PDF’s at very high energies: Effects of electroweak radiation

Work in collaboration with
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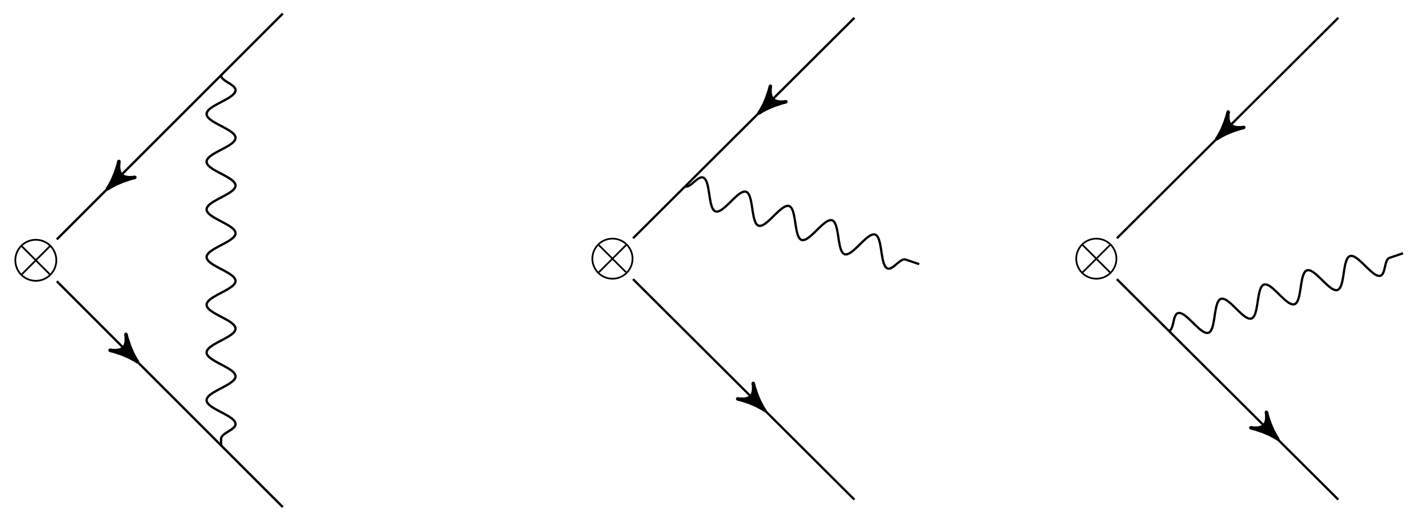
1. Quick overview of electroweak Sudakov logarithms
2. Relationship between DGLAP and ISR parton shower
3. DGLAP evolution in the full Standard model
4. Results

Exchange of massive gauge bosons gives rise double logarithmic sensitivity in both virtual and real diagrams

Consider example of qq production

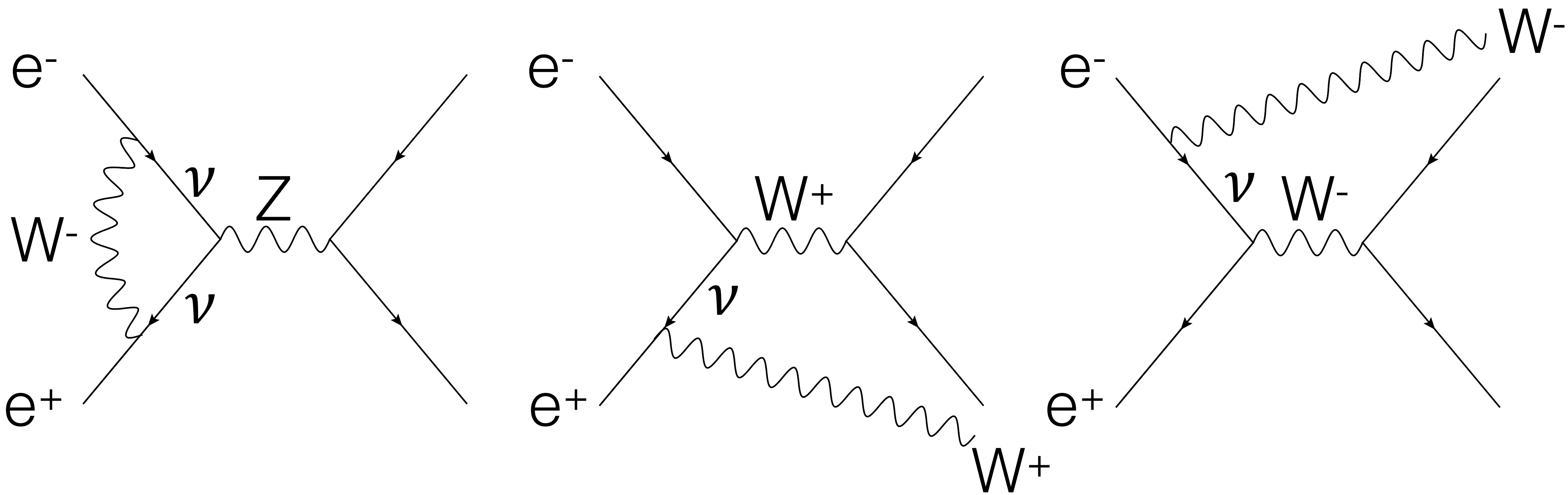


For massive W, IR "divergences" turn into $\log(m_W^2/s)$, and generally have two powers per power of alpha

Both virtual and real sensitive to $\log(m_W^2/s)$

Fully inclusive observables: real and virtual "divergences" cancel

Since no existing experiment collides SU(2) singlets, cancellation between virtual and real logs incomplete



Incomplete cancellation since the collider only collides electrons, not neutrinos.

For proton colliders, SU(2) breaking since $f_{u/p}(x,q) \neq f_{d/p}(x,q)$

Logarithmic effects in virtual corrections have been resummed in SCET quite a while ago

Chiu, Golf, Kelley, Manohar, ('08)

$$\mu = Q \quad \frac{\text{Full theory}}{\text{SCET}_{W,Z,\gamma}\ (M=0)} \quad \mathbf{C(Q,\mu)}$$

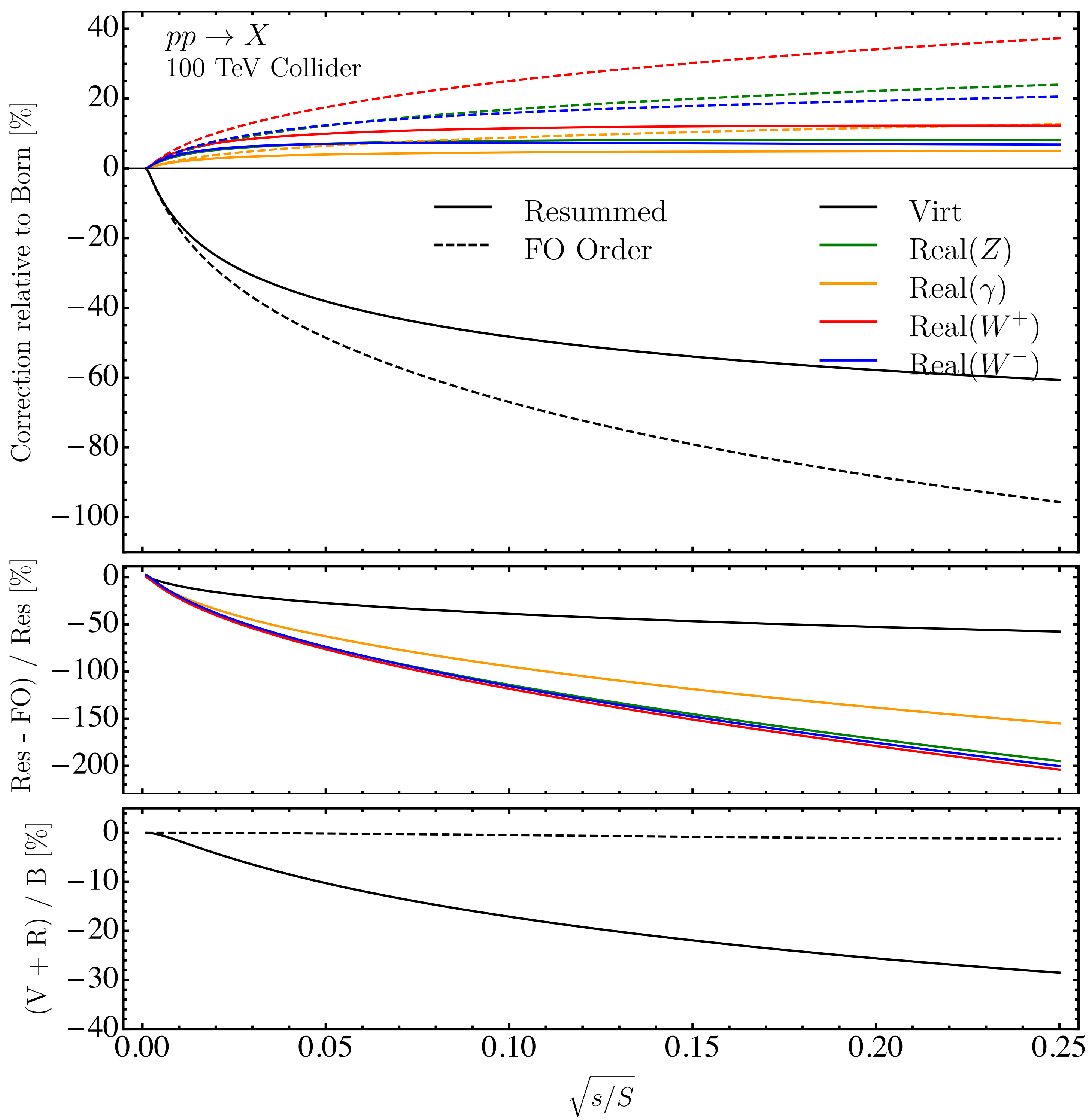
$$\Big\downarrow \ \gamma_{\text{SCET}}$$

$$\mu = m_V \quad \frac{\text{SCET}_{W,Z,\gamma}\ (M\neq 0)}{\text{SCET}_{\gamma}} \quad \mathbf{D(m_V,\mu)}$$

Problem is completely solved at NLL'
for any process

Logarithmic effects in real radiation were resummed recently, using analogy with parton shower

CWB, Ferland ('16)



To get understanding of Sudakov logarithms for general observable, need to have full parton shower

Full parton shower has many advantages over previous analytical results

- Can describe fully realistic observables
- Study multiple emissions of electroweak gauge bosons
- Study how double logarithmic sensitivity depends on inclusivity of observable

Since initial state effects are what cause non-cancellation of logs even for "inclusive" observables, study ISR shower first

As is common for a parton shower, one defines no-branching probability, and from that an emission probability

Starting from initial state particle at scale q with flavor f and momentum fraction x, no branching probability is given by

$$\Pi_i(t_1, t_2; x) = \frac{\Delta_i(t_1, t_0)}{\Delta_i(t_2, t_0)} \frac{f_{i/p}(x, t_2)}{f_{i/p}(x, t_1)}$$

Naive splitting probability given by

$$\tilde{P}_{ij}(z; x, t) = P_{ij}^R(z) \frac{f_j(x/z, t)}{f_i(x, t)}$$

Why does one have the ratio's of pdf's?

Initial state parton shower nothing but a “backward” version of DGLAP equations

Start from generic DGLAP equation

$$t\frac{\mathrm{d}}{\mathrm{d}t}f_{i/p}(x,t) = \frac{\alpha}{\pi}\left[P_i^V(t)f_{i/p}(x,t) + \sum_j C_{ij}^R \int \mathrm{d}z\, P_{ij}^R(z)\, f_{j/p}(x/z,t)\right]$$

Defining the Sudakov factor …

$$\Delta_i(t,t_0) = \exp\left\{-\frac{\alpha}{\pi}\int_{t_0}^{t}\frac{\mathrm{d}t'}{t'}P_i^V(t')\right\}$$

… can write DGLAP equation as

$$\Delta_i(t,t_0)\, t\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{f_{i/p}(x,t)}{\Delta_i(t,t_0)}\right] = \frac{\alpha}{\pi}\sum_j C_{ij}^R \int \mathrm{d}z\, P_{ij}^R(z) f_{j/p}(x/z,t)$$

Initial state parton shower nothing but a "backward" version of DGLAP equations

$$\Delta_i(t,t_0)\, t\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{f_{i/p}(x,t)}{\Delta_i(t,t_0)}\right] = \frac{\alpha}{\pi}\sum_j C_{ij}^R \int \mathrm{d}z\, P_{ij}^R(z) f_{j/p}(x/z,t)$$

"Solve" this by writing as integral equation

$$\frac{f_{i/p}(x,t)}{\Delta_i(t,t_0)} = \frac{f_{i/p}(x,t_0)}{\Delta_i(t_0,t_0)} + \frac{\alpha}{\pi}\int_{t_0}^{t}\frac{\mathrm{d}t'}{t'}\frac{1}{\Delta_i(t',t_0)}\sum_j C_{ij}^R\int \mathrm{d}z\, P_{ij}^R(z) f_{j/p}(x/z,t')$$

$$\Rightarrow 1 = \frac{\Delta_i(t,t_0)}{\Delta(t_0,t_0)}\frac{f_{i/p}(x,t_0)}{f_{i/p}(x,t)} + \frac{\alpha}{\pi}\int_{t_0}^{t}\frac{\mathrm{d}t'}{t'}\frac{\Delta_i(t,t_0)}{\Delta_i(t',t_0)}\sum_j C_{ij}^R\int \mathrm{d}z\, P_{ij}^R(z)\frac{f_{j/p}(x/z,t')}{f_{i/p}(x,t)}$$

$$= \Pi_i(t,t_0;x) + \frac{\alpha}{\pi}\int_{t_1}^{t_2}\frac{\mathrm{d}t'}{t'}\frac{\Delta_i(t,t_0)}{\Delta_i(t',t_0)}\sum_j C_{ij}^R\int \mathrm{d}z\, P_{ij}^R(z)\frac{\Delta_j(t',t_0)}{\Delta_j(t_0,t_0)}\frac{f_{j/p}(x/z,t_0)}{f_{i/p}(x,t)} + \ldots$$

$$= \Pi_i(t,t_0;x) + \frac{\alpha}{\pi}\int_{t_0}^{t}\frac{\mathrm{d}t'}{t'}\Pi_i(t,t';x)\sum_j C_{ij}^R\int \mathrm{d}z\, \tilde{P}_{ij}(z;x,t')\Pi_j(t',t_0;x/z)$$

By introducing the ratio's of pdf's (which is usually single log effect), ISR shower solves DGLAP exactly

To implement any ISR parton shower, one needs solution to DGLAP

Splitting functions and Sudakov factors depend on ratios of pdf’s at different scales, thus on solution to DGLAP

While these ratios are always single logarithmic, can be numerically large at low q since some pdf’s vanish at $q = m_W$

Solution to DGLAP already gives many interesting aspects of logarithmic effect in broken gauge theories

Will study DGLAP evolution of pdf’s for remainder of talk

Parton distribution functions are matrix elements of collinear bi-local operators

Parton distribution functions are matrix elements of collinear operators of field separated along the light-cone

$$f_i(x) = x \int \frac{dy}{2\pi}\, e^{-i\,2x\bar{n}\cdot p\, y} \langle p \big| \, \bar{\psi}^{(i)}(y)\, \bar{\not{n}}\, \psi^{(i)}(-y) \big| p \rangle$$

$$f_V(x) = \frac{2}{\bar{n}\cdot p} \int \frac{dy}{2\pi}\, e^{-i\,2x\bar{n}\cdot p\, y}\, \bar{n}_\mu \bar{n}^\nu \langle p \big| \, V^{\mu\lambda}(y) V_{\lambda\nu}(-y) \big| p \rangle \Big|_{\text{spin avg.}}$$

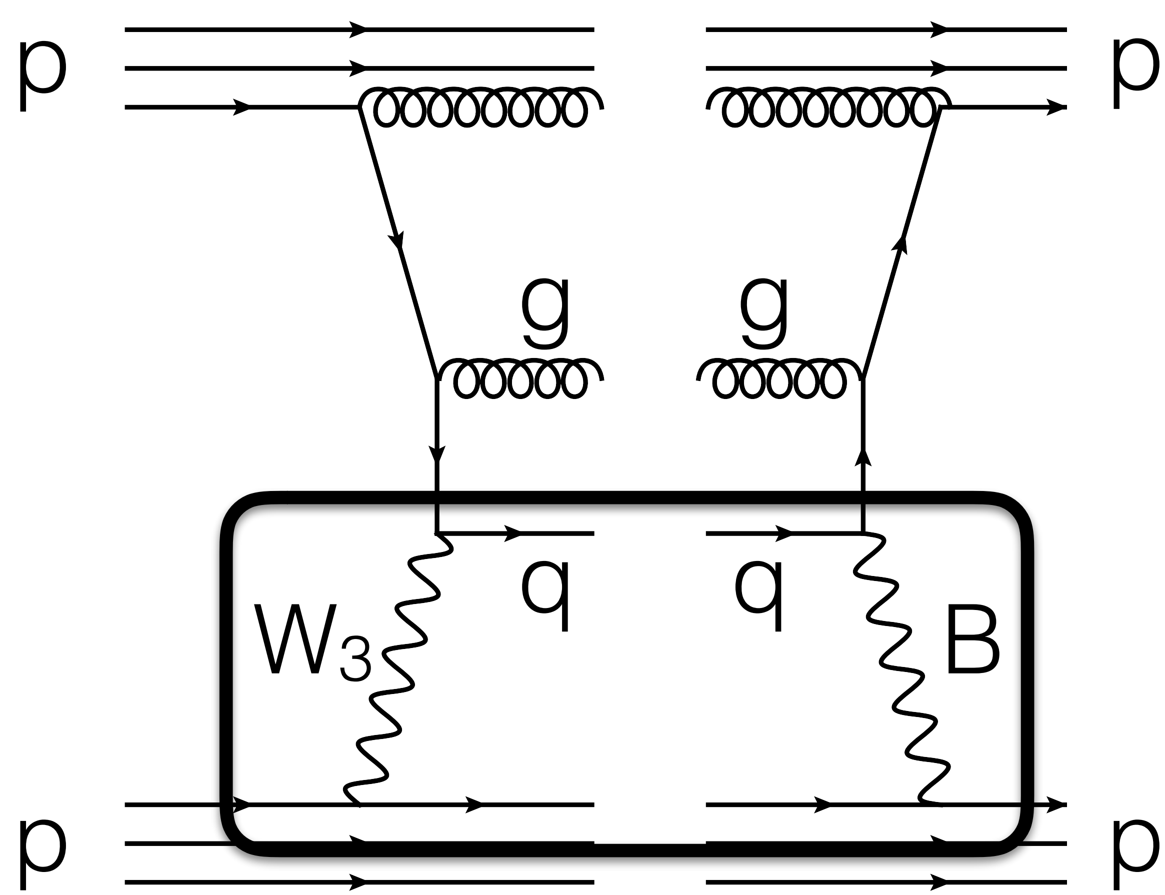
Diagramatically, can think of them as

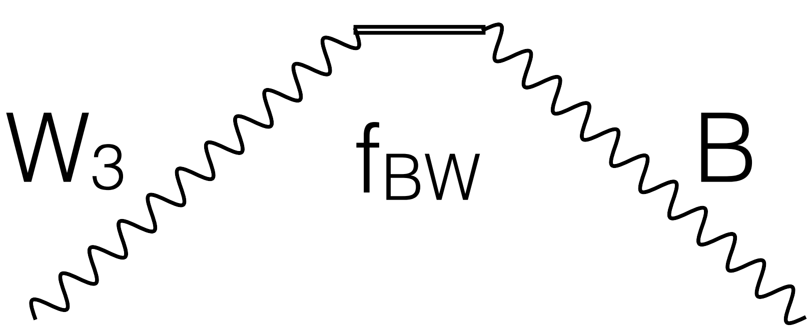
Once full SM evolution is considered, need pdf for every particle (including Higgs)

Besides these "standard" forward pdf's, one also needs to consider non-forward, mixed pdf's

$$f_{BW}(x) = \frac{1}{2}\left(\frac{2}{\bar{n}\cdot p}\int\frac{dy}{2\pi}\, e^{-i\,2x\bar{n}\cdot p\, y}\, \bar{n}_\mu \bar{n}^\nu \langle p|\, B^{\mu\lambda}(y) W^3_{\lambda\nu}(-y)|p\rangle\Big|_{\text{spin avg.}} + \text{h.c.}\right)$$

This pdf is required to describe mixed processes with Z or gamma in initial state

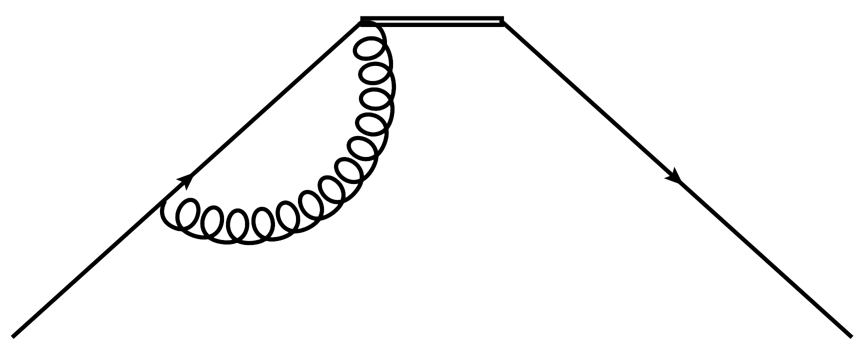


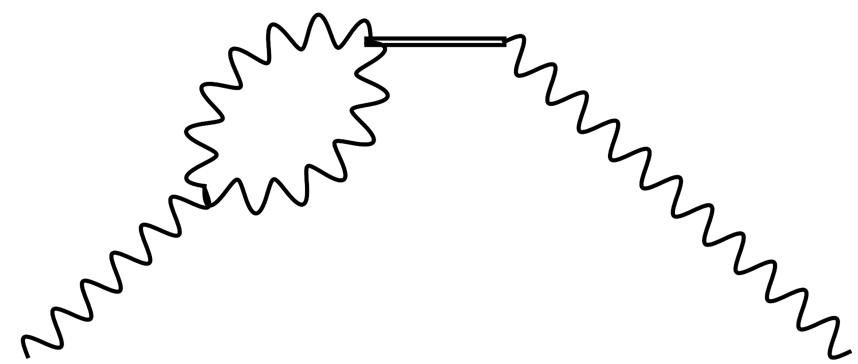


DGLAP equations are simply renormalization group equations of these operators

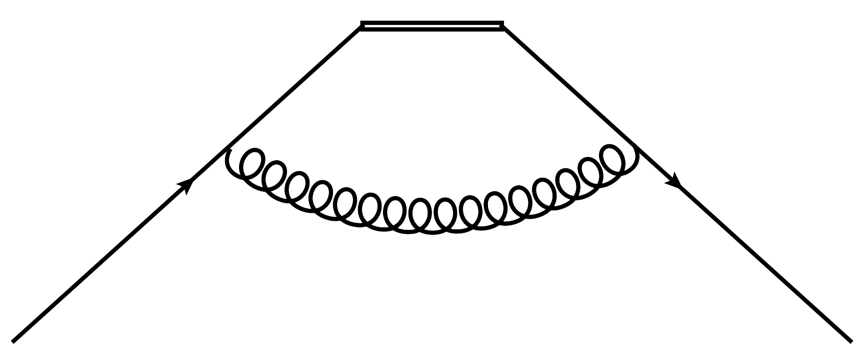
As for any operator in field theory depend on renormalization scale, and RGE is derived from divergent structure of loops

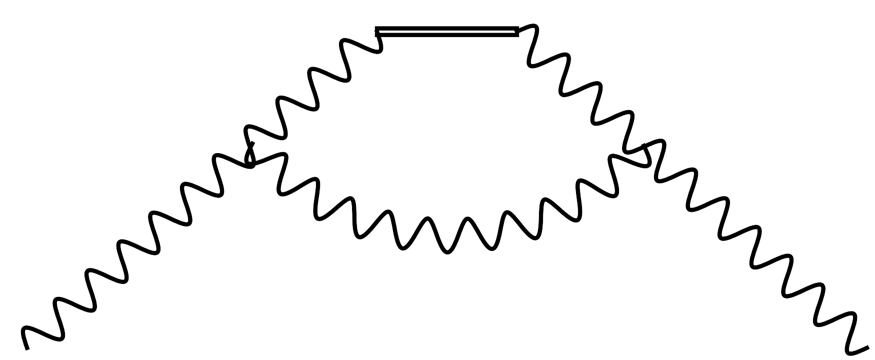
Virtual contributions have loop stay on same side of operator





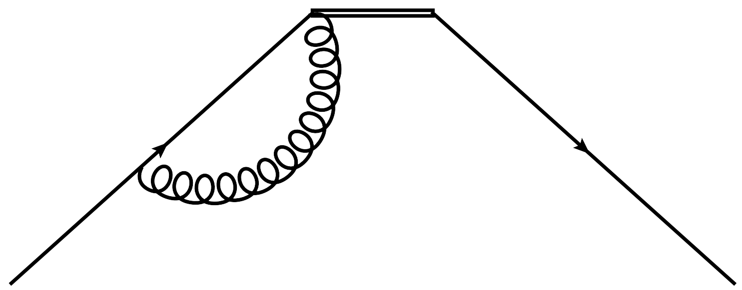
Real contributions have loop go from one side to other





For usual QCD evolution of PDF's solution to DGLAP is only single logarithmic

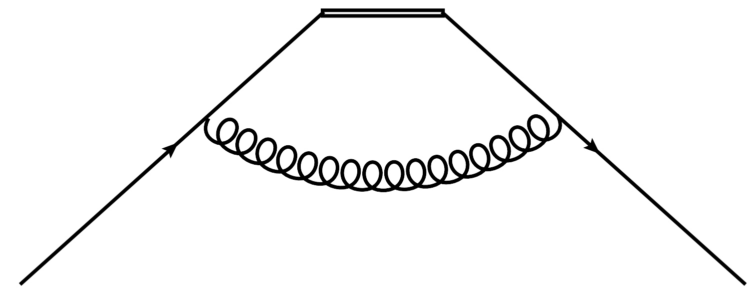
Consider evolution of quark pdf:



Virtual

$$t\frac{\mathrm{d}}{\mathrm{d}t}f_u(x,t) = \frac{\alpha\, C_F}{\pi} P_q^V(t) f_u(x,t)$$

$$P_q^V(t) = -\int_0^{z_{\max}(t)} \mathrm{d}z\, P_{qq}(z)$$



Real

$$t\frac{\mathrm{d}}{\mathrm{d}t}f_q(x,t) = \frac{\alpha\, C_F}{\pi}\int_x^{z_{\max}(t)} \mathrm{d}z\, P_{qq}(z) f_q(x/z,t)$$

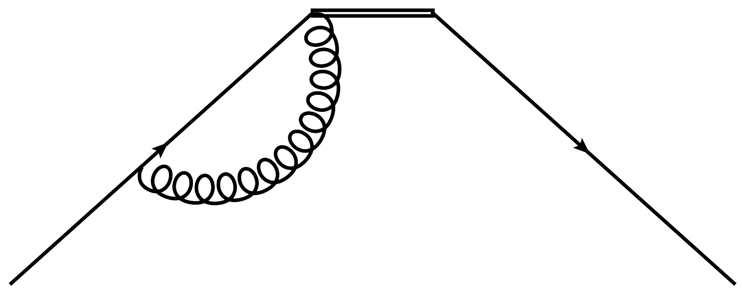
Combination

$$t\frac{\mathrm{d}}{\mathrm{d}t}f_q(x,t) = \frac{\alpha\, C_F}{\pi}\int_0^{z_{\max}(t)} \mathrm{d}z\, P_{qq}(z)\left[f_q(x/z,t) - f_q(x,t)\right] + \ldots$$

Logarithmic singularity as z→1 vanishes

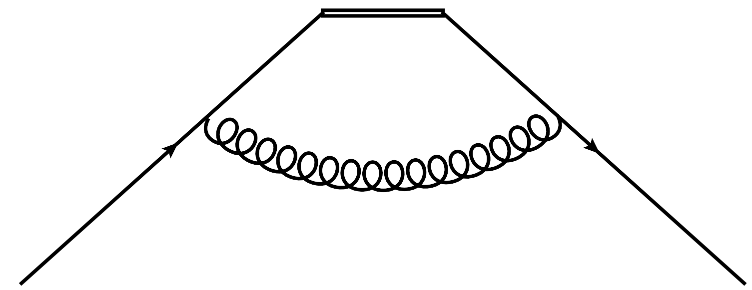
Since charged W bosons can change the flavor of the fermions, cancellation between virtual and real broken

Consider evolution of an up-type pdf:



Virtual

$$t\frac{\mathrm{d}}{\mathrm{d}t}f_u(x,t) = \frac{\alpha\, C_F}{\pi} P_q^V(t) f_u(x,t)$$



Real

$$t\frac{\mathrm{d}}{\mathrm{d}t}f_u(x,t) = \frac{\alpha\, C_F}{\pi}\int_0^{z_{\max}(t)} \mathrm{d}z\, P_{qq}(z) \times \left[\frac{2}{3}f_d(x/z,t) + \frac{1}{3}f_u(x/z,t)\right]$$

Since $f_u \neq f_d$ (the proton is not SU(2) singlet), real and virtual contributions do not cancel

Double logarithmic terms remain

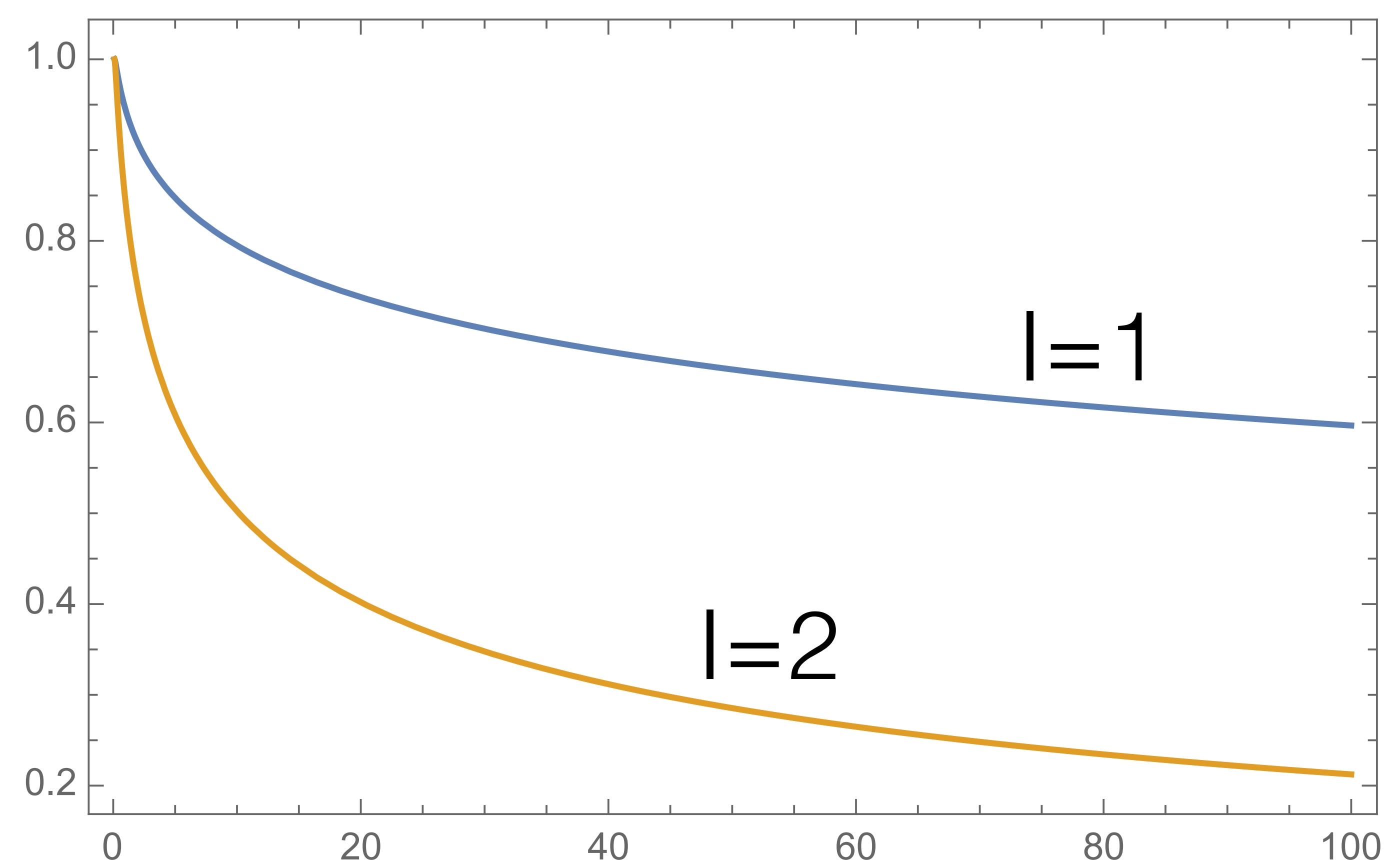
By studying the equations more carefully, one finds that the double logarithms restore electroweak symmetry breaking

By switching from a flavor basis to an isospin basis

$$f^0(x,t) = \frac{f_u(x,t) + f_d(x,t)}{2} \qquad f^1(x,t) = \frac{f_u(x,t) - f_d(x,t)}{2}$$

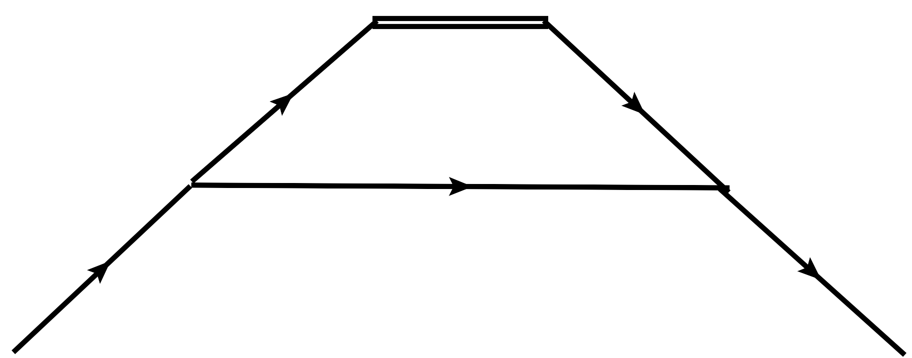
States with I ≠ 0 go double logarithmically to zero

$$f^I(x,t) \sim \exp\left[-\frac{I(I+1)}{2}\frac{\alpha_2}{\pi}\ln^2\frac{t}{m_V}\right]$$

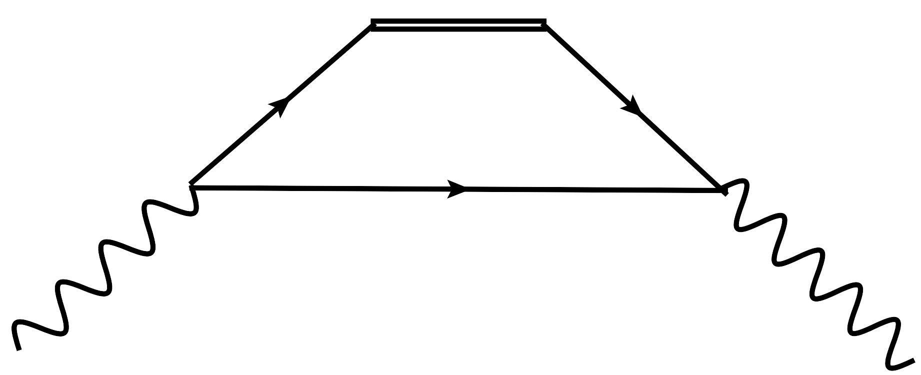


Before I show results, let me give the complete evolution of quark pdf as an example

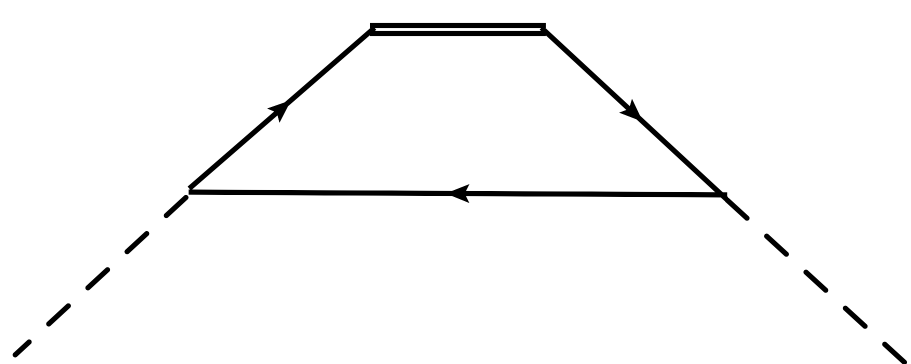
The possible diagrams one can draw are



q d/dq f = P ⊗ f



q d/dq f = P ⊗ V



q d/dq f = P ⊗ H

Before I show results, let me give the complete evolution of quark pdf as an example

U(1):

$$\left[\Delta_{f,1}\, q\frac{\partial}{\partial q}\frac{f_f}{\Delta_{f,1}}\right]_1 = \frac{\alpha_1}{\pi}Y_i^2\left[P^R_{ff,G}\otimes f_f + N_f P^R_{fV,G}\otimes f_B\right]$$

SU(2):

$$\left[\Delta_{f_L,2}\, q\frac{\partial}{\partial q}\frac{f_{u_L}}{\Delta_{f_L,2}}\right]_2 = \frac{\alpha_2}{\pi}\left\{P^R_{ff,G}\otimes\left[\frac{f_{d_L}}{2}+\frac{f_{u_L}}{4}\right]\right.$$
$$\left. + N_f P_{fV,G}\otimes\left[\frac{f_{W^+}}{2}+\frac{f_{W^3}}{4}\right]\right\}$$

SU(3):

$$\left[\Delta_{q,3}\, q\frac{\partial}{\partial q}\frac{f_q}{\Delta_{q,3}}\right]_3 = \frac{\alpha_3}{\pi}\left[C_F P^R_{ff,G}\otimes f_q + T_R P^R_{fV,G}\otimes f_g\right]$$

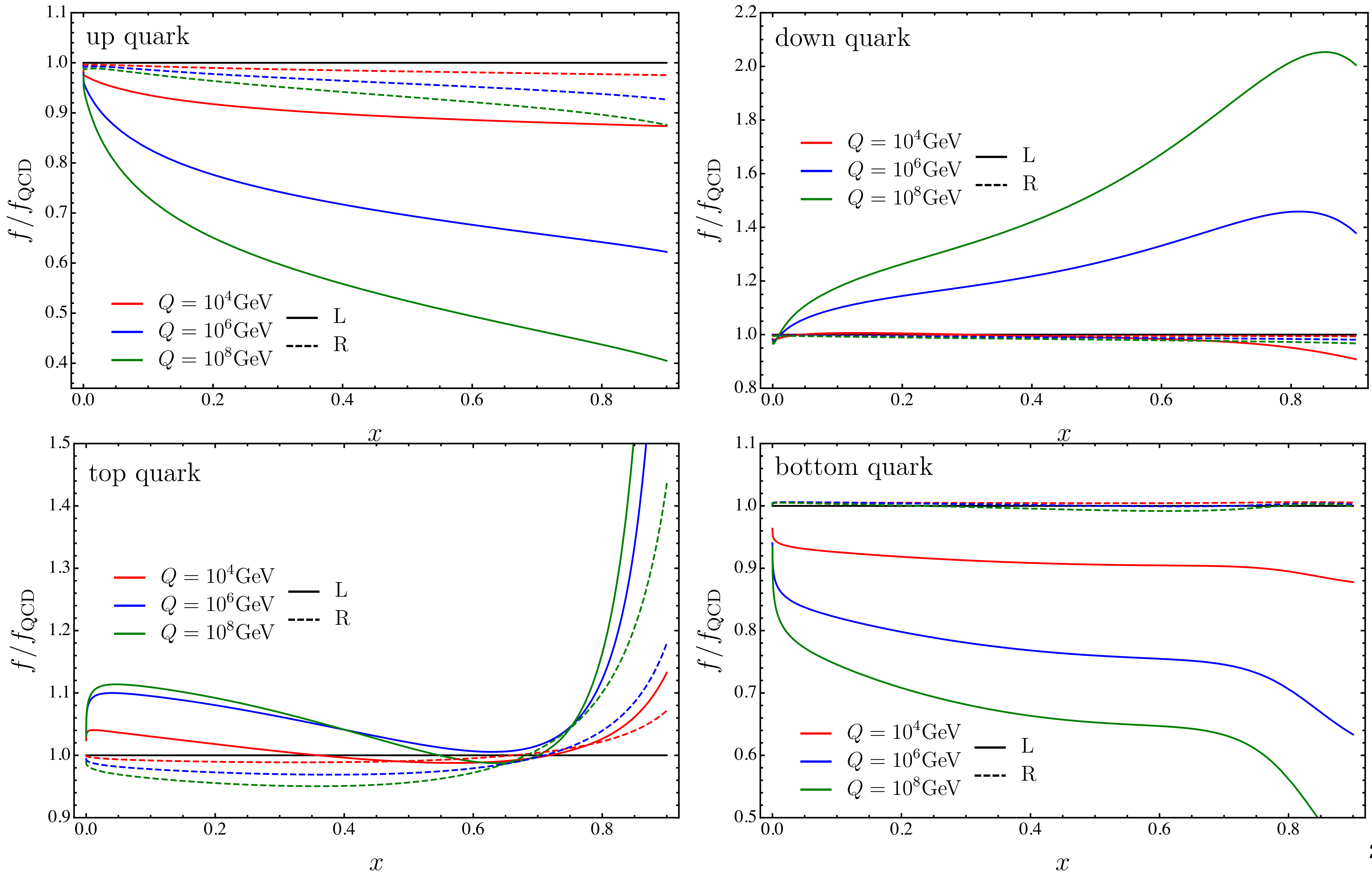
Yukawa:

$$\left[\Delta_{q^3_L,Y}\, q\frac{\partial}{\partial q}\frac{f_{t_L}}{\Delta_{q^3_L,Y}}\right]_Y = \frac{\alpha_Y}{\pi}\left\{P^R_{ff,Y}\otimes f_{t_R} + N_C P_{fH,Y}\otimes f_{\bar{H}^0}\right\}$$

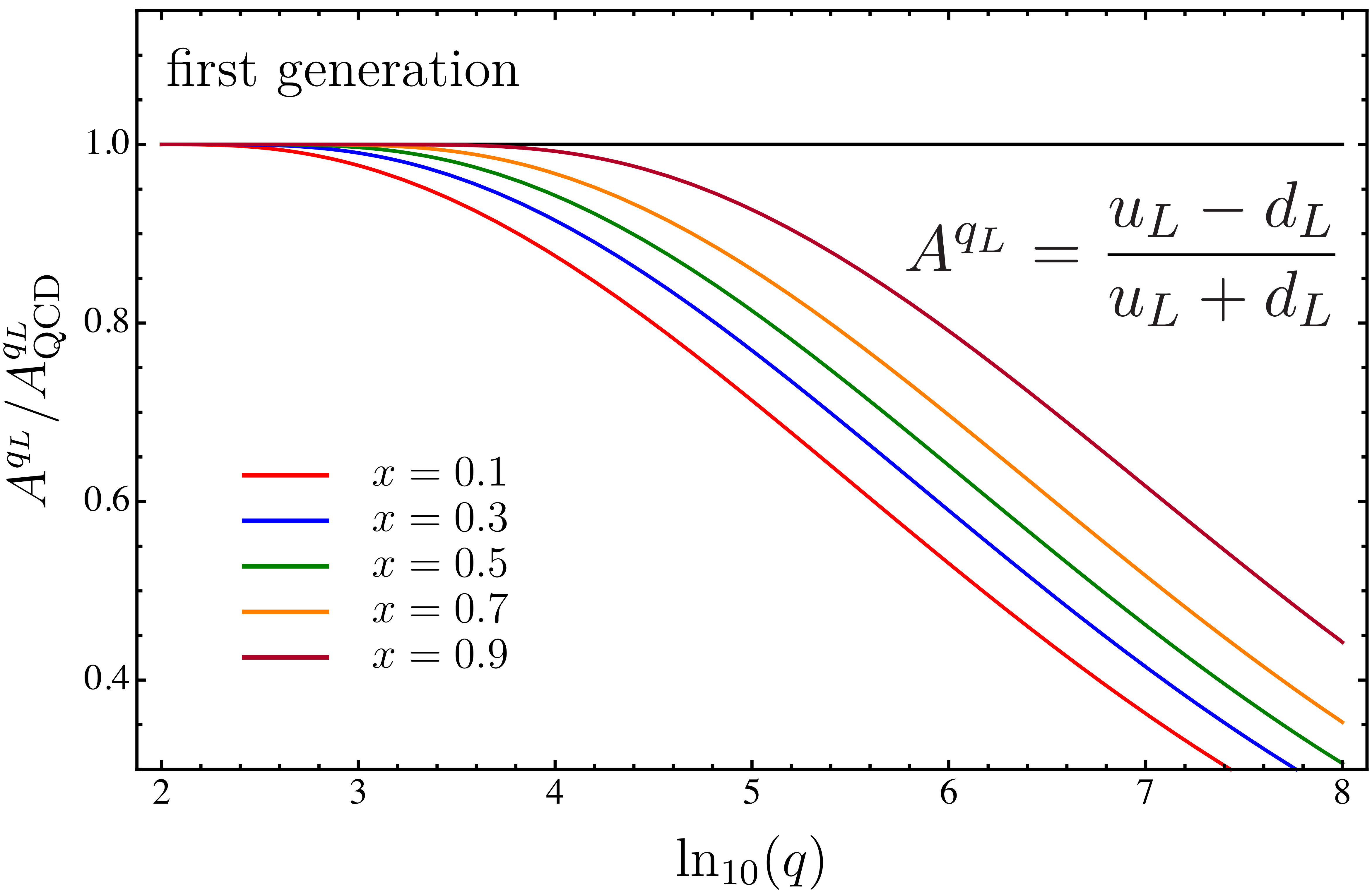
Mixed:

$$\left[q\frac{\partial}{\partial q}f_{f_u}\right]_M = \frac{\alpha_M}{\pi}\frac{Y_f}{2}N_f P^R_{fV,G}\otimes f_{BW}$$

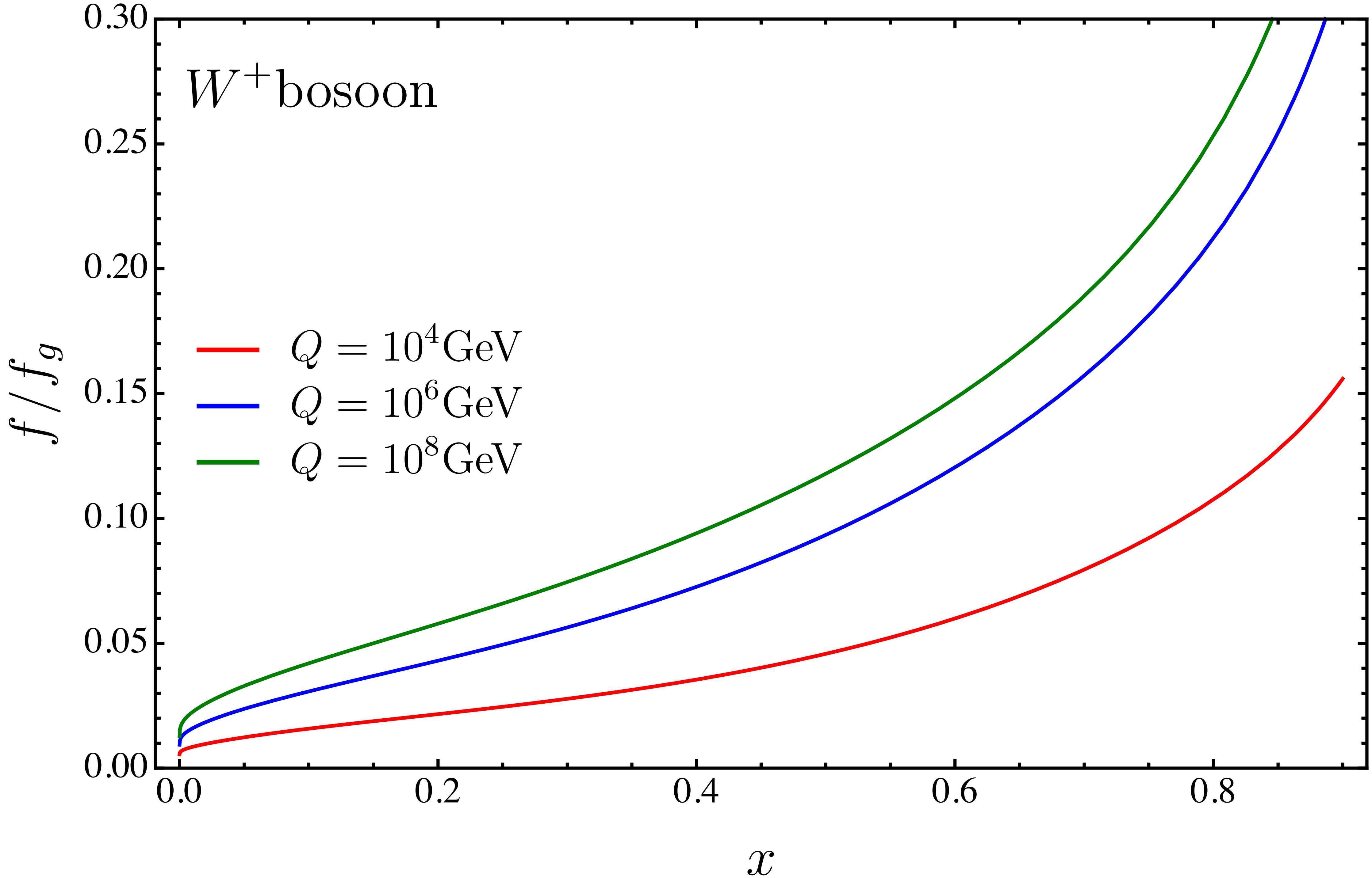
Quark pdf’s are modified from their value obtained with only QCD evolution included



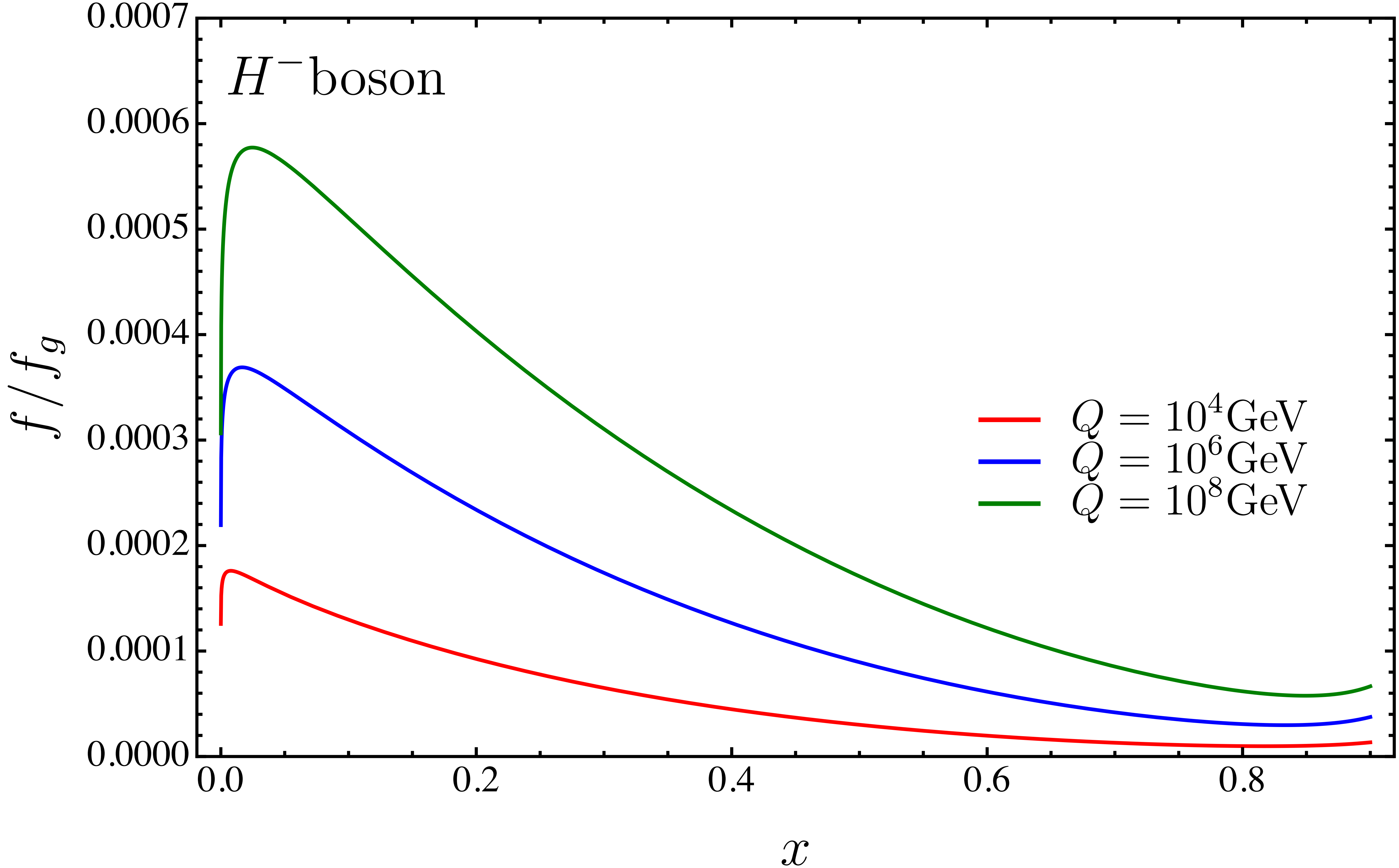
The isospin asymmetry is driven to zero, as predicted earlier



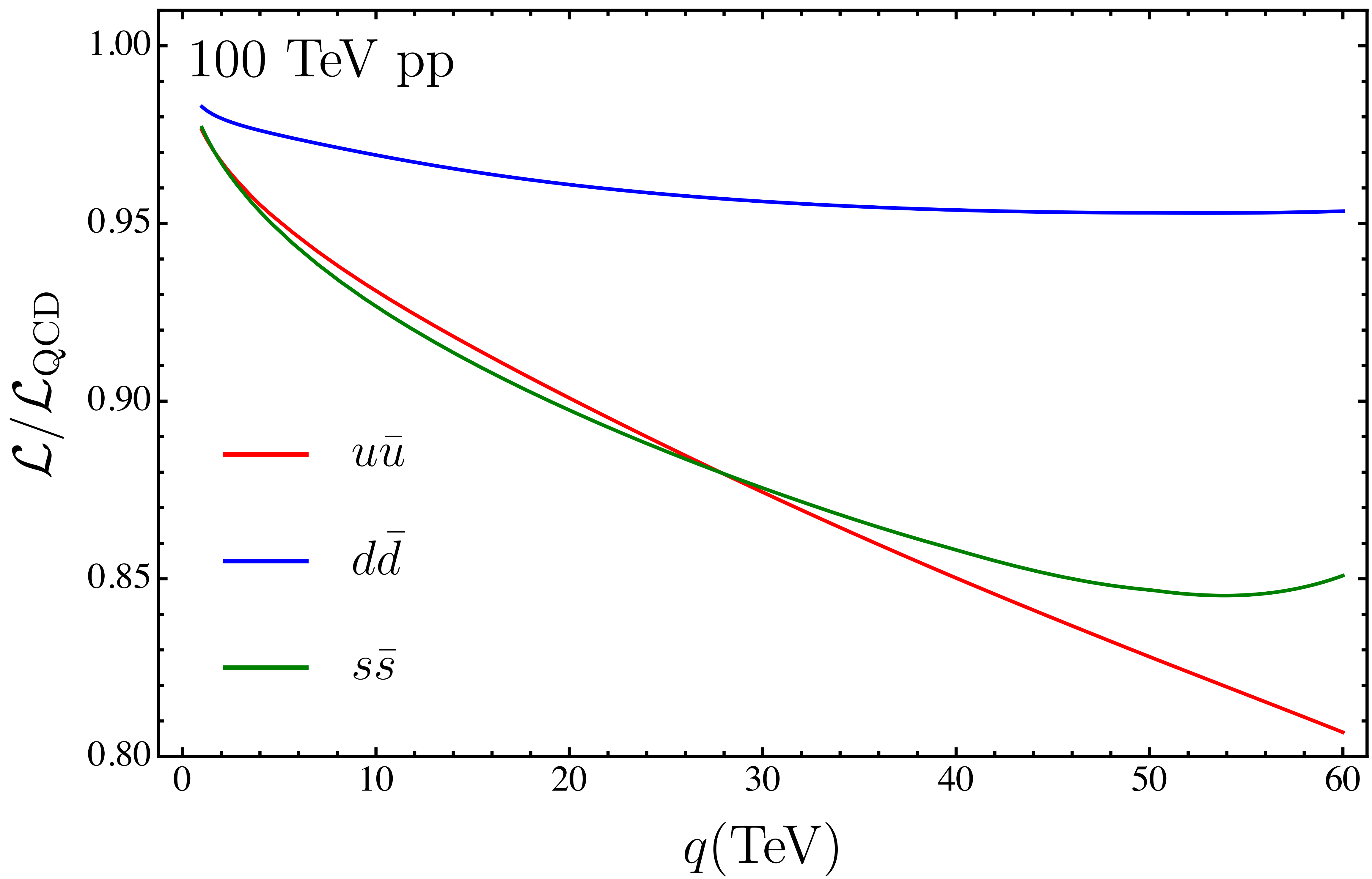
The probability of finding a vector boson in the proton becomes comparable to that to find a gluon



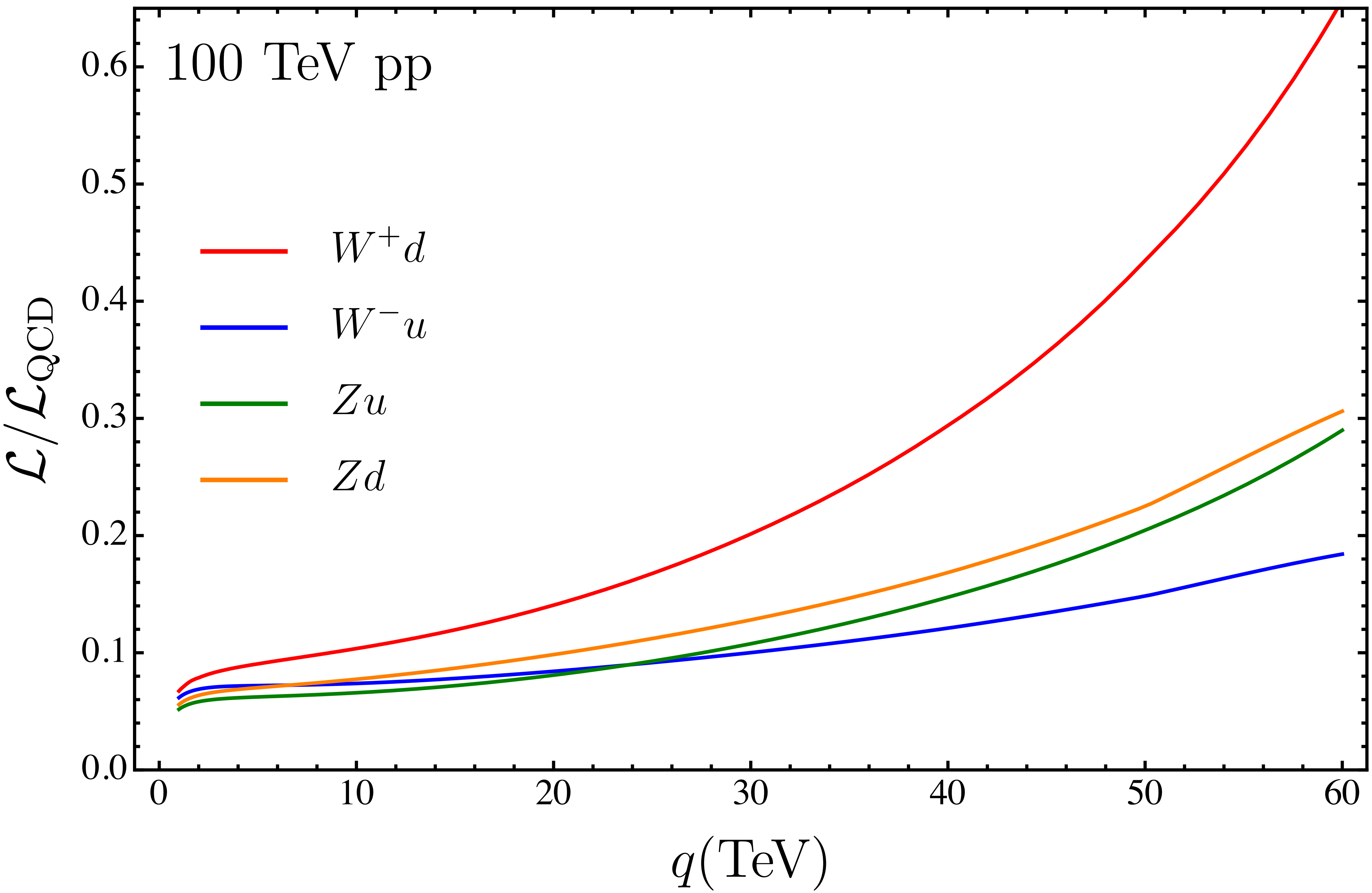
Even have probability of finding a Higgs bosons in proton, but much smaller than gluon



Luminosities at a 100TeV collider are changed noticeably from the values including only QCD running



Luminosities including vector bosons become a significant fraction of more standard luminosities



In conclusion, including all SM interactions can significantly alter DGLAP evolution

- "Regular" pdfs are affected significantly
- New luminosities are required
- DGLAP evolution is basis for ISR shower

Questions?