Dynamical spin effects in predicting pion observables

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Overview

1. Light-front wavefunction
2. Pion's special case
3. Dynamical spin wavefunction
4. meson LF Wavefunction with dynamical spin effects.
5. Predictions for radius, EM and transition form factor and DA
6. Summary

Meson holographic LFWF

The meson holographic LFWFs for massless quarks can thus be written in closed form:

$$\Psi_{nL}(\zeta, x, \phi) = e^{iL\phi}\sqrt{x(1-x)}(2\pi)^{-1/2}\kappa^{1+L}\sqrt{\frac{2n!}{(n+L)!}}\zeta^L \exp\left(\frac{-\kappa^2\zeta^2}{2}\right) L_n^L(x^2\zeta^2)$$

$$\Psi^{\pi}(x, \zeta^2) = \mathcal{N}\sqrt{x(1-x)} \exp\left[-\frac{\kappa^2\zeta^2}{2}\right] \exp\left[-\frac{m_f^2}{2\kappa^2 x(1-x)}\right]$$

where $\mathcal{N}$ is a normalization constant fixed by requiring that

$$\int \mathrm{d}^2\mathbf{b}\mathrm{d}x|\Psi^{\pi}(x,\zeta^2)|^2 = P_{q\bar{q}}$$

where $P_{q\bar{q}}$ is the probability that the meson consists of the leading quark-antiquark Fock state.

Universal fundamental AdS/QCD scale κ

AdS/QCD scale κ can be chosen to fit the experimentally measured Regge slopes

- $\kappa = 590$ MeV for pseudoscalar mesons and $\kappa = 540$ MeV for vector mesons. Brodsky, de Teramond, Dosch, Erlich, Phys. Rep. 584, 1 (2015)
- A fit to the HERA data on diffractive ρ electroproduction, with $m_{u/d} = 140$ MeV, gives $\kappa = 560$ MeV. Forshaw, Sandapen, PRL 109, 081601 (2012)
- $\kappa = 550$ MeV (with $m_{u/d}[m_s] = 46[140]$ MeV) leads to a good simultaneous description of the HERA data on diffractive ρ and ϕ electroproduction. Sandapen, Sharma, MA, PRD 94, 074018 (2016)

Pion's LFWF

Earlier applications of LFH with massless quarks, much lower κ were required to fit the pion data:

- $\kappa = 375$ MeV in order to fit the pion EM form factor data
  Brodsky, de Teramond, PRD 77, 056007 (2008)
- $\kappa = 432$ MeV (with $P_{q\bar{q}} = 0.5$) to fit the photon-to-pion transition form factor data simultaneously at large Q^2 and $Q^2 = 0$
  Brodsky, Cao, de Teramond, PRD 84, 075012 (2011)

Higher value of $\kappa = 787$ MeV is also used with $m_{u/d} = 330$ MeV leading to the prediction $P_{q\bar{q}} = 0.279$ from fit to data.
Vega, Schmidt, Branz, Gutsche, Lyubovitskij, PRD 80, 055014 (2009)
When a universal $\kappa = 550$ MeV is used, together with $m_{u/d} = 420$ MeV, $P_{q\bar{q}} = 0.6$ is fixed for the pion only: for the kaon, $P_{q\bar{q}} = 0.8$ and for all other mesons, $P_{q\bar{q}} = 1$
Branz, Gutsche, Lyubovitskij, Schmidt,Vega, PRD 82, 074022 (2010)

Pion's special treatment

More recently with $m_{u/d} = 330$ MeV, a universal $\kappa = 550$ MeV for all meson but fix the wavefunction normalization for the pion so as to fit the decay constant. Consequently, this implies that $P_{q\bar{q}} = 0.61$ only for the pion.

Swarnkar, Chakrabarti, PRD 92, 074023 (2015)

Observation:

- All these previous studies seem to indicate that a special treatment is required at least for the pion either by using a distinct AdS/QCD scale κ or/and relaxing the normalization condition on the holographic wavefunction, i.e. invoking higher Fock states contributions.
- Pion observables are predicted using the holographic wavefunction with the helicity dependence is always assumed to decouple from the dynamics, i.e. the helicity wavefunction is taken to be momentum-independent.

Dynamical spin wavefunction

It is possible to achieve a better description of the pion observables by using a universal AdS/QCD scale κ and without the need to invoke higher Fock state contributions by assuming the helicity dependence of the holographic wavefunction is given as

$$\Psi(x,\mathbf{k}) \to \Psi_{h\bar{h}}(x,\mathbf{k}) = S_{h\bar{h}}(x,\mathbf{k})\Psi(x,\mathbf{k})$$

where $S_{h\bar{h}}(x,\mathbf{k})$ corresponds to the helicity wavefunction for a point-like pseudoscalar meson-$q\bar{q}$ coupling and, in most general form, can be written as

$$S^{\pi}_{h\bar{h}}(x,\mathbf{k}) = \frac{\bar{v}_{\bar{h}}((1-x)P^{+},-\mathbf{k})}{\sqrt{1-x}}\left[(A\not{P} + BM_{\pi})\gamma^{5}\right]\frac{u_{h}(xP^{+},\mathbf{k})}{\sqrt{x}}$$

A and B are constants

Dynamical spin effects

Using the light-front spinors we obtain

$$S^{\pi}_{h\bar{h}}(x,\mathbf{k}) = \left\{ AM_{\pi}^2 + B\left(\frac{m_f M_{\pi}}{x(1-x)}\right)\right\}(2h)\delta_{-h\bar{h}} + B\left(\frac{M_{\pi} k e^{i(2h)\theta_k}}{x(1-x)}\right)\delta_{h\bar{h}}$$

with $\mathbf{k} = ke^{i\theta_k}$

If we take $B = 0$, the helicity wavefunction becomes momentum-independent:

$$S^{\pi}_{h\bar{h}}(x,\mathbf{k}) \to S^{\pi}_{h\bar{h}} = \frac{1}{\sqrt{2}}(2h)\delta_{-h\bar{h}}$$

normalized such that $\sum_{h\bar{h}} |S^{\pi}_{h,\bar{h}}|^2 = 1$

We shall refer to this case as the non-dynamical (momentum-independent) helicity wavefunction.

Spin improved wavefunction

A two-dimensional Fourier transform of our spin-improved wavefunction to impact space gives

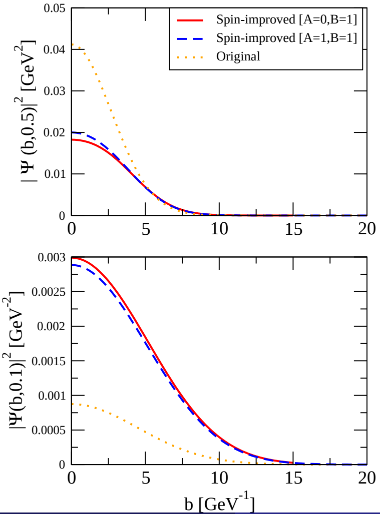
$$\Psi^{\pi}_{h\bar{h}}(x,\mathbf{b}) = \{(Ax(1-x)M_\pi^2 + Bm_f M_\pi)(2h)\delta_{-h\bar{h}} - BM_\pi i\partial_b \delta_{h\bar{h}}\} \frac{\Psi^{\pi}(x,\zeta^2)}{x(1-x)}$$

which can be compared to the original holographic wavefunction,

$$\Psi^{\pi[\mathrm{o}]}_{h\bar{h}}(x,\mathbf{b}) = \frac{1}{\sqrt{2}} h\delta_{-h\bar{h}} \Psi^{\pi}(x,\zeta^2)$$

$\Psi^{\pi}(x,\zeta^2)$ in both of the above equations, is the holographic wavefunction.

Results: Wavefunction



Results: Pion radius

The root-mean-square pion radius is given by:

$$\sqrt{\langle r_\pi^2 \rangle} = \left[\frac{3}{2} \int \mathrm{d}x \mathrm{d}^2\mathbf{b} [b(1-x)]^2 |\Psi^\pi(x, \mathbf{b})|^2 \right]^{1/2}$$

	$\sqrt{\langle r_\pi^2 \rangle}$ [fm]
Original	0.544
Spin-improved $(A = 0, B = 1)$	0.683
Spin-improved $(A = 1, B = 1)$	0.673
Experiment	0.672 ± 0.008

Table: Our predictions for the pion radius using the holographic wavefunction with $\kappa = 523$ MeV and $m_{u/d} = 330$ MeV. Experimental numbers are from PDG 2014

Decay constant

$$\langle 0|\bar{\Psi}_d \gamma^\mu \gamma_5 \Psi_u|\pi^+\rangle = f_\pi P^\mu$$

Taking $\mu = +$ and expanding the left-hand-side we obtain

$$\langle 0|\bar{\Psi}_d \gamma^+ \gamma^5 \Psi_u|\pi^+\rangle = \sqrt{4\pi N_c} \sum_{h,\bar{h}} \int \frac{\mathrm{d}^2\mathbf{k}}{16\pi^3} \mathrm{d}x \Psi^\pi_{h,\bar{h}}(x,\mathbf{k}) \left\{ \frac{\bar{v}_{\bar{h}}}{\sqrt{1-x}} (\gamma^+\gamma^5) \frac{u_h}{\sqrt{x}} \right\}$$

The light-front matrix element in curly brackets can readily be evaluated:

$$\left\{ \frac{\bar{v}_{\bar{h}}}{\sqrt{1-x}} (\gamma^+\gamma^5) \frac{u_h}{\sqrt{x}} \right\} = 2P^+(2h)\delta_{-h\bar{h}}$$

$$\Rightarrow f_\pi = 2\sqrt{\frac{N_c}{\pi}} \int \mathrm{d}x \{A((x(1-x)M_\pi^2) + Bm_f M_\pi\} \frac{\Psi^\pi(x,\zeta)}{x(1-x)}\bigg|_{\zeta=0} .$$

Prediction for pion's decay constant

	f_π [MeV]
Original	161
Spin-improved ($A = 0, B = 1$)	135
Spin-improved ($A = 1, B = 1$)	138
Experiment	$130.4 \pm 0.04 \pm 0.2$

Table: Our predictions for the pion decay constant using the holographic wavefunction with $\kappa = 523$ MeV and $m_{u/d} = 330$ MeV. The datum is from PDG 2014.

Pion EM form factor

Pion EM form factor defined as

$$\langle \pi^+ : P' | J^\mu_{\text{em}}(0) | \pi^+ : P \rangle = 2(P + P')^\mu F_\pi(Q^2)$$

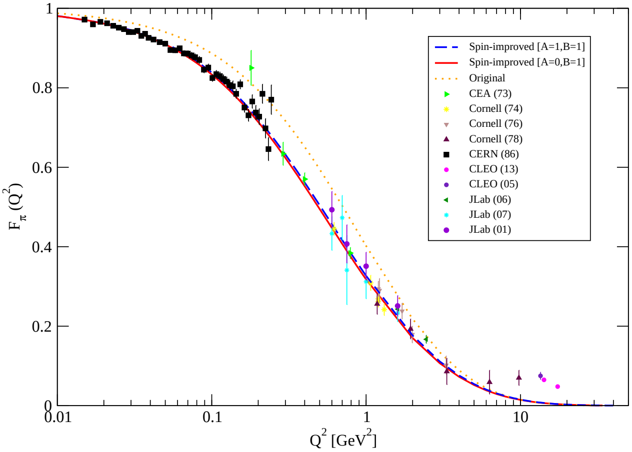
$P' = P + q$, $Q^2 = -q^2$ and the EM current $J^\mu_{\text{em}}(z) = \sum_f e_f \bar{\Psi}(z)\gamma^\mu \Psi(z)$ with $f = \bar{d}, u$ and $e_{\bar{d},u} = 1/3, 2/3$.
The EM form factor can be expressed in terms of the pion LFWF using the Drell-Yan-West formula:

$$F_\pi(Q^2) = 2\pi \int \mathrm{d}x \mathrm{d}b \; b \; J_0[(1-x)bQ] \; |\Psi^\pi(x, \mathbf{b})|^2$$

Note that the above equation implies that $F_\pi(0) = 1$ and that the slope of the EM form factor at $Q^2 = 0$ is related to the mean radius of the pion via

$$\langle r_\pi^2 \rangle = -\frac{6}{F_\pi(0)} \frac{\mathrm{d}F_\pi}{\mathrm{d}Q^2}\bigg|_{Q^2=0}$$

Pion EM form factor-predictions



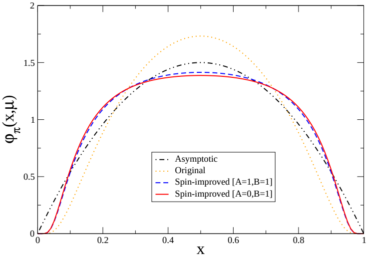
Pion's distribution amplitude

Twist-2 holographic pion DA is defined (at $z^2 = 0$) as

$$\langle 0|\bar{\Psi}_d(z)\gamma^+\gamma_5\Psi_u(0)|\pi^+\rangle = f_\pi P^+ \int \mathrm{d}x e^{ix(P\cdot z)}\varphi_\pi(x,\mu)$$

Proceeding in the same manner as for the decay constant, we can show

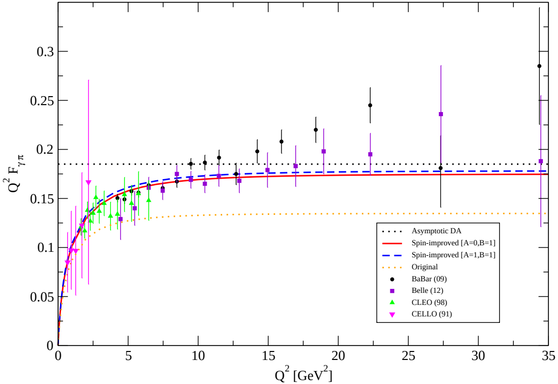
$$f_\pi\varphi_\pi(x,\mu) = 2\sqrt{\frac{N_c}{\pi}}\int \mathrm{d}b J_0(\mu b) b\{A((x(1-x)M_\pi^2) + B m_f M_\pi\}\frac{\Psi^\pi(x,\zeta)}{x(1-x)}\ .$$



Photon-to-pion transition form factor (TFF)

TFF, to leading order in pQCD, is given as

$$F_{\gamma\pi}(Q^2) = \frac{\sqrt{2}}{3} f_\pi \int_0^1 \mathrm{d}x \frac{\varphi_\pi(x, xQ)}{Q^2 x} \; .$$



Summary

- We have accounted for dynamical spin effects in the holographic pion light-front wavefunction and found a significant improvement in the description of pion radius, decay constant, EM form factor and photon-to-pion TFF.
- Our results suggest that it could be possible to have a unified treatment of all light mesons, including the pion, with a universal fundamental AdS/QCD scale which fits the baryon and meson Regge slopes.
- This suggests that $(A = 0, 1; B = 1)$, does bring a significant improvement in the description of all available experimental data without necessarily having to use a much smaller AdS/QCD scale and/or invoke higher Fock states contributions exclusively for the pion. Our findings thus support the idea of the emergence of a universal AdS/QCD confinement scale κ.

LFWF

$$H_{\mathrm{QCD}}^{\mathrm{LF}}|\Psi(P)\rangle = M^2|\Psi(P)\rangle$$

where $H_{\mathrm{QCD}}^{\mathrm{LF}} = P^+P^- - P_\perp^2$ is the LF QCD Hamiltonian and M is the hadron mass. At equal light-front time ($x^+ = 0$) and in the light-front gauge $A^+ = 0$, the hadron state $|\Psi(P)\rangle$ admits a Fock expansion, i.e.

$$|\Psi(P^+, \mathbf{P}_\perp, S_z)\rangle = \sum_{n,h_i} \int [\mathrm{d}x_i][\mathrm{d}^2\mathbf{k}_{\perp i}] \frac{1}{\sqrt{x_i}} \Psi_n(x_i, \mathbf{k}_{\perp i}, h_i)|n : x_i P^+, x_i \mathbf{P}_\perp + \mathbf{k}_{\perp i}, h_i\rangle$$

where $\Psi_n(x_i, \mathbf{k}_{\perp i}, h_i)$ is the LFWF of the Fock state with n constituents and the integration measures are given by

$$[\mathrm{d}x_i] \equiv \prod_i^n \mathrm{d}x_i \delta(1 - \sum_{j=1}^n x_j) \qquad [\mathrm{d}^2\mathbf{k}_i] \equiv \prod_{i=1}^n \frac{\mathrm{d}^2\mathbf{k}_i}{2(2\pi)^3} 16\pi^3 \delta^2(\sum_{j=1}^n \mathbf{k}_i) \, .$$

Holographic Schrödinger equation

The valence meson LFWF can then be written in a factorized form:

$$\Psi(\zeta, x, \phi) = e^{iL\phi}\mathcal{X}(x)\frac{\phi(\zeta)}{\sqrt{2\pi\zeta}}$$

It can then be shown that reduces to a 1-dimensional Schrödinger-like wave equation for the transverse mode of LFWF of the valence ($n = 2$ for mesons) state, namely:

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right)\phi(\zeta) = M^2\phi(\zeta)$$

the potential is given by

$$U(z_5, J) = \frac{1}{2}\varphi''(z_5) + \frac{1}{4}\varphi'(z_5)^2 + \left(\frac{2J-3}{4z_5}\right)\varphi'(z_5)$$

where $\varphi(z_5)$ is the dilaton field which breaks conformal invariance in AdS space. A quadratic dilaton, $\varphi(z_5) = \kappa^2 z_5^2$, profile results in a light-front harmonic oscillator potential in physical spacetime:

$$U(\zeta, J) = \kappa^4\zeta^2 + \kappa^2(J-1)$$

Solutions to holographic Schrödinger equation

With the confining potential specified, one can solve the holographic Schrödinger equation to obtain the meson mass spectrum,

$$M^2 = 4\kappa^2 \left(n + L + \frac{S}{2} \right)$$

which, as expected, predicts a massless pion. The corresponding normalized eigenfunctions are given by

$$\phi_{nL}(\zeta) = \kappa^{1+L} \sqrt{\frac{2n!}{(n+L)!}} \zeta^{1/2+L} \exp\left(\frac{-\kappa^2 \zeta^2}{2} \right) L_n^L(x^2 \zeta^2) \ .$$

To completely specify the holographic meson wavefunction, we need the analytic form of the longitudinal mode $\mathcal{X}(x)$. This is obtained by matching the expressions for the pion EM or gravitational form factor in physical spacetime and in AdS space. Either matching consistently results in $\mathcal{X}(x) = \sqrt{x(1-x)}$

Vector meson case

For vector mesons, the helicity wavefunction is similar to that of the point-like photon-$q\bar{q}$ coupling, i.e.

$$S^V_{h\bar{h}}(x,\mathbf{k}) = \frac{\bar{v}_{\bar{h}}((1-x)P^+,-\mathbf{k})}{\sqrt{(1-x)}}[\gamma\cdot\epsilon_V]\frac{u_h(xP^+,\mathbf{k})}{\sqrt{x}}$$

where ϵ^μ_V is the polarization vector of the vector meson

- Substituting ϵ^μ_V by the photon polarization vector leads to the well-known photon light-front wavefunctions.
- This assumption for the helicity structure of the vector meson is very common when computing diffractive vector meson production in the dipole model. PRD.94.074018(2016)

Pion's distribution amplitude-Comparison with lattice QCD and Sum Rules

$$\langle \xi_n \rangle = \int_0^1 \mathrm{d}x (2x-1)^n \varphi_\pi(x,\mu)$$

$$\langle x^{-1} \rangle = \int_0^1 \mathrm{d}x \frac{\varphi_\pi(x,\mu)}{x}$$

DA	μ [GeV]	$\langle \xi_2 \rangle$	$\langle \xi_4 \rangle$	$\langle x^{-1} \rangle$
Asymptotic	∞	0.2	0.085	3
LFH spin-improved $(A=1, B=1)$	~ 1	0.195	0.076	2.74
LFH spin-improved $(A=0, B=1)$	~ 1	0.199	0.078	2.76
LFH (original)	~ 1	0.151	0.050	2.50
LF Quark Model	~ 1	0.24[0.22]	0.11[0.09]	
Sum Rules	1	0.24	0.11	
Renormalon model	1	0.28	0.13	
Instanton vacuum	1	$0.22, 0.21$	$0.10, 0.09$	
Lattice	2	$0.2361(41)(39), 0.27 \pm 0.04$		
NLC Sum Rules	2	$0.248^{+0.016}_{-0.015}$	$0.108^{+0.05}_{-0.03}$	$3.16^{+0.09}_{-0.09}$
Sum Rules	2	0.343	0.181	4.25
Dyson-Schwinger[RL,DB]	2	$0.280, 0.251$	$0.151, 0.128$	$5.5, 4.6$
Platykurtic	2	$0.220^{+0.009}_{-0.006}$	$0.098^{+0.008}_{-0.005}$	$3.13^{+0.14}_{-0.10}$