# Simulating DIS events with SHERPA

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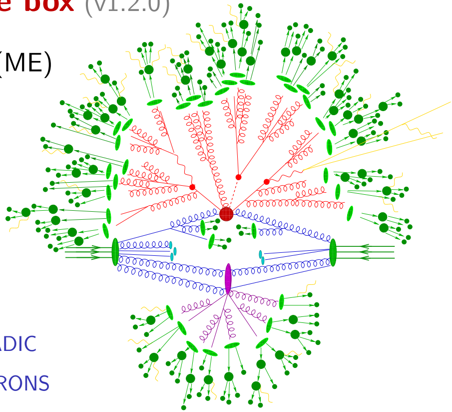
## Things that are currently in the box (v1.2.0)



- Two multi-purpose Matrix Element (ME) generators AMEGIC++ JHEP02(2002)044 and Comix JHEP12(2008)039
- A standard Parton Shower (PS) APACIC++ CPC174(2006)876 and the dipole-like PS CSS JHEP03(2008)038
- A multiple interaction simulation à la Pythia AMISIC++ hep-ph/0601012
- A cluster fragmentation module AHADIC
- A hadron and $\tau$ decay package HADRONS
- A photon radiation generator à la YFS PHOTONS JHEP12(2008)018

**Sherpa's traditional strength is the perturbative part of the event**

NLO real ME's consistently combined with PS à la JHEP05(2009)053

# Combining ME and PS

We know ...

- Methods to compute full ME's given order in coupling
  ME generators implement these to simulate hard processes
- Schemes to compute resummation effects given logarithmic accuracy
  PS generators implement these to simulate parton cascades

**To sensibly simulate *full events* we must combine the two !**

Strategy: Get the best of both !

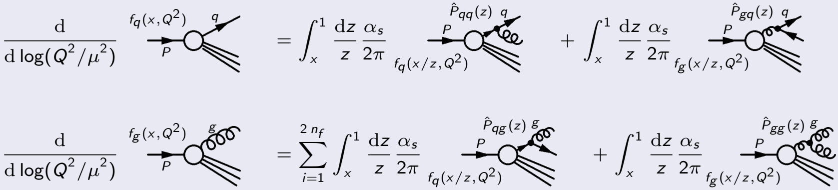
- Employ best possible ME for given analysis
  i.e. one that describes all investigated final states
  e.g. 3jet ME's in 3jet analyses ...
- Properly implement resummation using PS's
  i.e. fill remaining phasespace with softer emissions
  hardest, i.e. most important emissions should always be described by ME

# Introduction: Parton Showers

The basis of Parton Showers: QCD evolution equations DGLAP

$$\frac{\partial f_a(z,Q^2)}{\partial \log(Q^2/\mu^2)} = \int_x^1 \frac{\mathrm{d}z}{z}\,\frac{\alpha_s}{2\pi} \sum_{b=q,g} \hat{P}_{ba}(z)\, f_b\!\left(\frac{x}{z},Q^2\right)$$

Pictorially:




Can iterate these equations

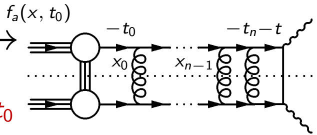
$\longrightarrow$ ladder-like structure of amplitude squared

with strong ordering in scales $t_0 < \ldots < t$

**Factorization now occurs at any stage above $t_0$**

can split emissions off ME one by one

$\longleftrightarrow$



In MC event generators, we employ *exclusive* evolution think of DGLAP again

$$\frac{\partial g_a(z,t)}{\partial \log(t/\mu^2)} = \int_z^{\zeta_{max}} \frac{d\zeta}{\zeta} \sum_{b=q,g} \mathcal{K}_{ba}(\zeta,t)\, g_b\Big(\frac{z}{\zeta},t\Big) - g_a(z,t) \int_{\xi_{min}}^{\xi_{max}} d\xi \sum_{b=q,g} \xi\, \mathcal{K}_{ab}(\xi,t)$$

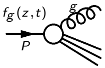
$\mathcal{K}_{ab}(\zeta,t)$ - evolution kernels of the scheme e.g. DGLAP kernels $\alpha_s/2\pi\ P_{ab}(z)$

$\xi_{min}$, $\xi_{max}/\zeta_{max}$ - resolution criteria separate resolved from unresolved emissions

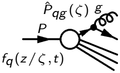
Example: DGLAP gluon-PDF evolution

inclusive form, i.e. $\xi_{min} \to 0$, $\xi/\zeta_{max} \to 1$ (no loss term) recover example from intro

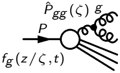
$$\frac{d}{d\log(t/\mu^2)}$$




$$= \int_z^1 \frac{d\zeta}{\zeta} \frac{\alpha_s}{2\pi} \sum_{i=1}^{2n_f}$$




$$+ \int_z^1 \frac{d\zeta}{\zeta} \frac{\alpha_s}{2\pi}$$




Employ Sudakov form factors $\Delta_a(\mu^2,t)$ to rewrite want to absorb loss term

Interpretation: unconditional probability that parton $a$ does not branch between scales $\mu^2$ and $t$

$$\Delta_a(\mu^2,t) = \exp\left\{ -\int_{\mu^2}^{t} \frac{d\bar{t}}{\bar{t}} \int_{\xi_{\rm min}}^{\xi_{\rm max}} d\xi \sum_{b=q,g} \frac{1}{2} \mathcal{K}_{ab}(\xi,\bar{t}) \right\}$$

# Introduction: Parton Showers

Using Sudakovs, QCD evolution reads loss term absorbed into derivative of Sudakov

$$\frac{\partial}{\partial\log(t/\mu^2)}\,\frac{g_a(z,t)}{\Delta_a(\mu^2,t)} = \frac{1}{\Delta_a(\mu^2,t)}\int_z^{\zeta_{\max}}\frac{d\zeta}{\zeta}\sum_{b=q,g}\mathcal{K}_{ba}(\zeta,t)\,g_b\Big(\frac{z}{\zeta},t\Big)$$

Integrate this equation ...

$$\frac{\Delta_a(\mu^2,t')\,g_a(z,t)}{\Delta_a(\mu^2,t)\,g_a(z,t')} = \exp\left\{-\int_t^{t'}\frac{d\bar{t}}{\bar{t}}\,\mathcal{I}(z,\bar{t})\right\}\,,\quad \mathcal{I}(z,\bar{t}) = \int_z^{\zeta_{\max}}\frac{d\zeta}{\zeta}\sum_{b=q,g}\mathcal{K}_{ba}(\zeta,\bar{t})\,\frac{g_b(z/\zeta,\bar{t})}{g_a(z,\bar{t})}$$

Compare to radioactive decay: $\frac{\mathcal{N}(\tau)}{\mathcal{N}(\tau_0)} = \exp\left\{-\int_{\tau_0}^{\tau}d\tau'\,f(\tau')\right\}$

**Integral of evolution equations is conditional no-branching probability !**

probability not to radiate anything resolveable from parton $a$ of energy fraction $z$ between $t$ and $t'$

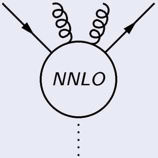
$$\mathcal{P}^{(B)}_{\mathrm{no},\,a}(z,t,t') = \frac{\Delta_a(\mu^2,t')\,g_a(z,t)}{\Delta_a(\mu^2,t)\,g_a(z,t')}$$

**This is the basic equation of any Parton Shower !**

Used to generate emission at $t$ from parton at $t'$ by solving $1-\mathcal{P}^{(B)}_{\mathrm{no},a}(z,t,t') \stackrel{!}{=} R$ for $t$ with $R$ - random number

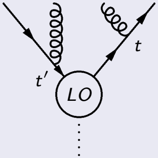
# Combining ME & PS: The Problem

Problem: Matrix elements and parton showers deal with the same physics !




- Coherent sum of real NLO corrections
- No resummation

$\longleftrightarrow$




- Incoherent sum
- Proper resummation in parts of phase space

**How do we run a parton shower on a $N^xLO$ tree-level matrix element ?**

1. Find suitable starting conditions for the parton shower
   i.e. find a tree-structure corresponding to the full ME
   which can be used by the parton shower as a branching history
2. Make sure not to double-count or miss out emissions
   i.e. eventually populate the whole available real emission
   phase space with *either* matrix elements *or* the parton shower

# Solution part 1: Defining PS histories

Basic idea: **Interpret ME as if PS had produced it**

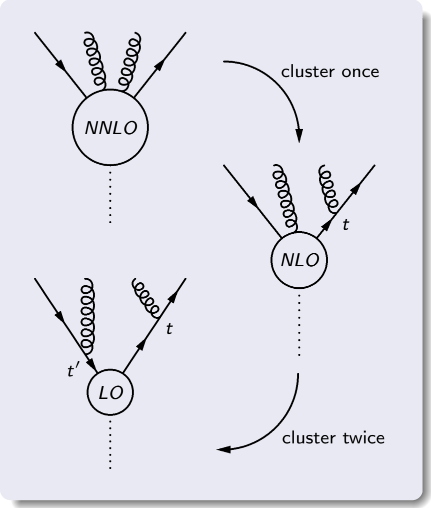
- Identify most likely splitting acc. to PS branching probability
- Combine partons into mother parton acc. to inverse PS kinematics
- Continue until 2 → 2 core process

→ Cluster algorithm similar to $k_T$ algo

PS starts at core process and possibly radiates additional partons on intermediate lines i.e. "between" ME partons

ME branchings must be respected
evolution-, splitting- & angular variable preserved

→ **Truncated shower** see later
universal concept for ME-PS merging



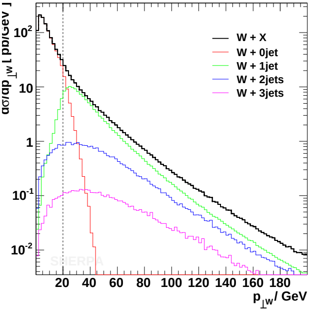
# Solution part 2: Slicing the phasespace

Basic idea: **Separate phasespace into "hard" and "soft" region**

- Matrix elements populate hard domain
- Parton shower populates soft domain

need criterion to define "hard" & "soft" as above and below a certain cut value
$\longrightarrow$ **Jet criterion** $Q$ e.g. $k_T$-jet measure

$\longleftrightarrow$




First replace kernels in QCD evolution equations with note that $\mathcal{K}_{ab} = \mathcal{K}_{ab}^{\rm ME} + \mathcal{K}_{ab}^{\rm PS}$

$$\mathcal{K}_{ab}^{\rm ME}(\xi, \bar{t}) = \mathcal{K}_{ab}(\xi, \bar{t})\,\Theta\left[Q_{ab}(\xi, \bar{t}) - Q_{\rm cut}\right] \qquad \mathcal{K}_{ab}^{\rm PS}(\xi, \bar{t}) = \mathcal{K}_{ab}(\xi, \bar{t})\,\Theta\left[Q_{\rm cut} - Q_{ab}(\xi, \bar{t})\right]$$

Then replace PS evolution kernel in ME domain with full ME

$$\mathcal{K}_{ab}^{\rm ME}(z, t) \;\to\; \frac{1}{\sigma_a^{(N)}(\Phi_N)}\,\frac{{\rm d}^2\sigma_b^{(N+1)}(z, t; \Phi_N)}{{\rm d}\log(t/\mu^2)\,{\rm d}z}$$

# ME & PS: Theory in a nutshell

Recall: Sudakov form factor $\Delta_a(\mu^2,t) \;=\; \exp\left\{\; -\int_{\mu^2}^{t}\frac{\mathrm{d}\bar{t}}{\bar{t}}\int_{\xi_{\min}}^{\xi_{\max}}\mathrm{d}\xi\sum_{b=q,g}\frac{1}{2}\,\mathcal{K}_{ab}(\xi,\bar{t})\;\right\}$

Evolution kernels $\mathcal{K}_{ab}(\xi,\bar{t}) = \mathcal{K}^{\rm ME}_{ab}(\xi,\bar{t}) + \mathcal{K}^{\rm PS}_{ab}(\xi,\bar{t})$

$\longrightarrow$ Factorization of Sudakovs follows trivially !

$$\Delta_a(\mu^2,t) = \Delta^{\rm ME}_a(\mu^2,t)\,\Delta^{\rm PS}_a(\mu^2,t)$$

Conditional no-branching probabilities factorize almost identically

$$\mathcal{P}^{(B)}_{{\rm no},\,a}(z,t,t') \;=\; \frac{\Delta^{\rm ME}(\mu^2,t')}{\Delta^{\rm ME}(\mu^2,t)}\,\frac{\Delta^{\rm PS}_a(\mu^2,t')\,g_a(z,t)}{\Delta^{\rm PS}_a(\mu^2,t)\,g_a(z,t')} \;=:\; \mathcal{P}^{\rm ME}_{{\rm no},\,a}(t,t')\,\mathcal{P}^{(B)\,{\rm PS}}_{{\rm no},\,a}(z,t,t')$$

Remaining task: **Interpret this equation !**

Note the scheme independence: decomposition of $\mathcal{P}^{(B)}_{{\rm no},\,a}(z,t,t')$ works independent of the definition of $\mathcal{K}_{ab}$
$\longrightarrow$ can be implemented for any parton shower and any jet measure $Q$

# ME & PS: The need for truncated showers

## How to interpret $\mathcal{P}^{(B)\,\mathrm{PS}}_{\mathrm{no},\,a}(z,t,t')$ ?

Assume predefined branchings at $t$ and $t' > t$

$$\mathcal{K}_{ab}(\xi,\bar{t})\ \Theta\left[Q_{\mathrm{cut}} - Q_{ab}(\xi,\bar{t})\right]$$

means running a vetoed shower

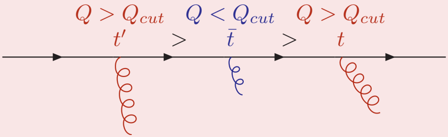
emission phase space is limited from above by $Q_{\mathrm{cut}}$

$$\mathcal{P}^{(B)\,\mathrm{PS}}_{\mathrm{no},\,a}(z,t,t')$$

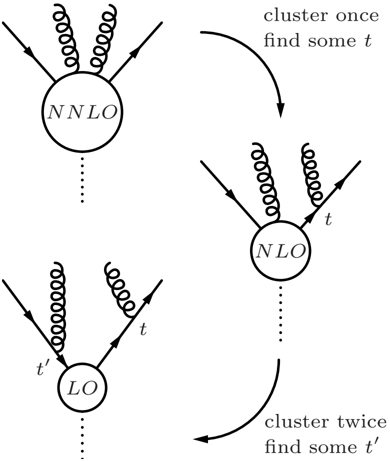
means running a truncated shower

$t$ is larger than global shower cutoff $t_0$

## What is the catch of it ?




Example branching history



# ME & PS: Sudakov suppression and cross sections

How to interpret $\mathcal{P}^{\rm ME}_{{\rm no},\,a}(t,t')$ ?

Assume predefined branchings at $t$ and $t' > t$

$$\mathcal{K}_{ab}(\xi,\bar{t})\,\Theta\left[Q_{ab}(\xi,\bar{t}) - Q_{\rm cut}\right]$$

means running a vetoed shower

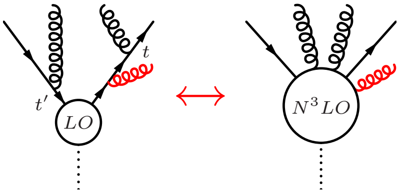
emission phase space is limited from below by $Q_{\rm cut}$

$$\mathcal{P}^{\rm ME}_{{\rm no},\,a}(t,t')$$

means running a truncated shower

$t$ is larger than global shower cutoff $t_0$

Example emission




What happens if we emit something ?

Emission must be implemented to preserve full QCD evolution, i.e. $\mathcal{P}^{(B)}_{{\rm no},\,a}(z,t,t')$

**But we want matrix elements to take care of such emissions !**

To avoid double-counting, the complete event must be rejected

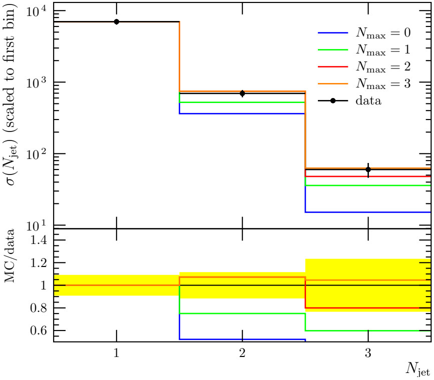
**Event is lost $\Rightarrow$ rejection reduces initial cross section $\sigma$ to $\sigma\cdot\mathcal{P}^{\rm ME}_{{\rm no},\,a}(t,t')$**

"Gap" is filled by higher order ME$\otimes$PS $\Rightarrow$ $\sigma$ preserved at LO

# ME & PS Example: Jet rates

Example: DY-pair production

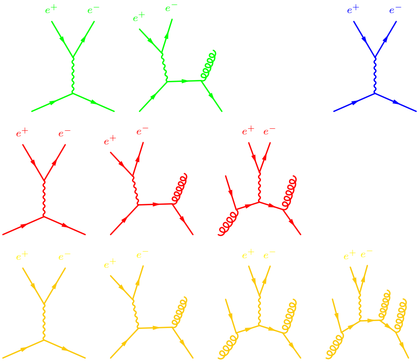
$\sigma$ @ Tevatron JHEP05(2009)053




Consequence of the method:

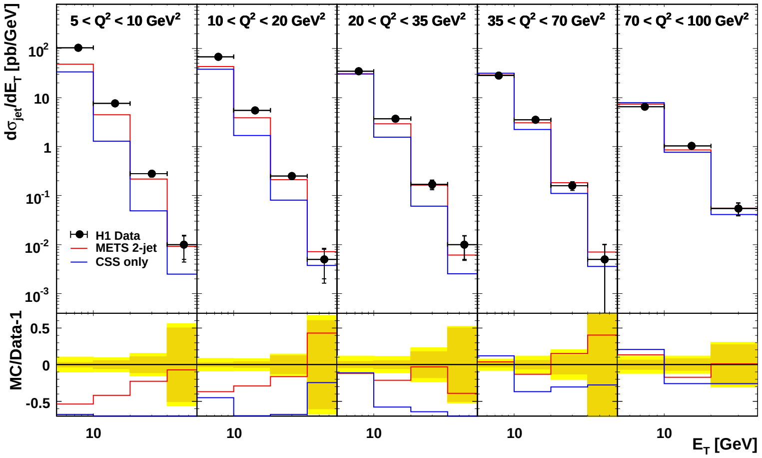
Jet rates and -spectra improved compared to pure PS simulation
due to usage of NLO real ME's

Note: minor corrections to total cross section might still have big effect on rare events !



# ME & PS results: Inclusive jets in DIS

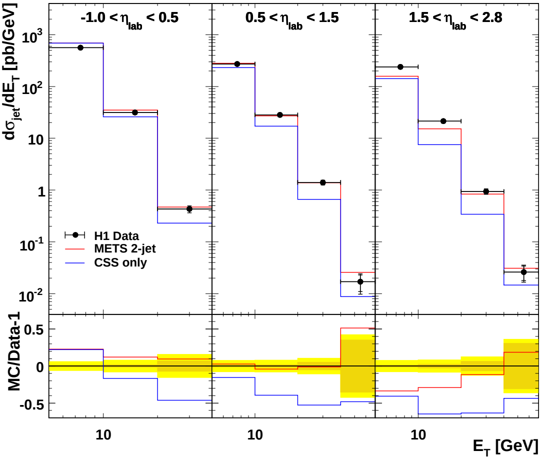
Jet-$E_T$ spectra in $Q^2$-bins PLB542(2002)193 [2]
ME⊗PS vs. parton shower




[2] All plots: SHERPA ⊕ HZTool

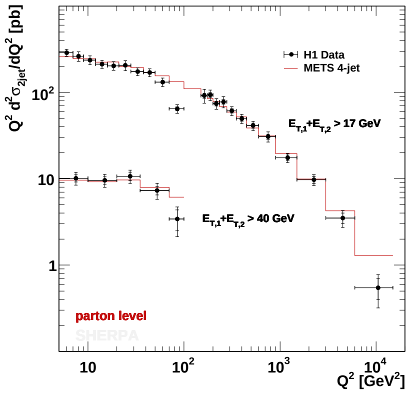
# ME & PS results: Inclusive jets in DIS

Jet-$E_T$ spectra in $\eta_{lab}$-bins PLB542(2002)193
ME⊗PS vs. parton shower

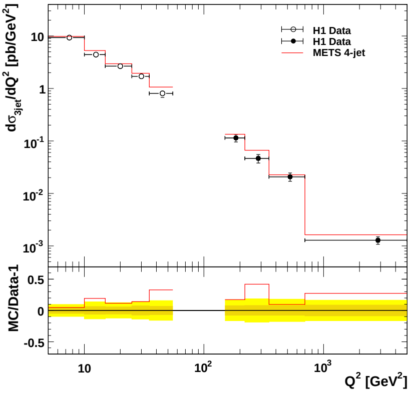


# ME & PS results: Inclusive di-/trijets in DIS

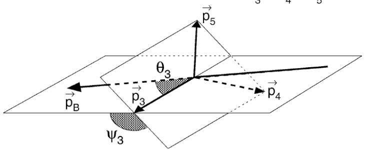
$Q^2$ spectra in $E_T$-bins EPJC19(2001)289



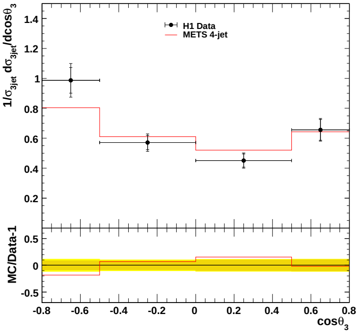

$Q^2$ spectrum PLB515(2001)17



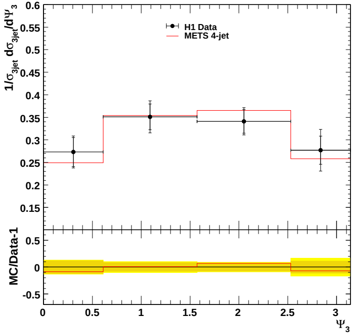
# ME & PS results: Inclusive trijets in DIS




$\cos\theta_3$ PLB515(2001)17 $Q^2 > 150$ GeV$^2$



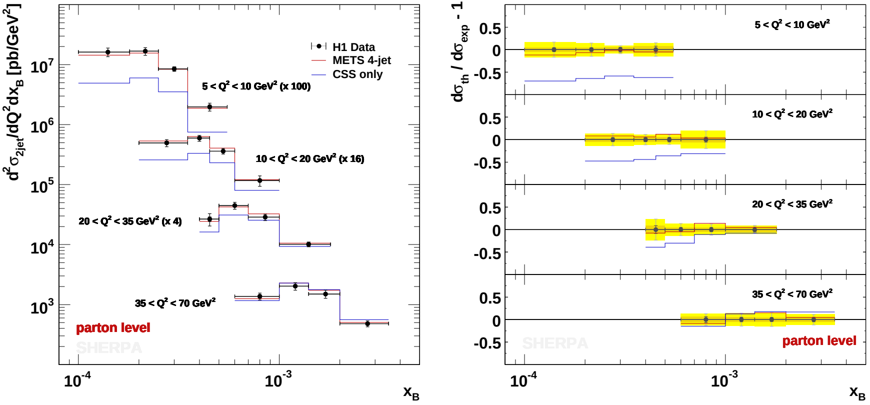

$\Psi_3$ PLB515(2001)17 $Q^2 > 150$ GeV$^2$



# ME & PS results: Inclusive dijets in DIS

$x_B$ spectra EPJC19(2001)289 low-$Q^2$

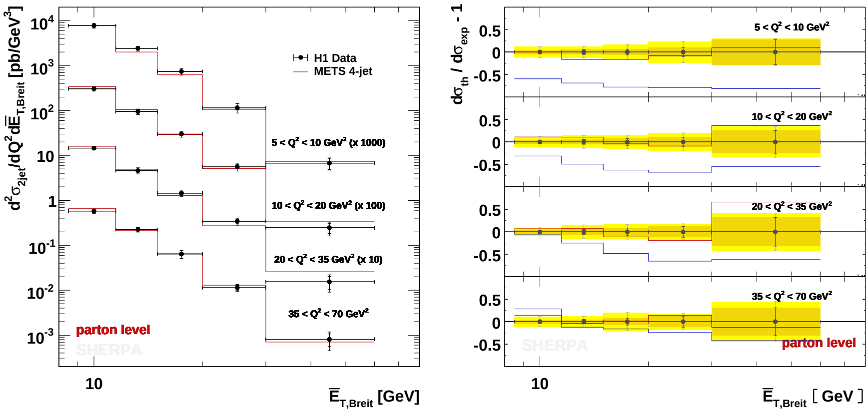
ME⊗PS vs. parton shower



# ME & PS results: Inclusive dijets in DIS

$\overline{E}_{T,Breit}$ spectra EPJC19(2001)289 low-$Q^2$

ME⊗PS vs. parton shower



# Summary

Things already done ...

- SHERPA including ME$\otimes$PS set up for DIS
  framework stable, promising first results
- HZTool steering included in SHERPA
  $\rightarrow$ "any" existing HZTool analysis can be done

**Things to be done ...**

- More tests and validations
  forward jets, 4-jets, ...
- Resolved photons
- Multiparton events

**Looking forward to meet the challenge !**

# How to run a Parton Shower

**What does the form of $\mathcal{P}^{(B)}_{\mathrm{no},\,a}(z,t,t')$ imply ?**

- Exclusive radiation patterns, i.e. $\xi_{min} > 0$, $\xi_{max}/\zeta_{max} < 1$
  PS's deconvolute PDF's and FF's to generate N$^x$LL-corrections to LO kinematics
- Preservation of hard cross section $\hat{\sigma}$ through unitarity & sum rules
  either splitting or no splitting $\rightarrow$ no additional events & no rejections,
  just "dress" hard ME with additional partons

**The Monte Carlo implementation of it is called a Parton Shower:**

- Generate emission scale $t$ from parton $a$ at $t'$ via

$$1 - \mathcal{P}^{(B)}_{\mathrm{no},\,a}(z,t,t') \overset{!}{=} R\,, \qquad \text{where} \qquad R \in [0,1] \text{ random}$$

  nonzero probability for unresolved emission ($\zeta > \zeta_{max}$) $\rightarrow$ nonzero probability for no splitting at all
- Dice splitting variable $\zeta$ and flavour $b$ according to $\mathcal{K}_{ba}(\zeta,t)\,g_b(z/\zeta,t)/g_a(z,t)$

**Various choices for evolution and splitting variable possible**, e.g.

- Virtuality $t$ and energy fraction $z$ PYTHIA, ISAJET, APACIC++
- Angle $\theta$ and light cone momentum fraction $z$ fHERWIG, HERWIG++
- Soft eikonal $k_T^2$ and rapidity $y$ dipole showers ARIADNE

# Dipole-like showers

Construction of the parton shower along the lines of DGLAP case

- Identify $\mathcal{K}_{ab} \to \frac{\alpha_s}{2\pi} \langle V_{ij,k} \rangle$, where $\langle V_{ij,k} \rangle$ - spin-averaged CS dipole functions
- Employ evolution variable $t$ according to dipole type
  - Transverse momentum in cms of splitting dipole for final state splitters
  - $2\tilde{p}_{ai} p_b \, \tilde{v}_i \, \frac{1-x_{i,ab}}{x_{i,ab}}$ for initial-initial dipoles ( $\tilde{v}_i = \frac{p_i p_a}{p_a p_b}$, $x_{i,ab} = 1 - \frac{p_i(p_a+p_b)}{p_a p_b}$ )
    Similar for inital-final dipoles ($\tilde{v}_i \leftrightarrow u_i$, $p_b \leftrightarrow p_k$)
- Evaluate $\alpha_s$ at value of evolution variable (+ possible scale factor)

CS subtraction respects all possible color correlations
$\longrightarrow$ must project onto leading $1/N_C$ terms for PS
reduces correlations to connection with *at most two* dipole partners
emissions occur with probability proportional $T_R$ or Casimir operators $C_A$ & $C_F$

**What are the advantages of this PS model?**

- CS dipoles give excellent approximation of real NLO ME
- QCD coherence is respected by the specific form of $V_{ij,k}$
- Recoil from parton splitting compensated *locally* by color partner