

# **Charm decay studies with threshold data**

**-some selected topics of (semi-)leptonic decays, hadronic structure and quantum correlation**

**Hailong Ma (IHEP&BESIII)**

**XXXVII International Symposium on Physics in Collision,  
Prague, Czech Republic, September 4-8 2017**

# Introduction

Leptonic and hadronic decays of charmed hadrons ( $D^0$ ,  $D^+$ ,  $D_s^+$  and  $\Lambda_c^+$ ) provide an ideal test-bed to explore weak and strong effects

## ➤ D leptonic decays

$f_{D(s)^+}$ ,  $f_{K(\pi)^+}^{K(\pi)}$ : better calibrate LQCD

$|V_{cs(d)}|$ : better test on CKM unitarity

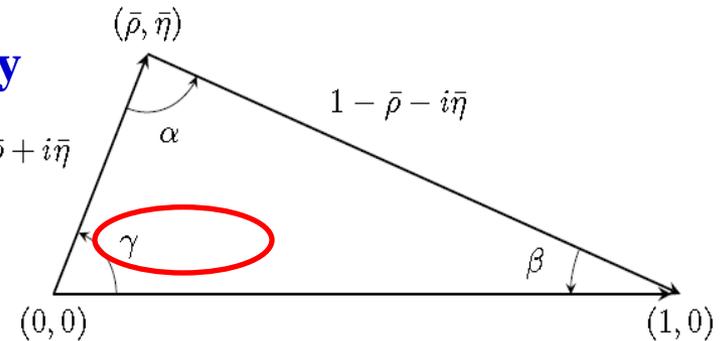
$$U = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix}$$

## ➤ D hadronic decays

Hadronic decays: structure, SU(3) symmetry

$D^0\bar{D}^0$  mixing parameters and CP violation

Strong phase in  $D^0$  decays: Constraint on  $\gamma/\phi_3$  measurement in B decays



## ➤ D rare decays → Search for new physics

## ➤ Absolute BFs of $\Lambda_c^+$ decays

No absolute BF measurements of  $\Lambda_c^+$  using near threshold data before BESIII

# Contents

- Data at threshold

- (Semi-)leptonic  $D_{(s)}$  decays

$$D_{(s)}^+ \rightarrow l^+ \nu & Kl^+ \nu$$

- Hadronic  $D_{(s)}$  decays

structure & quantum correlation

- $\Lambda_c^+$  decays at BESIII

- Summary



Some selected  
topics at CLEO-  
c and BESIII

# Experiments near threshold

- $e^+e^- \rightarrow \psi(3770) \rightarrow D^0\bar{D}^0, D^+D^- @ \sim 3.773$  GeV from MARKIII, BESII, CLEO-c and BESIII
- $e^+e^- \rightarrow \psi(4030) \rightarrow D_s^+D_s^- @ \sim 4.03$  GeV from BESII and BESIII
- $e^+e^- \rightarrow \psi(4170) \rightarrow D_s^{*+}D_s^- + c.c. @ \sim 4.17$  GeV from MARKIII, CLEO-c and BESIII
- $e^+e^- \rightarrow \Lambda_c^+\bar{\Lambda}_c^- + c.c. @ \sim 4.6$  GeV at BESIII

**Absolute  
measurements**

**ST yield:**

$$N_{ST}^i = 2 \times N_{D\bar{D}} \times B_{ST}^i \times \mathcal{E}_{ST}^i$$

**DT yield:**

$$N_{DT}^i = 2 \times N_{D\bar{D}} \times B_{ST}^i \times B_{sig} \times \mathcal{E}_{ST \text{ vs. sig}}^i$$

**Branching fraction:**

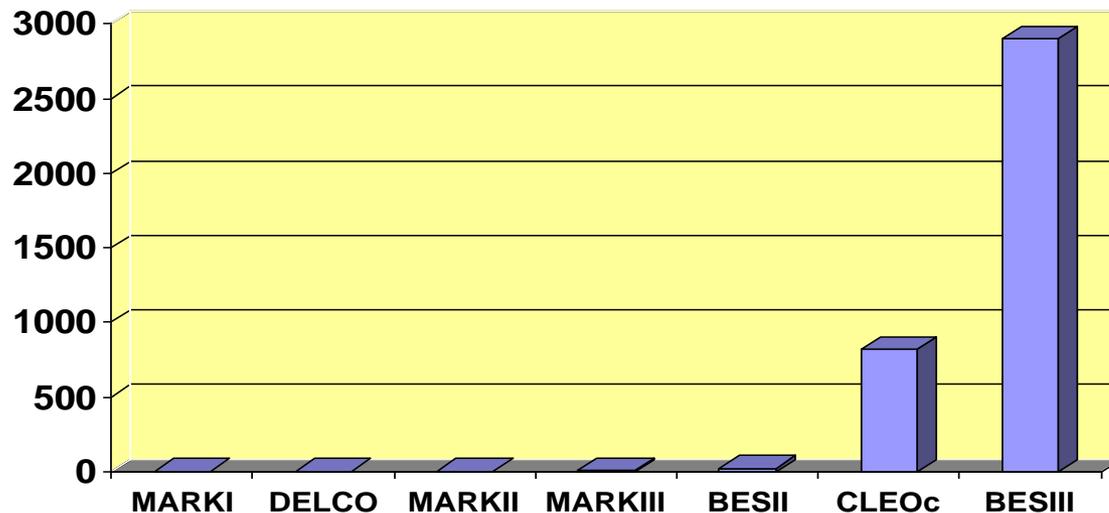
$$B_{sig} = \frac{N_{DT}^{tot}}{N_{ST}^{tot} \times \bar{\mathcal{E}}_{sig}}$$

**Average efficiency:**

$$\bar{\mathcal{E}}_{sig} = \sum_{i=1}^N (N_{ST}^i \times \mathcal{E}_{ST \text{ vs. sig}}^i / \mathcal{E}_{ST}^i) / \sum_{i=1}^N N_{ST}^i$$

# $D^{0(+)}, D_s^+, \Lambda_c^+$ samples ( $\text{pb}^{-1}$ ) at BESIII

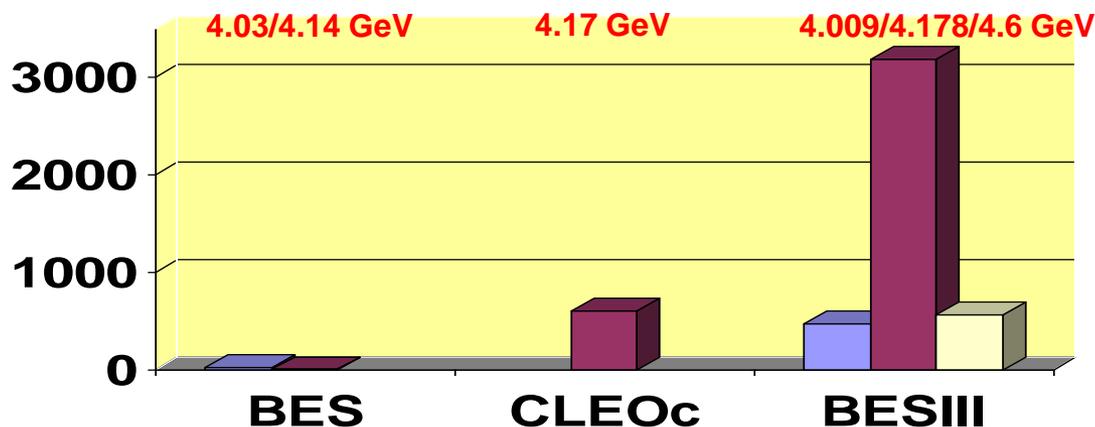
## ➤ $D^{0(+)}$ samples



Focus on some selected topics from CLEO-c and BESIII

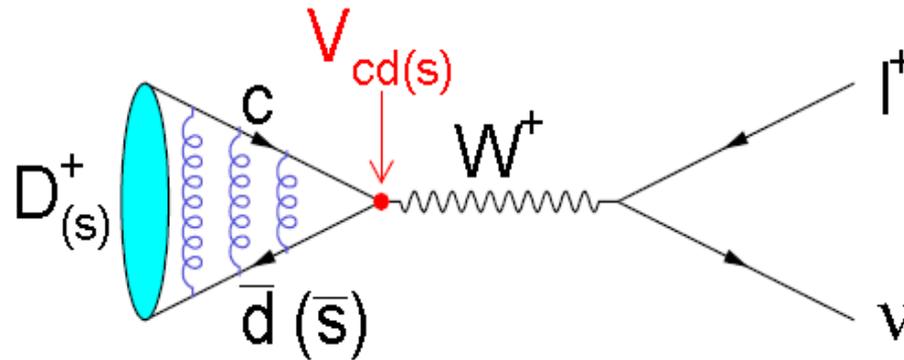
A series of results were published using a portion of data at CLEO-c. Final results are chosen

## ➤ $D_s^+ / D_s^- / \Lambda_c^+$ samples



# **(Semi)leptonic $D_{(s)}$ decays**

# $D_{(s)}^+$ leptonic decays



In the SM:

$$\Gamma(D_{(s)}^+ \rightarrow l^+ \nu_l) = \frac{G_F^2 f_{D_{(s)}^+}^2}{8\pi} |V_{cd(s)}|^2 m_l^2 m_{D_{(s)}^+} \left(1 - \frac{m_l^2}{m_{D_{(s)}^+}^2}\right)^2$$

Bridge to precisely measure

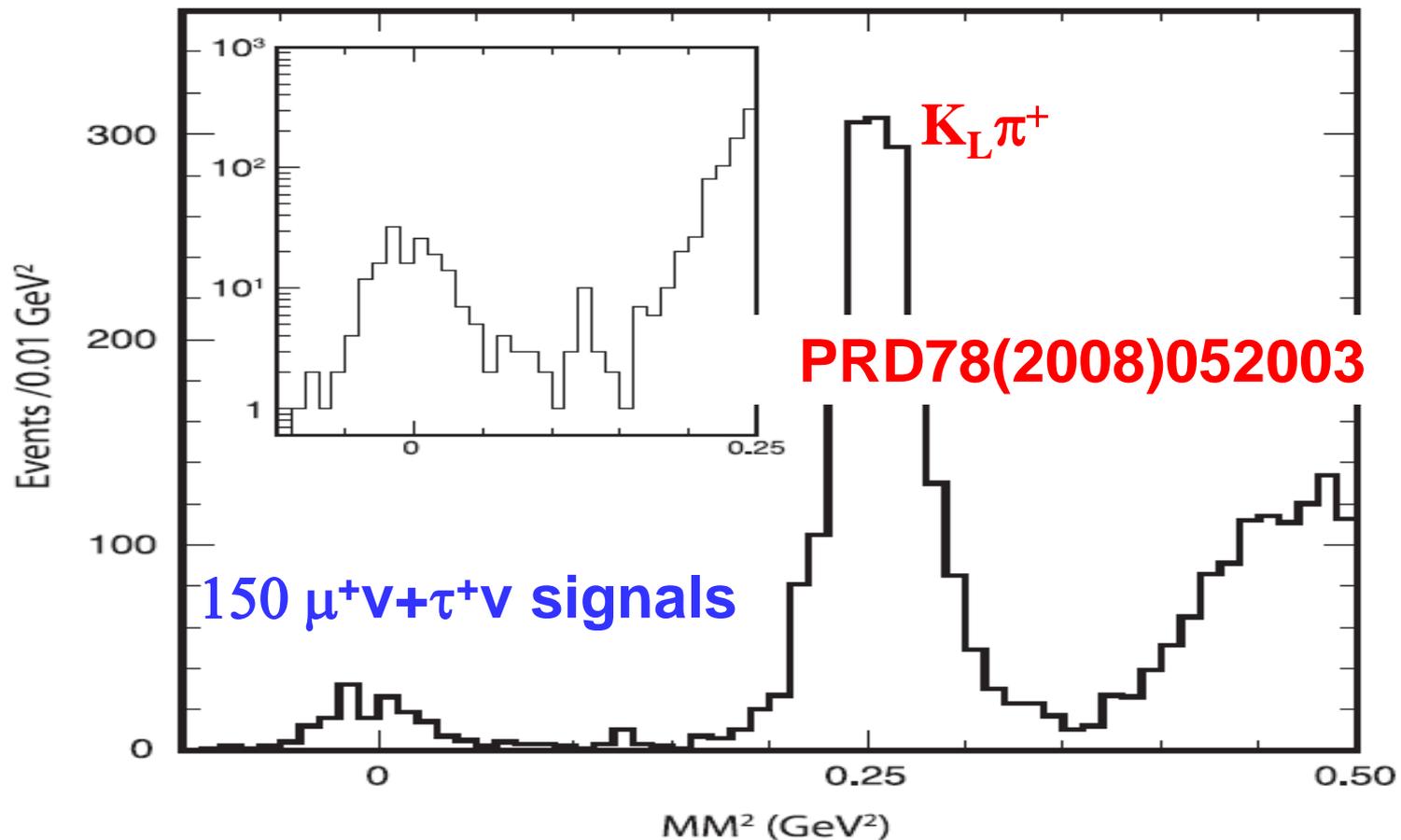
- Decay constant  $f_{D_{(s)}^+}$  with input  $|V_{cd(s)}|^{\text{CKMfitter}}$
- CKM matrix element  $|V_{cd(s)}|$  with input  $f_{D_{(s)}^+}^{\text{LQCD}}$

# Measurement of $f_{D^+}$ at CLEO-c

818 pb<sup>-1</sup> around  $\psi(3770)$  (2004–2008)

$$B[D^+ \rightarrow \mu^+ \nu] = (3.82 \pm 0.32 \pm 0.09) \times 10^{-4}$$

$$f_{D^+} = 205.8 \pm 7.5 \pm 2.5 \text{ MeV}$$

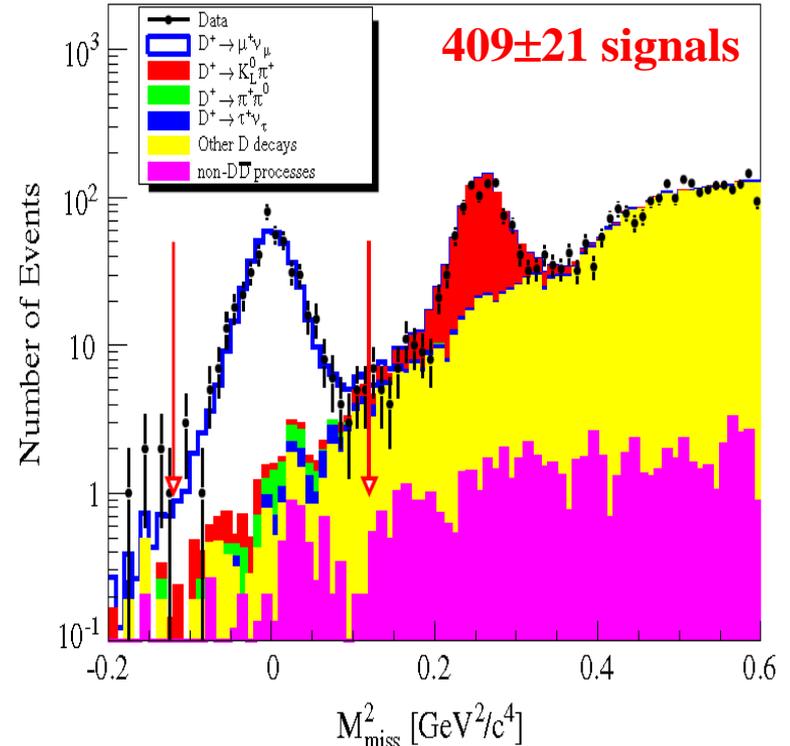
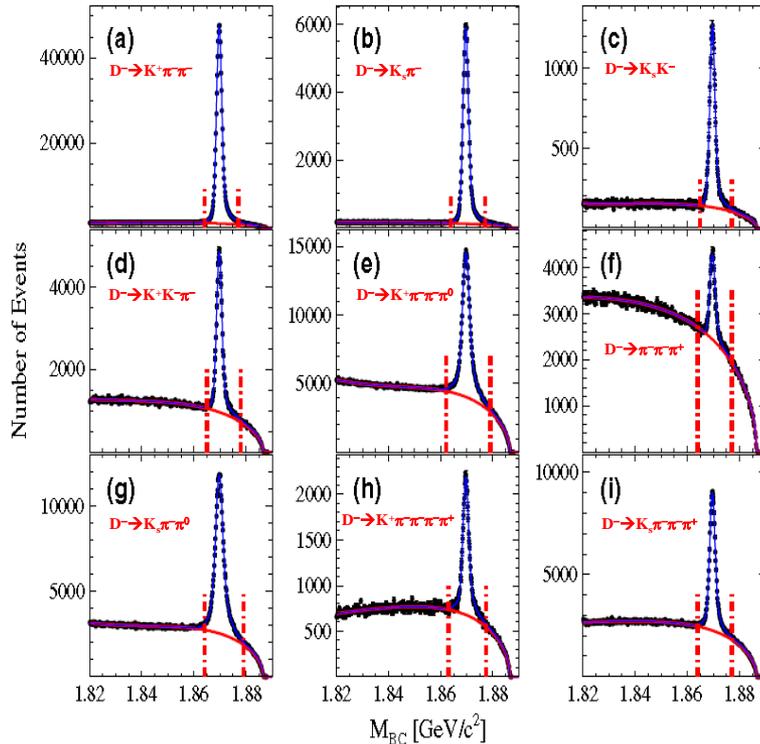


# Improved $B[D^+ \rightarrow \mu^+ \nu]$ , $f_{D^+} |V_{cd}|$ at BESIII

$$e^+e^- \rightarrow \psi(3770) \rightarrow D^+D^-$$

2.93 fb<sup>-1</sup> data@ 3.773 GeV

PRD89(2014)051104R



$$N_{D^+_{\text{tag}}} = (170.31 \pm 0.34) \times 10^4$$

$$B[D^+ \rightarrow \mu^+ \nu] = (3.71 \pm 0.19 \pm 0.06) \times 10^{-4}$$

Input  $t_{D^+}$ ,  $m_{D^+}$ ,  $m_{\mu^+}$  on PDG  
and  $|V_{cd}|$  of CKM-Fitter

BESIII

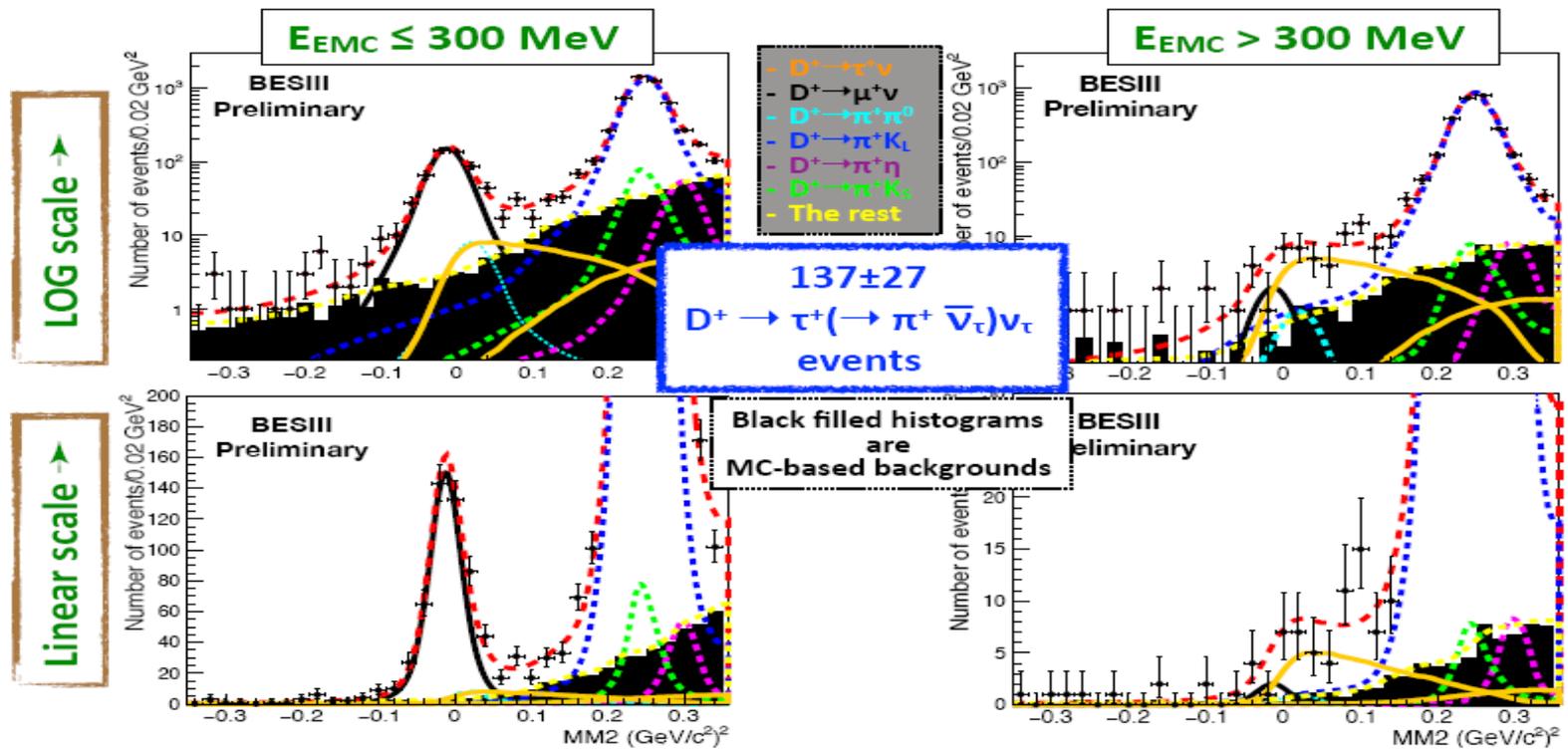
Input  $t_{D^+}$ ,  $m_{D^+}$ ,  $m_{\mu^+}$  on PDG and  
LQCD calculated  $f_{D^+} = 207 \pm 4$   
MeV [PRL100(2008)062002]

$$f_{D^+} = (203.2 \pm 5.3 \pm 1.8) \text{ MeV}$$

$$|V_{cd}| = 0.2210 \pm 0.0058 \pm 0.0047$$

# Evidence for $D^+ \rightarrow \tau^+(\pi^+ \nu) \nu$ ( $4\sigma$ ) at BESIII

With 6 dominant  $D^-$  single tag **Fitting to DATA**



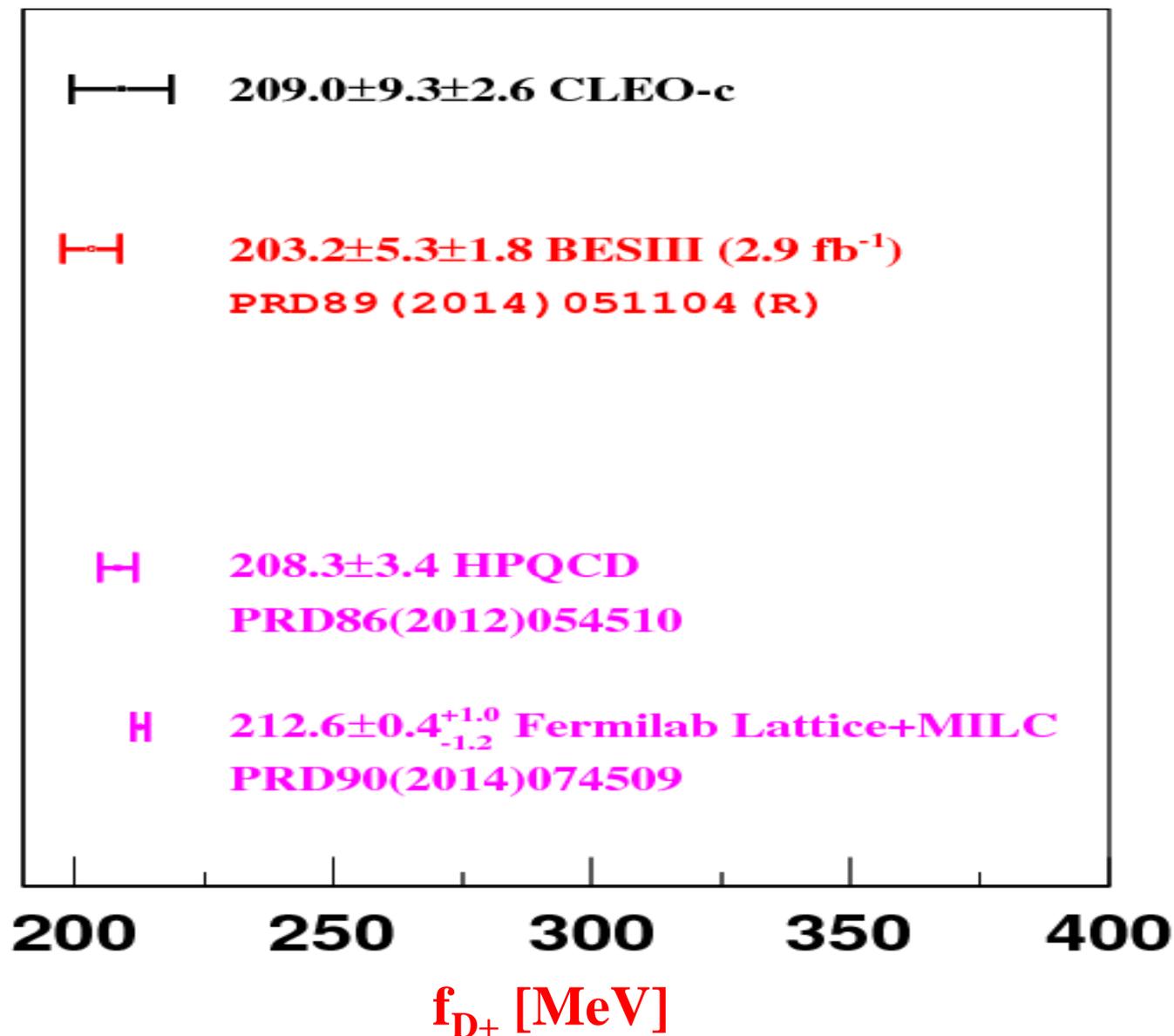
$$B[D^+ \rightarrow \tau^+ \nu] = (1.20 \pm 0.24_{\text{stat.}}) \times 10^{-3}$$

**SM prediction:  $2.66 \pm 0.01$**

**BESIII:  $3.21 \pm 0.64$**

$$R \equiv \frac{\Gamma(D^+ \rightarrow \tau^+ \nu)}{\Gamma(D^+ \rightarrow \mu^+ \nu)} = \frac{m_{\tau^+}^2 \left(1 - \frac{m_{\tau^+}^2}{M_{D^+}^2}\right)^2}{m_{\mu^+}^2 \left(1 - \frac{m_{\mu^+}^2}{M_{D^+}^2}\right)^2}$$

# Comparison of $f_{D^+}$



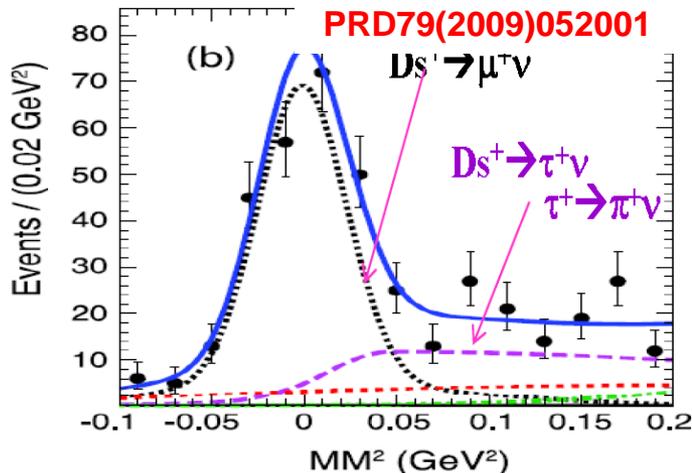
# Measurements of $f_{D_{S^*+}}$ at CLEO-c

$D_{S^*+}D_{S^-}$ , 600 pb<sup>-1</sup> @ 4.17 GeV

Absolute measurement gives significantly improved statistical and systematic errors [697 signal]

235 ± 14  $D_{S^*+} \rightarrow \mu^+\nu + \tau^+\nu$  signals

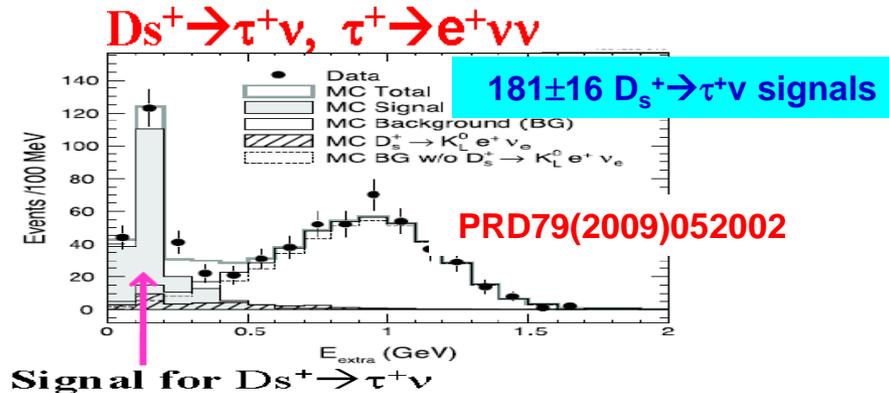
126 ± 16  $D_{S^*+} \rightarrow \tau^+\nu$  signals



$$B[D_{S^*+} \rightarrow \mu^+\nu] = (5.65 \pm 0.45 \pm 0.17) \times 10^{-3}$$

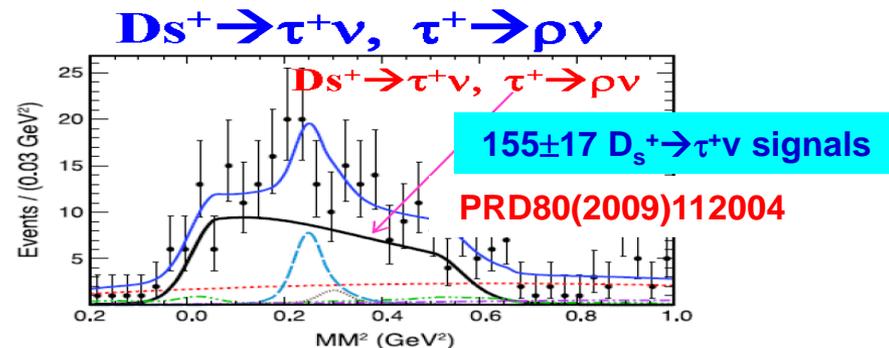
$$B[D_{S^*+} \rightarrow \tau^+\nu] = (6.42 \pm 0.81 \pm 0.18)\%$$

$$f_{D_{S^*+}} = 263.3 \pm 8.2 \pm 1.9 \text{ MeV}$$



$$B[D_{S^+} \rightarrow \tau^+\nu] = (5.30 \pm 0.47 \pm 0.21)\%$$

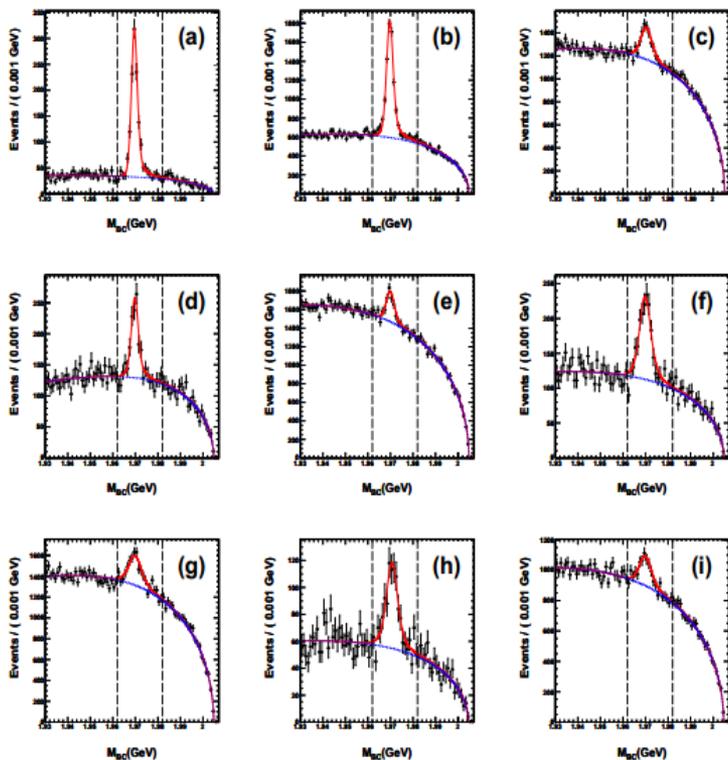
$$f_{D_{S^+}} = 252.2 \pm 11.1 \pm 5.2 \text{ MeV}$$



$$B[D_{S^+} \rightarrow \tau^+\nu] = (5.52 \pm 0.57 \pm 0.22)\%$$

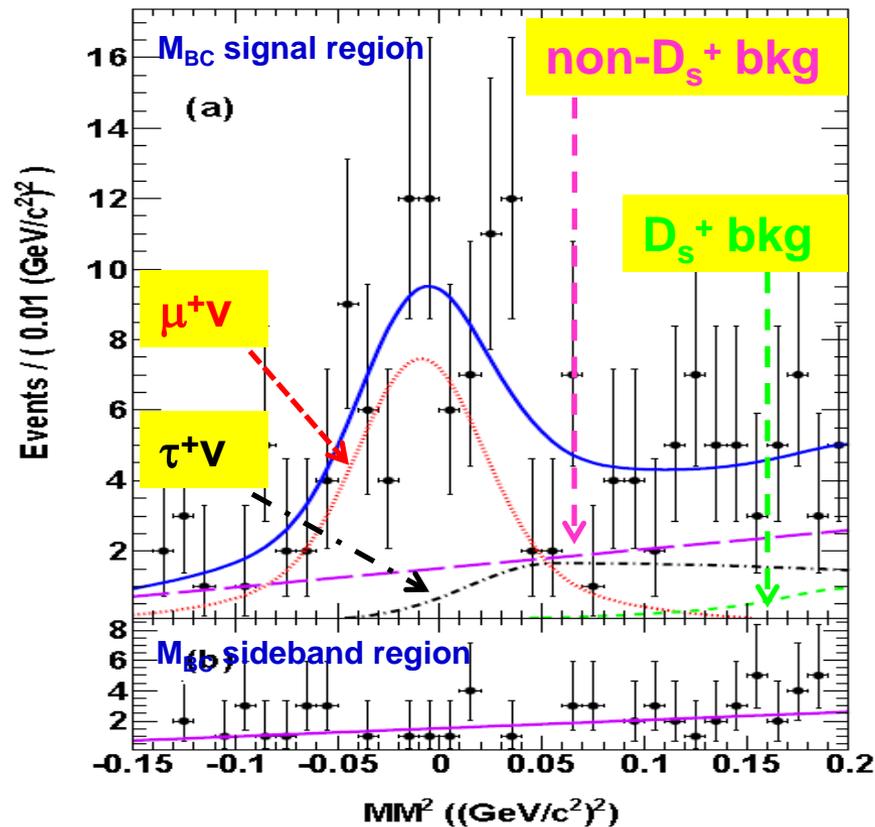
$$f_{D_{S^+}} = 257.8 \pm 13.3 \pm 5.2 \text{ MeV}$$

# $f_{D_{S^+}}$ at 4.009 GeV at BESIII



$$N_{D_s^+ \text{ tag}} = 15127 \pm 312$$

PRD94(2016)072004



$$B[D_s^+ \rightarrow \mu^+ \nu] = (0.495 \pm 0.067 \pm 0.026)\%$$

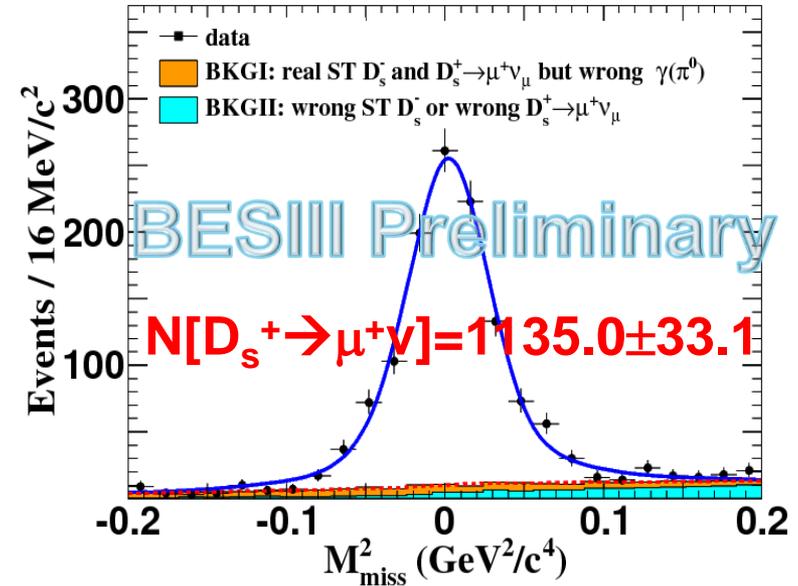
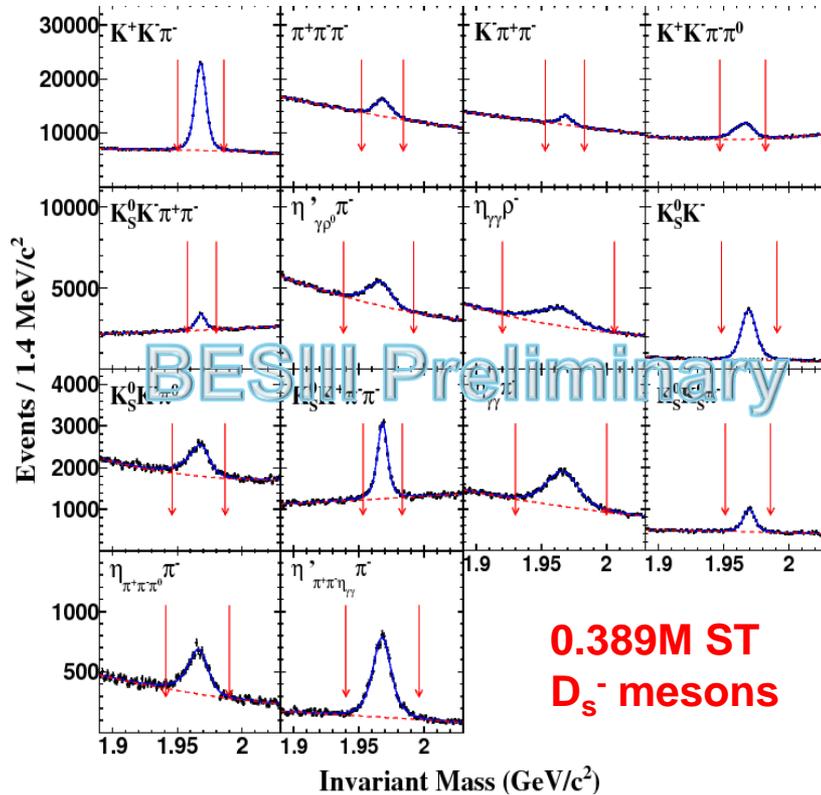
$$B[D_s^+ \rightarrow \tau^+ \nu] = (4.83 \pm 0.65 \pm 0.26)\%$$

$$f_{D_{S^+}} = (241.0 \pm 16.3 \pm 6.6) \text{ MeV}$$

# Improved $B[D_s^+ \rightarrow \mu^+ \nu]$ , $f_{D_s^+} |V_{cs}|$ at BESIII

3.19 fb<sup>-1</sup> data taken at 4.178 GeV in 2016

Use  $\mu$  counter to suppress background



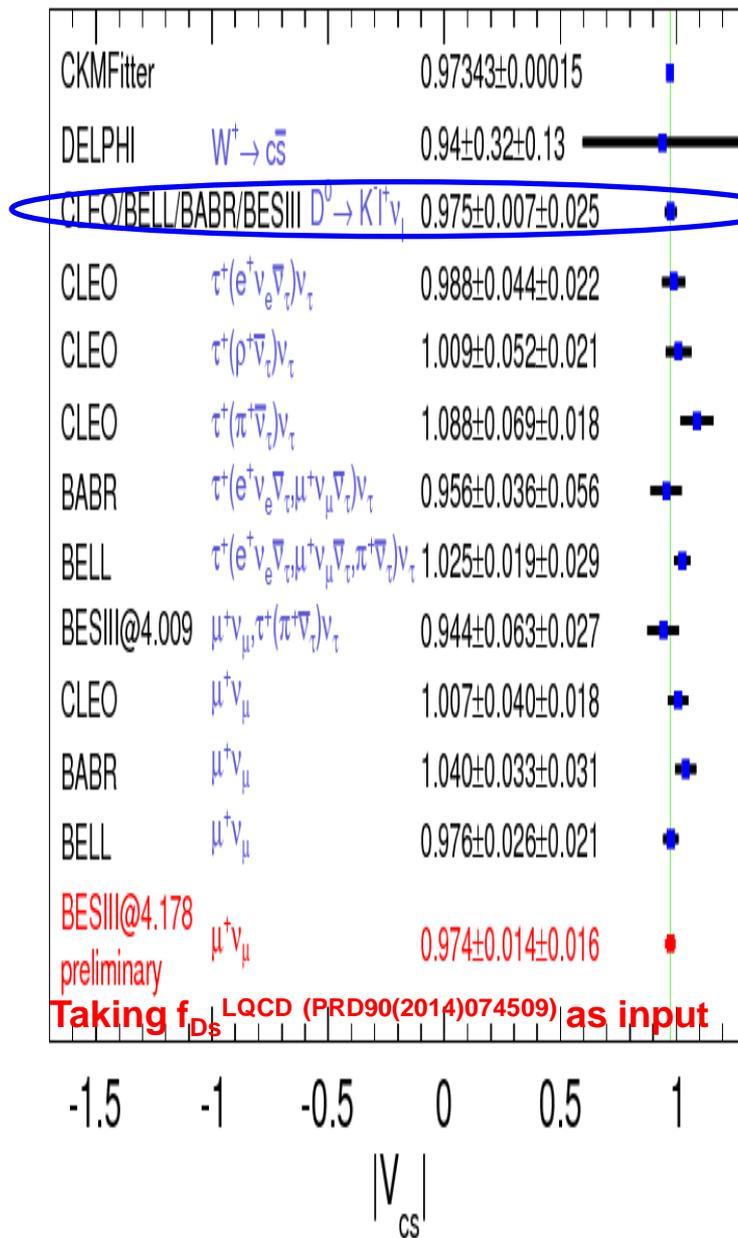
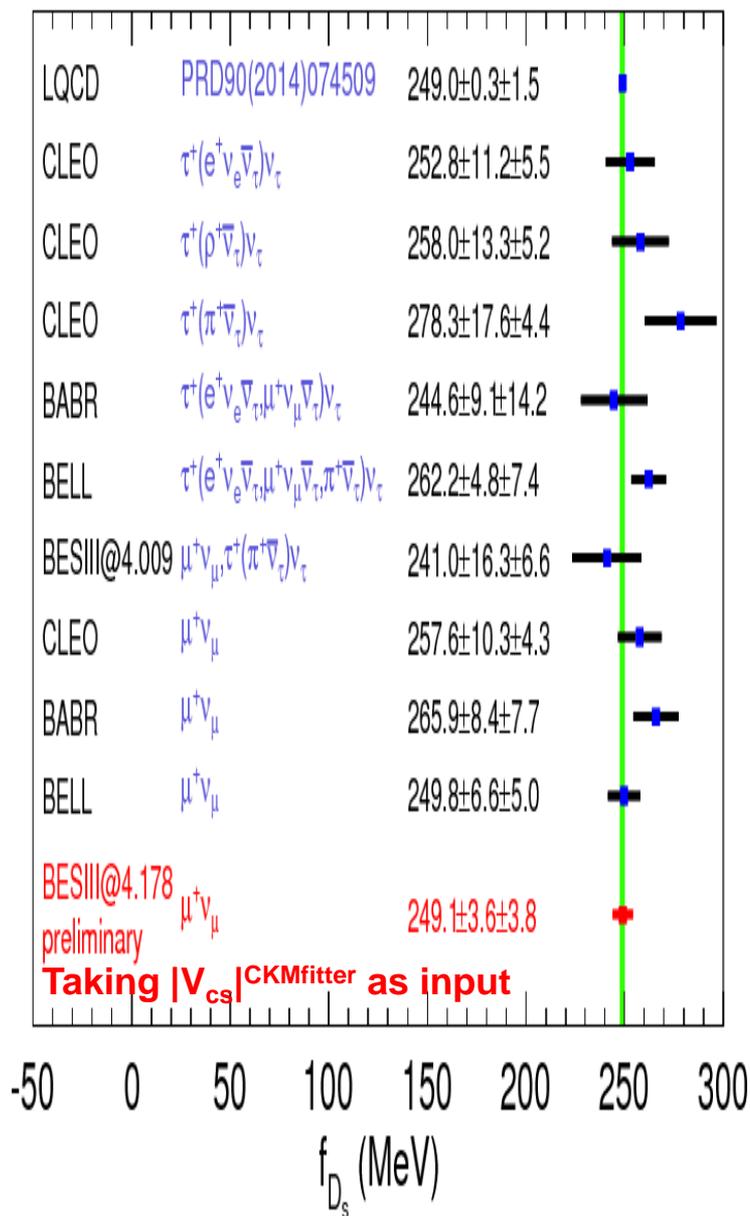
$$B[D_s^+ \rightarrow \mu^+ \nu] = (5.28 \pm 0.15_{\text{stat}} \pm 0.14_{\text{syst}}) \times 10^{-3}$$

$$f_{D_s^+} |V_{cs}| = 242.5 \pm 3.5_{\text{stat}} \pm 3.7_{\text{syst}} \text{ MeV}$$

1. Constraining signal/BKGI ratio via signal MC
2. Fixing BKGII via inclusive MC

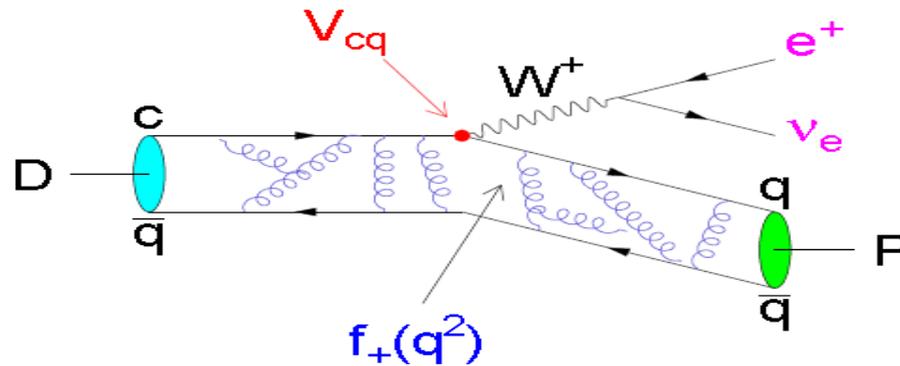
Source	Uncertainties(%)
Number of $D_s^-$	0.8
$\Delta E$ cut	0.5
$\gamma(\pi^0)$ match rate	1.1
$\gamma(\pi^0)$ efficiency	1.0
$\mu^+$ tracking	0.5
$\mu^+$ PID	0.8
$E_{\text{extra}}^{\text{max}} \gamma$ cut	0.3
$N_{\text{char}}^{\text{extra}} = 0$ cut	0.9
Signal shape	0.2
Fit range	0.6
Background estimation	0.6
MC statistics	0.4
$BF(D_s^{*+} \rightarrow e^+ e^- D_s^+)$	0.2
Radiative correction	1.0
Tag bias	0.6
Total	2.7

# Comparisons of $f_{D_{s^+}}$ and $|V_{cs}|$



Taken from PDG, and the SL method suffers about 2.4% uncertainty from LQCD

# Semi-leptonic decay $D \rightarrow K(\pi)e^+\nu$



Differential rates: 
$$\frac{d\Gamma}{dq^2} = X \frac{G_F^2 |V_{cd(s)}|^2}{24\pi^3} p^3 |f_+(q^2)|^2$$

Bridge to precisely measure:

■ **Form factors  $f_+^{D \rightarrow K(\pi)}(0)$  with input  $|V_{cd(s)}|^{\text{CKMfitter}}$**

– Single pole form

$$f_+(q^2) = \frac{f_+(0)}{1 - \frac{q^2}{M_{\text{pole}}^2}}$$

– Modified pole model

$$f_+(q^2) = \frac{f_+(0)}{\left(1 - \frac{q^2}{M_{\text{pole}}^2}\right) \left(1 - \alpha \frac{q^2}{M_{\text{pole}}^2}\right)}$$

– ISGW2 model

$$f_+(q^2) = f_+(q_{\text{max}}^2) \left(1 + \frac{r_{\text{ISGW2}}^2}{12} (q_{\text{max}}^2 - q^2)\right)^{-2}$$

– Series expansion model

$$f_+(t) = \frac{1}{P(t)\Phi(t, t_0)} a_0(t_0) \left(1 + \sum_{k=1}^{\infty} r_k(t_0) [z(t, t_0)]^k\right)$$

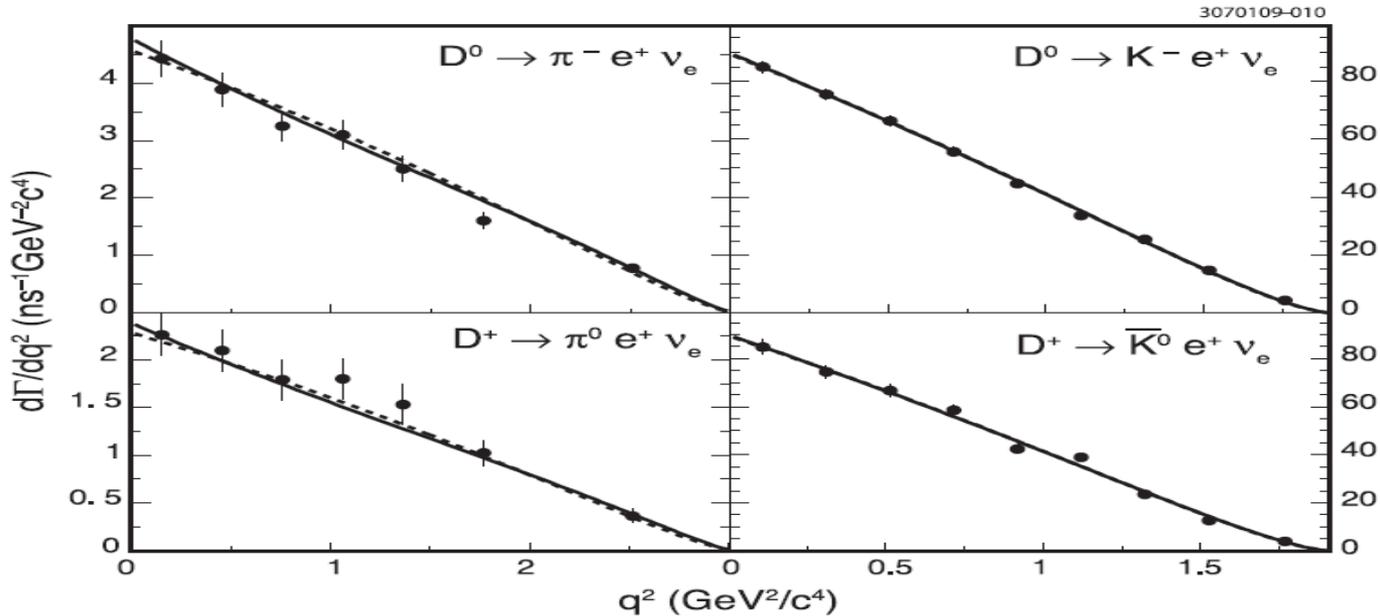
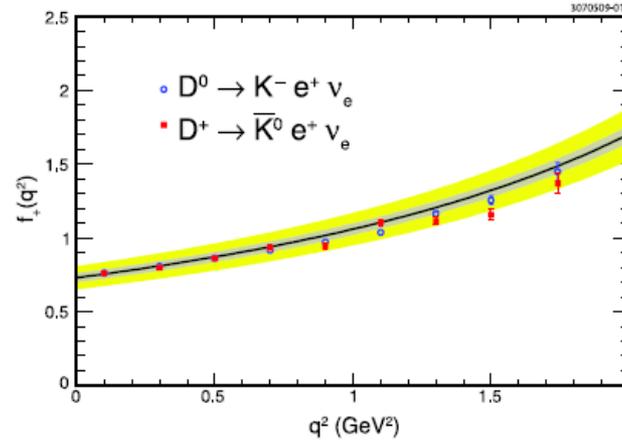
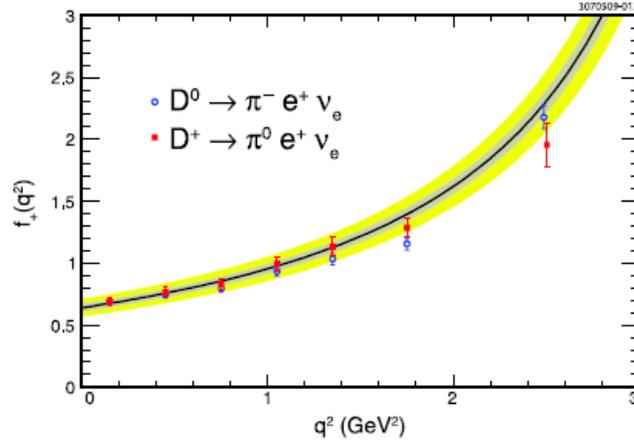
■ **CKM matrix element  $|V_{cs(d)}|$  with input  $f_+^{\text{LQCD}, D \rightarrow K(\pi)}(0)$**

# Tagged analyses of $D \rightarrow K(\pi)e^+\nu$ at CLEO-c

818 pb<sup>-1</sup> data at 3.773 GeV

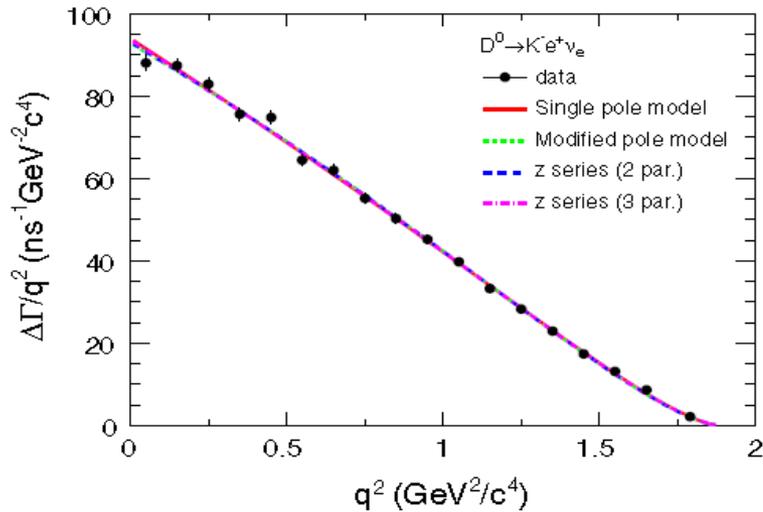
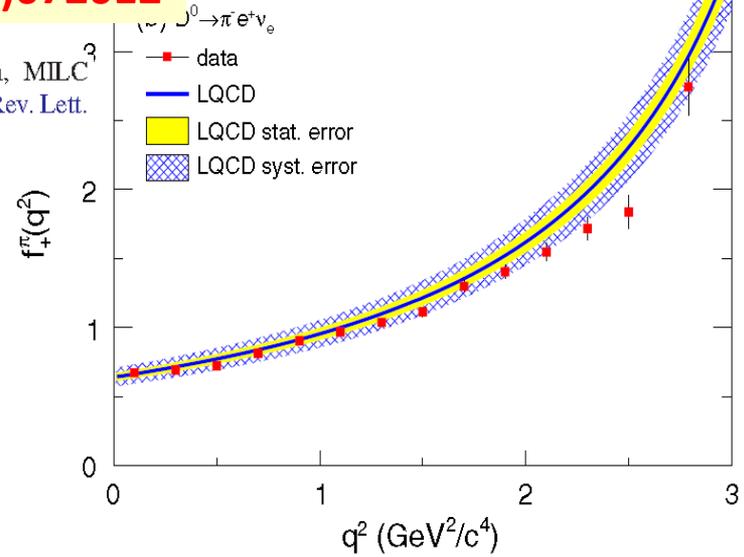
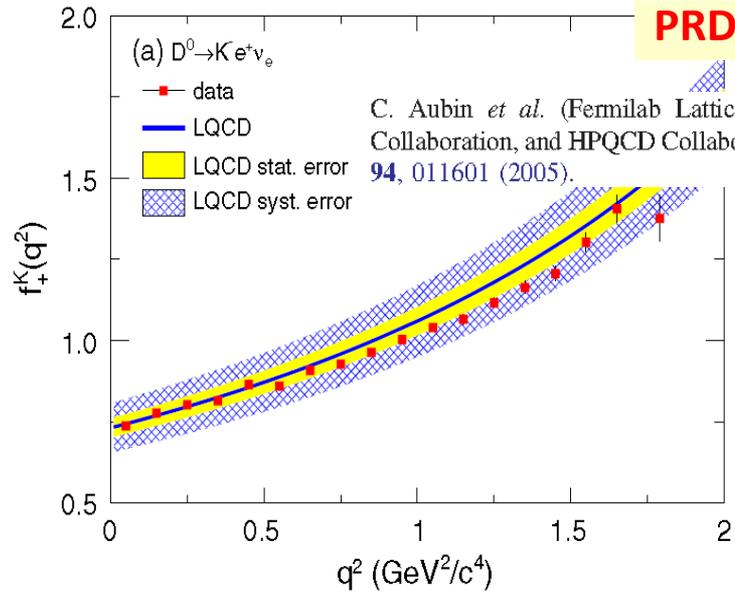
PRD80(2009)032005

IMPROVED MEASUREMENTS OF  $D$  MESON ...

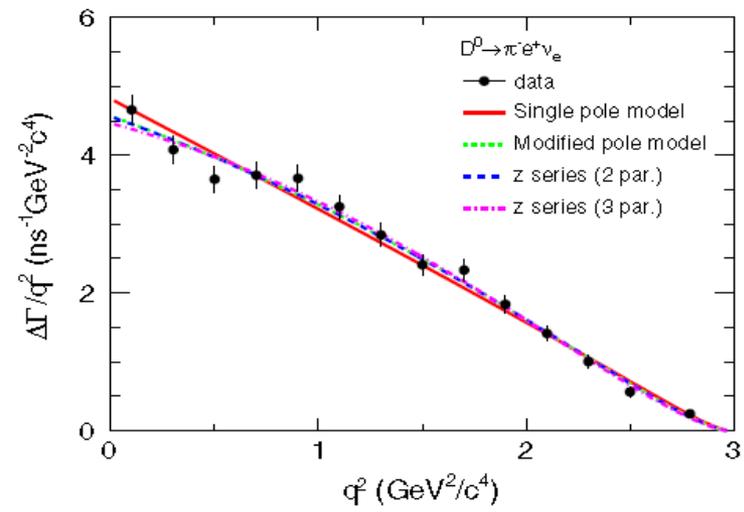


# Analyses of $D \rightarrow K(\pi)e^+\nu$ dynamics at BESIII

PRD92(2015)072012

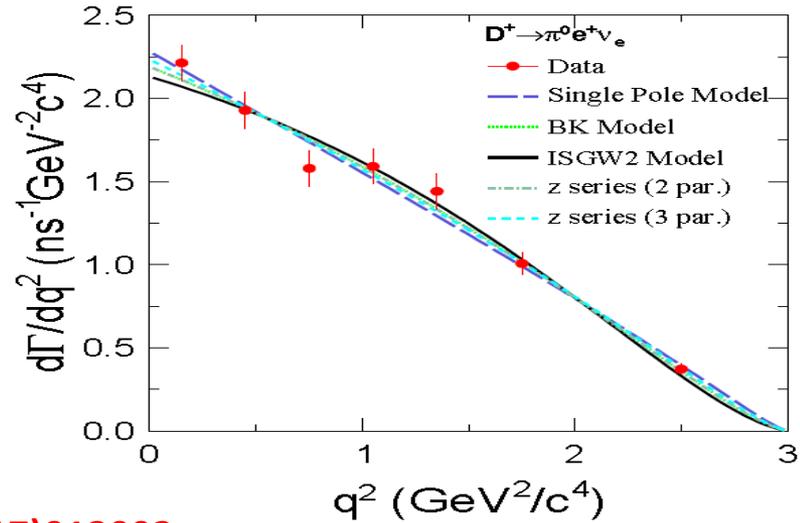
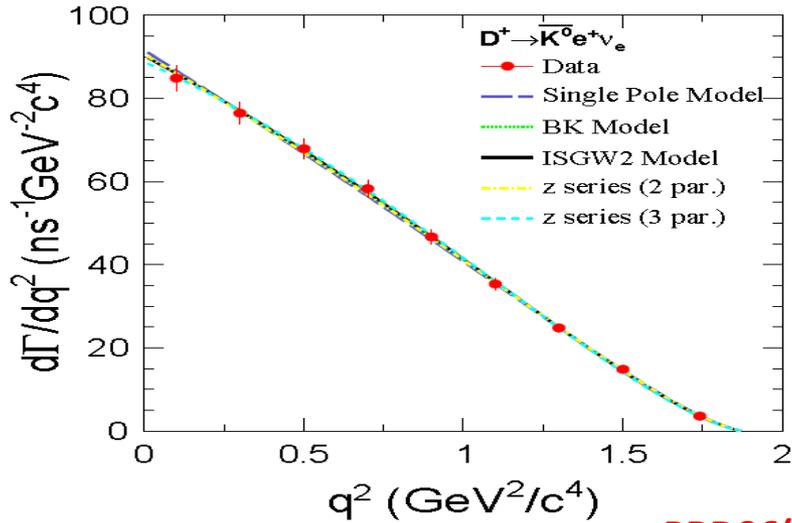


$D^0 \rightarrow K^- e^+ \nu$

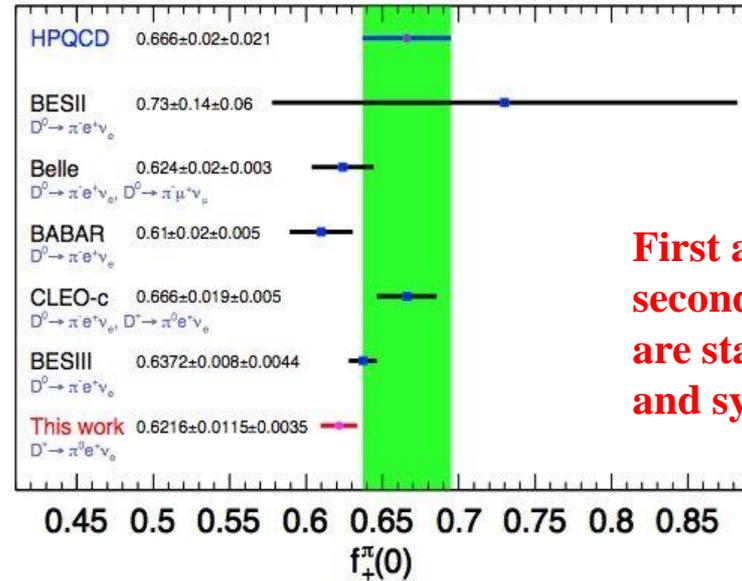
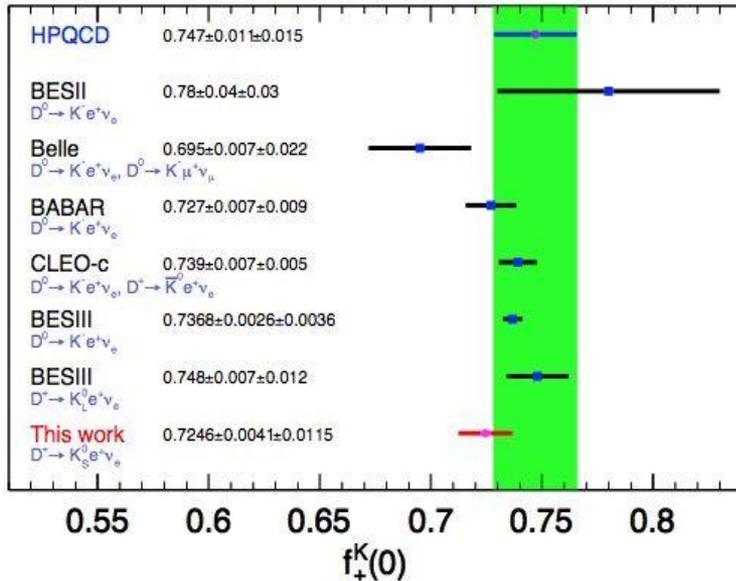


$D^0 \rightarrow \pi^- e^+ \nu$

# Comparisons of FFs by $D^+ \rightarrow \bar{K}^0(\pi^0)e^+\nu$

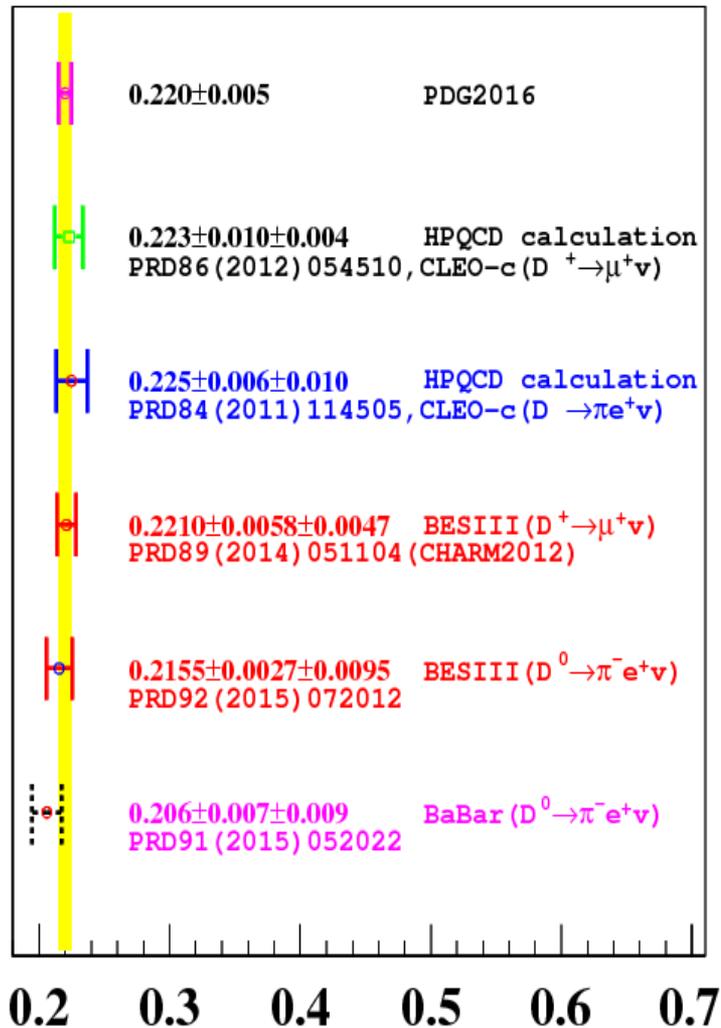


PRD96(2017)012002

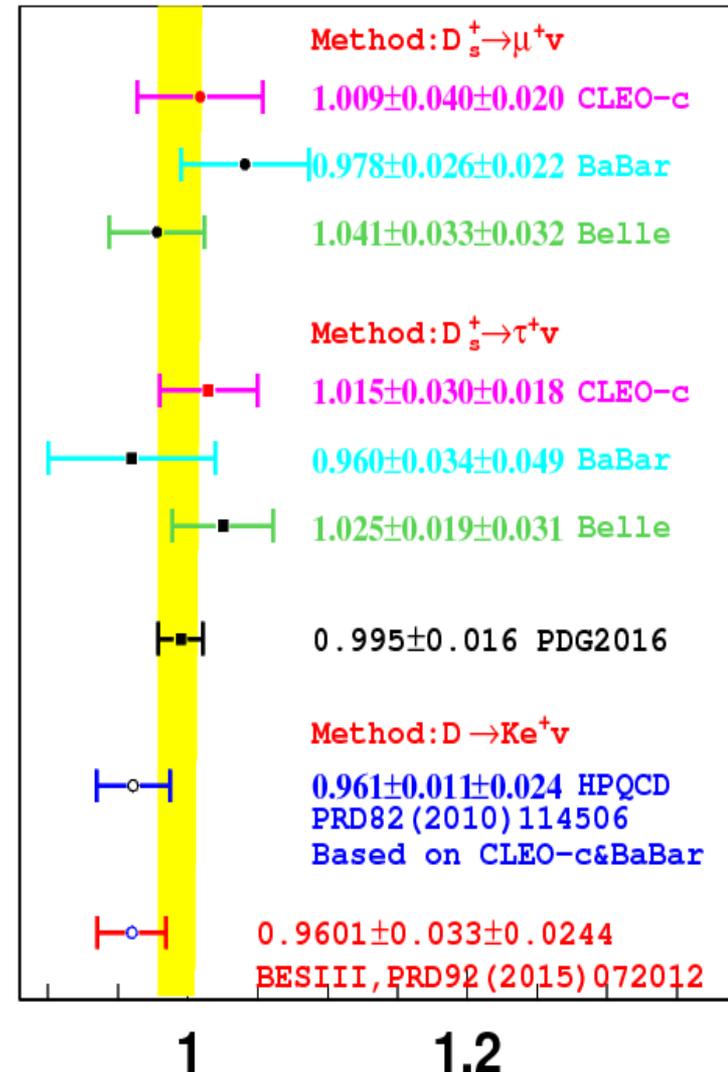


First and second errors are statistical and systematic

# Comparison of $|V_{cd(s)}|$



$|V_{cd}|$



$|V_{cs}|$

# Analysis of $D^+ \rightarrow K_L e^+ \nu$ at BESIII

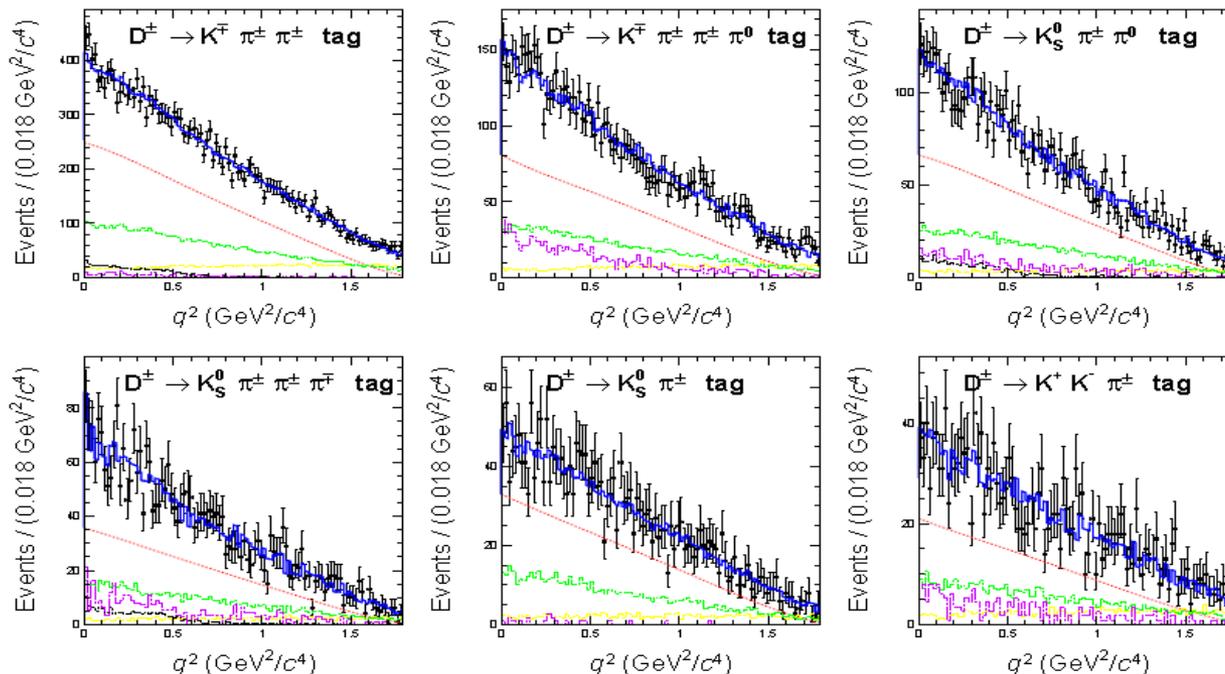
- Regardless of long flight distance,  $K_L$  interact with EMC and deposit part of energy, thus giving position information
- After reconstructing all other particles,  $K_L$  can be inferred with position information and constraint  $U_{\text{miss}} \rightarrow 0$

$$\overline{B}(D^+ \rightarrow K_L e^+ \nu) = (4.482 \pm 0.027 \pm 0.103)\%$$

$$A_{CP} \equiv \frac{\mathcal{B}(D^+ \rightarrow K_L^0 e^+ \nu_e) - \mathcal{B}(D^- \rightarrow K_L^0 e^- \bar{\nu}_e)}{\mathcal{B}(D^+ \rightarrow K_L^0 e^+ \nu_e) + \mathcal{B}(D^- \rightarrow K_L^0 e^- \bar{\nu}_e)}$$

$$A_{CP}^{D^+ \rightarrow K_L e^+ \nu} = (-0.59 \pm 0.60 \pm 1.50)\%$$

Simultaneous fit to event density  $I(q^2)$  with 2-par. series Form Factor



$D^+ \rightarrow K_L e^+ \nu$  is measured for the first time

PRD92(2015)112008

With 6 dominant  $D^-$  single tag

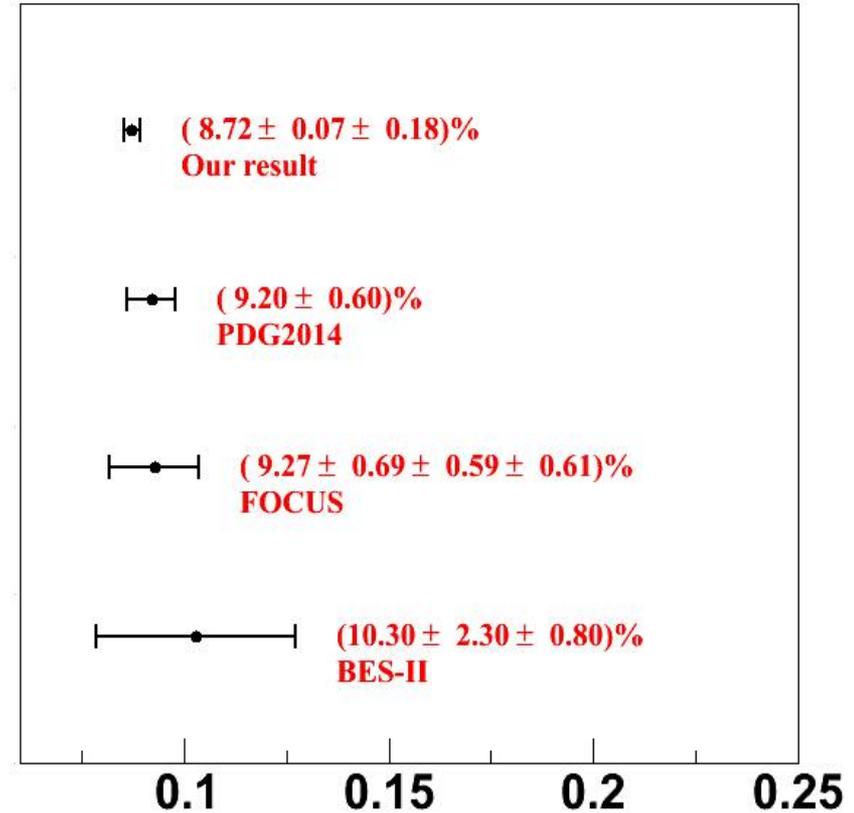
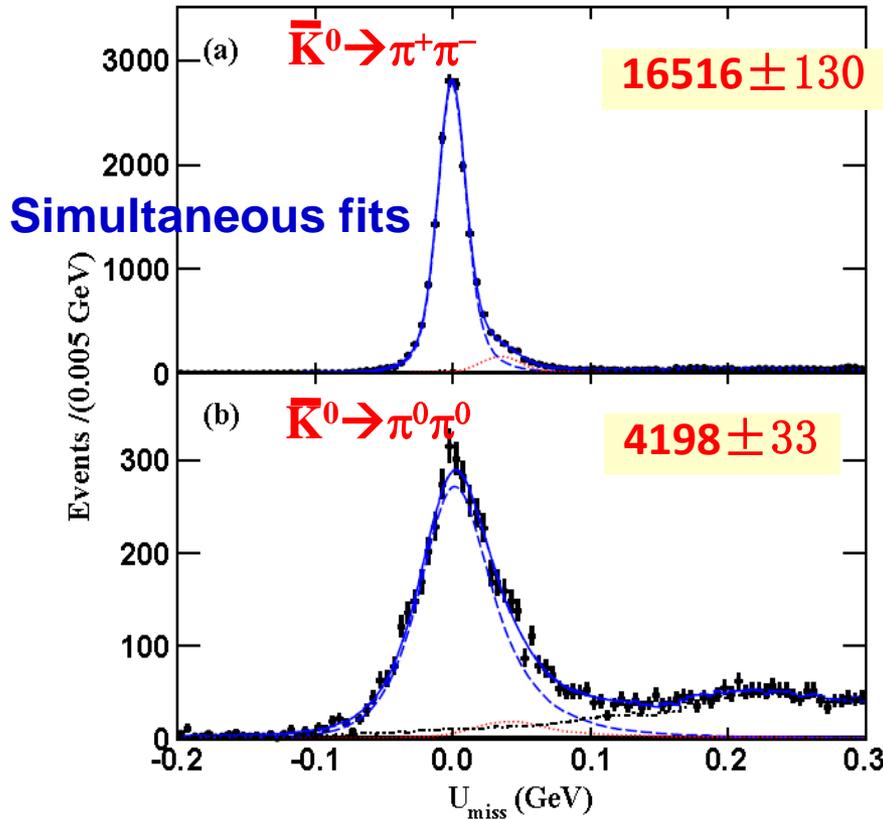
$$f_{+}^{K}(0)|V_{cs}| = 0.728 \pm 0.006 \pm 0.011$$

$$r_1 = a_1/a_0 = -1.91 \pm 0.33 \pm 0.24$$

# Improved BF for $D^+ \rightarrow \bar{K}^0 \mu^+ \nu$ at BESIII

With 6 dominant  $D^-$  single tag

EPJC76(2016)369



Taking  $B[D^0 \rightarrow \bar{K}^- \mu^+ \nu]$   
and  $B[D^+ \rightarrow \bar{K}^0 e^+ \nu]$   
from the PDG as input

$$\frac{\Gamma[D^0 \rightarrow \bar{K}^- \mu^+ \nu]}{\Gamma[D^+ \rightarrow \bar{K}^0 \mu^+ \nu]} = 0.963 \pm 0.044$$

$$\frac{\Gamma[D^+ \rightarrow \bar{K}^0 \mu^+ \nu]}{\Gamma[D^+ \rightarrow \bar{K}^0 e^+ \nu]} = 0.988 \pm 0.033$$

Support isospin conservation in  
these two decays within errors

Consistent with theory  
prediction 0.97 within error

# Lepton universality in $D^{0(+)} \rightarrow \pi^{-(0)} l^+ \nu$ at BESIII

## Lepton universality (LU)

$$R_{LU}^{0(+)} = \frac{B(D^{0(+)} \rightarrow \pi^{-(0)} \mu^+ \nu)}{B(D^{0(+)} \rightarrow \pi^{-(0)} e^+ \nu)} \sim 0.97$$

Expectations based on ZPC46 (1990)93, PRD69 (2004)074025, PLB633(2006)61 and PDG16

BFs on PDG16:

$$R_{LU}^0 = 0.82 \pm 0.08 \quad (\sim 2.0\sigma)$$

$$B(D^0 \rightarrow \pi^- \mu^+ \nu) = (0.237 \pm 0.024)\%$$

Large error in  $B[D^0 \rightarrow \pi^- \mu^+ \nu]$  and no measure of  $B[D^+ \rightarrow \pi^0 \mu^+ \nu]$ . Precision measurements are desired

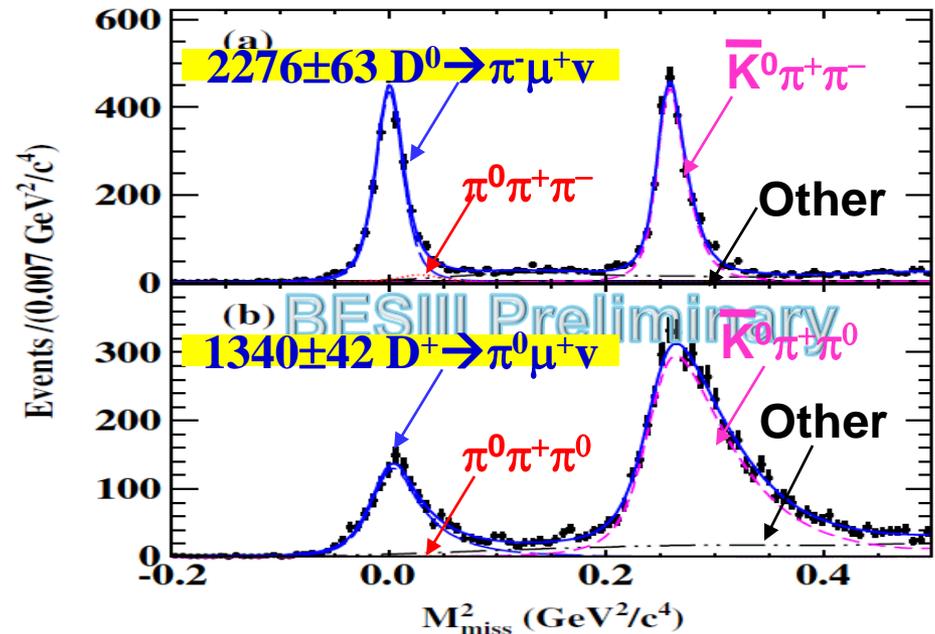
## Isospin symmetry (IS)

$$R_{IS}^e = \frac{\Gamma(D^0 \rightarrow \pi^- \ell^+ \nu)}{2\Gamma(D^+ \rightarrow \pi^0 \ell^+ \nu)} \sim 1$$

PDG16:  $R_{IS}^e = 0.911 \pm 0.043 \quad (2.1\sigma)$

BESIII:  $R_{IS}^e = 1.03 \pm 0.03 \pm 0.02$

With 3(6) dominant  $D^{0(-)}$  single tag



BFs:  $B(D^0 \rightarrow \pi^- \mu^+ \nu) = (0.267 \pm 0.007 \pm 0.007)\%$

agrees with PDG and with better precision

$B(D^+ \rightarrow \pi^0 \mu^+ \nu) = (0.342 \pm 0.011 \pm 0.010)\%$

measured for the first time

LU:  $R_{LU}^0 = 0.918 \pm 0.036 \quad R_{LU}^+ = 0.921 \pm 0.045$

agree with expectation in 1.5(1.1)  $\sigma$

IS:  $R_{IS}^{\mu} = 0.990 \pm 0.054$

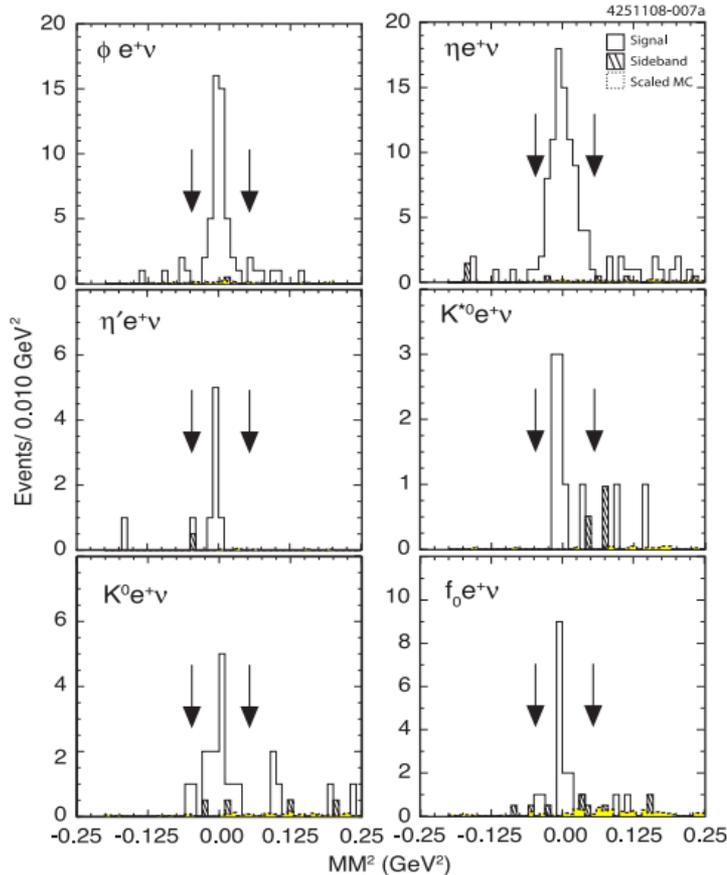
agree with IS prediction within uncertainty

# Semileptonic $D_s^+$ decays at CLEO-c

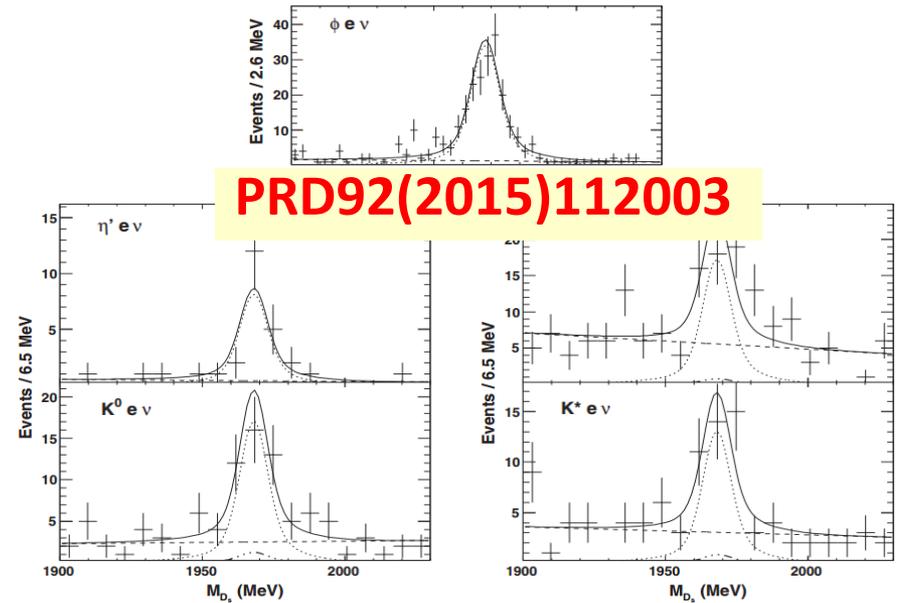
J. Hietala,<sup>1,\*</sup> D. Cronin-Hennessy,<sup>1,†</sup> T. Pedlar,<sup>2</sup> and I. Shipsey<sup>3</sup>

with 310 pb<sup>-1</sup> data @4.17 GeV

with 600 pb<sup>-1</sup> data @4.17 GeV



PRD80(2009)052007



Signal mode	BABAR (%)	CLEO-c (%)	This analysis (%)
$D_s \rightarrow \phi e \nu$	$2.61 \pm 0.03 \pm 0.08 \pm 0.15$	$2.36 \pm 0.23 \pm 0.13$	$2.14 \pm 0.17 \pm 0.08$
$D_s \rightarrow \eta e \nu$	...	$2.48 \pm 0.29 \pm 0.13$	$2.28 \pm 0.14 \pm 0.19$
$D_s \rightarrow \eta' e \nu$	...	$0.91 \pm 0.33 \pm 0.05$	$0.68 \pm 0.15 \pm 0.06$
$D_s \rightarrow f_0 e \nu, f_0 \rightarrow \pi \pi$	Seen	$0.20 \pm 0.03 \pm 0.01$	$0.13 \pm 0.03 \pm 0.01$
$D_s \rightarrow K_S e \nu$	...	$0.19 \pm 0.05 \pm 0.01$	$0.20 \pm 0.04 \pm 0.01$
$D_s \rightarrow K^* e \nu$	...	$0.18 \pm 0.07 \pm 0.01$	$0.18 \pm 0.04 \pm 0.01$

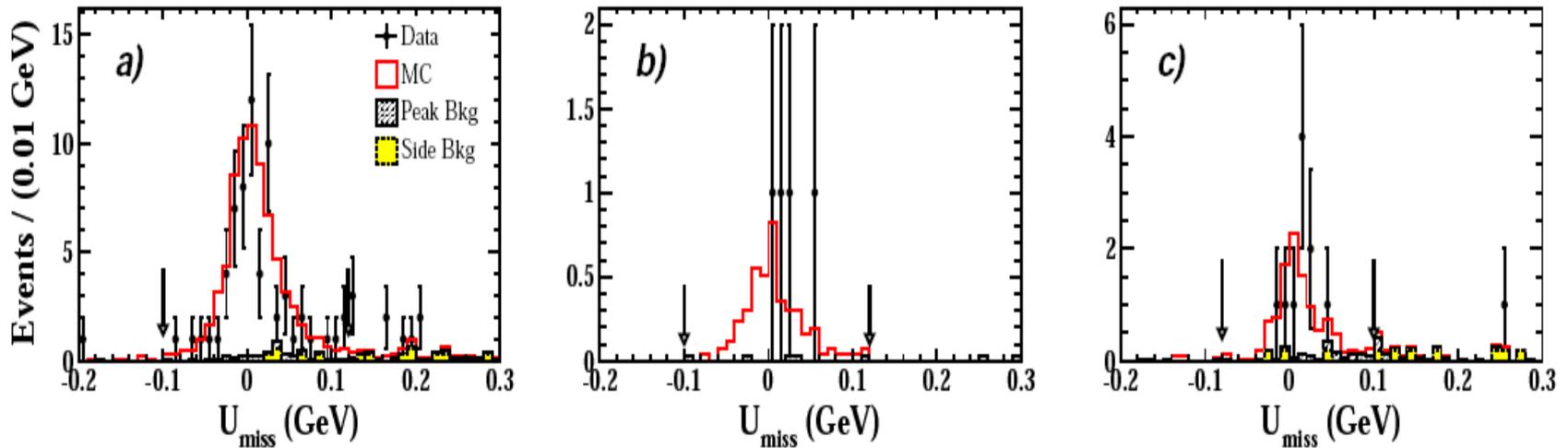
$$\theta(\eta-\eta' \text{ mixing angle}) = (42 \pm 2 \pm 2)^0$$

$$\theta(f_0\text{-ss mixing angle}) = (20^{+32}_{-20})^0$$

# Measurements of $D_s^+ \rightarrow \eta^{(\prime)} e^+ \nu$ at 4.009 GeV

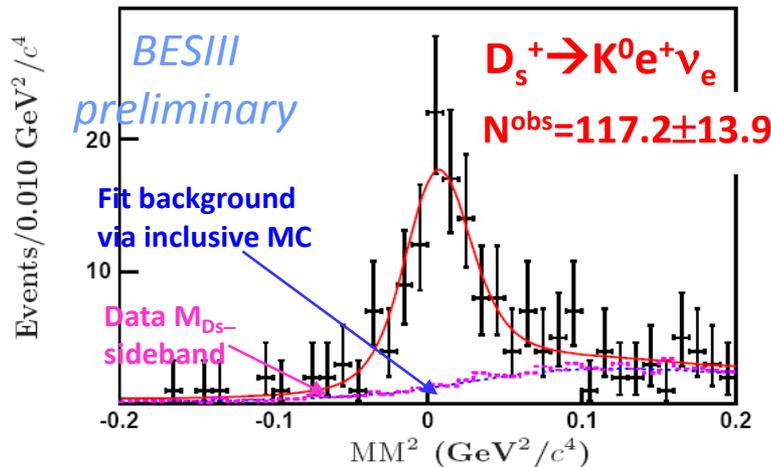
- Benefit the understanding of the source of difference of inclusive decay rates of  $D^{0(+)}$  and  $D_s^+$
- Complementary information to understand  $\eta$ - $\eta'$  mixing

PRD94(2016)112003



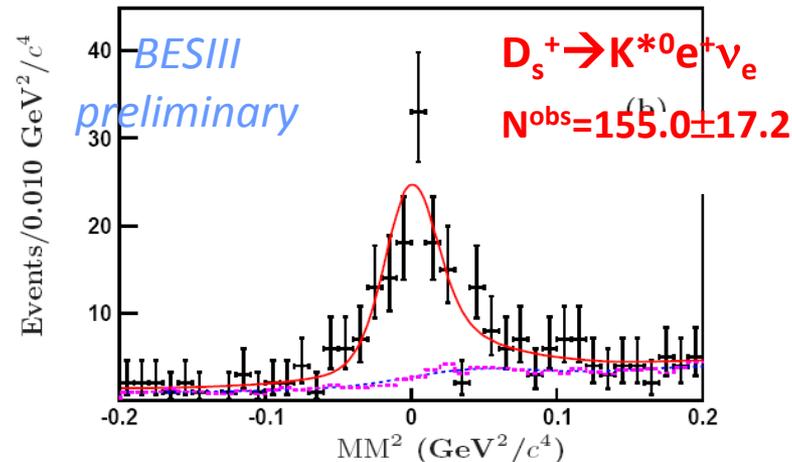
	BESIII	CLEOII 95	CLEOc09	CLEOc15	PDG [4]
$B(D_s^+ \rightarrow \eta e^+ \nu_e)$ [%]	$2.30 \pm 0.31 \pm 0.08$	—	$2.48 \pm 0.29 \pm 0.13$	$2.28 \pm 0.14 \pm 0.20$	$2.67 \pm 0.29$
$B(D_s^+ \rightarrow \eta' e^+ \nu_e)$ [%]	$0.93 \pm 0.30 \pm 0.05$	—	$0.91 \pm 0.33 \pm 0.05$	$0.68 \pm 0.15 \pm 0.06$	$0.99 \pm 0.23$
$\frac{B(D_s^+ \rightarrow \eta' e^+ \nu_e)}{B(D_s^+ \rightarrow \eta e^+ \nu_e)}$	$0.40 \pm 0.14 \pm 0.02$	$0.35 \pm 0.09 \pm 0.07$	—	—	—

# Studies of $D_s^+ \rightarrow K^{(*)0} e^+ \nu$ at 4.178 GeV



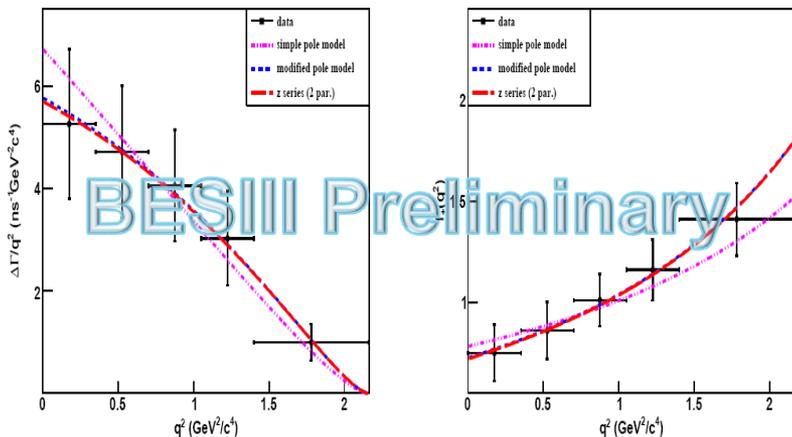
$$B[D_s^+ \rightarrow K^0 e^+ \nu_e] = (3.25 \pm 0.38_{\text{stat}} \pm 0.14_{\text{syst}}) \times 10^{-3}$$

$$(3.9 \pm 0.9) \times 10^{-3} \text{ [PDG17]}$$



$$B[D_s^+ \rightarrow K^{*0} e^+ \nu_e] = (2.38 \pm 0.26_{\text{stat}} \pm 0.12_{\text{syst}}) \times 10^{-3}$$

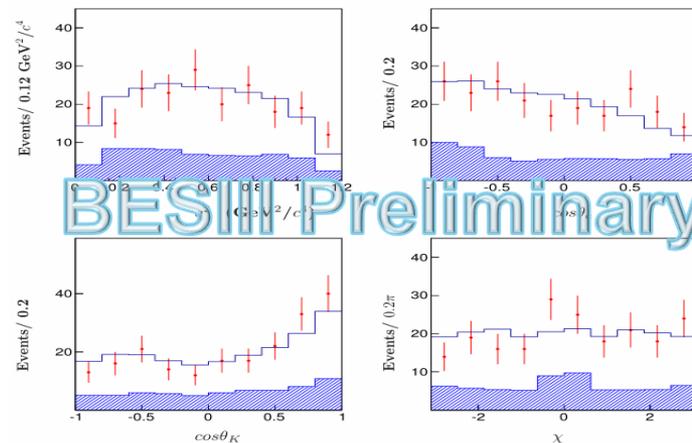
$$(1.8 \pm 0.4) \times 10^{-3} \text{ [PDG17]}$$



Model	Parameter	Value	$f_+(0)$
Simple pole	$f_+(0) V_{cd} $	$0.175 \pm 0.010 \pm 0.001$	$0.778 \pm 0.044 \pm 0.004$
Modified pole model	$f_+(0) V_{cd} $	$0.163 \pm 0.017 \pm 0.003$	$0.725 \pm 0.076 \pm 0.013$
	$\alpha$	$0.45 \pm 0.44 \pm 0.02$	
Series two parameters	$f_+(0) V_{cd} $	$0.162 \pm 0.019 \pm 0.003$	$0.720 \pm 0.084 \pm 0.013$
	$r_1$	$-2.94 \pm 2.32 \pm 0.14$	

Taking  $|V_{CKM}^{\text{fitter}}{}_{cd}|$  as input

Four dimensional un-binned likelihood fit is performed.  $K^*$  parameters are fixed



$$r_V = 1.67 \pm 0.34 \pm 0.16$$

$$r_2 = 0.77 \pm 0.28 \pm 0.07$$

# Hadronic structure and quantum correlation

# D hadronic decays

- Better inputs for beauty physics
- Window into strong final-state interactions
- Quantum correlated  $D^0\bar{D}^0$  decays:
  - CP asymmetry in mixing and decays
  - Interference  $\rightarrow$  strong phase parameters  $c_i$  and  $s_i \rightarrow$  Impact on  $\gamma/\phi_3$ , which is important for CKM UT

## Direct measurement

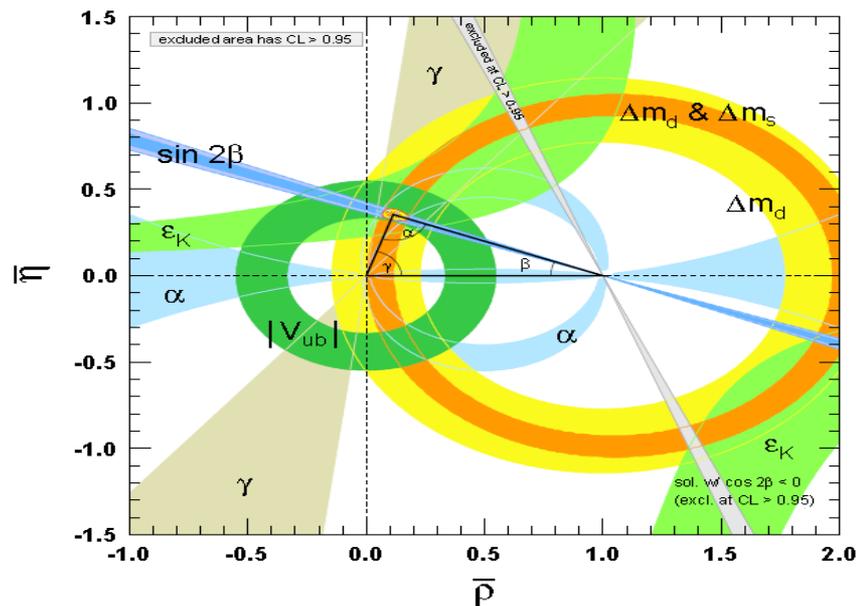
$$\alpha/\phi_2 = \left( 85.4^{+4.0}_{-3.9} \right)^\circ$$

$$\beta/\phi_1 = \left( 21.38^{+0.79}_{-0.77} \right)^\circ$$

$$\gamma/\phi_3 = \left( 68^{+8.0}_{-8.5} \right)^\circ$$

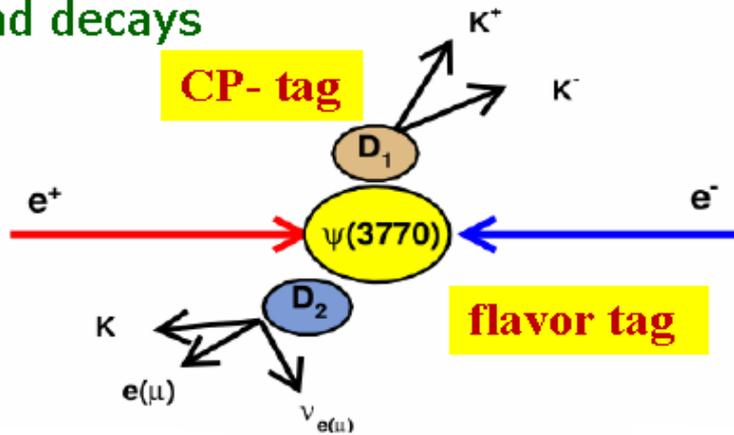
$\gamma$  is the worst measured angle, mostly due to systematic error

Significant deviation from UT will imply NP beyond SM



# $D^0\bar{D}^0$ mixing parameter $y_{CP}$ at BESIII

We measure the  $y_{CP}$  using CP-tagged semi-leptonic D decays allow to access CP asymmetry in mixing and decays



For D decay to CP eigenstates:

$$R_{CP\pm} \propto |A_{CP\pm}|^2 (1 \mp y_{CP})$$

$$y_{CP} = \frac{1}{2} [y \cos\phi (|\frac{q}{p}| + |\frac{p}{q}|) - x \sin\phi (|\frac{q}{p}| - |\frac{p}{q}|)]$$

For CP tagged semileptonic D decays:

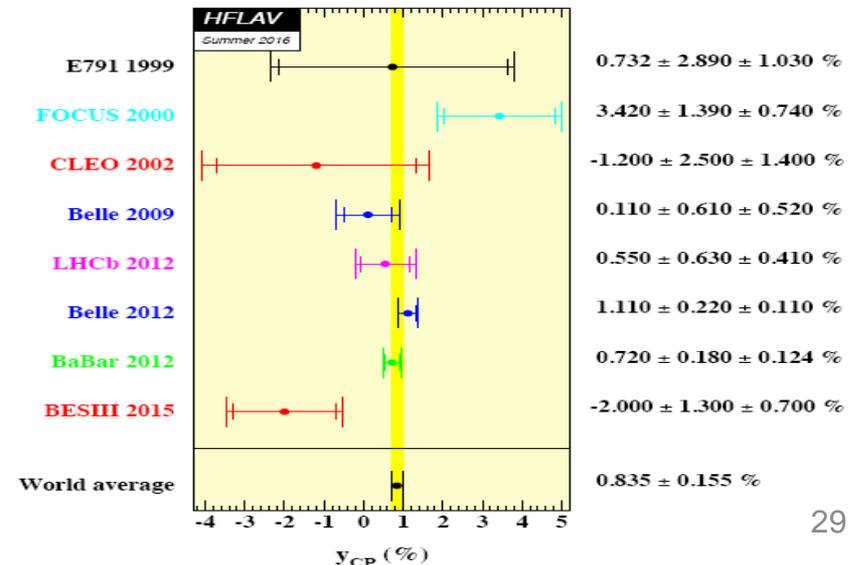
$$R_{l,CP\pm} \propto |A_l|^2 |A_{CP\pm}|^2$$

$$y_{CP} \approx \frac{1}{4} \left( \frac{R_{l;CP+} R_{CP-}}{R_{l;CP-} R_{CP+}} - \frac{R_{l;CP-} R_{CP+}}{R_{l;CP+} R_{CP-}} \right)$$

Type	Modes
$CP^+$	$K^+K^-, \pi^+\pi^-, K_S\pi^0\pi^0$
$CP^-$	$K_S^0\pi^0, K_S^0\omega, K_S^0\eta$
$l^\pm$	$Ke\nu, K\mu\nu$

$$y_{CP} = (-2.1 \pm 1.3 \pm 0.7)\%$$

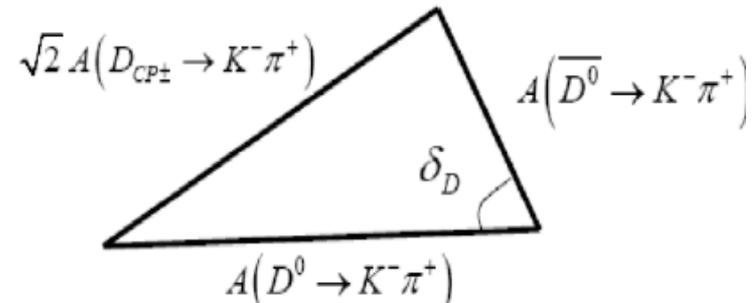
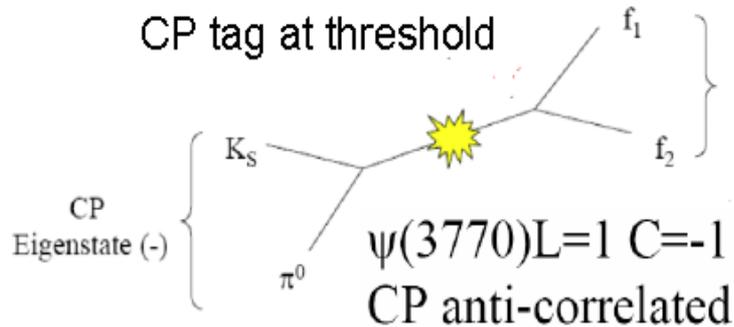
PLB 744(2015)339



# Strong phase difference $\delta_{K\pi}$ at BESIII

Quantum correlation  $\rightarrow$  Interference  $\rightarrow$  access strong phase!

If CP violation in charm is neglected: mass eigenstates = CP eigenstates



$\delta_{K\pi}$  is important to relate to mixing parameters  $x$  and  $y$  from  $x'$  and  $y'$

$$\mathcal{A}_{CP \rightarrow K\pi} = \frac{\mathcal{B}_{D_2 \rightarrow K^- \pi^+} - \mathcal{B}_{D_1 \rightarrow K^- \pi^+}}{\mathcal{B}_{D_2 \rightarrow K^- \pi^+} + \mathcal{B}_{D_1 \rightarrow K^- \pi^+}}$$

$$2r \cos \delta_{K\pi} + y = (1 + R_{WS}) \cdot \mathcal{A}_{CP \rightarrow K\pi},$$

$$|D_1\rangle \equiv \frac{|D^0\rangle + |\overline{D}^0\rangle}{\sqrt{2}} \quad |D_2\rangle \equiv \frac{|D^0\rangle - |\overline{D}^0\rangle}{\sqrt{2}}$$

$$A_{CP}^{K\pi} = (12.7 \pm 1.3 \pm 0.7)\%$$

Type	Mode
Flavored	$K^- \pi^+, K^+ \pi^-$
CP+	$K^+ K^-, \pi^+ \pi^-, K_S^0 \pi^0 \pi^0, \pi^0 \pi^0, \rho^0 \pi^0$
CP-	$K_S^0 \pi^0, K_S^0 \eta, K_S^0 \omega$

With external inputs of the parameters in HFAG2013 and PDG

$$R_D = 3.47 \pm 0.06\%, \quad y = 6.6 \pm 0.9\% \quad R_{WS} = 3.80 \pm 0.05\%$$

$$\cos \delta_{K\pi} = 1.02 \pm 0.11 \pm 0.06 \pm 0.01$$

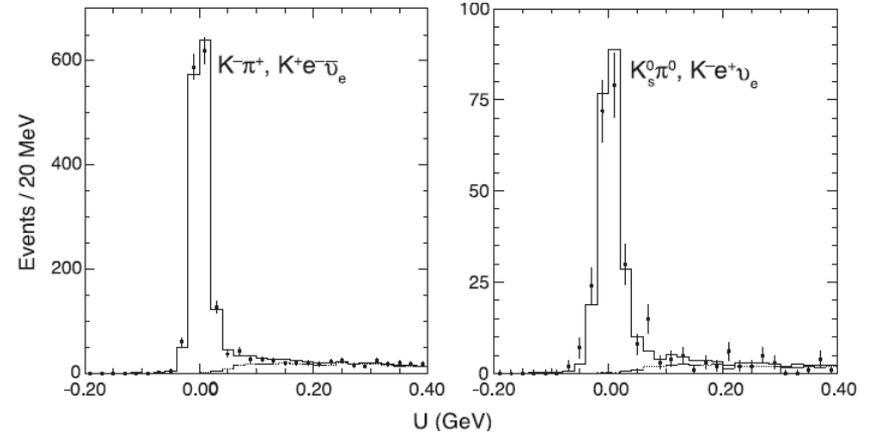
most precise to date

PLB734(2014)227

# Strong phase difference $\delta_{K\pi}$ at CLEO-c

PRD86(2012)112001

Type	Reconstruction	Final states
$f$	Full	$K^- \pi^+, Y_0 - Y_7$
$\bar{f}$	Full	$K^+ \pi^-, \bar{Y}_0 - \bar{Y}_7$
$S_+$	Full	$K^+ K^-, \pi^+ \pi^-, K_S^0 \pi^0 \pi^0$
$S_+$	Partial	$K_L^0 \pi^0, K_L^0 \eta, K_L^0 \omega$
$S_-$	Full	$K_S^0 \pi^0, K_S^0 \eta, K_S^0 \omega$
$S_-$	Partial	$K_L^0 \pi^0 \pi^0$
$\ell^+$	Partial	$K^- e^+ \nu_e, K^- \mu^+ \nu_\mu$
$\ell^-$	Partial	$K^+ e^- \bar{\nu}_e, K^+ \mu^- \bar{\nu}_\mu$



Mode	Correlated	Uncorrelated
$K^- \pi^+$	$1 + R_{WS}$	$1 + R_{WS}$
$S_+$	2	2
$S_-$	2	2
$Y_k$	$1 + Q_k$	$1 + Q_k$
$K^- \pi^+, K^- \pi^+$	$R_M[(1 + R_{WS})^2 - 4r \cos \delta (r \cos \delta + y)]$	$R_{WS}$
$K^- \pi^+, K^+ \pi^-$	$(1 + R_{WS})^2 - 4r \cos \delta (r \cos \delta + y)$	$1 + R_{WS}^2$
$K^- \pi^+, S_+$	$1 + R_{WS} + 2r \cos \delta + y$	$1 + R_{WS}$
$K^- \pi^+, S_-$	$1 + R_{WS} - 2r \cos \delta - y$	$1 + R_{WS}$
$K^- \pi^+, \ell^-$	$1 - ry \cos \delta - rx \sin \delta$	1
$K^- \pi^+, \ell^+$	$r^2(1 - ry \cos \delta - rx \sin \delta)$	$R_{WS}$
$K^- \pi^+, \bar{Y}_i$	$(1 + R_{WS})(1 + Q_i) - r^2 - \rho_i^2 - 2(r \cos \delta + y)(\rho_i c_i + y) + 2r \sin \delta \rho_i s_i$	$1 + R_{WS} Q_i$
$K^- \pi^+, Y_i$	$(1 + R_{WS})(1 + Q_i) - 1 - r^2 \rho_i^2 - 2(r \cos \delta + y)(\rho_i c_i + y) - 2r \sin \delta \rho_i s_i$	$R_{WS} + Q_i$
$S_+, S_+$	0	1
$S_-, S_-$	0	1
$S_+, S_-$	4	2
$S_+, \ell^-$	$1 + y$	1
$S_-, \ell^-$	$1 - y$	1
$S_+, Y_i$	$1 + Q_i + 2\rho_i c_i + y$	$1 + Q_i$
$S_-, Y_i$	$1 + Q_i - 2\rho_i c_i - y$	$1 + Q_i$
$Y_i, \ell^-$	$1 - \rho_i y c_i - \rho_i x s_i$	1
$Y_i, \ell^+$	$\rho_i^2(1 - \rho_i y c_i - \rho_i x s_i)$	$Q_i$
$Y_i, \bar{Y}_j$	$(1 + Q_i)(1 + Q_j) - \rho_i^2 - \rho_j^2 - 2(\rho_i c_i + y)(\rho_j c_j + y) + 2\rho_i s_i \rho_j s_j$	$1 + Q_i Q_j$
$Y_i, Y_j$	$(1 + Q_i)(1 + Q_j) - 1 - \rho_i^2 \rho_j^2 - 2(\rho_i c_i + y)(\rho_j c_j + y) - 2\rho_i s_i \rho_j s_j$	$Q_i + Q_j$

$$\cos \delta = 0.81^{+0.22+0.07}_{-0.18-0.05}$$

$$\sin \delta = -0.01 \pm 0.41 \pm 0.04$$

$$\delta = (10^{+28+13}_{-53-0})^0$$

If including external inputs on mixing parameters in the fit,

$$\cos \delta = 1.15^{+0.19+0.00}_{-0.17-0.08}$$

$$\sin \delta = 0.56^{+0.32+0.21}_{-0.31-0.20}$$

$$\delta = (18^{+11}_{-17})^0$$

# Strong phase difference $\delta_{K\pi\pi^0/K3\pi}$ at CLEO-c

PRD80(2009)031105

Mode	$\mathcal{B}$
$D^0 \rightarrow CP$	$A_{CP}^2(1 - \lambda_{\pm}y)$
$D^0 \rightarrow F$	$A_F^2$
$D^0 \rightarrow \bar{F}$	$A_{\bar{F}}^2[1 - (y/r_D^F)R_F \cos\delta_D^F + (x/r_D^F)R_F \sin\delta_D^F + (y^2 + x^2)/2(r_D^F)^2]$
$D^0 \rightarrow K^- \pi^+$	$A_{K^- \pi^+}^2$
$D^0 \rightarrow K^+ \pi^-$	$A_{K^+ \pi^-}^2[1 - (y/r_D^{K\pi}) \cos\delta_D^{K\pi} + (x/r_D^{K\pi}) \sin\delta_D^{K\pi} + (y^2 + x^2)/2(r_D^{K\pi})^2]$

Mode	$K^{\pm} \pi^{\mp} \pi^{\mp} \pi^{\pm}$	$K^{\pm} \pi^{\mp} \pi^0$	$K^{\pm} \pi^{\mp}$
$K^{\mp} \pi^{\pm} \pi^{\pm} \pi^{\mp}$	$4,044 \pm 64$		
$K^{\pm} \pi^{\mp} \pi^{\mp} \pi^{\pm}$	$29.1 \pm 5.9$		
$K^{\mp} \pi^{\pm} \pi^0$	$9,594 \pm 99$	$7,342 \pm 87$	
$K^{\pm} \pi^{\mp} \pi^0$	$63.6 \pm 8.8$	$12.5 \pm 4.1$	
$K^{\mp} \pi^{\pm}$	$5,206 \pm 72$	$7,155 \pm 85$	
$K^{\pm} \pi^{\mp}$	$35.6 \pm 6.2$	$7.3 \pm 3.3$	
$K^+ K^-$	$536 \pm 23$	$764 \pm 28$	
$\pi^+ \pi^-$	$246 \pm 16$	$336 \pm 18$	
$K_S^0 \pi^0 \pi^0$	$283 \pm 18$	$406 \pm 21$	$221 \pm 15$
$K_L^0 \pi^0$	$827 \pm 30$	$1,236 \pm 38$	$689 \pm 28$
$K_L^0 \omega$	$296 \pm 18$	$449 \pm 22$	$251 \pm 17$
$K_S^0 \pi^0$	$705 \pm 27$	$891 \pm 30$	$473 \pm 22$
$K_S^0 \omega$	$319 \pm 19$	$389 \pm 21$	$183 \pm 14$
$K_S^0 \phi$	$53.0 \pm 7.5$	$90.9 \pm 9.9$	$42.8 \pm 6.9$
$K_S^0 \eta(\gamma\gamma)$	$128 \pm 12$	$116 \pm 11$	$65.5 \pm 8.3$
$K_S^0 \eta(\pi^+ \pi^- \pi^0)$	$35.9 \pm 6.5$	$36.3 \pm 7.2$	$27.2 \pm 5.4$
$K_S^0 \eta'$	$35.7 \pm 6.0$	$60.6 \pm 7.8$	$30.0 \pm 5.5$

Significant coherence

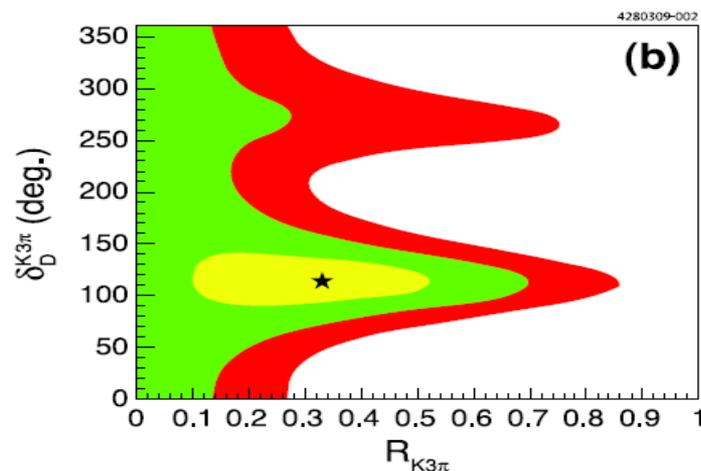
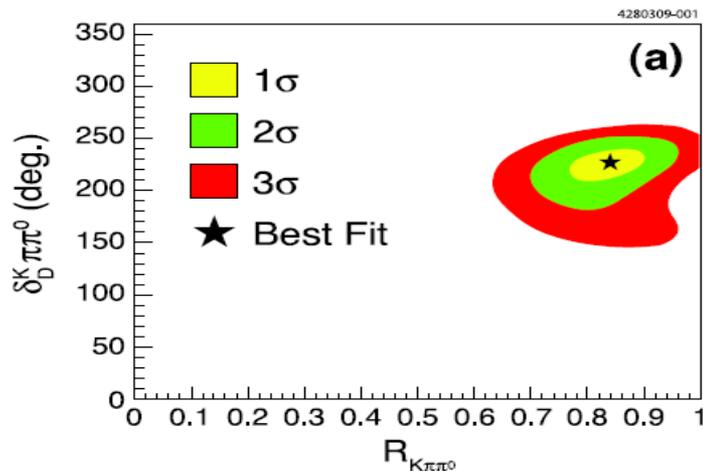
Lower coherence

$$\cos_{\mathcal{D}}^{K\pi\pi^0} = (227_{-17}^{+14})^0$$

$$\cos_{\mathcal{D}}^{K3\pi} = (114_{-23}^{+26})^0$$

$$R_{K\pi\pi^0} = 0.84 \pm 0.07$$

$$R_{K3\pi} = 0.33_{-0.30}^{+0.20}$$



# Strong phase difference in $D^0/\bar{D}^0 \rightarrow K_S^0 \pi^+ \pi^-$

Help to reduce systematic/model uncertainty of CKM UT angle  $g/f_3$  (GGSZ method [PRD68\(2003\)054018](#))

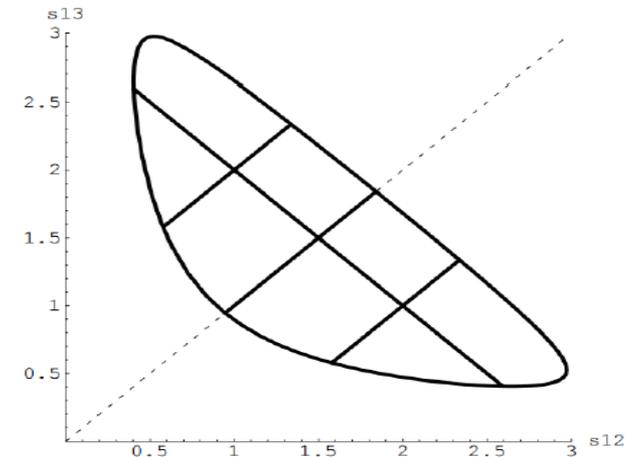
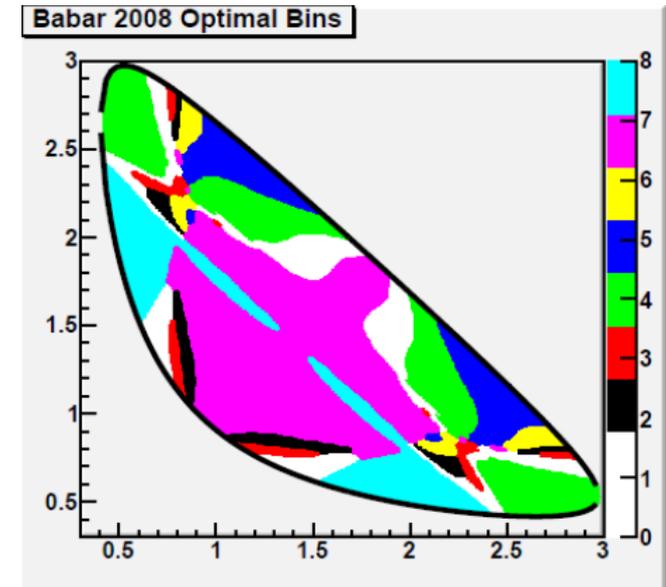
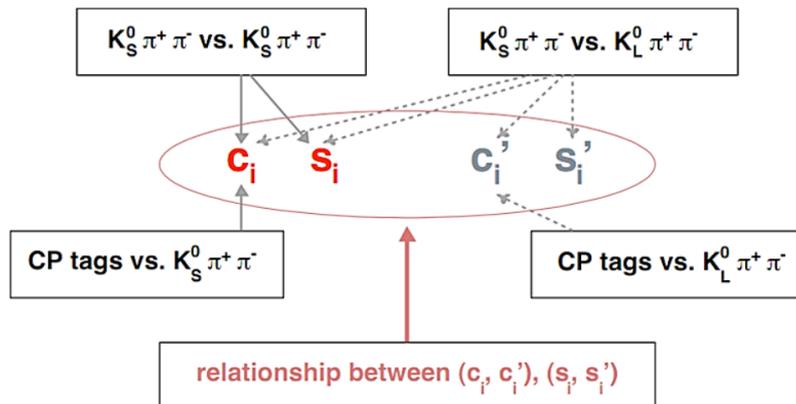
Binned decay rate:

$$\Gamma_i^\pm \equiv \int_i d\Gamma(B^\pm \rightarrow (K_S^0 \pi^- \pi^+)_D K^\pm) \\ = T_i + r_B^2 T_{\bar{i}} \pm 2r_B \sqrt{T_i T_{\bar{i}}} [\cos(\delta_B + \gamma) c_i - \sin(\delta_B + \gamma) s_i]$$

- $T_i$ : Bin yield measured in flavor decays
- $r_B$ : color suppression factor  $\sim 0.1$
- $\delta_B$ : strong phase of B decay
- $c_i, s_i$ : weighted average of  $\cos(\Delta\delta_D)$  and  $\sin(\Delta\delta_D)$  respectively where  $\Delta\delta_D$  is the difference between phase of  $D^0$  and  $\bar{D}^0$

Measured at B-Factories

Through  $D^0 \rightarrow K_S \pi^+ \pi^-$  analysis



Mirrored binning over  $x=y$  makes it so  $c_i = c_i$  and  $s_i = -s_i$

# Strong phase difference in $D^0/D^0 \rightarrow K_S^0 h^+ h^-$

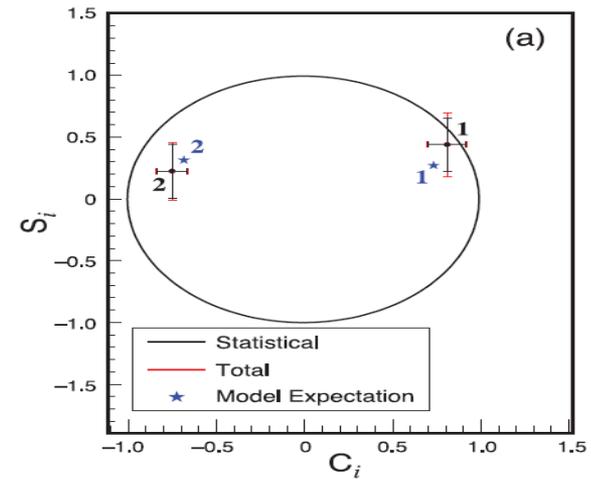
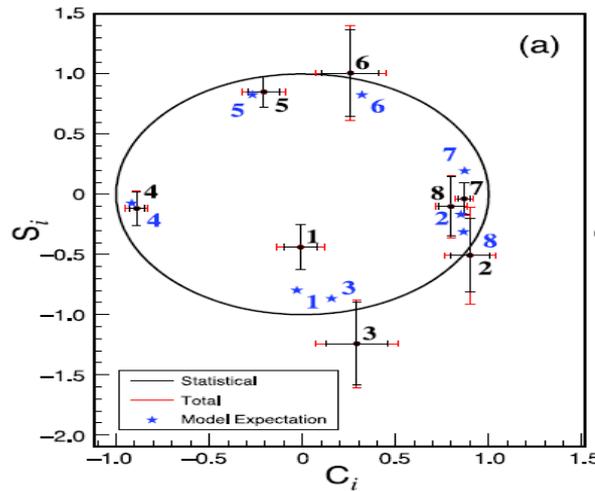
PRD82(2010)112006

$K_S^0 \pi^+ \pi^-$

$K_S^0 K^+ K^-$

Optimal			
$i$	$c_i$		$s_i$
1	$-0.009 \pm 0.088 \pm 0.094$	$-0.438 \pm 0.184 \pm 0.045$	
2	$0.900 \pm 0.106 \pm 0.082$	$-0.490 \pm 0.295 \pm 0.261$	
3	$0.292 \pm 0.168 \pm 0.139$	$-1.243 \pm 0.341 \pm 0.123$	
4	$-0.890 \pm 0.041 \pm 0.044$	$-0.119 \pm 0.141 \pm 0.038$	
5	$-0.208 \pm 0.085 \pm 0.080$	$0.853 \pm 0.123 \pm 0.035$	
6	$0.258 \pm 0.155 \pm 0.108$	$0.984 \pm 0.357 \pm 0.165$	
7	$0.869 \pm 0.034 \pm 0.033$	$-0.041 \pm 0.132 \pm 0.034$	
8	$0.798 \pm 0.070 \pm 0.047$	$-0.107 \pm 0.240 \pm 0.080$	

$i$	$c_i$		$s_i$
$\mathcal{N} = 2$ equal $\Delta\delta_D$ bins			
1	$0.818 \pm 0.107 \pm 0.037$	$0.445 \pm 0.215 \pm 0.143$	
2	$-0.746 \pm 0.083 \pm 0.035$	$0.229 \pm 0.220 \pm 0.079$	
$\mathcal{N} = 3$ equal $\Delta\delta_D$ bins			
1	$0.793 \pm 0.063 \pm 0.038$	$0.431 \pm 0.222 \pm 0.142$	
2	$-0.566 \pm 0.092 \pm 0.034$	$0.413 \pm 0.234 \pm 0.094$	
3	$-0.096 \pm 0.329 \pm 0.131$	$-0.461 \pm 0.432 \pm 0.175$	
$\mathcal{N} = 4$ equal $\Delta\delta_D$ bins			
1	$0.858 \pm 0.059 \pm 0.034$	$0.309 \pm 0.248 \pm 0.180$	
2	$0.176 \pm 0.223 \pm 0.091$	$0.992 \pm 0.473 \pm 0.403$	
3	$-0.819 \pm 0.095 \pm 0.045$	$0.307 \pm 0.267 \pm 0.201$	
4	$0.376 \pm 0.329 \pm 0.157$	$-0.133 \pm 0.659 \pm 0.323$	



Uncertainty in  $\gamma/\phi_3$  due to strong phase difference error is expected to be  $1.7^\circ$ - $3.9^\circ$

Uncertainty in  $\gamma/\phi_3$  due to strong phase difference error is expected to be  $3.2^\circ$ - $3.9^\circ$

# Improved measurement is expected at BESIII

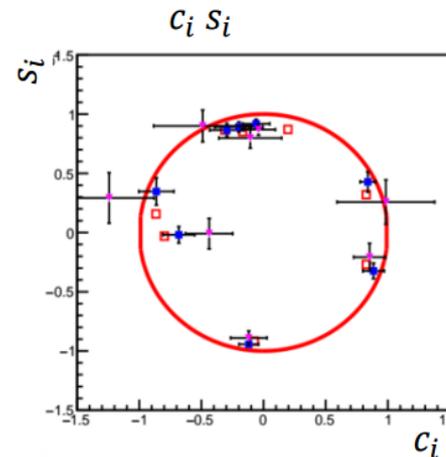
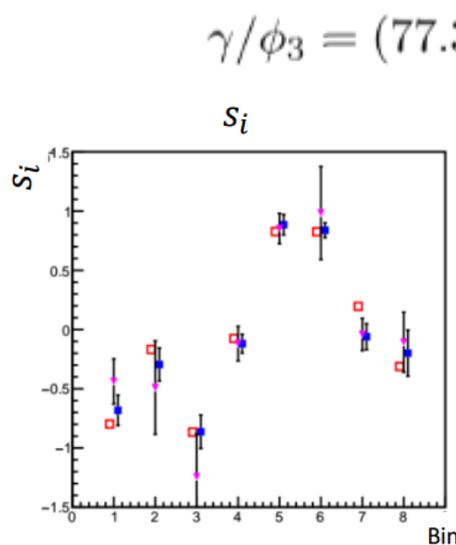
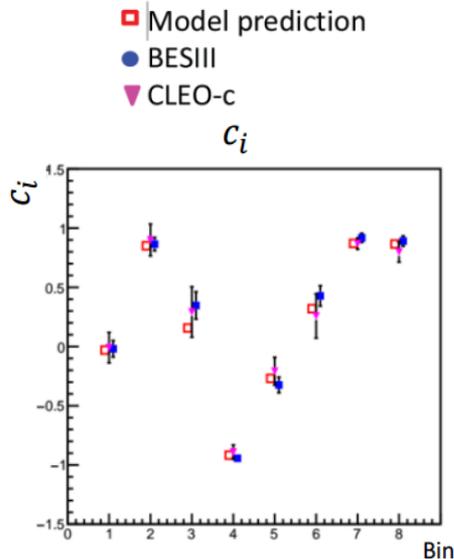
## BESIII preliminary

Bins	$c_i$		$s_i$	
	BES-III	CLEO-c	BES-III	CLEO-c
1	$0.066 \pm 0.066$	$-0.009 \pm 0.088$	$-0.843 \pm 0.119$	$-0.438 \pm 0.184$
2	$0.796 \pm 0.061$	$0.900 \pm 0.106$	$-0.357 \pm 0.148$	$-0.490 \pm 0.295$
3	$0.361 \pm 0.125$	$0.292 \pm 0.168$	$-0.962 \pm 0.258$	$-1.243 \pm 0.341$
4	$-0.985 \pm 0.017$	$-0.890 \pm 0.041$	$-0.090 \pm 0.093$	$-0.119 \pm 0.141$
5	$-0.278 \pm 0.056$	$-0.208 \pm 0.085$	$0.778 \pm 0.092$	$0.853 \pm 0.123$
6	$0.267 \pm 0.119$	$0.258 \pm 0.155$	$0.635 \pm 0.293$	$0.984 \pm 0.357$
7	$0.902 \pm 0.017$	$0.869 \pm 0.034$	$-0.018 \pm 0.103$	$-0.041 \pm 0.132$
8	$0.888 \pm 0.036$	$0.798 \pm 0.070$	$-0.301 \pm 0.140$	$-0.107 \pm 0.240$

BESIII only statistical error

CLEO-c PRD82,112006

- Consistent with CLEO-c, better stat. err
- Reduction of contribution to uncertainty of  $\gamma$  meas. of 40%(80% for  $20\text{fb}^{-1}$ )
- Improved stat. from B factories could place uncertainty from  $c_i, s_i$  contribution @1%



$$\gamma/\phi_3 = (77.3_{-14.9}^{+15.1}(\text{stat.}) \pm 4.1(\text{syst.}) \pm 4.3(c_i/s_i))^{\circ}$$

↓ **CLEO-c**

$$\pm 2.5(0.9)(c_i/s_i)$$

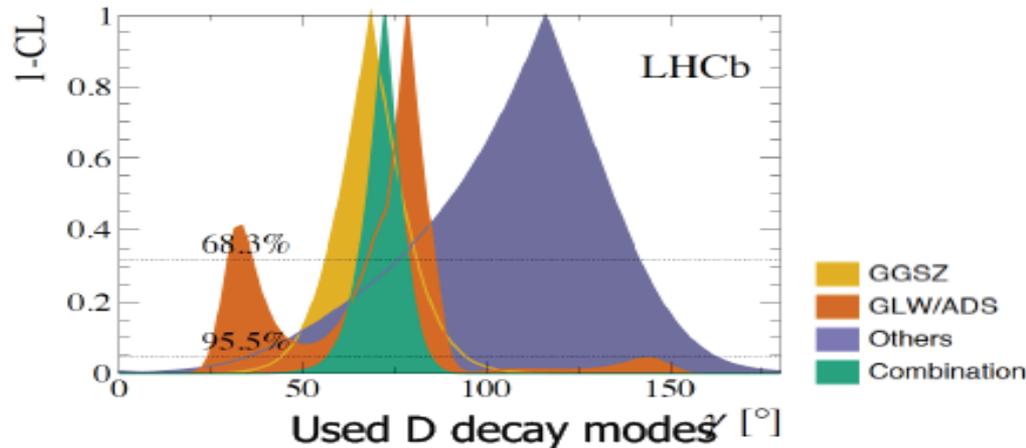
# Constraint on $\gamma/\phi_3$ measurement

taken from Liming Zhang's talk at FPCPV2016

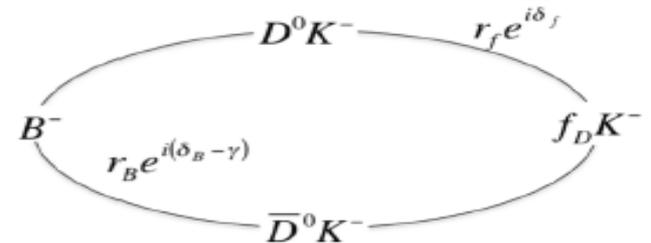


## $\gamma$ combination at LHCb

Determine  $\gamma$  from CPV measurements  
LHCb-PAPER-2016-032



<b>GLW:</b> $D \rightarrow K^+ K^-$ $\pi^+ \pi^-$ $K_S^0 \pi^0$	<b>ADS:</b> $D \rightarrow \pi^+ K^-$	<b>quasi-ADS</b> $D \rightarrow \pi^+ K^- \pi^+ \pi^-$ $\pi^+ K^- \pi^0$
<b>GGSZ</b> $D \rightarrow K_S^0 \pi^+ \pi^-$ $K_S^0 K^+ K^-$	<b>quasi-GLW</b> $D \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ $K^+ K^- \pi^0$ $\pi^+ \pi^- \pi^0$	<b>GLS</b> $D \rightarrow K_S^0 K^+ \pi^+$ $K_S^0 \pi^+ K^+$



$$\gamma = (72.2_{-7.3}^{+6.8})^\circ \text{ syst. included}$$

BaBar:  $\gamma = (70 \pm 18)^\circ$

Belle:  $\gamma = (73^{+13}_{-15})^\circ$

### Prospects

Sample	$\sigma_{\text{stat}}(\gamma)^\circ$
Run 1	8
Run 2	4
Upgrade	$\sim 1$
Future upgrade	$< 0.5$

- Current one syst.  $\sim 2^\circ$  from CLEO strong phase measurements
- 15-20  $\text{fb}^{-1}$   $\psi(3370)$  data from BESIII are desired to avoid syst. limitation for upgrade scenario

More  $\psi(3770)$  data at BESIII will better constrain on  $\gamma/\phi_3$  36

# Amplitude analysis of $D^0 \rightarrow K^+ K^- \pi^+ \pi^-$ at CLEO-c

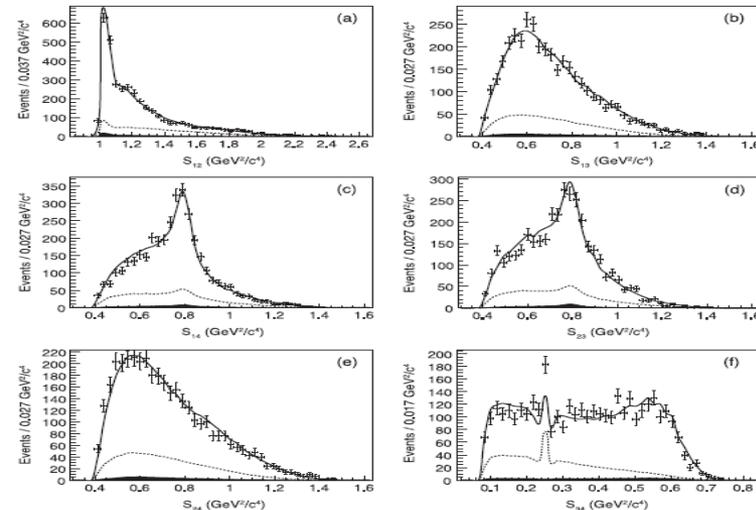
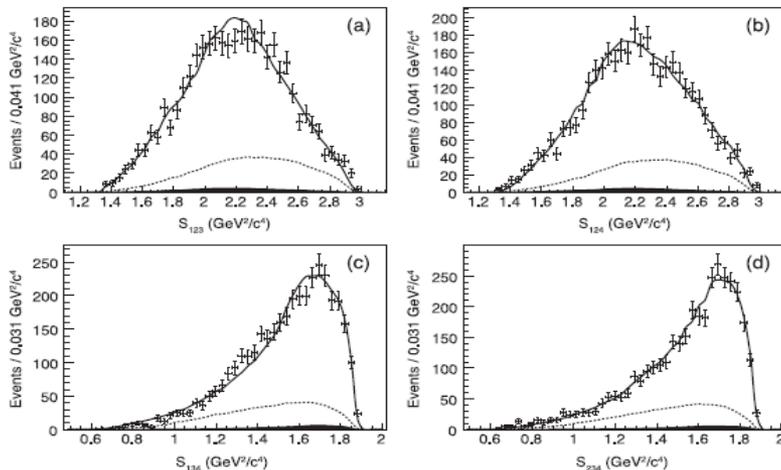
PRD85(2012)122002

818  $\text{pb}^{-1}$  data @ 3.774 GeV  
 600  $\text{pb}^{-1}$  data @ 4.17 GeV  
 9  $\text{fb}^{-1}$  data @ ~10 GeV with CLEO II.V  
 15.3  $\text{fb}^{-1}$  data @ 7-11.2 GeV with CLEO III

Previous analyses from E691 and FOCUS  
 Search for CPV in the SCS decay  
 Strong phase to benefit  $\gamma/\phi_3$

Sample	Yield	Purity (%)	Mistag rate (%)
Flavor tags			
CLEO II.V	279	$74 \pm 3$	$0.64 \pm 0.32$
CLEO III	1225	$89.2 \pm 0.4$	$0.64 \pm 0.05$
CLEO-c 3770	1396	$84.1 \pm 0.6$	$4.5 \pm 0.5$
CLEO-c 4170	739	$65.7 \pm 1.1$	$7.5 \pm 0.7$
CP tags			
CP eigenstates	81	$71.9 \pm 3.1$	...
Admixture	150	$82.1 \pm 1.5$	...

2959 flavor-tagged and 181 CP-tagged true signals



Component	Fit fraction (%)		Difference ( $\sigma$ )
	CLEO II.V/III	CLEO-c	
$K_1(1270)^+(K^{*0}\pi^-)K^-$	$4.4 \pm 0.9 \pm 0.8$	$9.8 \pm 1.2 \pm 4.6$	1.1
$K_1(1270)^-(\bar{K}^{*0}\pi^-)K^+$	$3.6 \pm 0.9 \pm 1.1$	$0.2 \pm 0.2 \pm 2.3$	1.2
$K_1(1270)^+(\rho^0 K^+)K^-$	$2.6 \pm 0.8 \pm 0.5$	$8.5 \pm 1.4 \pm 4.2$	1.3
$K_1(1270)^-(\rho^0 K^-)K^+$	$7.9 \pm 1.2 \pm 0.8$	$3.5 \pm 1.1 \pm 3.4$	1.2
$K^*(1410)^+(K^{*0}\pi^+)K^-$	$4.5 \pm 0.9 \pm 0.6$	$4.5 \pm 1.1 \pm 1.2$	0.0
$K^*(1410)^-(\bar{K}^{*0}\pi^-)K^+$	$5.5 \pm 1.0 \pm 0.8$	$4.6 \pm 0.9 \pm 1.1$	0.4
$K^{*0}\bar{K}^{*0} S$ wave	$7.5 \pm 1.8 \pm 1.8$	$5.2 \pm 1.0 \pm 2.0$	0.7
$\phi\rho^0 S$ wave	$39.8 \pm 2.7 \pm 1.4$	$36.9 \pm 3.2 \pm 4.0$	0.5
$\phi\rho^0 D$ wave	$4.7 \pm 1.0 \pm 1.1$	$1.7 \pm 0.8 \pm 2.3$	1.0
$\phi\{\pi^+\pi^-\}_S$	$8.3 \pm 1.2 \pm 0.5$	$10.9 \pm 1.7 \pm 3.0$	0.7
$\{K^-\pi^+\}_\rho\{K^+\pi^-\}_S$	$14.7 \pm 1.8 \pm 4.3$	$7.8 \pm 1.4 \pm 5.3$	1.1

The sensitive to the CP-violating parameter  $\gamma/\phi_3$  from 2000  $B^+ \rightarrow D^0(K^+K^-\pi^+\pi^-)K^+$  decay is estimated to be  $(11.3 \pm 0.3)^0$

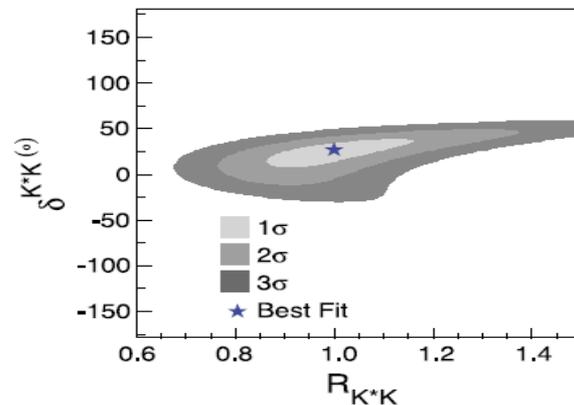
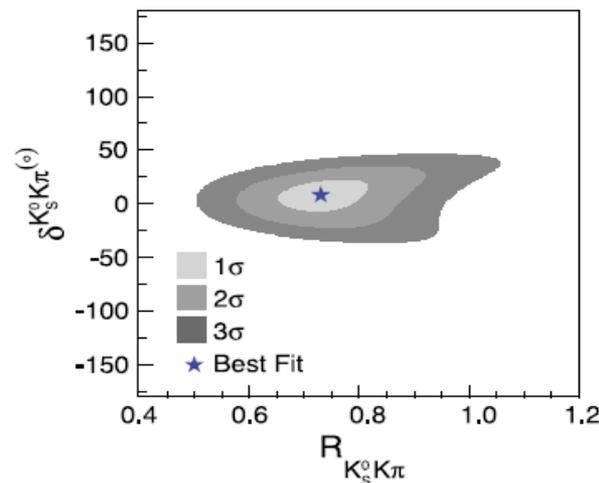
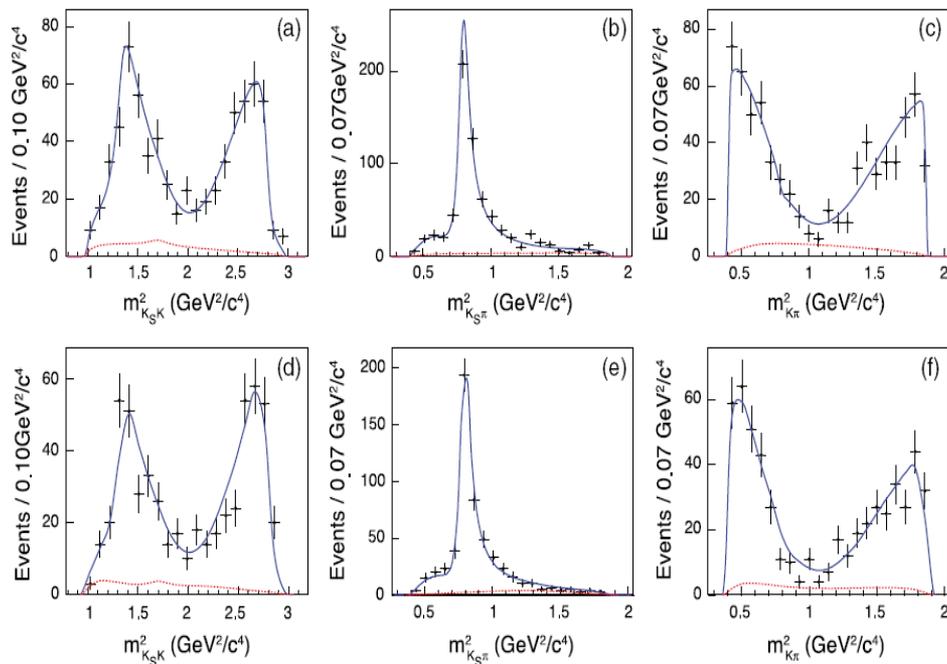
# Amplitude analysis of $D^0 \rightarrow K_S K^- \pi^+ / K^+ \pi^-$ at CLEO-c

PRD89(2012)092016

818  $\text{pb}^{-1}$  data@3.774 GeV

15.3  $\text{fb}^{-1}$  data@7-11.2 GeV with CLEOIII

Tag	$D^0 \rightarrow K_S^0 K^- \pi^+$		$D^0 \rightarrow K_S^0 K^+ \pi^-$	
	Yield	Purity %	Yield	Purity %
$K \pi$	$122.0 \pm 11.5$	$93.2 \pm 0.5$	$80.6 \pm 9.0$	$99.5 \pm 0.1$
$K \pi \pi \pi$	$161.8 \pm 13.4$	$89.8 \pm 0.5$	$101.5 \pm 10.2$	$97.6 \pm 0.3$
$K \pi \pi^0$	$221.4 \pm 15.6$	$91.0 \pm 0.4$	$120.8 \pm 11.1$	$97.4 \pm 0.2$



$$R_{K_S K \pi} = 0.73 \pm 0.08$$

$$\delta_D^{K_S K \pi} = (8.3 \pm 15.2)^\circ$$

$$R_{K^* K} = 1.00 \pm 0.16$$

$$\delta_D^{K^* K} = (26.5 \pm 15.8)^\circ$$

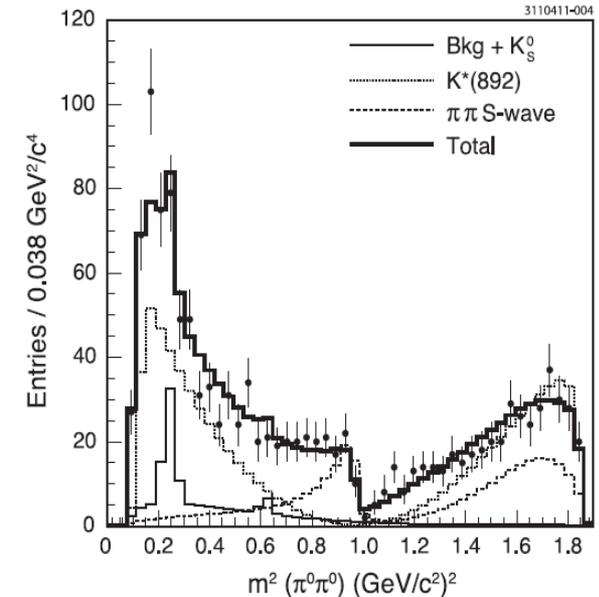
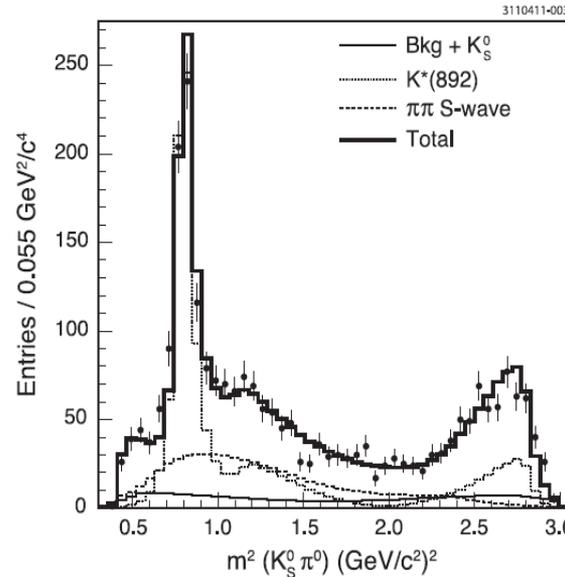
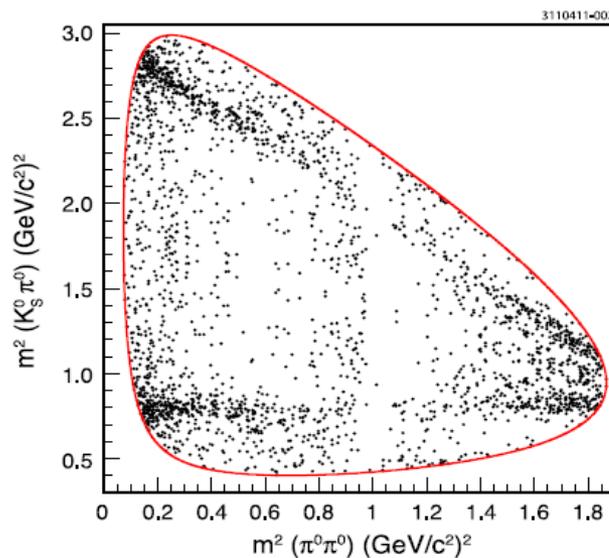
useful inputs for charm-mixing measurement

# Amplitude analysis of $D^0 \rightarrow K_S \pi^0 \pi^0$ at CLEO-c

PRD84(2011)092005

1259 tagged events with about 7.5% backgrounds

Ideal to study  $\pi\pi$  S-wave contribution without  $\rho$  amplitude



Component	Fit fraction (%)
$\pi\pi$ S-wave	$28.9 \pm 6.3 \pm 3.1$
$K^*(892)$	$65.6 \pm 5.3$
$K_2^*(1430)$	$0.49 \pm 0.45 \pm 2.5 \pm 0.23$
$K^*(1680)$	$11.2 \pm 2.7 \pm 2.5$
$f_2(1270)$	$2.48 \pm 0.91 \pm 0.78$
$K_S^0$	$3.46 \pm 0.92 \pm 0.66$

$$\alpha_\tau = 1 + 2r \cos \delta_\tau + R_{WS}^\tau + y \equiv 1 + R_{WS}^\tau + \Delta_{QC}^\tau$$

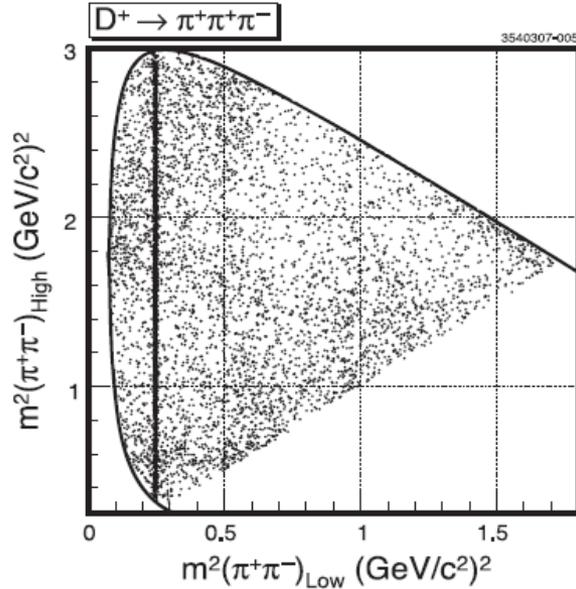
Tag Mode	$\mathcal{B}(D^0 \rightarrow K_S^0 \pi^0 \pi^0)$ (%)
$K^+ \pi^0$	$1.030 \pm 0.069 \pm 0.094$
$K^+ \pi^- \pi^0$	$1.061 \pm 0.054 \pm 0.110$
$K^+ \pi^- \pi^+ \pi^-$	$1.099 \pm 0.084 \pm 0.115$
Average	$1.058 \pm 0.038 \pm 0.073$

# Dalitz plot analysis of $D^+ \rightarrow \pi^+ \pi^- \pi^+$ at CLEO-c

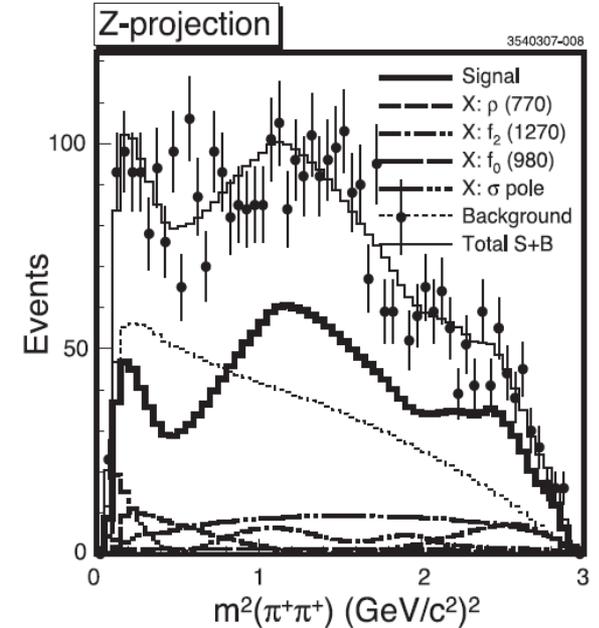
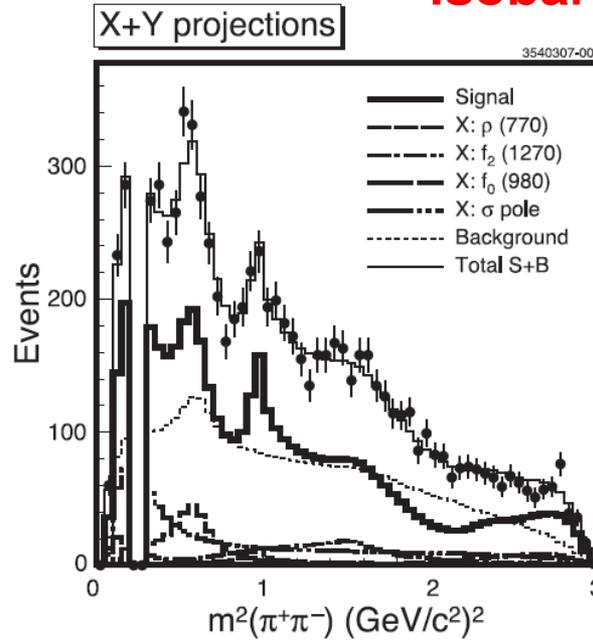
PRD86(2012)112001

6991 candidates with 2159 backgrounds  
with single tag method from 281 pb<sup>-1</sup> data

Dalitz plot



isobar model fit



Mode	Amplitude (a.u.)	Phase (°)	Fit fraction (%)
$\rho(770)\pi^+$	1 (fixed)	0 (fixed)	$20.0 \pm 2.3 \pm 0.9$
$f_0(980)\pi^+$	$1.4 \pm 0.2 \pm 0.2$	$12 \pm 10 \pm 5$	$4.1 \pm 0.9 \pm 0.3$
$f_2(1270)\pi^+$	$2.1 \pm 0.2 \pm 0.1$	$-123 \pm 6 \pm 3$	$18.2 \pm 2.6 \pm 0.7$
$f_0(1370)\pi^+$	$1.3 \pm 0.4 \pm 0.2$	$-21 \pm 15 \pm 14$	$2.6 \pm 1.8 \pm 0.6$
$f_0(1500)\pi^+$	$1.1 \pm 0.3 \pm 0.2$	$-44 \pm 13 \pm 16$	$3.4 \pm 1.0 \pm 0.8$
$\sigma$ pole	$3.7 \pm 0.3 \pm 0.2$	$-3 \pm 4 \pm 2$	$41.8 \pm 1.4 \pm 2.5$

Large S-wave contribution  
is found at low  $\pi\pi$  mass

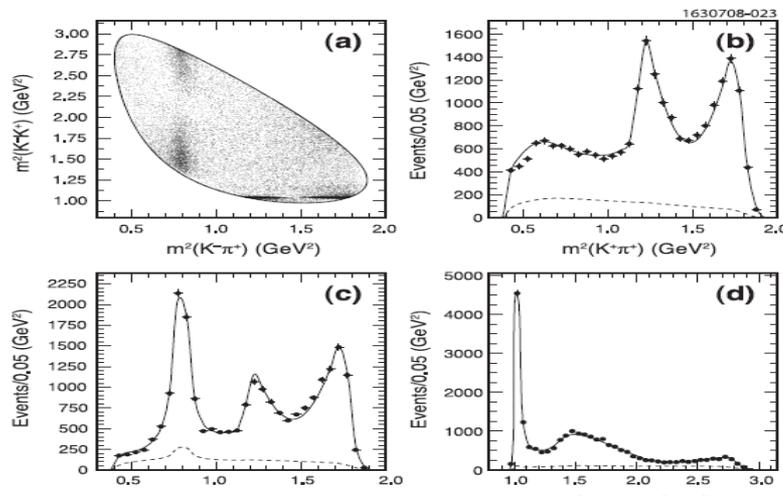
# Dalitz plot analysis of $D^+ \rightarrow K^+ K^- \pi^+$ at CLEO-c

PRD78(2008)072003

9757  $D^+$  and 9701  $D^-$  signals

$$A_{CP} = (-0.03 \pm 0.84 \pm 0.29)\%$$

CPV is also searched in different amplitudes



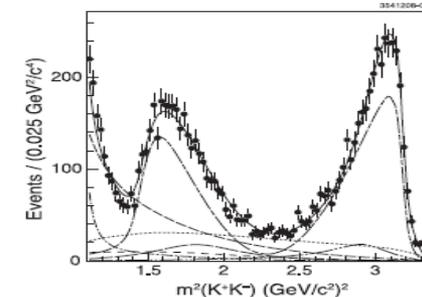
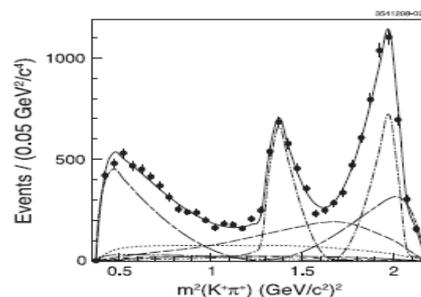
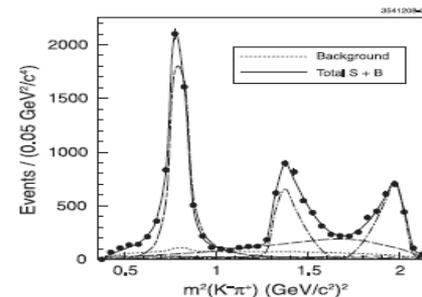
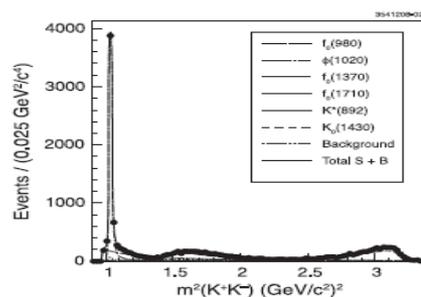
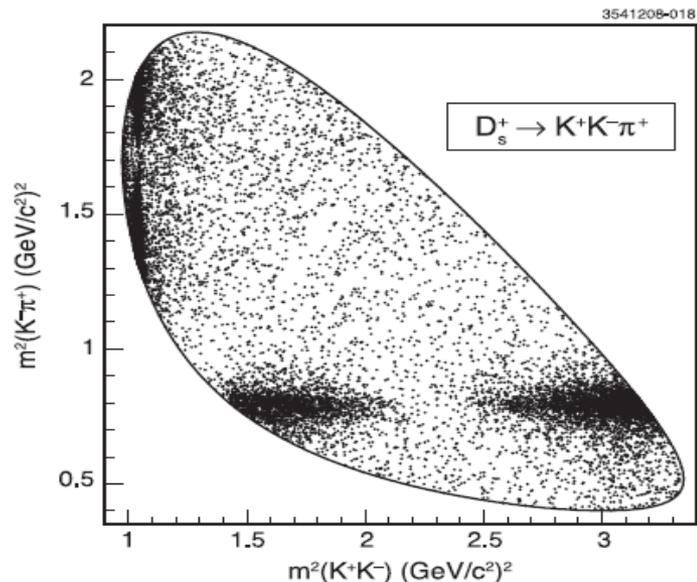
$r$	$b/a$ (%)	$\phi$ ( $^\circ$ )	FF asymmetry (%)
$\bar{K}^{*0}$	0(fixed)	0(fixed)	$-0.4 \pm 2.0^{+0.2+0.6}_{-0.5-0.3}$
$\bar{K}_0^*(1430)^0$	$4 \pm 3^{+1+2}_{-0-1}$	$-1 \pm 6^{+0+6}_{-3-1}$	$8 \pm 6^{+1+4}_{-1-1}$
$\phi$	$-0.7 \pm 1.3^{+0.2+0.3}_{-0.1-0.2}$	$3 \pm 3^{+0+3}_{-1-1}$	$-1.8 \pm 1.6^{+0.0+0.2}_{-0.4-0.1}$
$a_0(1450)^0$	$-10 \pm 7 \pm 2^{+6}_{-3}$	$4 \pm 3^{+1+2}_{-2-1}$	$-19 \pm 12^{+5+6}_{-3-11}$
$\phi(1680)$	$-4 \pm 11^{+5+6}_{-4-4}$	$3 \pm 6 \pm 2^{+3}_{-2}$	$-9 \pm 22^{+10+9}_{-7-12}$
$\bar{K}_2^*(1430)^0$	$23^{+12+1+3}_{-11-7-7}$	$5^{+5+1+3}_{-4-3-1}$	$43 \pm 19^{+1+5}_{-13-12}$
$\kappa(800)$	$-6 \pm 6^{+3+1}_{-1-5}$	$3 \pm 6^{+2+1}_{-2-4}$	$-12 \pm 11^{+0+14}_{-6-2}$

	Magnitude	Phase ( $^\circ$ )	Fit fraction (%)
Fit A [ $\chi^2/d.o.f. = 898/708$ ]			
$\bar{K}^{*0}$	1(fixed)	0(fixed)	$25.0 \pm 0.6^{+0.4+0.2}_{-0.3-1.2}$
$\bar{K}_0^*(1430)^0$	$3.7 \pm 0.5^{+0.5+1.0}_{-0.1-1.0}$	$73 \pm 9^{+6+15}_{-1-38}$	$12.4 \pm 3.3^{+3.4+7.3}_{-0.7-5.8}$
$\phi$	$1.189 \pm 0.015^{+0.000+0.028}_{-0.001-0.010}$	$-179 \pm 4^{+3+13}_{-1-5}$	$28.1 \pm 0.6^{+0.1+0.2}_{-0.3-0.4}$
$a_0(1450)^0$	$1.72 \pm 0.10^{+0.11+0.81}_{-0.11-0.28}$	$123 \pm 3^{+1+9}_{-1-15}$	$5.9 \pm 0.7^{+0.7+6.7}_{-0.6-1.8}$
$\phi(1680)$	$1.9 \pm 0.2^{+0.0+1.3}_{-0.1-0.7}$	$-52 \pm 8^{+0+10}_{-5-16}$	$0.51 \pm 0.11^{+0.02+0.85}_{-0.04-0.12}$
$K_2^*(1430)^0$	$6.4 \pm 0.9^{+0.5+1.9}_{-0.4-2.8}$	$150 \pm 6^{+1+28}_{-2-13}$	$1.2 \pm 0.3^{+0.2+0.8}_{-0.1-0.6}$
NR	$5.1 \pm 0.3^{+0.4+0.8}_{-0.3-0.2}$	$53 \pm 7^{+5+18}_{-5-11}$	$14.7 \pm 1.8^{+0.2+3.9}_{-1.6-1.5}$
Total Fit Fraction = (88.7 $\pm$ 2.9)%			
Fit B [ $\chi^2/d.o.f. = 895/708$ ]			
$\bar{K}^{*0}$	1(fixed)	0(fixed)	$25.7 \pm 0.5^{+0.4+0.1}_{-0.4-0.7}$
$\bar{K}_0^*(1430)^0$	$4.56 \pm 0.13^{+0.10+0.42}_{-0.01-0.39}$	$70 \pm 6^{+1+16}_{-6-23}$	$18.8 \pm 1.2^{+0.6+3.2}_{-0.1-3.4}$
$\phi$	$1.166 \pm 0.015^{+0.001+0.025}_{-0.009-0.009}$	$-163 \pm 3^{+1+14}_{-1-5}$	$27.8 \pm 0.4^{+0.1+0.2}_{-0.3-0.4}$
$a_0(1450)^0$	$1.50 \pm 0.10^{+0.09+0.92}_{-0.06-0.31}$	$116 \pm 2^{+1+7}_{-1-14}$	$4.6 \pm 0.6^{+0.5+7.2}_{-0.2-2.4}$
$\phi(1680)$	$1.86 \pm 0.20^{+0.02+0.23}_{-0.08-0.77}$	$-112 \pm 6^{+3+19}_{-4-12}$	$0.51 \pm 0.11^{+0.01+0.37}_{-0.04-0.15}$
$K_2^*(1430)^0$	$7.6 \pm 0.8^{+0.5+2.4}_{-0.3-2.8}$	$171 \pm 4^{+0+24}_{-2-11}$	$1.7 \pm 0.4^{+0.3+1.7}_{-0.1-0.7}$
$\kappa(800)$	$2.30 \pm 0.13^{+0.01+0.52}_{-0.11-0.29}$	$-87 \pm 6^{+2+15}_{-3-10}$	$7.0 \pm 0.8^{+0.0+3.5}_{-0.6-1.9}$
Total Fit Fraction = (86.1 $\pm$ 1.1)%			
Fit C [ $\chi^2/d.o.f. = 912/710$ ]			
$\bar{K}^{*0}$	1(fixed)	0(fixed)	$25.3 \pm 0.5^{+0.2+0.2}_{-0.4-0.7}$
LASS	$3.81 \pm 0.06^{+0.05+0.13}_{-0.05-0.46}$	$25.1 \pm 2^{+1+6}_{-1-14}$	$40.6 \pm 0.8^{+0.4+1.6}_{-0.1-9.1}$
$\phi$	$1.193 \pm 0.015^{+0.003+0.021}_{-0.010-0.011}$	$-176 \pm 2^{+0+8}_{-2-8}$	$28.6 \pm 0.4^{+0.2+0.1}_{-0.3-0.5}$
$a_0(1450)^0$	$1.73 \pm 0.07^{+0.14+0.68}_{-0.03-0.38}$	$122 \pm 2^{+1+8}_{-1-10}$	$6.0 \pm 0.4^{+0.9+5.4}_{-0.2-2.4}$
$\phi(1680)$	$1.71 \pm 0.16^{+0.02+0.41}_{-0.02-0.77}$	$-72 \pm 8^{+2+10}_{-2-11}$	$0.42 \pm 0.08^{+0.02+0.19}_{-0.01-0.16}$
$\bar{K}_2^*(1430)^0$	$4.9 \pm 0.7^{+0.1+2.2}_{-0.4-2.3}$	$146 \pm 9^{+0+34}_{-7-11}$	$0.7 \pm 0.2^{+0.7+1.7}_{-0.1-0.3}$
Total Fit Fraction = (101.5 $\pm$ 0.8)%			

# Dalitz plot analysis of $D_s^+ \rightarrow K^+ K^- \pi^+$ at CLEO-c

PRD79(2008)072008

14400 candidates with  $\sim 15\%$  background  
method from  $586 \text{ pb}^{-1}$  data

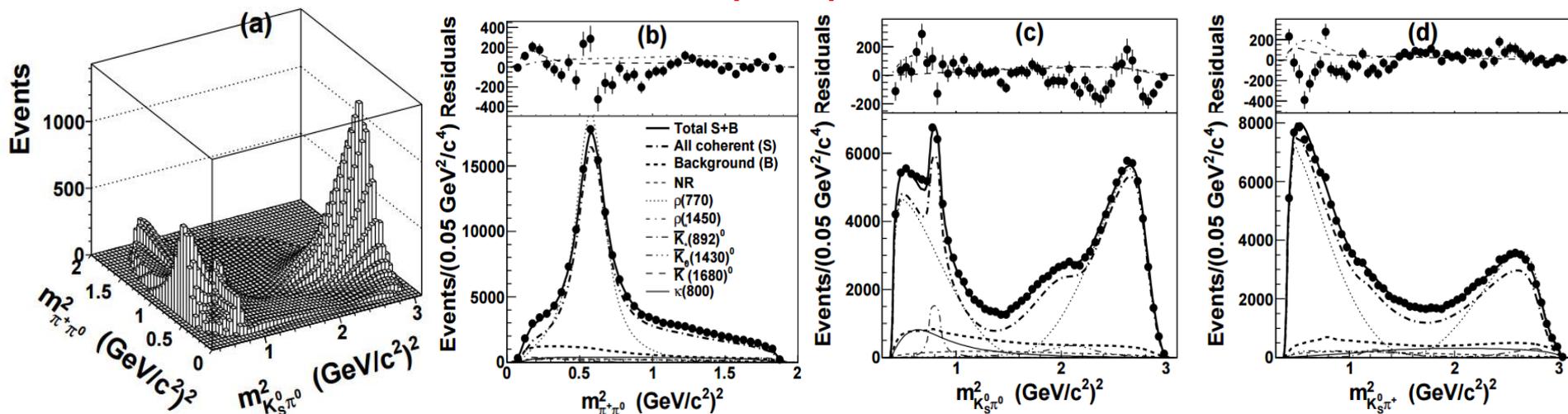


Parameter	E687 model	$f_2(1270)$	$a_2(1320)$	$f_0(1370)$	$f_0(1500)$	$f_2(1525)$	$a_0(1450)$	$\phi(1680)$
$m_{K^*(892)}$	$895.8 \pm 0.5$	-0.4	-0.1	-0.9	-0.5	0.0	-0.8	0.1
$\Gamma_{K^*(892)}$	$44.2 \pm 1.0$	2.3	2.4	1.5	0.6	0.6	1.0	1.2
$a_{K_s^*(1430)}$ (a.u.)	$1.76 \pm 0.12$	0.11	0.08	-0.25	-0.03	-0.16	-0.22	-0.18
$\phi_{K_s^*(1430)}$ ( $^\circ$ )	$145 \pm 8$	-32	-28	1.0	-15	1.7	-15	18
$a_{f_0(980)}$ (a.u.)	$3.67 \pm 0.13$	0.29	0.26	1.05	0.52	0.03	1.09	0.20
$\phi_{f_0(980)}$ ( $^\circ$ )	$156 \pm 3$	-2	-1.6	1.3	2.3	0.22	3.8	10.5
$a_{\phi(1020)}$ (a.u.)	$1.15 \pm 0.02$	-0.03	-0.04	-0.02	-0.003	-0.02	-0.007	-0.012
$\phi_{\phi(1020)}$ ( $^\circ$ )	$-15 \pm 4$	-7	-6.3	7.2	-0.6	1.5	4.3	13.2
$a_{f_0(1710)}$ (a.u.)	$1.27 \pm 0.07$	0.08	0.07	-0.16	0.17	-0.04	0.03	-0.018
$\phi_{f_0(1710)}$ ( $^\circ$ )	$102 \pm 4$	7	4.7	-13	-4.1	-3.8	-17	5.3
$a_{\text{add}}$ (a.u.)		$0.64 \pm 0.09$	$0.45 \pm 0.06$	$1.15 \pm 0.09$	$0.50 \pm 0.05$	$0.50 \pm 0.07$	$1.32 \pm 0.10$	$1.04 \pm 0.17$
$\phi_{\text{add}}$ ( $^\circ$ )		$17 \pm 9$	$40 \pm 8$	$53 \pm 5$	$132 \pm 7$	$173 \pm 10$	$103 \pm 5$	$-4 \pm 11$
$\chi^2/\nu$	278/119	237/117	237/117	178/117	229/117	249/117	192/117	256/117

Comparing to E687 model, involving an additional  $f_0(1370)$  amplitude gives the best fit quality and involving other amplitude does not

# Dalitz plot analysis of $D^+ \rightarrow K_S \pi^+ \pi^0$ at BESIII

PRD89(2014)052001



Partial branching fractions calculated by combining fit fractions with PDG's  $D^+ \rightarrow K_S \pi^+ \pi^0$  branching ratio

Mode	Partial Branching Fraction (%)
$D^+ \rightarrow K_S^0 \pi^+ \pi^0$ Non Resonant	$0.32 \pm 0.05 \pm 0.25^{+0.28}_{-0.25}$
$D^+ \rightarrow \rho^+ K_S^0, \rho^+ \rightarrow \pi^+ \pi^0$	$5.83 \pm 0.16 \pm 0.30^{+0.45}_{-0.15}$
$D^+ \rightarrow \rho(1450)^+ K_S^0, \rho(1450)^+ \rightarrow \pi^+ \pi^0$	$0.15 \pm 0.02 \pm 0.09^{+0.07}_{-0.11}$
$D^+ \rightarrow \bar{K}^*(892)^0 \pi^+, \bar{K}^*(892)^0 \rightarrow K_S^0 \pi^0$	$0.250 \pm 0.012 \pm 0.015^{+0.025}_{-0.024}$
$D^+ \rightarrow \bar{K}_0^*(1430)^0 \pi^+, \bar{K}_0^*(1430)^0 \rightarrow K_S^0 \pi^0$	$0.26 \pm 0.04 \pm 0.05 \pm 0.06$
$D^+ \rightarrow \bar{K}^*(1680)^0 \pi^+, \bar{K}^*(1680)^0 \rightarrow K_S^0 \pi^0$	$0.09 \pm 0.01 \pm 0.05^{+0.04}_{-0.08}$
$D^+ \rightarrow \bar{\kappa}^0 \pi^+, \bar{\kappa}^0 \rightarrow K_S^0 \pi^0$	$0.54 \pm 0.09 \pm 0.28^{+0.36}_{-0.19}$
$NR + \bar{\kappa}^0 \pi^+$	$1.30 \pm 0.12 \pm 0.12^{+0.12}_{-0.30}$
$K_S^0 \pi^0$ S-wave	$1.21 \pm 0.10 \pm 0.16^{+0.19}_{-0.27}$

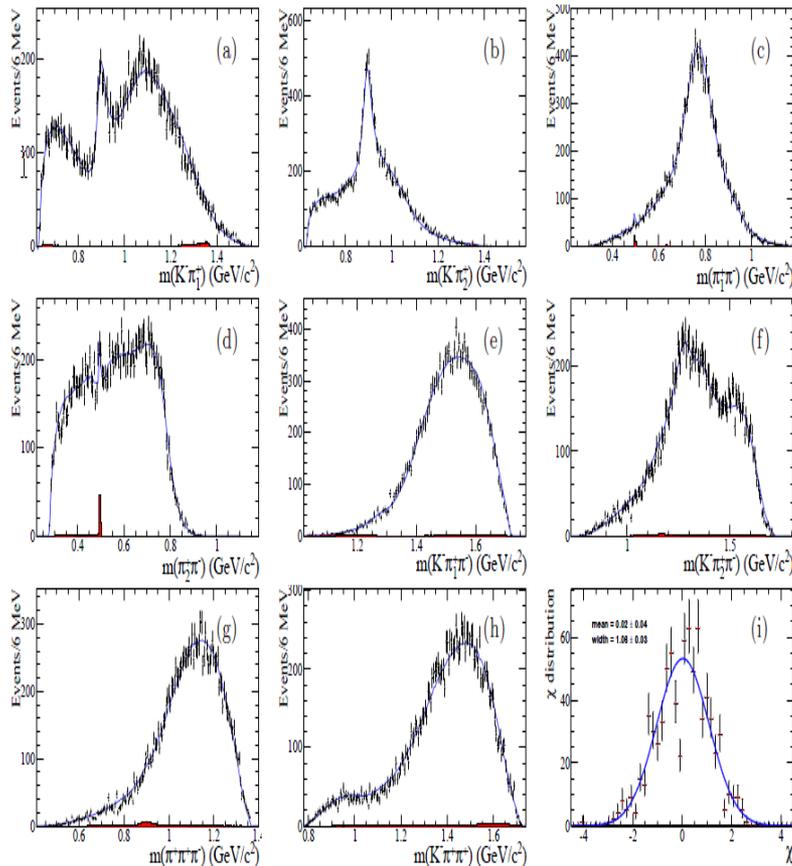
provide rich information about sub-resonance and strong phase

# Amplitude analysis of $D^0 \rightarrow K^- \pi^+ \pi^+ \pi^-$ at BESIII

PRD95(2017)072010

Help to determine the absolute BF, strong phase, benefit  $\gamma/\phi_3$

Previous analyses only from MarkIII and E691

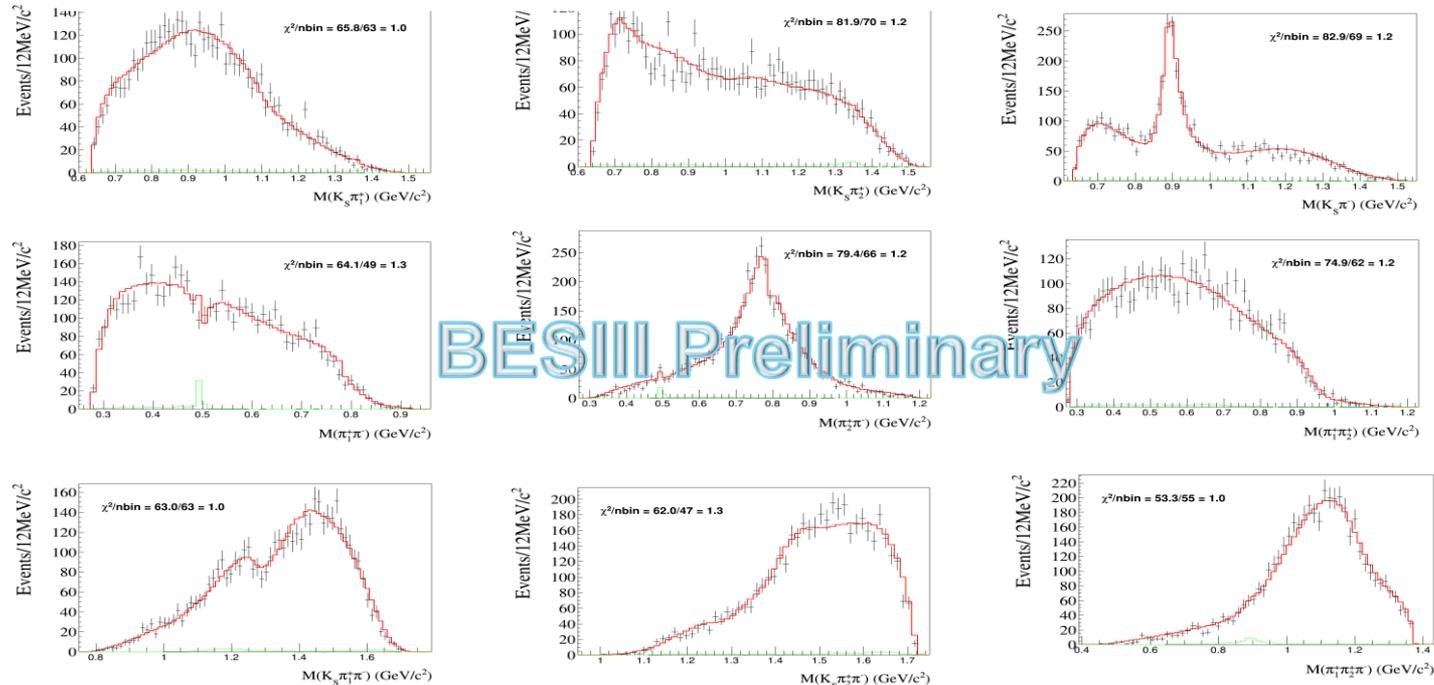


Amplitude	$\phi_i$	Fit fraction (%)
$D^0[S] \rightarrow K^* \rho^0$	$2.35 \pm 0.06 \pm 0.18$	$6.5 \pm 0.5 \pm 0.8$
$D^0[P] \rightarrow K^* \rho^0$	$-2.25 \pm 0.08 \pm 0.15$	$2.3 \pm 0.2 \pm 0.1$
$D^0[D] \rightarrow K^* \rho^0$	$2.49 \pm 0.06 \pm 0.11$	$7.9 \pm 0.4 \pm 0.7$
$D^0 \rightarrow K^- a_1^+(1260), a_1^+(1260)[S] \rightarrow \rho^0 \pi^+$	0(fixed)	$53.2 \pm 2.8 \pm 4.0$
$D^0 \rightarrow K^- a_1^+(1260), a_1^+(1260)[D] \rightarrow \rho^0 \pi^+$	$-2.11 \pm 0.15 \pm 0.21$	$0.3 \pm 0.1 \pm 0.1$
$D^0 \rightarrow K_1^-(1270) \pi^+, K_1^-(1270)[S] \rightarrow K^{*0} \pi^-$	$1.48 \pm 0.21 \pm 0.24$	$0.1 \pm 0.1 \pm 0.1$
$D^0 \rightarrow K_1^-(1270) \pi^+, K_1^-(1270)[D] \rightarrow K^{*0} \pi^-$	$3.00 \pm 0.09 \pm 0.15$	$0.7 \pm 0.2 \pm 0.2$
$D^0 \rightarrow K_1^-(1270) \pi^+, K_1^-(1270) \rightarrow K^- \rho^0$	$-2.46 \pm 0.06 \pm 0.21$	$3.4 \pm 0.3 \pm 0.5$
$D^0 \rightarrow (\rho^0 K^-)_A \pi^+, (\rho^0 K^-)_\Lambda [D] \rightarrow K^- \rho^0$	$-0.43 \pm 0.09 \pm 0.12$	$1.1 \pm 0.2 \pm 0.3$
$D^0 \rightarrow (K^- \rho^0)_P \pi^+$	$-0.14 \pm 0.11 \pm 0.10$	$7.4 \pm 1.6 \pm 5.7$
$D^0 \rightarrow (K^- \pi^+)_S \rho^0$	$-2.45 \pm 0.19 \pm 0.47$	$2.0 \pm 0.7 \pm 1.9$
$D^0 \rightarrow (K^- \rho^0)_V \pi^+$	$-1.34 \pm 0.12 \pm 0.09$	$0.4 \pm 0.1 \pm 0.1$
$D^0 \rightarrow (K^{*0} \pi^-)_P \pi^+$	$-2.09 \pm 0.12 \pm 0.22$	$2.4 \pm 0.5 \pm 0.5$
$D^0 \rightarrow K^{*0} (\pi^+ \pi^-)_S$	$-0.17 \pm 0.11 \pm 0.12$	$2.6 \pm 0.6 \pm 0.6$
$D^0 \rightarrow (K^{*0} \pi^-)_V \pi^+$	$-2.13 \pm 0.10 \pm 0.11$	$0.8 \pm 0.1 \pm 0.1$
$D^0 \rightarrow ((K^- \pi^+)_S \pi^-)_\Lambda \pi^+$	$-1.36 \pm 0.08 \pm 0.37$	$5.6 \pm 0.9 \pm 2.7$
$D^0 \rightarrow K^- ((\pi^+ \pi^-)_S \pi^+)_\Lambda$	$-2.23 \pm 0.08 \pm 0.22$	$13.1 \pm 1.9 \pm 2.2$
$D^0 \rightarrow (K^- \pi^+)_S (\pi^+ \pi^-)_S$	$-1.40 \pm 0.04 \pm 0.22$	$16.3 \pm 0.5 \pm 0.6$
$D^0[S] \rightarrow (K^- \pi^+)_V (\pi^+ \pi^-)_V$	$1.59 \pm 0.13 \pm 0.41$	$5.4 \pm 1.2 \pm 1.9$
$D^0 \rightarrow (K^- \pi^+)_S (\pi^+ \pi^-)_V$	$-0.16 \pm 0.17 \pm 0.43$	$1.9 \pm 0.6 \pm 1.2$
$D^0 \rightarrow (K^- \pi^+)_V (\pi^+ \pi^-)_S$	$2.58 \pm 0.08 \pm 0.25$	$2.9 \pm 0.5 \pm 1.7$
$D^0 \rightarrow (K^- \pi^+)_T (\pi^+ \pi^-)_S$	$-2.92 \pm 0.14 \pm 0.12$	$0.3 \pm 0.1 \pm 0.1$
$D^0 \rightarrow (K^- \pi^+)_S (\pi^+ \pi^-)_T$	$2.45 \pm 0.12 \pm 0.37$	$0.5 \pm 0.1 \pm 0.1$

Component	Branching fraction (%)	PDG value (%)
$D^0 \rightarrow \bar{K}^{*0} \rho^0$	$0.99 \pm 0.04 \pm 0.04 \pm 0.03$	$1.05 \pm 0.23$
$D^0 \rightarrow K^- a_1^+(1260) (\rho^0 \pi^+)$	$4.41 \pm 0.22 \pm 0.30 \pm 0.13$	$3.6 \pm 0.6$
$D^0 \rightarrow K_1^-(1270) (K^{*0} \pi^-) \pi^+$	$0.07 \pm 0.01 \pm 0.02 \pm 0.00$	$0.29 \pm 0.03$
$D^0 \rightarrow K_1^-(1270) (K^- \rho^0) \pi^+$	$0.27 \pm 0.02 \pm 0.04 \pm 0.01$	
$D^0 \rightarrow K^- \pi^+ \rho^0$	$0.68 \pm 0.09 \pm 0.20 \pm 0.02$	$0.51 \pm 0.23$
$D^0 \rightarrow \bar{K}^{*0} \pi^+ \pi^-$	$0.57 \pm 0.03 \pm 0.04 \pm 0.02$	$0.99 \pm 0.23$
$D^0 \rightarrow K^- \pi^+ \pi^+ \pi^-$	$1.77 \pm 0.05 \pm 0.04 \pm 0.05$	$1.88 \pm 0.26$

# Amplitude analysis of $D^+ \rightarrow K_S \pi^+ \pi^+ \pi^-$ at BESIII

## Help to understand $D \rightarrow AP$ decay and the mixing between $K_1(1270)$ and $K_1(1400)$



Component	BESIII	Mark III	E691
$D^+ \rightarrow K_S^0 a_1(1260)^+ (\rho^0 \pi^+)$	$0.567 \pm 0.020 \pm 0.044$	$0.539 \pm 0.057 \pm 0.070$	$0.830 \pm 0.140 \pm 0.200$
$D^+ \rightarrow K_S^0 a_1(1260)^+ (f_0(500) \pi^+)$	$0.050 \pm 0.006 \pm 0.007$		
$D^+ \rightarrow \bar{K}_1(1400)^0 \pi^+$	$0.372 \pm 0.015 \pm 0.016$	$0.277 \pm 0.047 \pm 0.080$	
$D^+ \rightarrow \bar{K}_1(1270)^0 \pi^+$	$0.036 \pm 0.004 \pm 0.002$		
$D^+ \rightarrow \bar{K}(1460)^0 \pi^+$	$0.014 \pm 0.004 \pm 0.003$		
$D^+ \rightarrow K_S^0 \pi^+ \rho^0$	$0.044 \pm 0.005 \pm 0.005$		$0.070 \pm 0.040 \pm 0.060$
$D^+ \rightarrow K^{*-} \pi^+ \pi^+$	$0.139 \pm 0.012 \pm 0.020$		$0.330 \pm 0.060 \pm 0.140$
$D^+ \rightarrow K_S^0 \pi^+ \pi^+ \pi^-$ nonresonant	$0.074 \pm 0.005 \pm 0.008$	$0.170 \pm 0.056 \pm 0.100$	$0.100 \pm 0.040 \pm 0.060$

Component	Branching fraction(%)
$D^+ \rightarrow K_S^0 a_1(1260)^+, a_1(1260)^+ \rightarrow \rho^0 \pi^+$	$1.769 \pm 0.062 \pm 0.136 \pm 0.062$
$D^+ \rightarrow K_S^0 a_1(1260)^+, a_1(1260)^+ \rightarrow f_0(500) \pi^+$	$0.156 \pm 0.019 \pm 0.022 \pm 0.006$
$D^+ \rightarrow \bar{K}_1(1400)^0 \pi^+, \bar{K}_1(1400)^0 \rightarrow K^{*-} \pi^+, K^{*-} \rightarrow K_S^0 \pi^-$	$1.161 \pm 0.047 \pm 0.051 \pm 0.041$
$D^+ \rightarrow \bar{K}_1(1270)^0 \pi^+, \bar{K}_1(1270)^0 \rightarrow K_S^0 \rho^0$	$0.112 \pm 0.012 \pm 0.007 \pm 0.004$
$D^+ \rightarrow \bar{K}(1460)^0 \pi^+, \bar{K}(1460)^0 \rightarrow K_S^0 \rho^0$	$0.044 \pm 0.012 \pm 0.011 \pm 0.002$
$D^+ \rightarrow K_S^0 \pi^+ \rho^0$ three-body	$0.137 \pm 0.016 \pm 0.015 \pm 0.005$
$D^+ \rightarrow K^{*-} \pi^+ \pi^+$ three-body, $K^{*-} \rightarrow K_S^0 \pi^-$	$0.434 \pm 0.037 \pm 0.062 \pm 0.015$
$D^+ \rightarrow K_S^0 \pi^+ \pi^+ \pi^-$ nonresonant	$0.231 \pm 0.016 \pm 0.024 \pm 0.008$

# $\Lambda_c^+$ decays at BESIII

# Studies of $\Lambda_c^+$ decays before 2014

- $\Lambda_c^+$  was observed in 1979
- Before 2014, all decays of  $\Lambda_c^+$  are measured relative to  $\Lambda_c^+ \rightarrow pK^-\pi^+$ , which suffer large error of 25%, with high energy data. No absolute measurement using data produced at  $\Lambda_c^+$  pair threshold
- Sum of BFs of known decays  $\Lambda_c^+$  is only about 60%
- In 2014, Belle reported improved measurement of  $B[\Lambda_c^+ \rightarrow pK^-\pi^+]$ , with a precision of ~5%

$\Lambda_c^+$ DECAY MODES	Fraction ( $\Gamma_i/\Gamma$ )	Scale factor/ Confidence level	$P$ (MeV/c)
<b>Hadronic modes with a <math>\rho</math>: <math>S = -1</math> final states</b>			
$\rho \bar{K}^0$	( 2.3 $\pm$ 0.6 ) %		873
$\rho K^-\pi^+$	[a] ( 5.0 $\pm$ 1.3 ) %		823
$\rho \bar{K}^*(892)^0$	[b] ( 1.6 $\pm$ 0.5 ) %		685
$\Delta(1232)^{++} K^-$	( 8.6 $\pm$ 3.0 ) $\times 10^{-3}$		710
$\Lambda(1520)\pi^+$	[b] ( 1.8 $\pm$ 0.6 ) %		627
$\rho K^-\pi^+$ nonresonant	( 2.8 $\pm$ 0.8 ) %		823
$\rho \bar{K}^0 \pi^0$	( 3.3 $\pm$ 1.0 ) %		823
$\rho \bar{K}^0 \eta$	( 1.2 $\pm$ 0.4 ) %		568
<b>Hadronic modes with a hyperon: <math>S = -1</math> final states</b>			
$\Lambda\pi^+$	( 1.07 $\pm$ 0.28 ) %		864
$\Lambda\pi^+\pi^0$	( 3.6 $\pm$ 1.3 ) %		844
$\Lambda\rho^+$	< 5 %	CL=95%	636
$\Lambda\pi^+\pi^+\pi^-$	( 2.6 $\pm$ 0.7 ) %		807
$\Sigma(1385)^+\pi^+\pi^-, \Sigma^{*+} \rightarrow$	( 7 $\pm$ 4 ) $\times 10^{-3}$		688
$\Lambda\pi^+$			
$\Sigma(1385)^-\pi^+\pi^+, \Sigma^{*-} \rightarrow$	( 5.5 $\pm$ 1.7 ) $\times 10^{-3}$		688
$\Lambda\pi^-\rho^0$	( 1.1 $\pm$ 0.5 ) %		524
$\Sigma(1385)^+\rho^0, \Sigma^{*+} \rightarrow \Lambda\pi^+$	( 3.7 $\pm$ 3.1 ) $\times 10^{-3}$		363
$\Lambda\pi^+\pi^+\pi^-\pi^0$ nonresonant	< 8 $\times 10^{-3}$	CL=90%	807
$\Lambda\pi^+\pi^+\pi^-\pi^0$ total	( 1.8 $\pm$ 0.8 ) %		757
$\Lambda\pi^+\eta$	[b] ( 1.8 $\pm$ 0.6 ) %		691
$\Sigma(1385)^+\eta$	[b] ( 8.5 $\pm$ 3.3 ) $\times 10^{-3}$		570
$\Lambda\pi^+\omega$	[b] ( 1.2 $\pm$ 0.5 ) %		517
$\Lambda\pi^+\pi^+\pi^-\pi^0$ , no $\eta$ or $\omega$	< 7 $\times 10^{-3}$	CL=90%	757
$\Lambda K^+\bar{K}^0$	( 4.7 $\pm$ 1.5 ) $\times 10^{-3}$	S=1.2	443
$\Xi(1690)^0 K^+, \Xi^{*0} \rightarrow \Lambda\bar{K}^0$	( 1.3 $\pm$ 0.5 ) $\times 10^{-3}$		286
$\Sigma^0\pi^+$	( 1.05 $\pm$ 0.28 ) %		825
$\Sigma^+\pi^0$	( 1.00 $\pm$ 0.34 ) %		827
$\Sigma^+\eta$	( 5.5 $\pm$ 2.3 ) $\times 10^{-3}$		713
$\Sigma^+\pi^+\pi^-$	( 3.6 $\pm$ 1.0 ) %		804
$\Sigma^+\rho^0$	< 1.4 %	CL=95%	575
$\Sigma^-\pi^+\pi^+$	( 1.7 $\pm$ 0.5 ) %		799
$\Sigma^0\pi^+\pi^0$	( 1.8 $\pm$ 0.8 ) %		803
$\Sigma^0\pi^+\pi^+\pi^-$	( 8.3 $\pm$ 3.1 ) $\times 10^{-3}$		763
$\Sigma^+\pi^+\pi^-\pi^0$	—		767
$\Sigma^+\omega$	[b] ( 2.7 $\pm$ 1.0 ) %		569
<b>Semileptonic modes</b>			
$\Lambda\ell^+\nu_\ell$	[c] ( 2.0 $\pm$ 0.6 ) %		871
$\Lambda e^+\nu_e$	( 2.1 $\pm$ 0.6 ) %		871
$\Lambda\mu^+\nu_\mu$	( 2.0 $\pm$ 0.7 ) %		867
<b>Inclusive modes</b>			
$e^+$ anything	( 4.5 $\pm$ 1.7 ) %		—
$p e^+$ anything	( 1.8 $\pm$ 0.9 ) %		—
$p$ anything	( 50 $\pm$ 16 ) %		—

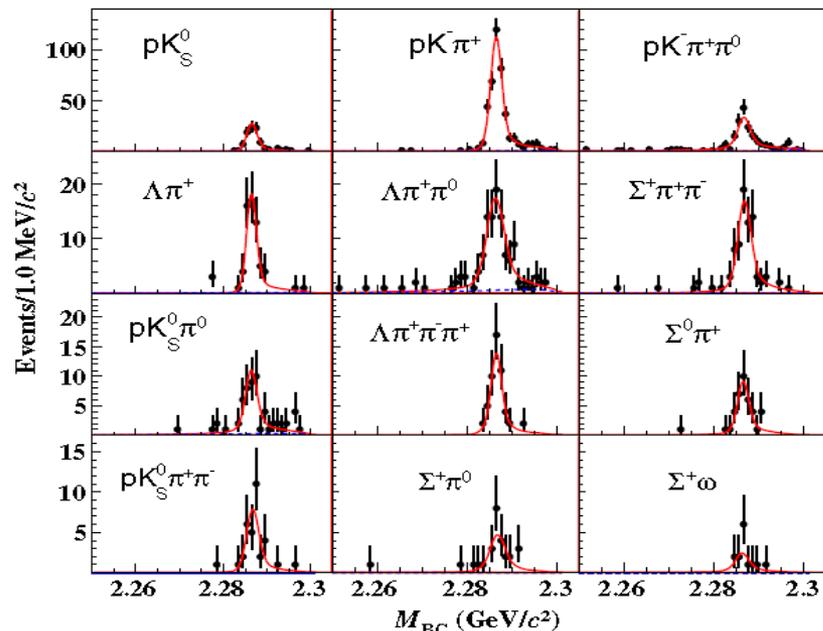
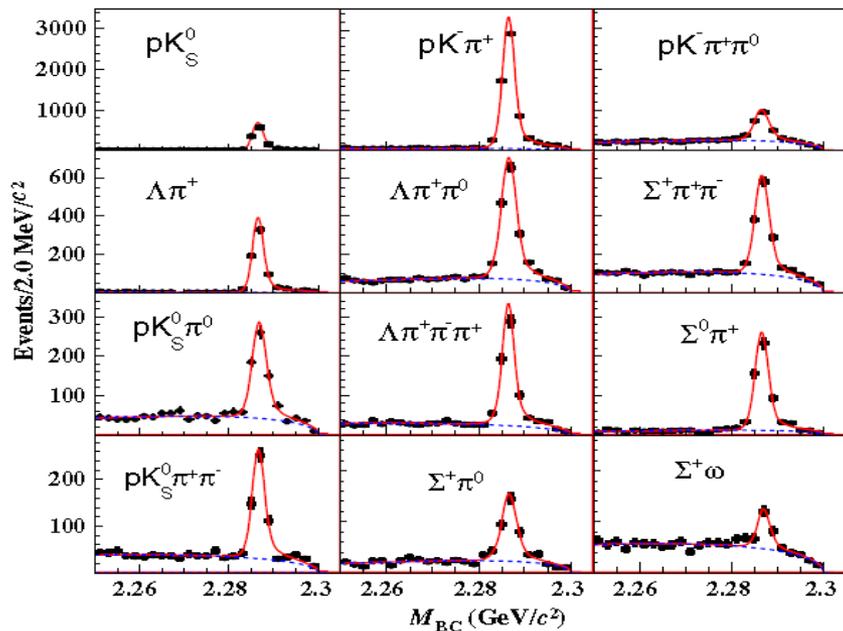
Systematic studies of  $\Lambda_c^+$ , search for new decays, absolute BF measurements are important to fully explore the  $\Lambda_c^+$  decay mechanisms

# Improved BF's of $\Lambda_c^+ \rightarrow$ hadronic decays

PRL116(2016)052001

ST:  $\sim 15000$

DT:  $\sim 1000$



$$N_j^{ST} = N_{\Lambda_c^+ \bar{\Lambda}_c^-} \mathcal{B}_j \epsilon_j$$

$$N_{ij}^{DT} = N_{\Lambda_c^+ \bar{\Lambda}_c^-} \mathcal{B}_i \mathcal{B}_j \epsilon_{ij}$$

$$\mathcal{B}_i = \frac{N_{ij}^{DT} \epsilon_j}{N_j^{ST} \epsilon_{ij}}$$

$$N_{i-}^{DT} = N_{\Lambda_c^+ \bar{\Lambda}_c^-} \sum_j \mathcal{B}_i \mathcal{B}_j \epsilon_{i-}^{DT}$$

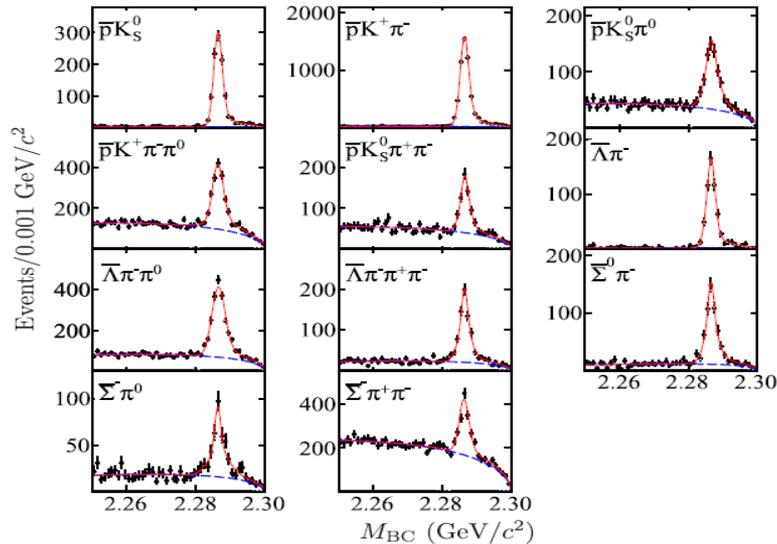
$$\epsilon_{i-}^{DT} \equiv [\sum_j (\mathcal{B}_j \epsilon_{ij}) / \sum_j \mathcal{B}_j]$$

Mode	This work (%)	PDG (%)
$pK_S^0$	$1.52 \pm 0.08 \pm 0.03$	$1.15 \pm 0.30$
$pK^- \pi^+$	$5.84 \pm 0.27 \pm 0.23$	$5.0 \pm 1.3$
$pK_S^0 \pi^0$	$1.87 \pm 0.13 \pm 0.05$	$1.65 \pm 0.50$
$pK_S^0 \pi^+ \pi^-$	$1.53 \pm 0.11 \pm 0.09$	$1.30 \pm 0.35$
$pK^- \pi^+ \pi^0$	$4.53 \pm 0.23 \pm 0.30$	$3.4 \pm 1.0$
$\Lambda \pi^+$	$1.24 \pm 0.07 \pm 0.03$	$1.07 \pm 0.28$
$\Lambda \pi^+ \pi^0$	$7.01 \pm 0.37 \pm 0.19$	$3.6 \pm 1.3$
$\Lambda \pi^+ \pi^- \pi^+$	$3.81 \pm 0.24 \pm 0.18$	$2.6 \pm 0.7$
$\Sigma^0 \pi^+$	$1.27 \pm 0.08 \pm 0.03$	$1.05 \pm 0.28$
$\Sigma^+ \pi^0$	$1.18 \pm 0.10 \pm 0.03$	$1.00 \pm 0.34$
$\Sigma^+ \pi^+ \pi^-$	$4.25 \pm 0.24 \pm 0.20$	$3.6 \pm 1.0$
$\Sigma^+ \omega$	$1.56 \pm 0.20 \pm 0.07$	$2.7 \pm 1.0$

Much better  
precision

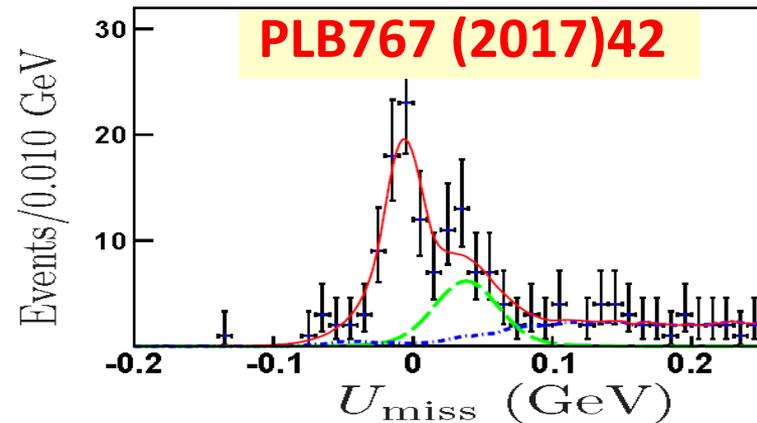
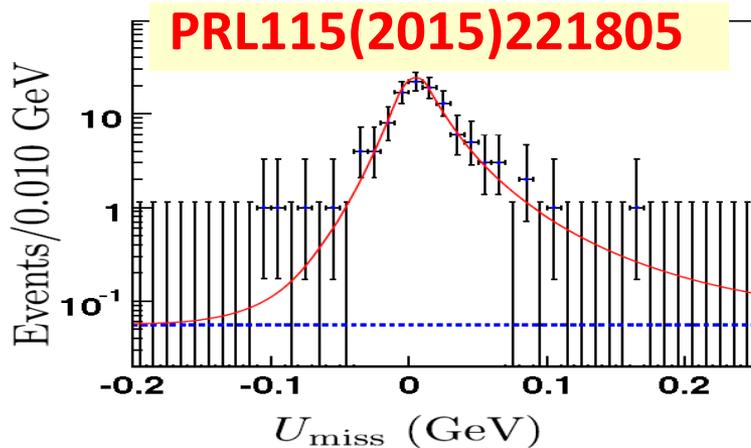
# First absolute BF's of $\Lambda_c^+ \rightarrow \Lambda l^+ \nu$

**Theory: (1.4-9.2)%**



Theoretical Models	predicated branching fraction for $\Lambda_c^+ \rightarrow \Lambda e^+ \nu_e$
MBM [1]	1.9%
NRQM [1]	2.6%
SU(4)-symmetry limit [2]	9.2%
RSQM [3]	4.4%
QCM [4]	5.62%
SQM [5]	1.96%
NRQM2 [6]	2.15%
NRQM3 [7]	1.42%
QCD SR1 [8]	$(3.0 \pm 0.9)\%$
QCD SR2 [9]	$(2.6 \pm 0.4)\%$
QCD SR3 [9]	$(5.8 \pm 1.5)\%$
STSR [10]	2.22% for $\Lambda_c^+ \rightarrow \Lambda l^+ \nu_l$
STNR [10]	1.58% for $\Lambda_c^+ \rightarrow \Lambda l^+ \nu_l$
HO SR [10]	4.72% for $\Lambda_c^+ \rightarrow \Lambda l^+ \nu_l$
HONR [10]	4.2% for $\Lambda_c^+ \rightarrow \Lambda l^+ \nu_l$
LCSR <sub>s</sub> [11]	$(3.0 \pm 0.3)\%$ for $\Lambda_c^+ \rightarrow \Lambda l^+ \nu_l$ (CZ-type)
PDG 2014 [14]	$(2.1 \pm 0.6)\%$
BESIII	$(3.63 \pm 0.38 \pm 0.20)\%$

**3 fb<sup>-1</sup> data help to explore FF studies**



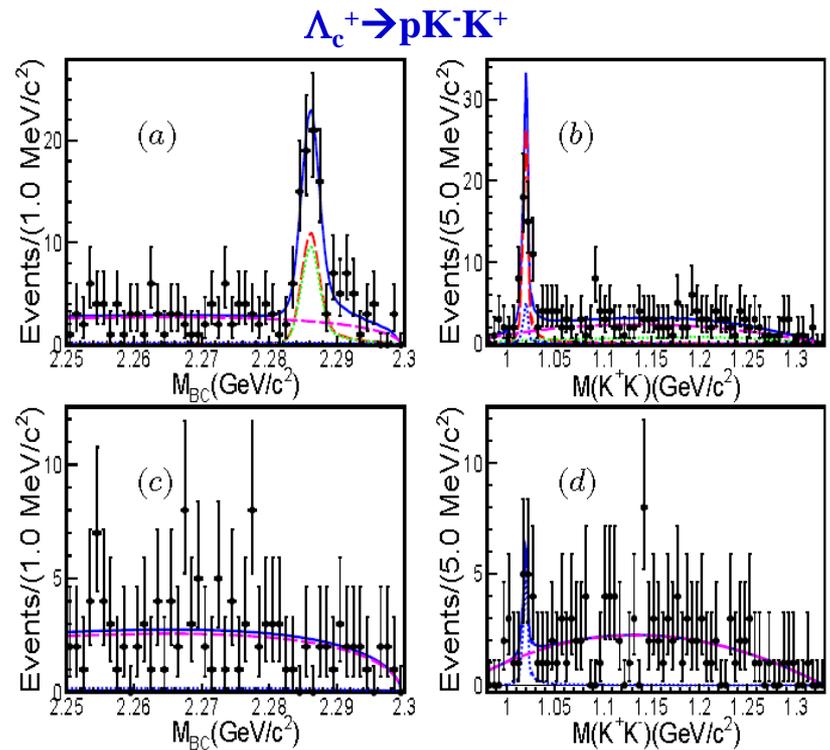
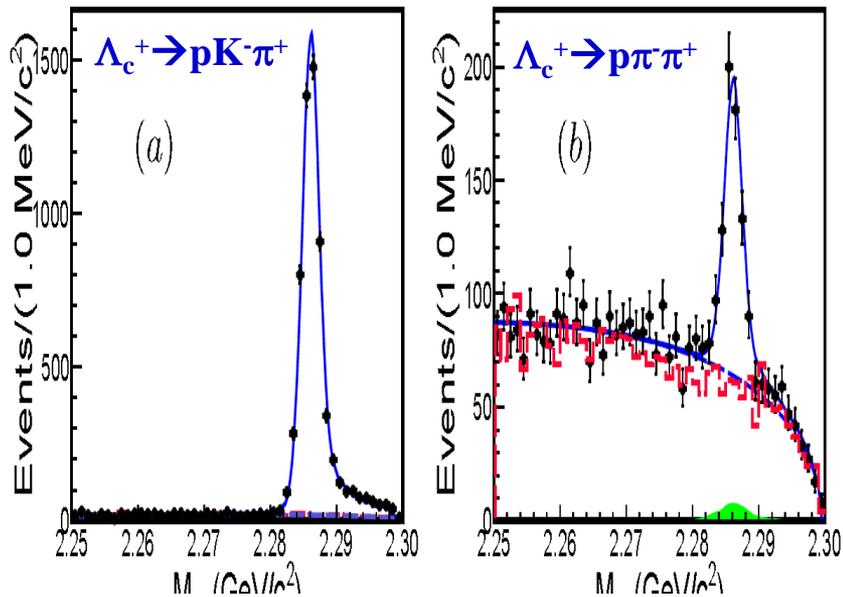
$$B[\Lambda_c^+ \rightarrow \Lambda e^+ \nu] = (3.63 \pm 0.38 \pm 0.20)\%$$

$$B[\Lambda_c^+ \rightarrow \Lambda \mu^+ \nu_\mu] = (3.49 \pm 0.46 \pm 0.26)\%$$

$$\Gamma[\Lambda_c^+ \rightarrow \Lambda \mu^+ \nu_\mu] / \Gamma[\Lambda_c^+ \rightarrow \Lambda e^+ \nu_e] = 0.96 \pm 0.16 \pm 0.04$$

# Measurements of SCS decays $\Lambda_c^+ \rightarrow pK^+K^- / \pi^+\pi^-$

PRL117(2016)232002

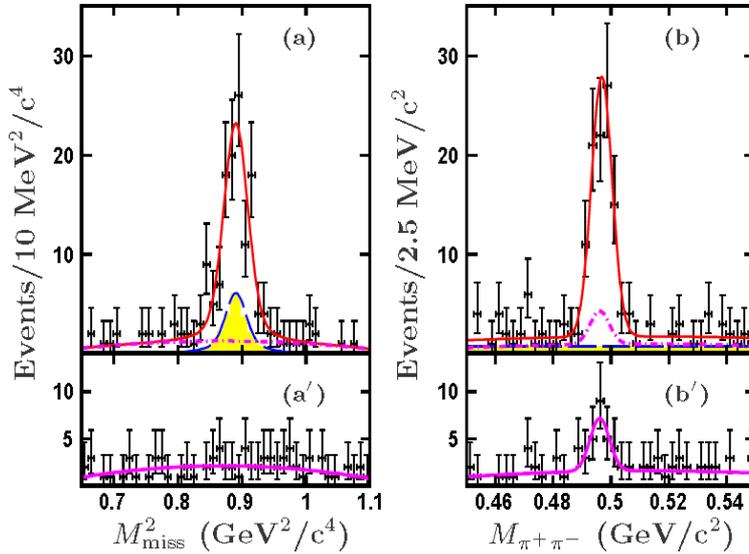


**These help to distinguish predictions from different theoretical models and understand contributions from factorizable effects**

Decay modes	$\mathcal{B}_{\text{mode}}/\mathcal{B}_{\text{ref.}}$	$\mathcal{B}_{\text{mode}}$	$\mathcal{B}(\text{PDG})$
$\Lambda_c^+ \rightarrow p\pi^+\pi^-$	$(6.70 \pm 0.48 \pm 0.25) \times 10^{-2}$	$(3.91 \pm 0.28 \pm 0.15 \pm 0.24) \times 10^{-3}$	$(3.5 \pm 2.0) \times 10^{-3}$
$\Lambda_c^+ \rightarrow p\phi$	$(1.81 \pm 0.33 \pm 0.13) \times 10^{-2}$	$(1.06 \pm 0.19 \pm 0.08 \pm 0.06) \times 10^{-3}$	$(8.2 \pm 2.7) \times 10^{-4}$
$\Lambda_c^+ \rightarrow pK^+K^-$ (non- $\phi$ )	$(9.36 \pm 2.22 \pm 0.71) \times 10^{-3}$	$(5.47 \pm 1.30 \pm 0.41 \pm 0.33) \times 10^{-4}$	$(3.5 \pm 1.7) \times 10^{-4}$

# Observation of $\Lambda_c^+ \rightarrow nK_S\pi^+$

PRL118(2017)112001



$$B[\Lambda_c^+ \rightarrow nK_S\pi^+] = (1.82 \pm 0.23 \pm 0.11)\%$$

$$\Gamma[\Lambda_c^+ \rightarrow n\bar{K}^0\pi^+] / \Gamma[\Lambda_c^+ \rightarrow pK^-\pi^+] = 0.62 \pm 0.09$$

$$\Gamma[\Lambda_c^+ \rightarrow n\bar{K}^0\pi^+] / \Gamma[\Lambda_c^+ \rightarrow p\bar{K}^0\pi^+] = 0.97 \pm 0.16$$

**First measurement of BF of  $\Lambda_c^+$  decay containing neutron**

$$\cos \delta = -0.24 \pm 0.08$$

$$|I^{(1)}| / |I^{(0)}| = 1.14 \pm 0.11$$

**Help to understand SU(3) and isospin symmetry and determine strong phase**

**Cai-Dian Lv et al, PRD93(2016)056008**

$\cos \delta$

$$= \frac{B(n\bar{K}^0\pi^+) - B(pK^-\pi^+)}{2\sqrt{B(p\bar{K}^0\pi^0)(B(pK^-\pi^+) + B(n\bar{K}^0\pi^+) - B(p\bar{K}^0\pi^0))}}$$

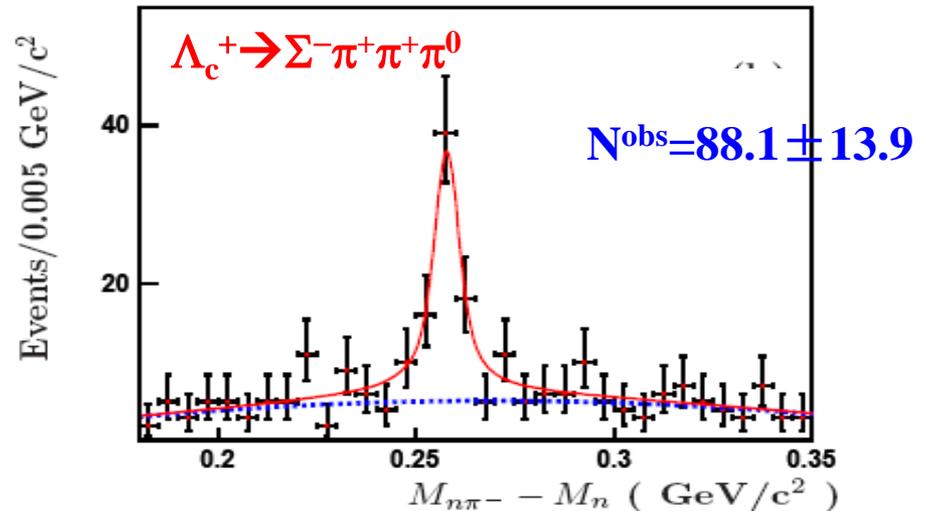
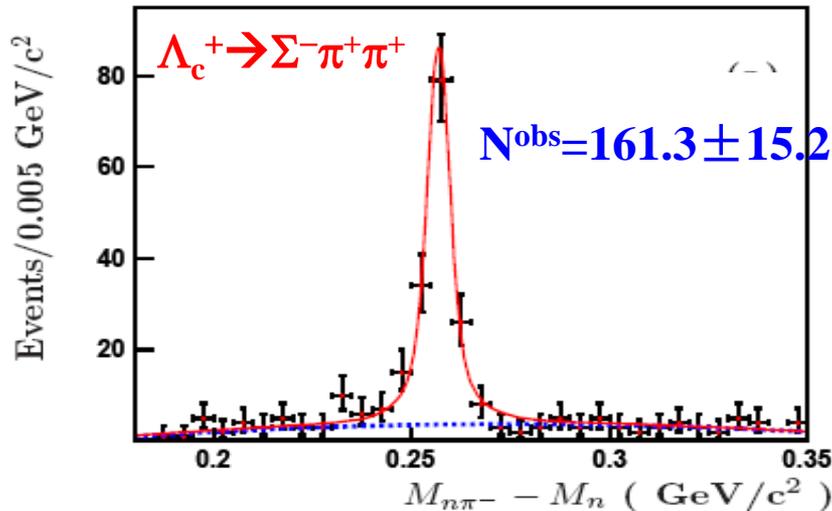
$$R_p = \frac{B(\Lambda_c \rightarrow p\bar{K}^0\pi^0)}{B(\Lambda_c \rightarrow pK^-\pi^+)}, \quad R_n = \frac{B(\Lambda_c \rightarrow n\bar{K}^0\pi^+)}{B(\Lambda_c \rightarrow pK^-\pi^+)}$$

involving a neutron. Under the isospin symmetry, its amplitude is related to those of the most favored proton modes  $\Lambda_c^+ \rightarrow pK^-\pi^+$  and  $\Lambda_c^+ \rightarrow p\bar{K}^0\pi^0$  as  $\mathcal{A}(n\bar{K}^0\pi^+) + \mathcal{A}(pK^-\pi^+) + \sqrt{2}\mathcal{A}(p\bar{K}^0\pi^0) = 0$ . Hence, precise measure-

[2,3]. In the three-body  $\Lambda_c^+$  decay to  $N\bar{K}\pi$ , the total decay amplitudes can be decomposed into two isospin amplitudes of the  $N\bar{K}$  system as isosinglet ( $I^{(0)}$ ) and isospin-one ( $I^{(1)}$ ). In the factorization limit, the color-allowed tree diagram, in which the  $\pi^+$  is emitted and the  $N\bar{K}$  is an isosinglet, dominates  $I^{(0)}$ , and  $I^{(1)}$  is expected to be small compared to  $I^{(0)}$  as it can only proceed through the color-suppressed tree diagrams. Though the factorization scheme is spoiled in

# Observation of $\Lambda_c^+ \rightarrow \Sigma^- \pi^+ \pi^+ \pi^0$

PLB772(2017)388



Preliminary results :

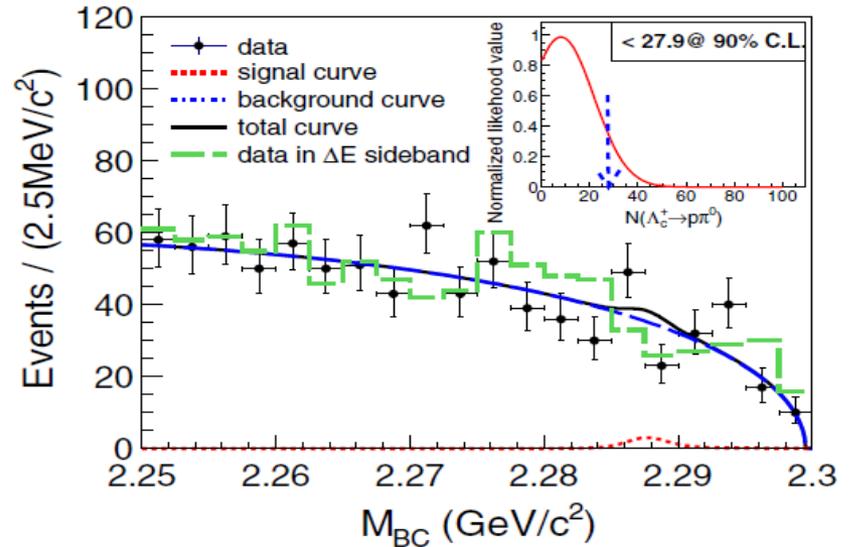
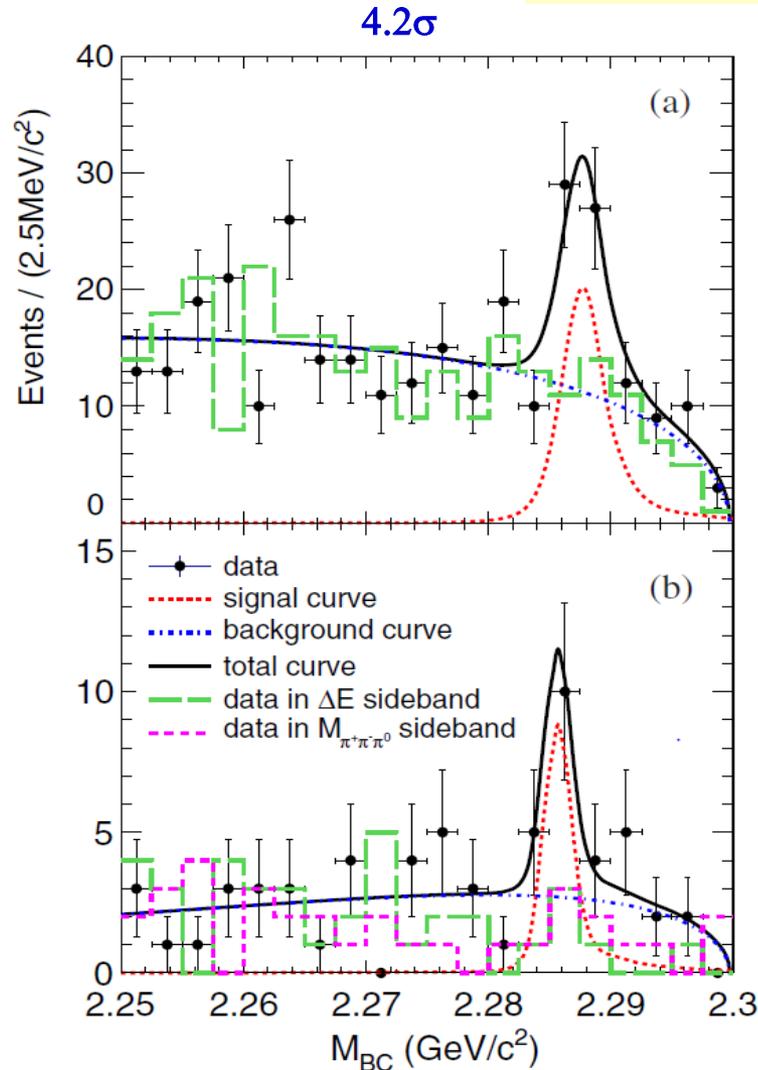
$$B[\Lambda_c^+ \rightarrow \Sigma^- \pi^+ \pi^+] = (1.81 \pm 0.17 \pm 0.09)\%$$

$$B[\Lambda_c^+ \rightarrow \Sigma^- \pi^+ \pi^+ \pi^0] = (2.11 \pm 0.33 \pm 0.14)\% \quad \text{[First observation]}$$

The previous one is consistent with and more precise than the PDG value of  $[\Lambda_c^+ \rightarrow \Sigma^- \pi^+ \pi^+] = (2.3 \pm 0.4)\%$ .

# Evidence of $\Lambda_c^+ \rightarrow p\eta$ and search of $\Lambda_c^+ \rightarrow p\pi^0$

PRD95(2017)111102(RC)



$B[\Lambda_c^+ \rightarrow p\pi^0] < 2.7 \times 10^{-4} \text{ 90\%CL}$

	$\Lambda_c^+ \rightarrow p\eta$	$\Lambda_c^+ \rightarrow p\pi^0$	$\frac{B_{\Lambda_c^+ \rightarrow p\pi^0}}{B_{\Lambda_c^+ \rightarrow p\eta}}$
BESIII	$1.24 \pm 0.29$	$< 0.27$	$< 0.24$
Sharma <i>et al.</i> [3]	$0.2^a(1.7^b)$	0.2	$1.0^a(0.1^b)$
Uppal <i>et al.</i> [4]	0.3	0.1–0.2	0.3–0.7
S. L. Chen <i>et al.</i> [12]	...	0.11–0.36 <sup>c</sup>	...
Cai-Dian Lü <i>et al.</i> [13]	...	0.45	...

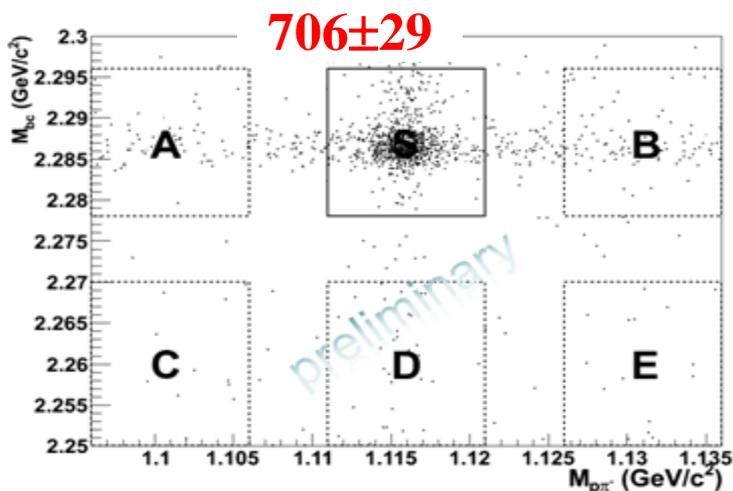
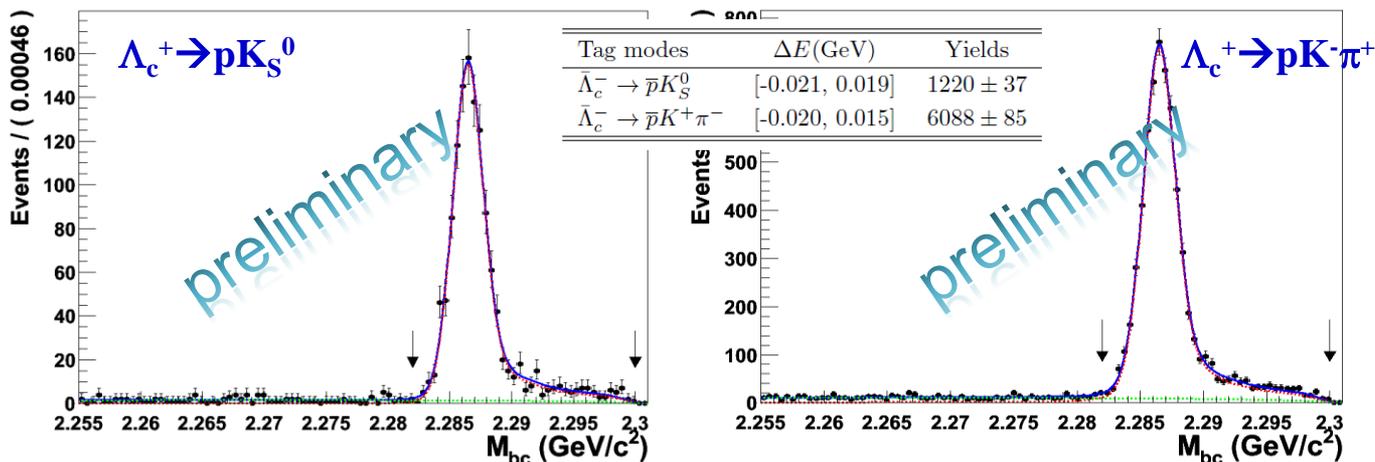
<sup>a</sup>Assumed to have a positive sign for the p-wave amplitude of  $\Lambda_c^+ \rightarrow \Xi^0 K^+$ .

<sup>b</sup>Assumed to have a negative sign for the p-wave amplitude of  $\Lambda_c^+ \rightarrow \Xi^0 K^+$ .

<sup>c</sup>Calculated relying on different values of parameters b and  $\alpha$ .

$B[\Lambda_c^+ \rightarrow p\eta] = (1.24 \pm 0.28 \pm 0.10) \times 10^{-3}$

# Inclusive decay $\Lambda_c^+ \rightarrow \Lambda X$



Help to explore the source of missing decays and search for new decay. Better input for charm baryon and B physics

$$N_{sig} = N_S - (N_A + N_B)/2 - r \cdot N_D + r \cdot (N_C + N_E)/2$$

$$\mathcal{B}(\Lambda_c^+ \rightarrow \Lambda + X) = (36.98 \pm 2.18)\% \quad \text{stat. only}$$

Agrees with PDG2015 value (35 ± 11)%,

$$\mathcal{A}_{CP} = \frac{\mathcal{B}(\Lambda_c^+ \rightarrow \Lambda + X) - \mathcal{B}(\bar{\Lambda}_c^- \rightarrow \bar{\Lambda} + X)}{\mathcal{B}(\Lambda_c^+ \rightarrow \Lambda + X) + \mathcal{B}(\bar{\Lambda}_c^- \rightarrow \bar{\Lambda} + X)}$$

Decay mode	Branching fraction(%)	$\mathcal{A}_{CP}$
$\Lambda_c^+ \rightarrow \Lambda + X$	$38.02 \pm 3.24$	$0.02 \pm 0.06$
$\bar{\Lambda}_c^- \rightarrow \bar{\Lambda} + X$	$36.70 \pm 3.04$	

# Summary

- In recent 10 years, rapid progress in charm hadron studies has been made with threshold data at CLEO-c and BESIII
- Studies of (semi-)leptonic D decays lead to improved decay constant  $f_{D^+}$  and form factor  $f_+^{D \rightarrow K(\pi)}(q^2)$ , which are important to test and calibrate LQCD calculations, CKM matrix element  $|V_{cs(d)}|$ , which are important to test the CKM matrix unitarity
- Studies of hadronic structure and quantum correction in  $\psi(3770) \rightarrow D^0 \bar{D}^0$  have been performed in wide range
- Significantly improved measurements of  $\Lambda_c^+$  decays with threshold data at 4.6 GeV have been made at BESIII
- Some preliminary results on  $D_s^+$  decays with  $3.19 \text{ fb}^{-1}$  data @ 4.178 GeV at BESIII are coming
- More results with improved precision (with more data) are expected

**Thank you!**

## Form factors in $D_s^+ \rightarrow K^{*0} e^+ \nu_e$

The differential decay rate for  $D_s^+ \rightarrow K^{*0} e^+ \nu_e$  can be expressed in terms of three helicity amplitudes ( $H_+(q^2)$ ,  $H_-(q^2)$  and  $H_0(q^2)$ )

$$\begin{aligned} \frac{d^5\Gamma}{dm_{K\pi} dq^2 d\cos\theta_K d\cos\theta_e d\chi} &= \frac{3}{8(4\pi)^4} G_F^2 |V_{cd}|^2 \frac{p_{K\pi} q^2}{M_{D_s}^2} \mathcal{B}(K^{*0} \rightarrow K^+ \pi^-) |\mathcal{BW}(m_{K\pi})|^2 \\ &\times [(1 + \cos\theta_e)^2 \sin^2\theta_K |H_+(q^2, m_{K\pi})|^2 \\ &+ (1 - \cos\theta_e)^2 \sin^2\theta_K |H_-(q^2, m_{K\pi})|^2 \\ &+ 4\sin^2\theta_e \cos^2\theta_K |H_0(q^2, m_{K\pi})|^2 \\ &+ 4\sin\theta_e (1 + \cos\theta_e) \sin\theta_K \cos\theta_K \cos\chi H_+(q^2, m_{K\pi}) H_0(q^2, m_{K\pi}) \\ &- 4\sin\theta_e (1 - \cos\theta_e) \sin\theta_K \cos\theta_K \cos\chi H_-(q^2, m_{K\pi}) H_0(q^2, m_{K\pi}) \\ &- 2\sin^2\theta_e \sin^2\theta_K \cos 2\chi H_+(q^2, m_{K\pi}) H_-(q^2, m_{K\pi})]. \end{aligned}$$

The helicity amplitudes of  $H_+(q^2)$ ,  $H_-(q^2)$  and  $H_0(q^2)$  take the form of

$$H_{\pm}(q^2) = (M_{D_s} + m_{K\pi}) A_1(q^2) \mp \frac{2M_{D_s} p_{K\pi}}{M_{D_s} + M_{K\pi}} V(q^2) \text{ and}$$

$$H_0(q^2) = \frac{1}{2m_{K\pi} q} [(M_{D_s}^2 - m_{K\pi}^2 - q^2)(M_{D_s} + m_{K\pi}) A_1(q^2) - \frac{4M_{D_s}^2 p_{K\pi}^2}{M_{D_s} + M_{K\pi}} A_2(q^2)],$$

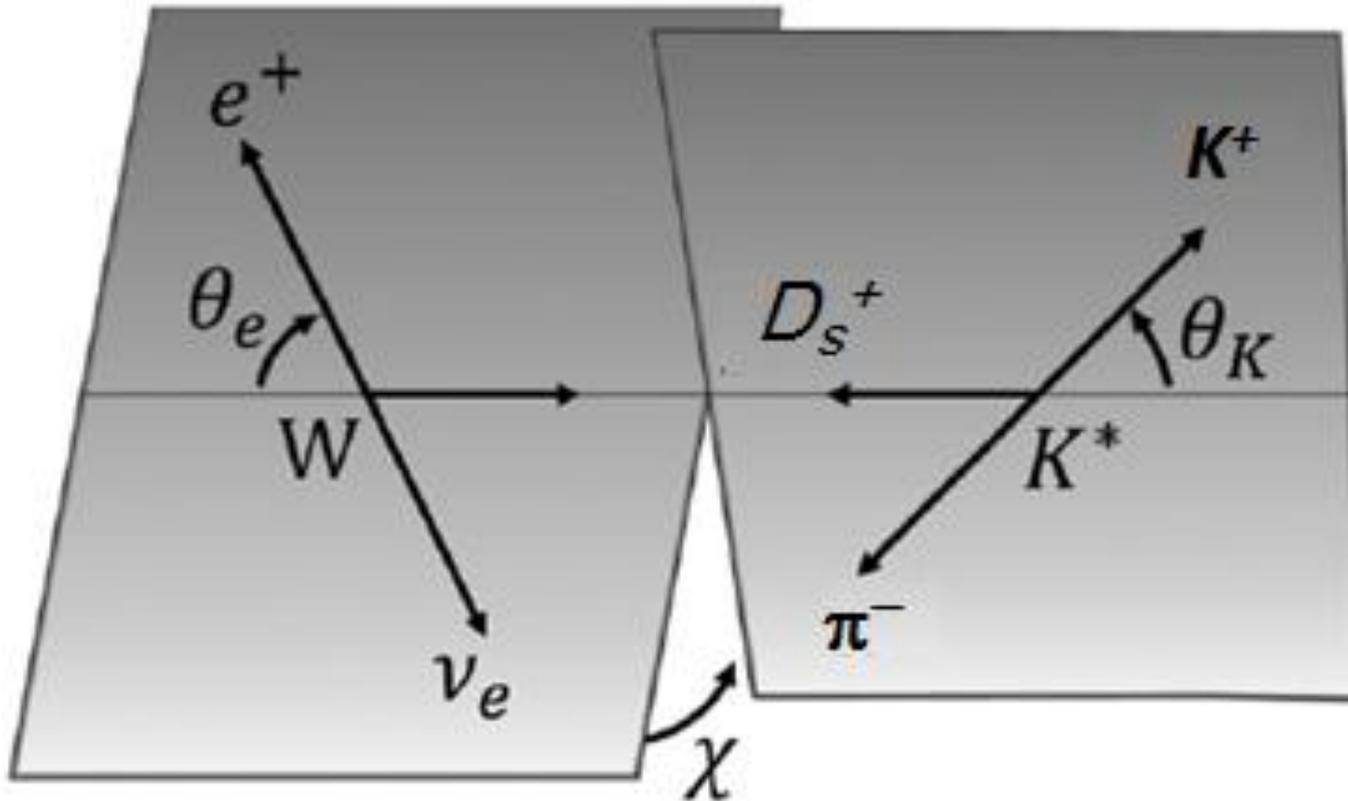
$$A_i(q^2) = \frac{A_i(0)}{1 - q^2/M_A^2} \text{ and } V(q^2) = \frac{V(0)}{1 - q^2/M_V^2}, \quad r_V = \frac{V(0)}{A_1(0)} \text{ and } r_2 = \frac{A_2(0)}{A_1(0)}.$$

The Breit-Wigner function of  $K^{*0}$  line shape takes the form as

$$\mathcal{BW}(M_{K\pi}) = \frac{\sqrt{m_0 \Gamma_0} (p/p_0)}{m_0^2 - m_{K\pi}^2 - i m_0 \Gamma(m_{K\pi})} \frac{B(p)}{B(p_0)}$$

$$\text{where } B(p) = \frac{1}{\sqrt{1 + R^2 p^2}} \text{ with } R = 3 \text{ GeV}^{-1} \text{ and } \Gamma(m_{K\pi}) = \Gamma_0 \left(\frac{p}{p_0}\right)^3 \frac{m_0}{m_{K\pi}} \left(\frac{B(p)}{B(p_0)}\right)^2.$$

## Definition of the angular variables<sup>[1,2]</sup>



### References:

- [1] BESIII Collaboration, M. Ablikim, *et al.*, Phys. Rev. D 94, 032001 (2016).
- [1] CLEO Collaboration, S. Dobbs, *et al.*, Phys. Rev. Lett. 110, 131802 (2013).