Manifestly Soft Gauge Invariant Formulation of *vNRQCD*

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SCET Workshop, Wayne State University, 2017

Introduction

- Non-relativistic QCD (NRQCD) [1] is an EFT for quarkonium which reproduces full QCD as an expansion in the relative velocity, $v << 1$, of the heavy quark (ψ) and antiquark (χ).
- The EFT has 3 scales quark mass $m >> p >> E >> \Lambda_{QCD}$.
- A systematic way to resum the logs in NRQCD was developed in a theory which is referred to as *vNRQCD* [2].
- The authors use velocity renormalization group (VRG) in-order to simultaneously sum the logs of $\mu_s = mv$ & $\mu_u = mv^2$.

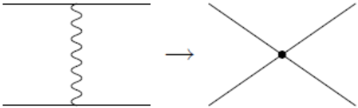
vNRQCD

- Modes in vNRQCD are classified as:

$$\text{potential } p^\mu \sim (mv^2, mv)$$
$$\text{soft } p_s^\mu \sim (mv, mv)$$
$$\text{ultrasoft (US) } p_{us}^\mu \sim (mv^2, mv^2).$$

- On-shell fermions have potential scaling.
- To have manifest power counting, we need two on-shell gluon fields: US gluons A_{us} and soft gluons A_s.
- Potential gluons are off-shell and integrated out.
- In this talk, I will discuss the soft sector of vNRQCD where a straight forward matching procedure leads to loss of manifest soft gauge invariance at the level of action.

Potentials in vNRQCD



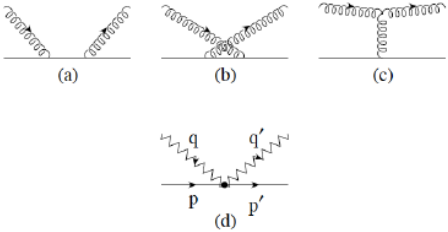
- Soft gluons are responsible for the VRG running of the leading order potential which comes from a t-channel potential gluon exchange between quark and anti-quark.

$$\mathcal{L}_{\rm p} = \frac{V_c}{|\vec{k}|^2}\psi^\dagger T^a \psi \chi^\dagger \overline{T}^a \chi + \ldots$$

- $\vec{k}$ is the momentum transfer and tree level matching gives

$$V_c(m) = 4\pi\alpha(m)$$

Old way of matching



- The leading order soft operator [4] (d) in the EFT comes from matching the Compton graphs (a, b, c) of full theory.
- The intermediate quark and gluon propagators are off-shell and are absorbed in the coefficients.

$$\mathcal{L}_s^{int} = 4\pi\alpha\psi^{\dagger}_{\vec{p}'}\Big(U^{(\sigma)}_{\mu\nu}[A^{\mu}_{\vec{q}}, A^{\nu}_{\vec{q}'}] + Y^{(\sigma)}[\bar{c}_{q'}, c_q]\Big)\psi_{\vec{p}}$$

- c_q are the soft ghosts and σ denotes order in v expansion.
- Energy conservation $q_0' = q_0$; momentum conservation $\vec{p}' - \vec{p} = \vec{q} - \vec{q}'$

Old way of matching

- The coefficients U's and Y's are complicated functions of momenta.

$$U_{ij}^{(0)} = -\delta_{ij}\frac{2q^0}{(\vec{\mathbf{p}}' - \vec{\mathbf{p}})^2},\; Y^{(0)} = -\frac{q_0}{(\vec{\mathbf{p}} - \vec{\mathbf{p}}')^2}$$

$$U_{i0}^{(1)} = -\frac{(\mathbf{p} + \mathbf{p}')^{\mathbf{i}}}{2m} - \frac{i[(\mathbf{p} - \mathbf{p}' + \mathbf{q}) \times \sigma]^{\mathbf{i}}}{2mq^0} + \frac{q^0(\mathbf{p} + \mathbf{p}')^{\mathbf{i}}}{2m(\vec{\mathbf{p}}' - \vec{\mathbf{p}})^2} + \frac{iq^0[(\mathbf{p} - \mathbf{p}') \times \sigma]^{\mathbf{i}}}{2m(\vec{\mathbf{p}}' - \vec{\mathbf{p}})^2}$$

$$U_{0i}^{(2)} = -\frac{\mathbf{p}\cdot\mathbf{q}(2\mathbf{p} + \mathbf{q})^{\mathbf{i}} + \mathbf{p}'\cdot\mathbf{q}(2\mathbf{p}' - \mathbf{q})^{\mathbf{i}}}{4m^2q_0^2} + i\frac{[\mathbf{q}\times\sigma]^{\mathbf{i}}(\mathbf{p} + \mathbf{p}')\cdot\mathbf{q}}{4m^2q_0^2} + \frac{(2\mathbf{p}' - 2\mathbf{p} - \mathbf{q})^{\mathbf{i}}}{4m^2}\left[\frac{1}{2} - \frac{i\sigma\cdot(\mathbf{p}\times\mathbf{p}')}{(\mathbf{p}' - \mathbf{p})^{\mathbf{2}}}\right] + \frac{\mathbf{p}'^{\mathbf{2}} - \mathbf{p}^{\mathbf{2}}}{4m^2(\mathbf{p}' - \mathbf{p})^{\mathbf{2}}}\left[(\mathbf{p} + \mathbf{p}')^{\mathbf{i}} + \mathbf{i}\sigma\cdot(\mathbf{p}\times\mathbf{p}')\right] + \ldots.$$

Soft Gauge Invariant Operator

- This formalism clearly gets more complicated as we go to higher order in v. Moreover its also not manifestly soft gauge invariant.
- Using the ideas recently proposed for Glauber modes [3] in SCET, introduce the soft gauge invariant building block

$$B_\mu = \frac{1}{g} S_v^\dagger(x,-\infty)(iD_\mu(x))S_v(x,-\infty)$$

- S_v is a soft Wilson line in the v^μ direction ($(1,\vec{0})$ in rest frame).

$$S_v(x,-\infty) = Pe^{ig\int_{-\infty}^{x} v\cdot A(\lambda v)d\lambda} = 1 + g\sum_{\mathbf{k}} \frac{v\cdot A_{\mathbf{k}}}{\mathbf{v}\cdot\mathbf{k} - i\epsilon} e^{i\mathbf{k}\cdot x} + \ldots$$

- $v \cdot B = 0$ as a consequence of the equations of motion (EOM) $(v\cdot D)S_v = 0$, obeyed by the soft Wilson line.

Soft Gauge Invariant Operator

- The B^μ operator starts off linear in the field and in momentum (label) space leads to (in the rest frame)

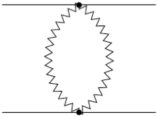
$$B^\mu_{\mathbf{k}} = A^\mu_{\mathbf{k}} - \mathbf{k}^\mu \frac{A^0_{\mathbf{k}}}{\mathbf{k}_0 - i\epsilon}$$

- Using the B operator, define a manifestly soft gauge invariant operator

$$\mathcal{O}_{ss} = -4\pi\alpha_s C \psi^\dagger_{\vec{\mathbf{p}}'} [B_{\mu,\mathbf{q}}, B^\mu_{-\mathbf{q}'}] \psi_{\vec{\mathbf{p}}}$$

- Despite its non-local nature at the scale mv, this operator is US gauge invariant, as the US gauge transformation acts locally, at x.

Soft Gauge Invariant Operator



- Like in the Glauber's case, we match on-shell by using EOM for soft gluons: $k^2 = 0$ and $k \cdot A_k = 0$.
- C is determined by matching the $\vec{A}$ polarizations of $\mathcal{O}_{ss}$ with full theory because $B_0 = 0$ due to EOM of Wilson lines.

$$C = U_{ij}^{(0)} = \delta_{ij} \frac{2q^0}{|\vec{p} - \vec{p}'|^2}$$

- On-shell, $\mathcal{O}_{ss}$ also reproduces the amplitude for A_0 polarizations.
- The 1-loop graph gives the running of the leading order potential in VRG [4]

$$\nu \frac{\partial}{\partial \nu} V_c = -2\beta_0 \alpha_s^2(m\nu)$$

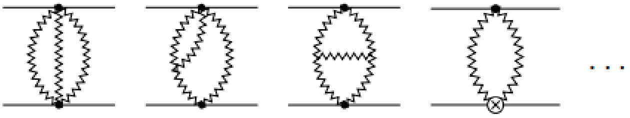
Soft Gauge Invariant Operator

- No longer need an operator coupling c_q to quarks. They only show up in soft gluon self energy diagrams.
- To any order σ, in v, the tree level contribution of soft graphs generates

$$\mathcal{O}_{ss}^{(\sigma)} = 4\pi\alpha\psi^\dagger\Big(U_{ij}^{(\sigma)}[B^i, B^j] + W_{ij}^{(\sigma)}\{B^i, B^j\}\Big)\psi$$

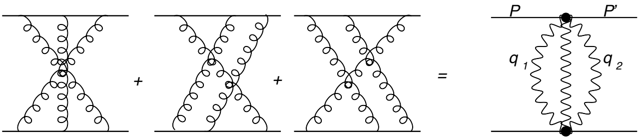
- Need just 1 coefficient U_{ij} instead of 4, at each order in v.

Choice of $i\epsilon$ prescription & 2 loop potential



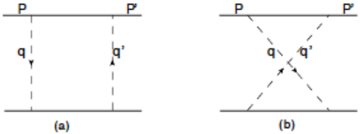
- For 2 loop running, don't need an operator like $\psi^\dagger BBB\psi$ unlike in the earlier formalism where there is a $\psi^\dagger AAA\psi$ operator .
- At any order in g_s, matching is required in our theory only if there are full theory diagrams with $\vec{A}_s$ gluons.
- The no. diagrams for 2 loop computation in the old and our formalism are the same but the Feynman rules in our formalism are much simpler.

Choice of $i\epsilon$ prescription & 2 loop potential



- The 2 loop potential gets a correction of $2\pi^2 C_A^2 C_F$ if $i\epsilon$ in eikonal propagators is properly taken into account.
- This term was missed in the original calculation [5] and was corrected later in [6].
- The problem of $i\epsilon$ in EFT is related to the fact that the operator B^μ is not globally gauge invariant, but transforms at infinity.

Choice of $i\epsilon$ prescription & 2 loop potential



- This issue is resolved by properly performing potential zero bin subtractions [7] of soft diagrams, which is necessary to avoid double counting the potential region of the theory.
- At 1 loop, the potential box diagram has pinch singularities and is non-vanishing while the cross box has no pinch singularities but vanishes.
- The 1 loop soft integral in EFT can correspond to either box or cross box integrand, by choosing a particular $i\epsilon$.

$$\frac{1}{k_0^2} \rightarrow \frac{1}{(k_0 - i\epsilon)(k_0 - i\epsilon)} \text{ or } \frac{1}{k_0^2} \rightarrow \frac{1}{(k_0 - i\epsilon)(k_0 + i\epsilon)}$$

Choice of iϵ prescription & 2 loop potential

- This pattern continues at higher orders as well because the diagrams which give rise to the running of the potential in full theory, are all free of pinch singularities.
- As long as the direction of soft Wilson lines does not lead to pinch singularities, we don't need to do potential zero bin subtractions.
- The iϵ prescription in soft loops does not correspond to a contribution of the potential region to the running of the V_c.
- In conclusion we can say that direction of soft Wilson lines does not matter as long we perform proper subtractions in the theory.

Conclusions

- By constructing a manifestly soft gauge invariant formulation of vNRQCD, we have reduced our operator basis significantly.
- For example we don't need a new operator to match 3 soft gluon emissions.
- Soft ghosts don't couple to potential fermions and the matching procedure is highly simplified.
- Our formalism also helps to elucidate some of the issue regarding the $i\epsilon$ prescription and the need for zero bins.

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THANK YOU