Track and vertex reconstruction

From classical to adaptive methods

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Outline

Introduction

What is it all about?

- Track and vertex reconstruction are essential steps in the data analysis chain
- Crucial factor in quality of physics analysis
- Growing importance for high-level trigger
- T **Track reconstruction:** determine location, direction and (inverse) momentum of charged tracks
- V **Vertex reconstruction:** determine location of interaction point and momenta of participating tracks
- T V Many basic features in common

Introduction

Ingredients

Track reconstruction

- ☺ Track model known (analytical or numerical)
- ☺ Observation errors known
- ☺ Process noise known (approximately)
- ☹ Assignment of observations to tracks unknown
- ☹ Hits from background tracks, electronic noise
- ☹ Mass not always known

Vertex reconstruction

- ☺ Track model known (analytical or numerical)
- ☺ Track errors known
- ☺ No process noise
- ☹ Assignment of tracks to vertices unknown
- ☹ Background tracks

Introduction

Three aspects

- **Pattern recognition**
  - T Find out which detector hits belong to the same track — highly detector dependent
  - V Find out which tracks are produced at the same interaction point — nearly detector independent
- **Estimation**
  - T Estimate track parameters at one or several points along the track
  - V Estimate vertex location and momenta of attached tracks
  - T V Can be formulated as extended Kalman filter in both cases
- **Test**
  - T Test track hypothesis and reject outlying detector hits
  - V Test vertex hypothesis and reject outlying tracks

Introduction

Classical vs. Adaptive

- **Classical approach**:
  - Do pattern recognition (track/vertex finding)
  - Submit track/vertex candidate to least-squares fit
  - Inspect test quantities, identify outlying hits/tracks
  - Remove outliers, repeat fit
  - ...
- **Adaptive approach**:
  - Do preliminary pattern recognition or none at all
  - Submit hit/track collection to adaptive fit
  - Inspect posterior weights of hits/tracks and remove outliers

Track finding

Global vs. local

- Rough distinction: **local/sequential** and **global/parallel** methods
- **Local** method: generate seeds and complete them to track candidates
- **Global** method: simultaneous clustering of detector hits into track candidates

Track finding: Local methods

Some local methods

- Track road
- Track following
- Progressive track finding

Track finding: Local methods

Track road

- Select two or three hits
- Compute approximative track plus tolerance
- Pick up hits inside the tolerance

Track following

- Construct initial track segment (seed)
- Extrapolate seed
- Pick up matching hits
- Repeat until last detector layer

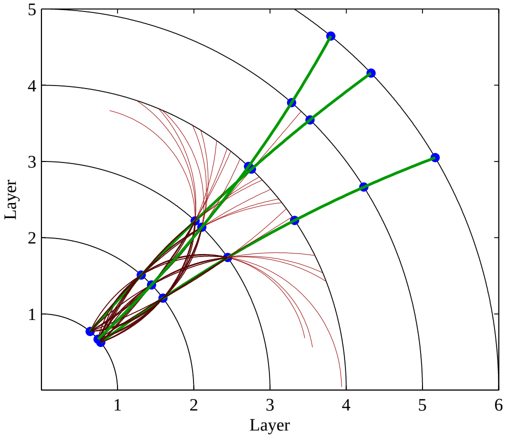
Track finding: Local methods

Progressive track finding

- Construct initial track segment (seed)
- Extrapolate seed
- Select best matching hit inside tolerance
- Update track parameters (weighted mean)
- Repeat until last detector layer
- Billoir and Qian (1990a), Billoir and Qian (1990b)

Track finding: Local methods

Example: progressive track finding



Track finding: Global methods

Some global methods

- Hough transform
- Legendre transform
- Hopfield network
- Elastic net
- Cellular automaton

Track finding: Global methods

Finding lines with the Hough transform

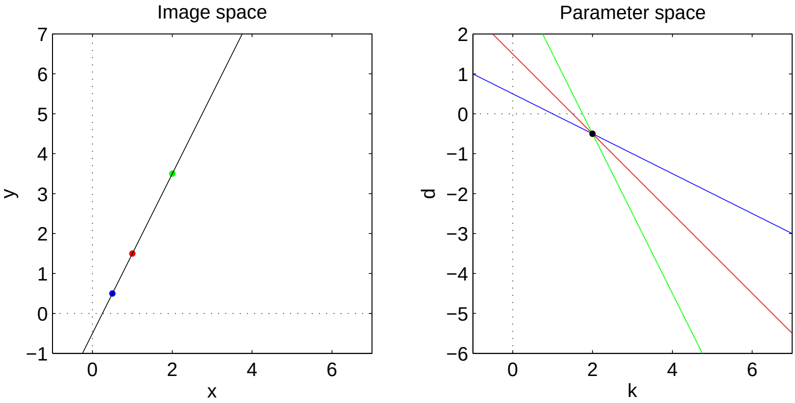
- Circles through the origin can be transformed to lines by a conformal transformation
- A point (x_0, y_0) in the image space (x/y) is transformed into a straight line

$$d = y_0 - kx_0$$

  in parameter space (k/d).
- Points on the straight line $y = k_0 x + d_0$ in image space are transformed into lines intersecting in the point (k_0/d_0) in parameter space
- If parameter space is discretized, intersections of lines can be found by histogramming

Track finding: Global methods

Finding lines with the Hough transform



Track finding: Global methods

Finding circles with the Hough transform

- A point (x_0, y_0) on a circle through the origin with center u/v is transformed into a straight line

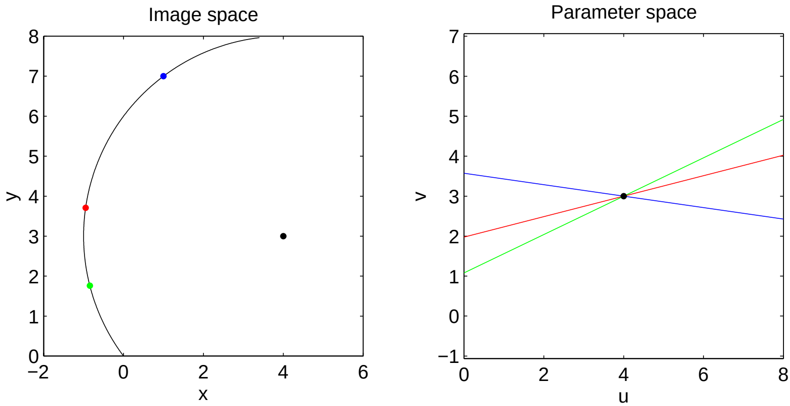
$$v = -\frac{x_0}{y_0}u + \frac{x_0^2 + y_0^2}{2y_0}$$

in parameter space (u/v)

- Points on the circle with center u_0/v_0 in image space are transformed into lines intersecting in the point (u_0/v_0) in parameter space
- If parameter space is discretized, intersections of lines can be found by histogramming

Track finding: Global methods

Finding circles with the Hough transform



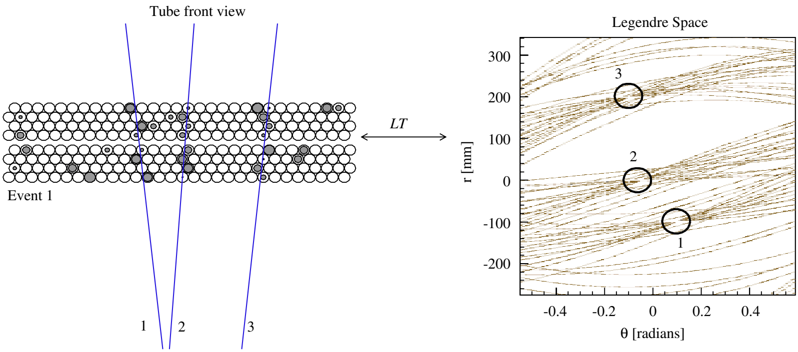
Track finding: Global methods

Finding tangents with the Legendre transform

- Track finding in drift tubes
- The drift circles are transformed by a Legendre transform to sine curves in polar coordinates
- Peaks at the intersections of several sine curves in this coordinate system correspond to common tangents to the drift circles
- See Alexopoulos (2008)

Track finding: Global methods

Example: Legendre transform



From: Alexopoulos *et al.*, NIM A 592 (2008) 456

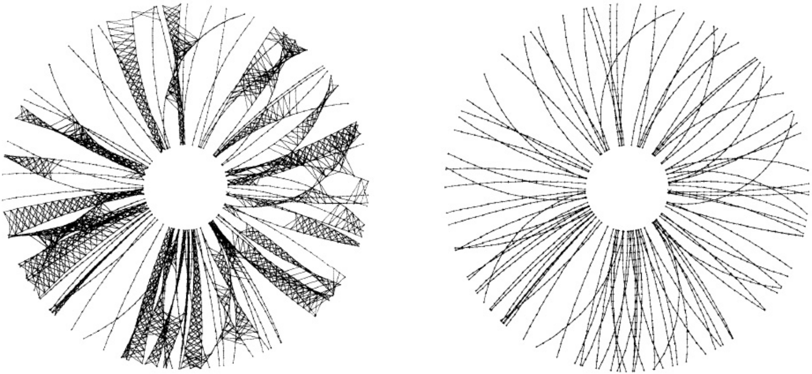
Track finding: Global methods

Track finding with a Hopfield network

- First adaptive approach to track reconstruction (Denby, 1988; Peterson, 1989)
- Track segments are neurons of a recursive ANN
- Network weights favor smooth tracks without bifurcations
- Energy function is minimized by gradient descent
- ☺ Deterministic annealing helps to find global optimum
- ☹ No physical track model

Track finding: Global methods

Example: Hopfield network



From: Stimpfl and Garrido, CPC 64 (1991) 46

Track finding: Global methods

Track finding with an elastic net

- Elastic net is a special type of neural network, related to Traveling Salesman Problem
- Neurons are attracted to the detector hits and to each other
- No physical track model
- See Kisel and Kovalenko (1996)
- Also used for ring finding in RICH detectors (see talk by S. Lebedev at this conference)

Track finding: Global methods

Example: elastic net

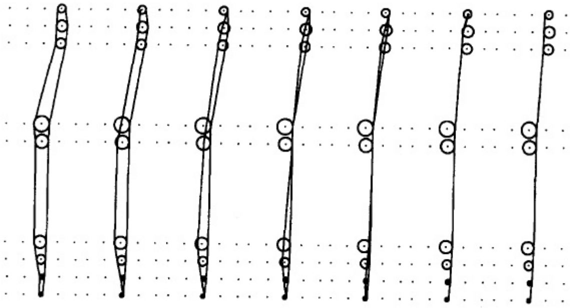


Fig. 7. Example of the elastic net evolution for a track with multiple scattering.

From: Kisel and Kovalenko, CPC 98 (1996) 45

Track finding: Global methods

Track finding with a cellular automaton

- Cells of the automaton are space points or track segments
- Find a sequence of cells that maximizes track length and track smoothness
- Optimization by appropriate update rules of the cells
- No physical track model
- See Kisel (1997), Abt (2002)

Track fitting: Traditional approach

Least-squares estimation

- Take track candidate and pass it to a least-squares estimator
- Three types: global, recursive, breakpoints
- **Global**: Set up regression model
- **Breakpoints**: Estimate track segments and multiple scattering angles
- **Recursive**: Interpret track as dynamic system and estimate with extended Kalman filter
- All three are **optimal** in the linear model with normal noise

Track fitting: Traditional approach

Regression

- In general non-linear model:

$$\boldsymbol{m} = \boldsymbol{h}(\boldsymbol{x}) + \boldsymbol{\epsilon}, \quad \mathsf{Cov}[\boldsymbol{\epsilon}] = \boldsymbol{V} = \boldsymbol{G}^{-1}$$

$\boldsymbol{m}$ ... Measurements $\boldsymbol{x}$ ... Initial track parameters
$\boldsymbol{h}$ ... Regression model $\boldsymbol{\epsilon}$ ... Measurement errors plus MS

- Minimize objective function:

$$M(\boldsymbol{x}) = (\boldsymbol{m} - \boldsymbol{h}(\boldsymbol{x}))^{\mathrm{T}} \boldsymbol{G} (\boldsymbol{m} - \boldsymbol{h}(\boldsymbol{x}))$$

Track fitting: Traditional approach

- Minimization methods: Gauss-Newton, Newton-Raphson, conjugate gradients, ...

$$\tilde{\boldsymbol{x}} = \arg\min M(\boldsymbol{x})$$

Test statistics

- Total χ^2

$$\chi^2 = M(\tilde{\boldsymbol{x}})$$

- Standardized residuals (pulls)

Track fitting: Traditional approach

Breakpoints

- Explicit estimation of multiple scattering angles
- Prior information about multiple scattering angles is used:

$$\mathsf{E}[\vartheta_{\mathrm{p}}] = 0, \quad \mathrm{var}[\vartheta_{\mathrm{p}}] = \sigma^2(m, p, d)$$

ϑ_{p} . . . Projected scattering angle

m . . . Mass of the particle

p . . . Momentum of the particle

d . . . Thickness of the material

Track fitting: Traditional approach

Kalman filter

- **Recursive**, no large matrices need to be inverted
- Estimated state vectors stay **close** to the actual track
- Track is described as **discrete dynamic system** (Frühwirth, 1987)
- **System equation**:

$$\boldsymbol{x}_k = \boldsymbol{f}_k(\boldsymbol{x}_{k-1}) + \boldsymbol{\delta}_k, \quad \mathsf{Cov}[\boldsymbol{\delta}_k] = \boldsymbol{Q}_k$$

$\boldsymbol{x}_k \ldots$ State vector in layer k (local track parameters)

$\boldsymbol{f}_k \ldots$ Local track model

$\boldsymbol{\delta}_k \ldots$ Local process noise (multiple scattering)

Track fitting: Traditional approach

- **Measurement equation**:

$$\boldsymbol{m}_k = \boldsymbol{h}_k(\boldsymbol{x}_k) + \boldsymbol{\epsilon}_k, \quad \mathsf{Cov}[\boldsymbol{\epsilon}_k] = \boldsymbol{V}_k$$

$\boldsymbol{m}_k$ ... Measurement in layer k

$\boldsymbol{h}_k$ ... Measurement model

$\boldsymbol{\epsilon}_k$ ... Measurement error

- Kalman filter proceeds **recursively** by alternating two steps
  1. Prediction
  2. Update

Track fitting: Traditional approach

Prediction

- **Propagate** state vector and covariance matrix to the next layer, increment covariance matrix by contributions from multiple scattering and energy loss

Update

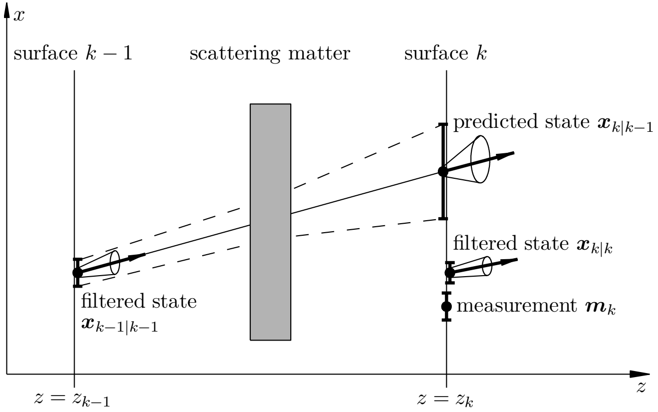
- Compute a **weighted mean** of the extrapolation and the observation

Test

- Local χ^2 statistic, ndf equals dimension of $\boldsymbol{m}_k$
- Total χ^2, sum of local χ^2 statistics

Track fitting: Traditional approach

Prediction and filter step



Track fitting: Traditional approach

Smoothing

- Optimal estimation of state vectors in **each layer**
- Standard algorithm, numerically unstable
- Combination of two filters (forward+backward) by a weighted mean, numerically stable

Test statistics

- Local χ^2s of the filter
- Total χ^2 of the filter (sum of local χ^2s)
- Local χ^2s of the smoother (correlated)

Track fitting: Adaptive approach

Problems of LS estimators

- ☹ The Kalman filter is LS-estimator, **not robust**
- ☹ Difficult to identify multiple outliers (bias)

Advantage of adaptive estimators

- ☺ **Concurrent** pattern recognition and estimation
  - Defer final decision to fitting stage
  - Complete information available
- ☺ **Automatic** suppression of background
  - Reduction of bias
  - No need to remove/add hits during fit
  - “Soft” assignment during entire fit
  - Can be made “hard” after optimal solution has been found

Track fitting: Adaptive approach

Various implementations

- **Elastic arms**, deformable templates: Ohlsson and Peterson (1992)
  - Based on neural network paradigm
- **Elastic tracking**: Gyulassi and Harlander (1991)
  - Inspired by Radon transform
- **Combinatorial Kalman filter**: Mankel (1997)
  - Full discrete combinatorial exploration
- **Gaussian-sum filter**: Frühwirth (1997)
  - Based on mixture models of noise
- **Deterministic annealing filter**: Frühwirth and Strandlie (1999)
  - Inspired by EM algorithm

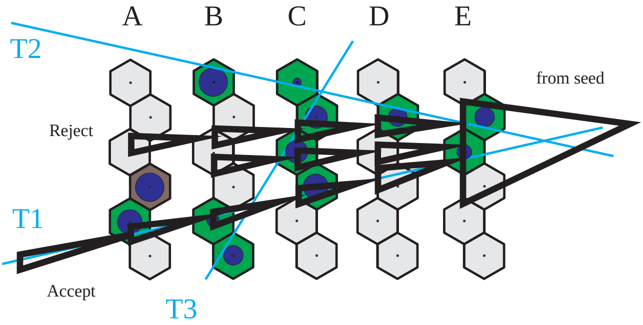
Track fitting: Adaptive approach

Combinatorial Kalman filter

- Extension of progressive track finding
- Full combinatorial exploration
- Several candidates are propagated in parallel
- Generate a branch for each compatible hit
- Generate a branch with a missing hit (optional)
- Limit growth by dropping branches
  - with bad total chi-square
  - with too many missing hits
  - which are subsets of other candidates
- Select "best" branch as final track

Track fitting: Adaptive approach

Example: Combinatorial Kalman filter



From: R. Mankel, NIM A 395 (1997) 169

Track fitting: Adaptive approach

Elastic arms, deformable templates

- First truly adaptive estimator
- Arms or templates are parameterized tracks
- Concurrent solution of two optimization problems
  - Continuous: minimize least-squares objective function
  - Discrete: decide which hit belongs to which template
- Discrete problem is transformed into a continuous one by **deterministic annealing**
- Minimization of the resulting non-quadratic energy function at each temperature

Track fitting: Adaptive approach

Deterministic annealing filter (DAF)

- Same principle as elastic arms
- Minimization by EM algorithm, implemented as **iterated re-weighted Kalman filter**
- Easy to deal with process noise
- Observations are assigned **weights**
- Iteration of two principal steps
  1. Full Kalman filter+smoother, using the current weights
  2. Calculation of weights, using current estimates
- The iteration ends when the weights are stable

Track fitting: Adaptive approach

Definition of the weights

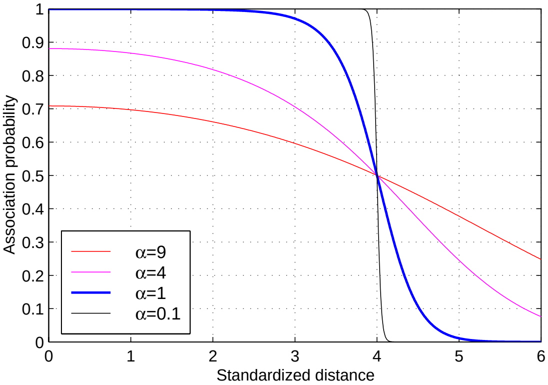
- Weight of observation i in layer k:

$$p_{ik} = \frac{\exp(-\chi^2_{ik}/2T)}{\exp(-\chi^2_{\mathrm{cut}}/2T) + \sum_j \exp(-\chi^2_{jk}/2T)}$$

- χ^2_{ik} measures the **distance** of observation i in layer k from the smoothed track state in layer k
- χ^2_{cut} is the **cut-off** parameter
- T is the **annealing** factor (temperature)
- For a single observation $p_{ik} = 0.5$ if $\chi^2_{ik} = \chi^2_{\mathrm{cut}}$

Track fitting: Adaptive approach

Weight function without competition

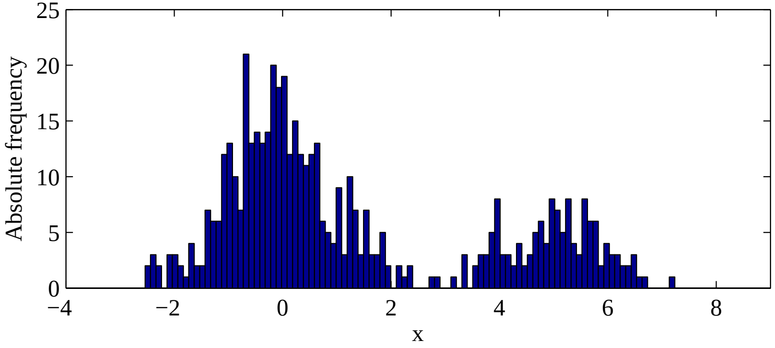


Weight function of an observation without competition

Track fitting: Adaptive approach

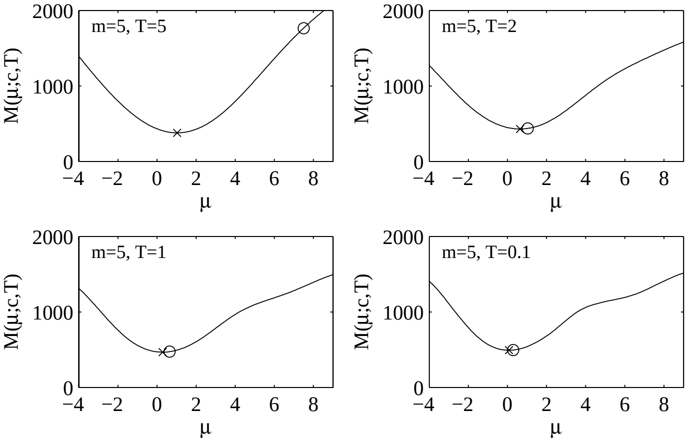
Example: 1D data with outliers

- Estimate location of the bulk of the data



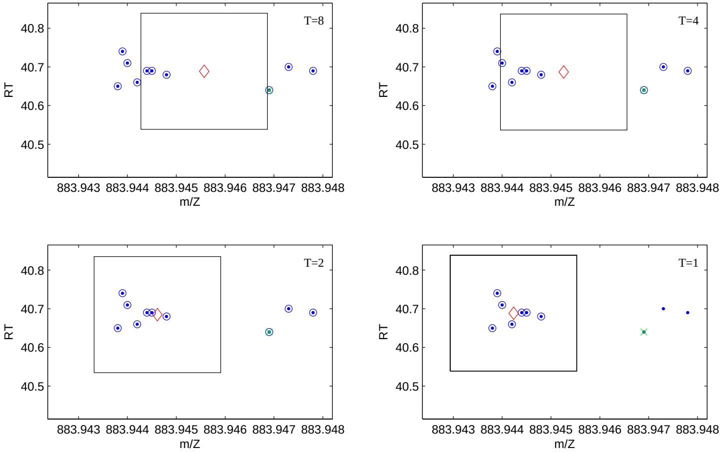
Track fitting: Adaptive approach

Evolution of the objective function



Track fitting: Adaptive approach

Example: 2D clustering of peptides



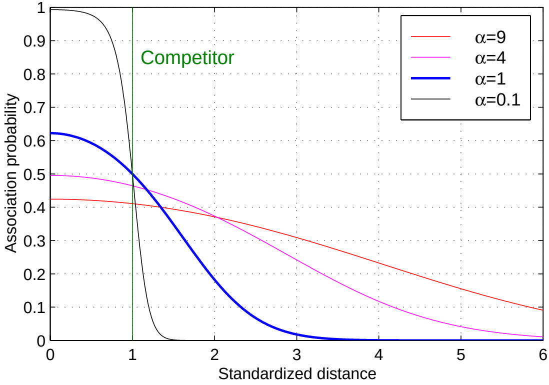
Track fitting: Adaptive approach

Weight function with competition

- If there are several observations in a detector layer, they may **compete** with each other
- A matching observations **suppresses** the other ones
- **Deterministic Annealing** helps to reach the optimal solution
  - At the start $T \gg 1$
  - During the iteration T is stepped down
  - The final value is $T = 1$
- Well-known technique of global optimization, e.g. with ANNs
- At $T > 0$ the association is "soft"
- "Cooling down" to $T = 0$ yields "hard" association. Not necessarily optimal!

Track fitting: Adaptive approach

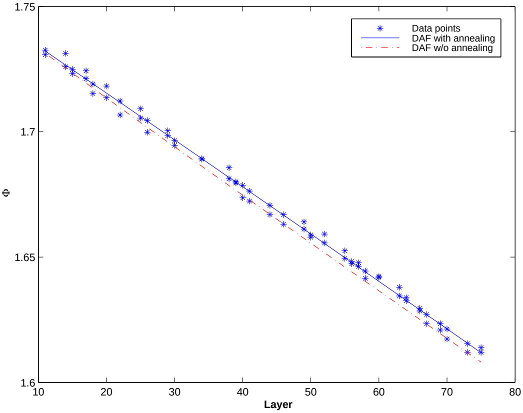
Weight function of the DAF



Weight function of an observation with competition

Track fitting: Adaptive approach

Example: DAF with and without annealing



From: Strandlie and Zerubia, CPC 123 (1999) 77

Track fitting: Adaptive approach

History of the DAF

- DAF was evaluated in CMS (Winkler, 2002)
- Studies with single tracks
- Studies in different physics contexts
- Implemented in CMS offline framework
- Implemented in ATLAS offline framework
- Studies of track finding with the DAF (Strandlie and Frühwirth, 2006)

Vertex reconstruction

Vertex finding

- Hierarchical clustering (agglomerative or divisive)
- Topological finding, Radon transform (Jackson, 1997)
- Minimum spanning tree (Hillert, 2008)
- Multi-layer perceptron (Lindsey and Denby, 1991)
- Adaptive vertex reconstructor (see below)

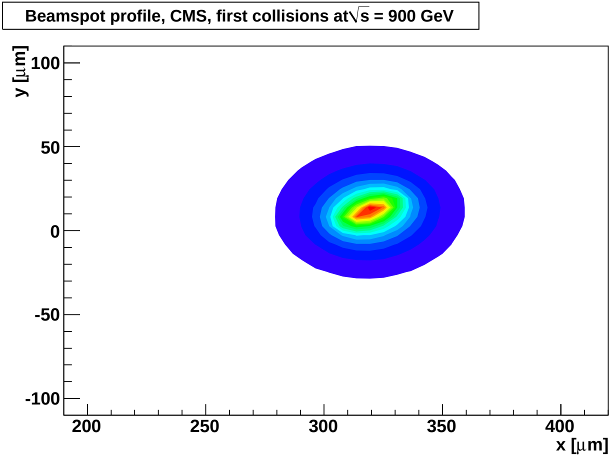
Vertex reconstruction

Adaptive vertex estimation

- Concepts of adaptive estimators can be transferred almost one-to-one from track to vertex fitting
- Algorithm is called **Adaptive Vertex Fitter (AVF)** (Waltenberger, 2004)
- Implemented as **iterated re-weighted Kalman filter**
- Outlying tracks are automatically down-weighted
- Resulting estimator is **highly robust**, but much easier to compute than other robust estimators such as LMS or LTS
- Extension to **Multi-Vertex Fitter (MVF)**: vertices compete for the tracks

Vertex reconstruction

Primary vertices estimated with AVF



Vertex reconstruction

Adaptive Vertex Reconstructor (AVR)

- Vertex finding by **iterated AVF** (Waltenberger, 2008)
  - Fit all tracks to a common vertex, using the AVF
  - Remove all tracks with weight above threshold
  - Fit all remaining tracks to a common vertex, using the AVF
  - Repeat until no valid vertex can be fitted
- Implemented and successfully validated in CMS offline software

Vertex reconstruction

RAVE Toolbox

- CMS algorithms (KF, AVF, AVR, ...) packed into detector-independent vertex reconstruction toolkit: **RAVE** (Waltenberger, 2007)
- Download from `http://projects.hepforge.org/rave`
- Used in new Belle2 framework
- Used for ILC studies (ILD, SiLC)

Conclusions and Outlook

- **Adaptive estimators** are useful tools for track and vertex reconstruction (Strandlie and Frühwirth, 2010)
- **Background** is automatically down-weighted, no need for iterative rejection of outliers
- **Competition** between hits or vertices is possible
- **Annealing** helps to reach the globally optimal solution
- **Implementations** are built on existing methods (Kalman filter)
- **Resistant** to high levels of noise, important for experiments at Super-LHC and the upgraded B-factory at KEK
- High-level trigger: Important work on **parallelization** going on (talks by M. McCool and I. Kisel at this conference)

Thanks to my colleagues:

Are Strandlie
Thomas Speer
Wolfgang Adam
Wolfgang Waltenberger
Matthias Winkler

Selected References (1)

- P. Billoir and S. Qian, NIM A 294 (1990) 219
- P. Billoir and S. Qian, NIM A 295 (1990) 492
- T. Alexopoulos *et al.*, NIM A 592 (2008) 456
- B. Denby, CPC 49 (1988) 429
- C. Peterson, NIM A 279 (1989) 537)
- G. Stimpfl-Abele and L. Garrido, CPC 64 (1991) 46
- I. Kisel and V. Kovalenko, CPC 98 (1996) 45
- I. Kisel *et al.*, NIM A 387 (1997) 433
- I. Abt *et al.*, NIM A 489 (2002) 389
- R. Frühwirth, NIM A 262 (1987) 444
- M. Ohlsson and C. Peterson, CPC 71 (1992) 77
- M. Gyulassi and M. Harlander, CPC 66 (1991) 31
- R. Mankel, NIM A 395 (1997) 169

Selected References (2)

- R. Frühwirth, CPC 100 (1997) 1
- R. Frühwirth and A. Strandlie, CPC 120 (1999) 197
- A. Strandlie and J. Zerubia, CPC 123 (1999) 77
- M. Winkler, PhD Thesis, Vienna Univ. of Technology (2002)
- A. Strandlie and R. Frühwirth, NIM A 566 (2006) 157
- D.J. Jackson, NIM A 388 (1997) 247
- S. Hillert, `http://arxiv.org/abs/0811.3904`
- C. Lindsey and B. Denby, NIM A 302 (1991) 217
- W. Waltenberger, PhD Thesis, Vienna Univ. of Technology (2004)
- W. Waltenberger *et al.*, NIM A 581 (2007) 549
- W. Waltenberger, CMS NOTE-2008/033, CERN, Geneva (2008)
- A. Strandlie and R. Frühwirth, Rev. Mod. Phys., to appear