Modelling The Gravitational Collapse Of Scalar Fields In AdS

Brad Cownden

May 31, 2017
CAP Congress, Queen's University

University of Manitoba
The University of Winnipeg

Motivation

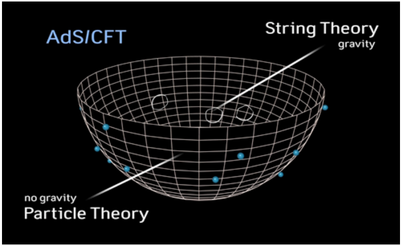
- AdS/CFT Correspondence $\rightarrow$ relates a gravitational theory in $(d+1)$-dimensions to quantum field theory (CFT) in d-dimensions
- Thermalization of strongly-coupled (conformal) field theories $\Leftrightarrow$ formation of black hole in Anti-de Sitter (AdS) space

Motivation

- AdS/CFT Correspondence $\rightarrow$ relates a gravitational theory in $(d+1)$-dimensions to quantum field theory (CFT) in d-dimensions
- Thermalization of strongly-coupled (conformal) field theories $\Leftrightarrow$ formation of black hole in Anti-de Sitter (AdS) space
- Massless scalar fields in AdS with generic boundary conditions [Bizoń & Rostworowski] $\rightarrow$ AdS is generically unstable, even at amplitude ϵ
- Expanded to $\mu \neq 0$ and complex fields [Buchel, Liebling, & Lehner] $\rightarrow$ stability based on initial data; "islands of stability" for single-mode data [Green, Maillard, *et al.*]
- Numerically evolve Einstein gravity + scalar field; expand on existing work [Deppe & Frey]
- Examine perturbative regime $\rightarrow$ use *Two-Time Formalism* (TTF); test range of validity

The AdS/CFT Correspondence

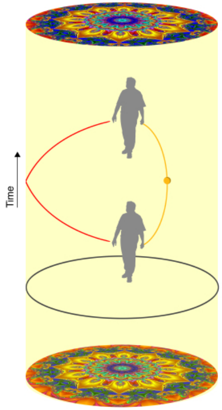
- Black holes in AdS ⇔ CFT in thermal equilibrium
- Scalar field at the boundary relates to operators in the CFT



learner.org

The AdS/CFT Correspondence

- Black holes in AdS $\Leftrightarrow$ CFT in thermal equilibrium
- Scalar field at the boundary relates to operators in the CFT
- AdS has a (conformal) boundary $\rightarrow$ light rays travel to ∞ and back in finite time
- Negative curvature: massive particles oscillate in gravitational well
- Couple $\phi(t, x)$ to gravity $\rightarrow$ modify AdS metric
- Examine $\mu = 0$, $\mu = 1$, $\mu > 1$; range of widths, σ



zeropoint.ca

Method

- Massive scalar minimally coupled to Einstein gravity
- Spherically symmetric, Schwarzschild-like coordinates $\to A(t,x)$, $\delta(t,x)$
- Evolution equations for the nonlinear system
- Horizon formation when $A(t_H, x_H) \leq 2^{7-n} \ll 1$
- Gaussian initial data

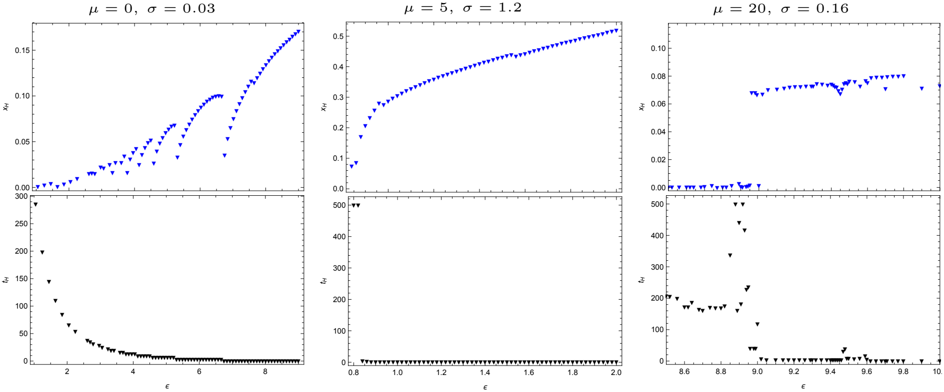
$$G_{ab} + \Lambda g_{ab} = 8\pi \left(\nabla_a \phi \nabla_b \phi - \frac{1}{2} g_{ab} (\nabla_c \phi \nabla^c \phi + \mu^2 \phi^2) \right)$$

$$ds^2 = \frac{\ell^2}{\cos^2(x/\ell)} \left(-A e^{-2\delta} dt^2 + A^{-1} dx^2 + \sin^2(x/\ell) d\Omega^{d-1} \right)$$

$$\Pi(t=0,x) = \epsilon \exp\left(-\frac{\tan^2(x)}{\sigma^2} \right)$$

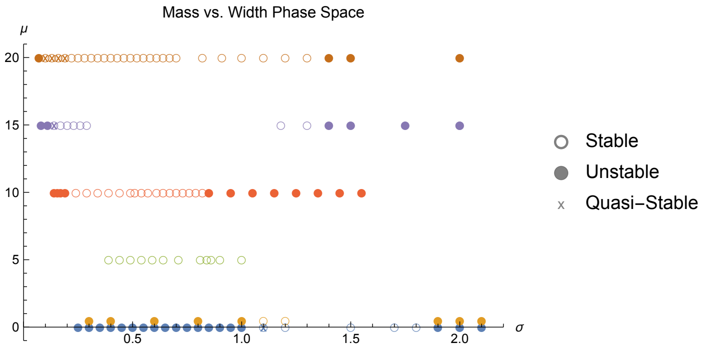
Results

- Observed: unstable at all amplitudes; stable below a critical amplitude; new, quasi-stable behaviour



In Progress: BC, Deppe, Frey

Results

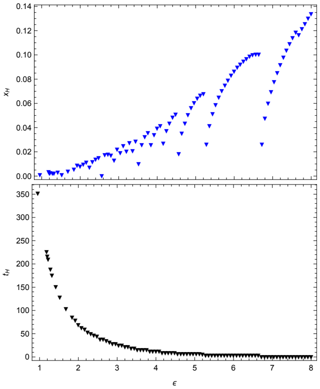


In Progress: BC, Deppe, Frey

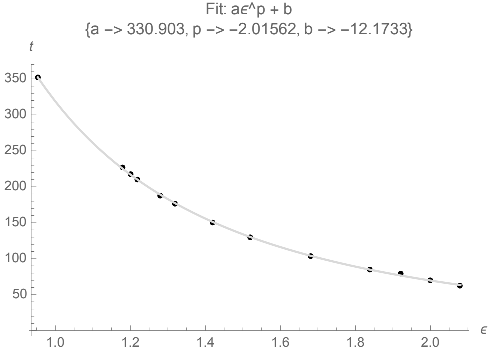
- Map the characteristic behaviours for a variety of masses and widths
- Confirm islands of stability; investigate quasi-stable regimes

Towards the Perturbative Regime

- Collapse of small amplitude $\mathcal{O}(\epsilon)$ approach power-law behaviour $\rightarrow$ fit $t_H = a\epsilon^p + b$ $\rightarrow$ look for $p \sim -2$, or unstable in different way?



$\mu = 0.5, \ \sigma = 0.3$



$\mu = 0.5, \ \sigma = 0.3$

In Progress: BC, Deppe, Frey

Two-Time Formalism

- Apply TTF to linearized system when $\mu = 0 \rightarrow$ spatial eigenfunctions, $e_j(x)$, are Jacobi polynomials; *slow time* $\tau \equiv \epsilon^2 t$; frequency $\omega_j = d + 2j$
- Slowly-modulating amplitudes allow for inverse cascade
- Exact solutions to non-linear dynamics up to times $1/\epsilon^2$

$$\phi(t, \tau, x) = \epsilon \sum_{l=0}^{\infty} \left(A_j(\tau) e^{-i\omega_j t} + \bar{A}_j(\tau) e^{i\omega_j t} \right) e_j(x)$$

Two-Time Formalism

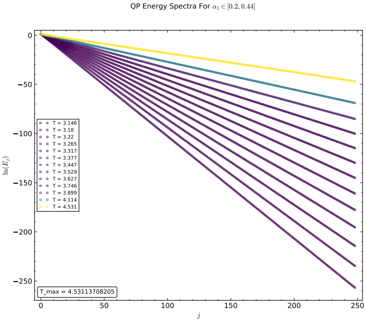
- Apply TTF to linearized system when $\mu = 0 \rightarrow$ spatial eigenfunctions, $e_j(x)$, are Jacobi polynomials; *slow time* $\tau \equiv \epsilon^2 t$; frequency $\omega_j = d + 2j$
- Slowly-modulating amplitudes allow for inverse cascade
- Exact solutions to non-linear dynamics up to times $1/\epsilon^2$
- Extend beyond known periodic solutions $\rightarrow$ quasi-periodic solutions (QP) $A_j(\tau) = \alpha_j \exp(-i\beta_j\tau)$ [Green, Maillard, *et al.*]
- Use conserved quantities E, N to fix two unknowns $\rightarrow$ families of solutions: $\alpha_0 = 1$, $\alpha_1 < \alpha_0$

$$-2\omega_l\alpha_l\beta_l = T_l\alpha_l^3 + \sum_{i\neq l} R_{il}\alpha_i^2\alpha_l + \sum_{i\neq l}\sum_{j\neq l}^{l\leq i+j} S_{ij(i+j-l)l}\alpha_i\alpha_j\alpha_{i+j-l}$$

$$E = 4\sum_j \omega_j^2\alpha_j^2\,, \qquad N = 4\sum_j \omega_j\alpha_j^2$$

TTF Solutions

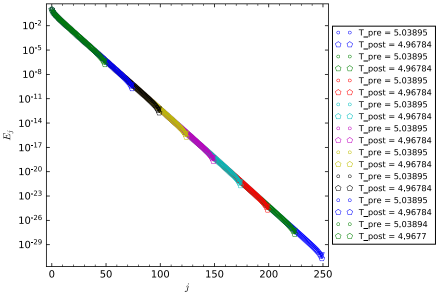
- Characterize solutions by $T = E/N$
- Necessary to truncate at some $j_{max} < \infty \rightarrow$ test robustness of QP solutions to truncation

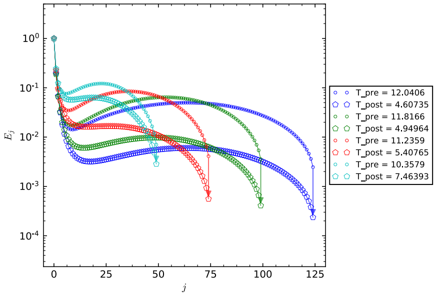


In Progress: BC, Deppe, Frey

TTF Solutions

- Characterize solutions by $T = E/N$
- Necessary to truncate at some $j_{max} < \infty \rightarrow$ test robustness of QP solutions to truncation
- Perturb to higher-temperature QP solutions $\rightarrow$ test robustness to QP solutions to temperature increases





In Progress: BC, Deppe, Frey

Summary

- Study injection of energy into strongly-coupled theory via AdS/CFT dual: massive scalar on AdS $\rightarrow$ black hole formation in gravity theory $\Leftrightarrow$ thermalization in conformal field theory
- Numerically solve nonlinear Einstein's equations $\rightarrow$ examine evolution of scalar fields of varying μ and σ
- Massive/massless scalars $\rightarrow$ explore a range of initial data $\rightarrow$ categorize into stable/unstable/quasi-stable solutions by comparing to $t_H \propto \epsilon^{-2}$
- Use TTF and construct high j_{max} solutions $\rightarrow$ dependence on truncation? Temperature?
- TTF solutions as initial data of nonlinear system $\rightarrow$ stable in the full theory?

Thanks

- CAP Congress & Queen's University
- Andrew Frey
- Nils Deppe (Cornell)
- Westgrid & Compute Canada