Automated Computation of One-Loop Amplitudes with the OPP Method

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LHC started its operation
The experimental collaborations will collect data at 7 GeV for 2 years
The theorist are in full "production mode"

- Year 2007-08 Refining Methods
- Year 2009 Calculations for the LHC
- Year 2010 ???

ACAT 2010 – Workshop on Advanced Computing and Analysis Techniques in Physics Research

– My talk will be about Algorithms and Techniques –

Outline

1. Motivation & Introduction
2. The OPP Algorithm
3. Implementation of the Method
4. Numerical Tests

LHC needs NLO

- The problem of an efficient and automated computation of scattering amplitudes for one-loop multi-leg processes is crucial for the analysis of the LHC data.
- The OPP method is an important building block towards a fully automated implementation of this type of calculations.
- I will discuss the ongoing efforts to target important issues such as stability, versatility and efficiency of the method.

Many thanks to:

Roberto Pittau, *Costas Papadopoulos*, *Andreas van Hameren*, *Pierpaolo Mastrolia*, *Thomas Binoth*, *Michal Czakon*, *Stefano Actis*, *Francesco Tramontano*, *Thomas Reiter*

NLO TROUBLES

Problems arising in NLO calculations:

- Large Number of Feynman diagrams
- Reduction to Scalar Integrals (or sets of known integrals)
- Numerical Instabilities (inverse Gram determinants, spurious phase-space singularities)
- We need regularization – the integrals are divergent in 4 dimensions
- Extraction of soft and collinear singularities (we need to combine virtual and real corrections)

Different Approaches

- **Numerical**
  fully numerical integration over "q"

- **Improved Tensorial Reduction** (improved PV)
  algebraic reduction to a set of known integrals

Denner, Dittmaier at al.
GOLEM collaboration
Zeppenfeld et al.
several talks at ACAT 2010

- **Unitarity-based Approach**
  direct extraction of the coefficients of a set of known integrals

see plenary talk of **Maitre**

State-of-the-art on $2 \to 4$

$pp \to W+$ **3 jets**

- **Berger et al**

  Blackhat + Sherpa

- **Ellis, Melnikov, Zanderighi**

  Rocket

$pp \to t\bar{t}b\bar{b}$

- **Bredenstein, Denner, Dittmaier, Pozzorini**

  “traditional” approach, tensorial reduction

- **Bevilacqua, Czakon, Papadopoulos, Pittau, Worek**

  CutTools + Helac1loop + Dipoles

Several methods/codes “available on the market”

$pp \to W+$ **3 jets**

- **Berger et al**

- **Elli**

NEW – arXiv:1002.4009

Bevilacqua, Czakon, Papadopoulos, Worek

$pp \to t\bar{t}$ **+ 2 jets**

pp

- **Bredenstein, Denner, Dittmaier, Pozzorini**

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Several methods/codes "available on the market"

OPP Method

Three years ago (Sept.2006), we proposed a new method for the numerical evaluation of scattering amplitudes, based on a decomposition at the **integrand level**.

Some of the advantages:

- Universal - applicable to any process
- Simple - based on basic algebraic properties
- Automatizable - easy to implement in a computer code

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Final Task

Produce a MULTI-PROCESS fully automatized NLO generator

“Standing on the shoulders of giants”

1. **Passarino-Veltman Reduction to Scalar Integrals**

$$\mathcal{M} = \sum_i d_i \,\mathrm{Box}_i + \sum_i c_i \,\mathrm{Triangle}_i + \sum_i b_i \,\mathrm{Bubble}_i + \sum_i a_i \,\mathrm{Tadpole}_i + \mathrm{R}\,,$$

   - Set the basis for our NLO calculations
   - Exploits the Lorentz structure

2. **Pittau/del Aguila Recursive Tensorial Reduction**
   - Express $q^\mu = \sum_i G_i \, \ell_i{}^\mu \;,\; \ell_i{}^2 = 0$
   - The generated terms might reconstruct denominators D_i or vanish upon integration

3. **“Cut-based” Techniques** (Bern, Dixon, Dunbar, Kosower in ’94) direct extraction of the coefficients of the scalar integral

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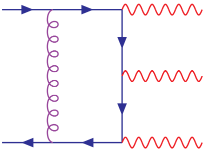

Any m-point one-loop amplitude can be written, before integration, as

$$A(\bar{q}) = \frac{N(\bar{q})}{\bar{D}_0 \bar{D}_1 \cdots \bar{D}_{m-1}}$$

where

$$\bar{D}_i = (\bar{q} + p_i)^2 - m_i^2 \quad , \quad \bar{q}^2 = q^2 + \tilde{q}^2 \quad , \quad \bar{D}_i = D_i + \tilde{q}^2$$

Our task is to calculate, for each phase space point:

$$\mathcal{M} = \int d^n\bar{q}\; A(\bar{q}) = \int d^n\bar{q} \frac{N(\bar{q})}{\bar{D}_0 \bar{D}_1 \ldots \bar{D}_{m-1}}$$

The traditional "master" formula

$$
\begin{aligned}
\int A \quad = \quad & \sum_{i_0<i_1<i_2<i_3}^{m-1} d(i_0 i_1 i_2 i_3) \int \frac{1}{\bar{D}_{i_0} \bar{D}_{i_1} \bar{D}_{i_2} \bar{D}_{i_3}} \\
+ \quad & \sum_{i_0<i_1<i_2}^{m-1} c(i_0 i_1 i_2) \int \frac{1}{\bar{D}_{i_0} \bar{D}_{i_1} \bar{D}_{i_2}} \\
+ \quad & \sum_{i_0<i_1}^{m-1} b(i_0 i_1) \int \frac{1}{\bar{D}_{i_0} \bar{D}_{i_1}} \\
+ \quad & \sum_{i_0}^{m-1} a(i_0) \int \frac{1}{\bar{D}_{i_0}} \\
+ \quad & \text{rational terms}
\end{aligned}
$$

The algebraic cartoon of "OPP integration"

Problem: we want to calculate

$$\int dx \, \frac{N(x)}{x^4}$$

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$$N(x) = a + b\ x + c\ x^2$$

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Example: if $N(0) = 3$, $N(1) = 10$, $N(-1) = 4$ *then* $a = 3$, $b = 3$, $c = 4$

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So we can calculate

$$\int dx \, \frac{N(x)}{x^4} = a \int dx \, \frac{1}{x^4} + b \int dx \, \frac{1}{x^3} + c \int dx \, \frac{1}{x^2}$$

where our "master integrals" are

$$\int dx \, \frac{1}{x^n}$$

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What is the "polynomial" structure of $N(q)$ for one-loop amplitudes??

OPP "MASTER" FORMULA - I

General expression for the **4-dim** $N(q)$ at the integrand level in terms of D_i

$$
\begin{aligned}
N(q) = & \sum_{i_0<i_1<i_2<i_3}^{m-1} \left[d(i_0 i_1 i_2 i_3) + \tilde{d}(q; i_0 i_1 i_2 i_3) \right] \prod_{i \neq i_0, i_1, i_2, i_3}^{m-1} D_i \\
+ & \sum_{i_0<i_1<i_2}^{m-1} \left[c(i_0 i_1 i_2) + \tilde{c}(q; i_0 i_1 i_2) \right] \prod_{i \neq i_0, i_1, i_2}^{m-1} D_i \\
+ & \sum_{i_0<i_1}^{m-1} \left[b(i_0 i_1) + \tilde{b}(q; i_0 i_1) \right] \prod_{i \neq i_0, i_1}^{m-1} D_i \\
+ & \sum_{i_0}^{m-1} \left[a(i_0) + \tilde{a}(q; i_0) \right] \prod_{i \neq i_0}^{m-1} D_i
\end{aligned}
$$

This is 4-dimensional Identity

Spurious Terms - I

– the recepy is not unique –

Following F. del Aguila and R. Pittau, arXiv:hep-ph/0404120

- Express any q in $N(q)$ as

$$q^\mu = -p_0^\mu + \sum_{i=1}^4 G_i\, \ell_i^\mu \;,\; {\ell_i}^2 = 0$$

$$k_1 = \ell_1 + \alpha_1 \ell_2\,, \quad k_2 = \ell_2 + \alpha_2 \ell_1\,,\; k_i = p_i - p_0$$
$${\ell_3}^\mu = < \ell_1|\gamma^\mu|\ell_2]\,,\; {\ell_4}^\mu = < \ell_2|\gamma^\mu|\ell_1]$$

- The resulting terms G_i either reconstruct denominators D_i or vanish upon integration

→ They give rise to d, c, b, a coefficients
→ They form the spurious $\tilde{d}$, $\tilde{c}$, $\tilde{b}$, $\tilde{a}$ coefficients

Spurious Terms - II

- $\tilde{d}$(q) term (only 1)

$$\tilde{d}(q) = \tilde{d}\, T(q)\,,$$

where $\tilde{d}$ is a constant (does not depend on q)

$$T(q) \equiv Tr[(\not{q} + \not{p}_0)\not{\ell}_1\not{\ell}_2\not{k}_3\gamma_5]$$

- $\tilde{c}$(q) terms (they are 6)

$$\tilde{c}(q) = \sum_{j=1}^{j_{max}} \left\{ \tilde{c}_{1j}[(q+p_0)\cdot\ell_3]^j + \tilde{c}_{2j}[(q+p_0)\cdot\ell_4]^j \right\}$$

In the renormalizable gauge, $j_{max} = 3$

- $\tilde{b}$(q) and $\tilde{a}$(q) give rise to 8 and 4 terms, respectively

OPP "MASTER" FORMULA - II

$$
\begin{aligned}
N(q) &= \sum_{i_0<i_1<i_2<i_3}^{m-1} \left[d(i_0 i_1 i_2 i_3) + \tilde{d}(q; i_0 i_1 i_2 i_3)\right] \prod_{i\neq i_0, i_1, i_2, i_3}^{m-1} D_i + \sum_{i_0<i_1<i_2}^{m-1} \left[c(i_0 i_1 i_2) + \tilde{c}(q; i_0 i_1 i_2)\right] \prod_{i\neq i_0, i_1, i_2}^{m-1} D_i \\
&+ \sum_{i_0<i_1}^{m-1} \left[b(i_0 i_1) + \tilde{b}(q; i_0 i_1)\right] \prod_{i\neq i_0, i_1}^{m-1} D_i + \sum_{i_0}^{m-1} \left[a(i_0) + \tilde{a}(q; i_0)\right] \prod_{i\neq i_0}^{m-1} D_i
\end{aligned}
$$

The quantities d, c, b, a are the coefficients of all possible scalar functions

The quantities $\tilde{d}$, $\tilde{c}$, $\tilde{b}$, $\tilde{a}$ are the "spurious" terms → vanish upon integration

IT IS NOW AN ALGEBRAIC PROBLEM:

Any N(q) just depends on a set of coefficients, to be determined!

OPP "master" formula - II

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\begin{aligned}
N(q) = & \sum_{i_0<i_1<i_2<i_3}^{m-1} \left[d(i_0 i_1 i_2 i_3) + \tilde{d}(q; i_0 i_1 i_2 i_3) \right] \prod_{i \neq i_0, i_1, i_2, i_3}^{m-1} D_i + \sum_{i_0<i_1<i_2}^{m-1} \left[c(i_0 i_1 i_2) + \tilde{c}(q; i_0 i_1 i_2) \right] \prod_{i \neq i_0, i_1, i_2}^{m-1} D_i \\
+ & \sum_{i_0<i_1}^{m-1} \left[b(i_0 i_1) + \tilde{b}(q; i_0 i_1) \right] \prod_{i \neq i_0, i_1}^{m-1} D_i + \sum_{i_0}^{m-1} \left[a(i_0) + \tilde{a}(q; i_0) \right] \prod_{i \neq i_0}^{m-1} D_i
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$$

The quantities d, c, b, a are the coefficients of all possible scalar functions

The quantities $\tilde{d}$, $\tilde{c}$, $\tilde{b}$, $\tilde{a}$ are the "spurious" terms → vanish upon integration

It is now an algebraic problem:

Any N(q) just depends on a set of coefficients, to be determined!

Choose $\{q_i\}$ wisely

by evaluating N(q) for a set of values of the integration momentum $\{q_i\}$ such that some denominators D_i vanish ("cuts")

$$N(q) = d + \tilde{d}(q) + \sum_{i=0}^{3} [c(i) + \tilde{c}(q;i)]\, D_i + \sum_{i_0<i_1}^{3} \left[b(i_0 i_1) + \tilde{b}(q; i_0 i_1)\right] D_{i_0} D_{i_1}$$

$$+ \sum_{i_0=0}^{3} [a(i_0) + \tilde{a}(q; i_0)]\, D_{i \neq i_0} D_{j \neq i_0} D_{k \neq i_0}$$

We look for a q such that

$$D_0 = D_1 = D_2 = D_3 = 0$$

→ there are two solutions $q_0^{\pm}$

Example: 4-particles process

$$N(q) \quad = \quad d + \tilde{d}(q)$$

Our "master formula" for $q = q_0^{\pm}$ is:

$$N(q_0^{\pm}) = [d + \tilde{d}\; T(q_0^{\pm})]$$

$\longrightarrow$ solve to extract the coefficients d and $\tilde{d}$

Example: 4-particles process

$$
\begin{aligned}
N(q) - d - \tilde{d}(q) &= \sum_{i=0}^{3} \left[c(i) + \tilde{c}(q;i) \right] D_i + \sum_{i_0<i_1}^{3} \left[b(i_0 i_1) + \tilde{b}(q; i_0 i_1) \right] D_{i_0} D_{i_1} \\
&+ \sum_{i_0=0}^{3} \left[a(i_0) + \tilde{a}(q; i_0) \right] D_{i \neq i_0} D_{j \neq i_0} D_{k \neq i_0}
\end{aligned}
$$

Then we can move to the extraction of c coefficients using

$$N'(q) = N(q) - d - \tilde{d}\, T(q)$$

and setting to zero three denominators (ex: $D_1 = 0$, $D_2 = 0$, $D_3 = 0$)

Example: 4-particles process

$$N(q) - d - \tilde{d}(q) = [c(0) + \tilde{c}(q;0)]\, D_0$$

We have infinite values of q for which

$$D_1 = D_2 = D_3 = 0 \quad \text{and} \quad D_0 \neq 0$$

$\longrightarrow$ Here we need 7 of them to determine $c(0)$ and $\tilde{c}(q;0)$

From 4 to N (part I - Denominators)

- We find the decomposition for $N(q)$

$$N(q) = \ldots + c_2 D_2 + \ldots$$

From 4 to N (part I - Denominators)

- We find the decomposition for $N(q)$, divide by the denominators

$$\frac{N(q)}{\bar{D}_0 \bar{D}_1 \bar{D}_2 \bar{D}_3} = \ldots + \frac{c_2 D_2}{\bar{D}_0 \bar{D}_1 \bar{D}_2 \bar{D}_3} + \ldots$$

From 4 to N (part I - Denominators)

- We find the decomposition for $N(q)$, divide by the denominators and finally integrate over q

$$\int \frac{N(q)}{\bar{D}_0 \bar{D}_1 \bar{D}_2 \bar{D}_3} = \ldots + \int \frac{c_2 D_2}{\bar{D}_0 \bar{D}_1 \bar{D}_2 \bar{D}_3} + \ldots$$

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- We have a mismatch $\rightarrow$ this is the origin of R_1

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$$\frac{D_2}{\bar{D}_2} = \left(1 - \frac{\tilde{q}^2}{\bar{D}_2}\right) \equiv \bar{Z}_2$$

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- Using the expression for $\bar{Z}_2$

$$\int \frac{N(q)}{\bar{D}_0 \bar{D}_1 \bar{D}_2 \bar{D}_3} = \ldots + \int \frac{c_2}{\bar{D}_0 \bar{D}_1 \bar{D}_3} + \int \frac{c_2\, \tilde{q}^2}{\bar{D}_0 \bar{D}_1 \bar{D}_2 \bar{D}_3} + \ldots$$

"Extra Integrals" for R_1

The "Extra Integrals" are of the form

$$I^{(n;2\ell)}_{s;\mu_1\cdots\mu_r} \equiv \int d^n q\, \tilde{q}^{2\ell} \frac{q_{\mu_1}\cdots q_{\mu_r}}{\bar{D}(k_0)\cdots\bar{D}(k_s)}\,,$$

where

$$\bar{D}(k_i) \equiv (\bar{q}+k_i)^2 - m_i^2\,, k_i = p_i - p_0$$

These integrals:

- **have dimensionality** $\mathcal{D} = 2(1+\ell-s)+r$
- **contribute only when** $\mathcal{D} \geq 0$**, otherwise are of** $\mathcal{O}(\epsilon)$

Pittau – arXiv:hep-ph/0406105
G.O., Papadopoulos, Pittau – arXiv:0802.1876

From 4 to N (part II - Numerators)

What if $N(q)$ develops an ϵ-dimensional part?

- Algebra of Dirac matrices
- $(\bar{q}.p)$ is 4-dim but $(\bar{q}.\bar{q}) = q^2 + \tilde{q}^2$

$\bar{N}(\bar{q})$ can be split into a 4-dim plus a ϵ-dimensional part

$$\bar{N}(\bar{q}) = N(q) + \tilde{N}(\tilde{q}^2, q, \epsilon)$$

$\tilde{N}(\tilde{q}^2, q, \epsilon)$ is responsible for the rational term R_2

A practical solution: **tree-level like Feynman Rules**

General idea and QED: G. O., Papadopoulos, Pittau - arXiv:0802.1876
Rules for QCD: Draggiotis, Garzelli, Papadopoulos, Pittau - arXiv:0903.0356
Full Standard Model: Garzelli, Malamos, Pittau - arXiv:0910.3130

Overview Rational Terms

$$R = R_1 + R_2$$

R_1 – The OPP expansion is written in terms of 4-dim D_i, while n-dim $\bar{D}_i$ appear in scalar integrals.

$$A(\bar{q}) = \frac{N(q)}{\bar{D}_0 \bar{D}_1 \cdots \bar{D}_{m-1}}$$

R_1 can be calculated in **two different ways**, both **fully automatized**.

R_2 – The numerator $\bar{N}(\bar{q})$ can be also split into a 4-dim plus a ϵ-dim part

$$\bar{N}(\bar{q}) = N(q) + \tilde{N}(\tilde{q}^2, q, \epsilon)\,.$$

Compute R_2 using **tree-level like Feynman Rules**.

One-Loop as a 3 step process

1) Compute the numerator $N(q)$ numerically at given q
2) Extract coefficients/rats with **OPP reduction**
3) Combine with scalar integrals

$$
\begin{aligned}
\mathcal{M} &= \sum_i d_i \, \mathrm{Box}_i + \sum_i c_i \, \mathrm{Triangle}_i \\
&+ \sum_i b_i \, \mathrm{Bubble}_i + \sum_i a_i \, \mathrm{Tadpole}_i + \mathrm{R}\,,
\end{aligned}
$$

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$$

One-Loop as a 3 step process

1) Compute the numerator $N(q)$ numerically at given q
2) Extract coefficients/rats with **OPP reduction** [`CutTools`]
3) Combine with scalar integrals [`OneLOop/QCDloop`]

$$\mathcal{M} = \sum_i d_i \,\mathrm{Box}_i + \sum_i c_i \,\mathrm{Triangle}_i + \sum_i b_i \,\mathrm{Bubble}_i + \sum_i a_i \,\mathrm{Tadpole}_i + \mathrm{R}\,,$$

To extract **all coefficients** d, c, b, and a we ONLY need to evaluate numerator N(q) **numerically** at **fixed given values of** q.

INTERMEZZO: CutTools

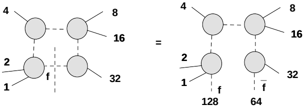
http://www.ugr.es/∼pittau/CutTools/

- **Initialization**
  - Choose or generate a phase-space point
  - Define denominators D_i: momenta and masses
- **Calculation of the Amplitude**
  - Write a routine that numerically evaluates $N(q)$ at any given q
  - Use **CutTools** to extract all coefficients + R_1
  - The calculation of the scalar integrals (via OneLOop or QCDloop) is incorporated
  - Add R_2 as tree-level construction
- **Repeat for a new PS point**

CutTools is available (and public!)

Numerators $N(q)$

- **Tree-Level Construction of $N(q)$ at fixed q**
  - After fixing the integration momentum q, any n-point one-loop amplitude is an $(n+2)$-point tree level amplitude



  - HELAC-1L reconstructs the one-loop amplitude as a tree-order calculation
- **One-Loop Algebraic Construction of $N(q)$**
  - Produce analytic expressions for the one-loop numerators (`Qgraf`, FORM, . . .)
  - Group numerators with similar structure (optimize their expressions)
  - Automatically feed the output to the reduction code

Tests on the Calculation

The aim is to detect numerically unstable points before using them

1. Tests on the reconstruction $\rightarrow$ **"$N = N$" test**

2. **Double** precision vs **Multiple** precision

3. Complete **cancellation of UV and IR poles**

4. **Stability test** on "special" configurations

Tests 1, 2, and 3 are universal (process-independent)

The $N \equiv N$ test

Our "master" formula again!

$$
\begin{aligned}
N(q) &= \sum_{i_0<i_1<i_2<i_3}^{m-1} \left[d(i_0 i_1 i_2 i_3) + \tilde{d}(q; i_0 i_1 i_2 i_3)\right] \prod_{i \neq i_0, i_1, i_2, i_3}^{m-1} D_i \\
&+ \sum_{i_0<i_1<i_2}^{m-1} \left[c(i_0 i_1 i_2) + \tilde{c}(q; i_0 i_1 i_2)\right] \prod_{i \neq i_0, i_1, i_2}^{m-1} D_i \\
&+ \sum_{i_0<i_1}^{m-1} \left[b(i_0 i_1) + \tilde{b}(q; i_0 i_1)\right] \prod_{i \neq i_0, i_1}^{m-1} D_i \\
&+ \sum_{i_0}^{m-1} \left[a(i_0) + \tilde{a}(q; i_0)\right] \prod_{i \neq i_0}^{m-1} D_i
\end{aligned}
$$

After determining all coefficients → this should hold for any q

Check UV and IR Poles

Example for $e^- e^+ \to e^- e^+ \gamma$

S. Actis, P. Mastrolia, and G. O. – arXiv:0909.1750

Output of our FORTRAN code at a given **phase space point**

$$
\begin{aligned}
\mathcal{I}^{\rm V}_{\rm NLO}(\mathcal{C}\mathcal{C}_4 + \mathcal{R}) &= +\frac{1}{\epsilon}\, 4.74506427003505 \cdot 10^{-2} + \ldots \\
\mathcal{I}^{\rm V}_{\rm NLO}(\mathcal{U}\mathcal{V}_{ct}) &= -\frac{1}{\epsilon}\, 5.28634805094576 \cdot 10^{-3} + \ldots \\
\mathcal{I}^{\rm V}_{\rm NLO} &= +\frac{1}{\epsilon}\, 4.21642946494047 \cdot 10^{-2} + \ldots
\end{aligned}
$$

- Results are expressed in GeV^{-2}
- All numbers have been obtained working in double precision

Check UV and IR Poles

Example for $e^- e^+ \to e^- e^+ \gamma$

S. Actis, P. Mastrolia, and G. O. – arXiv:0909.1750

Output of our FORTRAN code at a given **phase space point**

Test on the UV and IR poles!

$$
\begin{aligned}
\mathcal{I}^{\rm V}_{\rm NLO}(\mathcal{CC}_4 + \mathcal{R}) &= +\frac{1}{\epsilon}\, 4.74506427003505 \cdot 10^{-2} + \dots \\
\mathcal{I}^{\rm V}_{\rm NLO}(\mathcal{UV}_{ct}) &= -\frac{1}{\epsilon}\, 5.28634805094576 \cdot 10^{-3} + \dots \\
\mathcal{I}^{\rm V}_{\rm NLO} &= +\frac{1}{\epsilon}\, 4.21642946494047 \cdot 10^{-2} + \dots \\
\mathcal{I}^{\rm R}_{\rm NLO} &= -\frac{1}{\epsilon}\, 4.21642946495863 \cdot 10^{-2} + \dots
\end{aligned}
$$

- Results are expressed in GeV^{-2}
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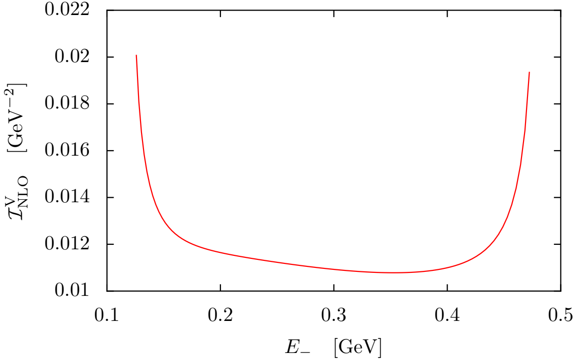
Stability test on **quasi-collinear** configuration

Example for $e^- e^+ \to \mu^- \mu^+ \gamma$
Virtual part $\mathcal{I}^{\rm V}_{\rm NLO}$ as a function of the energy E_- of the outgoing muon:
the muon is (almost) parallel or antiparallel to the photon momentum



There are no istabilities
(work done in double precision)

Conclusions

LHC requires NLO calculations!

- One-loop calculations are in fast **evolution**
- **OPP** is a now a **solid method** (and widely used!)
- Full **automatization** is under way (fast!!)

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`(what is still left to do??)`

- **New Codes**
- **Efficiency, Precision, and Stability**
- **Phenomenology - New processes for the LHC**

– work in progress –