

Absorbing systematic effects to obtain a better background model in a search for new physics

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For details please see: [S Caron et al 2009 JINST 4 P10009](#), [arXiv:0909.3718v2](#)

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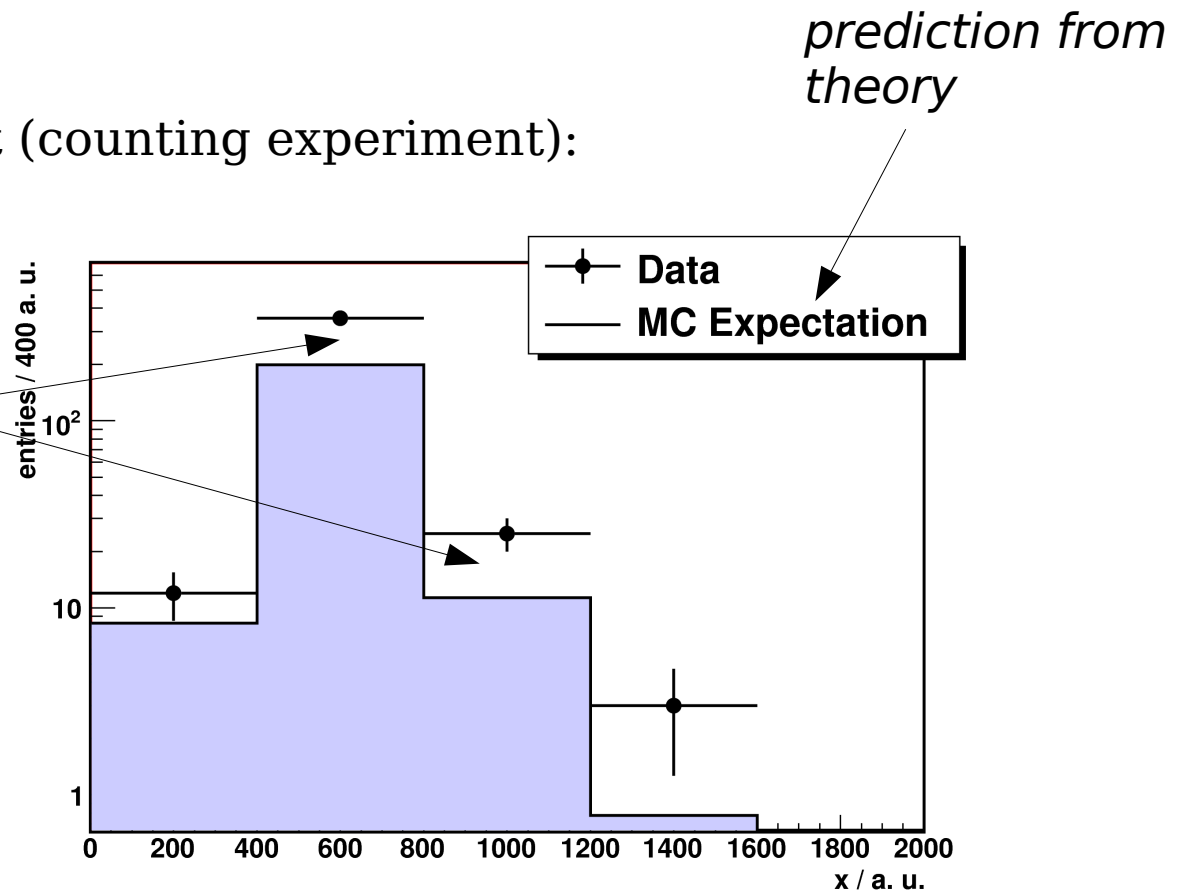
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Introduction

Sketch of a measurement (counting experiment):

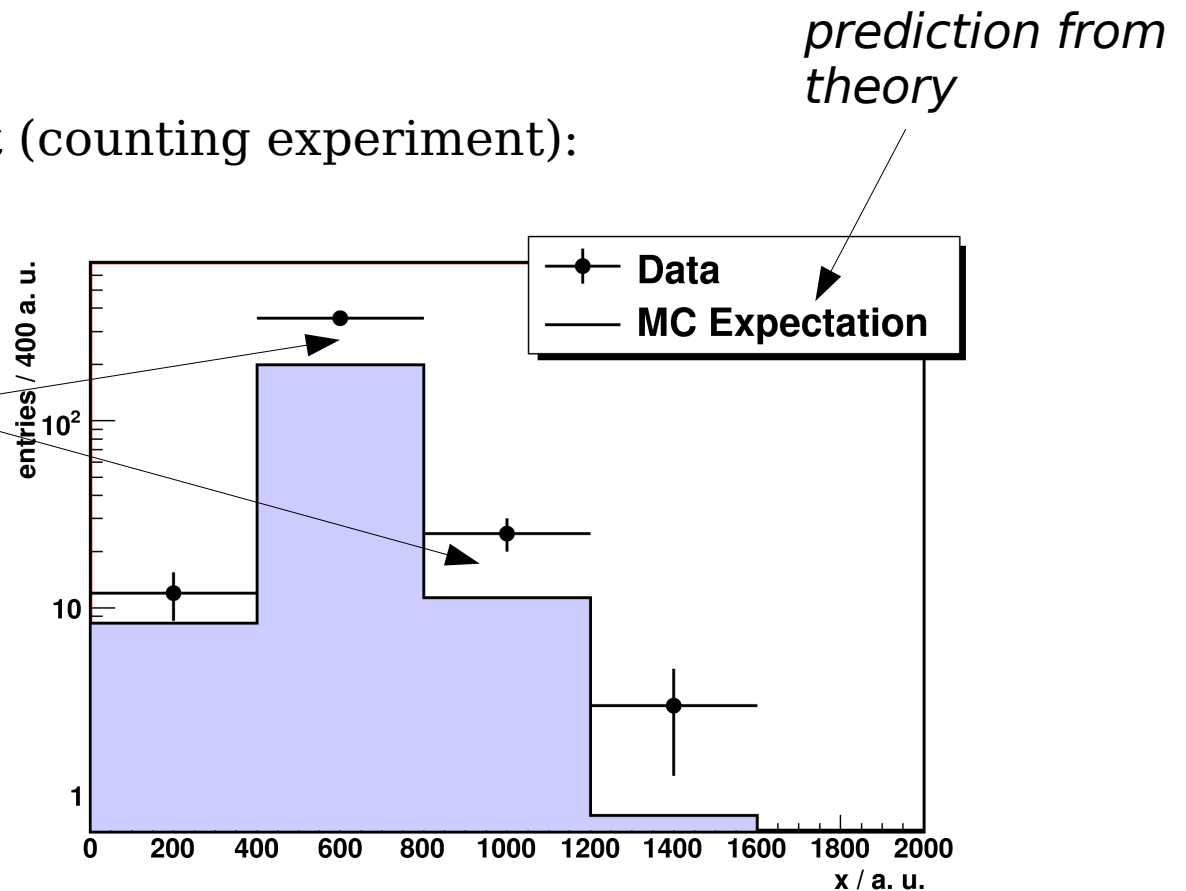
New physics or systematic effect?



Introduction

Sketch of a measurement (counting experiment):

New physics or systematic effect?



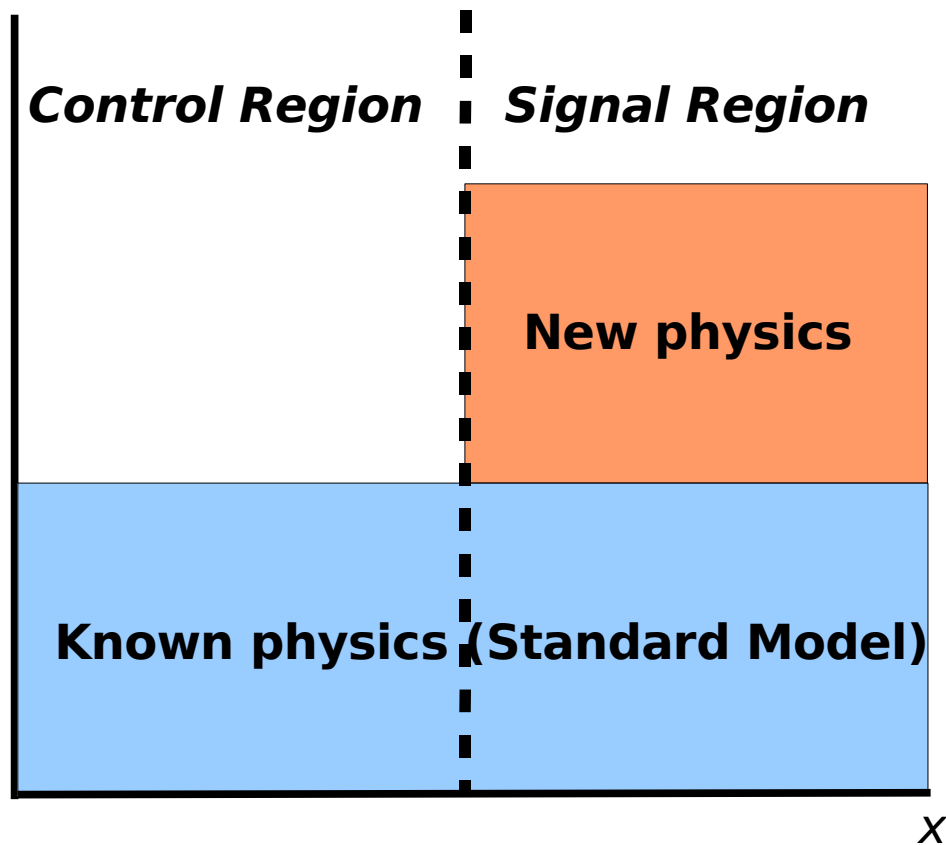
- × The **systematic effect** can arise from **shortcomings in modelling** (both in theory and detector simulation).
- × Therefore, the Monte Carlo (MC) **prediction** needs to be **verified** with data.

Introduction

- × To verify Monte Carlo find region in phase space, **Control Region**, satisfying:
 - ideally **only known physics (Standard Model)** present
 - observable of interest x : **similar physical meaning** and dependence on systematic effects in Control and Signal Region (“same” x)

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Desired scenario:

- new physics can appear in Signal Region only
- Same background (known physics) in Control and Signal Region

Introduction

Common approaches to obtain a background prediction for the Signal Region:

a) Use **data** from Control Region (CR) **as model** for Signal Region (SR)

Drawbacks: - data fluctuations induce bias
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b) **Divide data by MC template** in CR and use ratio as correction for SR

Drawbacks: - data fluctuations induce bias
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c) **Fit function to data** in CR and rescale it for SR

Drawbacks: - can be difficult to get shape right
- shapes in CR & SR must be the same

Introduction

Common approaches to obtain a background prediction for the Signal Region:

a) Use *D* on (SR)

Our proposal: Modify MC template with a correction function

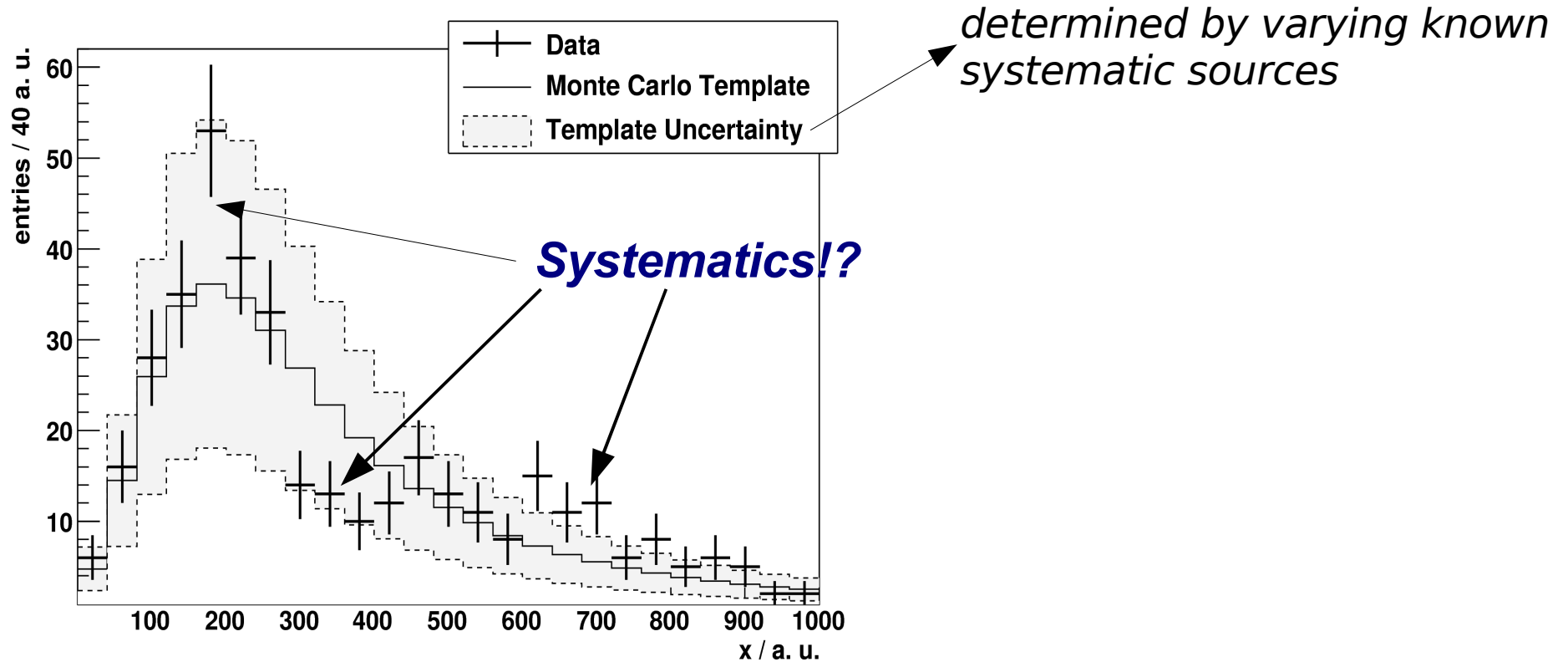
b) **Div** - Use **MC expectation as starting point**, since it is best estimate when no systematics present

- Assume that systematic effects can be described by simple functions

c) **Fit**
Draw
- shapes in CR & SR must be the same

Introducing the method

Toy example of a measurement in a control region:



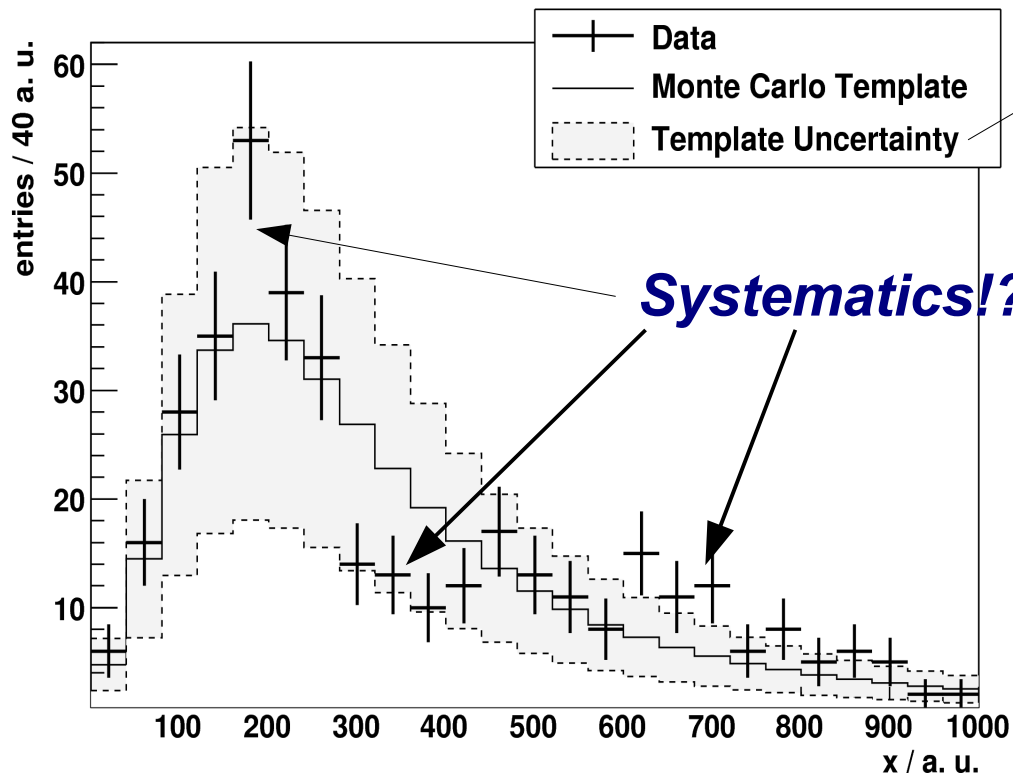
Compatibility with central prediction:

Probability $p = 0.002$

(Probability to observe such data or data less likely if MC template is true model)

Introducing the method

Toy example of a measurement in a control region:



determined by varying known systematic sources

1. Multiply the MC template with a correction function

$$Model_x = Template * Polynomial \text{ with } x \text{ parameters}$$

2. Fit the modified template to the data to determine parameters
3. Use successively more complex correction functions until satisfactory goodness-of-fit is reached (p -Value)

Compatibility with central prediction:

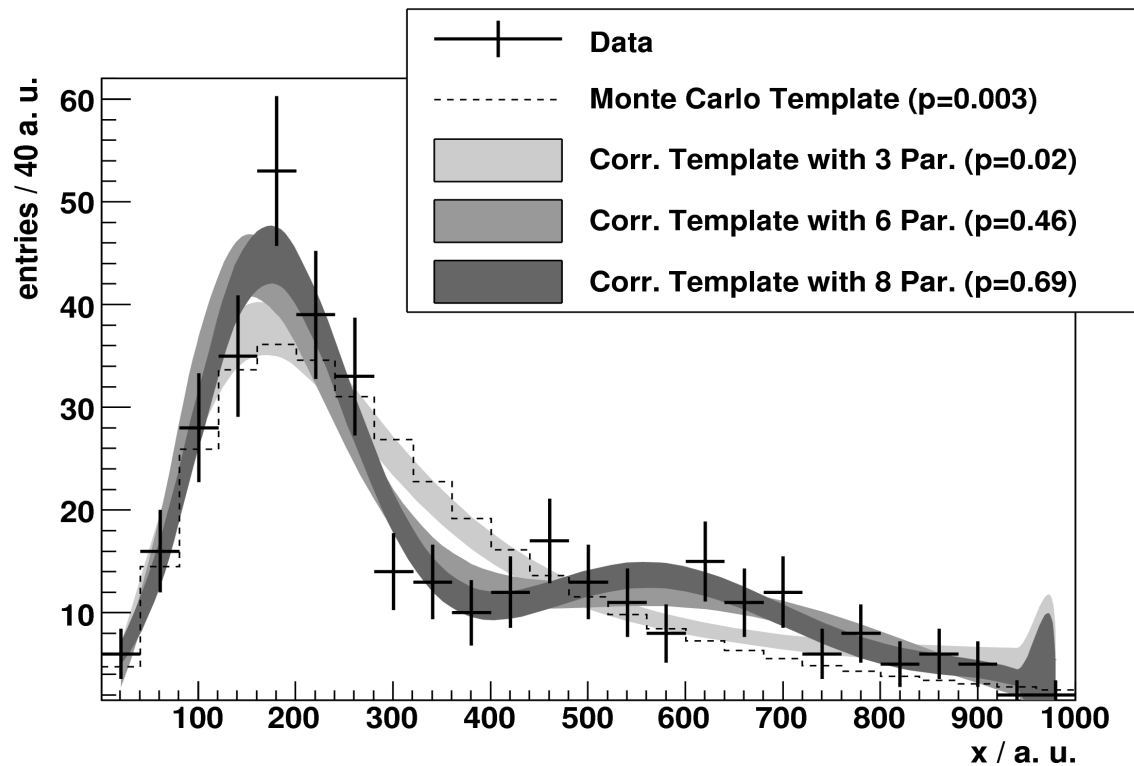
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Selecting a better model

Ordinary polynomials as correction functions:

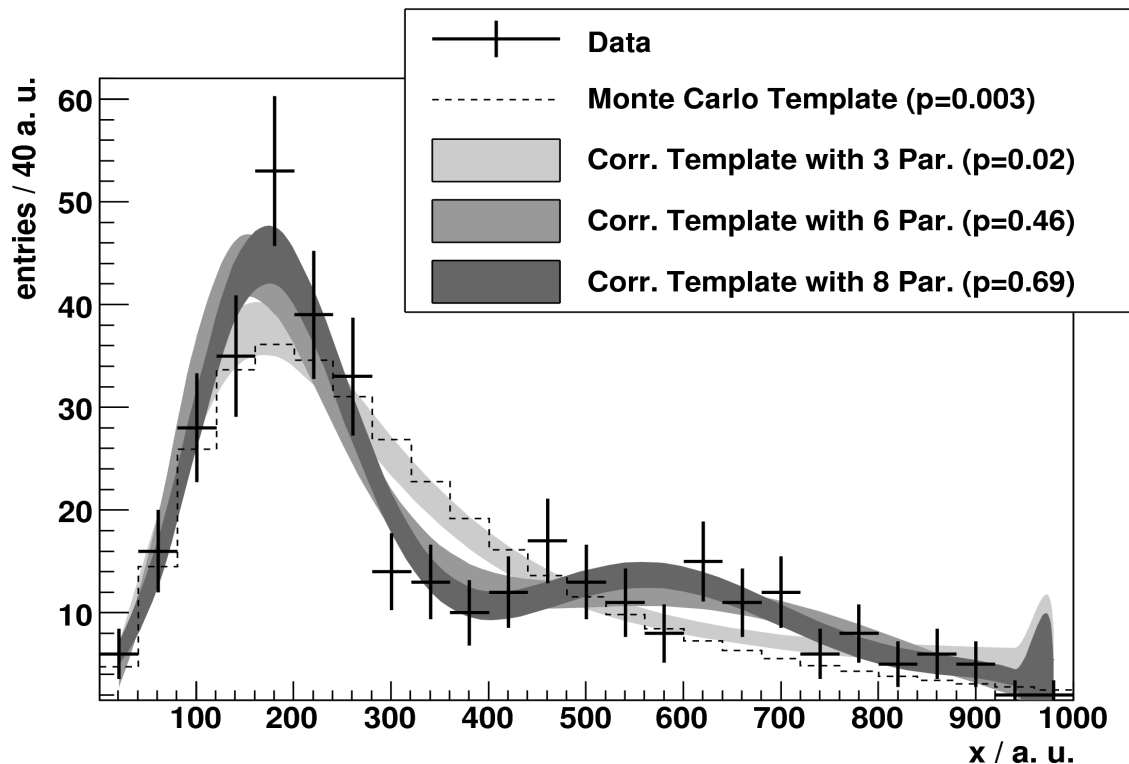
Model_x = Template * Polynomial with **x** parameters



Selecting a better model

Ordinary polynomials as correction functions:

Model_x = Template * Polynomial with **x** parameters



Absolute goodness-of-fit:

p(Model₀) = 0.0027

p(Model₁) = 0.0033

p(Model₅) = 0.33

p(Model₇) = 0.46

p(Model₈) = 0.69

p(Model₉) = 0.63

Relative goodness-of-fit:

p(Model₀ | Model₁) = 0.15

p(Model₇ | Model₈) = 0.04

p(Model₈ | Model₉) = 0.80



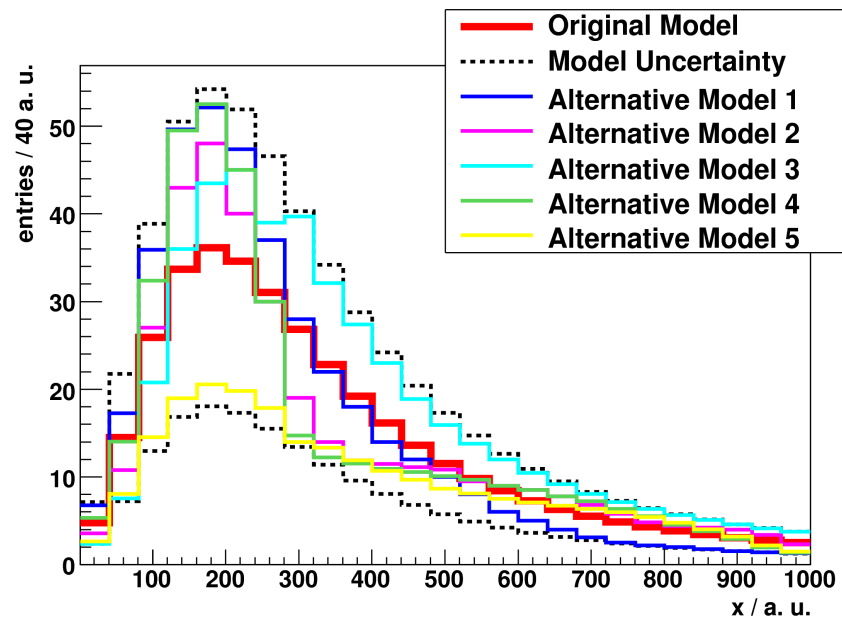
low number indicates improvement when going to the next model (see backup)

In this case several parameters needed due to large systematic effects (see next slide)

Shape uncertainty in starting template

In real case: vary Monte Carlo prediction according to known systematic effects to obtain alternative starting templates.

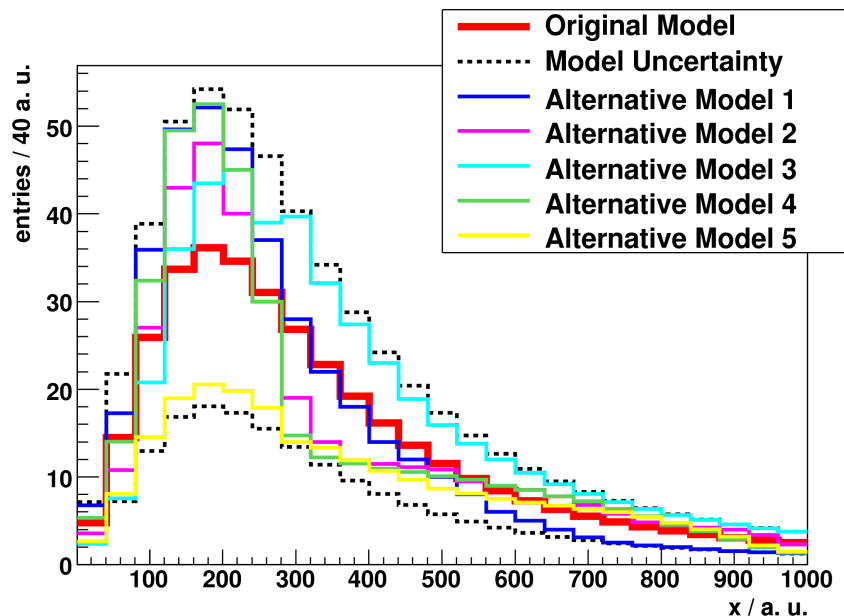
Before correction:



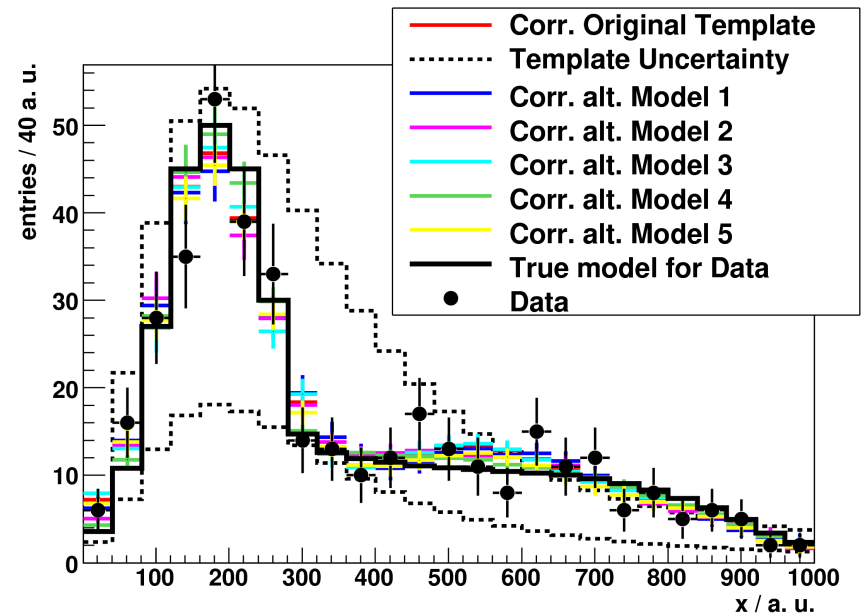
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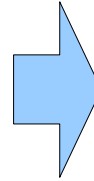
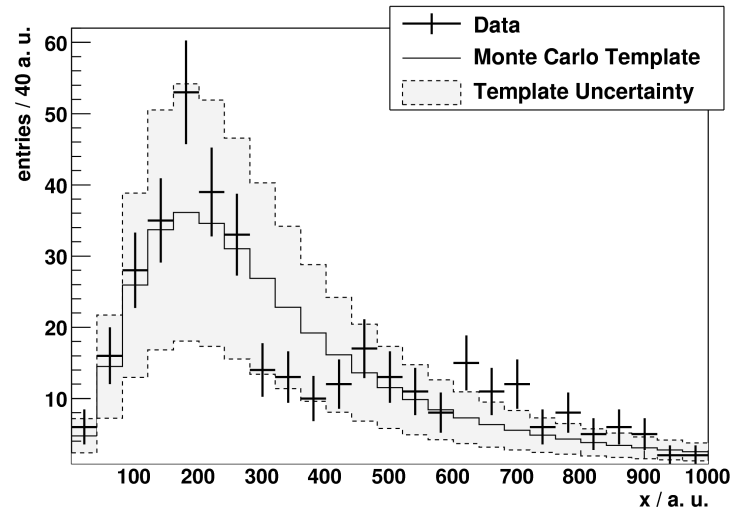
After correction:



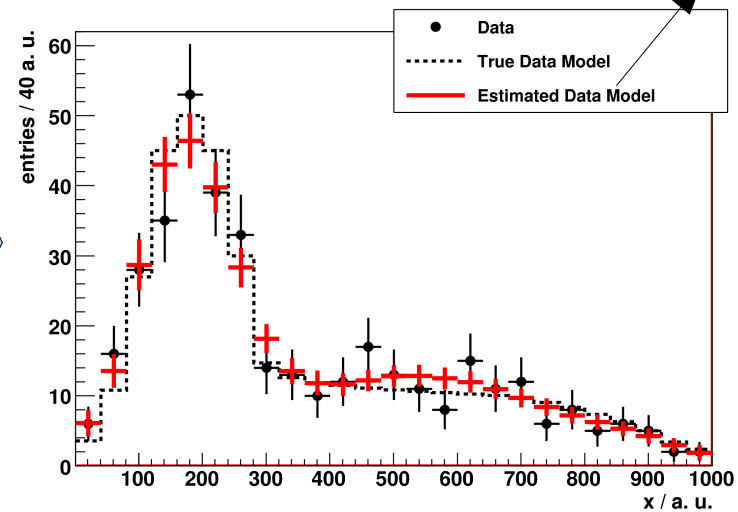
- × True model has large systematic deviations from original MC template, but they are absorbed into the new improved model
- × Furthermore, choice of the starting template has only little influence.

Average corrected models to obtain a best estimate →

Proposed Method applied:

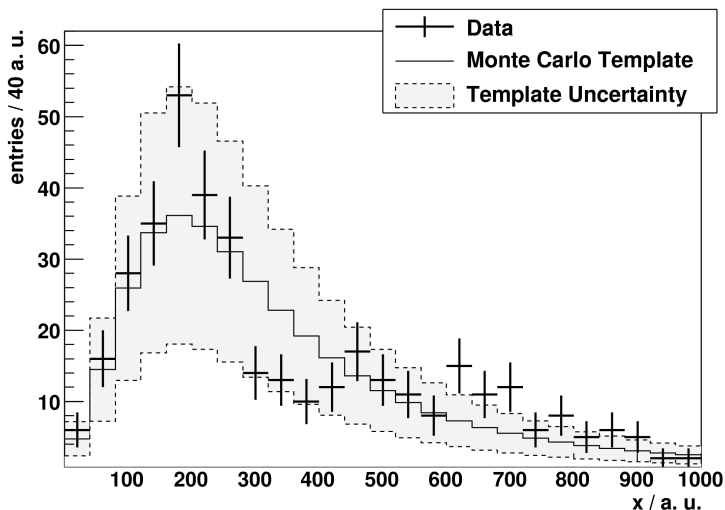


Errors determined using toy data sets generated from Estimated Model

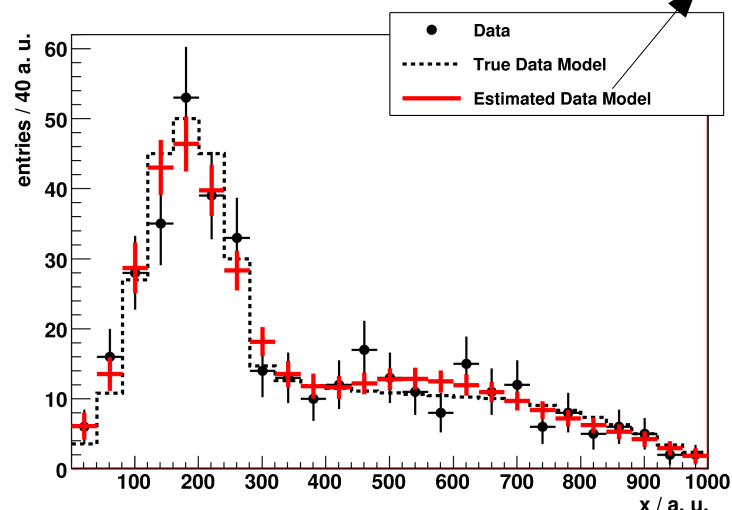


Large systematics absorbed and uncertainty reduced!

Proposed Method applied:

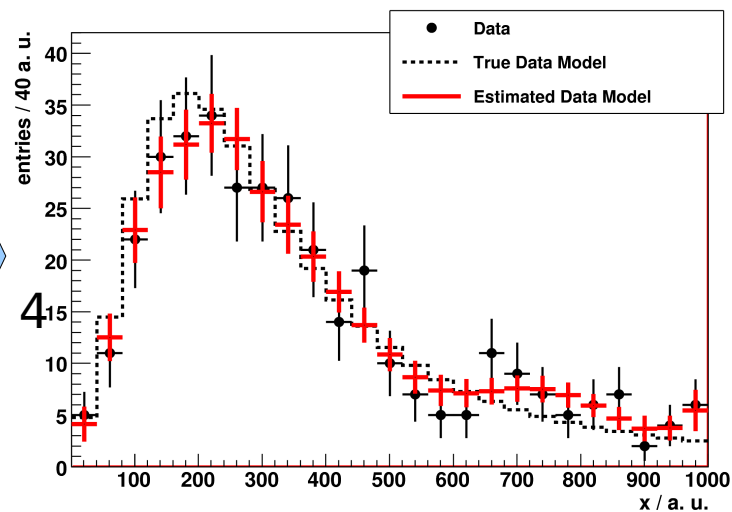
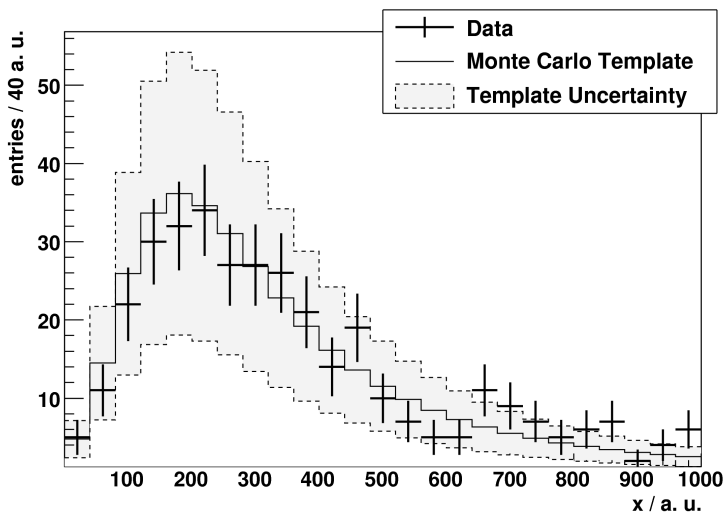


Errors determined using toy data sets generated from Estimated Model



Large systematics absorbed and uncertainty reduced!

Special test case: no systematic effects included



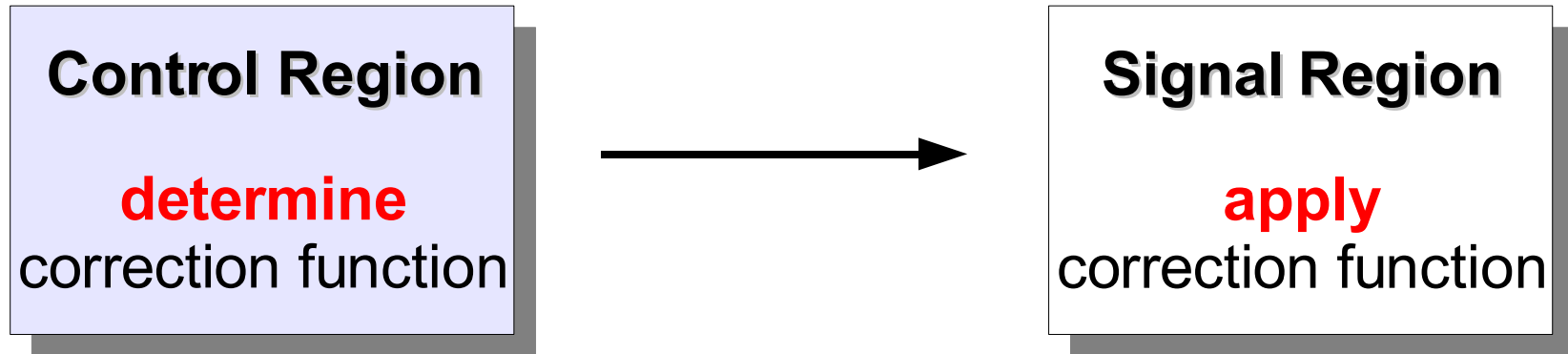
True model (= original MC prediction) reproduced!

Transfer to Signal Region:

After form of correction determined in Control Region, apply on Monte Carlo template for Signal Region

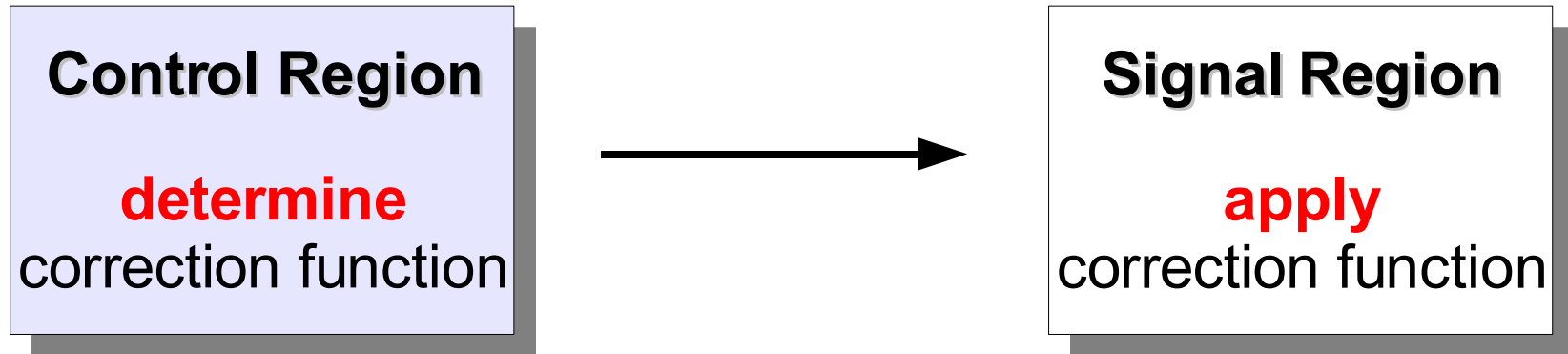
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Advantage of proposed method:

Data distributions don't need to have the same shapes in signal and control regions. Only the systematics have to affect them similarly.

Now look at Signal Region

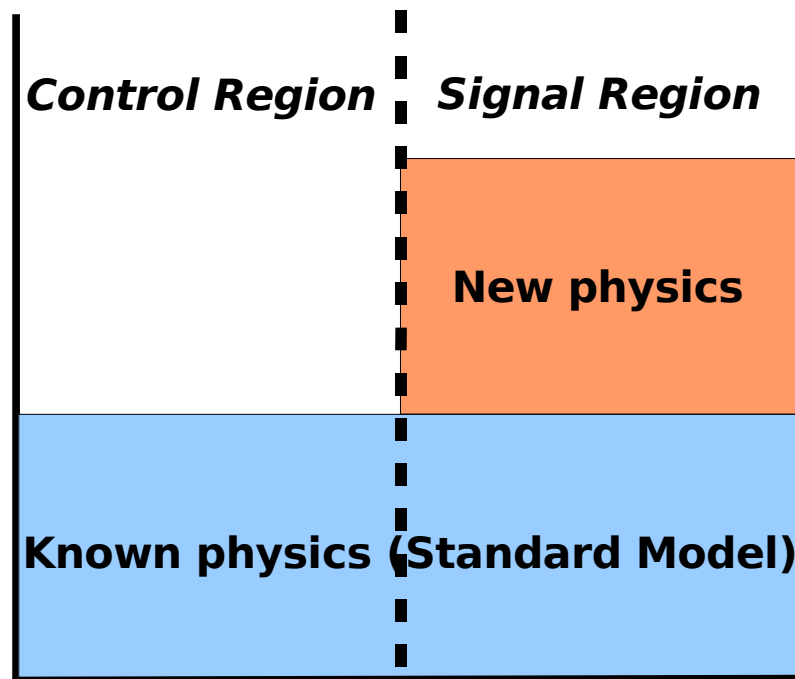
Consider simple case:

- × Shapes of MC templates in both regions the same
- × Event efficiency of Signal to Control Region taken to be unity

Now look at Signal Region

Consider simple case:

- x Shapes of MC templates in both regions the same
- x Event efficiency of Signal to Control Region taken to be unity

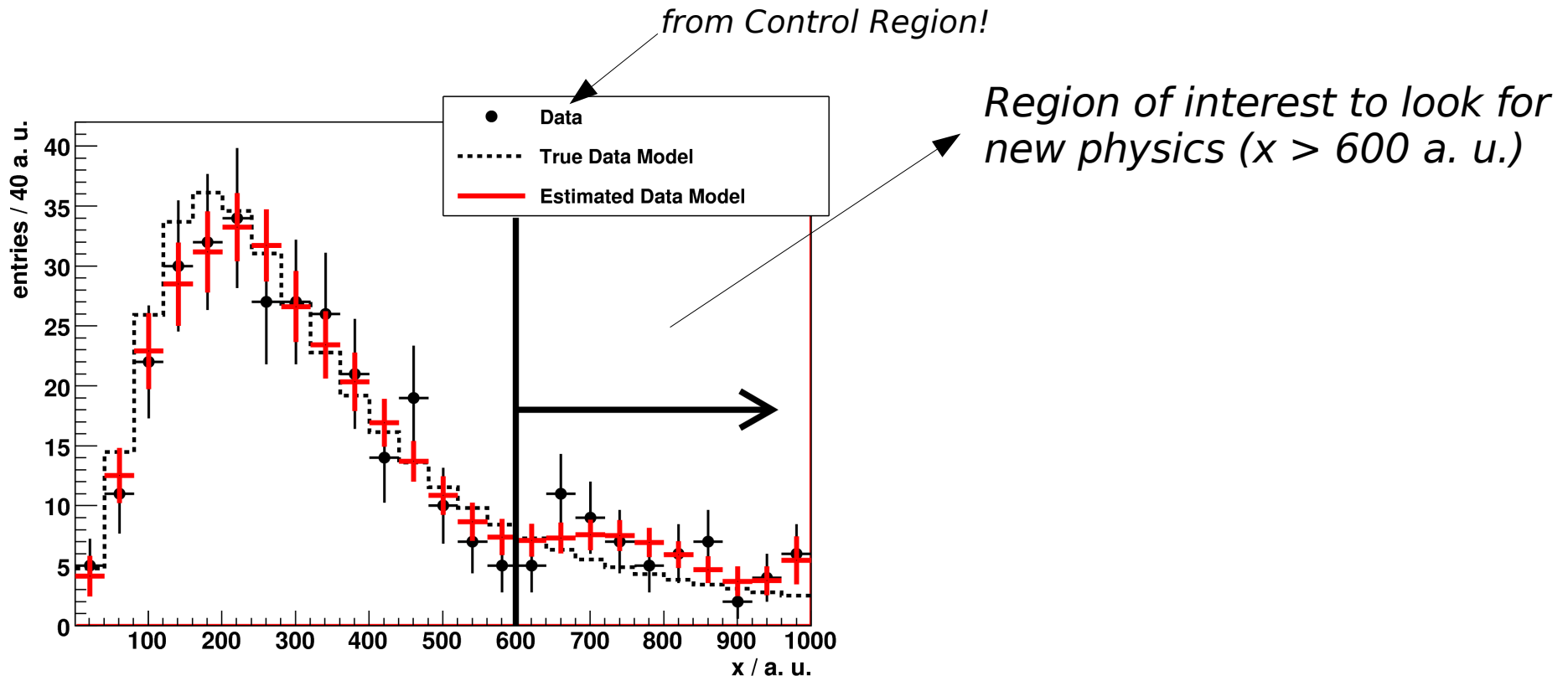


→ consider *scenario with no systematic effects* as a limiting case (original MC expectation = correct model) → next slide

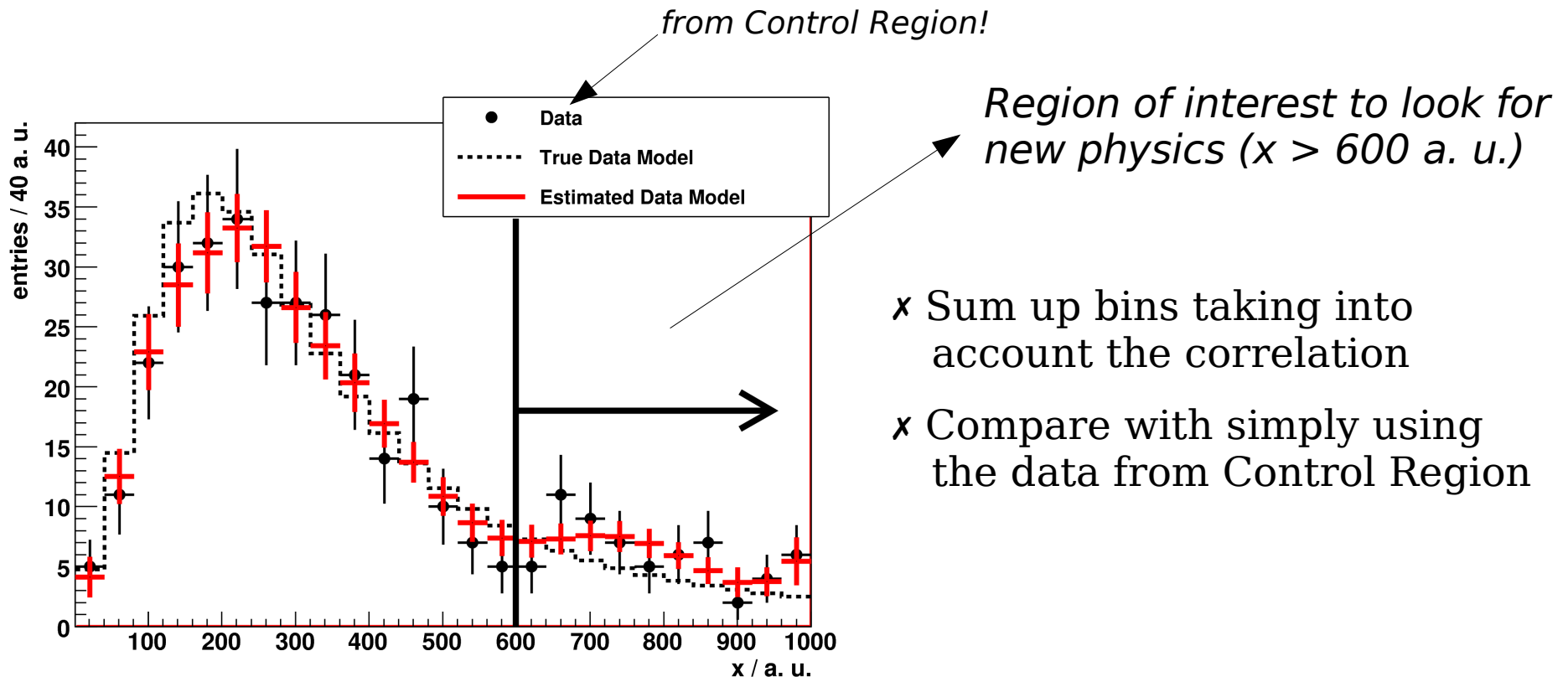
NOT accounted for here:

Systematic effects may affect regions differently → additional uncertainty

Expected background events in Signal Region



Expected background events in Signal Region



Model	Number of expected events	Relative error
Original prediction (MC template)	43.9 ± 21.9	50%
Corrected model	59.9 ± 7.6	12.7%
Data as model	62.0 ± 7.9	12.7%

But in general error of corrected model smaller than data error. →

Considering many experiments

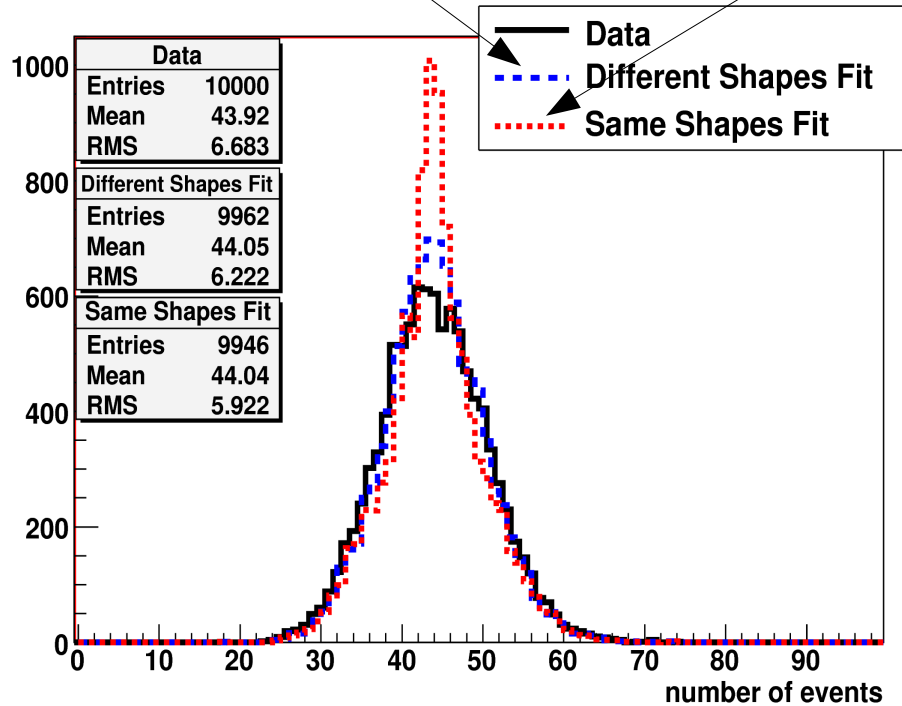
x Generate 10.000 toy data sets from true model and apply method

Considering many experiments

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Same starting templates as before

Templates differ from true model by scale only

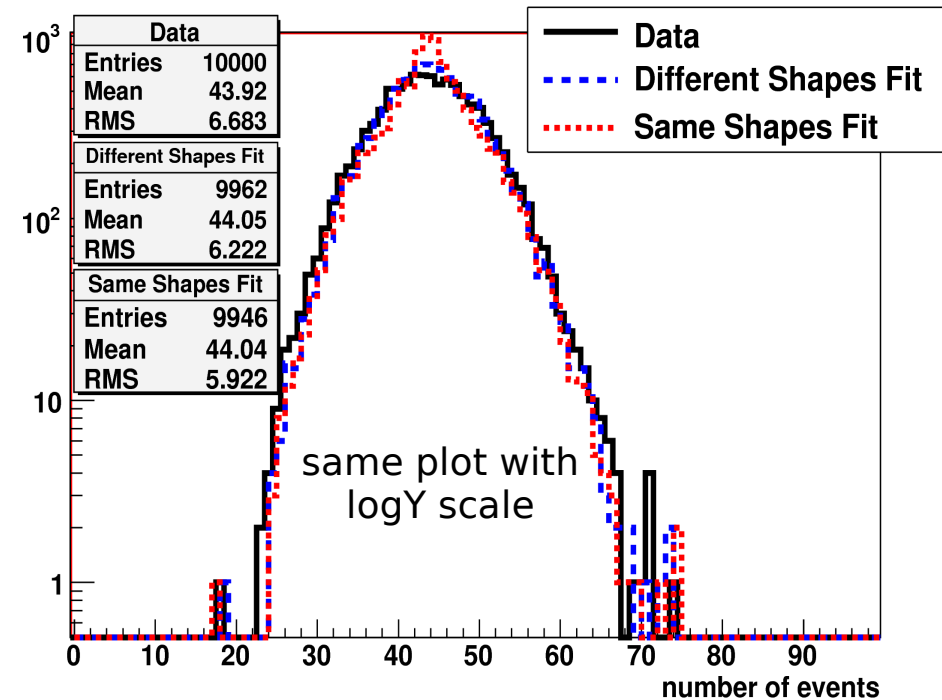
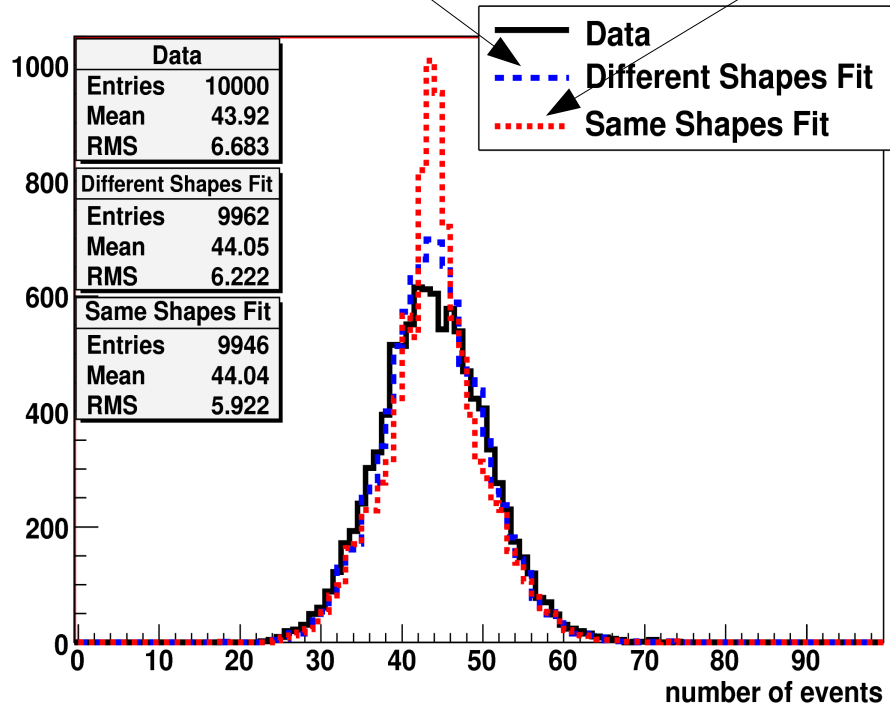


Considering many experiments

x Generate 10.000 toy data sets from true model and apply method

Same starting templates as before

Templates differ from true model by scale only



Method has smaller uncertainty than using the data as a model and reproduces true mean (43.89) within 2.6% of quoted error

Discovery Significance

Significance: convolute Poisson probability of a measurement with Gaussian priors for the background expectation (using the uncertainties from the previous slide):

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Assume the following measurements



	$x > 600$ a. u.: 99 events counted	
	Bgrd predicted: (true value 43.89)	Significance:
Data	43.92 ± 6.683	5.01
Different Shapes	44.05 ± 6.222	5.15
Same Shapes	44.04 ± 5.922	5.25



Equivalent to 4% luminosity increase

Discovery Significance

Significance: convolute Poisson probability of a measurement with Gaussian priors for the background expectation (using the uncertainties from the previous slide):

Assume the following measurements

	$x > 600$ a. u.: 99 events counted Bgrd predicted: (true value 43.89)	Significance:	$x > 800$ a. u. : 52 events counted Bgrd predicted: (true value 15.61)	Significance:
Data	43.92 ± 6.683	5.01	15.62 ± 3.933	5.10
Different Shapes	44.05 ± 6.222	5.15	15.57 ± 3.596	5.29
Same Shapes	44.04 ± 5.922	5.25	15.53 ± 3.446	5.38

Equivalent to 4% luminosity increase
12% lumi increase

**Improvement wrt. Data model even in this “optimal” scenario
(no systematic effects, shapes in CR & SR identical)**

Summary:

1. We propose to modify Monte Carlo predictions with correction functions to account for systematic effects.
2. Successively more complex functions are used until sufficient compatibility with data is reached.

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1. We propose to modify Monte Carlo predictions with correction functions to account for systematic effects.
2. Successively more complex functions are used until sufficient compatibility with data is reached.
3. Data distributions don't need to have the same shapes in signal and control regions.
Only the systematics have to affect them similarly.
4. Method not restricted to High Energy Physics!

Thank you for your attention

Backup slides

Statistical tests to determine the best model

Employ 2 likelihood ratios to assess the compatibility with data:

1. Absolute goodness-of-fit

Compare model i (polynomial i * template) with most flexible model where each bin can vary independently and will therefore take on the data values:

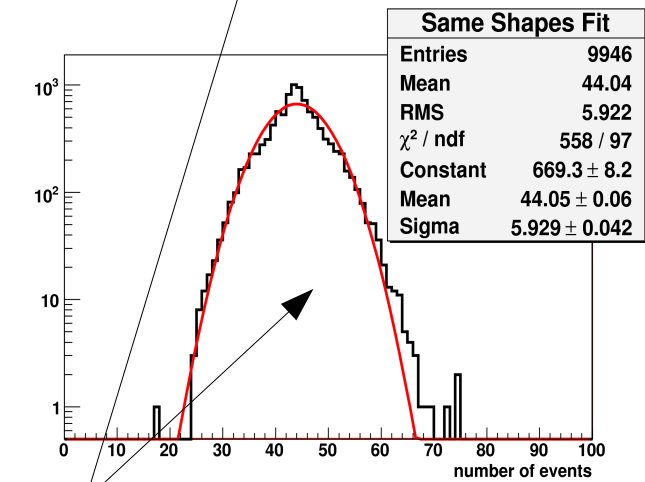
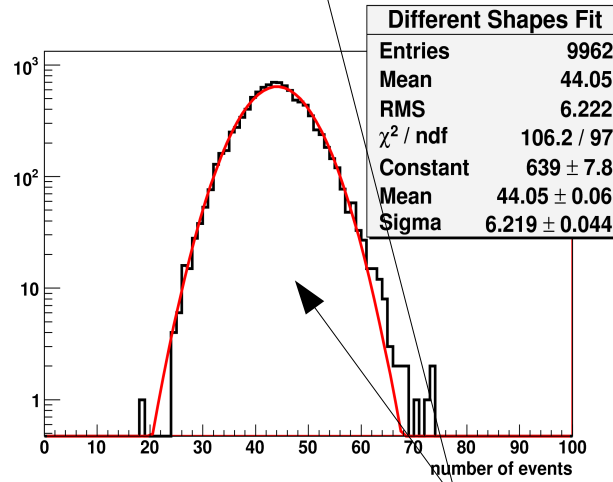
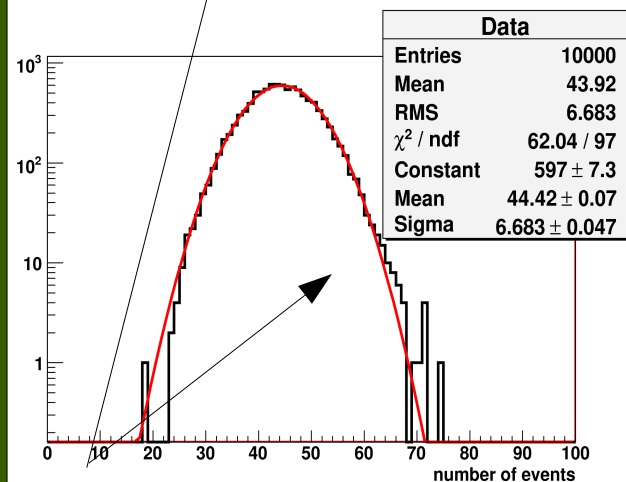
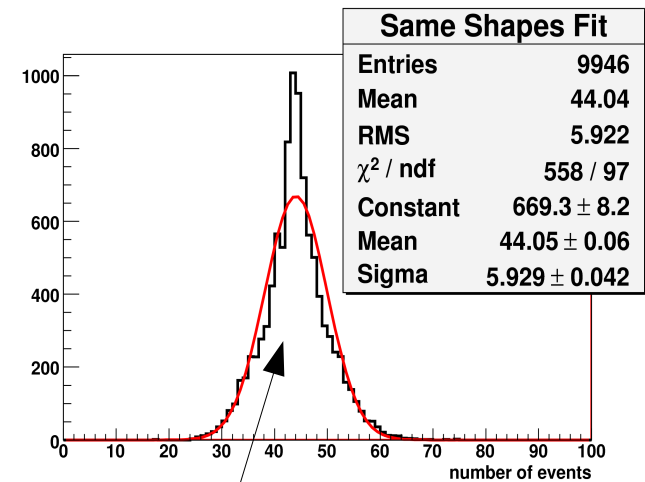
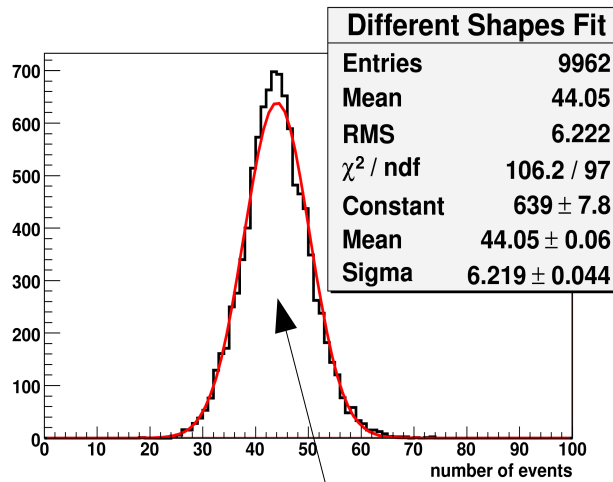
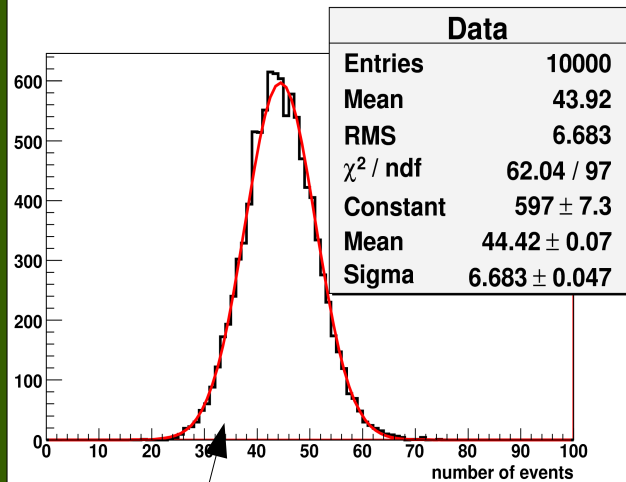
$$q_{\text{abs}} = -2 \ln \frac{\text{LH (Data | Model } i)}{\text{LH (Data | most flex. model = Data)}} \sim \chi^2$$

2. Does the next best model significantly improve the data description?

Compare model i with model $i+1$:

$$q_{\text{rel}} = -2 \ln \frac{\text{LH (Data | Model } i)}{\text{LH (Data | Model } i+1)}} \sim \chi^2$$

Considering many experiments - Gaussian fits



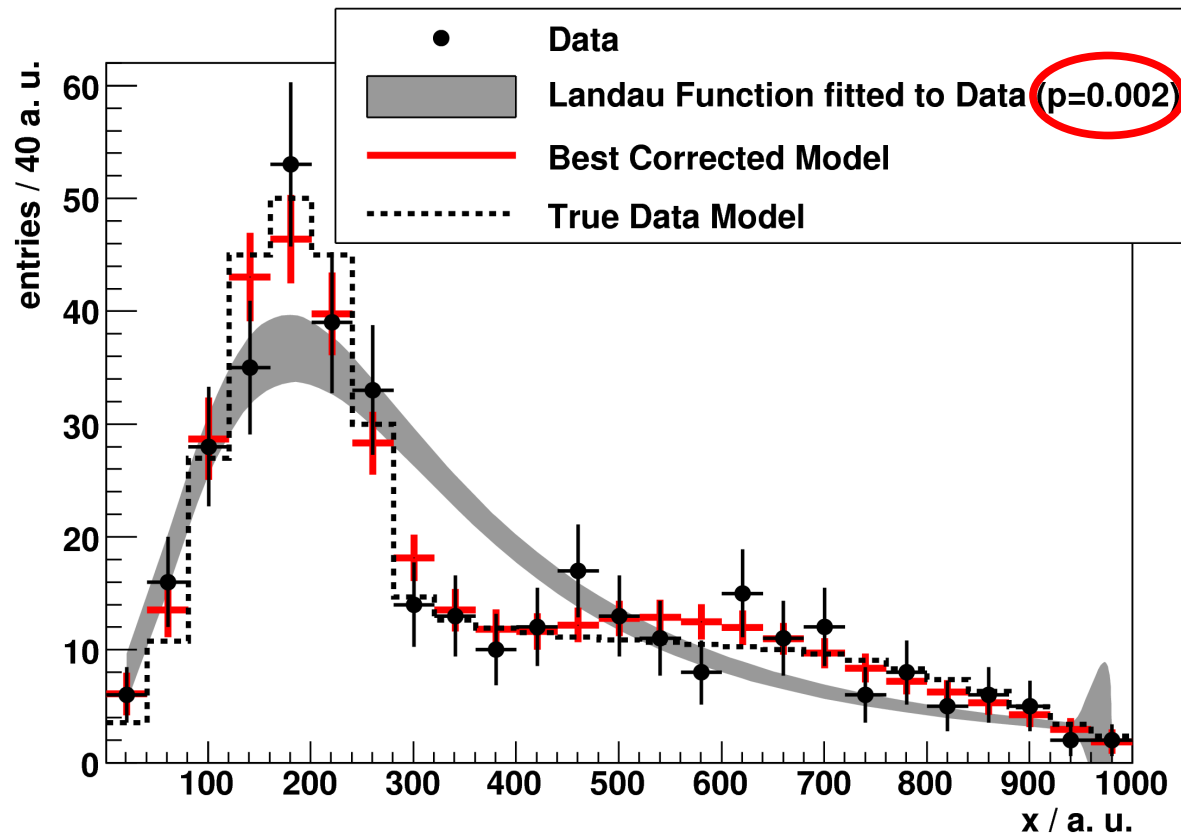
expect exact Poisson dist

Gaussian behavior desired

Expect Gaussian behavior to improve when including uncertainty for transfer from Control to Signal region.

Compare with: Parametrizing the Monte Carlo Template

Fit a **function inspired by the MC** to the data in the control region
(Original Template is a Landau Function)



This example: If systematics can't be compensated by adjustment of parameters data won't be nicely described.