# SCET and radiative corrections to lepton-nucleon scattering

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SCET workshop
Wayne State

14 March 2018

*based on 1605.02613*

# Overview

1) motivations: e-p, $\nu$-N,

2) some details on the Rydberg puzzle (aka proton rad. puzzle)

3) NLO analysis of radiative corrections

4) illustrative results

**FAQ**: *- aren't QED corrections tiny?*

No. In typical experimental configurations, large log enhancements: $\log(Q^2/m_e^2) \sim 15$, RC $\gtrsim 30\%$

*- weren't these computed in ancient times?*

Not really. Experimental implementations are based on old theory papers, often not addressing essential issues

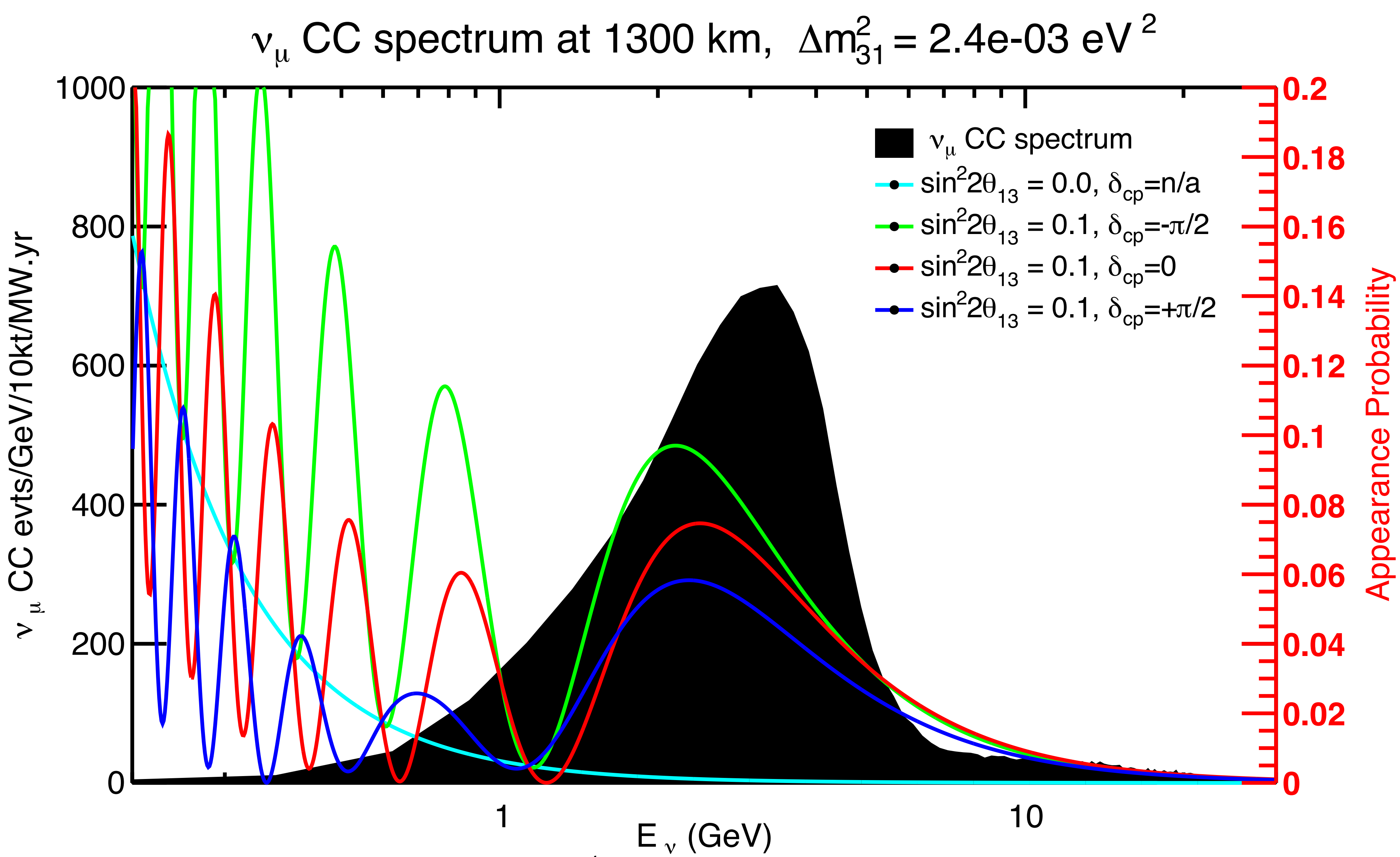
*- isn't this too easy? isn't this too hard?*

Not the right question. Compute what is computable, measure what is not. And nobody said that probing the GUT scale was easy.

- **<u>ν-N scattering</u>**: radiative corrections impact all cross sections, including critical $\nu_e/\nu_\mu$ ratios for long baseline program

*De Rujula, Petronzio & Savoy-Navarro, NPB 154, 394 (1979)*

- LL analysis of total inclusive cross section (but need exclusive, and beyond LL)




*LBNE, 1307.7335*

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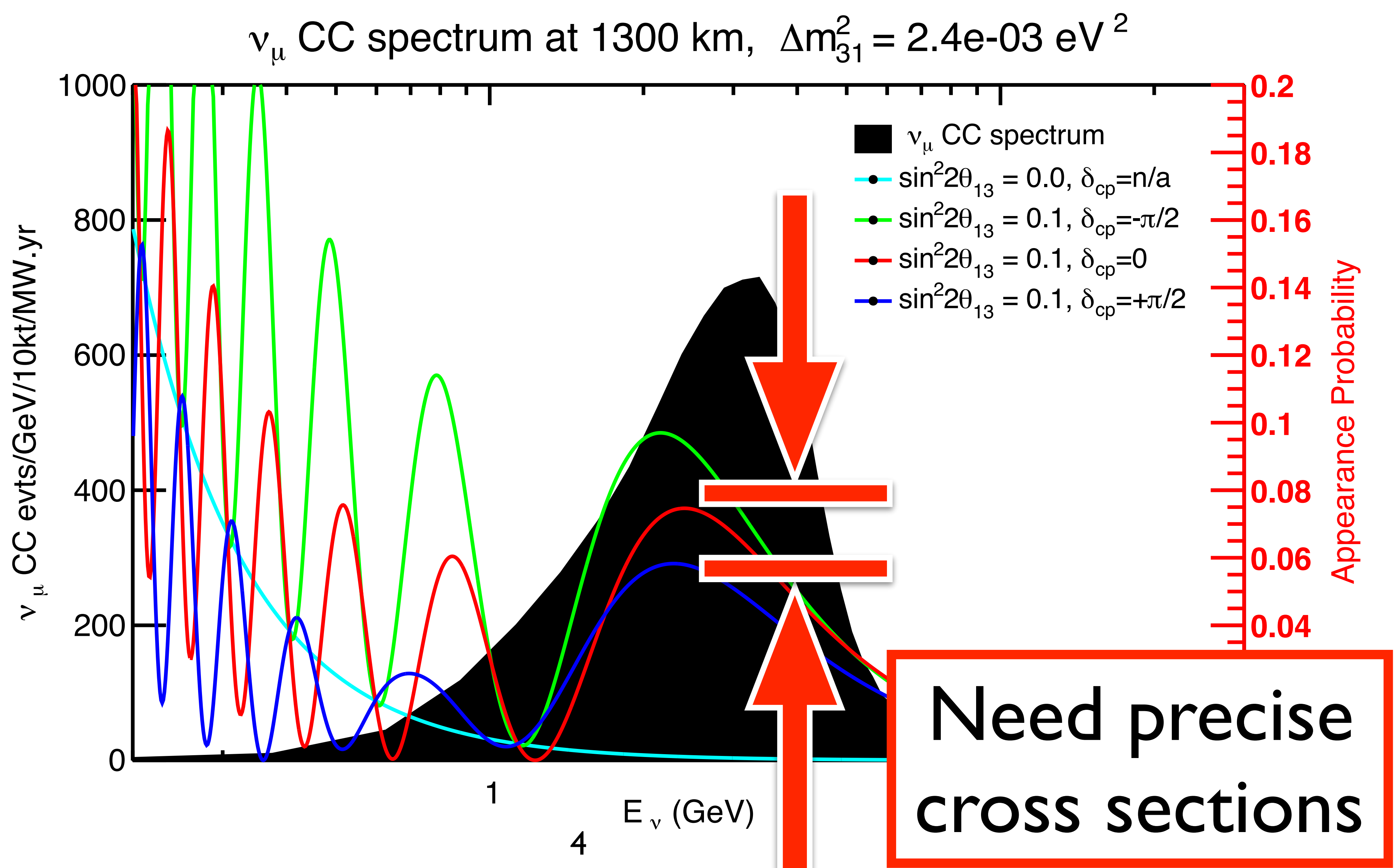
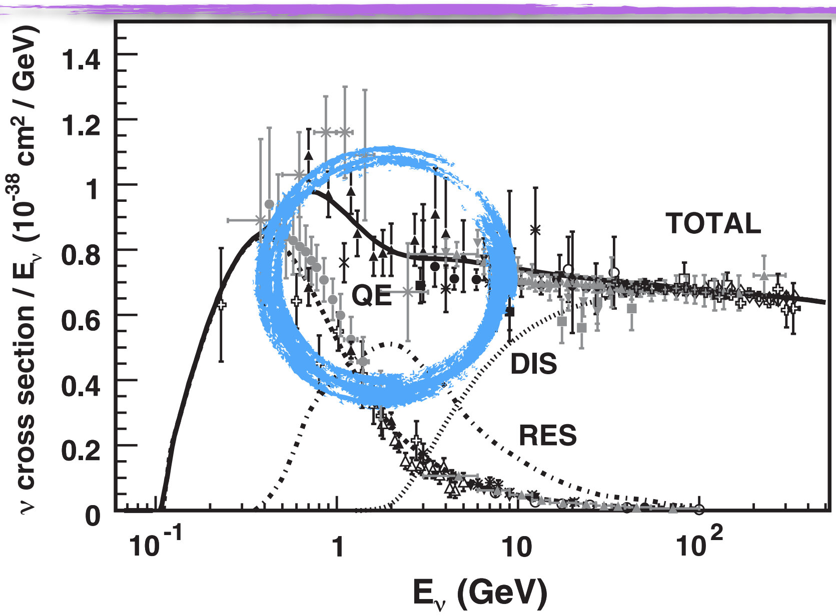



*LBNE, 1307.7335*

- **muon capture $\mu p \to \nu n$ from muonic hydrogen**: potential for best determination of nucleon axial radius, but radiative corrections need to be controlled at 0.1% level

*Sirlin, Phys.Rev. 164, 1767 (1967)*

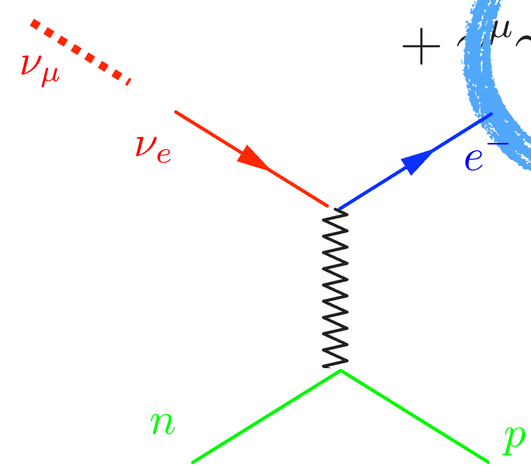
- factorization analysis for neutron beta decay (but $m_\mu \gg m_e$ , bound state corr.)




*Quasielastic dominance*

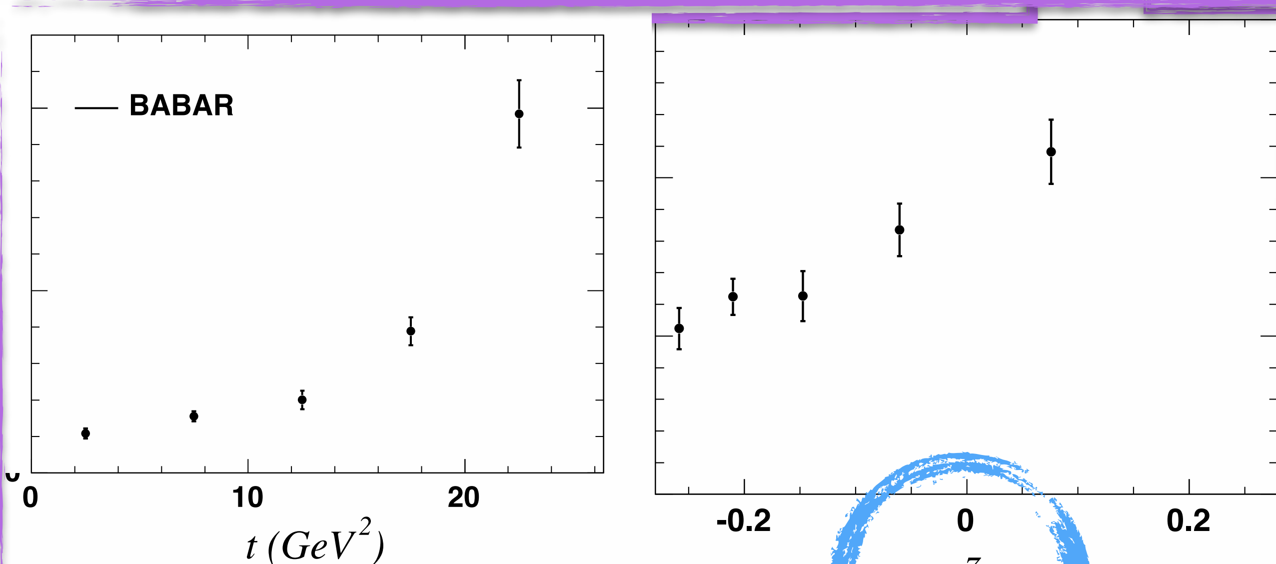
+

$$\langle p(p')|J_W^{+\mu}|n(p)\rangle \propto \bar{u}^{(p)}(p')\left\{\gamma^\mu F_1(q^2) + \frac{i}{2m_N}\sigma^{\mu\nu}q_\nu F_2(q^2) + \gamma^\mu\gamma_5 F_A(q^2) + \frac{1}{m_N}q^\mu\gamma_5 F_P(q^2)\right\}u^{(n)}(p)$$




*Unknown: axial-vector form nucleon form factor*

+



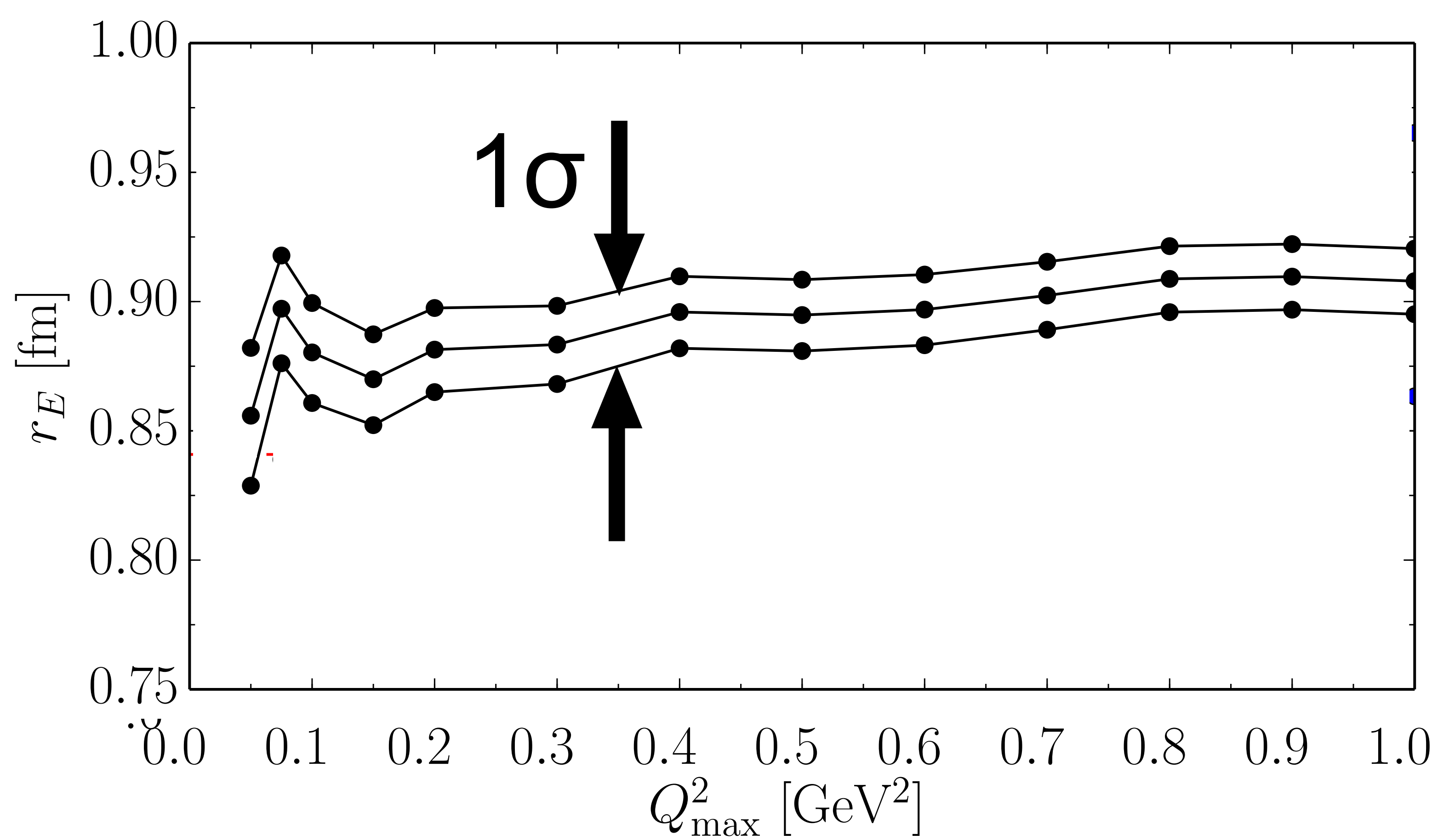

*$F_A$ linear in "z" variable*

$$\to \quad r_A^2 \equiv 6\frac{d}{dq^2}F_A(q^2)\Big|_{q^2=0}$$

**e-p scattering: probable(?) ~7 σ shift in Rydberg constant.**
Large contributor: radiative corrections in electron-proton scattering

*Yennie, Frautschi & Suura, Annals Phys. 13, 379 (1961)*

- exponentiation/cancellation of IR divergences (but need subleading logs)



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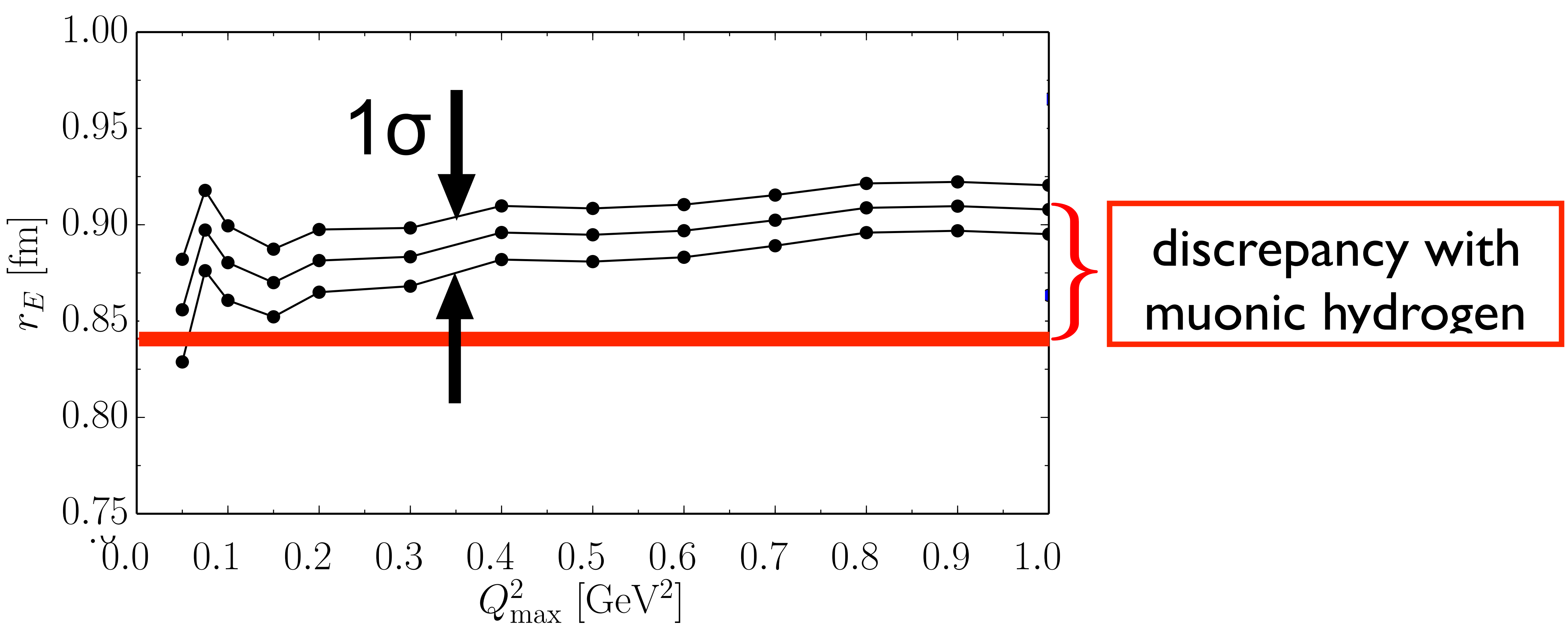

$\boxed{\text{the Rydberg or proton radius puzzle}}$

**Recall hydrogen spectrum:**

$$E_n \sim \frac{R_\infty}{n^2} + \frac{r_E^2}{n^3}$$

$$hcR_\infty = \frac{m_e c^2 \alpha^2}{2} \approx 13.6\,\mathrm{eV}$$

proton charge radius

Disentangle 2 unknowns, $R_\infty$ and $r_E$, using well-measured 1S-2S hydrogen transition *and*

electron-based measurements

muon-based measurements

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muon-based measurements

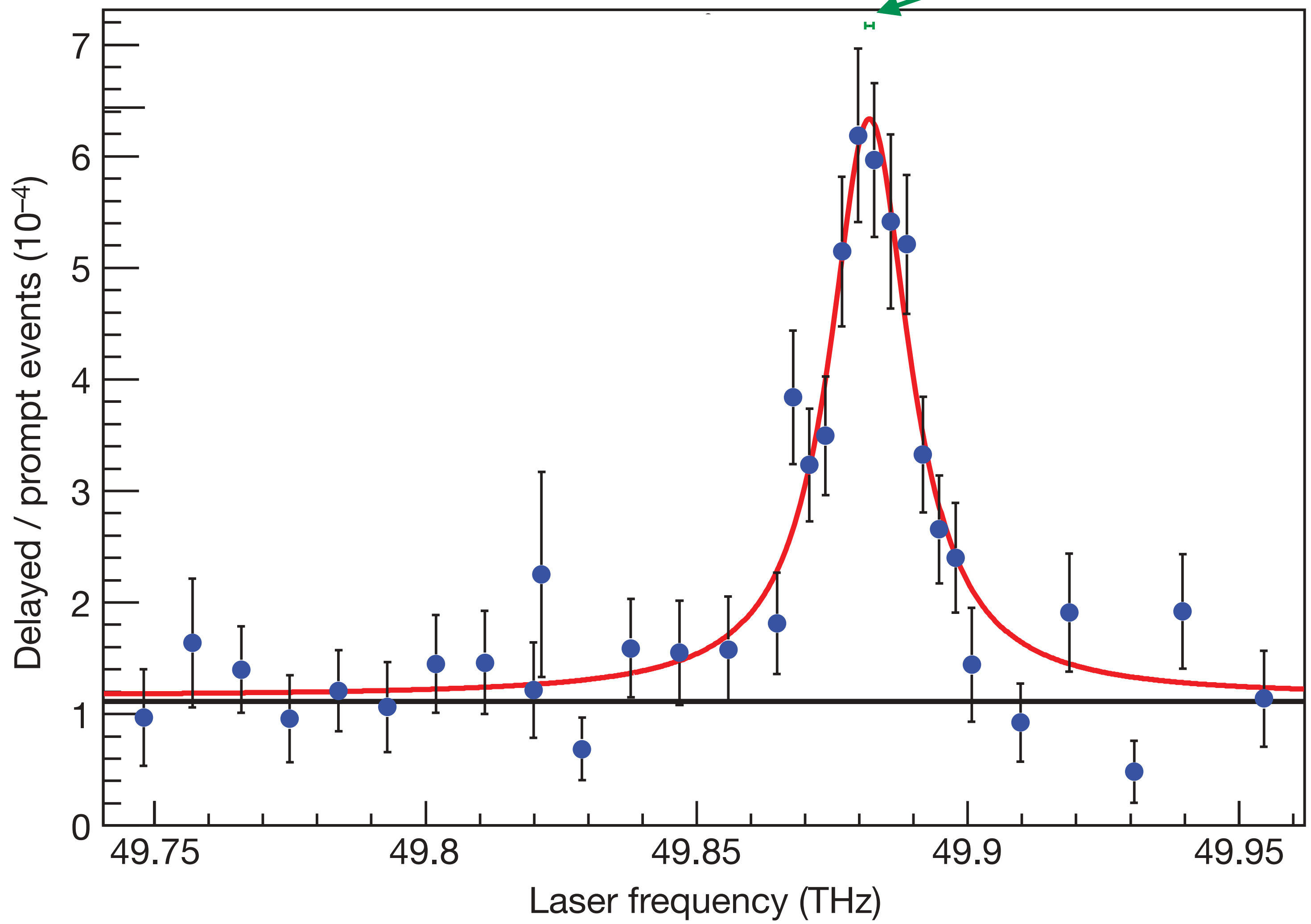
- a muonic hydrogen interval

7σ discrepancy between electron-based versus muon-based measurements

# muonic hydrogen Lamb shift measurement

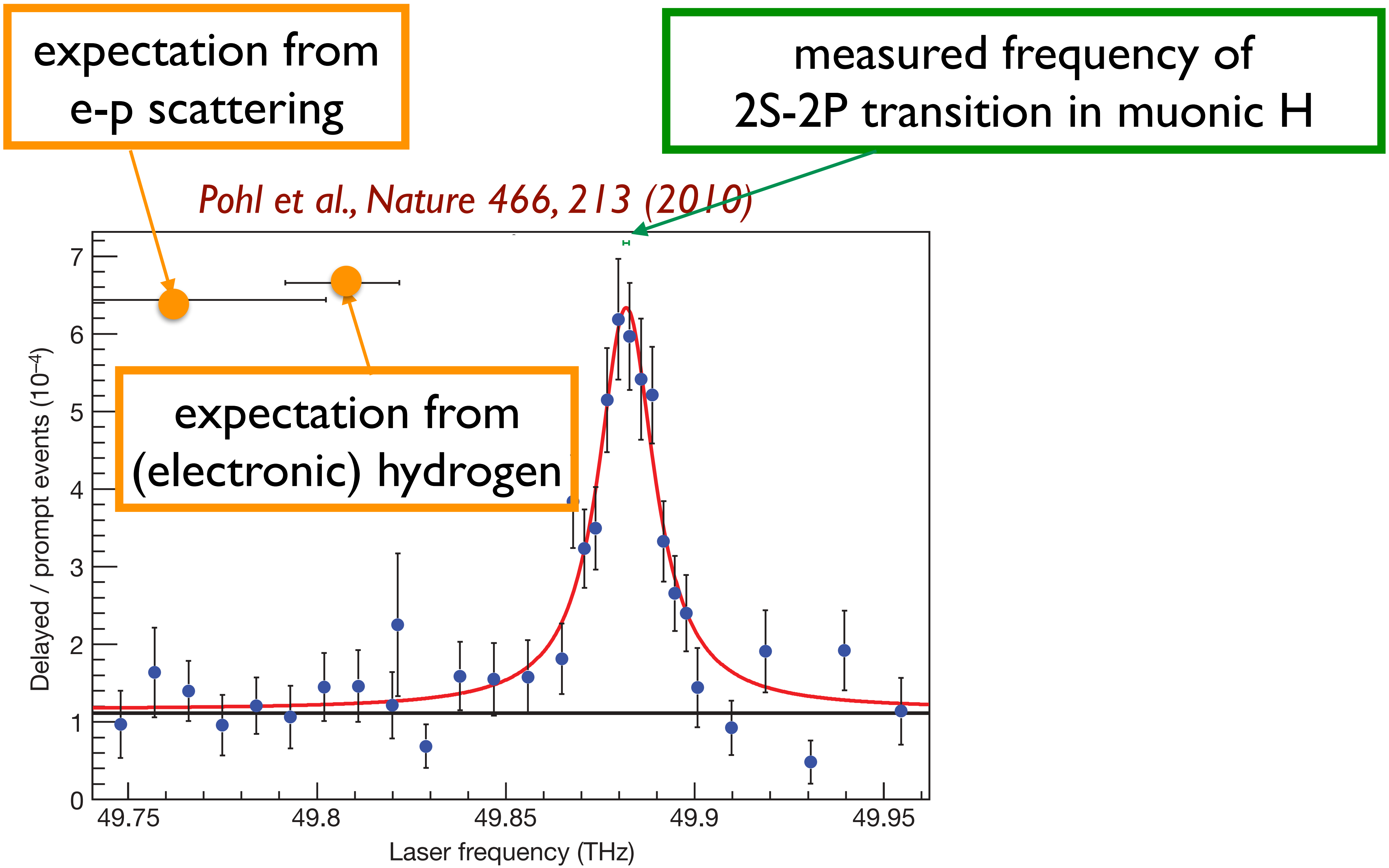
measured frequency of
2S-2P transition in muonic H

*Pohl et al., Nature 466, 213 (2010)*



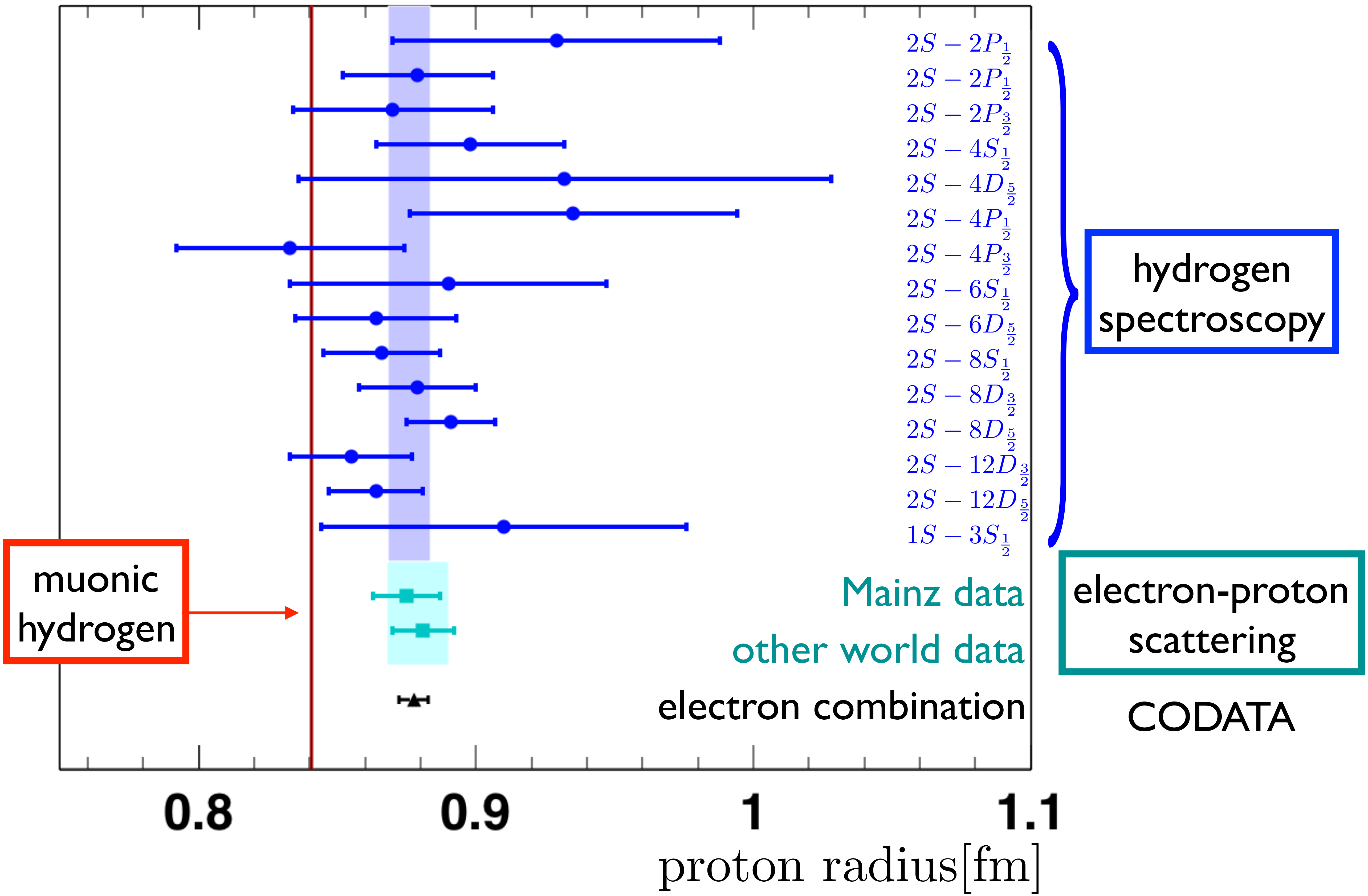

**new experimental capabilities: surprises and new insight ?**

# muonic hydrogen Lamb shift measurement




**new experimental capabilities: surprises and new insight ?**

# summary of electron- and muon- based measurements, circa 2010



form factor nonlinearities

## electron-proton scattering: theory issues

radius is defined as slope of form factor

*i) what are the constraints on nonlinearities?*

radiative corrections impact radius extraction and can be large (~30%)

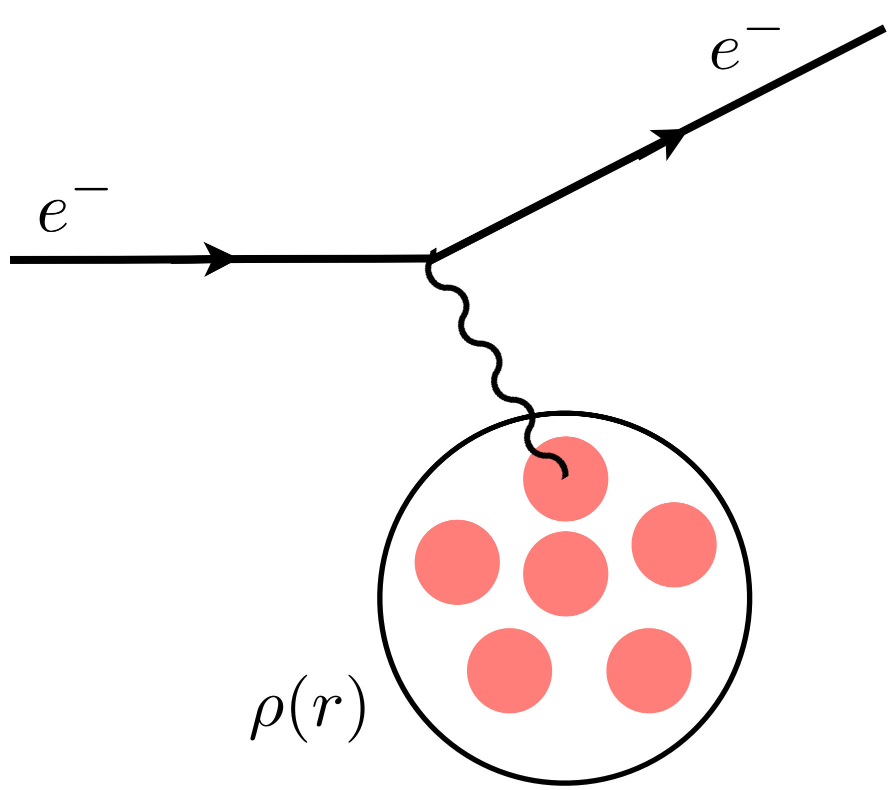
*ii) are radiative corrections controlled at the sub percent level?*

## *i) what are the constraints on nonlinearities?*

recall scattering from extended classical charge distribution:

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{pointlike}} |F(q^2)|^2$$

$$\begin{aligned} F(q^2) &= \int d^3r\, e^{i\boldsymbol{q}\cdot\boldsymbol{r}} \rho(\boldsymbol{r}) \\ &= \int d^3r \left[1 + i\boldsymbol{q}\cdot\boldsymbol{r} - \frac{1}{2}(\boldsymbol{q}\cdot\boldsymbol{r})^2 + \ldots\right] \rho(r) \\ &= 1 - \frac{1}{6}\langle r^2\rangle \boldsymbol{q}^2 + \ldots \end{aligned}$$




for the relativistic, QM, case, *define* radius as slope of form factor

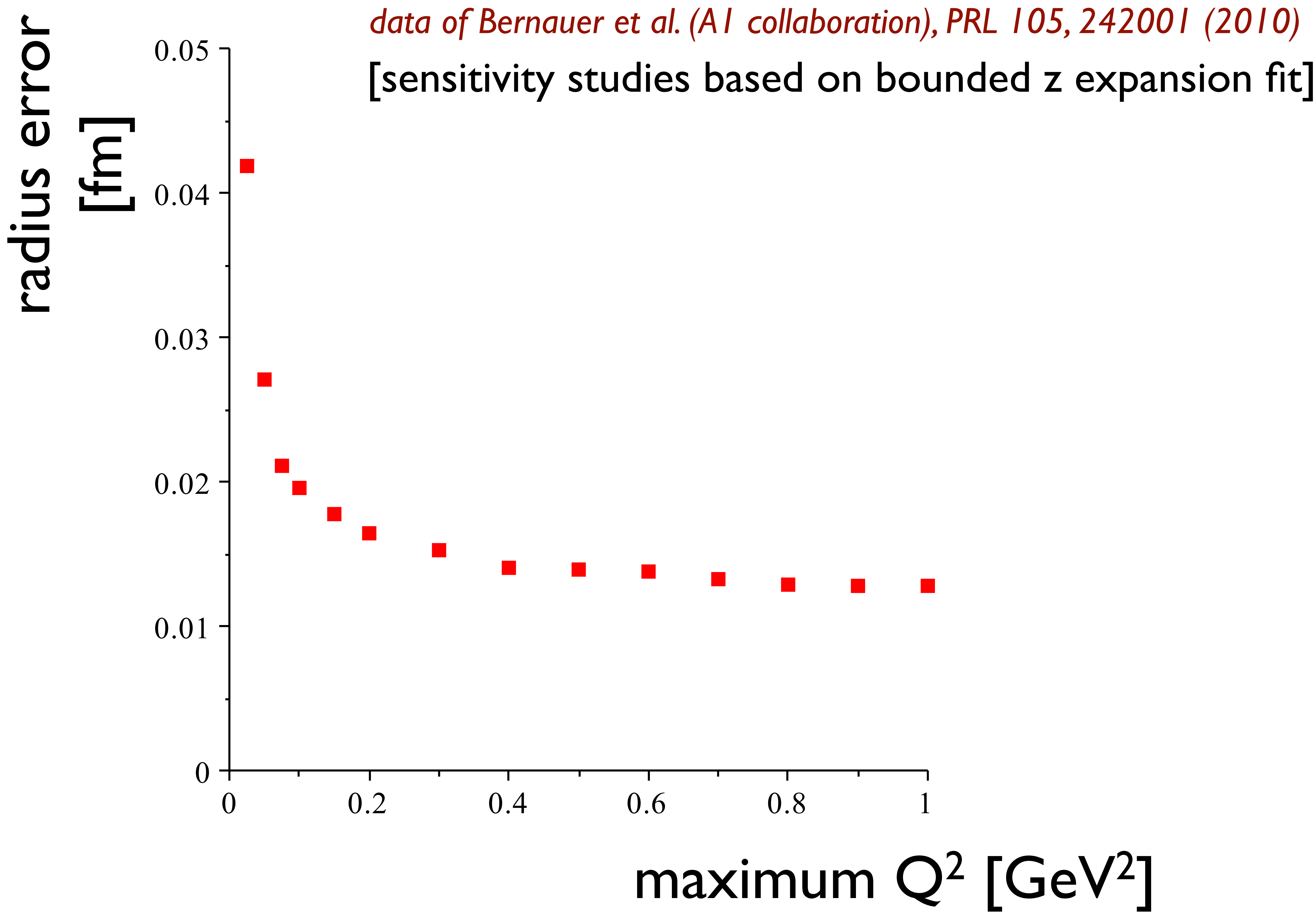
$$\langle J^\mu\rangle = \gamma^\mu F_1 + \frac{i}{2m_p}\sigma^{\mu\nu} q_\nu F_2$$

$$G_E = F_1 + \frac{q^2}{4m_p^2} F_2 \qquad G_M = F_1 + F_2$$

$$\boxed{r_E^2 \equiv 6 \frac{d}{dq^2} G_E(q^2)\Big|_{q^2=0}}$$

*(up to radiative corrections)*

## Radius extraction requires data over a $Q^2$ range where a simple Taylor expansion of the form factor is invalid



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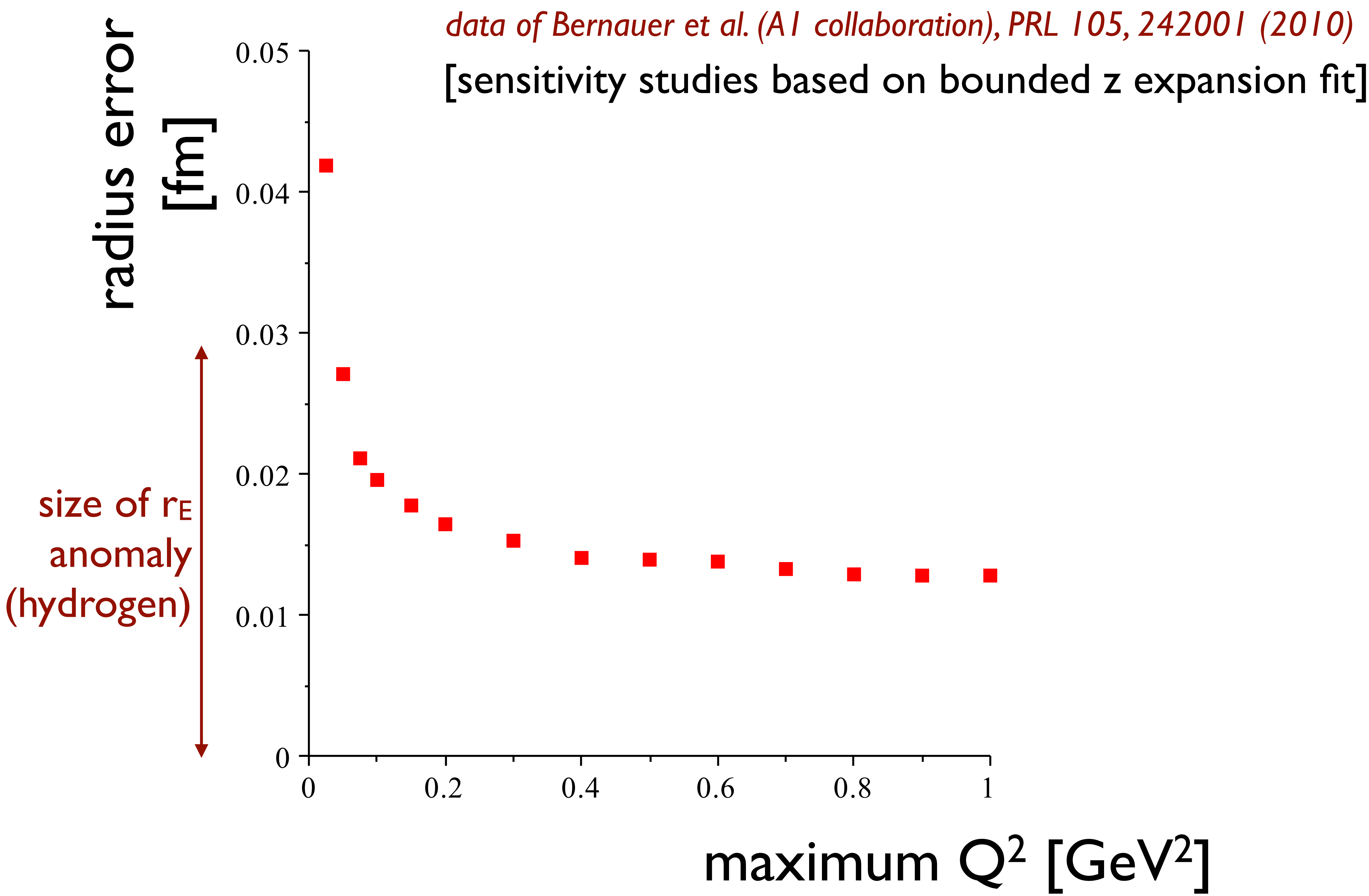

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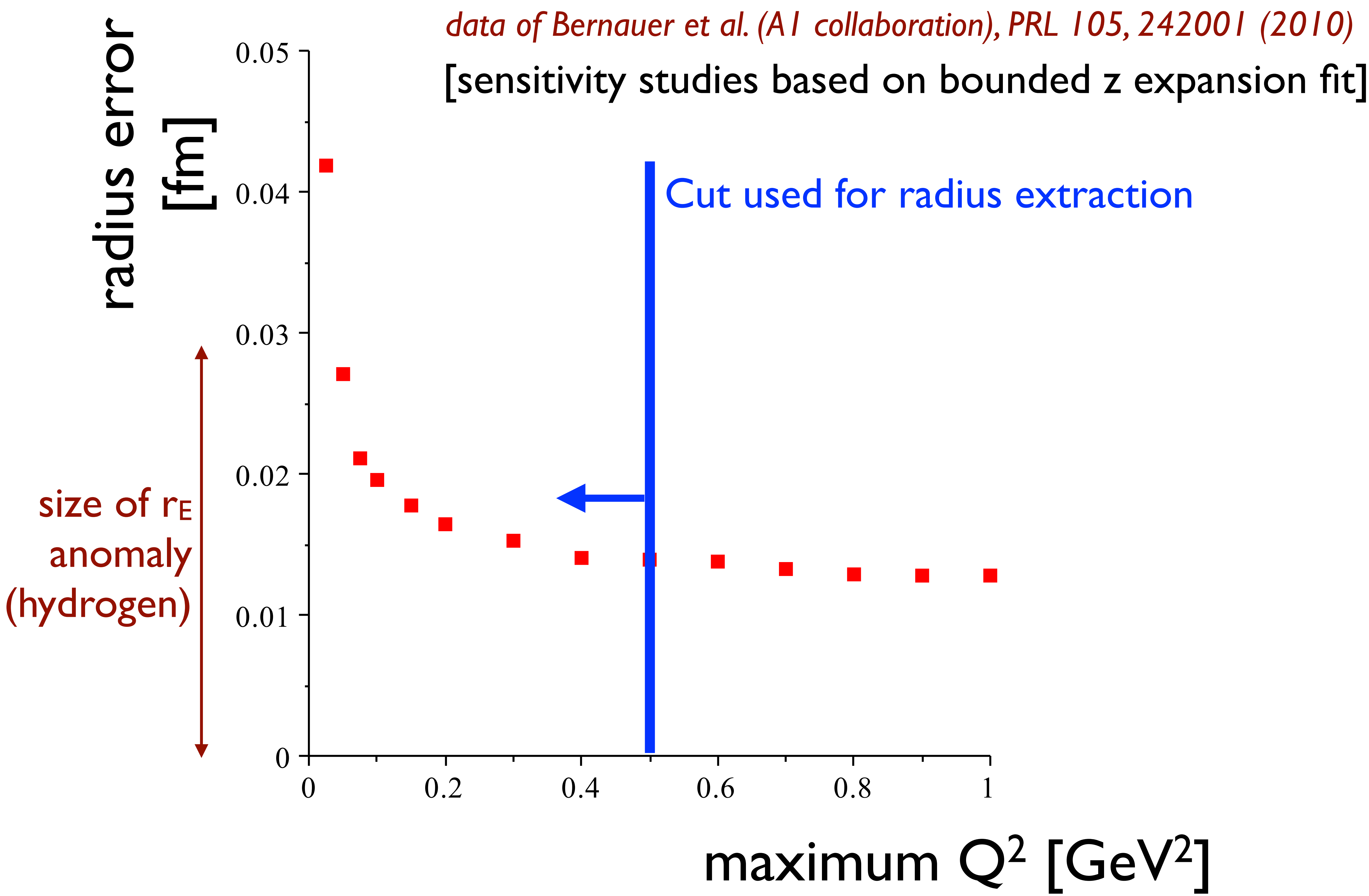

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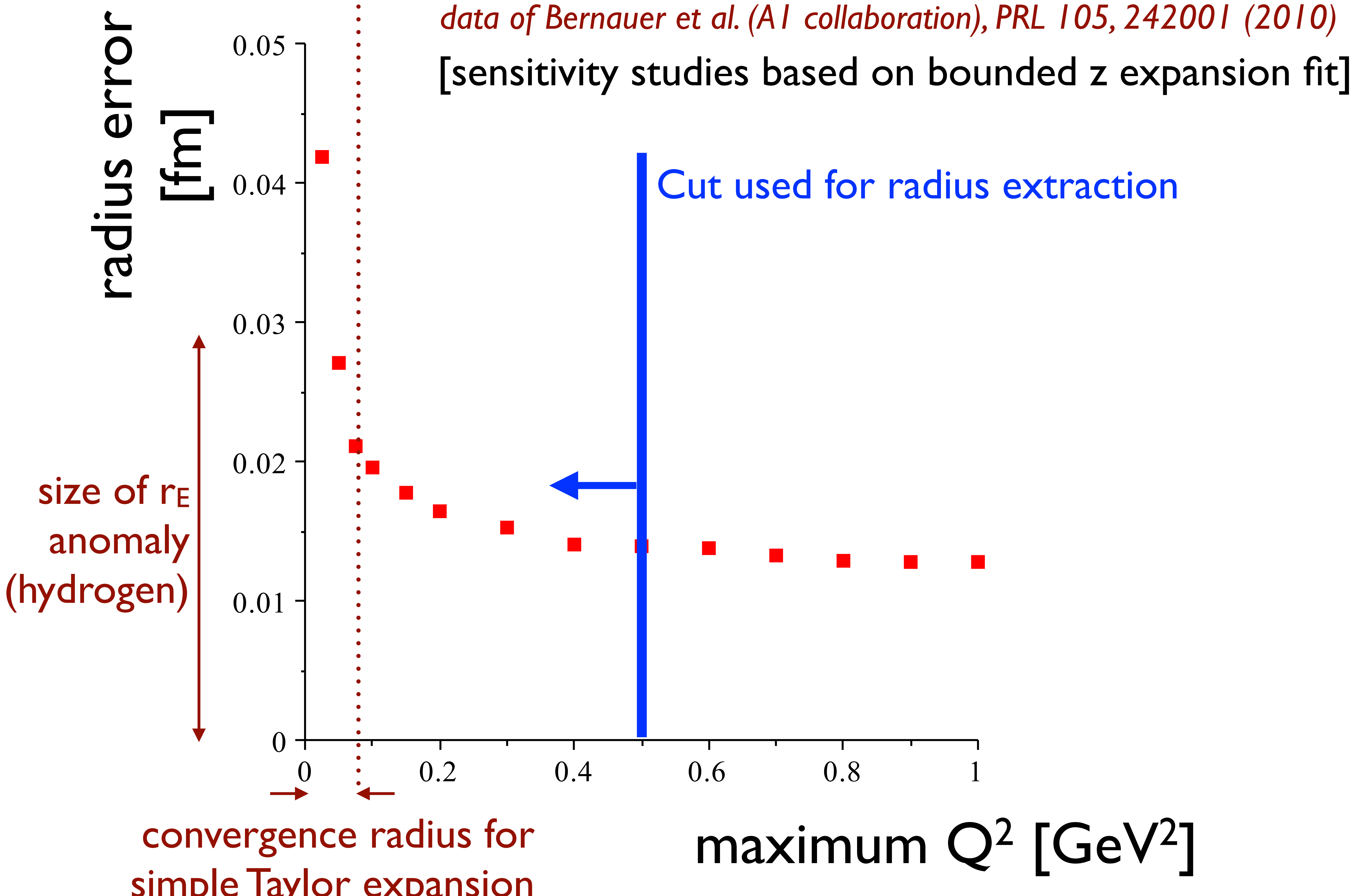
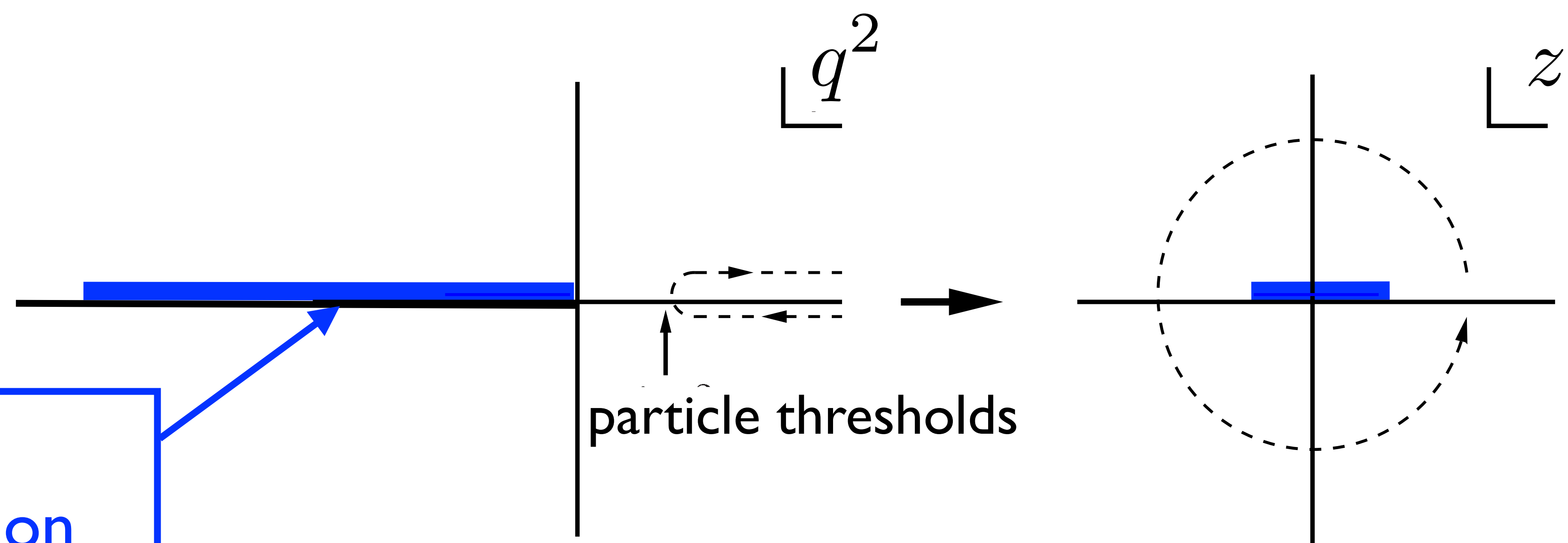


# That's ok: underlying QCD tells us that Taylor expansion of form factor in appropriate variable is convergent




experimental kinematic region

$$z(q^2, t_{\rm cut}, t_0) = \frac{\sqrt{t_{\rm cut} - q^2} - \sqrt{t_{\rm cut} - t_0}}{\sqrt{t_{\rm cut} - q^2} + \sqrt{t_{\rm cut} - t_0}}$$

$$F(q^2) = \sum_k a_k [z(q^2)]^k$$

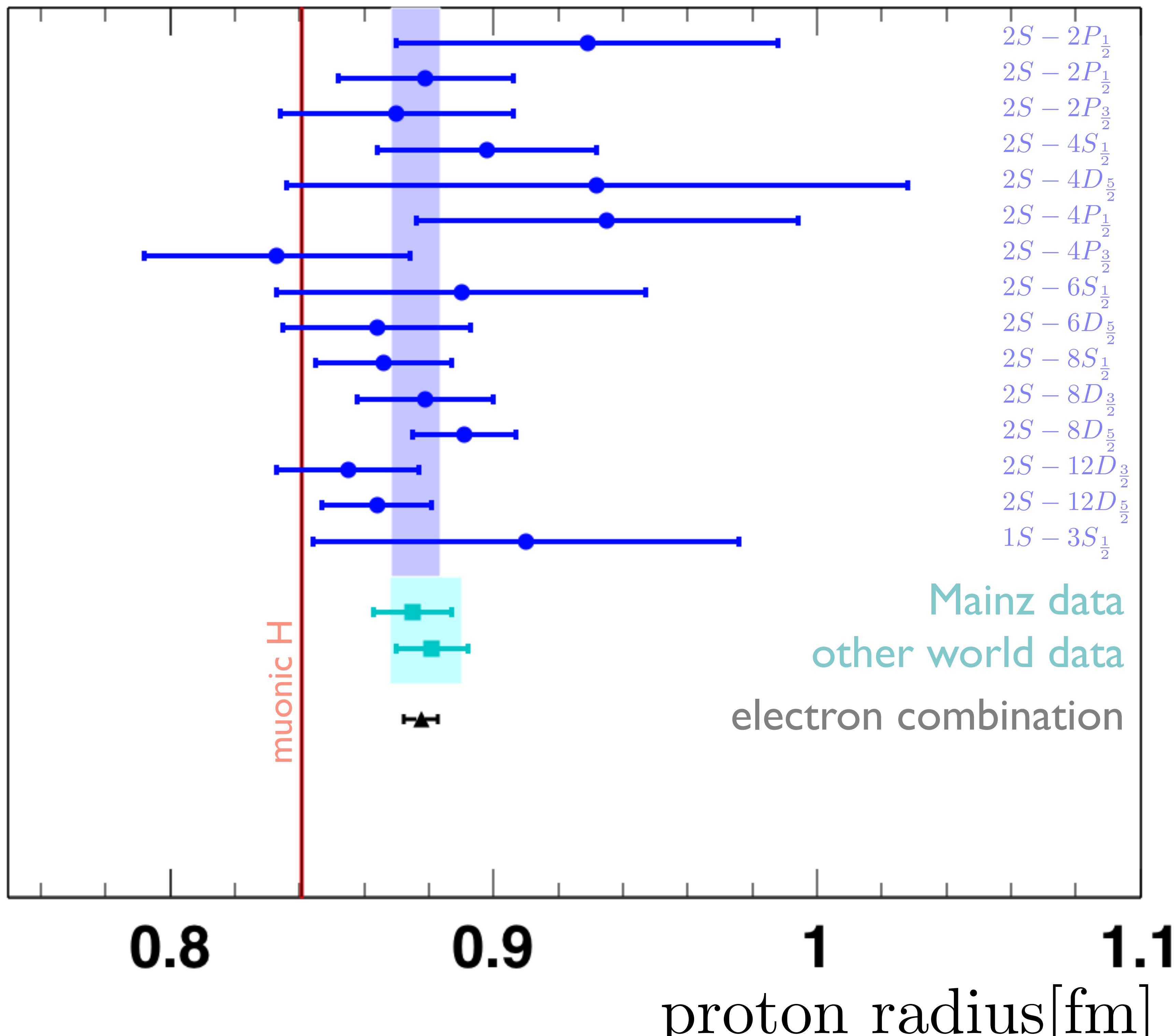
coefficients in rapidly convergent expansion encode nonperturbative QCD

...

*RJH, G. Paz (2010)*

...

# Reanalysis of scattering data reveals strong influence of shape assumptions




**Errors larger, but discrepancy remains**

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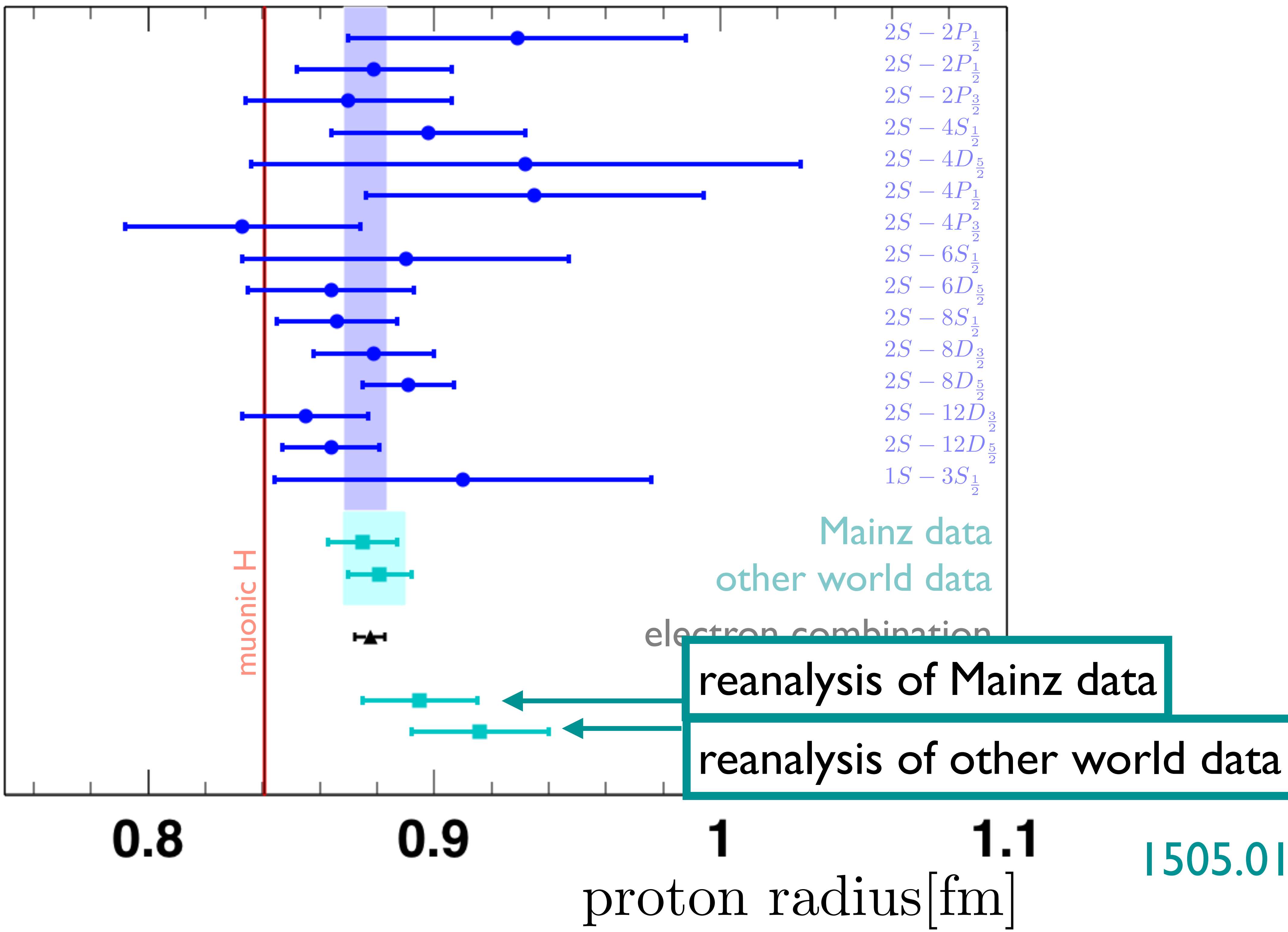


1505.01489

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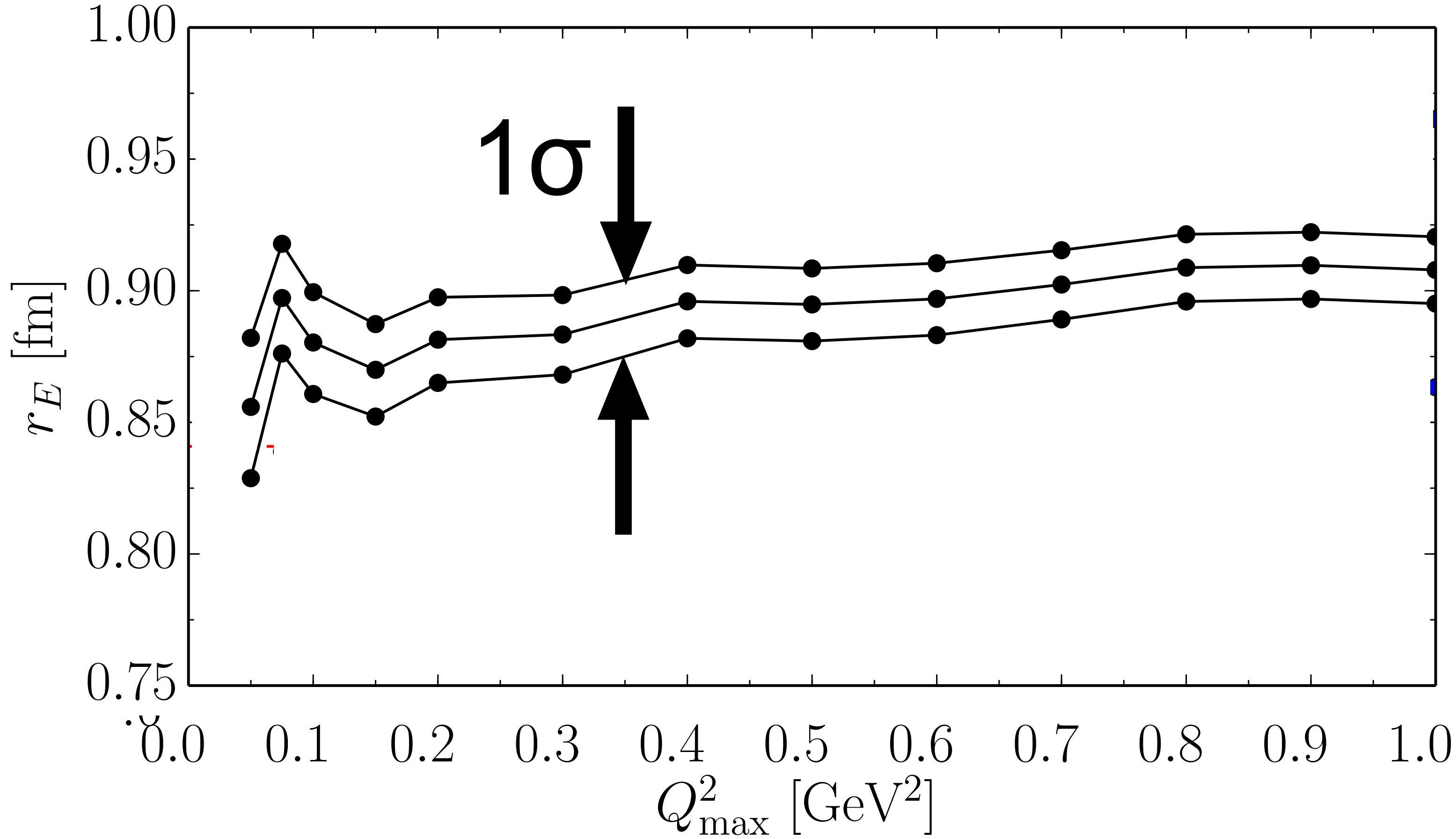
## Reanalysis of scattering data also reveals potential dependence of radius on chosen $Q^2$ range




To reconcile e-p scattering with muonic hydrogen, could:

- consider only small $Q^2$ data (less data ⇒ larger error)
- overrule scattering data with other data or assumptions, e.g. spectral function model *Belushkin, Hammer, Meissner (2007)* *Lorenz, Meissner, Hammer, Dong (2015)*

These options would avoid, but not resolve, the radius puzzle from electron scattering. Is there an unaccounted systematic effect impacting especially large $Q^2$ data?

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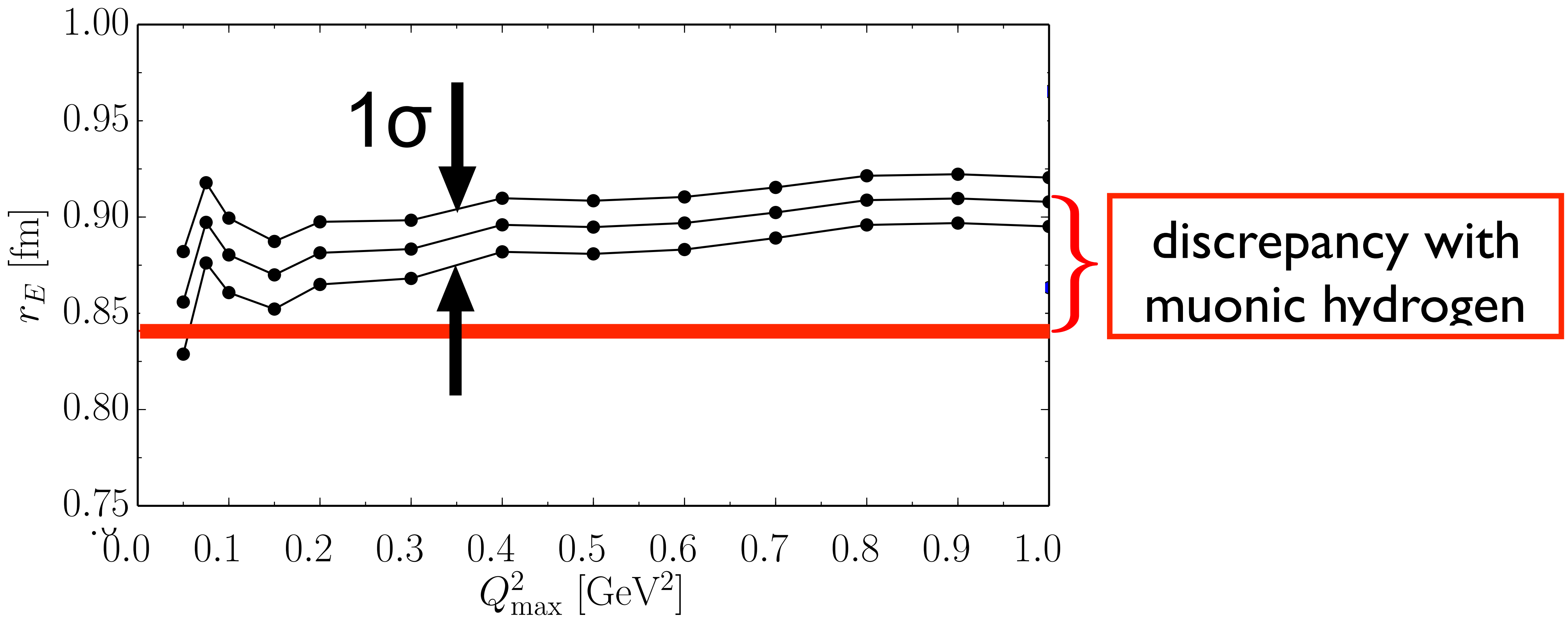


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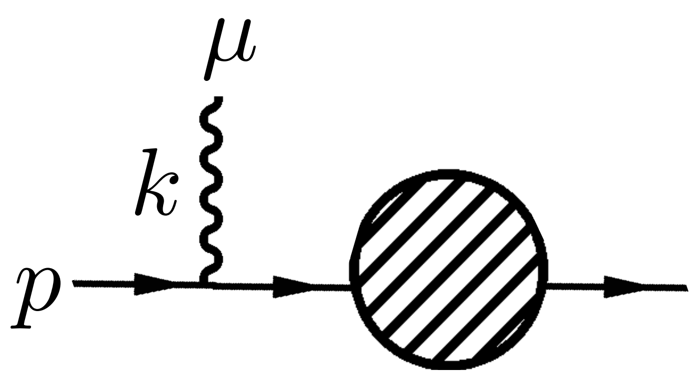
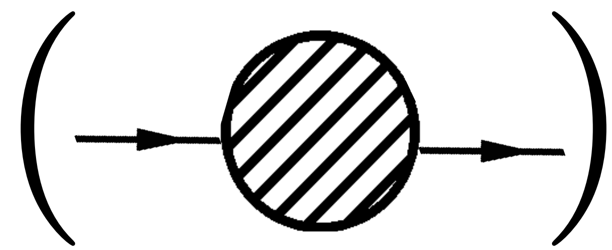
large logs




$$= e\,\frac{p^\mu}{p\cdot k}$$



$$+\dots$$

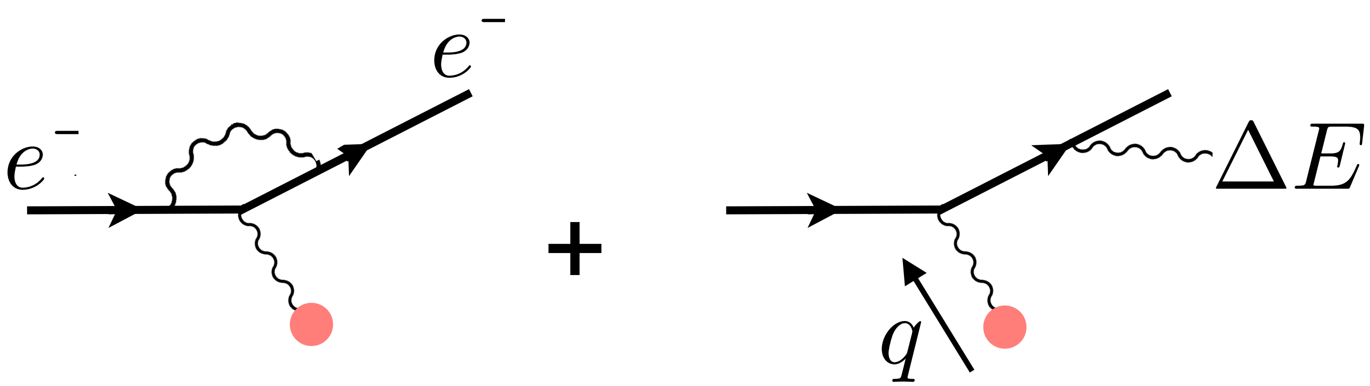
- eikonal coupling
- factorization of soft region
- proof by induction *Yennie, Frautschi, Suura (1961)*

⇒ exponentiation of IR divergences, cancellation between real and virtual

But exponentiation of IR divergences does not imply exponentiation of the entire first order correction

## Large logarithms spoil QED perturbation theory when $-q^2=Q^2\sim\text{GeV}^2$

$$|F(q^2)|^2 \to |F(q^2)|^2\left(1-\underbrace{\frac{\alpha}{\pi}\log\frac{Q^2}{m_e^2}\log\frac{E^2}{(\Delta E)^2}}_{\approx 0.5}+\dots\right)$$




## Experimental ansatz sums exponentiates 1st order:

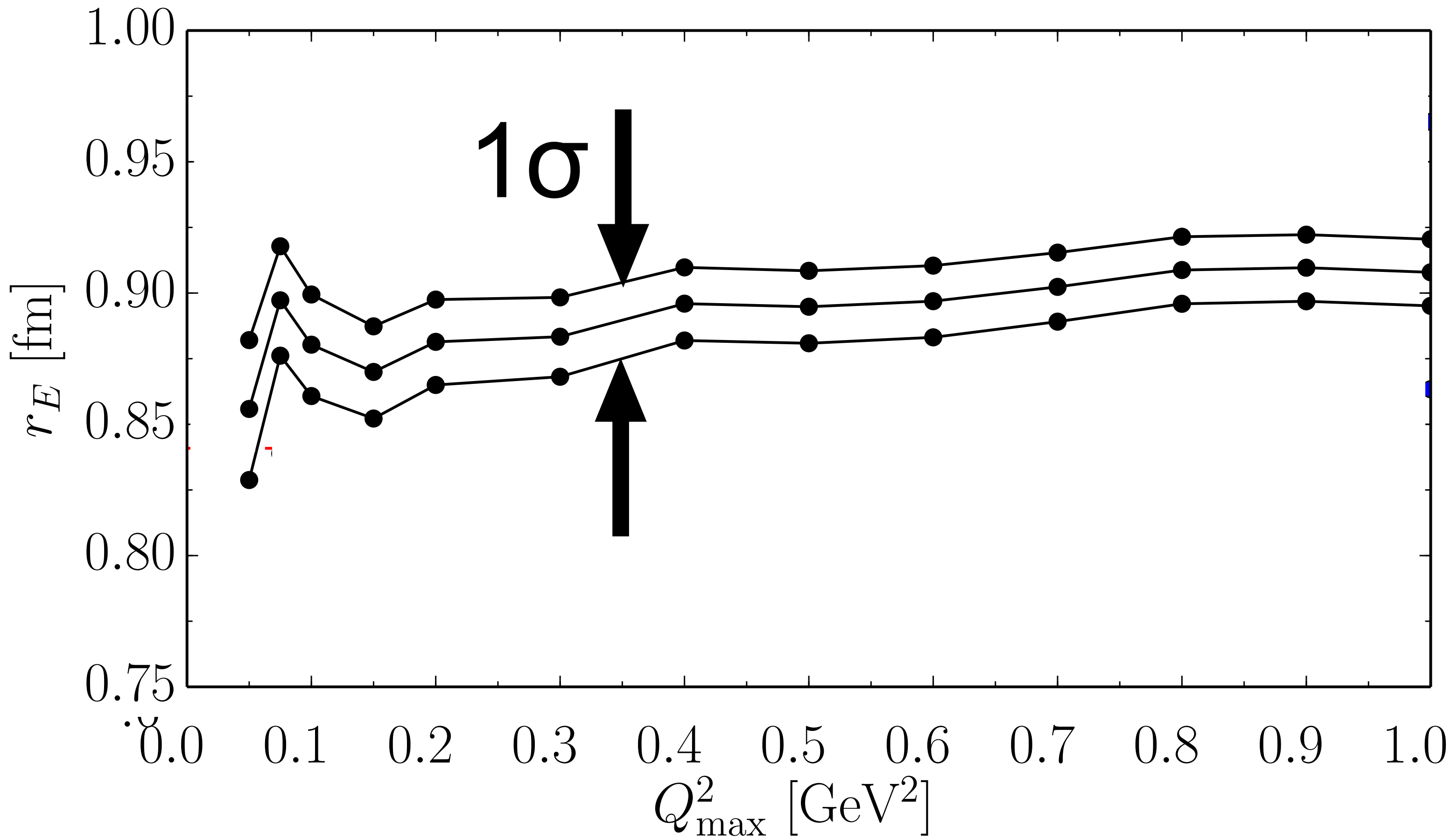
$$|F(q^2)|^2\left(1-\frac{\alpha}{\pi}\log\frac{Q^2}{m_e^2}\log\frac{E^2}{(\Delta E)^2}+\dots\right)\to|F(q^2)|^2\exp\left[-\frac{\alpha}{\pi}\log\frac{Q^2}{m_e^2}\log\frac{E^2}{(\Delta E)^2}\right]$$

## Captures leading logarithms when

$$Q\sim E\,,\quad \Delta E\sim m_e$$

## As consistency check, error budget should contain the difference from resumming:

$$\log^2\frac{Q^2}{m_e^2}\qquad \text{vs.}\quad \log\frac{Q^2}{m_e^2}\log\frac{E^2}{(\Delta E)^2}$$

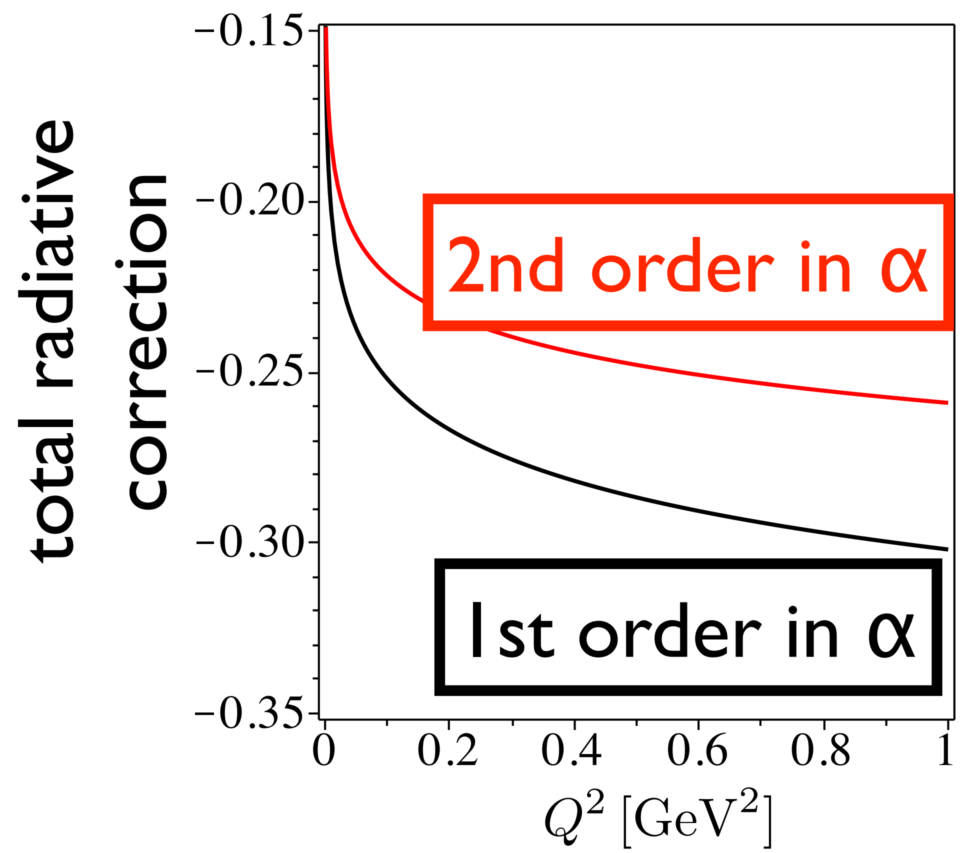



*electron energy:*

$$E = 1\,\mathrm{GeV}$$

*electron energy loss cut:*

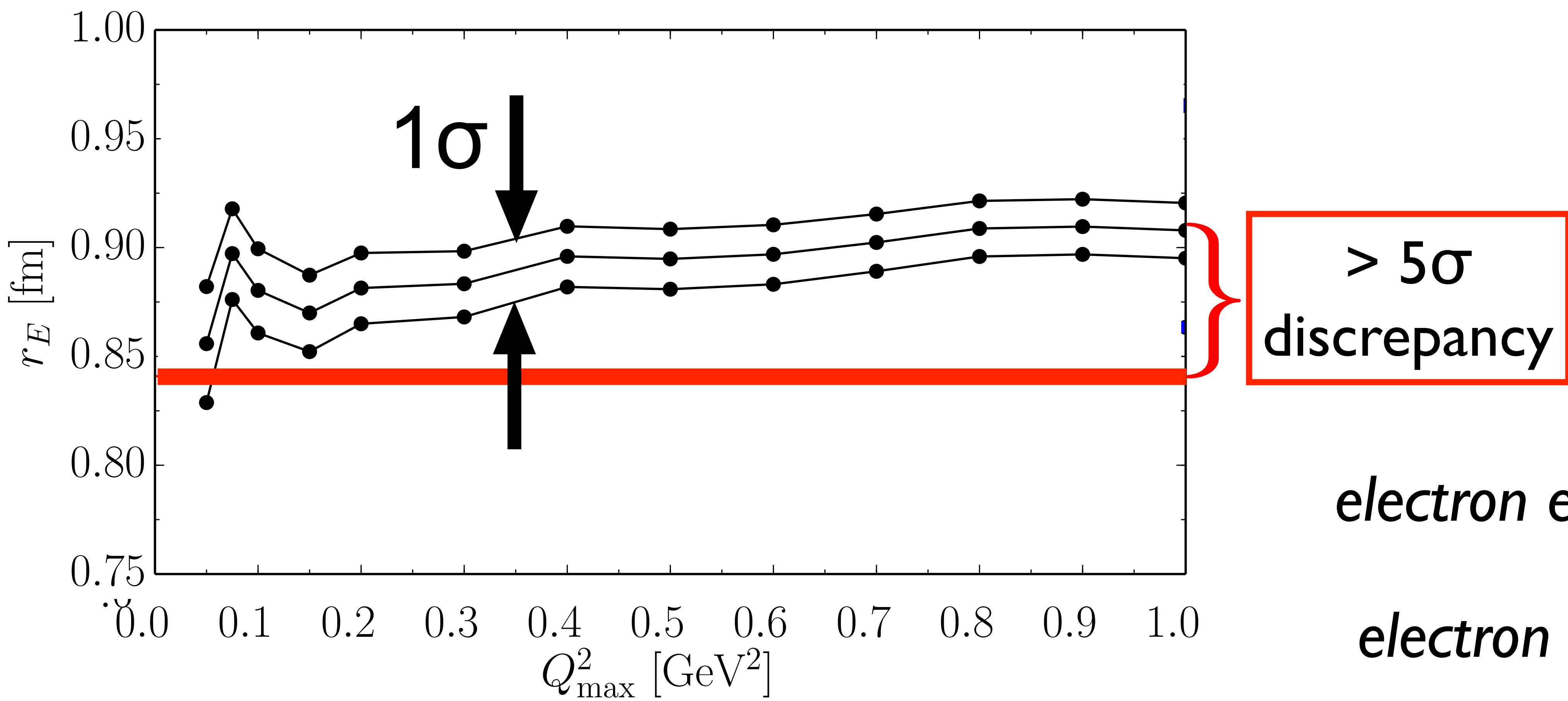
$$\Delta E = 5\,\mathrm{MeV}$$




- quoted systematics in A1 electron-proton scattering data are 0.2-0.5 %

- leading order radiative corrections ~30%

- need to systematically account for subleading logarithms, recoil, nuclear charge and structure

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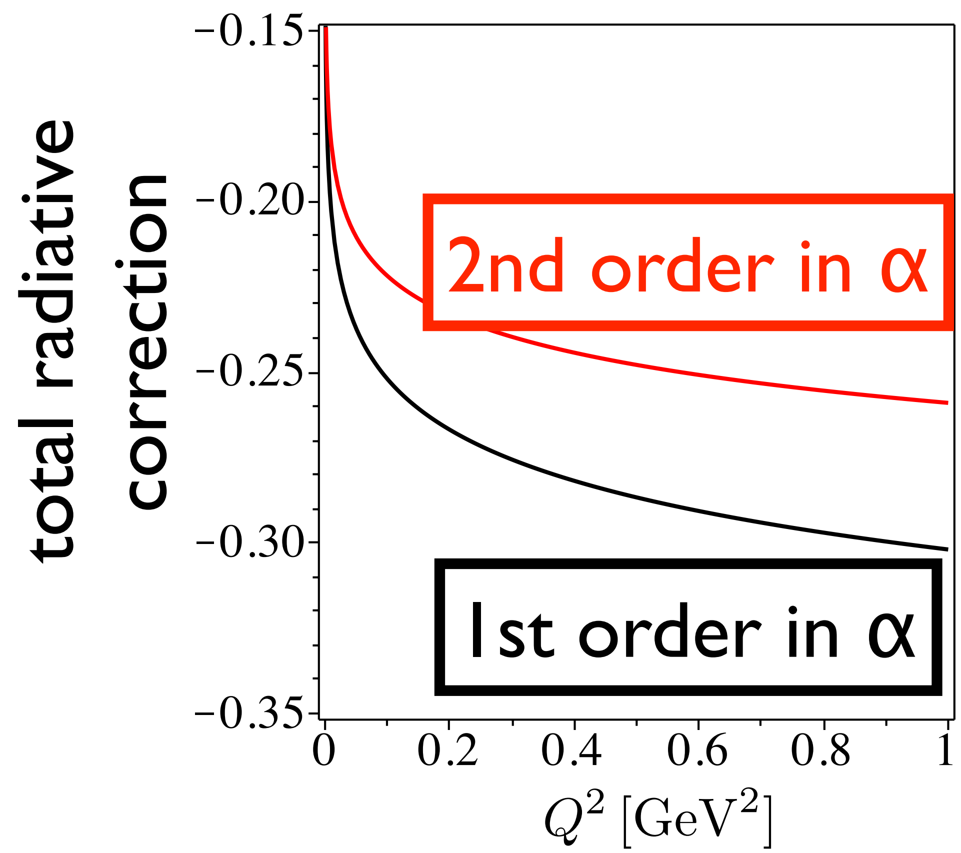


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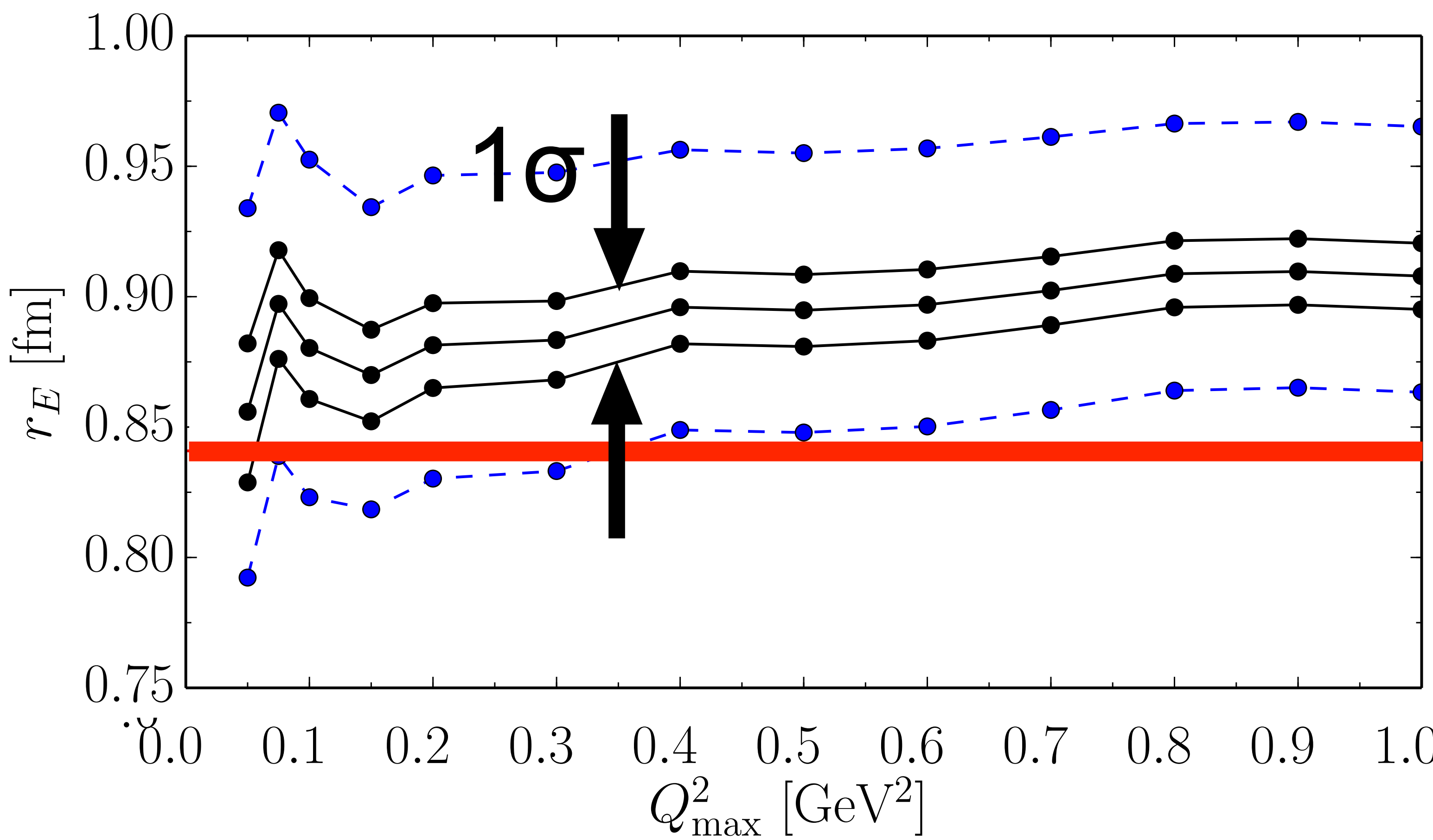


potentially large uncertainty from radiative corrections

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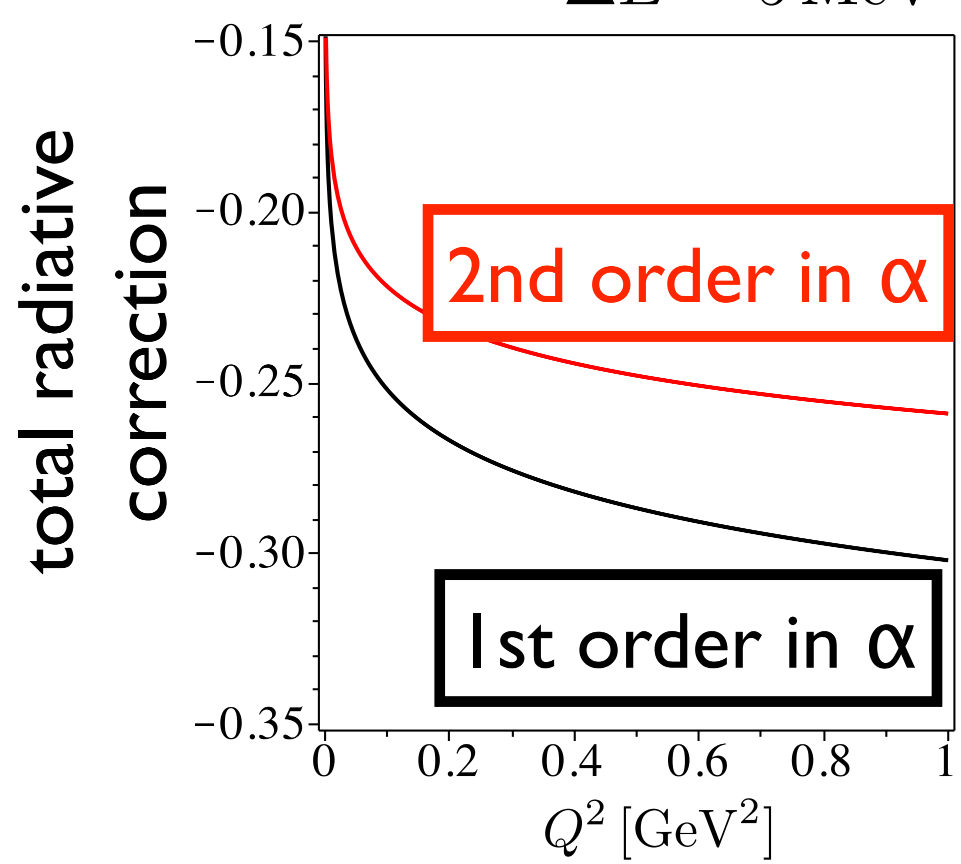


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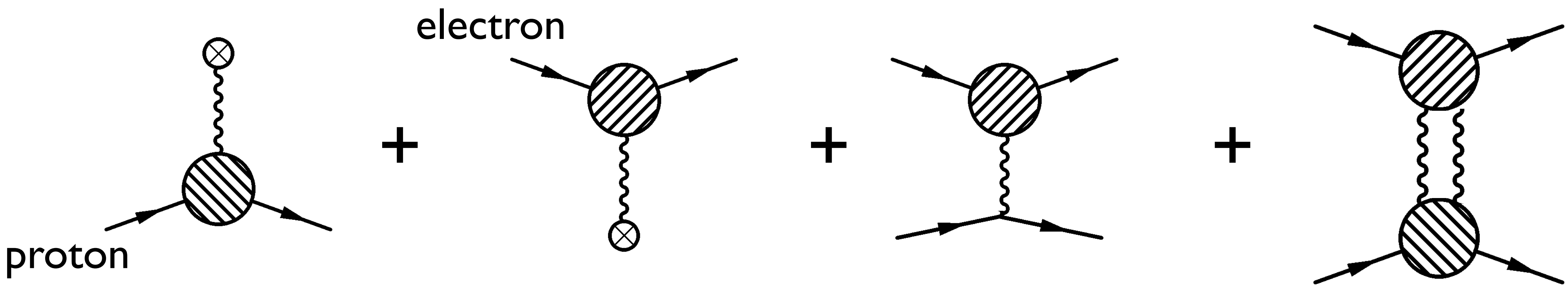
# treat the problem in stages:



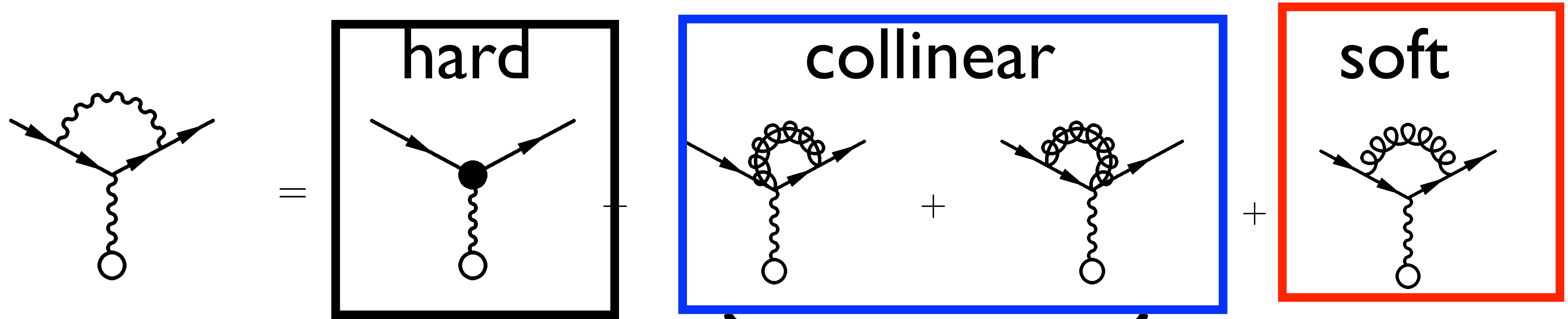

static source, nonrel. limit

static source, rel. limit

with recoil corrections

with nuclear charge corrections (two photon exchange)

- factorization




hadron structure
(Born form factors, …)

$$d\sigma \propto \boxed{H\left(\frac{Q^2}{\mu^2}\right)}\,\boxed{J\left(\frac{m^2}{\mu^2}\right) R\left(\frac{m^2}{\mu^2},\frac{p\cdot p'}{m^2}\right)}\,\boxed{S\left(\frac{\Delta E}{\mu},\frac{p\cdot p'}{m^2},\frac{E}{m},\frac{E'}{m}\right)}$$

[ remainder function starting
at 2-loop (collinear anomaly/rapidity logs) ]

*Becher, Melnikov (2007)*

*Chiu, Golf, Kelley, Manohar (2007)*

*Becher, Neubert (2010)*

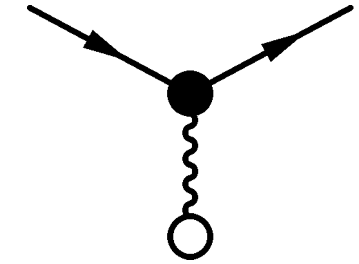
*Chiu, Jain, Neill, Rothstein (2012)*

…

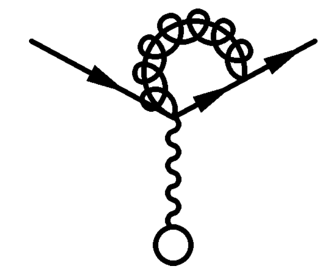
- physical electron mass regulates collinear divergences
- R given by ratio of Wilson loop matrix elements in m≠0/m=0

<u>Sudakov form factor at one loop:</u>

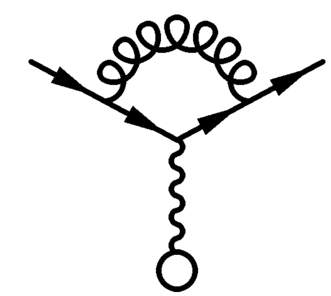
**Hard**
$$F_H(\mu) = 1 + \frac{\alpha}{4\pi}\left[-\log^2\frac{Q^2}{\mu^2} + 3\log\frac{Q^2}{\mu^2} - 8 + \frac{\pi^2}{6}\right]$$



**Collinear**
$$F_J(\mu) = 1 + \frac{\alpha}{4\pi}\left[\log^2\frac{m^2}{\mu^2} - \log\frac{m^2}{\mu^2} + 4 + \frac{\pi^2}{6}\right]$$



**Soft**
$$F_S(\mu) = 1 + \frac{\alpha}{4\pi}\left[2\log\frac{\lambda^2}{\mu^2}\left(\log\frac{Q^2}{m^2} - 1\right)\right]$$

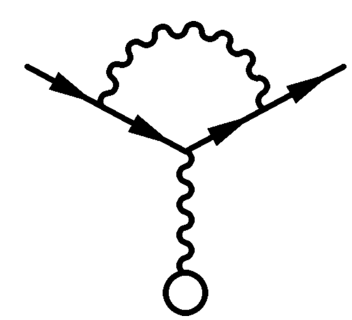


Large logarithms regardless of choice for μ

$F_S$: exponentiates (evaluate at any scale)

$F_J$: evaluate at μ~m

$F_H$: evaluate at μ~M~Q



*(two-loop matching, real+virtual see 1605.02613)*

$$F = F_H F_J F_S$$

## Two photon exchange

- Nuclear charge corrections introduce new spin structures (helicity counting: 3 amplitudes at leading power in $m_e/Q$)

$$F_H(\mu)\gamma^\mu \otimes \gamma_\mu \to \sum_{i=1}^{3} c_i(\mu)\, \Gamma_i^{(e)} \otimes \Gamma_i^{(p)}$$

- In principle, can use e+ and e- data to separately determine 1-photon exchange and 2-photon exchange contributions to $c_i$

- However, with available data, measure combination of 1-photon and 2-photon contributions.

- Regardless of treatment of hard coefficients, remaining radiative corrections are universal

$$d\sigma = H(M) \times \underbrace{\frac{H(\mu)}{H(M)} \times J(\mu) \times S(\mu)}$$

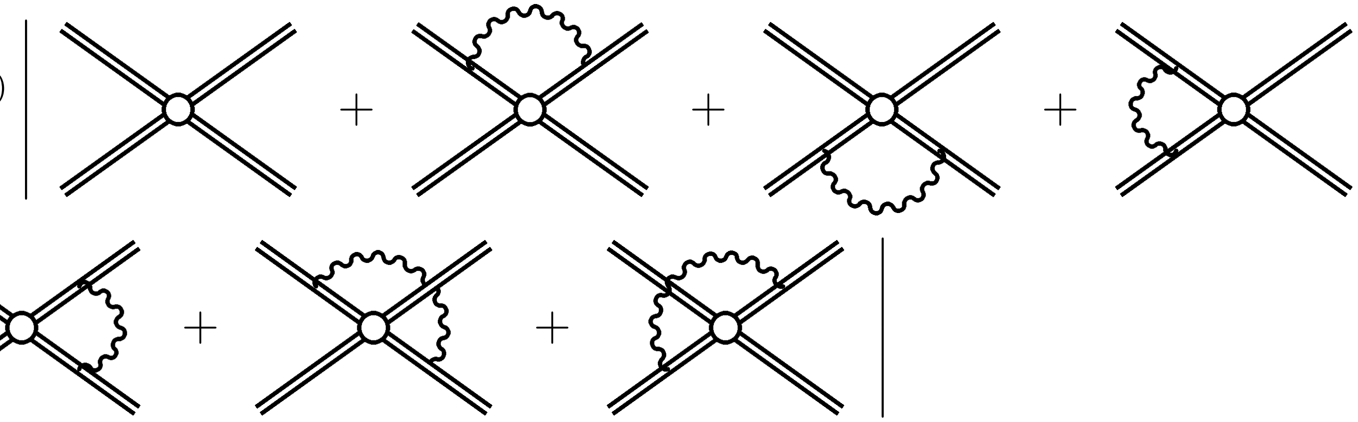
want to extract this

correct data by this factor

- J: refers to collinear region, same as before

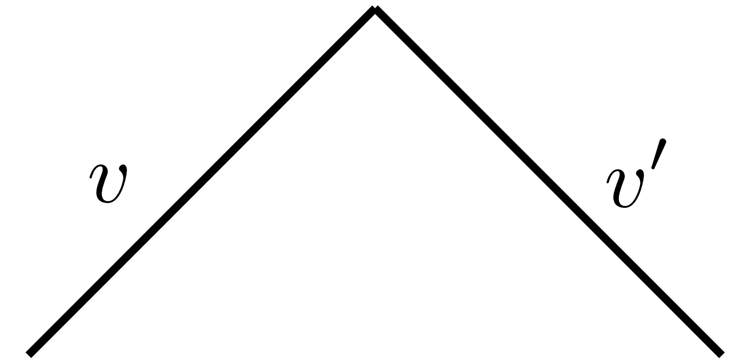
- S: include nuclear charge for general soft function (computed through 2-loop order)

$$\sqrt{S(\mu, \Delta E = 0)} = Z_h^{(e)} Z_h^{(p)}$$



- H(μ)/H(M): must now account for large logs in this factor

- resumming




governed by Wilson loops with cusps:

$$\bar{h} iv\cdot D h \to \bar{h}^{(0)} S_v^\dagger iv\cdot D S_v h^{(0)} = \bar{h}^{(0)} iv\cdot\partial h^{(0)}\,, \quad S_v(x) = P\exp\left[ i\int_{-\infty}^0 ds\, v\cdot A_s(x+sv)\right]$$

renormalization of hard function of interest:

$$\frac{d\log H}{d\log\mu} = 2\left[\gamma_{\rm cusp}(\bar\alpha)\log\frac{Q^2}{\mu^2} + \gamma_{\rm cusp}(v\cdot v', \bar\alpha) + 2\gamma_{\rm cusp}(\bar\alpha)\log\frac{v\cdot p'}{-v\cdot p - i0} + \gamma(\bar\alpha)\right]\,.$$

universal functions

$$\text{electron}: \; p^\mu$$
$$\text{proton}: \; Mv^\mu$$

solution, summing large logarithms:

$$\log\frac{H(\mu_L)}{H(\mu_H)} = -\frac{\alpha}{2\pi}\log^2\frac{\mu_H^2}{\mu_L^2} + \dots$$

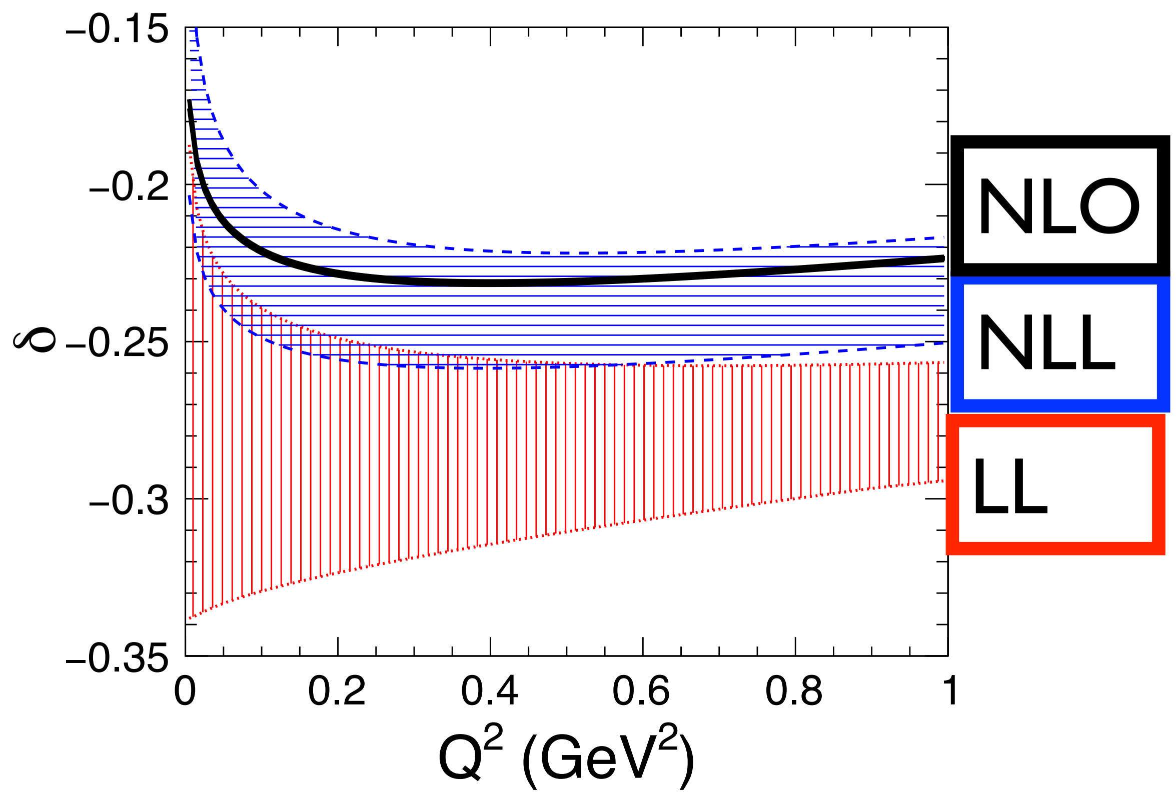
$$d\sigma = H(M) \times \underbrace{\frac{H(\mu)}{H(M)} \times J(\mu) \times S(\mu)}_{\text{total radiative correction}}$$

numerically: $\alpha L^2 = \alpha \log^2 \frac{Q^2}{m^2} \sim 1 \quad \Rightarrow \quad \alpha L \sim \alpha^{\frac{1}{2}}$ , etc.

*electron energy:* $E = 1\,\mathrm{GeV}$

*electron energy loss cut:* $\Delta E = 5\,\mathrm{MeV}$

total radiative correction



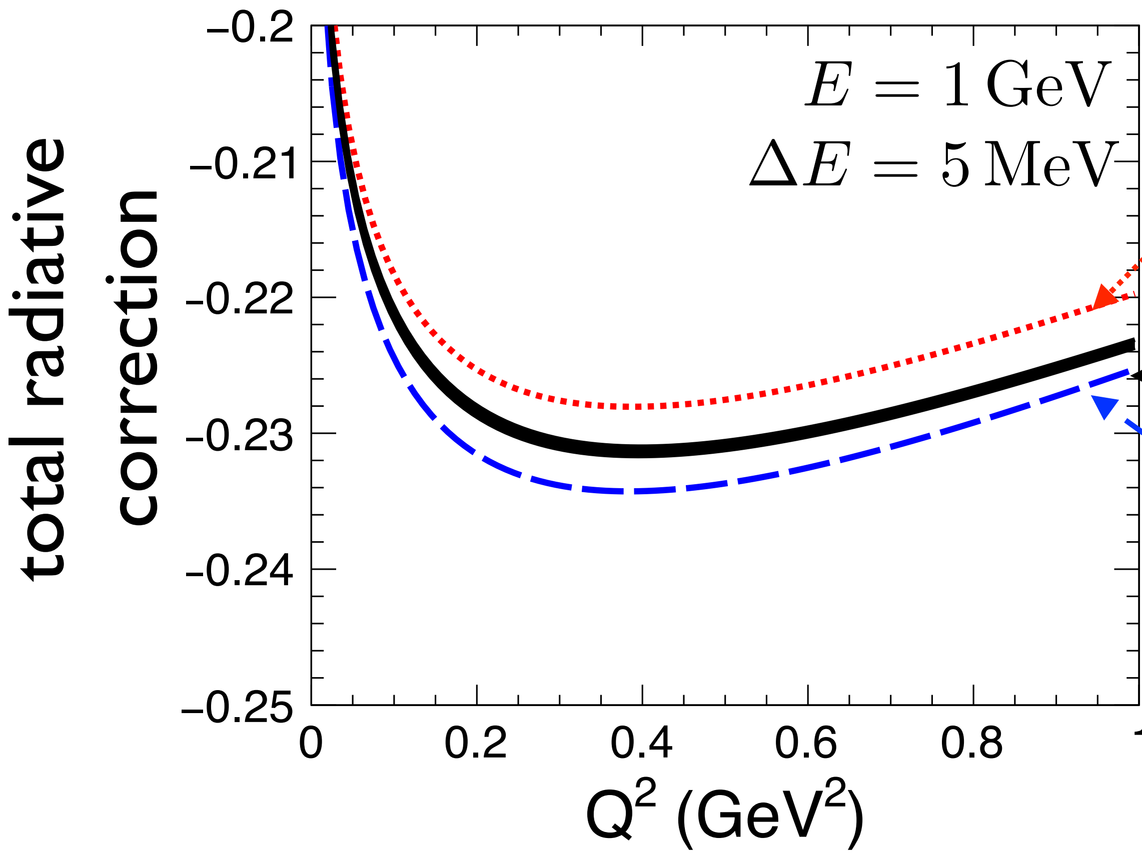

correct through:

NLO $\mathcal{O}(\alpha)$

NLL $\mathcal{O}(\alpha^{\frac{1}{2}})$

LL $\mathcal{O}(1)$

Comparison to previous implementations of radiative corrections, e.g. in A1 analysis of electron-proton scattering data




naive exponentiation of 1-loop, ($\mu^2=M^2$ in two-photon piece)

resummed EFT result

naive exponentiation of 1-loop, ($\mu^2=Q^2$ in two-photon piece)

- discrepancies at 0.5-1% compared to currently applied radiative correction models (cf. 0.2-0.5% systematic error budget of A1 experiment)

- conflicting implicit scheme choices for 1PE and 2PE

- complete analysis: account for floating normalizations, correlated shape variations when fitting together with backgrounds

EFT analysis clarifies several issues involving conflicting and/or implicit conventions and scheme choices

**1) Scheme choice and definition of radius and "Born" form factors**

**2) Scheme dependence of two-photon exchange**

**3) Limitations of naive exponentiation**

## 1) Scheme choice and definition of radius and "Born" form factors

$$\langle J^\mu \rangle = \bar{u}_{v'} \left[ \tilde{F}_1 \gamma^\mu + \tilde{F}_2 \frac{i}{2} \sigma^{\mu\nu} (v'_\nu - v_\nu) \right] u_v$$

Massive particle form factor (e.g. for proton):

$$\tilde{F}_i = F_H F_S$$

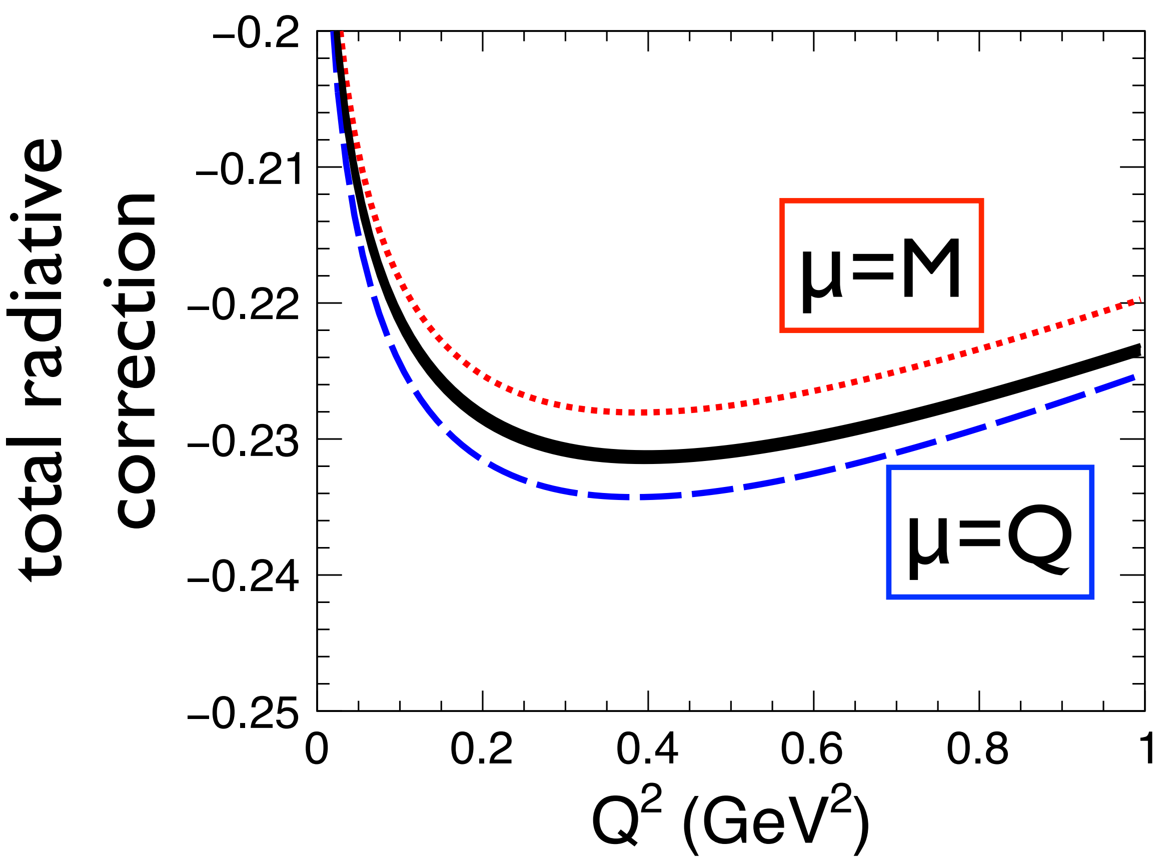
$$F_H(q^2, \mu = M) \equiv F_i(q^2)^{\text{Born}} \equiv \tilde{F}_i(q^2) F_S^{-1}(w, \mu = M)$$

hard coefficient          soft function

Multiple conventions in the literature. Different "Born" form factors, different radii (differences typically below current precision)

**2) Scheme dependence of two-photon exchange**

As for form factors, define hadronic functions in the general 2→2 scattering process as the hard component in the factorization formula at factorization scale μ=M




Prevailing conventions have used conflicting μ=M for 1 photon exchange, μ=Q for 2 photon exchange

A scale-variation estimate of uncertainty in the 2 photon exchange subtraction

## 3) Limitations of naive exponentiation

- Renormalization analysis for subleading logs :

$$\log \frac{H(\mu_L)}{H(\mu_H)} = -\frac{\alpha}{2\pi} \log^2 \frac{\mu_H^2}{\mu_L^2} + \dots$$

⇒ New terms at order $\alpha^2 L^3$, $\alpha^2 L^2$, $\alpha^3 L^4$, …

- *Total* versus *individual* real photon energy below $\Delta E$ :

$$S^{(2)} = \frac{1}{2!}[S^{(1)}]^2 - \frac{16\pi^2}{3}(L-1)^2 \qquad S = \sum_n \left(\frac{\alpha}{4\pi}\right)^n S^{(n)}$$

⇒ New terms at order $\alpha^2 L^2$

complete analysis: account for floating normalizations, correlated shape variations when fitting together with backgrounds. stay tuned

# Summary

- lepton-nucleon scattering: new arena for SCET

- exponentiation Ansatz a la Yennie et al. fails at the level of current experimental precision

- systematic error from missing soft+collinear radiative corrections potential explanation for proton radius puzzle

- complete calculation of soft+collinear factors: leaves 2-3 sigma radius tension between e-p and μH extractions

- SCET does not determine the hard matching coefficients (nonperturbative inputs), accessible from e+/e- ratios

- formalism applies to critical neutrino applications.  stay tuned (and hop aboard).