Fourier Transforms as a Tool for Analysis of Hadron-Hadron collisions

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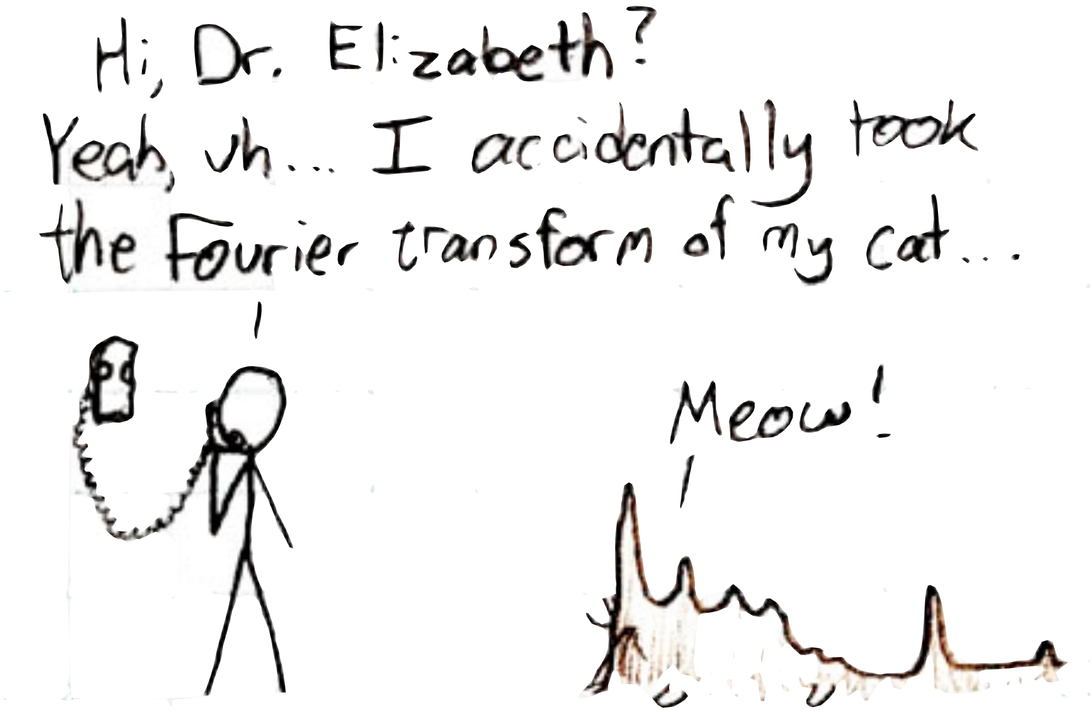
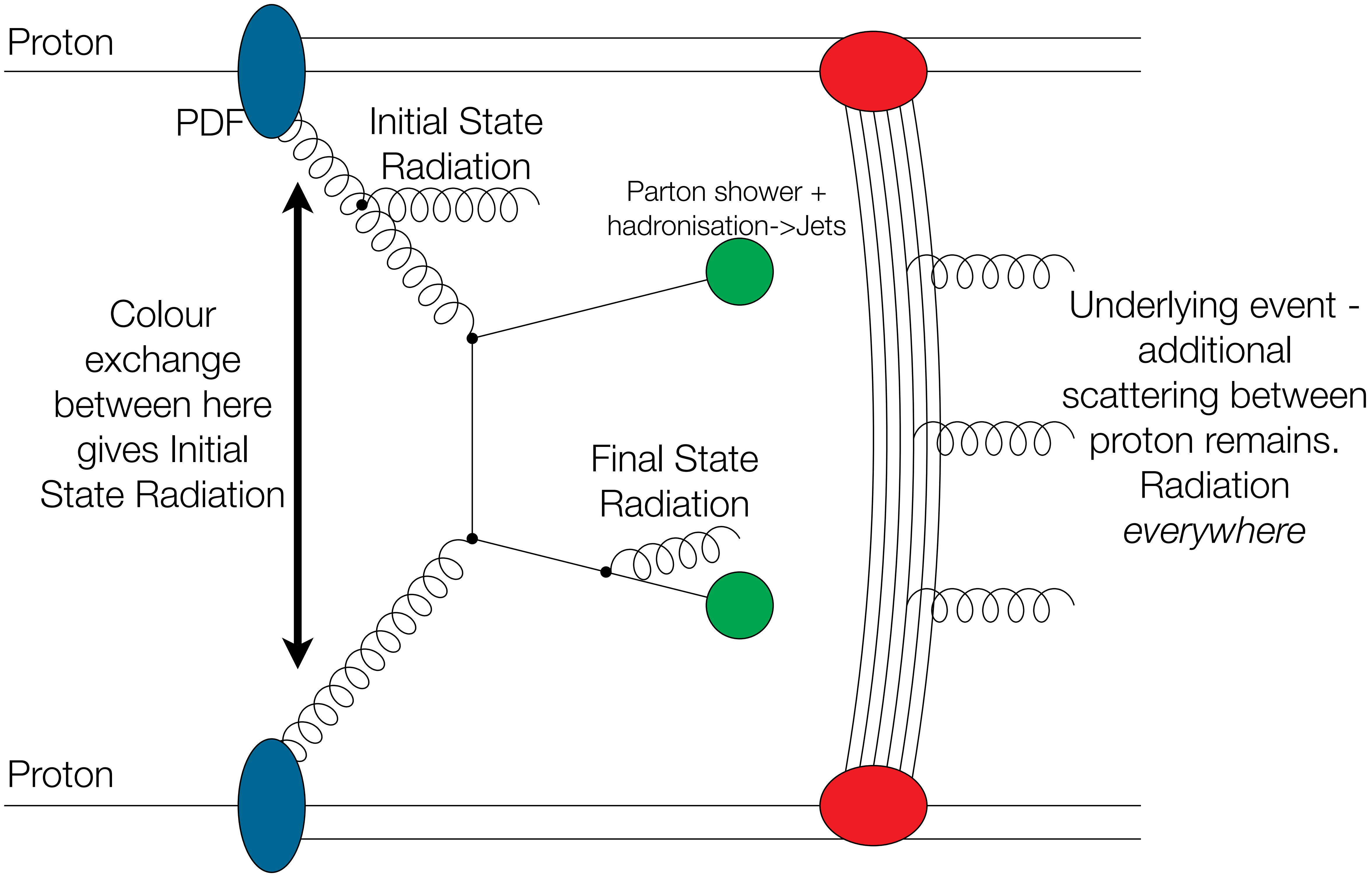


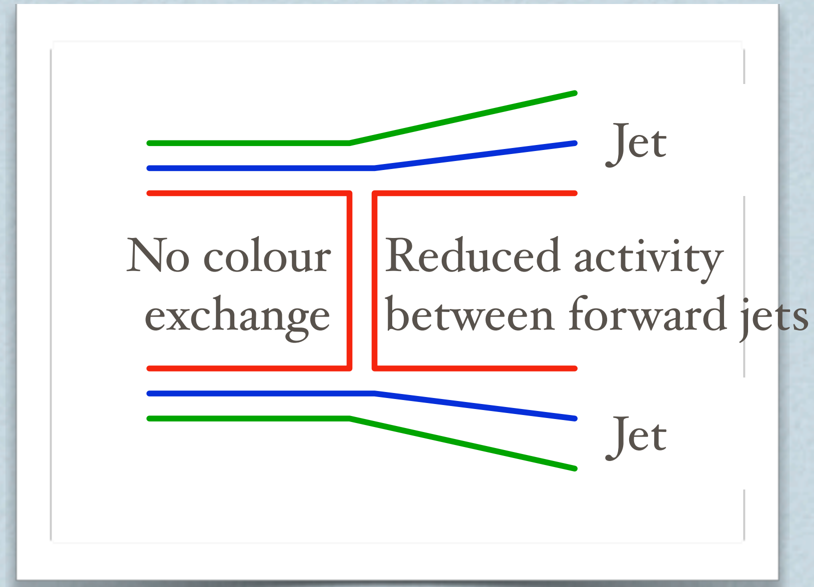
(xkcd.com)

Terminology

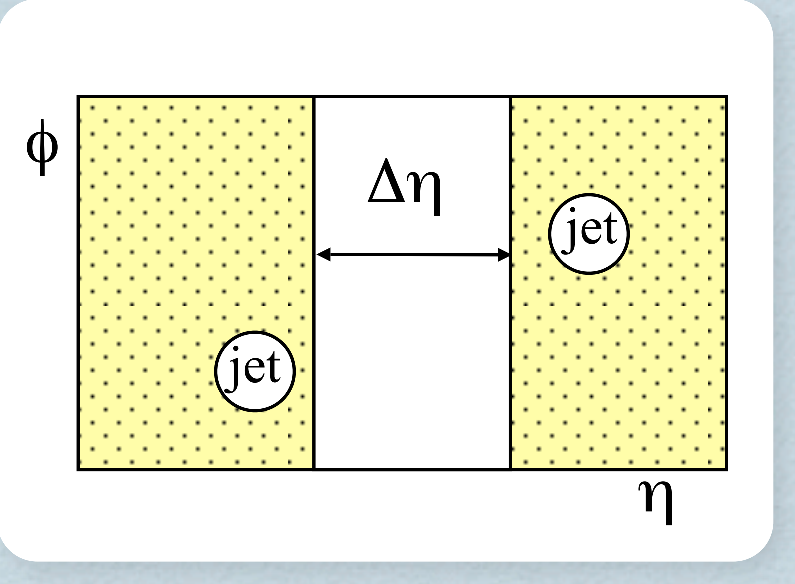


Introduction to Gaps Between Jets

- Pomeron has quantum numbers of the vacuum (colourless, chargeless, spin 0)

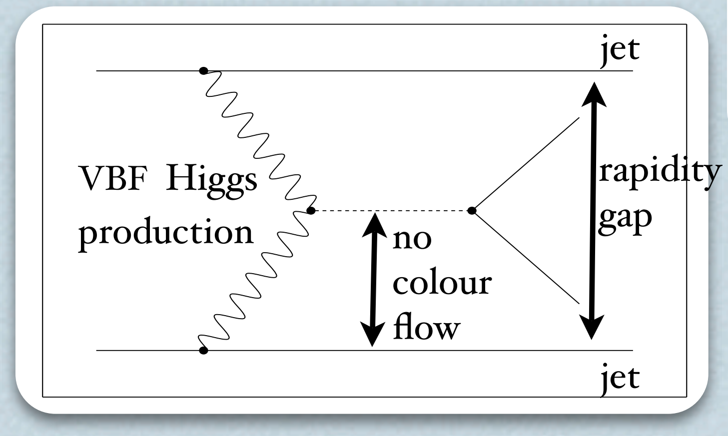


- Lack of colour means no QCD radiation off a pomeron.
- Experimental signature for pomeron exchange (diffraction) is a “gap” in the detector between the outgoing proton remnants



Jets separated by $\Delta\eta$

- Similar to vector boson fusion processes



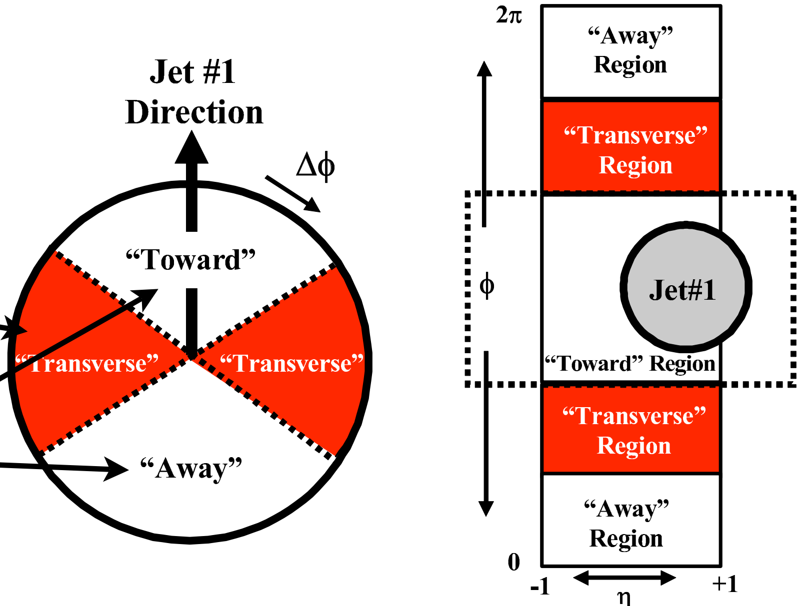
Gap selection

- Selection of gap events typically uses a veto on a third jet between the leading two forward jets, or on the ET sum in the gap region.

- Underlying event and pile up spoils the gap - additional radiation in the gap that fails the selection criteria.

- Started thinking about looking in specific parts of the inter-jet region

- Motivated by the transverse/away/towards regions originally used in CDF min bias analysis e.g. Phys. Rev. D65, 092002



Underlying event in this region

Hadronisation/showering in this region

Monte Carlo event selection

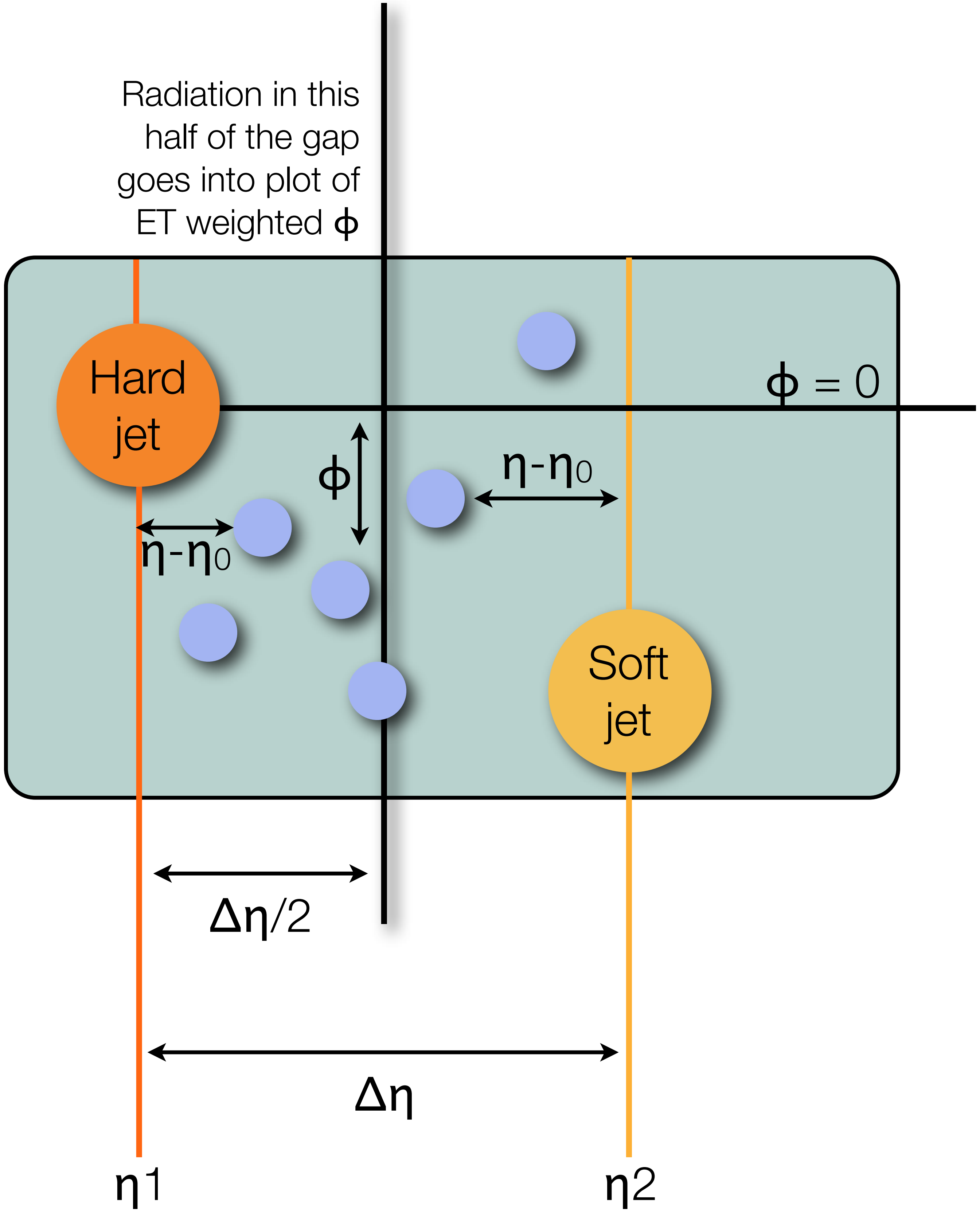
- Herwig, with and without Jimmy underlying event. QCD 2->2 events and colour singlet exchange (IPROC 1500 and 2400, respectively). 10 TeV collision energy.

- Run KT jet finding with r param of 0.7. Select events with two jets with ET > 30GeV and are separated by $\Delta\eta > 4$. No rejection of events based on radiation between the lead jets. Use region $|\eta| < 5$, consistent with LHC detectors.

- Run KT jet algorithm a second time with r param of 0.1 and minimum ET cut of 1 GeV. Approximately the size of a cell/tower in a detector. Use these smaller jets to inspect radiation patterns and still remain IR safe.

Radiation patterns between the jets

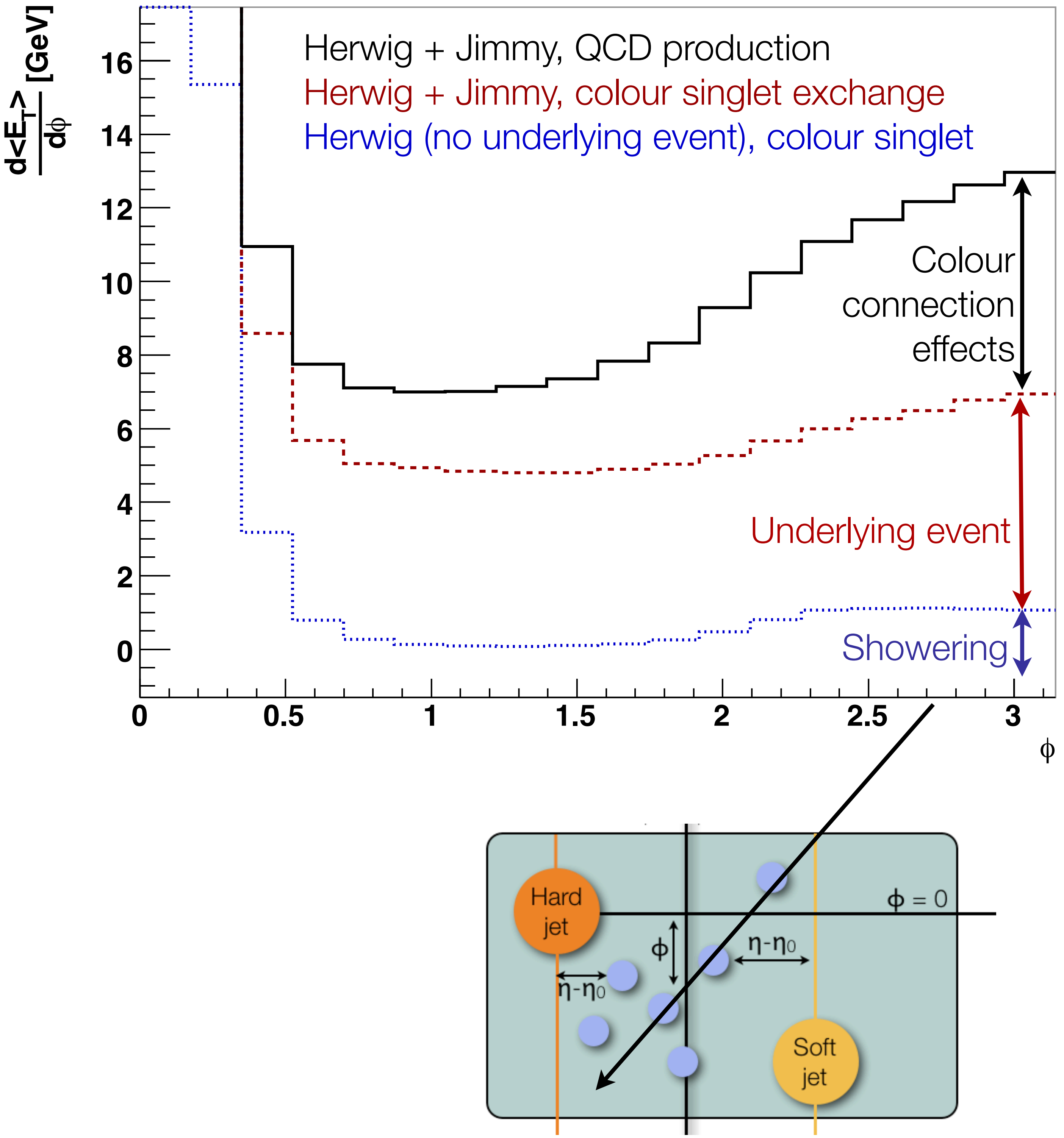
- Define ϕ = 0 at the hardest edge jet
- Split the gap in half to define a region that is nearer the hardest boundary jet than the softest boundary jet
- Plot the ET weighted distribution of radiation in ϕ in that half-gap region nearer the hardest boundary jet



Radiation patterns between the jets

- Shows the effect of turning on first the underlying event, then the colour connection between the leading jets.
- Rise as ϕ->π is caused by geometry - region at ϕ=π is simply nearer the “softer” jet on the other side of the “gap.”
- Softer jet hadronises/showers into the region we are looking at.
- Colour connection **enhances** this showering effect

Shows the ET weighted ϕ distribution of the KT0.1 jets



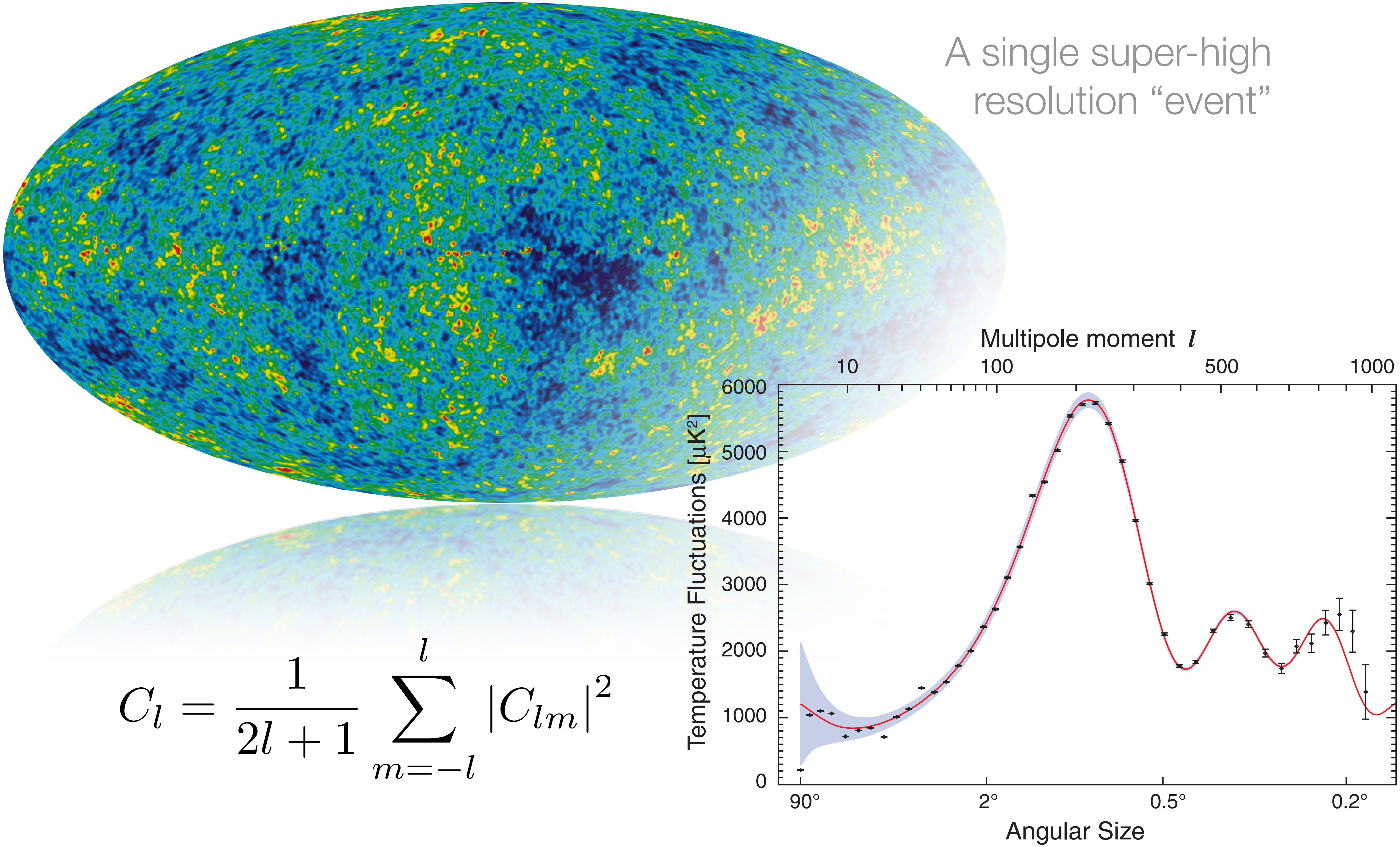
Different scales

- So there are objects of (at least) three different scales:
  - The jets (R~0.5)
  - The underlying event (fills the whole event)
  - Colour connection effects plus showering from one side of the gap into the other - an intermediate scale

We're trying to distinguish structures of different scale. Does that remind us of anything?

(probably lots of things, but this is what first came to my mind!)

A single super-high resolution “event”



$$C_l = \frac{1}{2l+1} \sum_{m=-l}^{l} |C_{lm}|^2$$

CMB decomposed into spherical harmonics

WMAP data

http://map.gsfc.nasa.gov/resources/featured_images_5yr_release.html

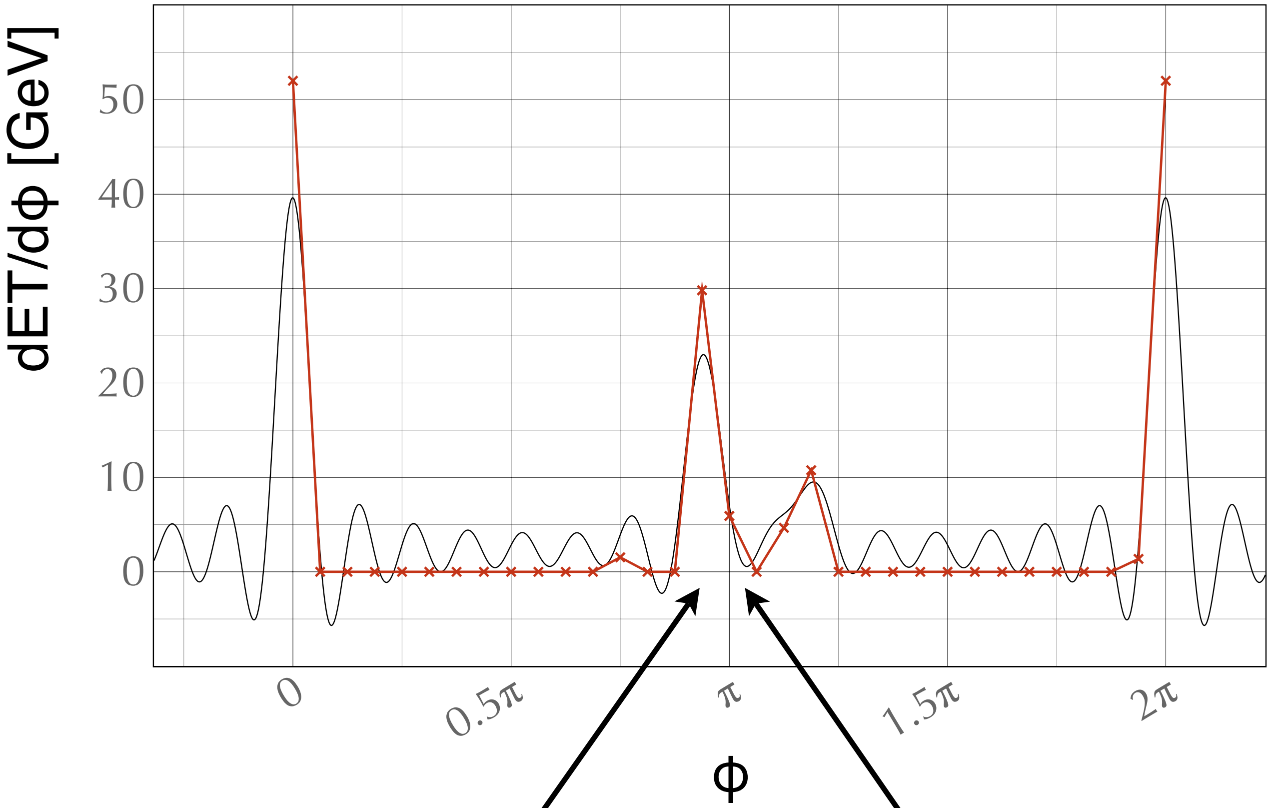
Applying something like that to hadron colliders

- Spherical harmonics no good for detectors - they are cylindrical. Use cylindrical co-ordinates instead.
- Work in 1 dimension to begin with. ɸ is bounded but η is not, so we started by just decomposing the ɸ ET flow.
- Slice 2π into 32 segments around the “detector.” In each segment/bin calculate the ET sum from the KT 0.1 jets
- **Important:** Choose the +ve ɸ direction to be such that the second jet is always between {0, π}. This means each event has the same origin/phase.
- Run fast Fourier transform routine on the ET values in the 32 bins in ɸ
- First look at a single event to make sure we get the Fourier transform correct

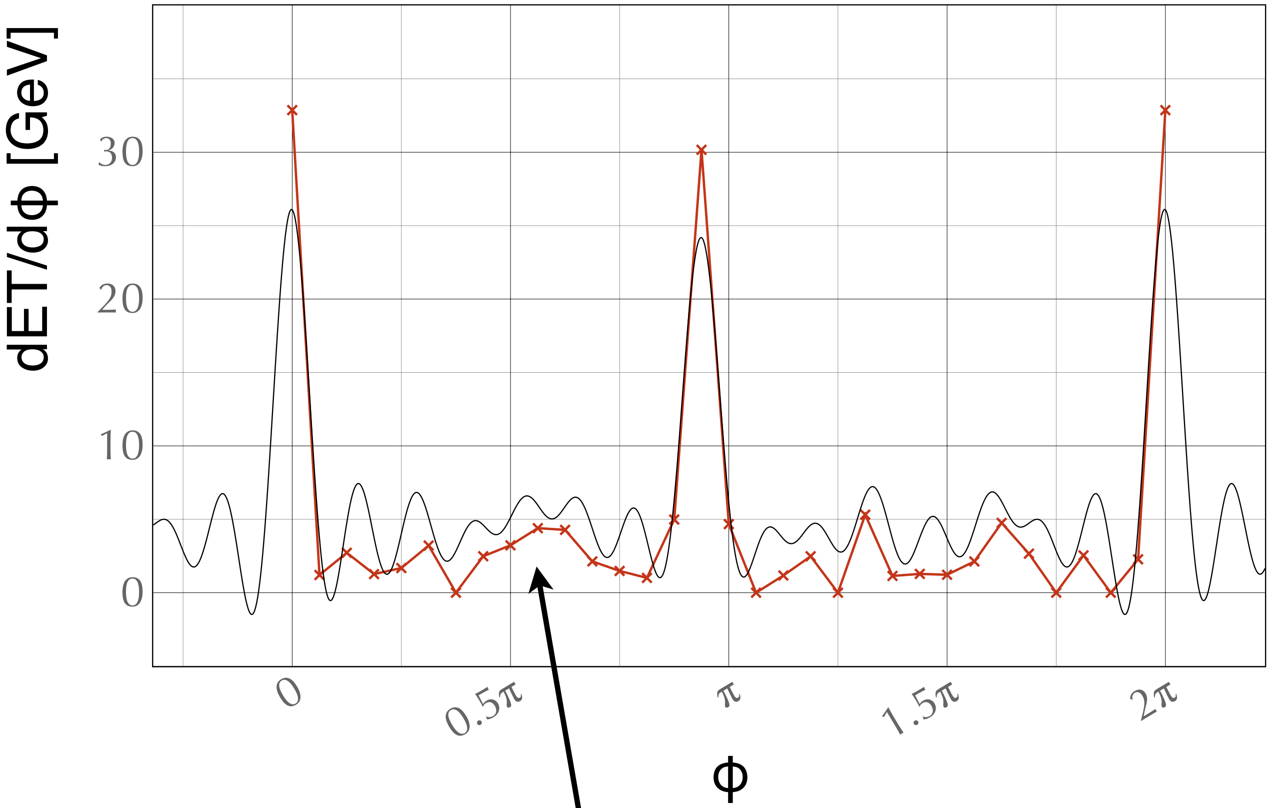
Fourier decomposition of a single event

Plot curve obtained from summation of Fourier terms (black) over input grid of ET bins (red)

Colour singlet exchange, no underlying event



QCD 2->2 with underlying event



Note second jet is
always in region ϕ < π

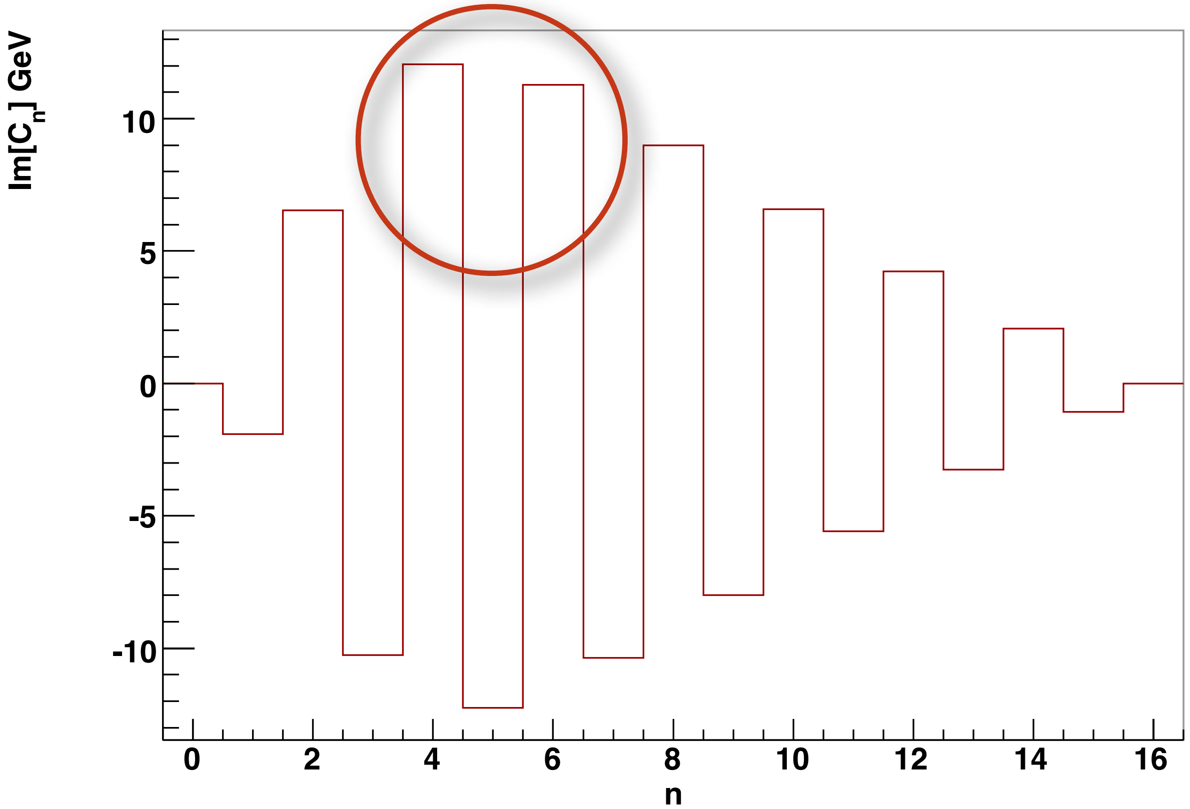
Second jet split in two

FT output matches
radiation between jets

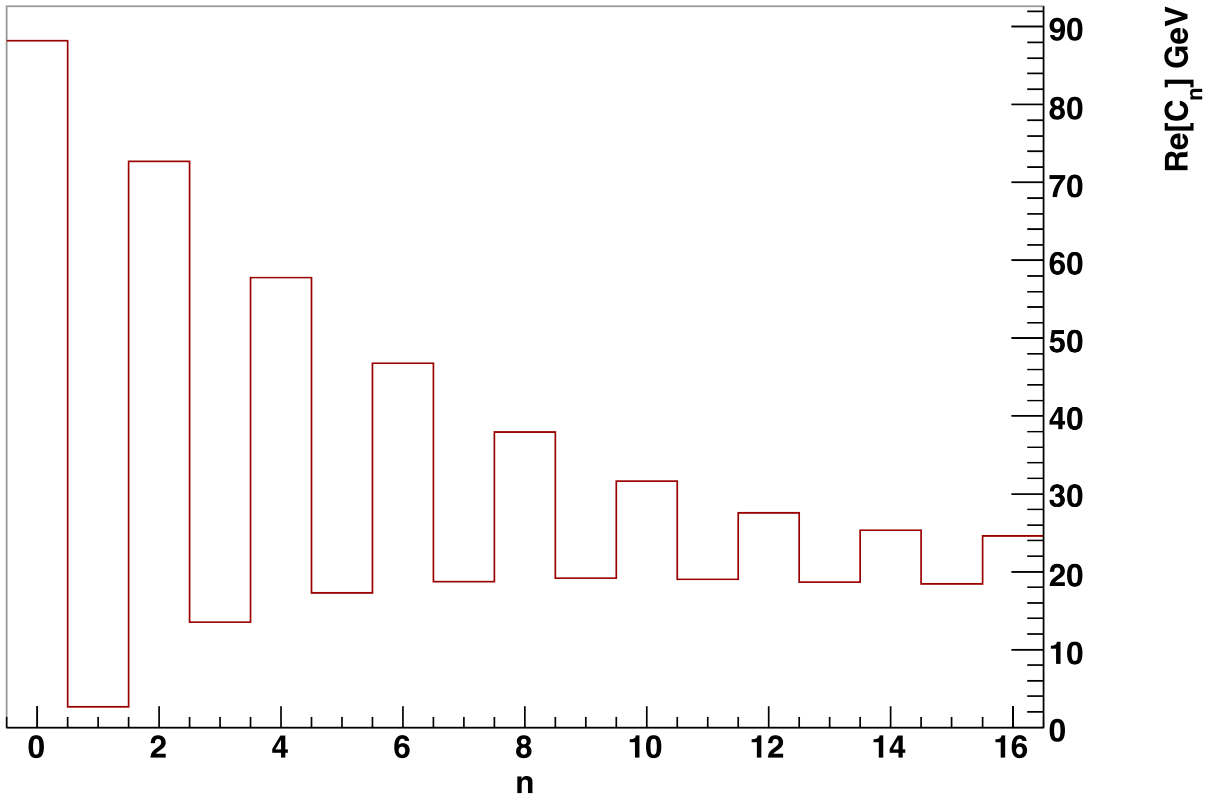
Shows Fourier Transform works for events with little inter-jet radiation or with radiation populating the inter-jet region

Look at the average of the coefficients over many (5M generated) events

Imaginary part of the coefficients



Real part of the coefficients



Simplest pure di-jet case. Colour singlet exchange, no underlying event.

There is a symmetry $C_{N-n} = C_n^*$ (n>16 not shown here)

The odd terms are suppressed.

Symmetry between n and N-n coefficient

- If there are N coefficients (N=32 here) then $C_n = C_{N-n}^*$

$$C_n = \sum_{l=0}^{N} E_T\left(\frac{2\pi l}{N}\right) e^{\left(in\frac{2\pi l}{N}\right)}$$

$$C_{N-n} = \sum_{l=0}^{N} E_T\left(\frac{2\pi l}{N}\right) e^{\left(-in\frac{2\pi l}{N}\right)} \underbrace{e^{\left(iN\frac{2\pi l}{N}\right)}}_{1}$$

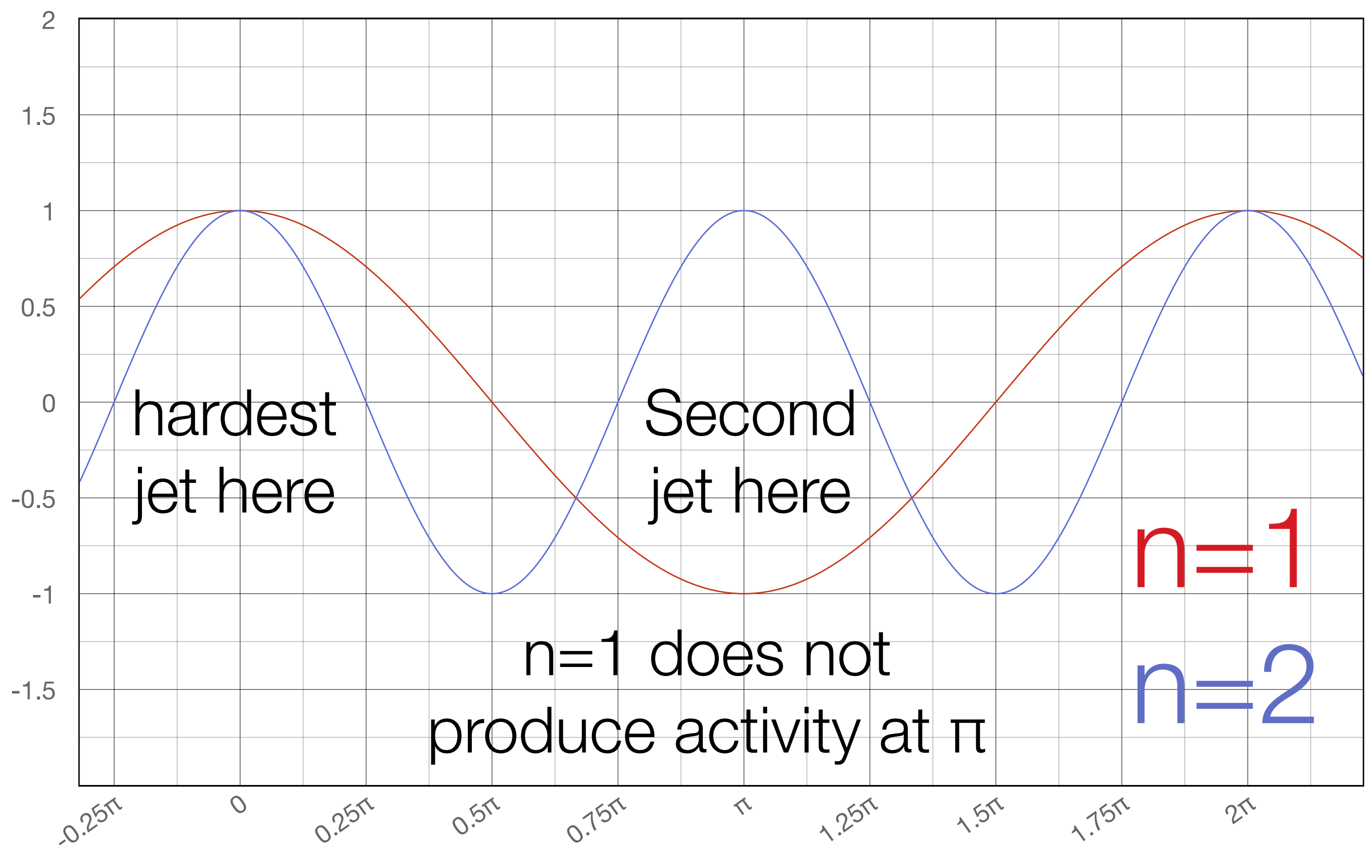
$$= C_n^*$$

So there are 16 independent complex coefficients

Suppression of odd coefficients

- The n^{th} coefficient corresponds to features of size ~π/n

- The odd coefficients can never have a peak at both 0 and π



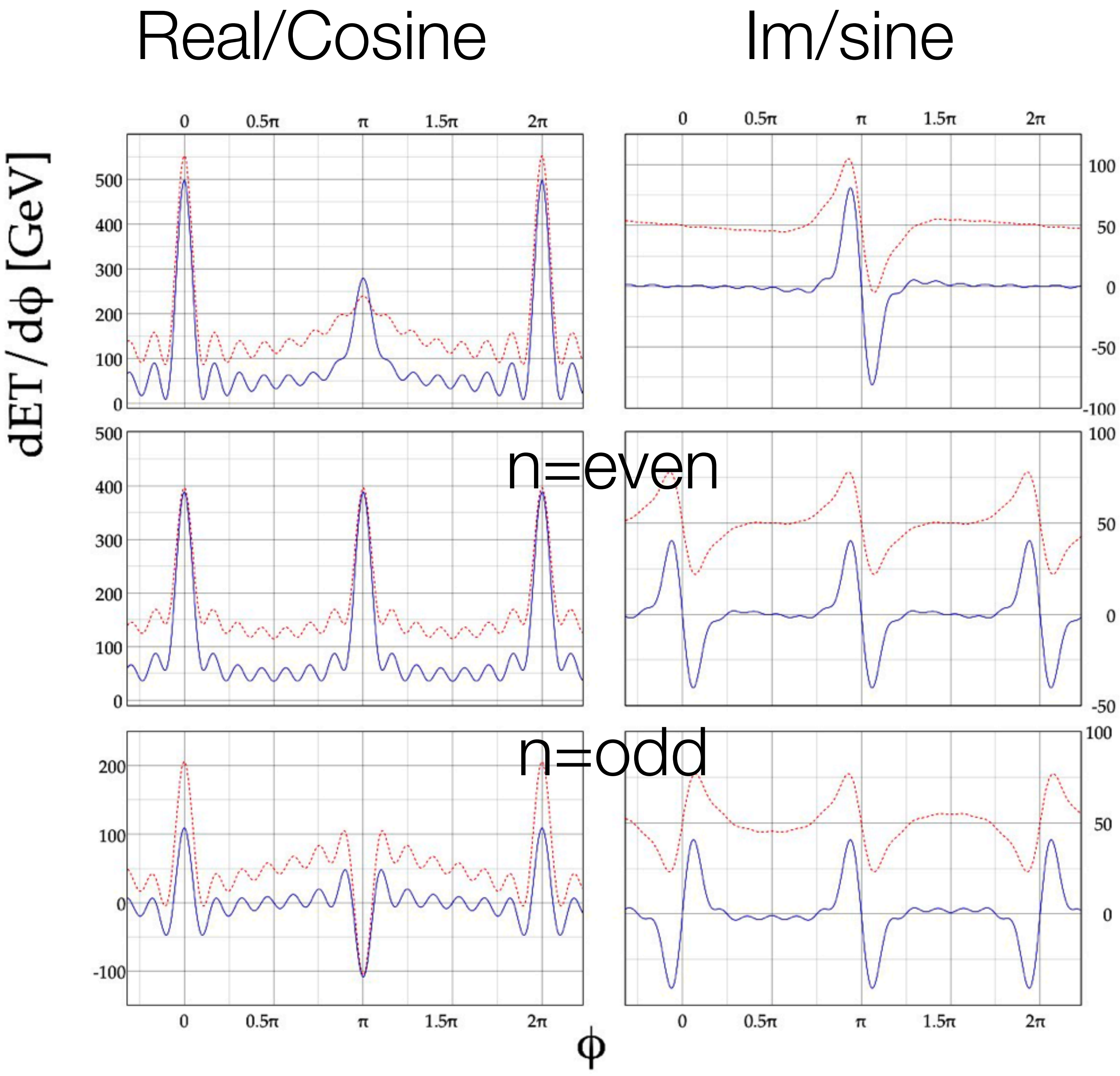
hardest jet here

Second jet here

n=1 does not produce activity at π

So the small and odd coefficients correspond to features that are large and not di-jet like!

in this case CSE with no underlying event is very di-jet like, so the odd coefficients are suppressed

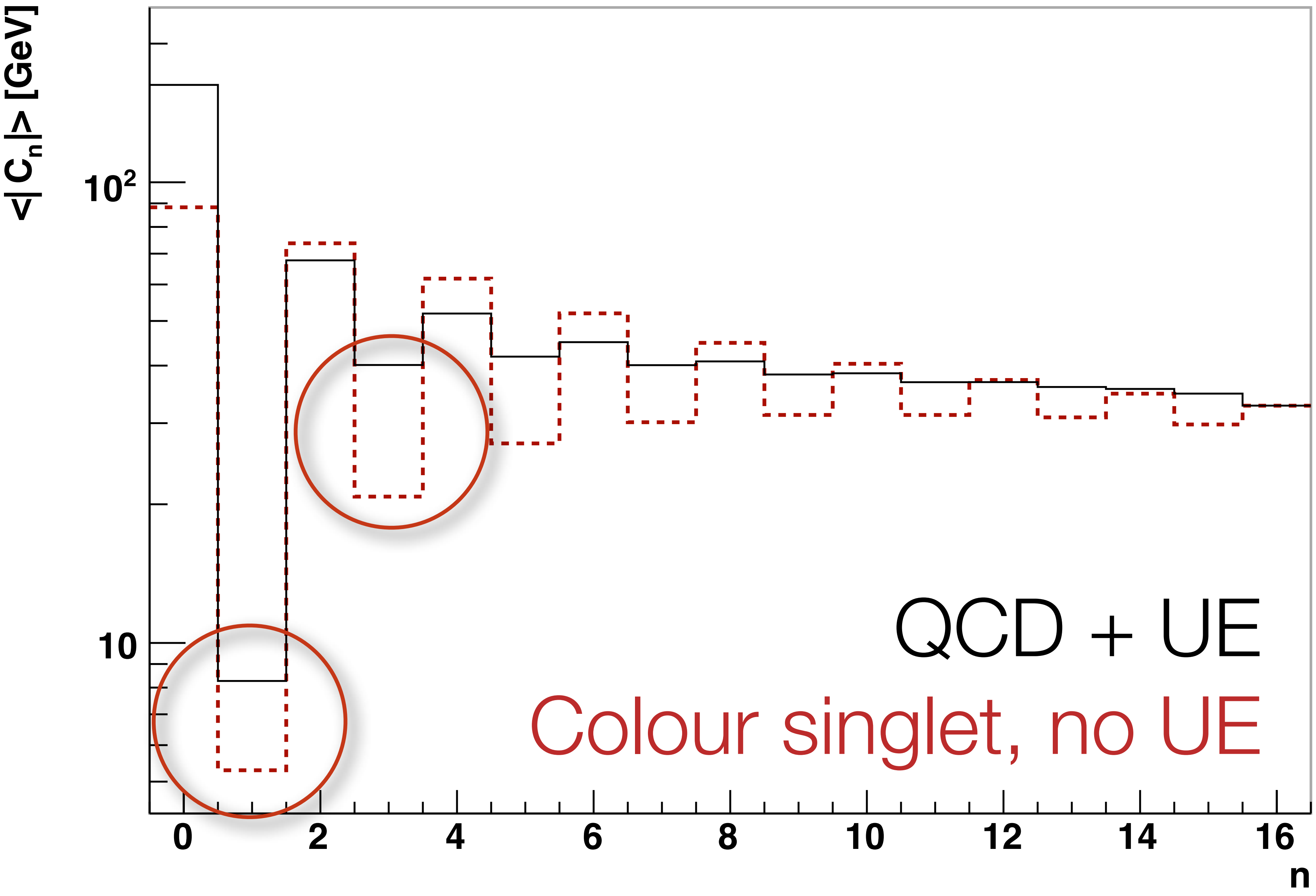


Colour singlet, no UE

QCD + UE

- See that the real and even part corresponds to exact back-to-back dijets
- The real and odd part is the difference in ET between the leading and second jet. Also broadens the second jet w.r.t the leading jet.
- The imaginary/sine terms **only** affect the second jet!
- The separate even and odd imaginary terms correspond to symmetric/anti-symmetric inter-jet radiation

Applying the inverse transform to separate real/imaginary/even/odd terms



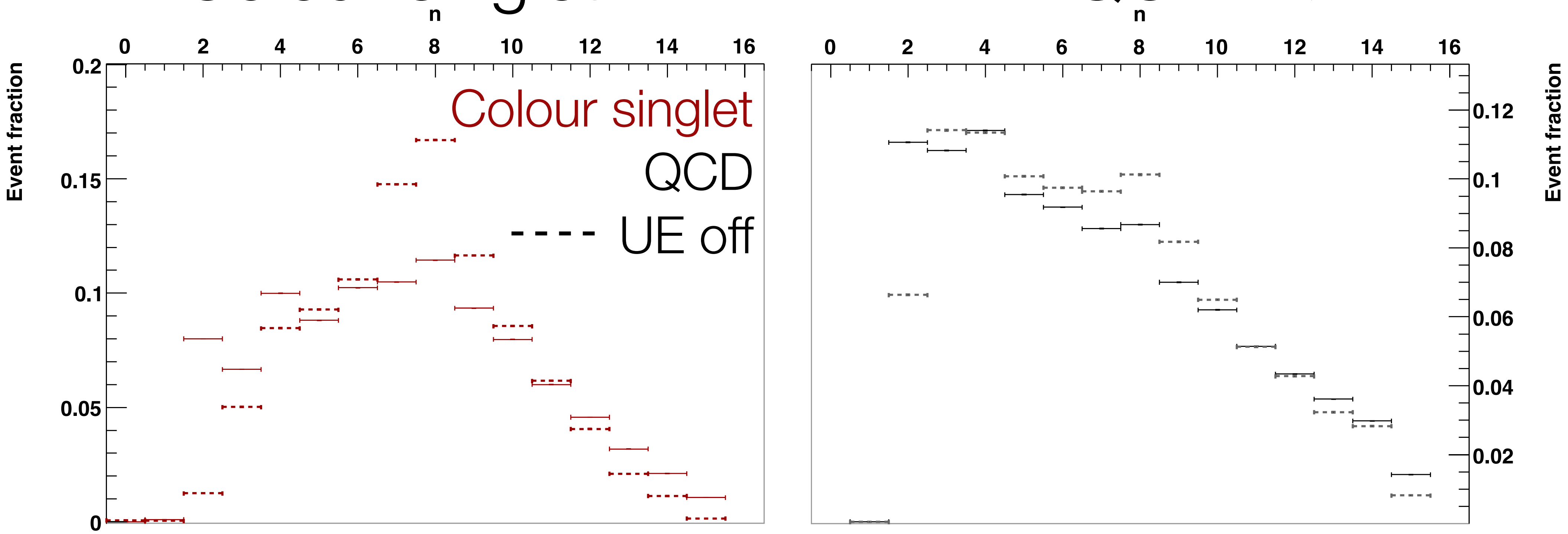
Comparing the average magnitude of the Fourier coefficients for QCD and colour singlet

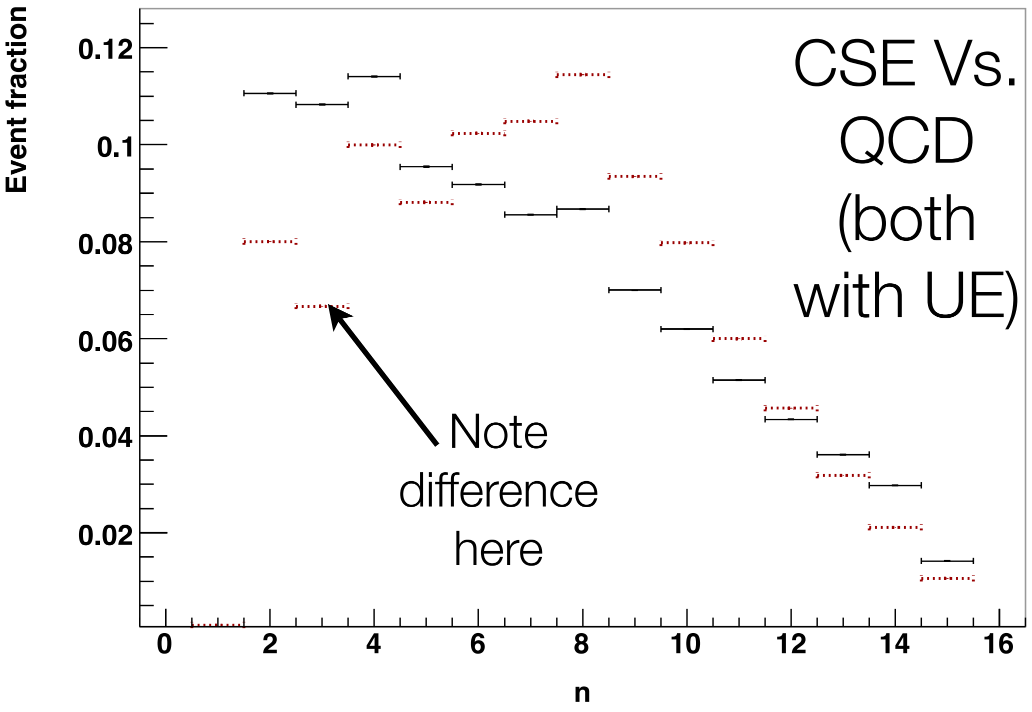
Less suppression of the odd coefficients with QCD 2->2 indicates those events are less like exact di-jets

Coefficient with the largest imaginary component

Colour singlet

QCD 2->2





Note difference here

Imaginary part = away from hardest jet

Largest imaginary part = dominant feature away from leading jet

Size of feature with n=8 is R~0.4

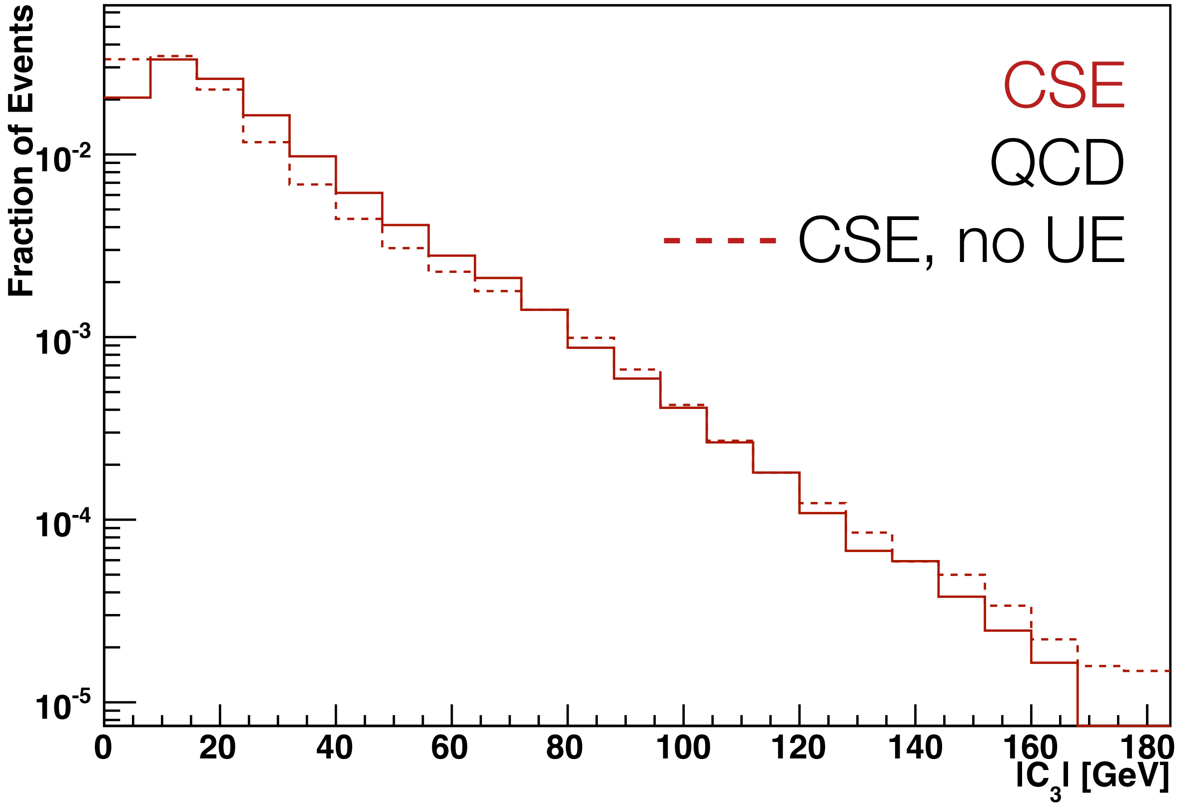
QCD showering and UE favour lower coefficients

3rd and 5th coefficients

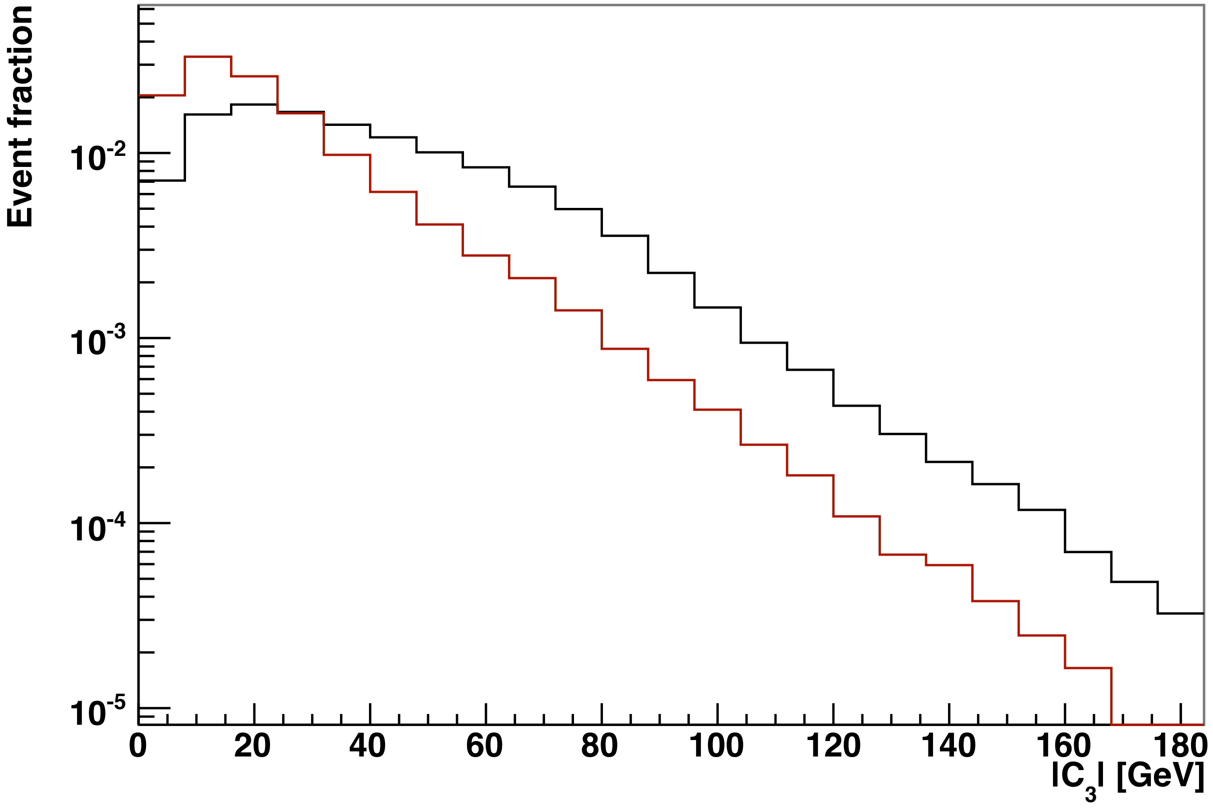
- Note the relative **depletion** in the **3rd** and **5th** coefficients for the colour singlet + UE sample.
- Remember the odd coefficients are **non-dijet** like
- The **n=1** coefficient will likely be populated by the **underlying event**, but the **n=3** and **n=5** *should* be less affected by underlying event yet hopefully still show **differences** from **hadronisation/colour connection** etc.
- Lets take a close look at the **3rd Coefficient...**

Magnitude of the 3rd Fourier coefficient

Colour singlet exchange
with and without UE



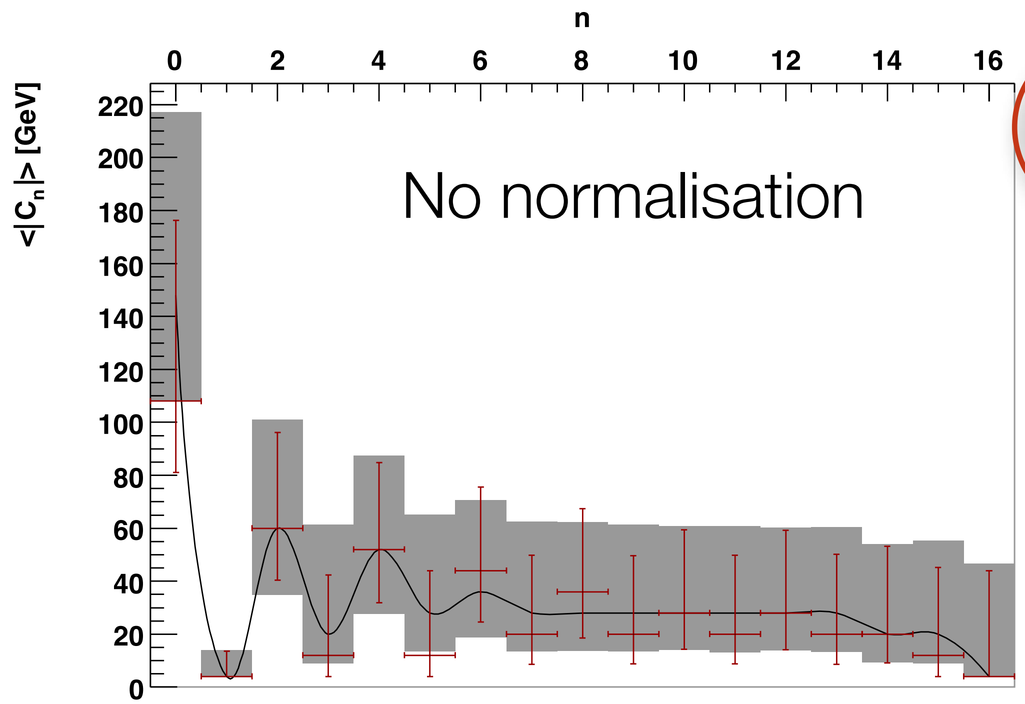
Colour singlet exchange
compared to QCD jets

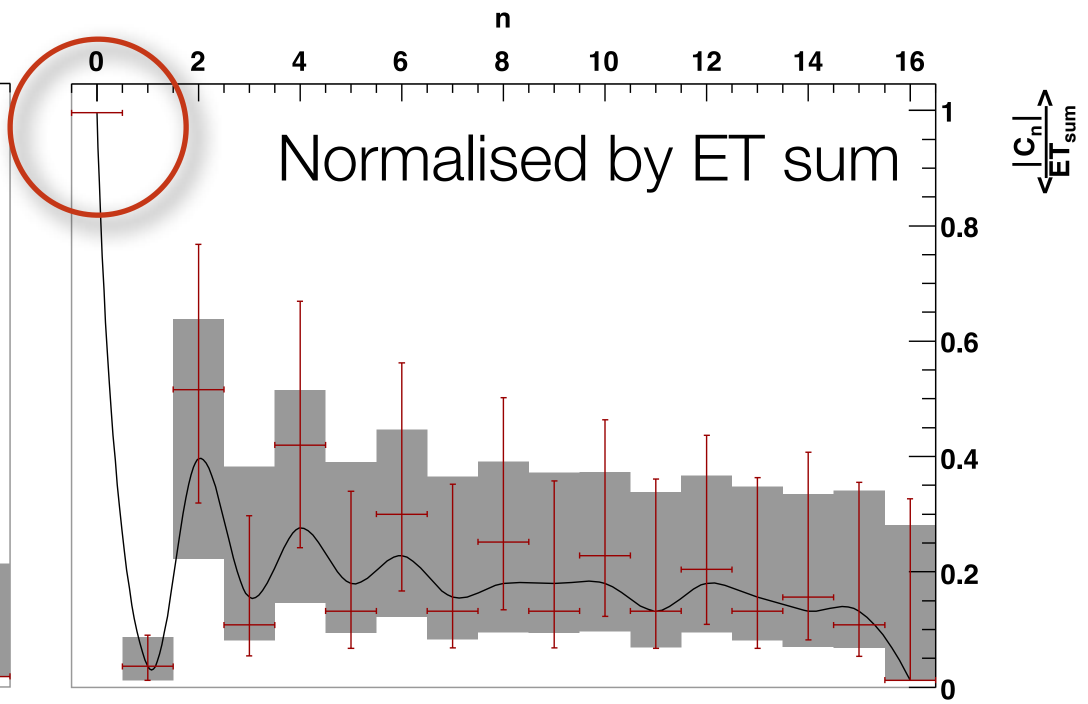


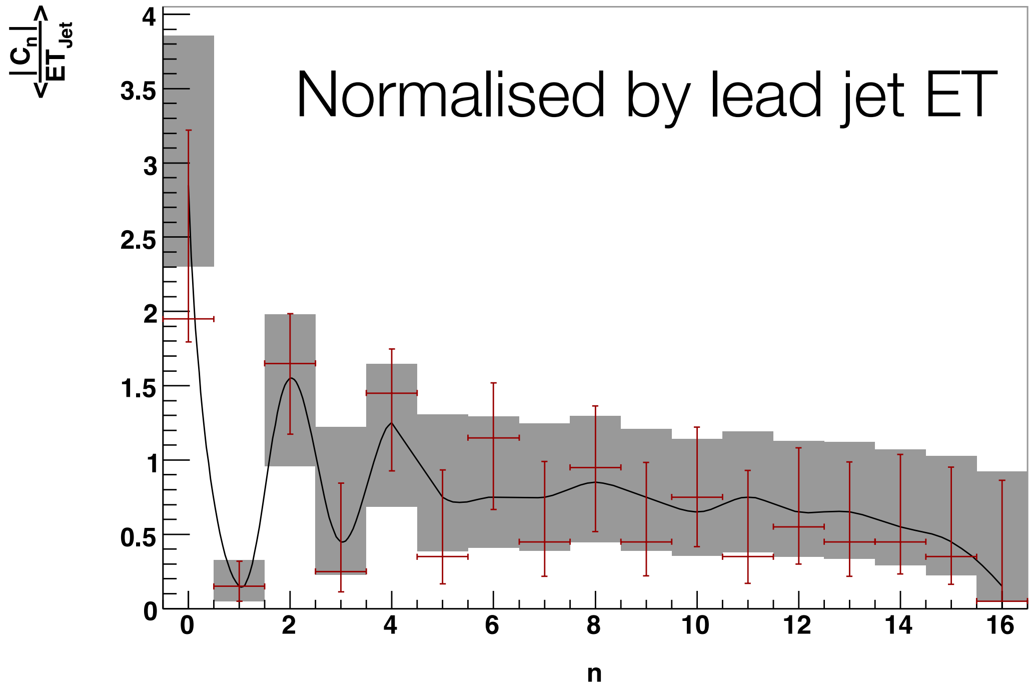
Note that the magnitude of the third coefficient barely changes with the addition of underlying event activity, but there is a clear difference between colour singlet and QCD jet production. QCD jet production, with the colour connection between the jets, favours a larger C3 due to the colour connection enhancing the radiation.

C3 is not very sensitive to the underlying event,
but can show differences in hadronisation models!

Accounting for the spread of ET







The spread of coefficients about the median value is partly due to differences in event shape, partly because there is a spread of jet ET values.

Can divide the coefficients by either the ET sum in the event or the ET of the leading jet to try and focus only on the effect of different event shapes.

Likelihood ratio

- From the previous distributions determine the probability distribution function for each coefficient for signal and background. P_n(signal) and P_n (BG)

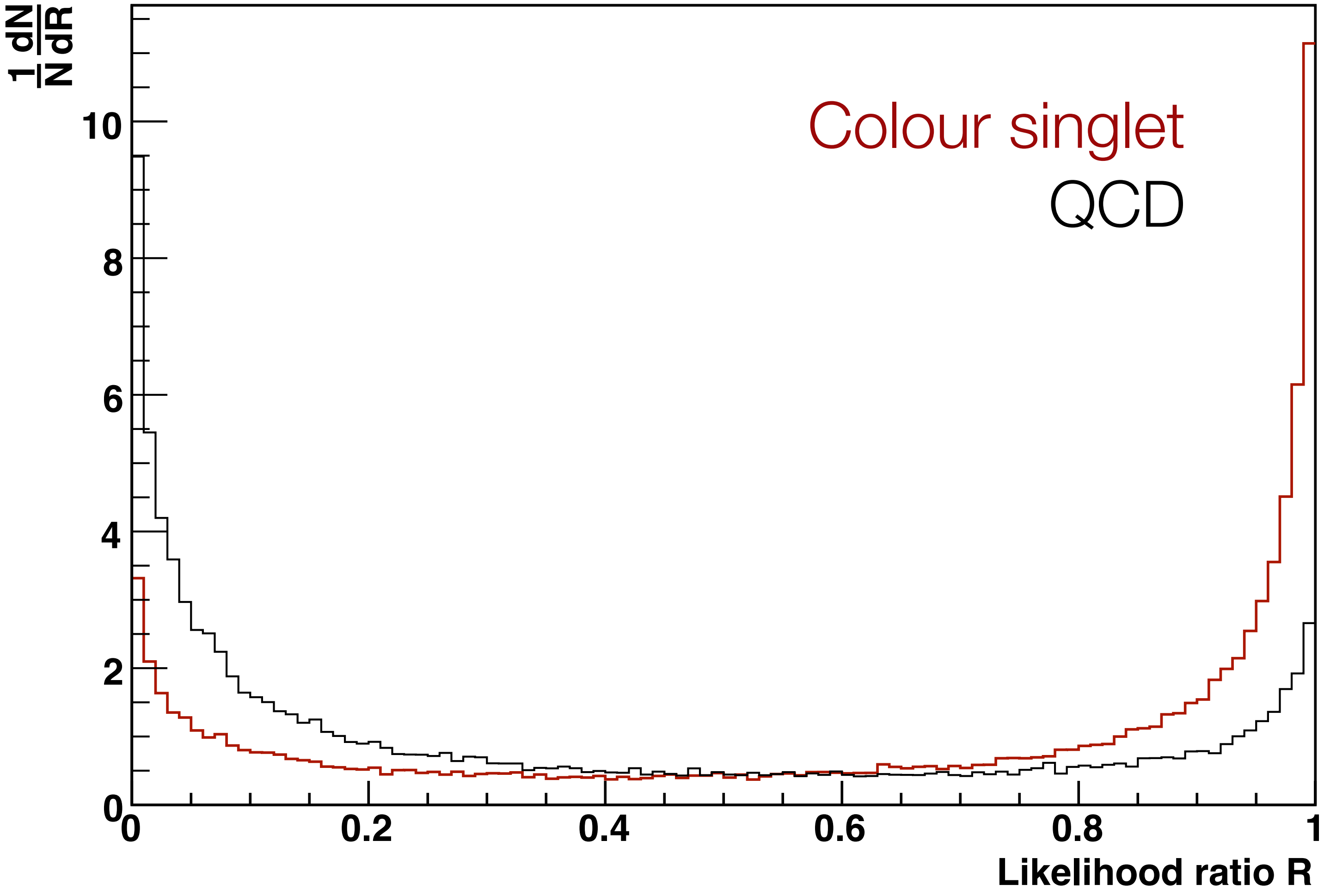
- On an independent sample of events determine for each event:

$$P(\text{signal}) = \prod_{n=0}^{16} P_n(\text{signal})$$

$$P(\text{BG}) = \prod_{n=0}^{16} P_n(\text{BG})$$

$$R = \frac{P(\text{signal}}{P(\text{signal}) + P(\text{BG})}$$

Likelihood ratio for the distribution of **magnitudes** of the 16 Fourier coefficients



Both **with** underlying event

Conclusion

- Identifying diffractive events is a problem of identifying features of different physical scale in the event.
- Fourier decomposition is a natural way to separate objects of different sizes.
- The effect of different features such as underlying event, hadronisation, jets and mini-jets are confined to separate regions of the coefficient space.
- “Gaps” present in diffractive events should appear as a depletion in certain coefficients, which are not necessarily the ones affected by pile-up or underlying event.
- To unlock the full power, suspect a 2D decomposition is needed (the gap is in η!), but there are issues with that (fixing phase of event in an unbounded η co-ordinate).
- Likely applications way beyond diffraction - underlying event studies, generator tuning, high jet multiplicity events.