On the convection of ionospheric density features

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Assumptions

- 2-D plasma
- Closed field lines
- Steady state
- Cold plasma
- Currents don’t make significant changes in $B$
- Single ion species
- Fully-magnetised electrons
- No neutral drift (or $E$ measured in neutral frame)
meridional plane

perpendicular plane

2-D

perpendicular plane
\[ \rho_1 = n \rho_0 \]

\[ \delta E \]

\[ E_0 \]

\[ J \]

\[ E_1 \]
\[ \rho_s \hat{n} \cdot (v_s - v_b) \bigg|_{\text{ext}} = \rho_s \hat{n} \cdot (v_s - v_b) \bigg|_{\text{int}} \]

\[ \hat{n} \cdot v_b = \hat{n} \cdot \left( n v_{s,\text{int}} - v_{s,\text{ext}} \right) \frac{1}{(n - 1)} \]
\[ v_b = \frac{\cos \phi_s}{B} R_s E_0 - \frac{(n+1) \eta \sin \phi_s \cos \phi_s}{(n-1)B \sigma_P} M_s H^{-1} \left[ \sigma \right] E_0 \]

\[ = v_{s,0} - \frac{\sin \phi_s \cos \phi_s}{B \sigma_P} M_s H^{-1} J_0 \]  

(14)

where the first term on the RHS is the background drift of the species, and we have used \( J = [\sigma] E \). We now use \( J = q_i n_i (v_i - v_e) \); \( \sigma_P = q_i n_i \kappa_i / B (1 + \kappa_i^2) \); and \( \kappa_i = \cot \phi_i \) to get (dropping the subscript 0 now since all quantities except \( \eta \) are background values)

\[ v_b = v_s - \frac{\sin \phi_s \cos \phi_s}{\sin \phi_i \cos \phi_i} M_s H^{-1} (v_i - v_e) \]

\[ = v_s + \frac{\sin \phi_s \cos \phi_s}{\sin \phi_i \cos \phi_i} \left[ \begin{array}{cc} 1 & \eta \kappa_s \\ -\eta \kappa_s & 1 \end{array} \right] \left[ \begin{array}{cc} 1 & -\eta / \kappa_i \\ \eta / \kappa_i & 1 \end{array} \right]^{-1} (v_e - v_i) \]

\[ = v_s + \frac{\sin \phi_s \cos \phi_s}{\sin \phi_i \cos \phi_i} \left[ \begin{array}{cc} 1 & \eta \kappa_s \\ -\eta \kappa_s & 1 \end{array} \right] \frac{\kappa_i}{\kappa_i^2 + \eta^2} \left[ \begin{array}{cc} \kappa_i & \eta \\ -\eta & \kappa_i \end{array} \right] (v_e - v_i) \]

\[ = v_s + \frac{\sin \phi_s \cos \phi_s}{\sin^2 \phi_i (\kappa_i^2 + \eta^2)} \left[ \begin{array}{cc} 1 & \eta \kappa_s \\ -\eta \kappa_s & 1 \end{array} \right] \left[ \begin{array}{cc} \kappa_i & \eta \\ -\eta & \kappa_i \end{array} \right] (v_e - v_i) \]  

(15)

This equation ought to yield the same answer for either ions or electrons. We first demonstrate this for \(|\eta| \ll 1\).
\[ \alpha \approx \frac{(n - 1)}{\sin 2\phi_i} \]
\[ \eta = \frac{n - 1}{n + 1} \]
\[ \tan \beta = \frac{\eta}{\kappa_i} \]
\[ \kappa_{\text{eff.}} = \frac{\Sigma_P}{\Sigma_H} \]

- \( \kappa_i = 0.1 \)
- \( \kappa_i = 0.5 \)
- \( \kappa_i = 1 \)
- \( \kappa_i = 2 \)
- \( \kappa_i = 10 \)
Removing neutral collisions

• Next two figures:

• Slab geometry: same result for $E$ as before, although for different reasons.

• Circular geometry: the patch (or depletion) still suffers a polarisation, but the dipole axis is exactly aligned with $E$. Otherwise identical results for patch drift.
\[ \chi_e = \frac{\rho_m}{\varepsilon_0 B^2} = \frac{c^2}{V_A^2} \]
Conclusions

• The treatment of a collisionless plasma as having an electric susceptibility yields results consistent with the $\kappa \rightarrow \infty$ limit of a conducting plasma.

• While an open flux tube may have a uniform electric field “imposed” on it, plasma on closed flux tubes will experience a structuring of the electric field that depends on mass density features.

• A non-conducting plasma can have a free-charge distribution, concomitant with an arbitrary initial 2-D flow field. But a conducting plasma has a unique steady state.

• For a circular density feature, a dipolar surface charge with appropriate magnitude and orientation yields a divergence-free current field.

• The boundary of a circular density feature retains a circular shape, and its electrons, although it does not “own” a particular parcel of ions.

• The boundary should convect with a velocity given by the expression found – always slower than ambipolar for an enhancement and usually faster for a depletion.
Questions?
detail from Zhang et al. 2013 Fig. 2.C.