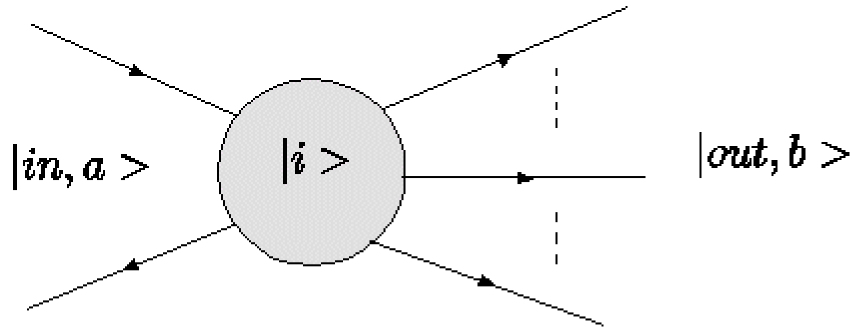
INFRARED SUBTRACTION SCHEMES

1. IR subtraction schemes
2. NNLO IR subtraction schemes
3. Conclusions and outlooks

Paolo Bolzoni – ACAT 2010, Jaipur, 26th Feb. 2010

1. IR SUBTRACTION SCHEMES

THE ORIGIN OF IR SINGULARITIES



Here the Hamiltonian is given by

$$H = H_0 + gH_1$$

$$|\text{in}, \text{a}> \; = \; (U_-^\dagger)_{ai}|i> = (U_-^\dagger)_{ai}(U_+)_{ib}|\text{out}, \text{b}> \equiv \text{S}_{\text{ab}}|\text{out}, \text{b}>$$

$$(U_+)_{ij} = \delta_{ij} + g\frac{<\text{out}, \text{j}|\text{H}_1|\text{out}, \text{i}>}{E_i - E_j}(1 - \delta_{ij}) + O(g^2)$$

(for U_- substitute |out> with |in>)

Divergences in ε=0 arises when it happens that for some finale stafe |j> , $E_j = E_i$ i.e. when one or more states are degenerate

One handles these singularities introducing a regulator ε such that

$$E_j(\epsilon) \neq E_i(\epsilon)$$

for every ε ≠ 0 and j ≠ i

THE KINOSHITA-LEE-NAUENBERG THEOREM

[T.Kinoshita (1962); T.D. Lee, M. Nauenberg (1963)]

The transition probability is obtained performing the square modulus of the amplitute (S matrix):

$$P_{a\to b} = |S_{ab}|^2 = \sum_{ij}[(U^*_-)_{ia}(U_-)_{ja}][(U_+)_{ib}(U^*_+)_{jb}]$$

This is not IR safe and hence not a physical quantity

The KLN theorem states that an IR safe physical quantity can be obtained summing over all degenerate states at the same order in perturbation theory

For example let us concentrate on the finale state, defining D(E_b) as the set of states with degenerate energy E_b and summing over it

$$P^{phys}_{a\to b} = \sum_{D(E_b)} P_{a\to b} = \sum_{i,j}[(U^*_-)_{ia}(U_-)_{ja}] \underbrace{\sum_{D(E_b)} [(U_+)_{ib}(U^*_+)_{jb}]}$$

$$= \sum_{D(E_b)} \{\delta_{ib}\delta_{jb} + g(H_1)_{ib}[\frac{\delta_{jb}(1-\delta_{ib})}{E_i(\epsilon)-E_b(\epsilon)} + \frac{\delta_{ib}(1-\delta_{jb})}{E_j(\epsilon)-E_b(\epsilon)}]\} = \ldots$$

A FIRST ORDER PROOF

$$\sum_{D(E_b)} [(U_+)_{ib}(U_+^*)_{jb}] = \sum_{D(E_b)} \{\delta_{ib}\delta_{jb} + g(H_1)_{ib}[\frac{\delta_{jb}(1-\delta_{ib})}{E_i(\epsilon)-E_b(\epsilon)} + \frac{\delta_{ib}(1-\delta_{jb})}{E_j(\epsilon)-E_b(\epsilon)}]\} =$$

$i = j$ → $$= \sum_{D(E_b)} \delta_{ib}$$

$i \neq j$:

- $i \in D(E_b), j \notin D(E_b)$ → $$= g\frac{(H_1)_{ij}}{E_j(\epsilon)-E_i(\epsilon)}$$
- $i \notin D(E_b), j \in D(E_b)$ → $$= g\frac{(H_1)_{ij}}{E_i(\epsilon)-E_j(\epsilon)}$$
- $i \in D(E_b), j \in D(E_b)$ → $$= g\frac{(H_1)_{ij}}{E_i(\epsilon)-E_j(\epsilon)} + g\frac{(H_1)_{ij}}{E_j(\epsilon)-E_i(\epsilon)} = 0$$

In all cases the limit in ε=0 is well defined. In the last case you can see explicitely the cancellation of divergences

OBSERVATIONS

1. **We saw the first order but the argument proove can be extended at any order of perturbation theory**

   [T.Kinoshita (1962); T.D. Lee, M. Nauenberg (1963)]

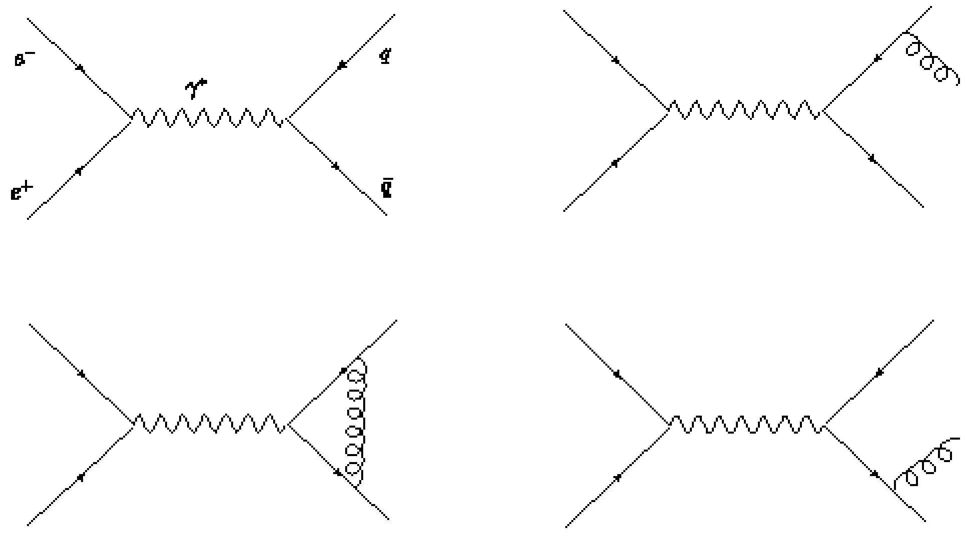
2. **Quantum field theory is nothing else but quantum mechanics applied to an infinite number of degrees of freedom (each spacial point of a field labels a dynamical quantum variable) and KLN theorem has a general validity**

3. **In QFT the case where you can have degenerary accurs when you have a virtual or a collinerar/soft real emission**

THE KLN THEOREM IN QUANTUM FIELD THEORY

$$\sigma = \sigma^{LO} + \alpha_s \sigma^{NLO} + \dots$$

Consider for example the NLO QCD contributions to the process: $e^+e^- \to Q\bar{Q}$



1. According to the KLN theorem the mass singularities cancel out at a fixed order in perturbation theory

2. The singularities from virtual quantum corrections are cancelled by the real emission contributions in the region where extra partons are unresoved

$$\sigma^{NLO} = \sigma^V + \int d\sigma^R$$

Even if the two contributions are separately divergent, their sum is finite and this is a numerical problem.

A TOY EXAMPLE

Consider the following toy example where the PS of the real emitted parton is parametrized by a single parameter x$\in$ (0,1):

$$\sigma^V = \frac{S(0)}{\epsilon} + F^V \qquad\qquad d\sigma^R(x) = x^{-1-\epsilon}\, S(x)\, dx$$

The NLO correction is given by the virtual correction plus the real correction integrated over the phase space of the radiated parton

$$\sigma^{NLO} \;=\; \sigma^V + \int_0^1 d\sigma^R(x)$$

What a substraction scheme wants to do is to make the cancellation explicit and to render the integral finite and well defined in the limit to ε=0

WHAT A SUBTRACTION SCHEME DO

$$\sigma^{NLO} = \sigma^V + \int_0^1 d\sigma^R(x) = \frac{S(0)}{\epsilon} + F + \int_0^1 x^{-1-\epsilon} S(x)\, dx$$

$$= \frac{S(0)}{\epsilon} + F + \int_0^1 dx\, x^{-1-\epsilon} [S(x) - S(0)] + \int_0^1 dx\, x^{-1-\epsilon} S(0)$$

$$= \frac{S(0)}{\epsilon} + F + \int_0^1 \left[\frac{S(x) - S(0)}{x}\right] dx - \frac{S(0)}{\epsilon} + O(\epsilon)$$

$$= F + \int_0^1 dx\, S(x) \left[\frac{1}{x}\right]_+ dx + O(\epsilon),$$

Now the two terms that survive at ε=0 are both finite and the integral can be computed using standard numerical techniques

This has been achieved adding and subtracting a counterterm $d\sigma^A$ (in this example $d\sigma^A = x^{-1-\epsilon} S(0)$) in the second line and integrating it explicitely in the third.

In general the choice of the counterterms is arbitrary under these constaints:

(i) The singular soft and/or collinear behavior of the real cross section is reproduced
(i) Their integration over the PS of the unresolved partons is factorized
(ii) It respects the delicate mechanism of cancellation between real and virtual contributions

Each choice of the counterterms defines an IR subtraction scheme and choosen one their integration can be done once and for all

Finally acording to the observable one is interested in one can also introduce a jet function

2. NNLO IR SUBTRACTION SCHEMES
(State of art)

WHY LOOKING FOR NEW SUBTRACTION SCHEMES?

Existing subtraction methods can not straightforwardly be generalized to NNLO

[S. Weinzierl; M. Grazzini, S. Frixione];
[A. Gehrmann-De Ridder, T. Gehrmann, E.W.N. Glover, G. Heinrich]

This new subtraction scheme is worked out for colorless incoming particle at NNLO and for hadronic initiated processes at NLO in an NNLO compatible way

[Z. Trocsanyi, G. Somogyi, V. Del Duca, Z. Nagy]

The proposed scheme can be generalized to any order in perturbation theory and is based on simple separation of soft and collinear singularities due to new phase space mappings

[Z. Trocsanyi, G. Somogyi, Z. Nagy]

NNLO computations are very long and complicated and we wish to have an independent method of evaluation of these challenging observables (e.g. for the NNLO $e^+e^- \to$ 3 jets)

[S. Weinzierl; A. Gehrmann-De Ridder, T. Gehrmann, E.W.N. Glover, G. Heinrich]

THE NNLO COUNTERTERMS IN THE TS SCHEME

$$\sigma^{NNLO} = \int_{m+2} d\sigma^{RR}_{m+2} J_{m+2} + \int_{m+1} d\sigma^{RV}_{m+1} J_{m+1} + \int_{m} d\sigma^{VV}_{m} J_{m}$$

$-d\sigma^{RR,A_1}_{m+2}$	**Regularizes the doubly-real cross section in the singly-unresolved region of the phase space**
$-d\sigma^{RR,A_2}_{m+2} + d\sigma^{RR,A_{12}}_{m+2}$	**Regularizes the doubly-real cross section in the doubly-unresolved region of the phase space and avoid the double subtraction**
$-d\sigma^{RV,A_1}_{m+1}$	**Regularizes the real-virtual cross section in the singly-unresolved region of the phase space**
$-\left(\int_1 d\sigma^{RR,A_1}_{m+2}\right)^{A_1}$	**Regularizes the first counterterm integrated over one-unresolved parton when the other one becomes also unresolved**

THE INTEGRATION OF THE COUNTERTERMS:
the state of art of analytic computation

$$\int_1 d\sigma_{m+2}^{RR,A_1}$$

Collinear, soft and collinear-soft integrals already computed analytically

[Z. Trocsanyi, G. Somogyi, V. Del Duca]

$$\int_1 d\sigma_{m+1}^{RV,A_1}$$

Collinear, soft and collinear-soft integrals: computed

[U. Aglietti, V. Del Duca, C. Duhr, G. Somogyi, Z. Trocsanyi; P.Bolzoni]

$$\int_1 \left(\int_1 d\sigma_{m+2}^{RR,A_1}\right)^{A_1}$$

Nested integrals: analytic computation finished

[P. Bolzoni, S. Moch, G. Somogyi, Z. Trocsany]

Numerical evaluation of all the singly-unresolved integrals computed numerically using sector decomposition **(T. Binoth, G. Heinrich)**

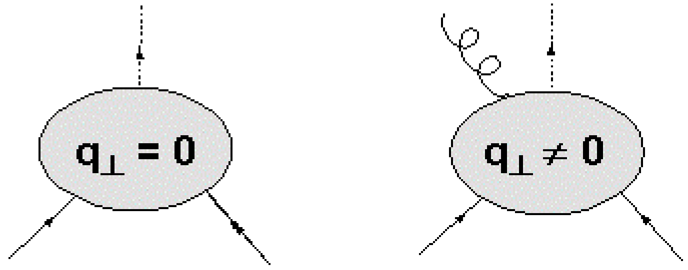
[Z. Trocsanyi, G. Somogyi,]

$$\int_2 \left[d\sigma_{m+2}^{RR,A_2} - d\sigma_{m+2}^{RR,A_{12}}\right]$$

Work in progress

[P. Bolzoni, G. Somogyi]

ANOTHER SUBTRACTION SCHEME UNDER INVESTIGATION:



Consider for example the transverse momentum distribution for the Higgs boson production in the s-channel. Here we use the $q_\perp$ to regulate the IR divergences

[S. Catani, M. Grazzini]

At LO $d\sigma_{\mathrm{LO}} = \frac{\sigma_0}{\pi}\delta(q_\perp^2)d^2q_\perp$ **and hence the transverse momentum of the Higgs is exaclty zero**

$$\Rightarrow \left\{ \begin{array}{l} d\sigma^H_{\mathrm{NLO}} = d\sigma^{H+1\mathrm{jet}}_{\mathrm{LO}} \\ d\sigma^H_{\mathrm{NNLO}} = d\sigma^{H+1\mathrm{jet}}_{\mathrm{NLO}} \end{array} \right\}$$

for $q_\perp \neq 0$

If we suppose that the IR divergences in $d\sigma^{H+1\mathrm{jet}}_{\mathrm{NLO}}$ **are treated using available NLO subtraction methods at NNLO we have only to peform a second subtraction and to add a contribution at $q_\perp = 0$ to obtain the correct total cross section:**

$$d\sigma^H_{\mathrm{NNLO}} = H\, d\sigma_{\mathrm{LO}} + [d\sigma^{H+1\mathrm{jet}}_{\mathrm{NLO}} - d\sigma^{CT}]$$

The counterterm can be constructed expanding the transverse resummation xpressions which encode the singular behavior at $q_\perp$=0

3. CONCLUSIONS AND OUTLOOKS

CONCLUSIONS AND OUTLOOKS

1. We have established a systematic method for the integration of counterterms in the TS scheme

   - MB representation and subsequent summation (up to triple sums)
   - Real-virtual and iterated integrated counterterms computed
   - Doubly-real integrated counterterms is on the way

3. We study couterterms at NNLO in the universal process-independent TS subtraction scheme and with a $q_{\perp}$-ispired scheme to have hopefully a more efficient set up than other existing methods

5. A NNLO subtraction scheme for hadron-initiated QCD processes and for heavy quark production are the future natural steps

BACK-UP SLIDES

WHAT IS A SUBTRACTION SCHEME

To summarize what we did in the previous example was:

1. The subtraction from the contribution of the real emission of the COUNTERTERM:

$$d\sigma^A(x) = x^{-1-\epsilon}\, S(0)\, dx$$

which reproduces the sigular behaviour in the strict unresold (soft and/or collinear) limit of the emitted parton:

2. The readdition of the same counterterm integrated over the phase space of the emitted parton:

$$\int d\sigma^A(x) = \int_0^1 x^{-1-\epsilon} S(0)\, dx = -\frac{S(0)}{\epsilon} + O(\epsilon)$$

which therefore should also be integrable over the all phase space of the emittied parton

AN EXAMPLE

$$\mathcal{E}(x;\epsilon,d_0) = x^2 \int_0^1 d\alpha \, \frac{\alpha^{-1-\epsilon}\,(1-\alpha)^{2d_0}}{[\alpha+(1-\alpha)]^{-1-\epsilon}\,[2\alpha+(1-\alpha)x]^{-1}} =$$

$$= \int_{q_1-i\infty}^{q_1+\infty} \frac{dz_1}{2\pi i} \int_{q_2-i\infty}^{q_2+i\infty} \frac{dz_2}{2\pi i}\, 2^{z_2}\, x^{-\epsilon-z_1-z_2}$$

$$\times\Gamma\left(\begin{array}{c} -z_1,\, -z_2,\, 2d_0-1-\epsilon-z_1-z_2,\, 1+\epsilon+z_1,\, 1+z_2,\, -\epsilon+z_1+z_2 \\ 2d_0-1-2\epsilon,\, 1+\epsilon \end{array}\right)$$

In this case for $d_0 \geq 3$ a good choise is q_1=-1/4, q_2=-1/8,ε=-1/2

Computing residua to perform the analytic continuation to ε=0, expanding in ε and converting MB integrals into sums, one gets:

$$\mathcal{E}(x;\epsilon,d_0) = -\frac{1}{\epsilon} - \log(2)\Sigma_0(x,d_0) + \log(x)\Sigma_1(x,d_0) - \Sigma_2(x,d_0)$$

$\Sigma_0(x,d_0)$,$\Sigma_1(x,d_0)$and $\Sigma_2(x,d_0)$ are double sums → **The extraction of poles comes out in a convenient way**

SUMS

To have an ides of the sums involved let's look at one sum of the example

$$\Sigma_2(x,d_0) = \sum_{m,n=1}^{\infty} \left(\frac{x}{2}\right)^m x^n \binom{2d_0-2+m+n}{m+n} \left[S_1(2d_0-2+m+n) - S_1(m+n)\right]$$

If d_0 is chosen as a positive integer it happens that

$$\binom{2d_0-2+m+n}{m+n} \left[S_1(2d_0-2+m+n) - S_1(m+n)\right]$$

Is a polynomial in m and n, thus the sum can be expressed in terms of these functions

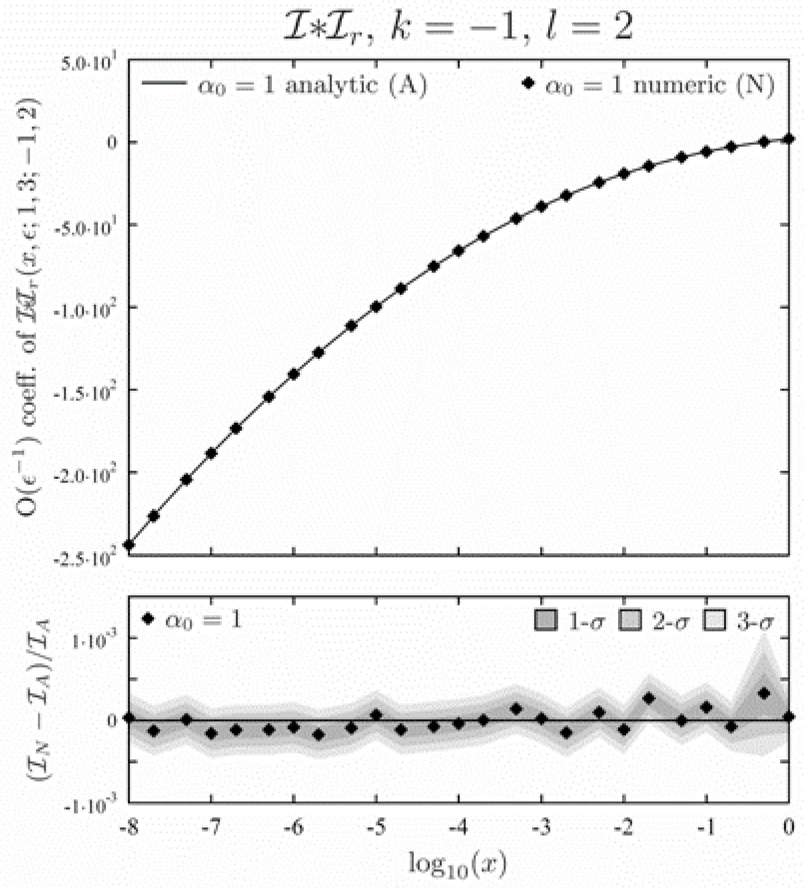
$$\sum_{n=1}^{\infty} \frac{x^n}{n^k} = \begin{cases} \mathrm{Li}_k(x) & \text{if } k \geq 0 \\ \frac{1}{(1-x)^{1-k}} \sum_{i=0}^{-k-1} \left\langle \begin{matrix} -k \\ i \end{matrix} \right\rangle x^{-k-i} & \text{if } k < 0 \end{cases}$$

This is a simple case, in general the sums are more complicated and we used XSummer

[XSummer: S. Moch, P. Uwer]

NESTED COLLINEAR INTEGRALS: Numerical and analytic results

Comparison between numeric and analytic results. The agreement is excellent. This analytic examples shows that the expansion coefficients of the integrals building the counterterms are are smooth functions



For practical purposes the integrals can be used in terms of interpolating tables

An available package SUMI.m evaluates all the singly-unresolved integrals from both the analytic results and from evaluation of MB integrals representations in the complex plane up to finite parts of the Laurent expansion

ANALYTIC AND NUMERICAL EVALUATION OF THE INTEGRATED COUNTERTERMS

1. The cancellations of all IR poles in this scheme is more convincingly once the structure of the integrated subtraction terms is exhibited analytically

2. The analytic results are very fast and very accurate compared to numerical ones

HOWEVER

we will see that the analytic results also show that the integrated counterterms consist of very smooth functions

HENCE

The final results for the integrated real-virtual counterterms can be conveniently given e.g. in the form of interpolating tables computed once and for all including efficiently also those cases (like the finite parts) for which the analytic results can not be carried out

COLLINEAR INTEGRALS:
analytic results

$$\mathcal{I}(x;\epsilon,d_0;1,k,\delta,g_I^{(\pm)}) = \frac{\delta_{k,-1}}{2(2-\delta)}\frac{1}{\epsilon^2} - \left[\frac{2\delta_{k,-1}\log(x)}{3-\delta} + \frac{1-\delta_{k,-1}}{2[1+k(1-\delta_{k,-1})]}\frac{1}{\epsilon}\right]$$

$$+\mathcal{G}_{I,k}^{(\pm)}(x) + \mathcal{F}(x;\epsilon,d_0,k) + O(\epsilon)$$

$I = C, D$

$k = -1, 0, 1, 2$

$$\mathcal{G}_{I,k}^{(\pm)}(x) = \begin{pmatrix} \left(\frac{2}{3}\pm\frac{1}{2}\right)\zeta_2 + \frac{1}{3}\log^2(x) & 1\pm\frac{1}{2} & \frac{1}{2}\pm\frac{3}{8} & \frac{13}{36}\pm\frac{11}{36} \\ \left(\frac{13}{36}\mp\frac{1}{16}\right)\zeta_2 + \left(\frac{1}{2}\pm\frac{1}{2}\right)\log^2(x) & 1\pm\frac{1}{2} & \frac{1}{2}\pm\frac{1}{8} & \frac{13}{36}\pm\frac{1}{18} \end{pmatrix}$$

$$\mathcal{F}(x;\epsilon,d_0=3,-1) = -\frac{3}{2}\zeta_2 + \log^2(x) - \frac{x(1-x)(35x^3-133x^2+188x-116)}{24(1-x)^5}$$
$$+\frac{x(25x^4-116x^3+212x^2-192x+96)}{12(1-x)^5}\log(x)$$
$$+\frac{(2-x)(x^4-3x^3+4x^2-2x+1)}{(1-x)^5}\mathrm{Li}_2(1-x)$$

$$\lim_{x\to 1}\mathcal{F}(x;\epsilon,d_0=3,-1) = -\frac{8731}{3600} - \frac{3}{2}\zeta_2$$

THE NLO COUNTERTERMS IN THE TS SCHEME

$$\sigma^{NLO} = \int_{m+1} d\sigma^R_{m+1} J_{m+1} + \int_m d\sigma^V_m J_m$$

$$\sigma^{LO} = \int_m d\sigma^B_m J_m$$

J_m is in general a function that defines the properties of the observed m-jets

$$-d\sigma^{R,A}_{m+1}$$

Regularizes the real emission cross section in Its unresolved region of the phase space

$$\left(\int_1 d\sigma^{R,A}_{m+1}\right)$$

Regularizes the virtual emission cross section

3. PHASE SPACE INTEGRALS:
Structure, computation and results
(We show only the collinear case as an illustrative example)

THE COLLINEAR and NESTED COLLINEAR INTEGRALS

$$x = \frac{2\tilde{p}_{ir} \cdot Q}{Q^2}$$

Kinematic variable that describes the splitting couple of partons

$k = -1, 0, 1, 2$ $\qquad \kappa = 0, 1$

δ	Function	$g_I^{(\pm)}(z)$
0	g_A	1
∓ 1	$g_B^{(\pm)}$	$(1-z)^{\pm\epsilon}$
0	$g_C^{(\pm)}$	$(1-z)^{\pm\epsilon}{}_2F_1(\pm\epsilon, \pm\epsilon, 1\pm\epsilon, z)$
± 1	$g_D^{(\pm)}$	${}_2F_1(\pm\epsilon, \pm\epsilon, 1\pm\epsilon, 1-z)$

$$\mathcal{I}(x;\epsilon, d_0, \kappa, k, \delta, g_I^{(\pm)}) = x\int_0^1 d\alpha\,(1-\alpha)^{2d_0-1}\,[\alpha(\alpha+(1-\alpha)x)]^{-1-(1+\kappa)\epsilon} \times \int_0^1 dv\,[v(1-v)]^{-\epsilon}\left(\frac{\alpha+(1-\alpha)xv}{2\alpha+(1-\alpha)x}\right)^{k+\delta\epsilon} g_I^{(\pm)}\left(\frac{\alpha+(1-\alpha)xv}{2\alpha+(1-\alpha)x}\right)$$

$$\mathcal{I}{*}\mathcal{I}_r(x;\epsilon,\alpha_0,d_0;k,l) = x\int_0^{\alpha_0} \mathrm{d}\alpha \int_0^1 \mathrm{d}v\,\alpha^{-1-\epsilon}\,(1-\alpha)^{2d_0-1}\,[\alpha+(1-\alpha)x]^{-1-\epsilon} \times [v(1-v)]^{-\epsilon}\left[\frac{\alpha+(1-\alpha)xv}{2\alpha+(1-\alpha)x}\right]^k \mathcal{I}\left(x\frac{\alpha+(1-\alpha)xv}{2\alpha+(1-\alpha)x};\epsilon,\alpha_0,d_0;0,l,0,1\right),$$

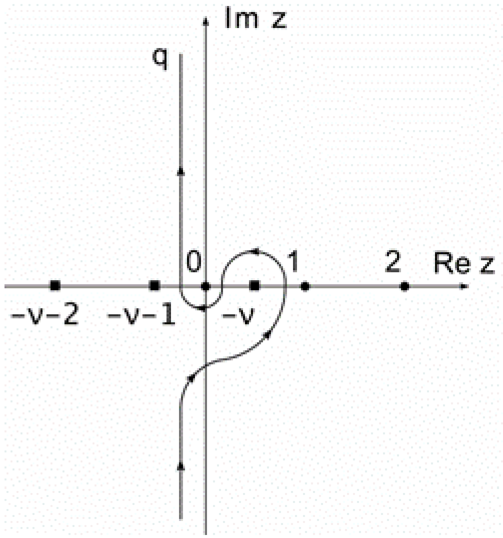
The integration variable v accounts for different fractions of momentmum that the parton p_r carries away from the splitting (ir)

The collinear counterterms to the cross section are consructed from these 'master' integrals

THE METHOD OF MB REPRESENTATION

1. Convert sums into products in the integrand

$$(a+b)^{-\nu} = \frac{1}{\Gamma(\nu)} \int_{q-i\infty}^{q+i\infty} \frac{dz}{2\pi i}\, a^{-\nu-z}\, b^{z}\, \Gamma(\nu+z)\Gamma(-z)$$



2. Integrate the over the real variables to obtain MB integrals

[V. A. Smirnov , J. B. Tausk]

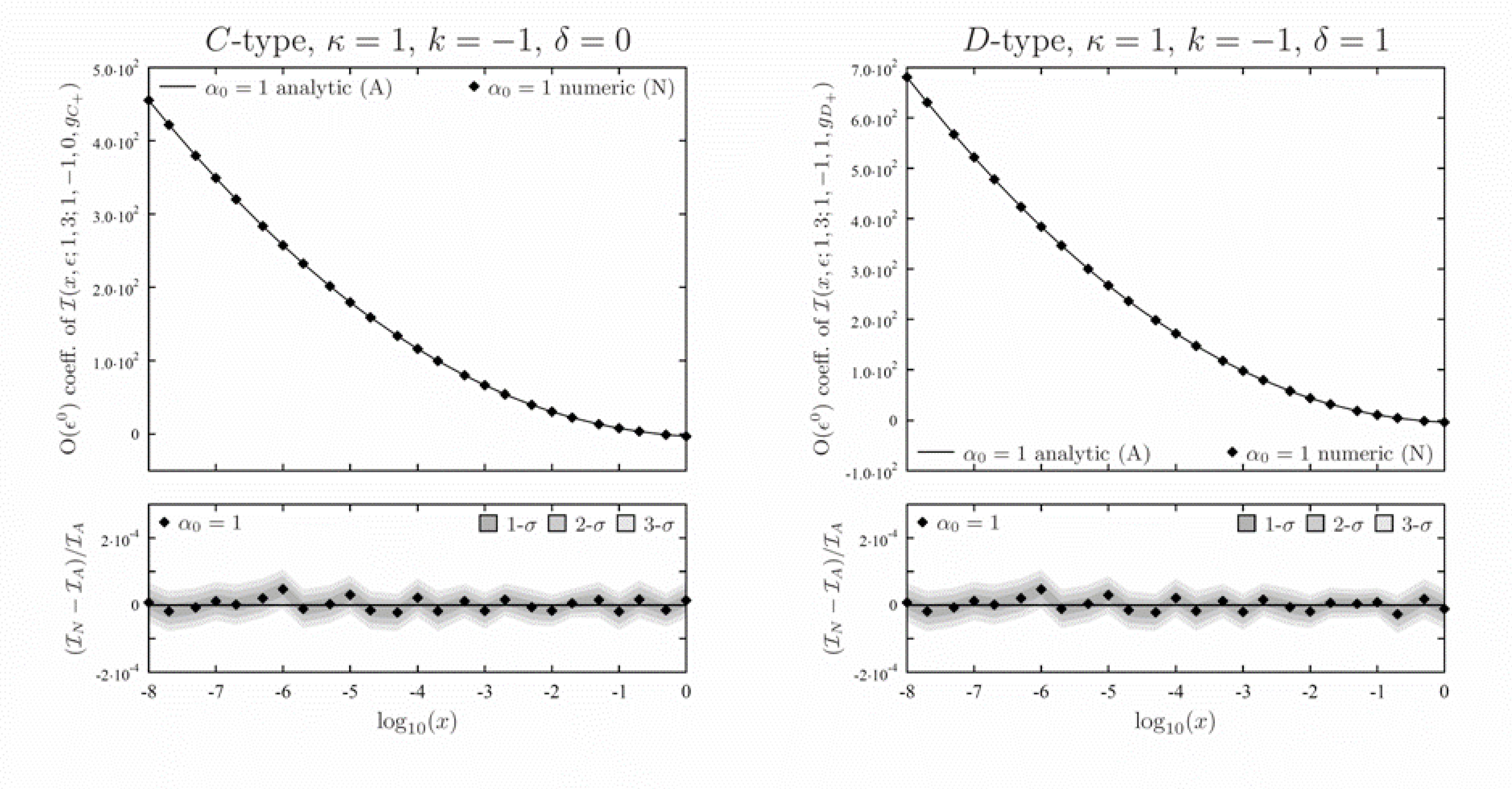
3. Compute the MB integrals converting them into sums over residua

[MB.m: M. Czakon; AMBRE.m: J. Gluza, F. Haas, K. Kajda, T. Riemann]

4. Perform the sums

[XSummer: S. Moch, P. Uwer]

COLLINEAR INTEGRALS: Numeric and analytic results



Comparison between numeric and analytic results. The numeric results have been obtained using standard residuum subtraction and a Monte Carlo integration program

NESTED COLLINEAR INTEGRALS:
an example of analytic result

$$
\begin{aligned}
\mathcal{I}{*}\mathcal{I}_r(x,\epsilon;1,3;-1,2) = & -\frac{1}{12}\,\frac{1}{\epsilon^3} + \left(-\frac{2}{9}+\frac{1}{3}\log(x)\right)\,\frac{1}{\epsilon^2} + \Bigg[\frac{1}{(1-x)^5}\,\Bigg(-\frac{1}{3}\,\zeta_2 - \frac{25}{36}\,\log(x) \\
& +\frac{1}{3}\,\log(1-x)\log(x) + \frac{1}{3}\,\mathrm{Li}_2(x)\Bigg) + \frac{1}{(1-x/2)^5}\,\left(\frac{1}{6}\log\left(\frac{x}{2}\right)\right) + \frac{1}{(1-x)^4}\,\left(-\frac{13}{36}+\frac{1}{6}\log(x)\right) \\
& +\frac{1/6}{(1-x/2)^4} + \frac{1}{(1-x)^3}\,\left(-\frac{7}{72}-\frac{1}{18}\,\log(x)\right) + \frac{1/12}{(1-x/2)^3} \\
& +\frac{1}{(1-x)^2}\,\left(-\frac{1}{6}-\frac{2}{9}\,\log(x)\right) + \frac{1/18}{(1-x/2)^2} + \frac{1}{(1-x)}\,\left(-\frac{25}{72}-\frac{7}{12}\,\log(x)\right) \\
& +\frac{1/24}{(1-x/2)} + \frac{31}{216} + \frac{1}{6}\,\log(2) + \frac{19}{9}\,\log(x) + \frac{2}{3}\,\log(1-x)\log(x) - \frac{2}{3}\log^2(x) \\
& +\frac{2}{3}\,\mathrm{Li}_2(x)\Bigg]\,\frac{1}{\epsilon} + \mathrm{O}(\epsilon^0).
\end{aligned}
$$

$$
\lim_{x\to 1}\mathcal{I}{*}\mathcal{I}_r(x,\epsilon;1,3;-1,2) = -\frac{1}{12}\,\frac{1}{\epsilon^3} - \frac{2}{9}\,\frac{1}{\epsilon^2} + \left(\frac{3091}{675}+\frac{2}{3}\,\zeta_2 - \frac{31}{6}\,\log(2)\right)\,\frac{1}{\epsilon} + \mathrm{O}(\epsilon^0)
$$

WHAT IS NEEDED TO DEFINE SUCH A SCHEME

To complete such a scheme three problems must be solved:

1. **The disentanglement of overlapping of soft and/or collinear configurations to avaoid multiple subtractions at any order**

   [Z. Nagy ,Z. Trocsanyi, G. Somogyi]

2. **From the strict unresolved limits to the whole phase space a proper mapping of momenta in needed to respect QCD factorization and the IR divergences cancellations**

   [Z. Trocsanyi, G. Somogyi, V. Del Duca]

3. **The computation of the integrated subtraction terms over the phase space of the unresolved partons**