \epsilon_2 \cos(2(\phi - \psi_2)) + \epsilon_3 \cos(3(\phi - \psi_3)) + \epsilon_4 \cos(4(\phi - \psi_4))\Big)$$

- More realistic: $\varepsilon_i \longrightarrow \left\langle \varepsilon_i \right\rangle_\psi$
- We ran simulations with a lot of ψ_2, ψ_3 ψ_4 $\rightarrow$ $\langle \varepsilon_i \rangle_\psi$

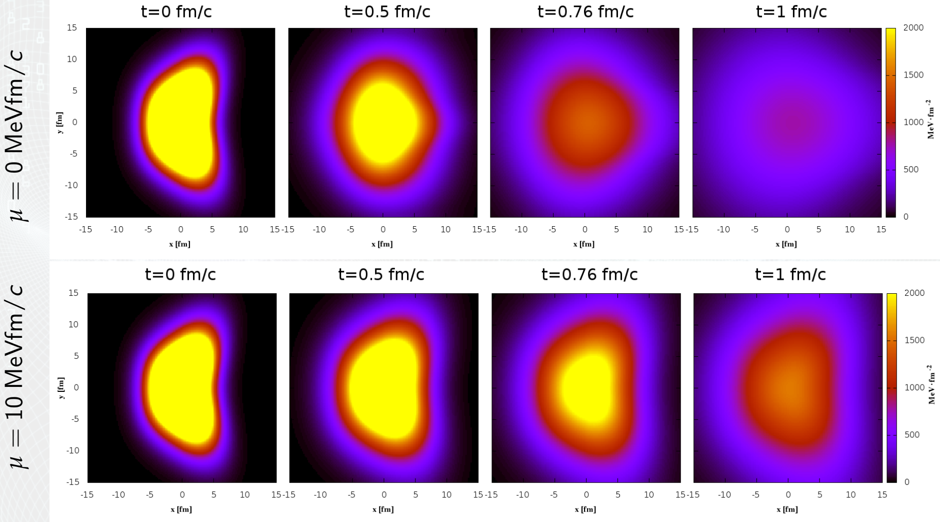
Effect of viscosity

- Energy- and number-distribution: slower disappearance
  - Viscosity: slower flow
- Velocity field: faster disappearance
  - Parts with big/small asymmetry feels different forces: big differences vanishes out fast
- Plot: ε_1 red, ε_2 green, ε_3 blue, ε_4 magenta

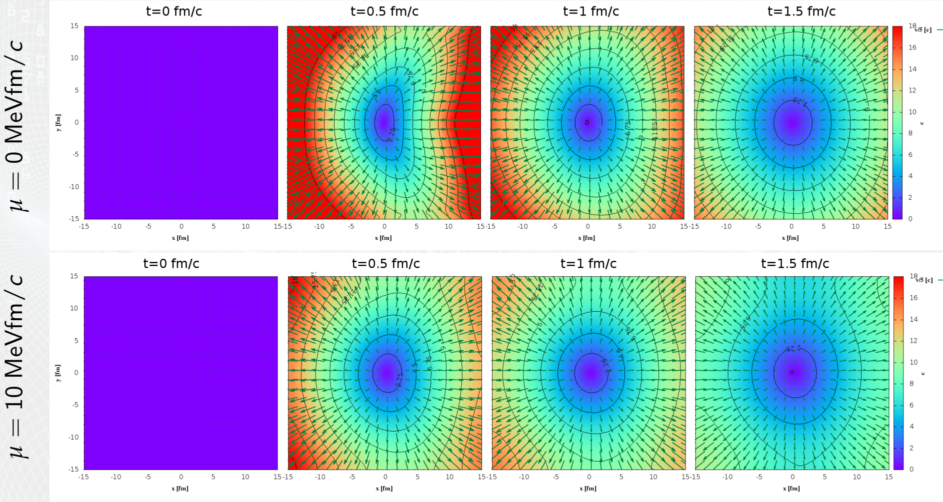




Effect of viscosity: time evolution of energy field

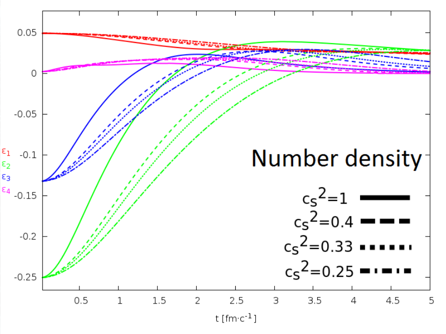


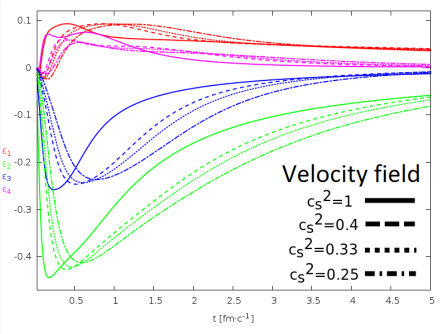
Effect of viscosity: time evolution of velocity field



Effect of speed of sound

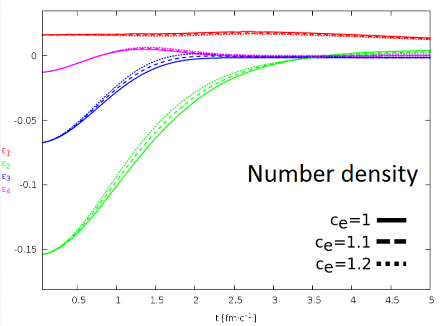
- In every distribution: time evolution of asymmetries gets slower
  - Speed of pressure waves decrease → equalization takes more time
- Speeds: $c_s^2 = 1$ or $0,4$ or $0,33$ or $0,25$

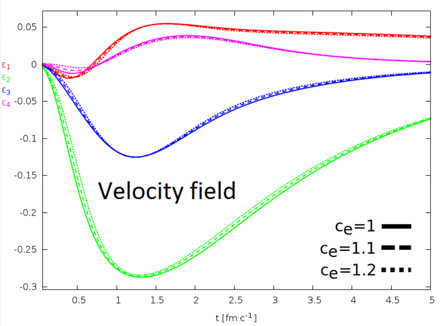




Pressure gradient

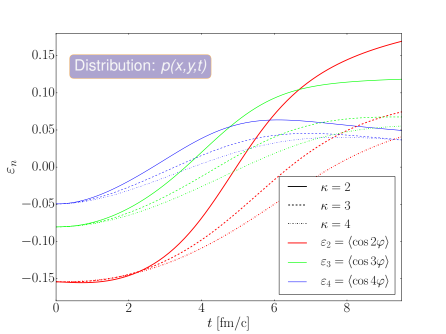
- Every distribution: asymmetries disappear faster
  - Bigger gradient: faster flow
- Number density $\propto \exp(-s)$
- Pressure $\propto \exp(-c_e \cdot s)$

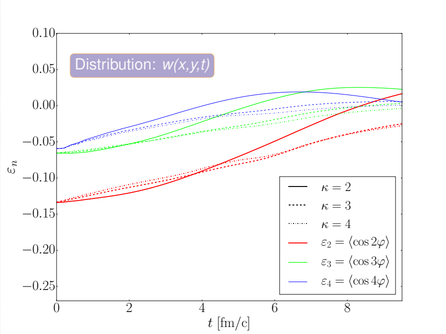




Effect of speed of sound

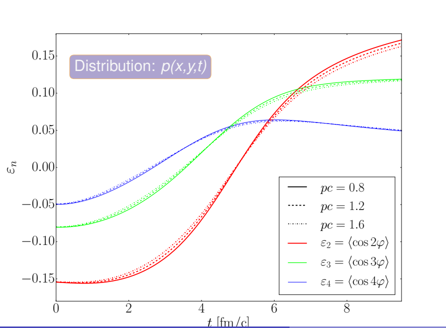
- In every distribution: time evolution of asymmetries gets slower
  - Speed of pressure waves decrease $\rightarrow$ equalization takes more time

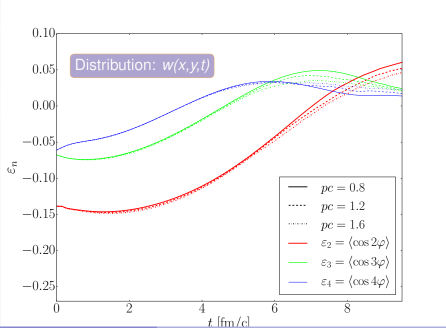




Effect of pressure

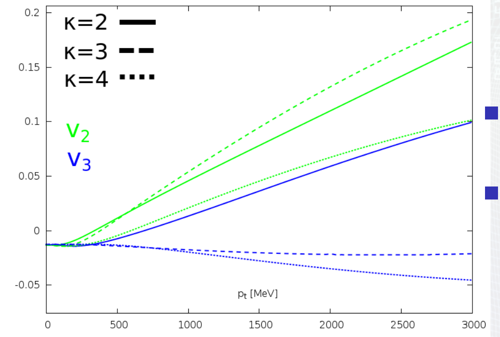
- Every distribution: asymmetries disappear faster
  - Bigger gradient: faster flow
- Number density $\propto \exp(-s)$
- Pressure $\propto \exp(-c_p \cdot s)$





Freeze-out

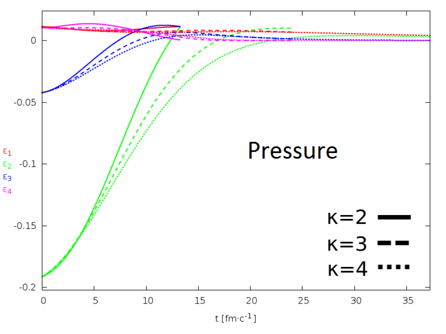
- Maxwell-Jüttner type source function:
  $$S(x,p)d^4x = \mathcal{N} n(x) \exp\left(-\frac{p_\mu u^\mu}{T(x)}\right) H(\tau) p_\mu d^3 \frac{u_\mu d^3 x}{u^0} d\tau$$
- Measurable quantity:
  $$v_n(p_t) = \langle \cos(n\varphi) \rangle_N = \frac{1}{N(p_t)} \int_0^{2\pi} N(p_t,\varphi)\cos(n\varphi)d\varphi$$

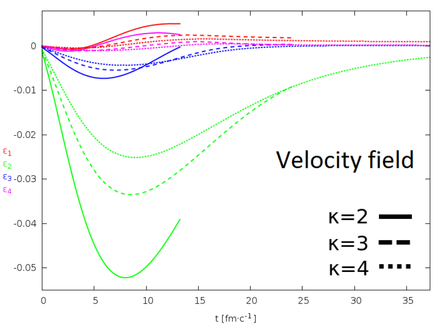


- Momentum space: speed of sound has a big effect
- Sensitive to speed of sound: time of freeze-out

Effect of speed of sound

- In every distribution: time evolution of asymmetries gets slower
  - Speed of pressure waves decrease → equalization takes more time





Summary

- Motivation: simple effects how affect the time evolution of asymmetries
- No much chance for analytic discussion, so we used numerical methods
- Initial condition is very close to analytic solution, but more realistic (with asymmetries)
- More realistic: cut the distributions $\rightarrow$ smoothing with convolution
- Decreasing the speed of sound $\rightarrow$ slower time evolution of asymmetries, freeze-out later
- Viscosity makes the time evolution slower in energy- and number-distribution, faster in velocity field
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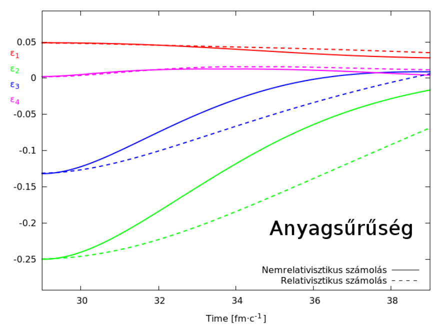
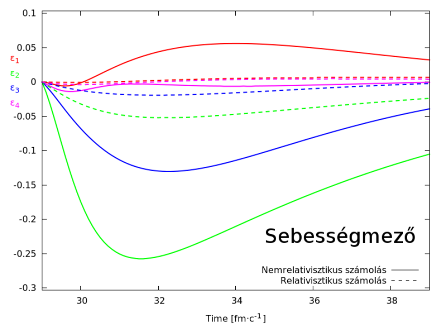
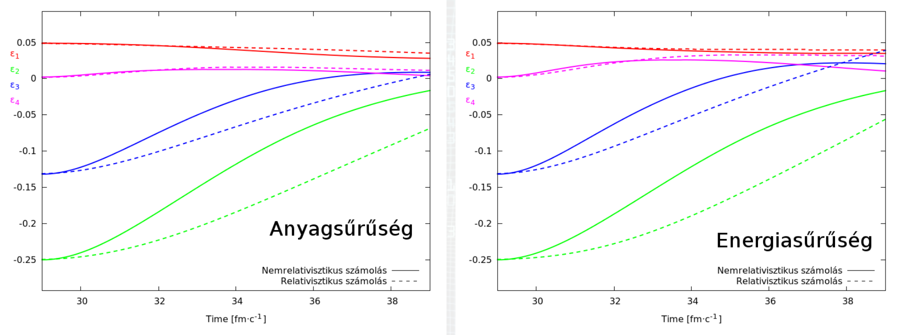
Relativistic versus nonrelativistic hydrodynamics

- Relativistic: asymmetry get washed out slower
- Nonrelativistic: bigger asymmetry in velocity field → bigger derivates → faster time evolution

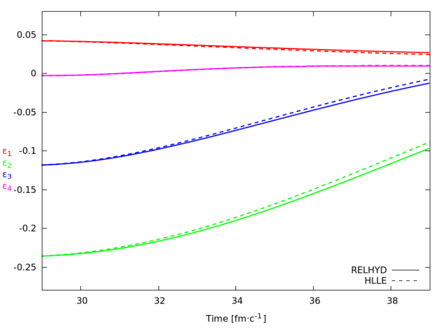


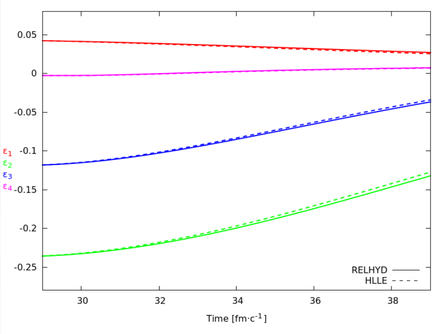


Relativistic versus nonrelativistic hydrodynamics

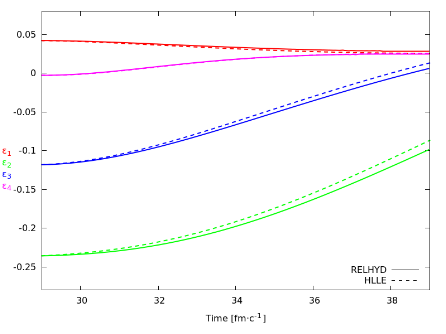


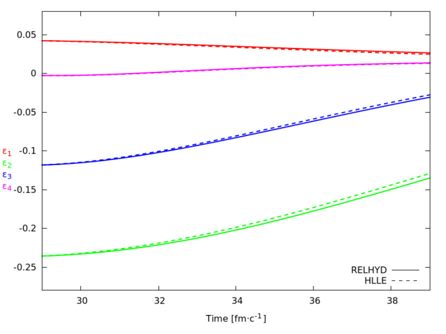
Relativistic code test: number-density $\kappa = 2$ and $\kappa = 4$



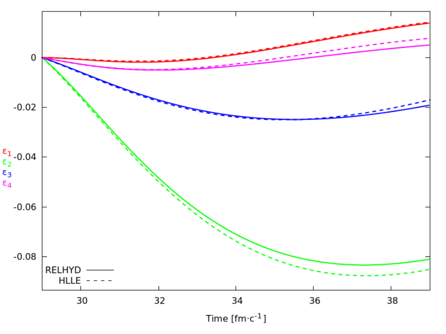


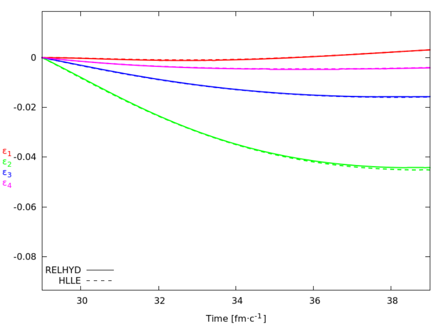
Relativistic code test: pressure $\kappa = 2$ and $\kappa = 4$





Relativistic code test: Velocity-field $\kappa = 2$ és $\kappa = 4$





Testing of Code

- Exact solution (Csörgő et al, PhysRevC67):

$$s = \frac{x^2}{X^2(t)} + \frac{y^2}{Y^2(t)}$$

$$\rho = \rho_0 \frac{V_0}{V} e^{-s}, \quad p = p_0 \left(\frac{V_0}{V}\right)^{1+\frac{1}{\kappa}} e^{-s}$$

$$\boldsymbol{v}(t, \boldsymbol{r}) = \left(\frac{\dot{X}}{X}x, \frac{\dot{Y}}{Y}y\right)$$

$$\ddot{X}X = \ddot{Y}Y = \frac{T_i}{m}\left(\frac{V_0}{V}\right)^{\frac{1}{\kappa}}, \quad V = X(t)Y(t)$$

Operator splitting

$$\partial_t u = Au + Bu$$

$$u(t + \Delta t) = e^{\Delta t(A+B)} u(t)$$

$$u_{\text{Lie}}(t + \Delta t) = e^{\Delta t A} e^{\Delta t B} u(t)$$

$$u_{\text{Strang}}(t + \Delta t) = e^{\frac{1}{2}\Delta t A} e^{\Delta t B} e^{\frac{1}{2}\Delta t A} e^{\Delta t B} u(t)$$

Viscous hydrodynamics

$$\partial_t Q + \partial_x F_{\mathrm{id}}(Q) + \partial_y G_{\mathrm{id}}(Q) + \partial_x F_{\mathrm{visc}}(Q, \partial Q) + \partial_y G_{\mathrm{visc}}(Q, \partial Q) = 0$$

$\Longrightarrow$ operator splitting

- Ideal step: $\partial_t Q + \partial_x F_{\mathrm{id}}(Q) + \partial_y G_{\mathrm{id}}(Q) = 0 \rightarrow Q^{\mathrm{id}}, \partial Q^{\mathrm{id}}$ $\rightarrow F_{\mathrm{visc}}, G_{\mathrm{visc}}$
- Viscous step: $\partial_t Q + \partial_x F_{\mathrm{visc}}(Q^{\mathrm{id}}, \partial Q^{\mathrm{id}}) + \partial_y G_{\mathrm{visc}}(Q^{\mathrm{id}}, \partial Q^{\mathrm{id}}) = 0$ $\rightarrow Q$