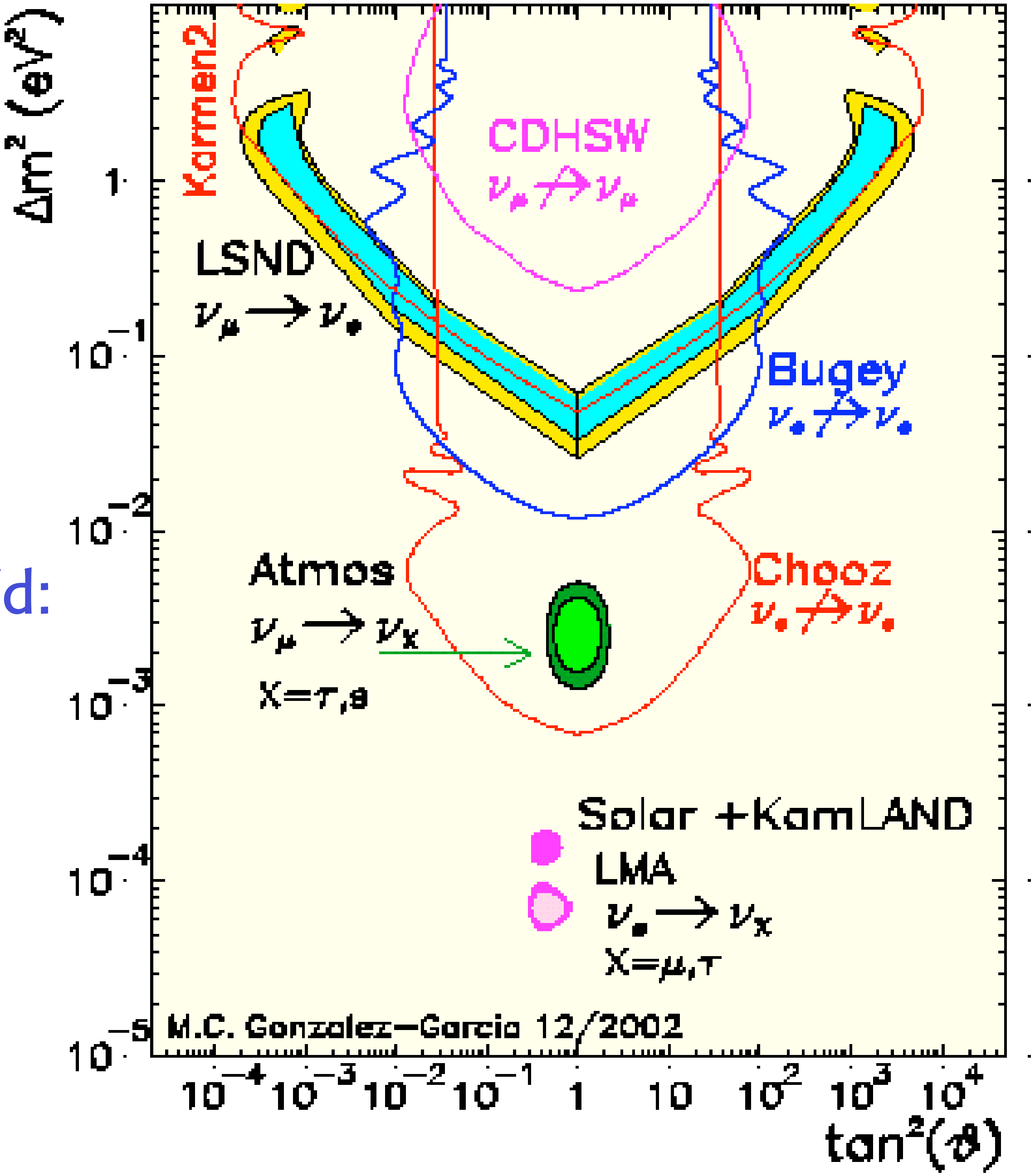
CERN, 1 October '09

Models of Neutrino Masses and Mixings

G. Altarelli
Universita' di Roma Tre/CERN

Where we stand. What we have learnt. Open problems

Evidence for solar and atmosph. ν oscillatn's confirmed on earth by K2K, KamLAND, MINOS...



Δm^2 values:

$\Delta m^2_{atm} \sim 2.5\ 10^{-3}\ eV^2$,

$\Delta m^2_{sol} \sim 8\ 10^{-5}\ eV^2$

and mixing angles measur'd:

θ_{12} (solar) large

θ_{23} (atm) large,~ maximal

θ_{13} (CHOOZ) small

Miniboone has not confirmed LSND

3 ν's are enough!

We do not need to add new neutrinos:

e.g. sterile neutrinos

The 3 known species are enough

We can assume CPT invariance

Additional ν's or CPT violations are not completely excluded but for economy we can assume that they do not exist

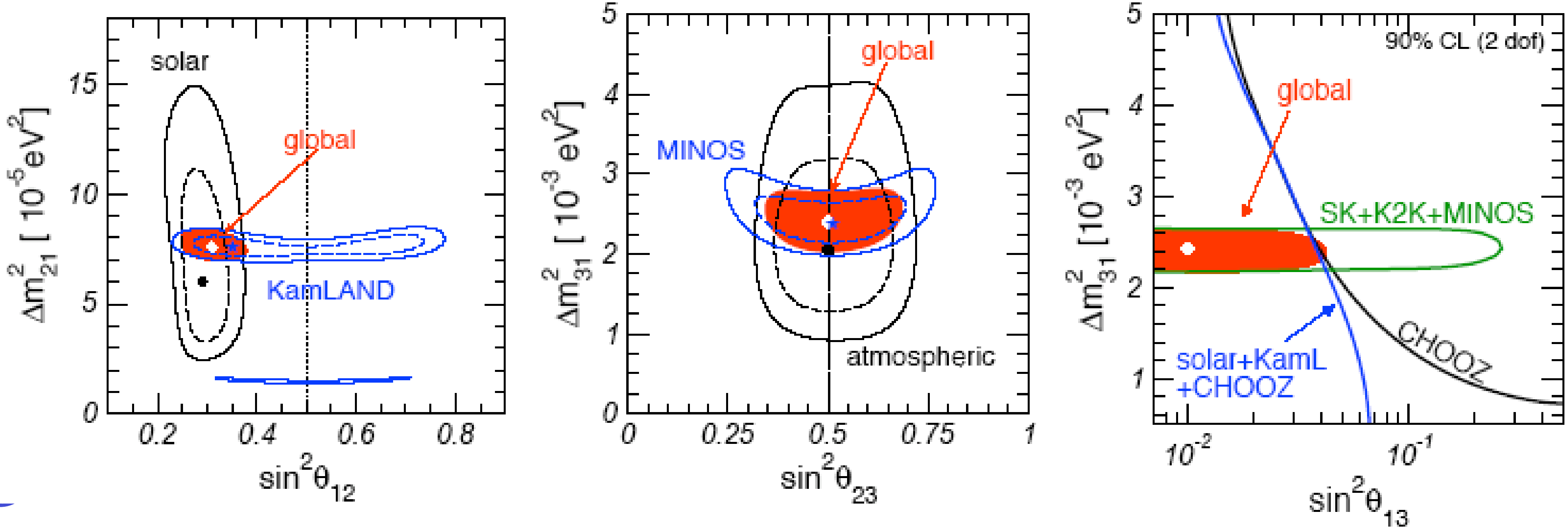
Neutrino oscillation parameters

- 2 distinct frequencies
- 2 large angles, 1 small

parameter	best fit	2σ	3σ
$\Delta m^2_{21}\ [10^{-5}\mathrm{eV}^2]$	$7.65^{+0.23}_{-0.20}$	7.25–8.11	7.05–8.34
$\lvert\Delta m^2_{31}\rvert\ [10^{-3}\mathrm{eV}^2]$	$2.40^{+0.12}_{-0.11}$	2.18–2.64	2.07–2.75
$\sin^2\theta_{12}$	$0.304^{+0.022}_{-0.016}$	0.27–0.35	0.25–0.37
$\sin^2\theta_{23}$	$0.50^{+0.07}_{-0.06}$	0.39–0.63	0.36–0.67
$\sin^2\theta_{13}$	$0.01^{+0.016}_{-0.011}$	$\lesssim 0.040$	$\lesssim 0.056$

Schwetz et al '08

Best measured angle

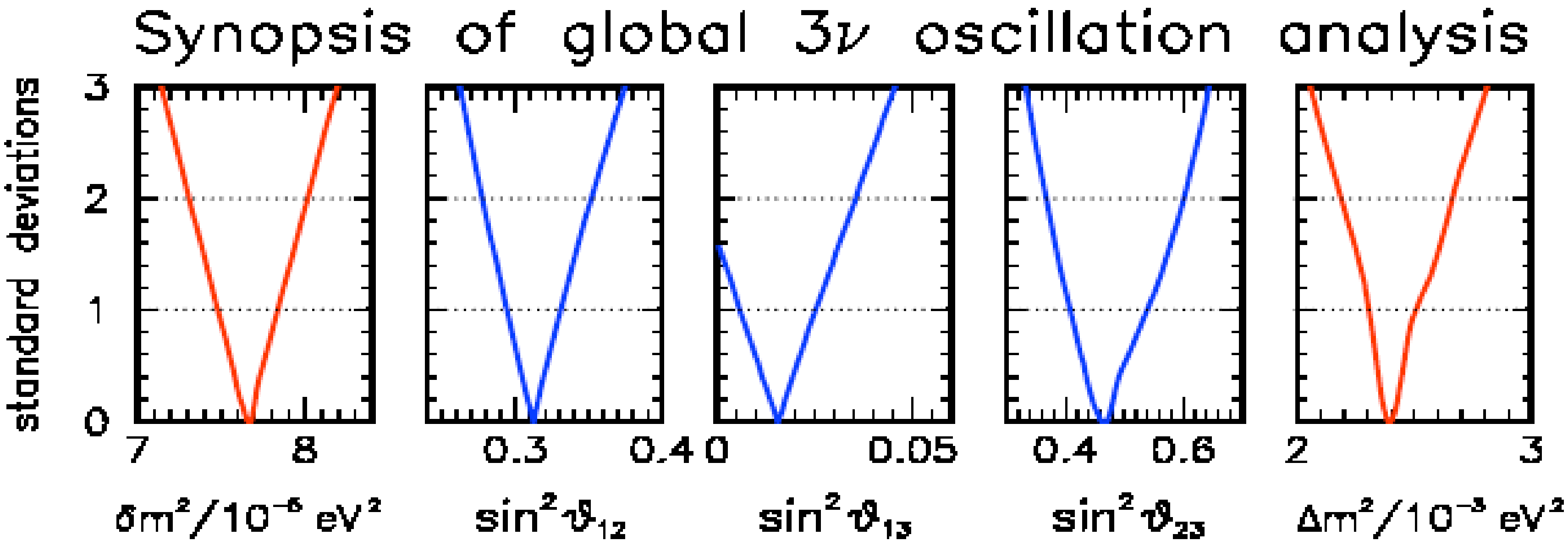


Different fits of the data agree

Fogli et al '08

Table 1: Global 3ν oscillation analysis (2008): best-fit values and allowed n_σ ranges, from Ref. [4].

Parameter	$\delta m^2/10^{-5}$ eV^2	$\sin^2\theta_{12}$	$\sin^2\theta_{13}$	$\sin^2\theta_{23}$	$\Delta m^2/10^{-3}$ eV^2
Best fit	7.67	0.312	0.016	0.466	2.39
1σ range	7.48 – 7.83	0.294 – 0.331	0.006 – 0.026	0.408 – 0.539	2.31 – 2.50
2σ range	7.31 – 8.01	0.278 – 0.352	$<$ 0.036	0.366 – 0.602	2.19 – 2.66
3σ range	7.14 – 8.19	0.263 – 0.375	$<$ 0.046	0.331 – 0.644	2.06 – 2.81

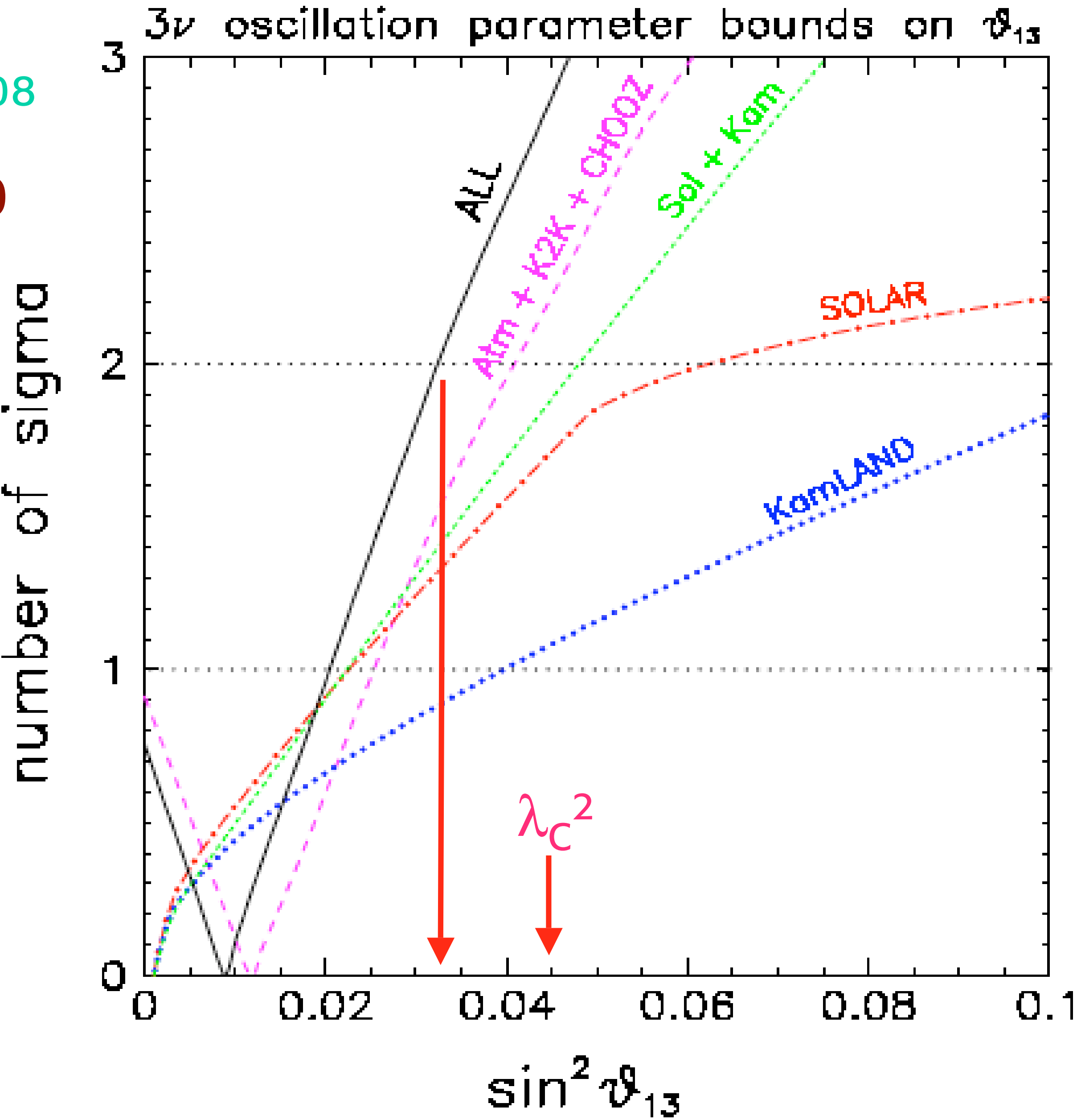


θ_{13} bounds

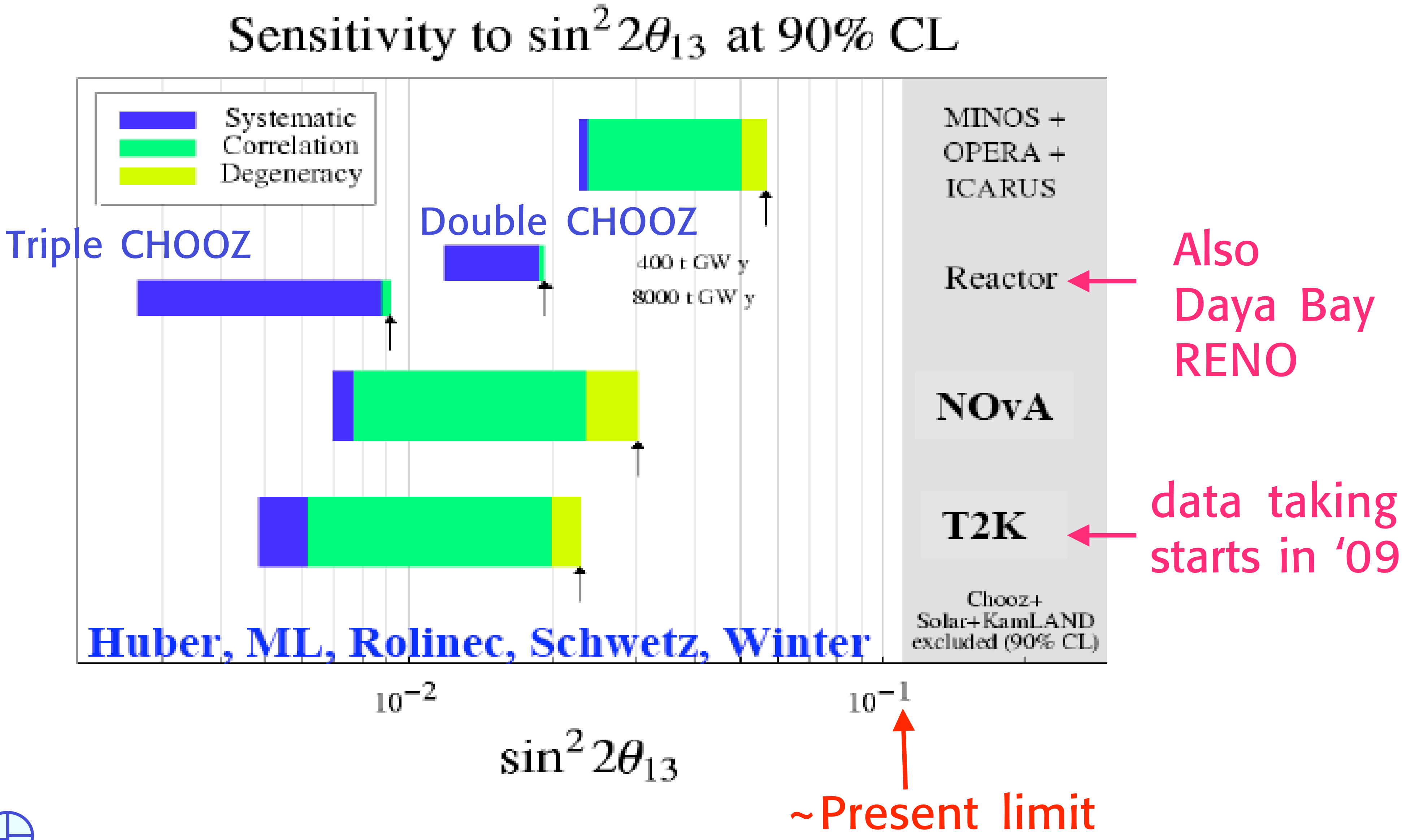
Fogli et al '08

$\sin^2\theta_{13}=0.016\pm0.010$

The 95% upper bound on $\sin\theta_{13}$ is close to $\lambda_C = \sin\theta_C$



Measuring θ_{13} is crucial for future ν-oscill's experiments (eg CP violation)



ν oscillations measure Δm^2. What is m^2?

$\Delta m^2_{atm} \sim 2.5\ 10^{-3}\ eV^2 = (0.05\ eV)^2$; $\Delta m^2_{sun} \sim 8\ 10^{-5}\ eV^2 = (0.009\ eV)^2$

- Direct limits

$m_{"\nu e"} < 2.2$ eV

$m_{"\nu\mu"} < 170$ KeV

$m_{"\nu\tau"} < 18.2$ MeV

End-point tritium β decay (Mainz, Troitsk)
Future: Katrin
0.2 eV sensitivity
(Karsruhe)

$m_{ee} = |\Sigma\, U_{ei}^2\, m_i|$

- $0\nu\beta\beta$ $m_{ee} < 0.2 - 0.7 - ?$ eV (nucl. matrix elmnts)

Evidence of signal? Klapdor-Kleingrothaus

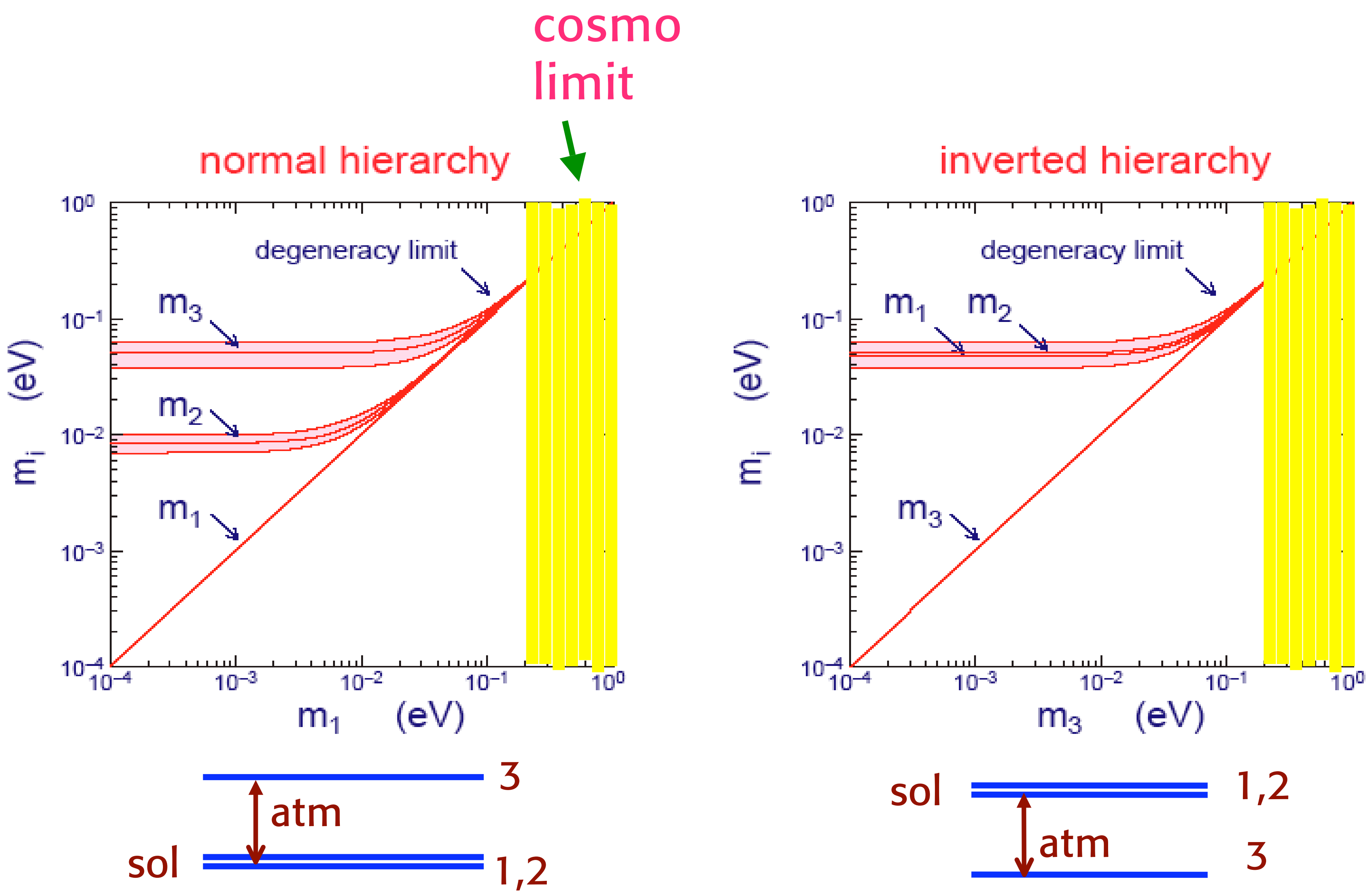
- Cosmology $\Omega_\nu\, h^2 \sim \Sigma_i m_i\ /94eV$ $(h^2 \sim 1/2)$

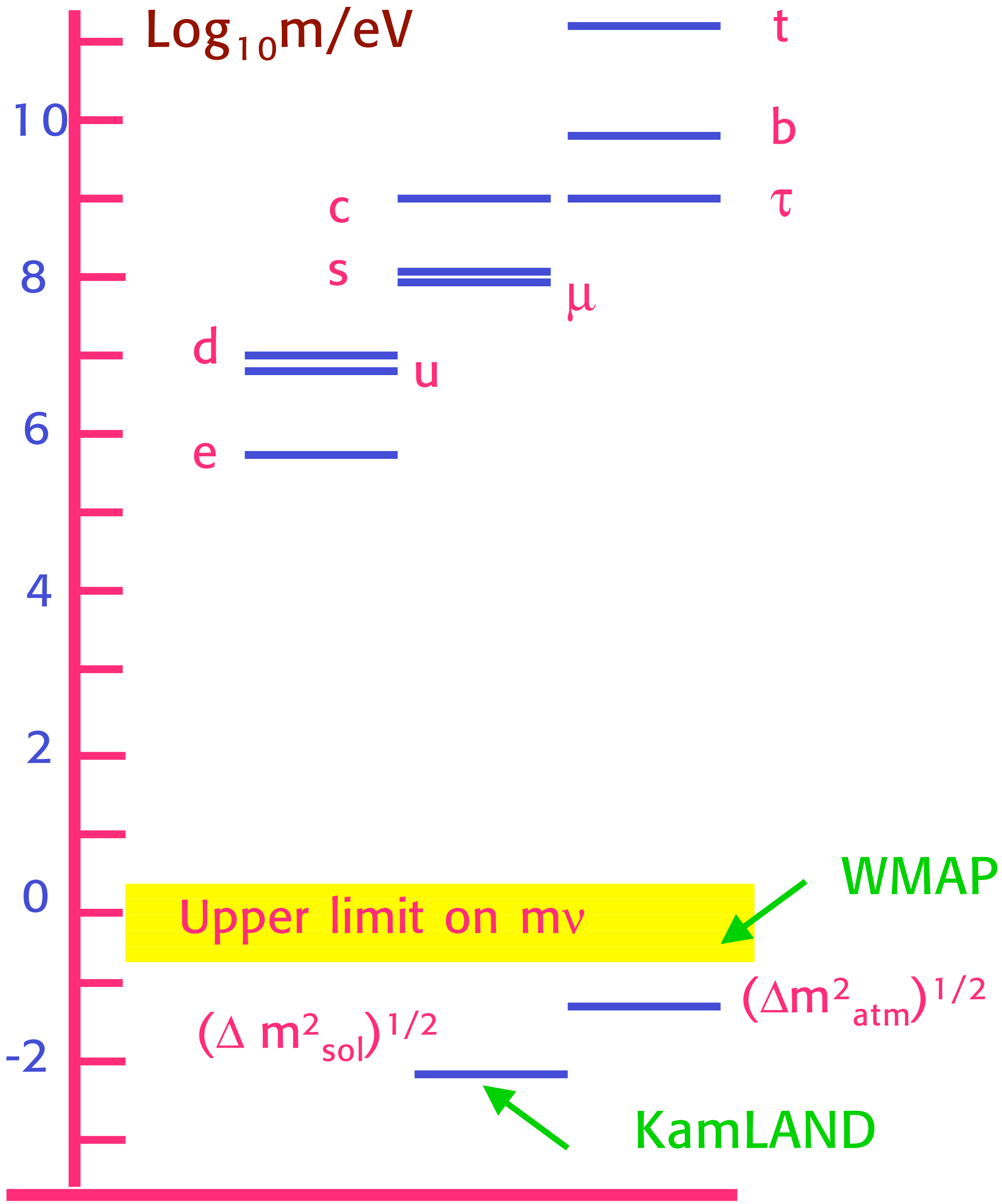
$\Sigma_i m_i < 0.2\text{-}0.7$ eV (dep. on data&priors)

WMAP, SDSS, 2dFGRS, Ly-α

→ Any ν mass $< 0.06 - 0.23 - 2.2$ eV

Only a moderate degeneracy allowed





Neutrino masses are really special!

$m_t/(\Delta m^2_{atm})^{1/2} \sim 10^{12}$

Massless ν's?

- no ν_R
- L conserved

Small ν masses?

- ν_R very heavy
- L not conserved

A very natural and appealing explanation:

ν's are nearly massless because they are Majorana particles and get masses through L non conserving interactions suppressed by a large scale M (the scale of ν_{RH} Majorana mass)

$$m_\nu \sim \frac{m^2}{M}$$

$m: \leq m_t \sim v \sim 200$ GeV

M: scale of L non cons.

Note:

$m_\nu \sim (\Delta m^2_{atm})^{1/2} \sim 0.05$ eV

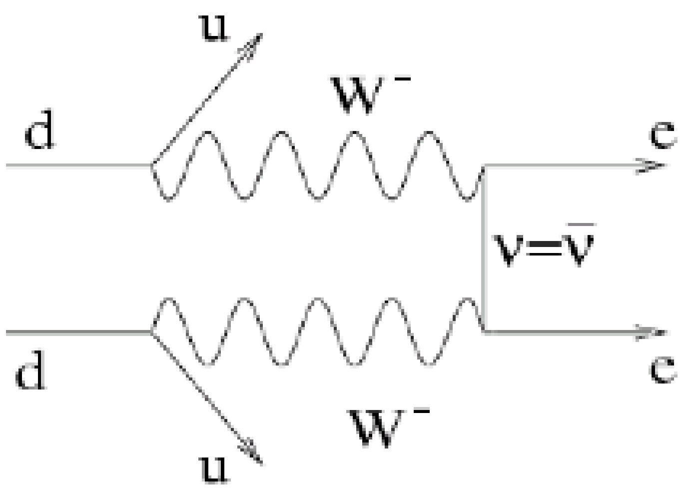
$m \sim v \sim 200$ GeV

$M \sim 10^{15}$ GeV

Neutrino masses are a probe of physics at M_{GUT} !

All we know from experiment on ν masses strongly indicates that ν's are Majorana particles and that L is not conserved (but a direct proof still does not exist).

Detection of 0νββ would be a proof of L non conservation. Thus a big effort is devoted to improving present limits and possibly to find a signal.



Heidelberg-Moscow
IGEX
Cuoricino
Nemo
Sokotvina
DAMA
•••••

0νββ = dd -> uue^-e^-

0νββ would prove that L is not conserved and ν's are Majorana
Also can tell degenerate, inverted or normal hierarchy

$|m_{ee}|=c_{13}^2\ [m_1 c_{12}^2+e^{i\alpha}m_2 s_{12}^2]+m_3 e^{i\beta}s_{13}^2$

Degenerate: $\sim|m||c_{12}^2+e^{i\alpha}s_{12}^2|\sim|m|(0.3\text{-}1)$

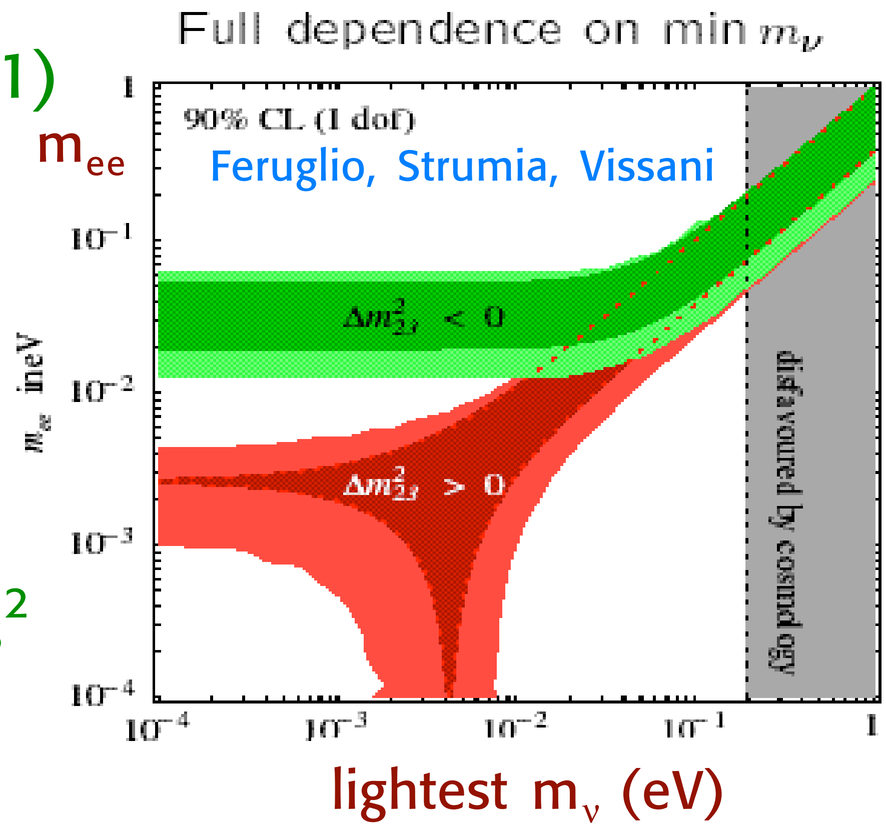
$|m_{ee}|\sim\ |m|\ (0.3\ \text{-}1)\leq\ 0.23\text{-}1$ eV

IH: $\sim(\Delta m^2_{atm})^{1/2}|c_{12}^2+e^{i\alpha}s_{12}^2|$

$|m_{ee}|\sim\ (1.6\text{-}5)\ 10^{-2}$ eV

NH: $\sim(\Delta m^2_{sol})^{1/2}s_{12}^2+(\Delta m^2_{atm})^{1/2}e^{i\beta}s_{13}^2$

$|m_{ee}|\sim$ (few) 10^{-3} eV



Present exp. limit: $m_{ee}<0.3\text{-}0.5$ eV
(and a hint of signal????? Klapdor Kleingrothaus)

Baryogenesis by decay of heavy Majorana ν's

BG via Leptogenesis near the GUT scale

$T \sim 10^{12\pm3}$ GeV (after inflation)

Buchmuller,Yanagida, Plumacher, Ellis, Lola, Giudice et al, Fujii et al ….

Only survives if Δ(B-L) is not zero
(otherwise is washed out at T_{ew} by instantons)

Main candidate: decay of lightest ν_R ($M \sim 10^{12}$ GeV)

L non conserv. in ν_R out-of-equilibrium decay:
B-L excess survives at T_{ew} and gives the obs. B asymmetry.

Quantitative studies confirm that the range of m_i from ν oscill's is compatible with BG via (thermal) LG

In particular the bound was derived for hierarchy

$m_i < 10^{-1}$ eV

Buchmuller, Di Bari, Plumacher; Giudice et al; Pilaftsis et al; Hambye et al Hagedorn et al

Can be relaxed for degenerate neutrinos
So fully compatible with oscill'n data!!

⊕

We cannot exclude that ν's are Dirac particles

We cannot exclude that ν masses arise at the EW scale

But if we believe in some form of GUT's and that L conservation is violated near the GUT scale

then it is very economical and natural to assume that ν's are Majorana particles and their mass is inversely related to the large scale of L non conservation.

This idea is supported by the observed values of the oscillation frequencies

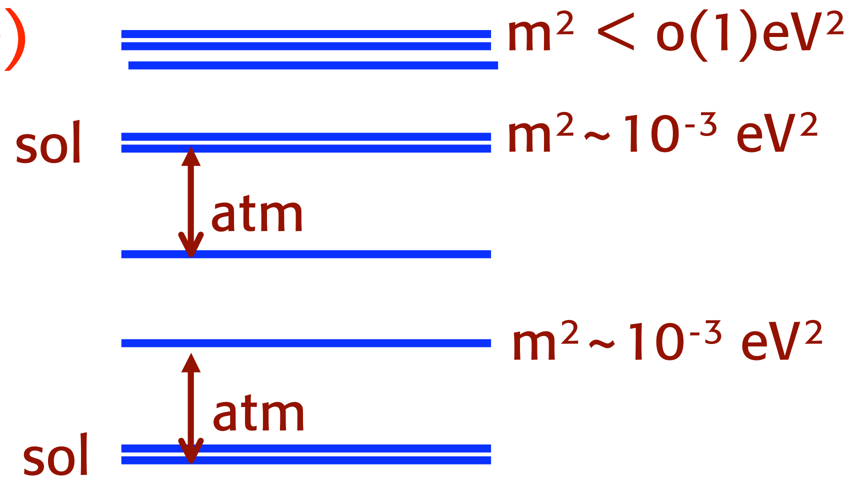
In turn ν's support GUT's

The current experimental situation on ν masses and mixings has much improved but is still incomplete

- what is the absolute scale of ν masses?
- value of θ_{13}......
- pattern of spectrum (sign of Δm^2_{atm})
- no detection of $0\nu\beta\beta$ (i.e. no proof that ν's are Majorana)
  see-saw?

3 light ν's are OK (MiniBoone)

- Degenerate ($m^2 >> \Delta m^2$)
- Inverse hierarchy
- Normal hierarchy



→ Different classes of models are still possible

General remarks

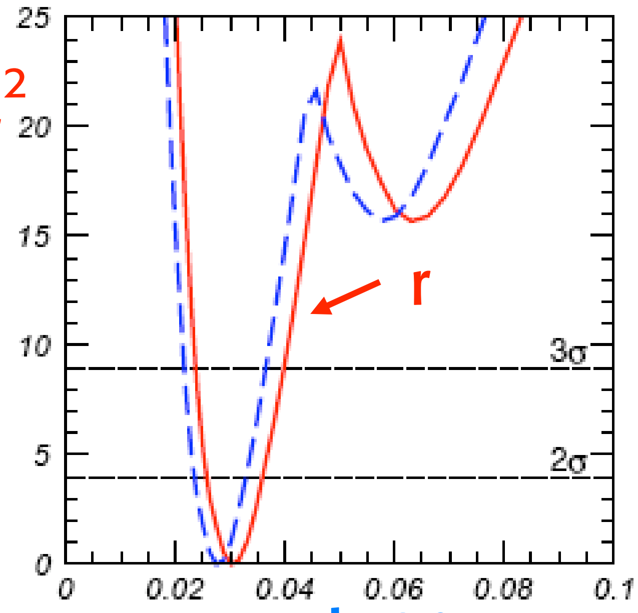
- After KamLAND, SNO and WMAP.... not too much hierarchy is found in ν masses:

$$r \sim \Delta m^2_{sol}/\Delta m^2_{atm} \sim 1/30$$

Only a few years ago could be as small as 10^{-8}!

Precisely at 3σ: $0.025 < r < 0.039$

Schwetz et al '08



or

$m_{heaviest} < 0.2 - 0.7$ eV

$m_{next} > \sim 8\ 10^{-3}$ eV

For a hierarchical spectrum: $\frac{m_2}{m_3} \approx \sqrt{r} \approx 0.2$

Comparable to $\lambda_C = \sin\theta_C$: $\lambda_C \approx 0.22$ or $\sqrt{\frac{m_\mu}{m_\tau}} \approx 0.24$

Suggests the same "hierarchy" parameters for q, l, ν (small powers of λ_C) → e.g. θ_{13} not too small!

- Still large space for non maximal 23 mixing

  2-σ interval $0.37 < \sin^2\theta_{23} < 0.60$ Fogli et al '08

  Maximal θ_{23} theoretically hard

- θ_{13} not necessarily too small

  probably accessible to exp.

  Very small θ_{13} theoretically hard

- θ_{12} is at present the best measured angle

  $\Delta\sin^2\theta_{12}/\sin^2\theta_{12} \sim 6\%$

For constructing models we need the data but also to decide which feature of the data is really relevant

Examples:

Is Tri-Bimaximal (TB) mixing really a significant feature or just an accident?

Is lepton-quark complementarity (LQC) a significant feature or just an accident?

Here we already see 3 different classes of models that can fit the data:
TB & LQC are accidents, TB is relevant, LQC is relevant

Accidents: a wide spectrum of models
Anarchy, Anarchy in 2-3 sector, Lopsided models,
$U(1)_{FN}$, ... GUT versions exist (SU(5), SO(10))
Typically there are parameters fitted to the angles

TB

$$U = \begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

TB mixing agrees with data at ~ 1σ

At 1σ:

G.L.Fogli et al '08

$\sin^2\theta_{12} = 1/3$: 0.29-0.33

$\sin^2\theta_{23} = 1/2$: 0.41-0.54

$\sin^2\theta_{13} = 0$: < ~0.02

A coincidence or a hint?

LQC

There is an intriguing empirical relation:

$$\theta_{12} + \theta_C = (47.0 \pm 1.2)^o \sim \pi/4$$

Raidal'04

A coincidence or a hint?

First consider models with $\theta_{13}= 0$ and θ_{23} maximal and θ_{12} generic

The most general mass matrix is given by (after ch. lepton diagonalization!!!) and it is 2-3 or $\mu-\tau$ symmetric

$$m_\nu = \begin{bmatrix} x & y & y \\ y & z & w \\ y & w & z \end{bmatrix}$$

Inspired models based on $\mu-\tau$ symmetry

Grimus, Lavoura..., Ma,....
Mohapatra, Nasri, Hai-Bo Yu ....

Neglecting Majorana phases it depends on 4 real parameters (3 mass eigenvalues and 1 mixing angle: θ_{12})

But actually θ_{12} is the best measured angle (after KamLAND, SNO....). And it is directly compatible with TB mixing.

A lot of model building has been devoted to TB mixing

By adding $\sin^2\theta_{12} \sim 1/3$ to $\theta_{13} \sim 0$, $\theta_{23} \sim \pi/4$:

Tribimaximal Mixing

$$m_\nu = \begin{bmatrix} x & y & y \\ y & z & w \\ y & w & z \end{bmatrix} \longrightarrow m = \begin{pmatrix} x & y & y \\ y & x+v & y-v \\ y & y-v & x+v \end{pmatrix}$$

$m_{11} + m_{12} = m_{22} + m_{23}$

$m_1 = x-y$
$m_2 = x+2y$
$m_3 = x-y+2v$

$$\sin^2 2\theta_{12} = \frac{8y^2}{(x-w-z)^2 + 8y^2}$$

$= 8/9$ for TB

The 3 remaining parameters are the mass eigenvalues

Tribimaximal Mixing

Harrison, Perkins, Scott

A simple mixing matrix compatible with all present data

$$U=\begin{bmatrix}\sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0\\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}}\\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}}\end{bmatrix}$$

In the basis of diagonal ch. leptons:

$m_\nu = U\mathrm{diag}(m_1, m_2, m_3)U^T$

$$m_\nu = \frac{m_3}{2}\begin{bmatrix}0 & 0 & 0\\ 0 & 1 & -1\\ 0 & -1 & 1\end{bmatrix} + \frac{m_2}{3}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix} + \frac{m_1}{6}\begin{bmatrix}4 & -2 & -2\\ -2 & 1 & 1\\ -2 & 1 & 1\end{bmatrix}$$

Eigenvectors: $m_3 \to \frac{1}{\sqrt{2}}\begin{bmatrix}0\\ 1\\ -1\end{bmatrix}$ $\quad m_2 \to \frac{1}{\sqrt{3}}\begin{bmatrix}1\\ 1\\ 1\end{bmatrix}$ $\quad m_1 \to \frac{1}{\sqrt{6}}\begin{bmatrix}-2\\ 1\\ 1\end{bmatrix}$

Note: mixing angles independent of mass eigenvalues

Compare with quark mixings $\lambda_C \sim (m_d/m_s)^{1/2}$

- For the TB mixing matrix all mixing angles are fixed to particularly symmetric values

Sparked interest in constructing models that can naturally produce this highly ordered structure

Models based on the A4 discrete symmetry (even permutations of 1234) offer a minimal solution

Ma...; GA, Feruglio, GA, Feruglio, Lin; hep-ph/0610165; GA, Feruglio, Hagedorn; Y. Lin; Csaki et al; GA, Meloni.......

Larger finite groups: T′, S4, $PSL_2(7)$ have also been studied

Feruglio et al; Chen, Mahanthappa;Frampton, Kephart; Lam; Bazzocchi et al , King et al

Alternative models based on $SU(3)_F$ or $SO(3)_F$ or their finite subgroups

Verzielas, G. Ross .... King ....

Discrete symmetries coupled with Sequential Dominance or Form Dominance

King ...., Chen, King......

Why discrete groups, in particular A4, work?

TB mixing corresponds to m
in the basis where
charged leptons are diagonal

$$m = \begin{pmatrix} x & y & y \\ y & x+v & y-v \\ y & y-v & x+v \end{pmatrix}$$

m is the most general matrix invariant under
$SmS = m$ and $A_{23} m A_{23} = m$ with:

$$S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$$

$$A_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$ 2-3 symmetry

$S^2 = A_{23}^2 = 1$

Charged lepton masses:
a generic diagonal matrix,
is invariant under T
(or ηT with η a phase):

$$m_l = v_T \frac{v_d}{\Lambda}\begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix}$$

$$m_l^+ m_l = T^+ m_l^+ m_l T$$

a possible T is

$$T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}$$

$S^2=T^3=(ST)^3=1$ define A4

$\omega^3=1$ --> $T^3=1$

Invariance under S and T can be made automatic in A4 while A_{23} is not in A4 (2<->3 exchange is an odd permutation)
But 2-3 symmetry happens in A4 if 1' and 1" flavons are absent.

S, T and A_{23} are all contained in S4

$S^4=T^3=(ST^2)^2=1$ define S4

Lam

A4 A4 is the discrete group of even perm's of 4 objects. (the inv. group of a tetrahedron). It has $4!/2 = 12$ elements.

A4 transformations can be written in terms of S and T

with: $S^2 = T^3 = (ST)^3 = 1$ as:

$1, T, S, ST, TS, T^2, TST, STS, ST^2, T^2S, T^2ST, TST^2$

An element is abcd which means 1234 --> abcd

C_1: $1 = 1234$

C_2: $T = 2314$ $ST = 4132$ $TS = 3241$ $STS = 1423$

C_3: $T^2 = 3124$ $ST^2 = 4213$ $T^2S = 2431$ $TST = 1342$

C_4: $S = 4321$ $T^2ST = 3412$ $TST^2 = 2143$

C_1, C_2, C_3, C_4 are equivalence classes

x, x' in same class if $[x' \sim gxg^{-1}]$ g: group element

Irr. reprent'ns 1, 1', 1", 3

L lepton doublet ~ 3

$e^c, \mu^c, \tau^c \sim 1, 1'', 1'$

Structure of A4 models

The model is invariant under the flavour group A4

There are flavons ϕ_T, ϕ_S , ξ... with VEV's that break A4:

ϕ_T down to G_T, the subgroup generated by 1, T, T^2, in the charged lepton sector

ϕ_S , ξ down to G_S, the subgroup generated by 1, S, in the neutrino sector

$$\langle \varphi_T \rangle = (v_T, 0, 0)$$
$$\langle \varphi_S \rangle = (v_S, v_S, v_S)$$
$$\langle \xi \rangle = u \quad , \quad \langle \tilde{\xi} \rangle = 0$$

ϕ_T, ϕ_S ~ 3

ξ ~ 1

The 2-3 symmetry occurs in A4 as 1′ and 1″ flavons are absent

The aligment occurs because is based on A4 group theory

TB mixing broken by higher dimension operators

Typically $\delta\theta \sim o(\lambda_C^2)$

Recent directions of research:

- Different (larger) finite groups

Ma;
Kobayashi et al;
Luhn, Nasri, Ramond [$\Delta(3n^2)$];
.....

- Extension to quark mixings

Carr, Frampton
Feruglio et al
Frampton, Kephart.....

- Construct GUT models with approximat TB mixing

it is indeed possible, also for A4!
GA, Feruglio, Hagedorn 0802.0090

Ma, Sawanaka, Tanimoto; Ma;
Morisi, Picarello, Torrente Lujan; Bazzocchi et al;
de Madeiros Verzielas, King, Ross [$\Delta(27)$];
King, Malinsky [$SU(4)_C xSU(2)_L xSU(2)_R$]; Antusch et al;
Chen, Mahanthappa. Bazzocchi et al [$\Delta(27)$]; .....

Extension to quarks

If we take all fermion doublets as 3 and all singlets as 1, 1′, 1″ (as for charged leptons): $Q_i \sim 3$; $u^c, d^c \sim 1$; $c^c, s^c \sim 1''$; $t^c, b^c \sim 1'$

Then u and d quark mass matrices, like for charged leptons, are BOTH diagonal in the T-diagonal basis

As a result V_{CKM} is unity: $V_{CKM} = U_u^+ U_d \sim 1$

So, in first approx. (broken by loops and higher dim operators), ν mixings are TB and quark mixings ~identity: NOT BAD

But the hierarchy of q mixing angles is difficult to be obtained. Those A4 transf. properties are not compatible with GUT's

From the q sector no confirmation of discrete flavour groups

Assume that LQC is a better guiding principle than TB

If θ_{13} is found near its present bound (e.g $o(\lambda_C)$) this would hint that TB is accidental and bimaximal mixing (BM) could be a better first approximation

There is an intriguing empirical relation:

$$\theta_{12} + \theta_C = (47.0 \pm 1.2)^o \sim \pi/4$$

Raidal'04

Suggests bimaximal mixing in 1st approximation, corrected by charged lepton diagonalization.

Recall that

$$\lambda_C \approx 0.22 \text{ or } \sqrt{\frac{m_\mu}{m_\tau}} \approx 0.24$$

$\lambda_C = \sin\theta_C$

While $\theta_{12} + o(\theta_C) \sim \pi/4$ is easy to realize,

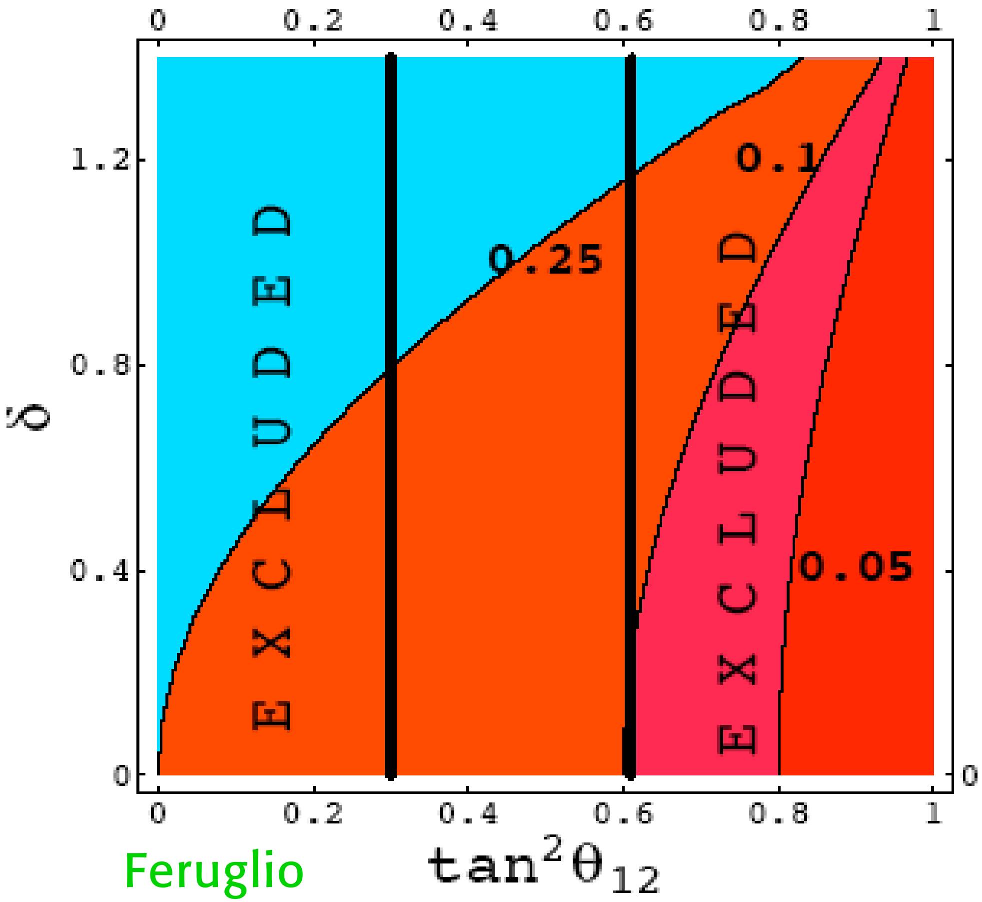
$\theta_{12} + \theta_C \sim \pi/4$ is more difficult: no compelling model

Minakata, Smirnov'04

Suggests that deviations from BM mixing arise from charged lepton diagonalisation (BM: $\theta_{12} = \theta_{23} = \pi/4 \quad \theta_{12} = 0$)

For the corrections from the charged lepton sector, typically $|\sin\theta_{13}| \sim (1 - \tan^2\theta_{12})/4\cos\delta \sim 0.15$

GA, Feruglio, Masina
Frampton et al
King
Antusch et al........



Feruglio

$$\bar{U}_{12} = -\frac{e^{-i(\alpha_1+\alpha_2)}}{\sqrt{2}} + \frac{s^e_{12}e^{-i\alpha_2} + s^e_{13}e^{i\delta_e}}{2}$$

$$\bar{U}_{13} = \frac{s^e_{12}e^{-i\alpha_2} - s^e_{13}e^{i\delta_e}}{\sqrt{2}}$$

$$\bar{U}_{23} = -e^{-i\alpha_2}\frac{1 + s^e_{23}e^{i\alpha_2}}{\sqrt{2}}$$

Corr.'s from s^e_{12}, s^e_{13} to U_{12} and U_{13} are of first order (2nd order to U_{23})

One can construct a model where BM mixing holds in 1st approximation and is then corrected by terms $o(\lambda_C)$

G.A., Feruglio, Merlo '09

BM mixing

$$\theta_{12} = \theta_{23} = \pi/4, \ \theta_{13} = 0$$

$$U_{BM} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

By adding $\sin^2\theta_{12} \sim 1/2$ to $\theta_{13} \sim 0$, $\theta_{23} \sim \pi/4$:

Bimaximal Mixing

$$m_\nu = \begin{bmatrix} x & y & y \\ y & z & w \\ y & w & z \end{bmatrix} \longrightarrow m_{\nu BM} = \begin{pmatrix} x & y & y \\ y & z & x-z \\ y & x-z & z \end{pmatrix}$$

$$\sin^2 2\theta_{12} = \frac{8y^2}{(x-w-z)^2 + 8y^2}$$

$$m_1 = x + \sqrt{2}y$$

$$m_2 = x - \sqrt{2}y$$

$$m_3 = 2z - x$$

The 3 remaining parameters are the mass eigenvalues

BM corresponds to $\tan^2\theta_{12}=1$
while exp.: $\tan^2\theta_{12} = 0.45 \pm 0.04$
so a large correction is needed

Bimaximal Mixing

In the basis of diagonal ch. leptons:

$$U_{BM} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$m_\nu = U \mathrm{diag}(m_1, m_2, m_3) U^T$

$$m_{\nu BM} = \left[\frac{m_3}{2} M_3 + \frac{m_2}{4} M_2 + \frac{m_1}{4} M_1 \right]$$

$$M_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix}, \quad M_2 = \begin{pmatrix} 2 & -\sqrt{2} & -\sqrt{2} \\ -\sqrt{2} & 1 & 1 \\ -\sqrt{2} & 1 & 1 \end{pmatrix}, \quad M_1 = \begin{pmatrix} 2 & \sqrt{2} & \sqrt{2} \\ \sqrt{2} & 1 & 1 \\ \sqrt{2} & 1 & 1 \end{pmatrix}$$

Eigenvectors: $(\sqrt{2}, 1, 1)/2, \; (-\sqrt{2}, 1, 1)/2 \; , \; (0, 1, -1)/\sqrt{2}$

In our model BM mixing is exact at LO

For the special flavon content chosen, only θ_{12} and θ_{13} are corrected from the charged lepton sector by terms of $o(\lambda_C)$ (large correction!) while θ_{23} gets smaller corrections (great!)
[for a generic flavon content also $\delta\theta_{23} \sim o(\lambda_C)$]

An experimental indication for this model would be that θ_{13} is found near its present bound at T2K

Conclusion

- No need for more than 3 light neutrinos or CPT violation
- Majorana ν's, the see-saw mechanism and $M \sim M_{GUT}$ explain the data (we expect L non cons. in GUT's)
  - needs confirmation from $0\nu\beta\beta$ decay
  - ν's support GUT's
- Different models can accommodate the data on ν mixing
  - e. g. TB mixing accidental or a hint?

Anarchy
Lopsided models
$U(1)_{FN}$,
••••••

discrete groups

- θ_{13}, sign Δm^2_{23}, CP phase δ, absolute m^2 scale.... ?????