Automation of multi-leg one-loop virtual amplitudes

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Program

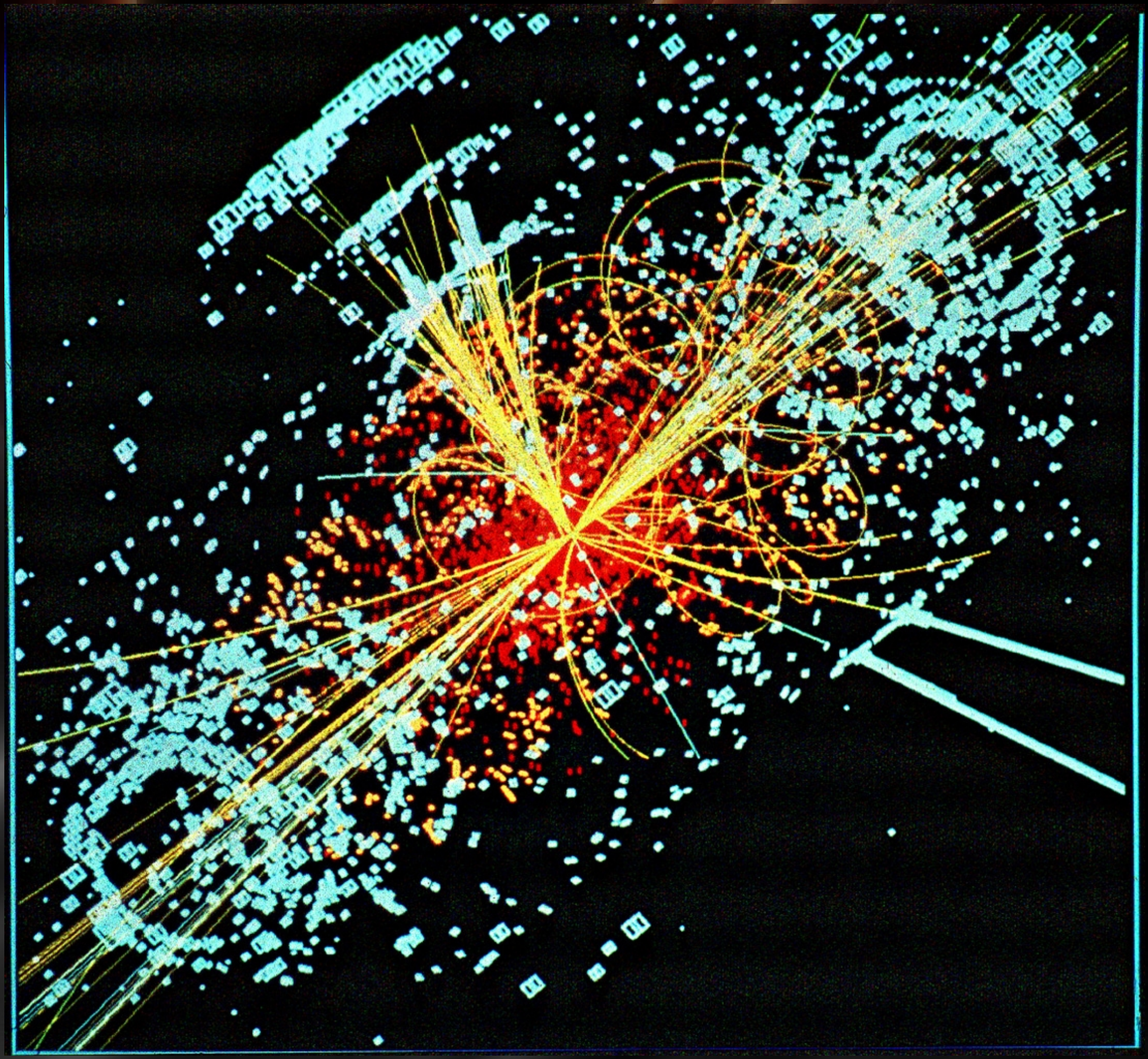
- NLO corrections
  - Real
  - Virtual
- Virtual part
  - Feynman diagrams
  - OPP
  - Unitarity based
- Les Houches Accord
- Computing aspects

Theory predictions

- Collider experiments need theory predictions
  - Signal
  - Background
- Many measurements limited by theory
- Good understanding of SM background mandatory

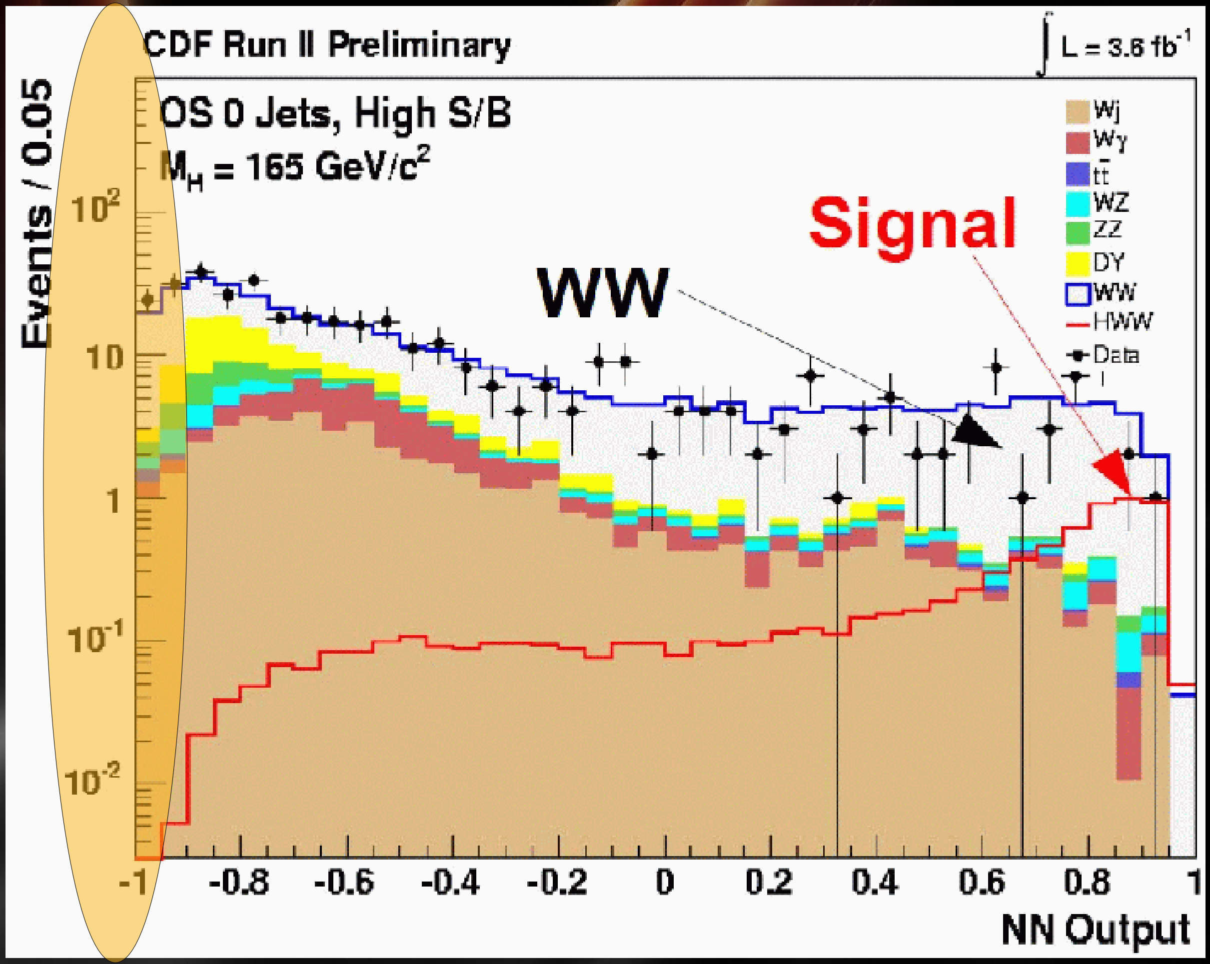
Signals are hard to see

- Large backgrounds



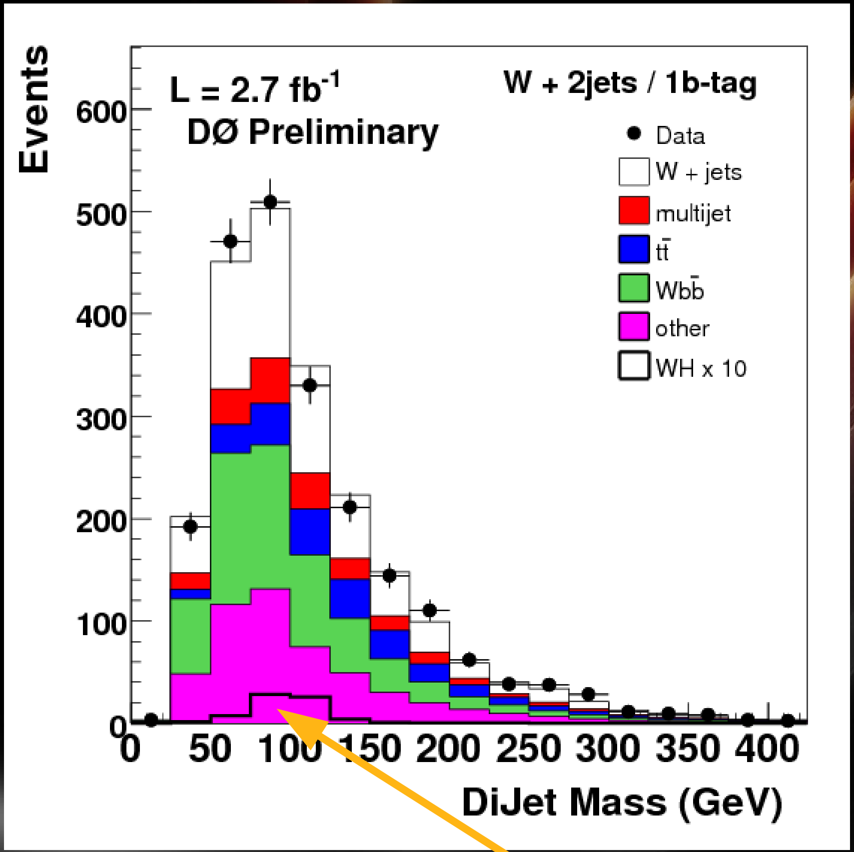
Motivation

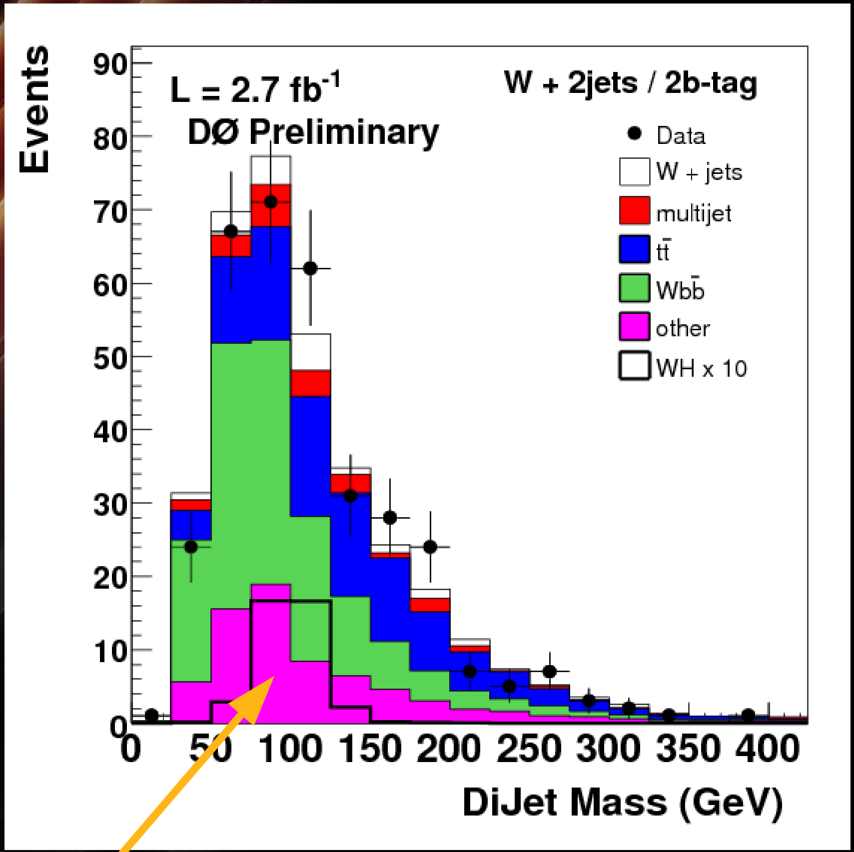
Higgs →WW search @ CDF



Motivation

Higgs associated production WH ($H \to b\bar{b}$)

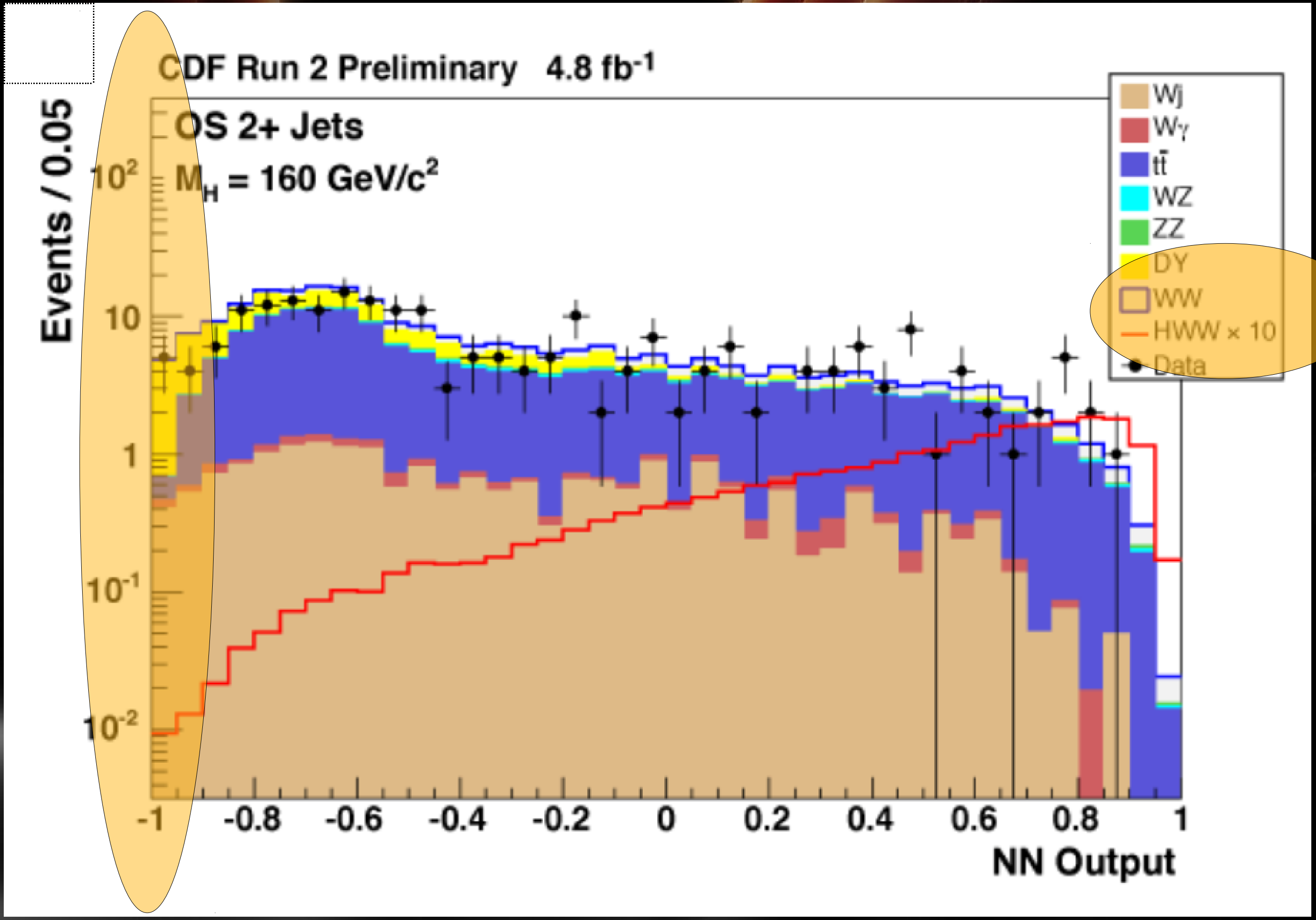




Signal x 10

Motivation

Higgs search HWW



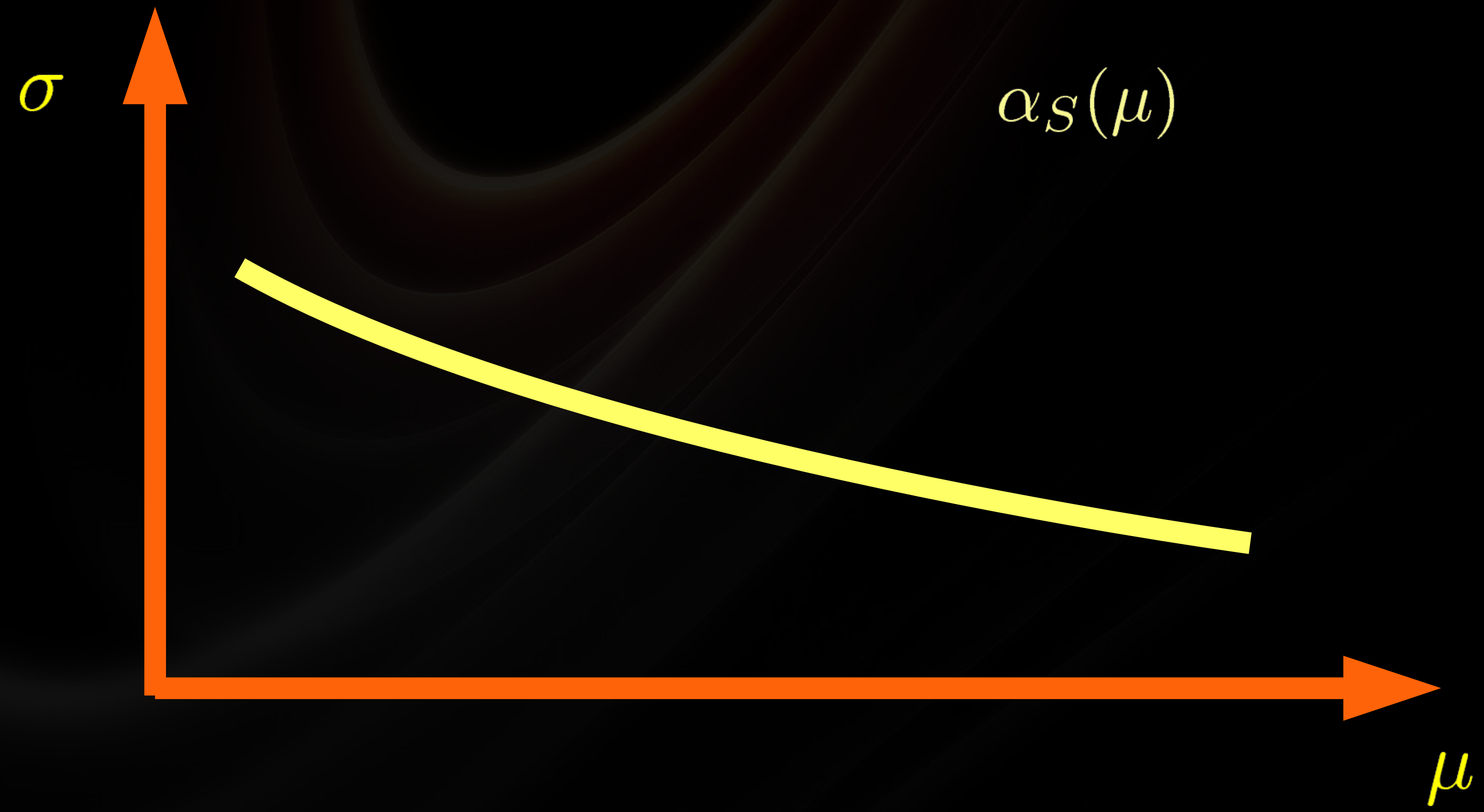
Leading order

Lots of good general tools for leading order cross sections (Madgraph, Herwig, Sherpa, Alpgen, Whizard, Pythia, …)

- Highly automated tools
- Possible improvements
  - Parton shower
  - Matrix element matching
  - Resummation (N....LL)
  - More orders in perturbation theory (N....LO)

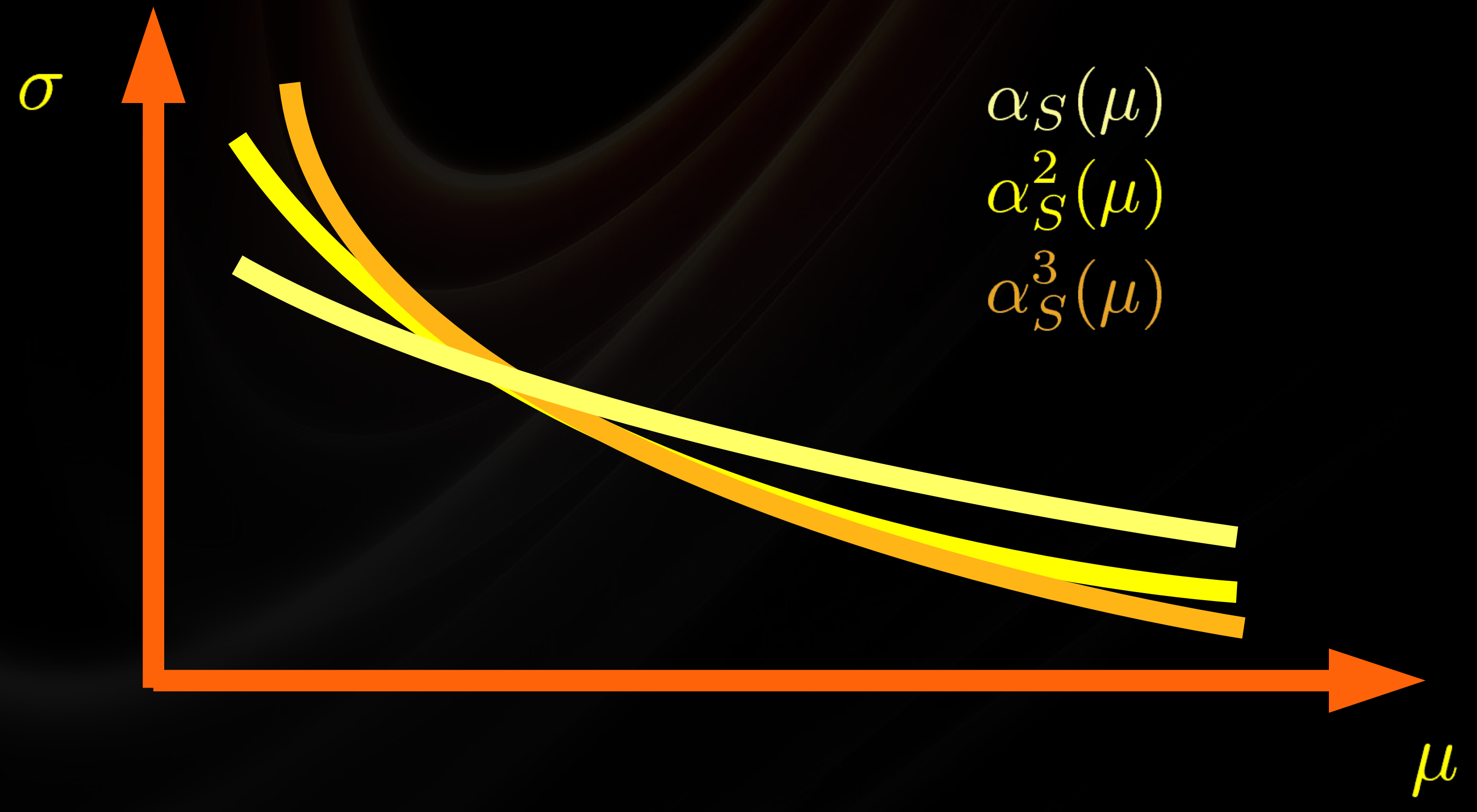
Renormalization scale dependence

- Coupling constant depends on an unphysical scale



Renormalization scale dependence

- Scale dependence increases with number of jets



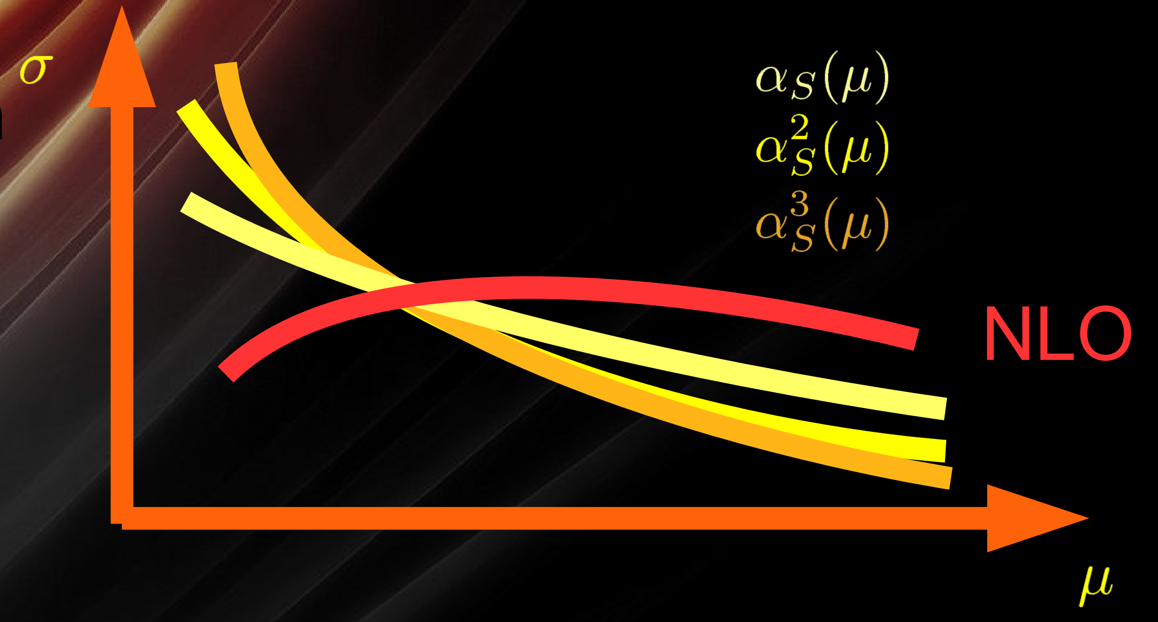
NLO Corrections

NLO corrections are needed for a good theoretical understanding of QCD processes

Improve theory prediction for

- Absolute normalization
- Reduce renormalization scale dependency

Number of jets	LO	NLO
1	16%	7%
2	30%	10%
3	42%	12%



- Corrections can be very large
- Shape of distributions

Theory prediction

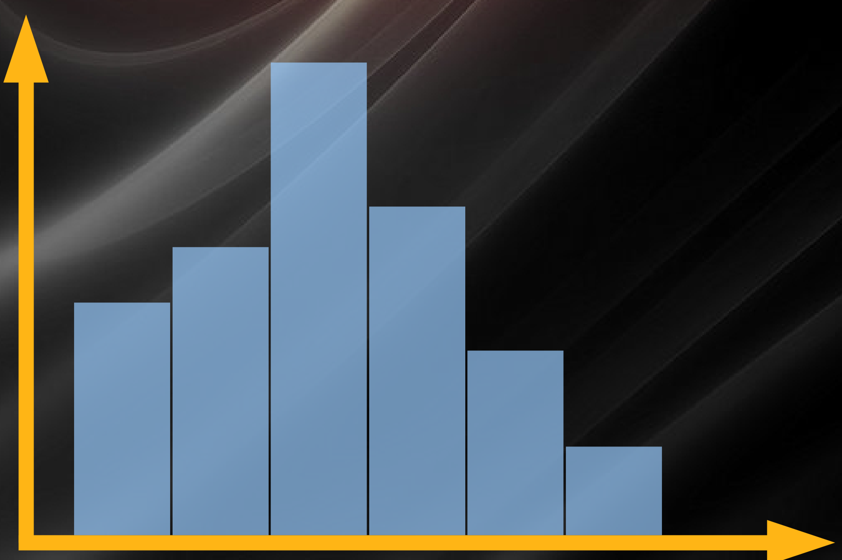
- Generate a phase-space configuration with n final state particles

$$p_1, \ldots, p_n$$

- Compute value of the observable(s) and weight

$O(p_1, \ldots, p_n)$ $\quad$ $W(p_1, \ldots, p_n)$

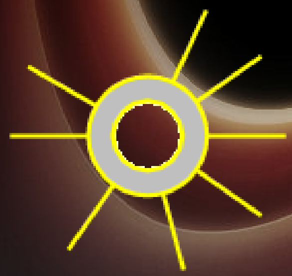
- Bin



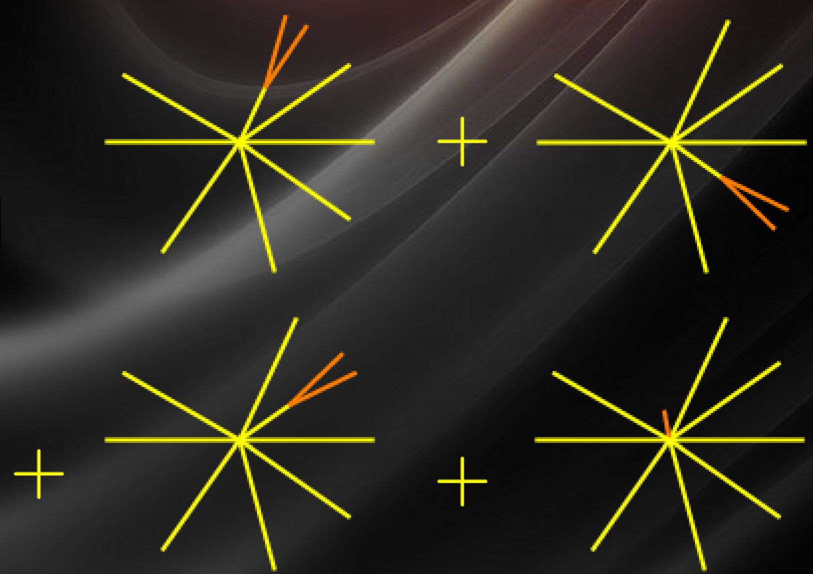
NLO Corrections

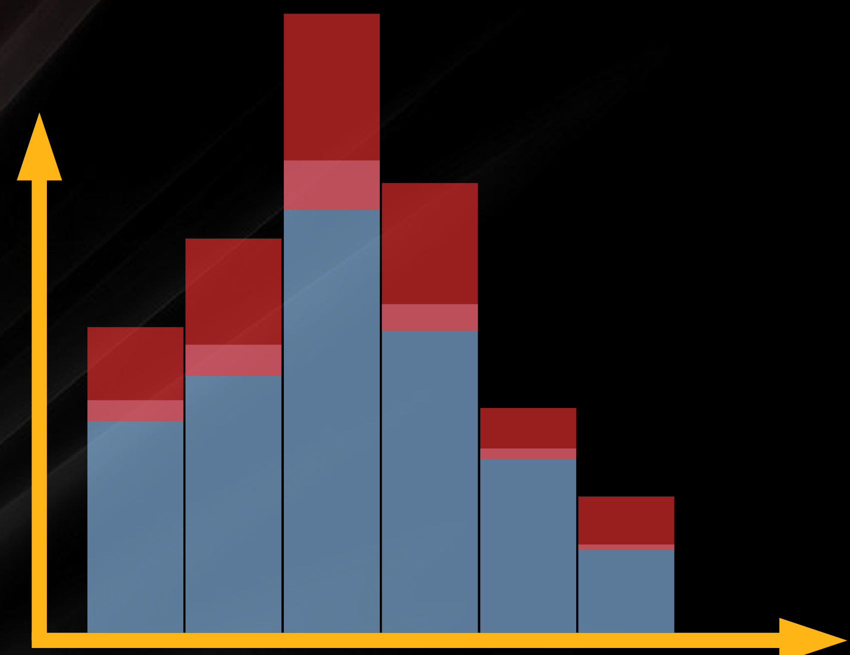
Consider (infrared safe) observable and add contributions that have an higher order in perturbation theory

Virtual



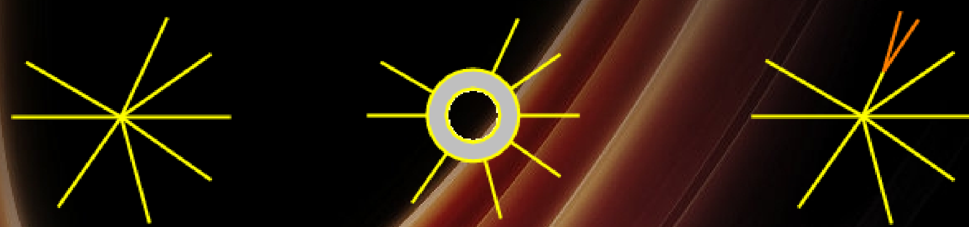
Real





NLO Corrections

NLO Cross section:



$$\sigma_n^{NLO} = \int_n \sigma_n^{tree} + \int_n \sigma_n^{virt} + \int_{n+1} \sigma_{n+1}^{real}$$

- Real & virtual corrections have infrared divergences
  - Virtual part has explicit divergences
  - Integral of the real part is divergent when particles become soft or collinear
- Combination is free of divergences

Real Correction

- Different techniques
  - Catani-Seymour
  - Frixione-Kunszt-Signer
  - Phase-space slicing
  - Antenna subtraction
  - …
- Automated

Automated implementations

- Different automated implementations
  - TevJet [Seymour,Tevlin]
  - Sherpa [Gleisberg,Krauss]
  - MadDipole [Frederix,Gehrmann,Greiner]
  - AutoDipole [Hasegawa,Moch,Uwer]
  - Dipoles [Czakon,Papadopoulos,Worek]
  - MadFKS [Frederix,Frixione,Maltoni,Stelzer]
  - POWEG BOX [Alioli,Oleari,Nason,Re]
  - ...

Virtual Correction

- Is the current bottleneck (from the automation point of view)
- Methods
  - Feynman Diagrams+tensor integral reduction
  - OPP
  - Unitarity

Standard integral reduction

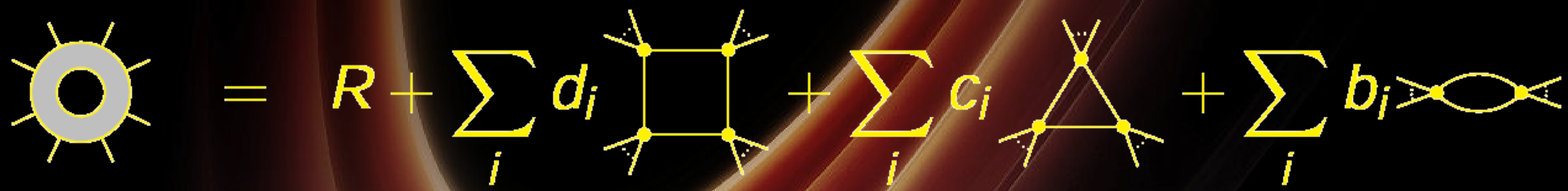
- The One-loop amplitude is the sum of a large number of Feynman diagrams
- Each of these Feynman diagrams is composed of a lot of tensor integrals
- Each tensor integral can be written in terms of scalar integrals
- To find the coefficients a lot of computer algebra has to be performed

Standard integral reduction

- Coefficients of the scalar integral are generally
  - Very large analytical expressions
  - Have numerical instabilities due to Gram determinants
- These problem can be addressed
- $pp \to t\bar{t}b\bar{b}$ [Bredenstein,Denner,Dittmaier,Pozzorini]
- $q\bar{q} \to b\bar{b}b\bar{b}$ [Golem: Binoth,Greiner,Guffanti,Guillet,Reiter,Reuter]
- ...

One-loop decomposition

A one-loop amplitude can be written in terms of scalar integrals



Scalar integrals are known

Coefficients are rational polynomials of spinor products

To compute one-loop integral, it is enough to compute the coefficients of the scalar integrals

OPP

- Reduction at the integrand level
  [del Aguila,Pittau;Ossola,Papadopoulos,Pittau]
- Form of the integrand is known →
  - Make an ansatz for the unintegrated amplitude

$$\mathcal{A} = \int dl \sum c_i T_i(l)$$

  - Do once for all the tensor reduction for the tensor structures *T*

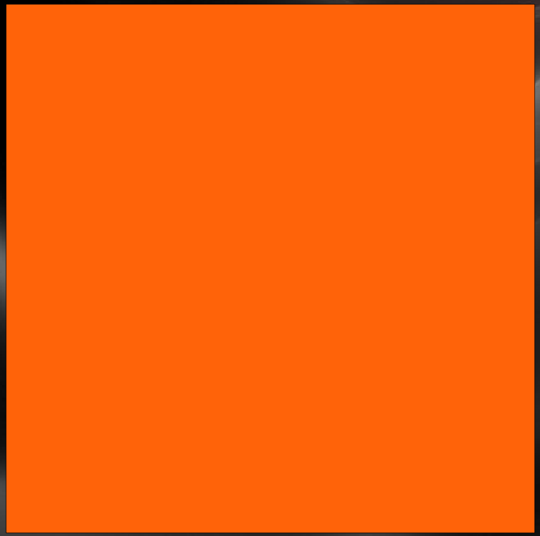
$$\int dl T_i(l) = \begin{cases} 0 \\ \sum_j S_j \end{cases}$$

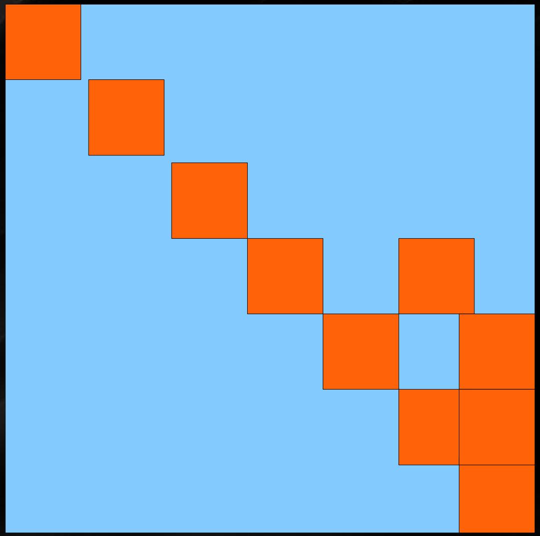
OPP [Ossola,Papadopoulos,Pittau]

- Evaluate the integrand at some points to find the coefficients of the ansatz

$$\mathcal{A} = \int dl \sum c_i T_i(l)$$

- Can choose the points in such a way that the system to solve is manageable





Application of the OPP

- pp $\rightarrow$ ZZZ,WWZ,WZZ,WWW [Binoth,Ossola,Papadopoulos,Pittau]
- $e^+e^- \rightarrow e^+e^-\gamma$ [Actis,Mastrolia,Ossola]
- HELAC-1L [van Hameren,Papadopoulos,Pittau]
  - 1 PS point for all NLO processes in the Les Houches Wishlist
  - $pp \rightarrow t\bar{t}b\bar{b}$ [Bevilacqua,Czakon,Papadopoulos,Pittau,Worek]
  - $pp \rightarrow t\bar{t}jj$ [Bevilacqua,Czakon,Papadopoulos,Worek]
- Cuttools [Ossola,Papadopoulos,Pittau]

Generalized Unitarity

- Can obtain the coefficient of the scalar integrals
- Use factorization properties of the amplitude
- Use complex momenta [Britto,Cachazzo,Feng]
- Compute coefficients with “cuts”
- Cut can be seen as a projector onto structures that have a given set of propagators

Unitarity cut

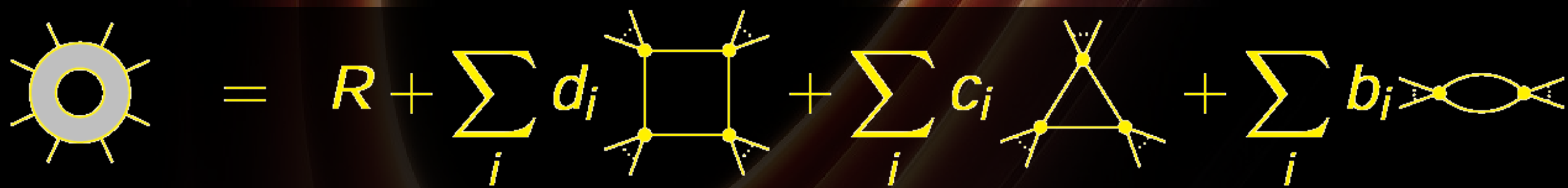
- Replacement under the loop integral propagator → delta function

$$\frac{1}{P^2} \to 2\pi i \delta(P^2)$$

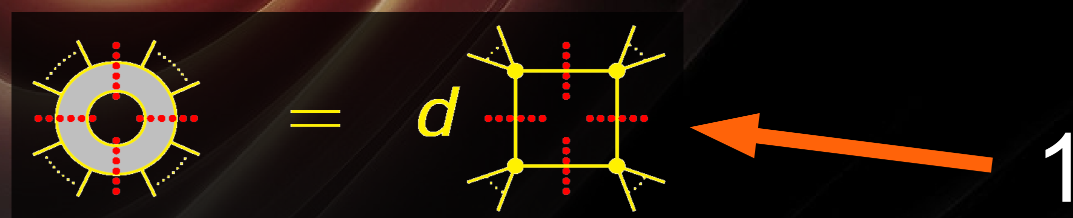
- Can apply more than one cut
  - Double cut
  - Triple cut
  - Quadruple cut
- Only possible in general with complex momenta

Unitarity cut

- One-loop decomposition



- Quadruple cut is a projector



- Quadruple Cut breaks the one-loop amplitudes in a product of tree amplitudes

Quadruple cut

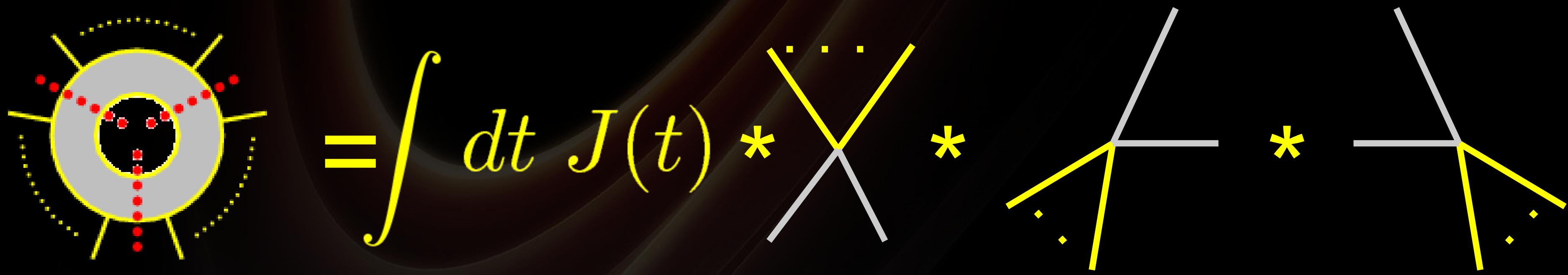
- The box coefficient is

$$d = \sum A_1 A_2 A_3 A_4$$

- Given in terms of on-shell trees
  - No gauge dependence
  - Compact expressions
  - Numerically stable

Triple cut

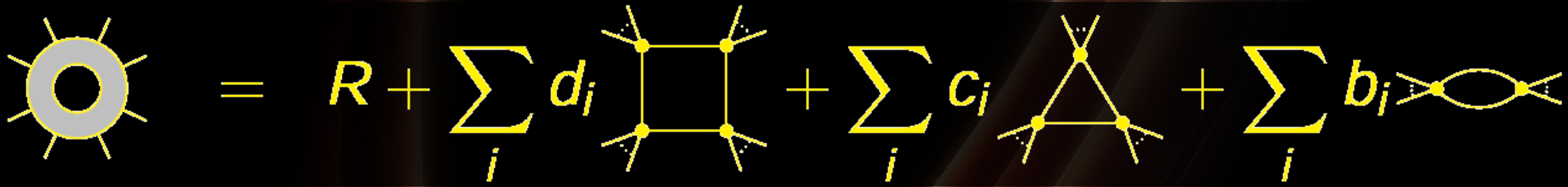
- Triple cut breaks the one-loop amplitudes in a product of tree amplitudes



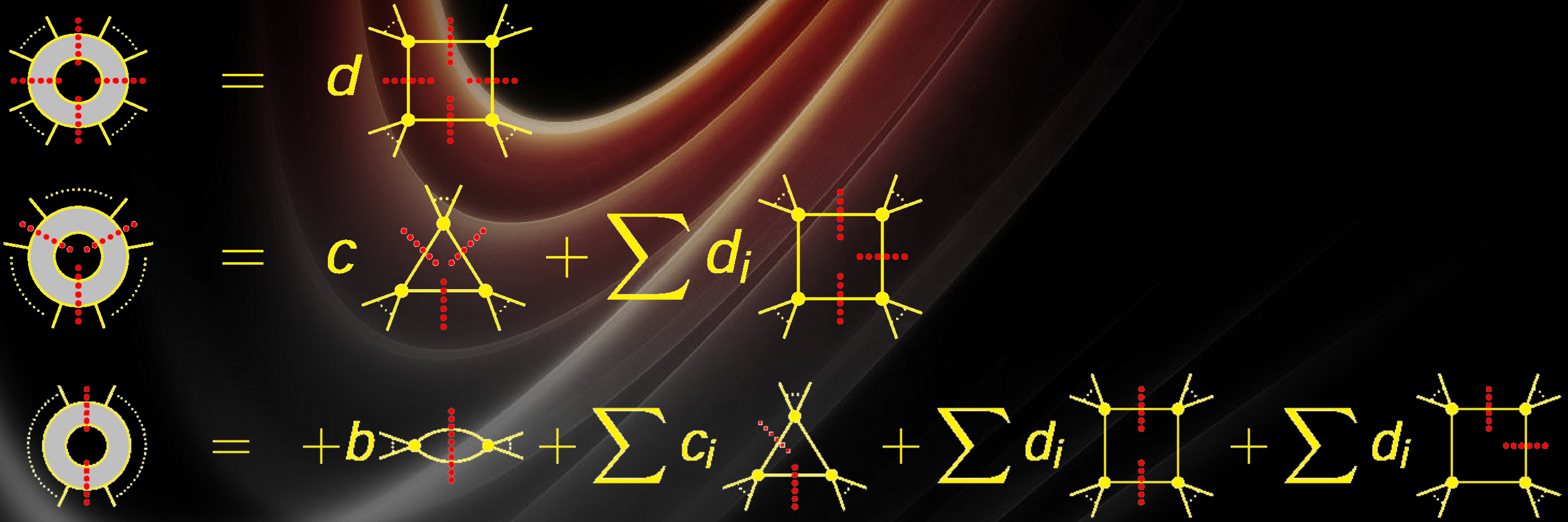
$$\int d^4k \; \delta(P_1)\delta(P_2)\delta(P_3) = \int dt J(t)$$

We know the structure of the integrand
→ can extract the relevant information by sampling different points (choices of *t*)
[Forde]

Generalized Unitarity



- Can obtain the coefficient of the scalar integrals



- Need to compute *R* by other means

Cuts in practice

Given external momenta configuration:

- Generate loop momenta configurations that satisfy the cut conditions (complex momenta)
- For each configuration, compute and multiply the trees at the corner of the cut diagram
- Combine the results appropriately

All the integral coefficients

effectively reduce a loop computation to tree computation

Different types of unitarity

- 4 Dimensional (A = C + R)
  - Recursion relations
  - Special Feynamn diagrams
    - [Draggiotis,Garzelli,Malamos,Papadopoulos,Pittau]
    - [Xiao,Yang,Zhu]
- D-Dimensional
  - Use different dimensions (C(D=D1) , C(D=D2))

    [Ellis,Giele,Kunszt,Melnikov,Zanderighi]
  - Stay in 4 Dimensions and emulate the additional dimensions as an additional mass in the propagators [Badger]

Recent applications

- W+3 jets
  - Full color, BlackHat+Sherpa [Berger,Bern,Dixon,Febres Cordero,Forde,Gleisberg,Ita,Kosower,DM]
  - Leading color approximation, ROCKET [Ellis,Melnikov,Zanderighi]
- $t\bar{t}$ + jet
  - ROCKET [Ellis,Giele,Kunszt,Melnikov]

Unitarity vs FD

Preferences (not restrictions)

- Feynman diagrams
  - More Masses
  - Less jets
  - More EW
- Unitarity
  - More massless
  - More jets
  - Less EW

Approaches are complimentary

Automation

- Real part already automated
- Virtual part automation
  - Golem [Binoth,Guffanti,Guillet,Heinrich,Karg,Kauer,Pilon,Reiter,Reuter]
  - Feynarts [Hahn]
  - ROCKET [Ellis,Kunszt,Melnikov,Zanderighi]
  - BlackHat [Berger,Bern,Dixon,Febres Cordero,Forde,Ita,Kosower,DM]
  - In fact all groups ...
- Les Houches Accord

Binoth Les Houches Accord

- Tree or tree-like

loop

Monte Carlo

Sherpa
MadFKS
POWHEG
MadEvent
...

Tree

Real part
subtraction
integrated subtraction

BLHA

Virtual

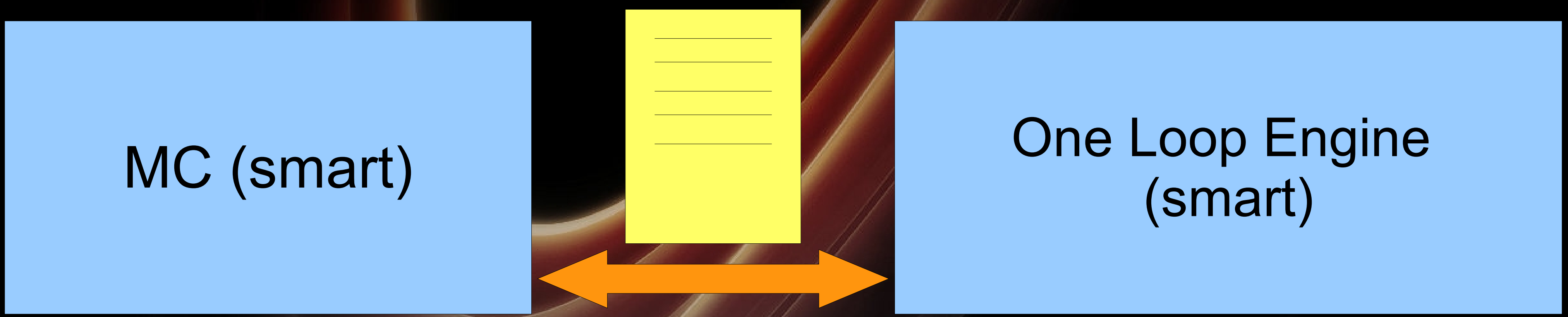
Aim: Standardise the communication

→ easier to use different 1-loop providers

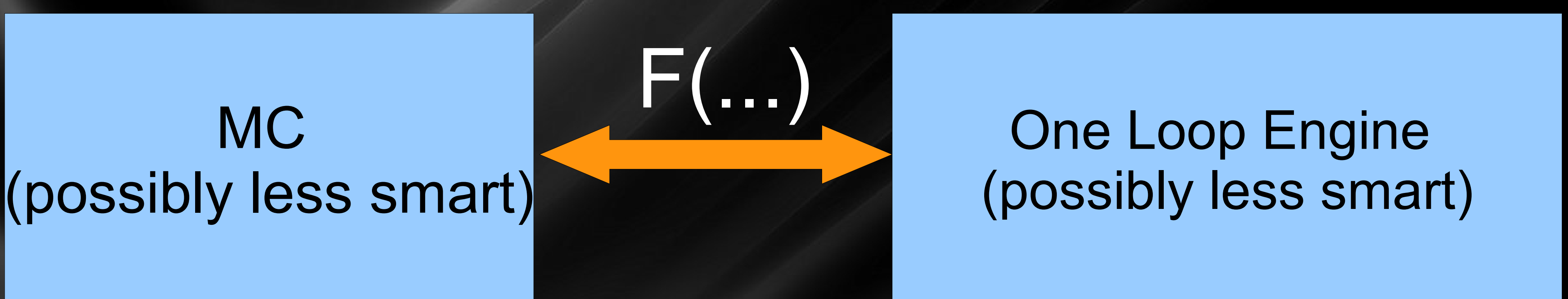
→ easier to compare 1-loop programs

Binoth Les Houches Accord

- Negotiation phase



- Run-time phase



Computer aspects

- Mostly for BlackHat+Sherpa, but issues are in general common to other automated methods

Challenges

- Real
  - More points
  - Larger multiplicity
  - Easier computation
- Virtual
  - Fewer points
  - Smaller multiplicity
  - More complicated computation

Computational needs

- Many separate runs

- Large number of PS points
  - Depend on precision
  - ~ 1G events for real part
  - ~ 100M events for virtual part

Embarrassingly parallel

Timing (Virtual)

- W+3 jets @ LHC or Tevatron
- Order of magnitude: ~10s per PS point
- Use approximation
  - LC: faster, ~90% of contribution
  - Full-LC: slower ~10% contribution
- Compute LC more often
  → less statistical error for fixed CPU time
  or
  less CPU time for fixed statistical error

Treatment of numerical instabilities

- Inherent numerical stabilities in 1-loop computations
- Unitarity:
  - Use higher precision library QD [Bailey,Hida,Li]
    - Use it only when necessary
    - Automatic diagnostic
    - Advantages
      - Same method, no need for special case
      - No need to know a priori when precision is endangered
      - Free accuracy test
- Feynam Diagrams: dedicated evaluation path

Conclusion

- A lot of progress has been made in the field of NLO computations
- Automation of one-loop amplitudes is getting close
- NLO-accuracy predictions for $2 \rightarrow 4$ processes are getting available