The automation of subtraction terms for NLO calculations in QCD

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- Available subtraction methods at NLO
  - Catani-Seymour dipoles
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LO calculations

- Many packages available to do tree-level (leading order) automatically
  Alpgen, CompHep/CalcHep, Helac, MadGraph/MadEvent, Sherpa, Whizard, ...
- Extremely useful:
  - Quick results for any imaginable process within any imaginable "New Physics model"
    (within reason)
- Main drawback:
  - LO calculations are not reliable enough for precision LHC & Tevatron physics -- in particular for total rates, but also for shapes of distributions

Goal

- The goal is to have the same flexibility for Next-to-Leading Order calculations

Contributions to NLO calculations

$$\sigma^{\text{NLO}} = \int_{m+1} d^{(d)}\sigma^{R} + \int_{m} d^{(d)}\sigma^{V} + \int_{m} d^{(4)}\sigma^{B}$$

‘Real emission’
NLO corrections

‘Virtual’ or ‘one-loop’
NLO corrections

This conference: G. Ossola & D. Maitre & others

‘Born’ or ‘LO’
contribution

Available ‘everywhere’

Binoth Les Houches Accord

"Dedicated to the memory of, and in tribute to, Thomas Binoth, who led the effort to develop this proposal for Les Houches 2009"

- Start-up phase

  Monte Carlo tool writes out an 'order file' with the process and parameters to calculate. Read by one-loop program (OLP) and confirmed (or rejected) in 'contract file'

- Run phase
  - initialization

    Contract file is read again by OLP
  - event-by-event

    Momenta, strong coupling & ren. scale are passed from MC to OLP. OLP returns the coefficients of the poles and finite terms of the virtual corrections

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IR divergence

$$\sigma^{\mathrm{NLO}} = \int_{m+1} d^{(d)}\sigma^{R} + \int_{m} d^{(d)}\sigma^{V} + \int_{m} d^{(4)}\sigma^{B}$$

- Real emission -> IR divergent
- (UV-renormalized) virtual corrections -> IR divergent
  - After integration, the sum of all contributions is finite (for infrared-safe observables)
- Relative straightforward to get explicit poles for virtual corrections, because loop integrals (scalar integrals) are done analytically

Subtraction terms

$$\sigma^{\text{NLO}} = \int_{m+1} d^{(d)}\sigma^R + \int_m d^{(d)}\sigma^V + \int_m d^{(4)}\sigma^B$$

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$$\sigma^{\text{NLO}} = \int_{m+1} d^{(d)}\sigma^R + \int_m d^{(d)}\sigma^V + \int_m d^{(4)}\sigma^B$$

$$\sigma^{\text{NLO}} = \int_{m+1} \left[d^{(4)}\sigma^R - d^{(4)}\sigma^A\right] + \int_m \left[d^{(4)}\sigma^B + \int_{\text{loop}} d^{(d)}\sigma^V + \int_1 d^{(d)}\sigma^A\right]_{\epsilon=0}$$

- Include subtraction terms to make real emission contributions and virtual contributions separately finite

- All can be integrated numerically

Subtraction terms

$$\sigma^{\text{NLO}} = \int_{m+1} d^{(d)}\sigma^R + \int_m d^{(d)}\sigma^V + \int_m d^{(4)}\sigma^B$$

$$\sigma^{\text{NLO}} = \int_{m+1} \left[d^{(4)}\sigma^R - d^{(4)}\sigma^A\right] + \int_m \left[d^{(4)}\sigma^B + \int_{\text{loop}} d^{(d)}\sigma^V + \int_1 d^{(d)}\sigma^A\right]_{\epsilon=0}$$

"subtraction term"

"(finite remainder of the) integrated subtraction term"

And this is what we would like to automate

Why automate?

- To check implementations done by hand
  All calculations should be double checked if possible

- To save time
  NLO calculations can take a long time. It would be nice to spend this time doing phenomenology instead.

- To reduce the number of bugs in the calculation
  Having a code that does everything automatically will be without* bugs once the internal algorithms have been checked properly.

- To have all processes within one framework
  To learn how to use a new code for each process is not something all our (experimental) colleagues are willing to do.

Automation of subtraction schemes

- Catani-Seymour dipole subtraction *Catani & Seymour 1997; Catani, Dittmaier, Seymour & Trocsanyi 2002.*
  - implemented by various groups *Seymour & Tevlin; RF, Gehrmann & Greiner; Hasegawa, Moch & Uwer; Gleisberg & Krauss; Czakon, Papadopoulos & Worek*
- Nagy-Soper dipoles *Nagy & Soper 2007;*
  - implementation in progress *Robens & Chung.*
- FKS subtraction *Frixione, Kunzst & Signer 1996.*
  - implemented in MadFKS *RF, Frixione, Maltoni & Stelzer* and the POWHEG BOX *Alioli, Nason, Oleari & Re.*
- No automation available for other methods (such as Antenna subtraction)

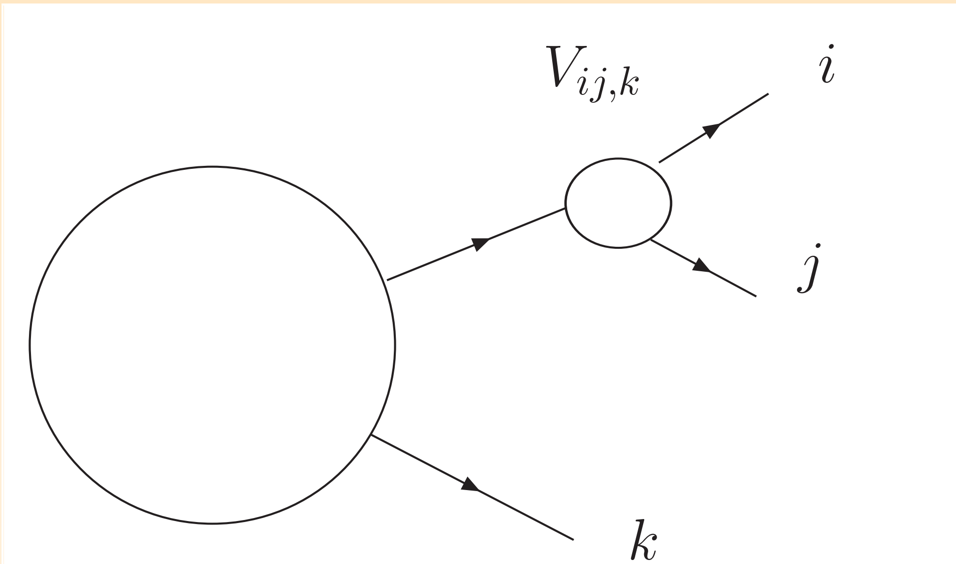
Catani-Seymour dipoles

- Originally by Stefano Catani and Mike Seymour (1997)
- Extended for massive QCD particles by Catani, Dittmaier, Seymour and Trocsanyi (2002)
- Most used method for NLO calculations
- Proven to be working for simple as well as (very) complicated cases:
  - all known NLO 2→4 particle processes use this method

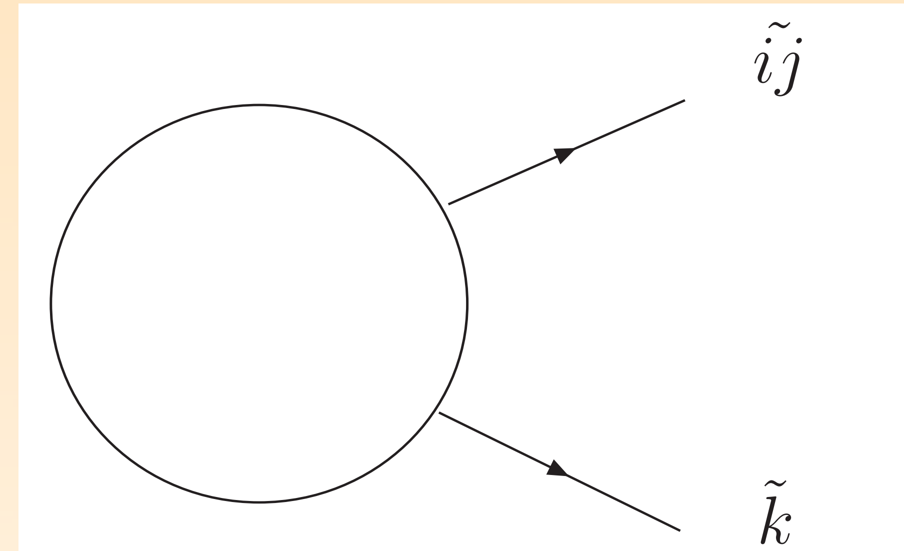
CS-dipoles

- Define subtraction terms with the same singularities
- Subtraction terms are defined by “emitter”, “unresolved” and the “spectator” particles
- Spectator is needed to keep all the momenta on-shell after phase space mapping

CS-dipoles



$$\rightarrow \quad \mathcal{D}_{ij,k} =$$



$$\mathcal{D}_{ij,k}\ (p_1, \ldots, p_{m+1}) = -\frac{1}{2p_i \cdot p_j}$$

$$\cdot\ {}_m\!< 1, .., \widetilde{ij}, .., \widetilde{k}, .., m+1 |\ \frac{\boldsymbol{T}_k \cdot \boldsymbol{T}_{ij}}{\boldsymbol{T}_{ij}^2}\ \boldsymbol{V}_{ij,k}\ |1, .., \widetilde{ij}, .., \widetilde{k}, .., m+1 >_m \ .$$

with emitter i and spectator k and dipole splitting function $V_{ij,k}$.

$$\widetilde{p}_k^\mu = \frac{1}{1-y_{ij,k}}\, p_k^\mu \quad , \quad \widetilde{p}_{ij}^\mu = p_i^\mu + p_j^\mu - \frac{y_{ij,k}}{1-y_{ij,k}}\, p_k^\mu , \quad y_{ij,k} = \frac{p_i p_j}{p_i p_j + p_j p_k + p_k p_i} .$$

CS-dipoles

- Number of dipoles scales like N^3, where N is the number of final state QCD partons
    - Can be improved by only subtracting when close to the singularity *(Nagy, Trocsanyi)*
- All dipoles have different kinematics due to the different phase-space mapping
- In general color factors for subtraction terms are different from the Born

FKS subtraction

- FKS subtraction: Frixione, Kunszt & Signer (1996). Standard subtraction method in MC@NLO and and automated in MadFKS and POWHEG BOX
- Also known as “residue subtraction”
- Based on using plus-distributions to regulate the infrared divergences of the real emission matrix elements

FKS SUBTRACTION

- Easiest to understand by starting from real emission:

$$d\sigma^R = |M^{n+1}|^2 d\phi_{n+1}$$

- $|M^{n+1}|^2$ blows up like $\frac{1}{\xi_i^2}\frac{1}{1-y_{ij}}$ with $\xi_i = E_i/\sqrt{\hat{s}}$, $y_{ij} = \cos\theta_{ij}$

- Partition the phase space in such a way that each partition has at most one soft and one collinear singularity

$$d\sigma^R = \sum_{ij} S_{ij}|M^{n+1}|^2 d\phi_{n+1} \qquad \sum_{ij} S_{ij} = 1$$

- Use plus distributions to regulate the singularities

$$d\tilde{\sigma}^R = \sum_{ij}\left(\frac{1}{\xi_i}\right)_+ \left(\frac{1}{1-y_{ij}}\right)_+ \xi_i(1-y_{ij})S_{ij}|M^{n+1}|^2 d\phi_{n+1}$$

FKS subtraction

$$d\tilde{\sigma}^R = \sum_{ij} \left(\frac{1}{\xi_i}\right)_+ \left(\frac{1}{1-y_{ij}}\right)_+ \xi_i (1-y_{ij}) S_{ij} |M^{n+1}|^2 d\phi_{n+1}$$

- Definition plus distribution

$$\int d\xi \left(\frac{1}{\xi}\right)_+ f(\xi) = \int d\xi \, \frac{f(\xi)-f(0)}{\xi}$$

- One event has maximally three counter events:
  - Soft: $\xi_i \to 0$
  - Collinear: $y_{ij} \to 1$
  - Soft-collinear: $\xi_i \to 0 \qquad y_{ij} \to 1$

FKS subtraction

$$d\tilde{\sigma}^R = \sum_{ij} \left(\frac{1}{\xi_i}\right)_{\xi_{cut}} \left(\frac{1}{1-y_{ij}}\right)_{\delta_O} \xi_i (1-y_{ij}) S_{ij} |M^{n+1}|^2 d\phi_{n+1}$$

- Definition plus distribution

$$\int d\xi \left(\frac{1}{\xi}\right)_{\xi_{cut}} f(\xi) = \int d\xi \, \frac{f(\xi) - f(0)\Theta(\xi_{cut} - \xi)}{\xi}$$

- One event has maximally three counter events:
  - Soft: $\xi_i \to 0$
  - Collinear: $y_{ij} \to 1$
  - Soft-collinear: $\xi_i \to 0 \qquad y_{ij} \to 1$

Subtraction terms

$$\sigma^{\text{NLO}} = \int_{m+1} \left[d^{(4)}\sigma^R - d^{(4)}\sigma^A \right] + \int_m \left[d^{(4)}\sigma^B + \int_{\text{loop}} d^{(d)}\sigma^V + \int_1 d^{(d)}\sigma^A \right]_{\epsilon=0}$$

- This defines the subtraction terms for the reals in the C.S. or FKS frameworks
- They need to be integrated over the one-parton phase space (analytically) and added to the virtual corrections
  - these are process-independent terms proportional to the (color-linked) Borns

FKS vs C.S. dipoles

	Catani-Seymour dipoles	FKS
Naive scaling with final state particles	$\sim N^3$	$\sim N^2$
Restrict subtraction to divergent regions	✓	✓
Symmetry to reduce # of divergent regions	✘	✓
Fixed helicity	✓	✓
Fixed color	✘	✘

Automation

- What is needed:
  - Bookkeeping!
  - Subtraction terms
    - Color-linked Borns
    - Off-diagonal helicity Borns
  - Phase-space integration

Available packages

	Type	Book-keeping	Color-linked/ Off-diagonal Borns	Integrated Subtr. terms	Phase Space	Authors
TevJet	C.S.	✓	✗	✓	✗	Seymour, Tevlin
AutoDipole	C.S.	✓	✓	✗	✗	Hasegawa *et al.*
MadDipole	C.S.	✓	✓	✗	✗	RF *et al.*
Sherpa	C.S.	✓	✓	✓	✓	Gleisberg, Krauss
Helac	C.S.	✓	✓	✓	✓	Czakon *et al.*
MadFKS	FKS	✓	✓	✓	✓	RF *et al.*
POWHEG Box	FKS	✓	✗	✓	✗	Alioli *et al.*

To conclude

- NLO corrections are needed for precision phenomenology and to understand all features of the experimental data
- Many packages available focussing on different users
  - To test calculations done by hand (TevJet, AutoDipole, MadDipole)
  - Complete NLO Monte Carlo (Sherpa, Helac, MadFKS)
  - Automated NLO + Parton shower calculation (POWHEG Box)
- We are getting close to a fully fledged, completely automatic Monte Carlo for NLO calculations for any given complexity