Unstable-particles pair production in modified perturbation theory in NNLO

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The problem:

A description of productions and decays of fundamental unstable particles for colliders subsequent to LHC (⟹ ILC) generally should be made with NNLO accuracy

(i) gauge cancellations and unitarity;
(ii) enough high accuracy of computation of resonant contributions

Existing methods: DPA, CMS, ... ⟹ NLO
Pinch-technique method ⟹ huge volume of extra calculations

Modified perturbation theory (MPT) implies direct expansion of the cross-section *in powers of the coupling constant* with the aid of distribution-theory methods

⇓

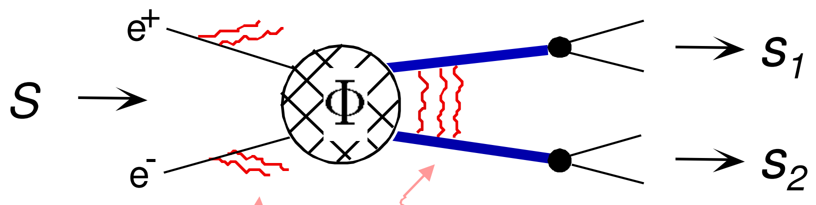
Gauge invariance should be maintained

Accuracy of description of resonant contributions = ?

To clear up this question, I do numerical simulation in the MPT up to the NNLO

/ M.Nekrasov, arXiv:0912.1025 /

Pair production and decay, double-resonant contributions



$$\sigma(s) = \int_{s_{\min}}^{s} \frac{\mathrm{d}s'}{s}\, \phi(s'/s; s)\, \hat{\sigma}(s')\,, \qquad \hat{\sigma}(s) = \iint \mathrm{d}s_1\, \mathrm{d}s_2\;\; \hat{\sigma}(s\,; s_1, s_2)\,(1+\delta_c)$$

$$\hat{\sigma}(s\,; s_1, s_2) = \frac{1}{s^2}\,\theta(\sqrt{s} - \sqrt{s_1} - \sqrt{s_2}\,)\,\sqrt{\lambda(s, s_1, s_2)}\;\Phi(s; s_1, s_2)\,\rho(s_1)\,\rho(s_2)$$

$$\rho(s_i) = \frac{M\Gamma}{\pi}\,\frac{1}{|s_i - M^2 + \alpha\Sigma(s_i)|^2}$$

MPT ⇨ expansion of the cross-section in powers of α

Basic ingredients of MPT

- Asymptotic expansion of BW factors in powers of α / F.Tkachov,1998 /

$$\rho(s) = \frac{M\Gamma_0}{\pi} \frac{1}{|s - M^2 + \Sigma(s)|^2} = \delta(s-M^2) + PV\,\mathcal{T}\,[\,\rho(s)\,] + \sum_n c_n(\alpha)\,\delta^{(n)}(s-M^2)$$

Taylor in α

Polynomial in α

3-loop

NNLO :

$$= \delta(s-M^2) + \frac{M\Gamma_0}{\pi}\left[PV\frac{1}{(s-M^2)^2} - PV\frac{2\alpha\,\mathrm{Re}\Sigma_1(s)}{(s-M^2)^3}\right] + \sum_{n=0}^{2} c_n(\alpha)\,\delta^{(n)}(s-M^2) + O(\alpha^3)$$

- Analytic regularization of the kinematic factor / M.Nekrasov,2007 /

$$\sqrt{\lambda(s,s_1,s_2)} \longrightarrow \lim_{\nu\to 1/2}\left\{\lambda(s,s_1,s_2)\right\}^{\nu}$$

analytic calculation of "singular" integrals

- Conventional-perturbation-theory for "test" function Φ

Coefficients $c_n(\alpha)$

NNLO:

$$\rho(s) = \delta(s-M^2) + \frac{M\Gamma_0}{\pi}\left[PV\frac{1}{(s-M^2)^2} - PV\frac{2\alpha\,\mathrm{Re}\Sigma_1(s)}{(s-M^2)^3}\right] + \sum_{n=0}^{2} c_n(\alpha)\,\delta^{(n)}(s-M^2)$$

$$I_n = \mathrm{Im}\Sigma_n(M^2)\,,\quad I'_n = \mathrm{Im}\Sigma'_n(M^2)\,,\ \cdots$$

$$R_n = \mathrm{Re}\Sigma_n(M^2)\,,\quad R'_n = \mathrm{Re}\Sigma'_n(M^2)\,,$$

$$\Sigma = \alpha\Sigma_1 + \alpha^2\Sigma_2 + \alpha^2\Sigma_3$$

OMS conventional : $R_n = R'_n = 0$

$$c_0 = -\alpha\frac{I_2}{I_1} + \alpha^2\left(\frac{I_2^2}{I_1^2} - \frac{I_3}{I_1} - \frac{1}{2}I_1 I_1''\right)$$

$$c_1 = -\alpha^2(I_1 I_1')\,,\qquad c_2 = -\alpha^2 I_1^2$$

$\overline{\text{OMS}}$: (pole scheme) / M.Nekrasov, 2002 / / B.Kniel & A.Sirlin, 2002 /

$$R_1 = R'_1 = 0\,,\qquad R_2 = -I_1 I'_1\,,\quad R'_2 = -I_1 I''_1/2$$

M – observable (pole) mass
gauge invariant

$$c_0 = -\alpha\frac{I_2}{I_1} + \alpha^2\left[\frac{I_2^2}{I_1^2} - \frac{I_3}{I_1} - (I'_1)^2\right]$$

$$c_1 = 0,\qquad c_2 = -\alpha^2 I_1^2$$

Unitarity: $\alpha I_1 = M\Gamma_0\,,\qquad \alpha^2 I_2 = M\alpha\Gamma_1\,,\qquad \alpha^3 I_3 = M\alpha^2\Gamma_2 + \Gamma_0^3/(8M)$

$$\Gamma = \Gamma_0 + \alpha\Gamma_1 + \alpha^2\Gamma_2 + \cdots$$

Singular integrals, scheme of calculations

Dimensionless variables

$s \to x$, $s_i \to x_i$

$$\sqrt{s} = 2M + \frac{M}{2}x\,, \quad \sqrt{s_i} = M_i + \frac{M}{2}x_i \qquad M \equiv \frac{M_1+M_2}{2}$$

$$\hat\sigma(x) = \iint dx_1\, dx_2\ (x-x_1-x_2)_+^\nu\ \rho(x_1)\,\rho(x_2)\ \Phi(x\,;x_1,x_2)$$

$$\left\{PV\frac{1}{x_1^{n_1}},\ \delta^{(n_1-1)}(x_1)\right\} \quad \left\{PV\frac{1}{x_2^{n_2}},\ \delta^{(n_2-1)}(x_2)\right\}$$

at given n_1 and n_2 :

$$\Phi(x\,;x_1,x_2) = \sum_{k_1=0}^{n_1-1}\sum_{k_2=0}^{n_2-1}\frac{x_1^{k_1}}{k_1!}\frac{x_2^{k_2}}{k_2!}\,\Phi^{(k_1,k_2)}(x\,;0,0) + \Delta\Phi(x\,;x_1,x_2)$$

$$x^k\,PV\frac{1}{x^n} = \frac{1}{x^{n-k}}$$

$$x^k\,\delta^{(n-1)}(x) \sim \delta^{(n-k-1)}(x)$$

$$0 \le k < n$$

at ν = 1/2

$$A^\nu_{l_1 l_2}(x) = \iint dx_1\, dx_2\ (x-x_1-x_2)_+^\nu\ \delta^{(l_1-1)}(x_1)\,\delta^{(l_2-1)}(x_2) \sim (x)_+^{5/2-l_1-l_2} + \text{`reg'}$$

$$B^\nu_{l_1 l_2}(x) = \iint dx_1\, dx_2\ (x-x_1-x_2)_+^\nu\ PV\frac{1}{x_1^{l_1}}\,\delta^{(l_2-1)}(x_2) \sim (-x)_+^{5/2-l_1-l_2} + \text{`reg'}$$

$$C^\nu_{l_1 l_2}(x) = \iint dx_1\, dx_2\ (x-x_1-x_2)_+^\nu\ PV\frac{1}{x_1^{l_1}}\,PV\frac{1}{x_2^{l_2}} \sim (x)_+^{5/2-l_1-l_2} + \text{`reg'}$$

$$\sigma(x) = \int dx'\ \phi(x',x)\,\hat\sigma(x')$$

$$\int dx\ x_+^\nu\,\varphi(x) \underset{def}{=} \int_0^\infty dx\ x^\nu \left\{\varphi(x) - \sum_{k=0}^{N-1}\frac{x^k}{k!}\varphi^{(k)}(0)\right\}$$

$$-N-1 < \mathrm{Re}\,\nu < -N$$

Numerical calculations & estimate of errors

- Fortran code with double precision
- Simpson method for calculating absolutely convergent integrals (relative accuracy $\delta_0 = 10^{-5}$)
- Linear patches for resolving 0/0-indeterminacies (x/x, x^2/x^2, …)

additional errors:

ε - actual size of the patch

integrand

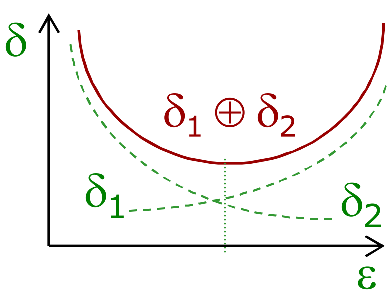
(1) due to patches themselves $\Rightarrow$ $\delta_1 \sim \varepsilon^2 \varphi''_0/\varphi_0$

(2) due to the loss of decimals near indeterminacy points:

x/x: $\frac{f(x)-f(0)}{x} \Rightarrow \frac{\varepsilon f'(0)}{\varepsilon}$ $\quad \varepsilon = 10^{-N}$ $\quad \delta_2 \sim 10^{-(D-N)} \frac{f_0}{f'_0}$

x^2/x^2: $\frac{f(x)-f(0)-xf'(0)}{x^2} \Rightarrow \frac{\varepsilon^2 f''(0)/2}{\varepsilon^2}$ $\quad \varepsilon^2 = 10^{-N}$ $\quad \delta_2 \sim 10^{-(D-N)} \frac{2f_0}{f''_0}$

…



minimization of errors $\delta_1 \oplus \delta_2$ (numerical estimate) $\Rightarrow$ $N = 8$ at $D = 15$ ← double precision

Overall error: $\delta = \delta_0 \oplus \delta_1 \oplus \delta_2 \cong 10^{-3}$ ← NNLO

Specific model for testing MPT

- Test function Φ :

  $e^+e^- \to \gamma, Z \to t\bar{t} \to W^+b\, W^-\bar{b}$ ⇐ Born approximation

- Breigt-Wigner factors :

  $\Sigma = \alpha\Sigma_1 + \alpha^2\Sigma_2 + \alpha^3\Sigma_3$ ⇐ three-loop contributions to self-energy

- Universal soft massless-particles contributions :

  Flux function in leading-log approximation:

$$\phi(z;s) = \beta_e(1-z)^{(\beta_e-1)} - \frac{1}{2}\beta_e(1+z)\,, \qquad \beta_e = \frac{2\alpha}{\pi}\left(\ln\frac{s}{m_e^2} - 1\right)$$

  Coulomb singularities through one-gluon exchanges:

$$\delta_c = \kappa\,\frac{\alpha_s\pi}{2\beta}\left[1 - \frac{2}{\pi}\arctan\left(\frac{|\beta_M|^2 - \beta^2}{2\beta\,\mathrm{Im}\beta_M}\right)\right]$$

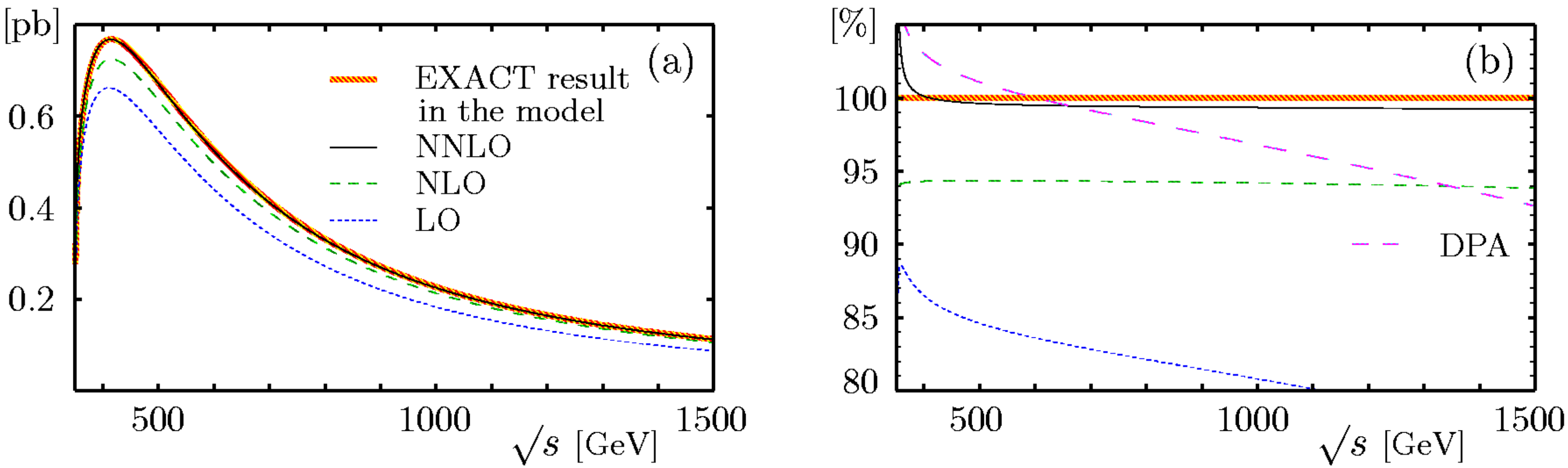
$$\beta = s^{-1}\sqrt{\lambda(s, s_1, s_2)}$$

$$\beta_M = \sqrt{1 - 4(M^2 - \mathrm{i}M\Gamma)/s}$$

Results

Total cross-section $\sigma(s)$:

M_t = 175 GeV
M_W = 80.4 GeV M_b = 0



$\sqrt{s}$ [GeV]	σ [pb]	σ_{LO}	σ_{NLO}	σ_{NNLO}
500	0.6724 100%	0.5687 84.6%	0.6344 94.3%	0.6698(7) 99.6(1)%
1000	0.2255 100%	0.1821 80.8%	0.2124 94.2%	0.2240(2) 99.3(1)%

Conclusion

In the case of pair production and decays of unstable particles:

- The existence of the MPT expansion practically has been shown (working FORTRAN code up to NNLO is written)

- MPT stably works at the energies near the maximum of the cross-section and higher

- At ILC energies NNLO in MPT provides a few per mile accuracy

Comment:

- The higher precision is possible if proceeding to NNNLO

  or if using NNLO of MPT for calculation loop contributions only

  (on the analogy of actual practice of application of DPA)