Universality of lowenergy Rashba scattering

Joel Hutchinson Joseph Maciejko

❖ Spin degeneracy is a consequence of time-reversal + inversion symmetry

$$
E(\mathbf{k} \uparrow) = E(-\mathbf{k} \downarrow) \qquad \text{Time-reve}
$$

$$
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$$

Time-reversal

$$
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$$

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e.g. surfaces, interfaces, quantum well with confining potential

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Spin and momentum are locked. Lots of potential applications!

Low-energy Rashba in two dimensions [1]

❖ 2D Hamiltonian:

$$
H(\boldsymbol{k}) = \frac{\boldsymbol{k}^2}{2m} + \lambda \hat{\boldsymbol{z}} \cdot (\boldsymbol{\sigma} \times \boldsymbol{k})
$$

 $\frac{1}{2}$ \mathbf{ent} scattering states at each angle. tities along the way. The natural starting point is the There are two different

the electron scattering energy by the dimensionless quan-

tor angle \overline{a}

tity ⌘ ^p¹ *[|]E|/E*0.

$$
E_0=\frac{1}{2}m\lambda^2
$$

Low-energy Rashba

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[◆] Basic question: Is there anything fundamentally different about Rashba scattering in this regime, independent of interactions and many-body physics?

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Example: Hard Disk

$$
V = \begin{cases} \infty & r < R \\ 0 & r > R \end{cases}
$$

❖ Wavefunction computed analytically from matching conditions.

$$
\Psi(r,\theta) = \sum_{l=-\infty}^{\infty} e^{il\theta} \left[a_l \left(\frac{H_l^+(k
$$

 \triangle Cross-sections and S-matrix extracted.

Example: Hard Disk abilities from the *S*-matrix elements for partial waves *l* = ⁰*,* ¹*,* ²*,* ³*,* ⁴, as a function of ⁼ ^p¹ *[|]E|/E*0. In both plots

 $U = 1$ and $U = 1$ the incident and scattered and scat conventional 2D system (no Rashba): ❖ Differential cross-section in

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J. Hutchinson and J. Maciejko, Phys. Rev. B 93, 245309 (2016).

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❖ Differential cross-section in Rashba system:

, (35) *E* $\frac{1}{2}$ *E* $\frac{1}{2}$ 0*.*01*E*0, *E* = 0*.*5*E*0, *E* = 0*.*99*E*0), and (c) *k*? states and ϵ is that the differential term in the differential that the differential cross ϵ and ϵ $\sum_{i=1}^{n}$ and $\sum_{i=1}^{n}$ becomes the theory becomes ❖ In the low energy limit, scattering looks 1D! \mathcal{O} , we use \mathcal{O}

$$
\left(\frac{d\sigma}{d\theta}\right)_{\geqslant}\bigg|_{E=-E_0} = \frac{2\pi}{k_0}\left[\delta^2(\theta) + \delta^2(\theta - \pi)\right]
$$

J. Hutchinson and J. Maciejko, Phys. Rev. B 93, 245309 $(2016).$ (2016).

Example: Hard Disk

where we have changed the integration variable using *q* ⌘ **koko we approximately defined the potential, we approximately defined the potential, we approximately defined the potential,** \mathbf{r} *k*⌫*, k*⁰ ˆ*kµ*), The energy dependence of the *T*-matrix is entirely detere: Hard Disk Example: Hard Disk

$$
V = \begin{cases} \infty & r < R \\ 0 & r > R \end{cases}
$$

 $\sum_{i=1}^{n}$ \bullet 5-mach $\sqrt{2}$ $\frac{1}{2}$ 1 + *im* ² [*^V ^l*(*k*0*, k*0) + *^V ^l*+1(*k*0*, k*0)] (61) ❖ S-matrix decomposed in partial *i* waves.

<u>Low energy limit:</u> $\frac{1}{2}$ <u>1m1t:</u> $(0, 1)$

$$
S^l = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}
$$

- $l = l = 2$ **c** Independent of l
- where we used the fact that angular momentum conservation of the fact that angular momentum conservation of the $\frac{1}{\sqrt{2}}$

J. Hutchinson and J. Maciejko, Phys. Rev. B 93, 245309 (2016). $(2010).$

Scattering Formalism Scattering Formaliem relation between the *T*-matrix and scattering amplitude: We will need to Fourier transform the *T*-matrix to Z *d*²*k*⁰ Z *d*²*k*˜ (2⇡)² *^T ^k*⁰ *k*˜ ⁰ *eⁱk*⁰ *·^reik*˜*·^r* 0 the *S*-matrix as the unitary transformation from asymptotic ingoing to asymptotic outgoing states. Schematicall and called the called the called trace of the called trac *T rr* 0 ⁰ = Z *d*²*k*⁰ $\overline{}$ α (2⇡)² *^T ^k*⁰ *k*˜ ⁰ *eⁱk*⁰ *·^reik*˜*·^r* 0 *.* (10) totic ingoing to asymptotic ingoing states. Schematically, (2⇡)² (2⇡)² *^T ^k*⁰ $\overline{1}$ Fullidii into (10) in the contract of $\overline{1}$ modified Lippinger equations and the contract of the contract

T rr

⁰ =

 $\frac{1}{2}$

⁰ *eⁱk*⁰

 \overline{z}

. (10)

lently the scattering amplitude), we use the definition of

>(*r*; *^E*) ⇠ in

·^reik˜*·^r*

 \overline{z}

> + *S>>*out

 $=$ p $=$ p p*kµk*⌫ * Relate T and S matrices through Lippmann-Schwinger equation: *>*(*r*; *^E*) ⇠ in modified Lippman-Schwinger equation In reference [2], the form of the *S*-matrix for lowerzer ec $\mathbf u$ *d*₂*r*¹ Z *d*²*k*⁰ ❖ Relate T and S matrices through Lippmann-Schwinger equation:

$$
\psi_{\boldsymbol{k}\sigma}(\boldsymbol{r};E) = \psi_{\boldsymbol{k}\sigma}^{\text{in}}(\boldsymbol{r};E) + \sum_{\sigma'\sigma''}\int d^2\boldsymbol{r}' \int \frac{d^2\boldsymbol{k}'}{(2\pi)^2} G_{\sigma\sigma'}^+(\boldsymbol{r},\boldsymbol{r}';E) \; T_{\sigma'\sigma''}^{\boldsymbol{k}'\boldsymbol{k}} e^{i\boldsymbol{k}'\cdot\boldsymbol{r}'} \eta_{\sigma''}^-(\theta_{k})
$$

 \bullet For negative energies: using *k> k<* = 2*k*0, and letting ✓ ⌘ ✓*^r* = ✓*^k*⌫ ✓*^k^µ* . ⇥*^T ^k*⁰ 000*eⁱk*⁰ galive energies.

T rr

0

Scattering Formalism ^p*k>eik>^r*⌘(✓*r*)⌘(✓*r*) *†T ^k>^k*⌘(0) $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ (21) <u> with an angle of </u>∠: which is the component of the helicity transformation of the helicity transform of $\overline{\mathcal{M}}$ volving only transitions within the negative helicity state. For the *k<* term, we use the fact that *k<* = *k*⁺ = *|k*+*|r*ˆ to write the eigenspinors as \overline{a} *m*² \blacksquare *k*⌫ *k*2 0² *[|]^T ^kµk*⌫ 1
1910 - John Barnett, amerikansk politik
1910 - John Barnett, amerikansk politik X $X \sim \text{R}^2$ The Lippman-Schwinger equation finally reads to \mathcal{L}_max wave *^µ* = 12 1151 2⇡*k^µ* I \overline{a} I = $\frac{1}{2}$ *kµ* $\overline{}$ *l*=1 *||B0rmal* km = 1
1 *kµ* \mathbf{C} 1
1 *l*=1 **∣FormalIsm** $P_{\text{A}} \sim t^{+}$ $\overline{}$ an an an an angle an angle an angle an angle an angle ang ⇢(*E*⌫) = ^Z ¹ *dk* (2⇡)² *^k*(*E*⌫ *^E*(*k*)) = *^m*

l=1

matrix (*|S^l*

❖ Cross-sections from Fermi's golden rule: *†T ^k>^k*⌘(0) = *T ^k>^k ,* (22) 1_{Ω} C_{1000} $\frac{1}{2}$ C ross-sections from Fermi's golden \bullet Cross-sections from Fermi s olden r

(25)

using *k> k<* = 2*k*0, and letting ✓ ⌘ ✓*^r* = ✓*^k*⌫ ✓*^k^µ* .

⌘*s*⌫ (✓*r*)*e ⁱ*⇡

⌘*^s*⌫ (✓*r*)*e ⁱ*⇡

p*k>eik>^r*⌘(✓*r*)⌘(✓*r*)

• Cross-sections from Fermi's golden rule:
$$
\frac{d\sigma}{d\theta}\Big|_{\mu\nu} = \frac{w_{\mu\to\nu}}{|\dot{\mathbf{j}}_{\mu}|}
$$

$$
\sigma_{\mu} = \frac{2}{k_{\mu}} \sum_{l=-\infty}^{\infty} (1 - \text{Re}(S_{\mu\mu}^{l}))
$$

where $\mathcal{L}_{\mathcal{L}}$ is the density of $\mathcal{L}_{\mathcal{L}}$ is the density of final states in the channel states in th

(2⇡)²

eil✓(*S^l*

*k*0

^µ⌫ *µ*⌫)

. (31)

$$
\text{ * \textit{Optical theorem}:} \quad \text{Im}(T_{--}^{\mathbf{k}_{\mu}\mathbf{k}_{\mu}}(\theta=0)) = -\frac{k_0 \delta}{2m} \sigma_{\mu}
$$

fµ⌫(✓*r*)*.* (26)

a a conoric for Is there a generic form for the T-matrix? index *µ*, and we used the unitarity condition of the *S*n for the 2*m n* T-matrix? \sim is there a generic form for the T-m *l*=1 \mathfrak{g} a ge. 1 Is there a generic form for the T-matrix?

Generic Rashba T-matrix c nashoa t-mi

energy Rashba *T*-matrix for any circularly symmetric,

 k^{old} finite range, spin-independent potential. ◆ Claim: The low-energy T-matrix takes a universal form for any circular-symmetric,

Generic Rashba T-matrix 1 2 \mathcal{L} 1 *l*=1 *V l* (*k, k*⁰)*eil*(✓*k*0✓*k*) (1 + *ijeⁱ*(✓*k*✓*k*⁰)) Fig. 1. (a) k-space contours and (b) low-energy spectrum sp *l*=1 1 ² **Z** 1 (*V ^l* (*k, k*⁰) + *ijV ^l*+1(*k, k*⁰))*eil*✓*k*0*^k ,*

allowed virtual transitions with *[|]^k ^k*0*[|] <* ⇤˜ to be incorpo-

◆ Claim: The low-energy T-matrix takes a universal form for any circular-symmetric, finite range, spin-independent potential. $2r$ *l*=1 (*V ^l* (*k, k*⁰) + *ijV ^l*+1(*k, k*⁰))*eil*✓*k*0*^k ,* ircular-symmetric, rated in the *T*-matrix. The orange lines show the continuum duced the partial wave component positive helicity branch.

l=1

1

1

Generic Rashba T-matrix The detailed derivation of the derivation of \mathbf{r}_i C. G. It is converted to define the convenient to define the set of the convenient of the convenient of the co (*k*0*, k*0) + *ijV ^l*+1(*k*0*, k*0)]*eil*✓ ⇤ *^l* ⌘ *m* μ /// **P** μ **2** μ **1** μ *natrix* ⁼ *^m* 2 ✓ *i* \overline{e} 2 ⇡⇤ ◆ [*V ^l* (*k*0*, k*0) + *V ^l*+1(*k*0*, k*0)] +*O*() + *O*(⇤)*.* (60)

The energy dependence of the *T*-matrix is entirely deter-

$$
V_{ji}(\boldsymbol{k}_{\boldsymbol{\nu}},\boldsymbol{k}_{\boldsymbol{\mu}}) \approx V_{ji}(k_0 \hat{k_{\nu}}, k_0 \hat{k_{\mu}}) + O(\delta)
$$

k⌫*, kµ*)*.* (52)

$$
T_{--}^l \approx \frac{1}{m} \frac{\delta_l^*}{1 + i \delta_l^* / \delta} = -\frac{i \delta}{m} + O(\delta^2)
$$

m

1 + *i*⇤

With With

where we have changed the integration variable using α

eter

is a crucial approximation. Since now the right hand side

is independent of this magnitude as well

With
$$
\delta_l^* \equiv \frac{m}{2} (V^l(k_0, k_0) + V^{l+1}(k_0, k_0))
$$

With
$$
\delta_l^* \equiv \frac{1}{2} (V^l(k_0, k_0) + V^{l+1}(k_0, k_0))
$$

$$
V^l(k, k') = \int_0^{2\pi} \frac{d\theta}{2\pi} \int_0^\infty dr r V(r) J_0(|\mathbf{k} - \mathbf{k}'|r) e^{-il\theta}
$$

Generic Rashba T-matrix nents just as we did with the potential, the posterior series of the potential, the potential, the Born series *Tl* = 1 million $\overline{}$ <u>L</u> [*V ^l* (*k*0*, k*0) + *ijV ^l*+1(*k*0*, k*0)]*eil*✓ The detailed derivation of this result is left for a proper C. To leading order in , we get [*V ^l* (*k*0*, k*0) + *ijV ^l*+1(*k*0*, k*0)]*eil*✓ C. To leading order in , we get

❖ Remarks: *ji*(*kµ*)*eil*✓ = X $\frac{1}{2}$ <u>Remarks:</u>

nents just as we did with the potential, the potential, the potential, the potential, the potential, the Born

1

ji(*kµ*)*eil*✓

 $\frac{1}{\sqrt{2}}$

simplifies to

simplifies to

1) To lowest order, T-matrix is independent of potential and partial wave! *q*(*V ^l n*) To lowest order. T-ma *nn*(*q*)*T^l ni*(*kµ*)*eil*✓*.*

2) We obtain previous S-matrix limit.

We will later argue that the error in this approximation \mathcal{C} and the error in this approximation \mathcal{C}

3) The energy dependence is fundamentally different than in conventional 2D scattering. ≈ 1 (*E*) $\sim \frac{1}{i-1} \ln(E/E_{c})$ $\overline{2}$ (*kµ*) ⇡ I V entronal *zD* scattering. p_{max} $\kappa \kappa' \approx T^0(E) \sim \frac{T/T^0}{T^0(E)}$ $\tau = \frac{1}{\pi} \ln (L/L_a)$ \overline{a} the s-wave term. $T^{\bm{k}\bm{k'}}\approx T^0(E)\sim$ 1*/m* $i - \frac{1}{\pi} \ln(E/E_a)$ <u>ا+</u> χ *l* and χ *l* χ *l* and χ *l* χ *l* and χ *l* χ *catter* pendence. Firstly, the *T*-matrix scales as the square root of \mathcal{I} $T^{kk'} \approx T^{0}(E) \sim \frac{1/m}{1}$ state $i - \frac{1}{\pi} \ln(E/E_a)$

4) The energy dependence is that of a 1D T-matrix! *k*¹) The er $\overline{\mathsf{P}}$ [*V ^l* (*k*0*, k*0) + *V ^l*+1(*k*0*, k*0)] +*I^l T^l* (*kµ*) + *^J^l* +*y* 111c ei $\overline{51}$ gy dependence is that of a II $\overline{1}$) The energy dependence is t

$$
T_{--}^l=-\frac{i\delta}{m}+O(\delta^2)
$$

⇡⇤)[*^V ^l*(*k*0*, k*0) + *^V ^l*+1(*k*0*, k*0)] ⁺ *^O*(²)*.*

$$
S^l=\begin{pmatrix}0&-1\\-1&0\end{pmatrix}
$$

person first later \mathcal{I} as the square root scales as the squar

$$
T_{1D} \approx \frac{i}{m} \sqrt{2mE}
$$

Example: Delta-shell V. DELTA-SHELL POTENTIAL POTENTIAL POTENTIAL POTENTIAL POTENTIAL POTENTIAL POTENTIAL POTENTIAL POTENTIAL POTEN

$$
V(r) = V_0 \delta(r - R)
$$

Example: Delta-shell V. DELTA-SHELL POTENTIAL POTENTIAL POTENTIAL POTENTIAL POTENTIAL POTENTIAL POTENTIAL POTENTIAL POTENTIAL POTEN

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$$
V(r) = V_0 \delta(r - R)
$$

Example: Circular Barrier

Conductivity $\sum_{n=1}^{\infty}$ ergy **E**ONOUC the reasons for it are not it are α

*I*⁰ = 2*mV*⁰

ⁱ⇡

 \pm

⇡

❖ Optical theorem gives low-energy cross section:

$$
\sigma \approx \frac{2}{k_0} \sum_{l=-\infty}^{\infty} \frac{\delta_l^{*2} / \delta^2}{1 + \delta_l^{*2} / \delta^2}
$$

 \bullet Semi-classical Boltzmann. ❖ Semi-classical Boltzmann:

$$
0 = \partial_t n_{\mathbf{k}} + \dot{\mathbf{k}} \cdot \nabla_{\mathbf{k}} n_{\mathbf{k}} + \mathbf{v} \cdot \nabla_{\mathbf{r}} n_{\mathbf{k}} - \left(\frac{\partial n_{\mathbf{k}}}{\partial t}\right)_{\text{collisions}}
$$

Conductivity $\sum_{n=1}^{\infty}$ ergy **E**ONOUC the reasons for it are not it are α

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⇡

❖ Optical theorem gives low-energy cross section:

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$$

partial waves (one for *l* and one for *l*) contribute to the ❖ Current density:

$$
\mathbf{j} = -e \sum_{\nu} \int d\phi \int dE \rho_{\nu}(E) n_{\mathbf{k}_{\nu}}(E) \mathbf{v}_{\nu}(E, \phi)
$$

 $\text{\textdegree{}}$ Conductivity: $\sigma_e =$

$$
\text{Conductivity:} \qquad \qquad \sigma_e = \frac{e^2 k_0}{2\pi n_i \sigma}
$$

Conductivity

Conductivity: $\frac{1}{2}$

$$
\sigma_e = \frac{e^2 k_0}{2\pi n_i \sigma}
$$

Summary

- ❖ The low energy limit of a Rashba system contains interesting physics not seen at energies above the Dirac point:
	- ❖ Change in the topology of the Fermi surface (Lifschitz transition).
- ❖ Low energy scattering quantities have a 1D character:
	- ❖ Differential cross sections become confined to a line (incident wave axis).
	- ❖ T matrix has an energy dependence inherent to 1D systems.
- ❖ Low energy T matrix is universal independent of potential features
- * Low energy \neq s-wave!
- ❖ Conductivity displays quantized plateaus.

Thank you!

$$
\psi_{\mu}(\mathbf{r};E) \approx \psi_{\mu}^{\text{in}}(\mathbf{r};E) - \frac{m}{(k_{>} - k_{<})} \sqrt{\frac{2i}{\pi r}} (\sqrt{k_{>}}e^{ik_{>}r}\eta^{-}(\theta_{r})\eta^{-}(\theta_{r})^{\dagger}T^{\mathbf{k}_{>}k}\eta^{-}(0) + i\sqrt{k_{<}}e^{-ik_{<}r}\eta^{+}(\theta_{r})\eta^{+}(\theta_{r})^{\dagger}T^{-\mathbf{k}_{<}\mathbf{k}}\eta^{-}(0))
$$

$$
\begin{aligned}\n\mathbf{r} \\
\mathbf{r} \\
\frac{\mathbf{r}}{2} \\
\frac{\mathbf{r}}{2}\n\end{aligned}
$$

$$
\Psi(r,\theta) = \sum_{l=-\infty}^{\infty} e^{il\theta} \left[a_l \left(\frac{H_l^+(k
$$

 δ