Monte-Carlo Top Quark Mass Calibrations: Update

Moritz Preisser

(University of Vienna / MIT)

Collaborators:

M. Butenschön, B. Dehnadi, A. Hoang, V. Mateu, I. Stewart

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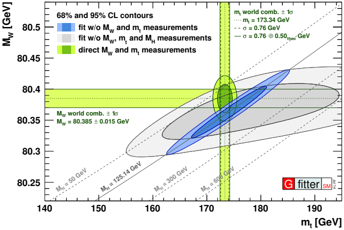
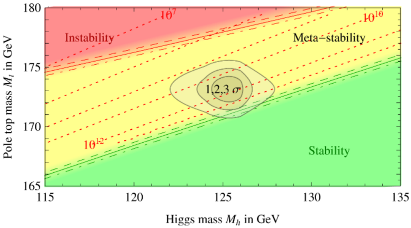
Outline

Motivation

- Top quark is the heaviest particle in the standard model
- Precise knowledge of top quark mass very important:
  - Electroweak precision tests of the SM
  - Stability of the SM vacuum
  - Top production important as background for BSM searches
  - ...



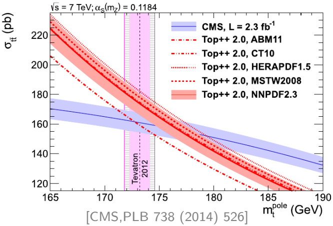
[GFitter,Phys. J. C (2014) 74]



[Degrassi et.al. 2012]

Top Mass Determinations

- Different methods available ($t\bar{t}$ production at hadron colliders)
  - total cross-section measurements
    $m_t^{\text{pole}} = 176.7^{+4.0}_{-3.4}$ GeV [K.A.Olive et.al. (PDG) 2014]
  - leptonic observables [Frixione, Mitov 2014; Kawabata 2016]
  - direct reconstruction measurements
  - alternative techniques [CMS 2016]
  - ...

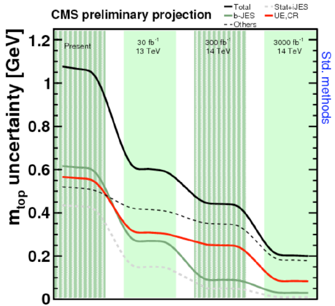


[CMS,PLB 738 (2014) 526]

- Direct reconstruction: standard technique
  - many individual measurements with uncertainty below 1 GeV → CMS combination reaches < 500 MeV
  - ➜ PDG quotes an uncertainty of ~ 900 MeV

$$m_{\text{t}} = 173.21 \pm 0.51(\text{stat}) \pm 0.71(\text{sys})\,\text{GeV}$$

  - relies on Monte Carlo (MC) generators e.g. PYTHIA to determine mass



Question: How should one interpret the "measured" top mass? What is m_t^{MC}?

Top Mass Determinations: MC Top Quark Mass

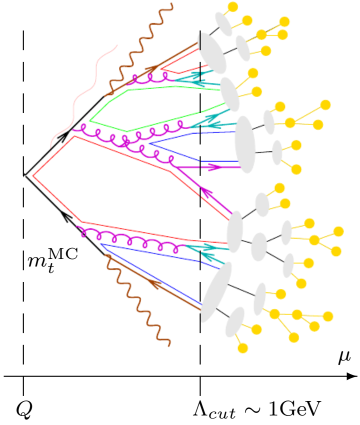
- Historically: all-order identification with m_t^{pole}
  - $\mathcal{O}(\Lambda_{\text{QCD}})$ renormalon ambiguity
  - Convergence issues when extracting the pole mass

- Steps in the MC:
  - Hard ME - $t\bar{t}$ production
  - Parton shower - evolution down to the shower cutoff $\Lambda_{cut} \sim 1\text{GeV}$
  - Hadronization - model dependent

➜ related to short distance mass: MSR mass

$$m_t^{\text{MC}} : m_t^{\text{MSR}}(R \simeq 1\text{GeV})$$

[Hoang, Stewart '08, Hoang '14]



[original picture D. Zeppenfeld]

Strategy

- **Strategy:** compare quark mass-sensitive hadron level QCD calculations with sample data from some MC
  - look into observables with strong kinematic mass sensitivity
  - get accurate hadron level QCD predictions ($\geq$NLO/NNLL) with full control over quark mass scheme dependence
  - fit QCD masses to different values of $m_t^{\rm MC}$

$$\boxed{m_t^{\rm MC} = m_t^{\rm MSR}(R \simeq 1{\rm GeV}) + \Delta_{\rm t,MC}^{\rm MSR}({\rm R} \simeq 1{\rm GeV})}$$

$$\boxed{m_t^{\rm MC} = m_t^{\rm pole} + \Delta_{t,{\rm MC}}^{\rm pole}} \qquad \Delta_{t,{\rm MC}} \simeq \mathcal{O}(1{\rm GeV})$$

Uncertainties we address in our e^+e^- study

- ▷ perturbative uncertainty
- ▷ scale uncertainties
- ▷ electroweak effects
- ▷ strong coupling α_s
- ▷ non-perturbative parameters

Additional pp systematics

- ▷ PS + UE
- ▷ color reconnection
- ▷ intrinsic uncertainty

Kinematic Situation & Massive Event Shapes

- *Simplification 1:* Look into **boosted tops** (c.o.m. energy $Q \gg m_t \sim$ high p_T)
- *Simplification 2:* Consider $e^+e^- \to t\bar{t} \to$ hadrons
  for progress on pp theory, see Aditya's talk
- *Observables:* (stable case) $\hat{m} = m/Q$
  - Jet Masses e.g. HJM

    $$\rho^{\text{peak}} = \hat{m}^2 + \mathcal{O}(\frac{\Lambda_{\text{QCD}}}{m})$$

  - 2-jettiness

    $$\tau_2^{\text{peak}} = 1 - \sqrt{1 - 4\hat{m}^2} + \mathcal{O}(\frac{\Lambda_{\text{QCD}}}{m})$$

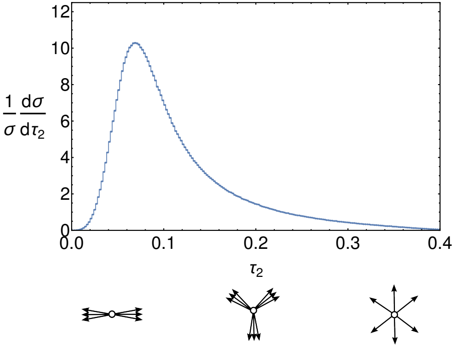
    $$\simeq 2\,\hat{m}^2 + \mathcal{O}(\hat{m}^4, \frac{\Lambda_{\text{QCD}}}{m})$$

  - C-jettiness: mass sensitive C-parameter

    $$C_{\text{J}}^{\text{peak}} = 12\hat{m}^2(1 - \hat{m}^2) + \mathcal{O}(\frac{\Lambda_{\text{QCD}}}{m})$$

    $$\simeq 12\,\hat{m}^2 + \mathcal{O}(\hat{m}^4, \frac{\Lambda_{\text{QCD}}}{m})$$

peak position is highly mass sensitive



First: focus on 2-jettiness

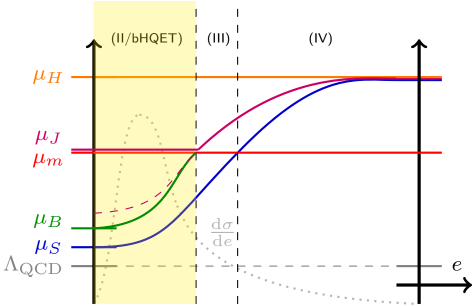
Theory Description - EFT treatment

- Boosted top jets: bHQET + SCET

[Fleming, Hoang, Mantry, Stewart 2007]

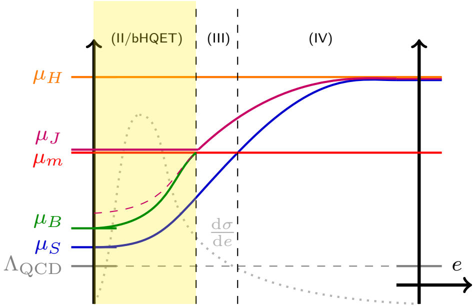
$$n_f = n_l + 1$$

$$\frac{d\sigma^{\text{bHQET}}}{d\tau} = Q\, H(Q, m, \mu_H) U_H^{(n_f)}(Q, \mu_H, \mu_m) H_m^{(n_f)}(Q, \mu_m) U_m^{(n_l)}(Q, m, \mu_m, \mu_B)$$
$$\times \int ds\, d\ell\; B_e^{(n_l)}(s, m, \mu_B) U_S^{(n_l)}(\ell, \mu_B, \mu_S) S_e^{(n_l)}\left(Q(\tau - \tau_{\min}) - \frac{s}{Q} - \ell, \mu_S\right)$$



Theory Description - EFT treatment

- Developments:
  - VFNS for final state jets (with massive quarks) [Gritschacher, Hoang, Jemos, Mateu, Pietrulewicz '13 '14]
  - MSR mass & R-evolution [Hoang, Jain, Scimemi, Stewart 2010]



- Non-perturbative power-corrections are included via a shape function

  [Korchemsky, Sterman 1999]
  [Hoang, Stewart 2007]
  [Ligeti, Stewart, Tackmann 2008]

$$\frac{d\sigma}{d\tau} = \frac{d\sigma^{\rm part}}{d\tau} \otimes F_{\rm mod}(\Omega_1, \Omega_2, \ldots)$$

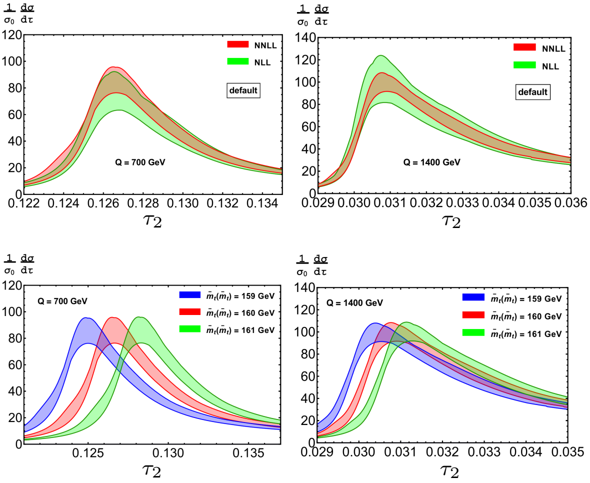
- Gap-scheme

NNLL + NLO
+ non-singular + hadronization
+ renormalon-subtraction
+ top quark decay

Convergence, Mass Sensitivity

- $\frac{d\sigma}{d\tau} = f(m_t^{\mathrm{MSR}}, \alpha_s(m_Z), \Omega_1, \Omega_2, \ldots, \mu_H, \mu_J, \mu_S, \mu_M, R, \Gamma_t)$

  any scheme non-perturbative renorm. scales finite lifetime



- Good convergence
- Reduction of scale uncertainty (NLL to NNLL)
- Control over whole distribution
- Higher mass sensitivity for lower Q
- Finite lifetime effects included
- Dependence on non-perturbative parameters

Preparing the Fits

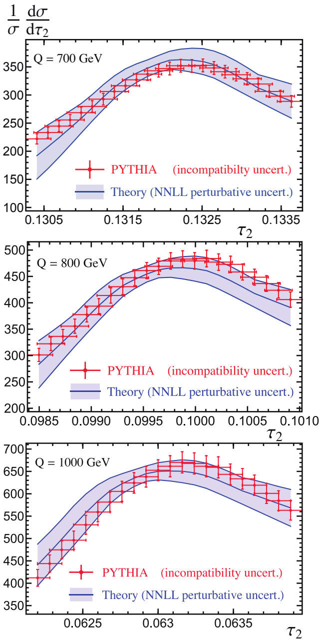
- $\frac{d\sigma}{d\tau} = f(m_t^{\mathrm{MSR}}, \alpha_s(m_Z), \Omega_1, \Omega_2, \ldots, \mu_H, \mu_J, \mu_S, \mu_M, R, \Gamma_t)$

  any scheme non-perturbative renorm. scales finite lifetime

- Generating `PYTHIA` 8.205 Samples:
  at different energies: $Q = 600, 700, 800, ..., 1400$ GeV
  - masses: $m_t^{\mathrm{MC}} = 170, 171, 172, 173, 174, 175$ GeV
  - width: $\Gamma_t = 1.4$ GeV
  - tune: 7 (Monash)
  - Statistics: 10^7 events for each set of parameters

- Feed MC data into **Fitting Procedure:** all ingredients are there
  Fit parameters: m_t^{MSR}, $\alpha_s(m_Z)$, $\Omega_1, \Omega_2, \ldots$
  - standard fit based on χ^2 minimization
  - analysis with 500 sets of profiles (τ_2 dependent renorm. scales) for the each MC sample
  - different Q-sets: 7 sets with energies between 600 - 1400 GeV
    different n-sets: 3 choices for fitranges - (xx/yy)% of maximum peak height

Fit Results: Pythia vs. Theory



- Good agreement of PYTHIA 8.205 with $N^2LL + NLO$ QCD description in peak region

- Perturbative uncertainties on theory side estimated via scale variations (profiles)

- MC incompatibility uncertainty estimate intrinsic difference between MC & theory via difference between different Q- & n-sets

Convergence & Stability: MSR vs Pole Mass

500 profiles; $\alpha_s = .118$; $\Gamma_t = 1.4$ GeV; tune 7;
$Q = 700, 1000, 1400$ GeV; peak$(60/80)\%$

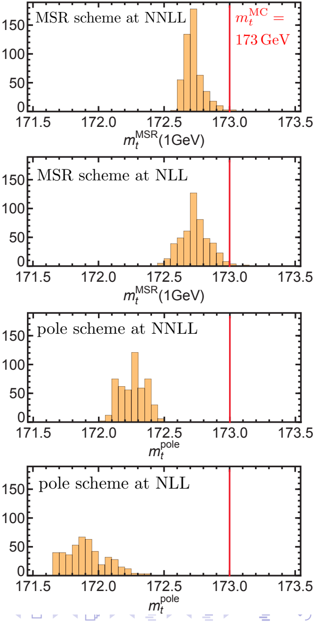
Input: $m_t^{\rm MC} = 173$ GeV

fit to find $m_t^{\rm MSR}(1{\rm GeV})$ or $m_t^{\rm pole}$

- Good convergence and stability for $m_t^{\rm MSR}(1{\rm GeV})$
- $m_t^{\rm MSR}(1{\rm GeV})$ numerically close to $m_t^{\rm MC}$

- Pole mass numerically not at all close to $m_t^{\rm MC}$
- $\sim 1100/700$ MeV difference at NLL/NNLL!
- $m_t^{\rm pole} \neq m_t^{\tt PYTHIA\ 8.2}$

Similar findings from the other 20 data sets



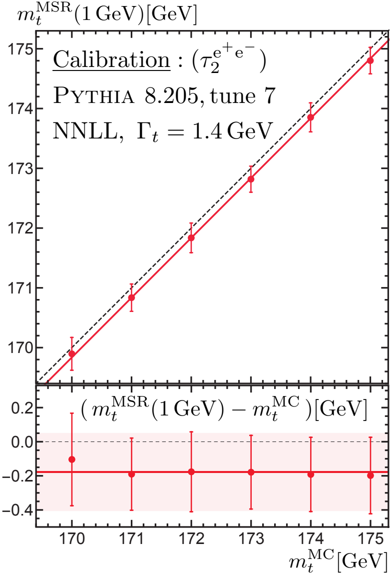
Final Results for m_t^{MSR}

- All investigated MC top mass values show consistent picture
- MC top quark mass is indeed closely related to MSR mass

  within uncertainties:
  $m_t^{\mathrm{MC}} \simeq m_t^{\mathrm{MSR}}(1\mathrm{GeV})$

$m_t^{\mathrm{MC}} = 173\,\mathrm{GeV}$ $\left(\tau_2^{e^+e^-}\right)$

mass	order	central	perturb.	incompatibility	total
$m_{t,1\,\mathrm{GeV}}^{\mathrm{MSR}}$	NLL	172.80	0.26	0.14	0.29
$m_{t,1\,\mathrm{GeV}}^{\mathrm{MSR}}$	$\mathrm{N^2LL}$	172.82	0.19	0.11	0.22
m_t^{pole}	NLL	172.10	0.34	0.16	0.38
m_t^{pole}	$\mathrm{N^2LL}$	172.43	0.18	0.22	0.28



Pole Mass Determinations

1. Pole mass implemented in code

$$m_t^{\text{pole}} < m_t^{\text{MSR}}(1\text{GeV}) < m_t^{\text{MC}}$$

$m_t^{\text{MC}} = 173\,\text{GeV}$ $\left(\tau_2^{e^+e^-}\right)$

mass	order	central	perturb.	incompatibility	total
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m_t^{pole}	NLL	172.10	0.34	0.16	0.38
m_t^{pole}	N^2LL	172.43	0.18	0.22	0.28

2. Pole mass determined from MSR mass

$$m_t^{\text{pole}} - m_t^{\text{MSR}}(1\text{GeV}) = \left[0.173 + 0.138 + 0.159 + 0.230 + \mathcal{O}(\alpha_s^5)\right]\,\text{GeV}$$

$\alpha_s(M_z) = 0.118;\ n_f = 5;$

$$m_t^{\text{MSR}}(1\text{GeV}) < m_t^{\text{pole}}$$

- Calibration in terms of pole mass involves large higher-order perturbative corrections
  $\rightarrow$ additional uncertainties for pole mass extraction (for $m_t^{\text{MC}} = 173$ GeV)

$(m_t^{\text{pole}})_{\text{NLL}} = 172.45 \pm 0.52$ GeV

$(m_t^{\text{pole}})_{\text{NNLL}} = 172.72 \pm 0.40$ GeV

Top Quark Decay & C-Jettiness / massive C-Parameter

C-Jettiness / massive C-Parameter

Second: consistency checks: look into different observables

- C-Jettiness is defined as:

$$C_{\mathrm{J}} = \frac{3}{2}\left[2 - \frac{1}{Q^2}\sum_{i\neq j}\frac{(p_i \cdot p_j)^2}{p_i^0 p_j^0}\right]$$

  - coincides with the C-parameter for the massless case
  - basically a *mass-sensitive* version of C-Parameter & very useful in the context of massive particles
    NLO FO with massive gluon [Gardi, Magnea 2003]; NLO FO with massive quarks [Hoang, Mateu, MP *not published yet*]

- Highly mass sensitive: at parton level we have

$$C_{\mathrm{J}}^{\mathrm{peak,stable}} = 12\,\hat{m}^2(1-\hat{m}^2)$$

$$C_{\mathrm{J}}^{\mathrm{peak,unstable}} \simeq 12\,\hat{m}^2(1-4\hat{m}^2) \qquad \text{(for } t \text{ to massless particles)}$$

even at high enough boost: big peak-shift!

Top quark case: highly decay sensitive!

Top Quark Decay: 2-Jettiness vs. C-Jettiness

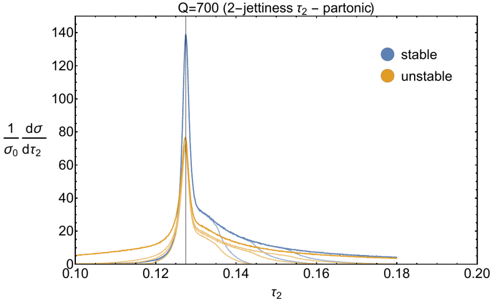
- Inclusive treatment essentially means dressing the distribution with a Breit-Wigner

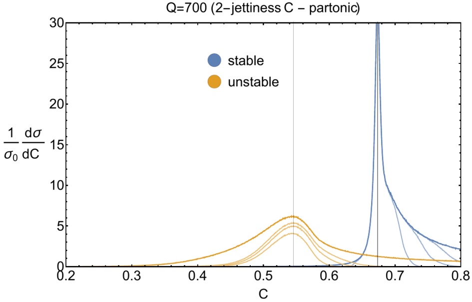
- 2-jettiness peak:
  - same position
  - new features below peak

  Inclusive setup is **sufficient** for peak position in unstable case!

- C-jettiness peak:
  - displaced
  - distorted

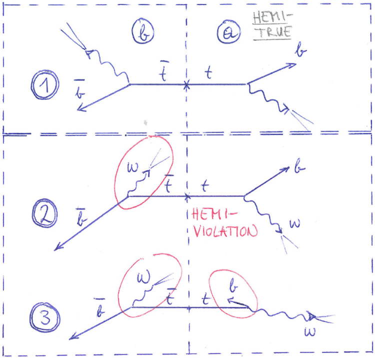
  Inclusive setup is **not sufficient** for unstable case!

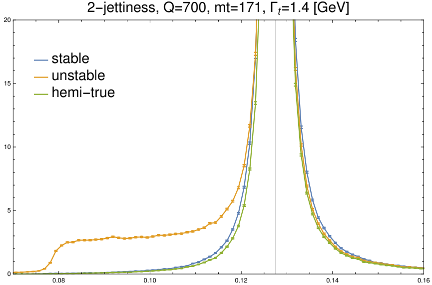


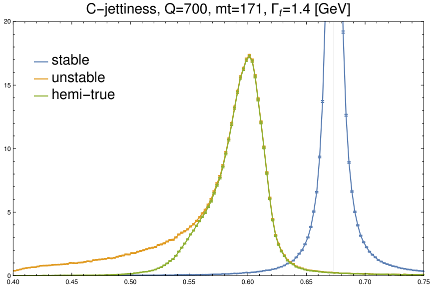


Top Quark Decay: Tree-level Kinematics

- Find compromise between:
  - mass sensitivity
  - high enough boost
- At finite boost:
  also hemisphere violating decays





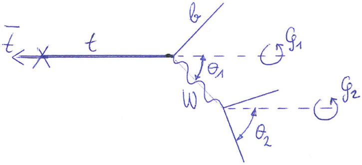


Simplifications & Decay Factorization

- To treat this we use some simplifications:
  - Top width small & $\Gamma_W \to 0$
  - Neglect spin correlations between top and decay products
  - Treat top decay at leading order in QCD

Interpretation: Top lives very long.

All radiation happens before decay of the top.



- With this we rewrite the differential QCD cross section for an event-shape e as:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}e} = \int \frac{\mathrm{d}\cos\hat{\theta}_1}{2}\frac{\mathrm{d}\hat{\phi}_1}{2\pi} \int \frac{\mathrm{d}\cos\hat{\theta}_2}{2}\frac{\mathrm{d}\hat{\phi}_2}{2\pi} \int \frac{\mathrm{d}\cos\hat{\theta}_3}{2}\frac{\mathrm{d}\hat{\phi}_3}{2\pi} \int \frac{\mathrm{d}\cos\hat{\theta}_4}{2}\frac{\mathrm{d}\hat{\phi}_4}{2\pi}$$

$$\sum_{n=0}^{\text{order}} \int \mathrm{d}\Pi_{n+2}\, S_n^{\text{incl}}(m_t, \Gamma_t, X_n)\, \delta(e - e^{\text{unstable}}(X_n, \hat{\phi}_i, \hat{\theta}_i))$$

Singular Piece: Treating the decay in bHQET+SCET

- Usual bHQET+SCET setup involves the following momenta/scalings (top version): $\Delta \sim \Gamma_t$
  - $k_{\rm uc} \sim \Delta(\frac{m_t}{Q}, \frac{Q}{m_t}, 1)$ ultra-collinear gluon interaction
  - $k_{\rm us} \sim \lambda^2 Q(1,1,1)$ ultra-soft gluon interaction with $Q^2\lambda^2 \overset{?}{\sim} m_t^2$

  top quark momentum: $p_t = m_t(\frac{m_t}{Q}, \frac{Q}{m_t}, 1) + k + \mathcal{O}(\frac{m_t^4}{Q^4}, \frac{m_t^3\Delta}{Q^4}, \lambda^4)$

$$e^{\rm part} = \overline{e}^{\rm stable} + \mathcal{O}(\frac{m_t^4}{Q^4}, \frac{m_t^3\Delta}{Q^4}, \frac{\Delta^2}{Q^2}, \lambda^4)$$

- Let's do a numerical comparison: (e.g. $m_t = 171$ GeV, $Q = 700$ GeV, $\Gamma_t = 1.4$ GeV)

$\frac{m^2}{Q^2} = 0.0597$ | $\frac{m^4}{Q^4} = 0.0036$ | $\frac{m^6}{Q^6} = 0.0002$

$\frac{m\Gamma_t}{Q^2} = 0.0005$ | $\frac{m^3\Gamma_t}{Q^4} = 0.00003$ | $\frac{\Gamma_t^2}{Q^2} = 0.000004$

- In the peak region $Q^2\lambda^2 \lesssim m_t\Gamma_t$ anyways holds for mentioned observables. Include full $\frac{m_t}{Q}$ dependence $\rightarrow$ full Boost-factors:

$$e^{\rm part} = \overline{e}^{\rm stable} + e^{(4)} + \mathcal{O}(\frac{m_t^3\Delta}{Q^4}, \frac{\Delta^2}{Q^2}, \lambda^4)$$

with $e^{(4)} \equiv e^{(4)}(\hat\theta_i, \hat\phi_i) \sim \mathcal{O}(\frac{m_t^4}{Q^4})$

Final Setup & Next Steps

- Final result:

$$\frac{\mathrm{d}\sigma^{\text{sing}}}{\mathrm{d}e} = \int \mathrm{d}\hat{e}\, \frac{\mathrm{d}\sigma^{\text{sing,incl}}}{\mathrm{d}e}(e-\hat{e})\; F^{\text{decay}}(\hat{e})$$

$$F^{\text{decay}}(\hat{e}) = \int \frac{\mathrm{d}\cos\hat{\theta}_1}{2}\frac{\mathrm{d}\hat{\phi}_1}{2\pi} \int \frac{\mathrm{d}\cos\hat{\theta}_2}{2}\frac{\mathrm{d}\hat{\phi}_2}{2\pi} \int \frac{\mathrm{d}\cos\hat{\theta}_3}{2}\frac{\mathrm{d}\hat{\phi}_3}{2\pi} \int \frac{\mathrm{d}\cos\hat{\theta}_4}{2}\frac{\mathrm{d}\hat{\phi}_4}{2\pi}\; \delta(\hat{e} - e^{(4)}(\hat{\theta}_i, \hat{\phi}_i))$$

easy to calculate using some MC code.

- Next Steps

1. Finish Theory implementation

   Include Top Decay: certain approximations needed ✓

   Profile functions for unstable C-Jettiness (work in progress)

2. MC Top Quark Mass Calibration using C-Jettiness

   `PYTHIA` sample incl. binning & ranges ✓

   → can be done as soon as theory implementation is finished.

Conclusion & Outlook

- First precise MC top quark mass calibration based on e^+e^- 2-jettiness
  PRL 117 (2016) 232001 (arXiv: 1608.01318 [hep-ph])

  QCD calculations at $\mathrm{NNLL} + \mathrm{NLO}$ based on an extension of the SCET approach to include massive quark effects

- Top mass calibration for `PYTHIA` 8.205 in terms of Pole and MSR mass.
  For $m_t^{\mathrm{MC}} = 173$ GeV at NNLL:
  - $m_t^{\mathrm{pole}} = 172.72 \pm 0.40\ \mathrm{GeV}$
  - $m_t^{\mathrm{MSR}}(1\mathrm{GeV}) = 172.82 \pm 0.22\ \mathrm{GeV}$

- Implementation of the top quark decay for more decay sensitive observables
  $\rightarrow$ future cross-check with C-jettiness

Outlook:

- Consistency, `CALIPER` & $(\mathrm{N}^3\mathrm{LL} + \mathrm{N}^2\mathrm{LO})$
- soft-drop for pp jet-mass at $\mathrm{N}^2\mathrm{LL}$ & calibration