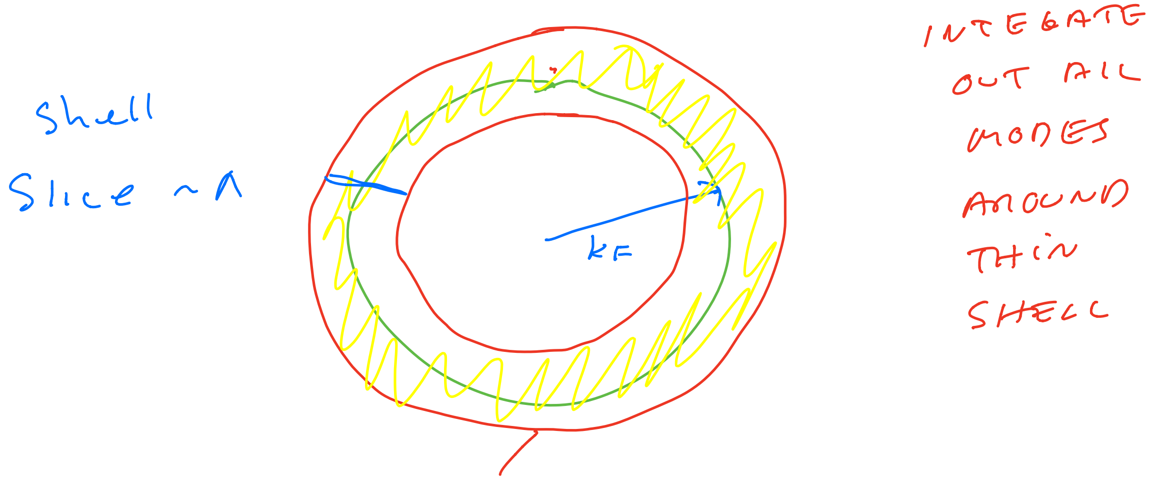
EFT of (Non) Fermi Liquid Behavior

Work Done with Anton Kapustin + Tristan McKinney

Fermi Liquid

shares much in common with SCET + NRQCD

Degenerate electron Gas



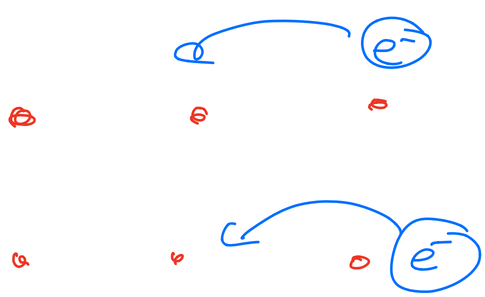
EXPAND FIELD AROUND FERMI SURFACE

EXPANSION PARAMETER

$$\lambda \sim \alpha / E_F$$

WHAT ARE THE RELEVANT D.O.F ?

U.V. THEORY: e^- interact with Lattice



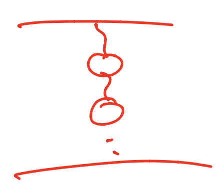
CAN'T ASSUME WEAK COUPLING.

IN U.V: electrons + phonon
+ photons
"full theory"

As long as ∃ no phase transition (QUANTUM CRITICAL)
THEN EXPECT CONTINUITY IN SPECTRUM:

IN I.R.:

A_μ : potential modes gapped due to SCREENING:

$V(r)$  $\sim \dfrac{e^{-mr}}{r}$

Radiation Modes: v/c down.

Φ^I: phonons

"potential" phonons:

$$\frac{1}{\omega^2 - V_s^2 \bar{k}^2}$$

$\omega < v_s k_F$ integrate out generates "local" potential: $\delta(x)$

"Radiation Phonon"

power suppressed due to deriv. coupling

Left with "e^-"

"quasiparticles" : $m^* \simeq 200\, m_e$

effective mass.

Action :

$$L = \int \psi^\dagger_{(k)}\, i\partial_t \psi_{(k)} + \psi^\dagger \psi\, \varepsilon(k)$$

unknown function of k

$$+\, g(k_i)\, \psi^\dagger_{k_1} \psi_{k_2} \psi^\dagger_{k_3} \psi_{k_4}$$

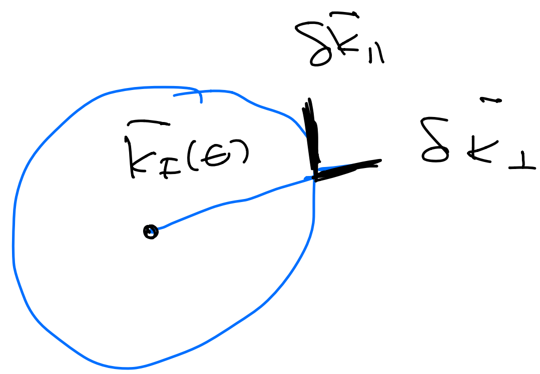
"coupling function"

NOTE: CAN <u>NOT</u> PERFORM DERIVATIVE EXPANSION (light-cone OPE)

Shankar,
Polchinski,
Benfatto
Gallavotti

POWER COUNTING

$$\vec{k} = \vec{k}_F(\theta) + \delta\vec{k}_\perp + \delta\vec{k}_\parallel$$



$$\delta\vec{k}_\perp \sim \lambda, \quad \delta\vec{k}_\parallel \sim 1$$

Break up surfaces
into "θ BINS"

$$\psi(x) = \sum_i e^{i k_F(\theta)\cdot\vec{x}}\, \psi_{\theta_i}(x)$$

NOTE: $\epsilon(k) \to \epsilon(k_\parallel)$ arbitrary function

Determine Relevant Int:

Field Scaling:

$$\int \psi^{\dagger} \partial_{\tau} \psi \, d^{d}x \, d\tau$$

$$\psi^{\dagger}\partial_\tau\psi \downarrow \lambda, \quad d^d x \downarrow \frac{1}{\lambda}, \quad d\tau \downarrow 1/\lambda$$

$$\psi(x) \sim \lambda^{1/2}, \quad \psi(p) \sim \lambda^{-1/2}$$

CONSIDER INTERACTION:

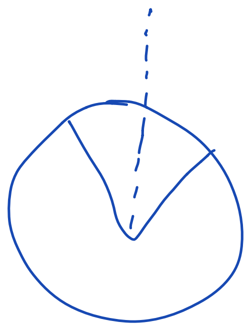
$$S = \int d\tau \prod_{i=1}^{4} d^2 p_i \, \psi^{\dagger}_{\theta_1} \psi_{\theta_2} \psi^{\dagger}_{\theta_3} \psi_{\theta_4} \, \delta^2\left(\sum p_i\right)$$

$$\boxed{d=2}$$

$$\sim \frac{1}{\lambda} (\lambda)^4 (\lambda^{-2}) \, 1 \sim \lambda$$

IRRELEVANT!

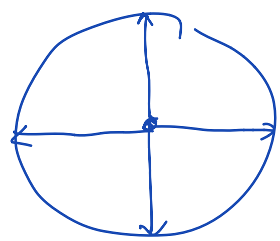
Special Kinematics



Forward scattering

or

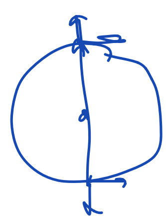
B.C.S



Assuming time reversal invariance.

Consider: $\delta^2(p_1 + p_2 + p_3 + p_4)$

$\vec{p}_1 = -\vec{p}_2$



Back-to-Back

$$\delta^{\parallel}(\vec{p}_3 + \vec{p}_4)\,\delta^{\perp}(\vec{p}_3 + \vec{p}_4)$$

$$\downarrow$$

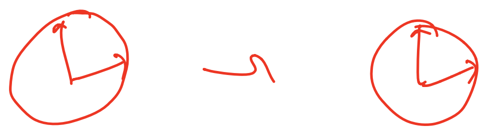
$$\frac{1}{\lambda}$$

now $\bar{\psi}\psi\psi^{\dagger}\psi$ is marginal.

analogous to power counting near end-points.

On FORWARD SCATTERING

(Glauber phonons)



So two possible interactions

$$L_{int} = \int d^4x \; g_{BCS}(\theta_1-\theta_2)\, \psi^\dagger_{\theta}\, \psi_{-\theta}\, \psi^\dagger_{\theta'}\, \psi_{-\theta'}$$

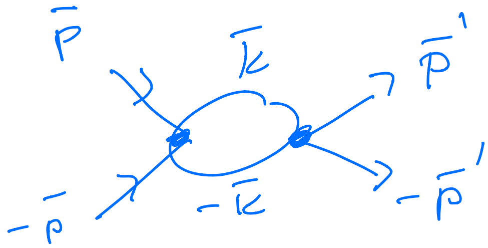
$$+ \int d^4x \; g_{F.S}(\theta_1-\theta_2)\, \psi^\dagger_{\theta}\, \psi_{\theta}\, \psi^\dagger_{\theta'}\, \psi_{\theta'}$$

coupling functions

analagous to P.D.F.

NOTE: 1-loop β-function

is EXACT



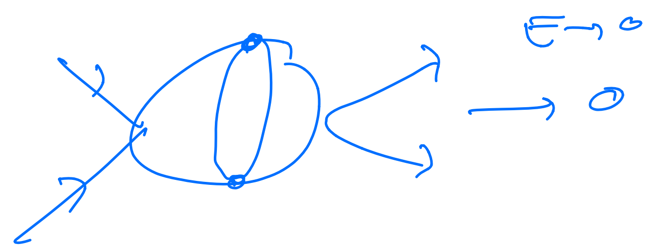
B.C.S

INTERACTION

Kinematics Don't Allow

For Corrections:

e.g.



NOTE: MORE INTERESTING

THAN $\delta(x)$ potential, since

IN NON-TRIVIAL BACKGROUND.

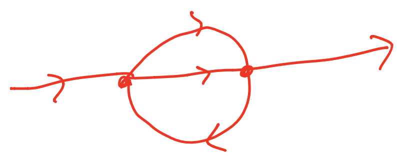
$$\boxed{\text{Canonical Fermi Liquid}}$$

$$L = i\psi^\dagger \partial_t \psi + \underbrace{\frac{\partial \varepsilon}{\partial k}}_{v_F}\cdot \vec{k}_\perp \psi_k^\dagger \psi_k + L_{int}$$

prediction for quasi-particle

Life time:

no contribution L.O.

Im 

power suppressed

$\sim E^2$

$\Gamma \sim E^2$ leads to

canonical prediction for

Resistivity $\rho(T) \sim \frac{1}{\sigma(T)}$

$$\sigma(T) \sim \frac{n\tau e^2}{m} \sim \frac{1}{T^2}$$

NOTE: ONE Hallmark of High-Tc

COMPOUNDS $\rho(T) \sim T$!

SO CANONICAL FERMI LIQUIDS

CAN'T EXPLAIN HIGH-Tc

Kane et al: 1987

"Marginal" Fermi Liquid

(MFL)

$\Gamma \sim E$ "theoretical"

underpinning

NEED DIFFERENT POWER

COUNTING!

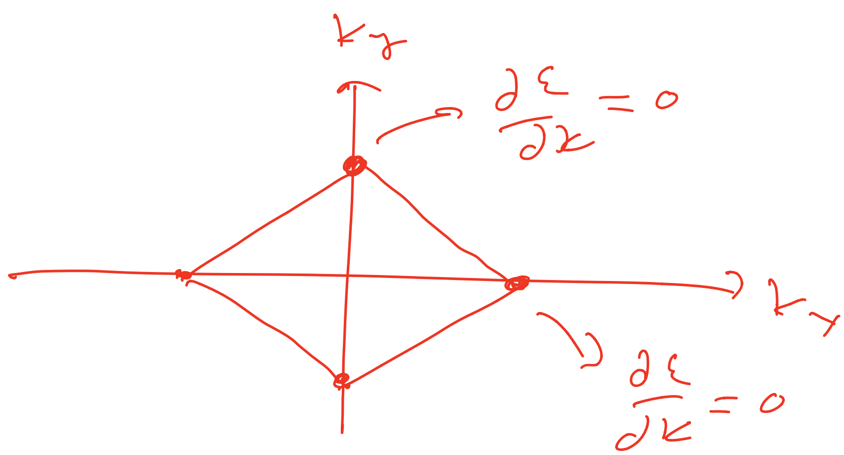
Hubbard Model

$$H = \sum_i t\, a_i^\dagger a_{i+1} + V (a_i^\dagger a_i)^2$$

continuum Limit

$$\varepsilon(k) = t(\cos k_x, \cos k_y)$$

@ Half-filling

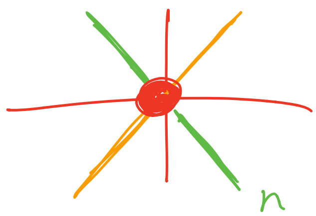


"Van Hove"

singularity

density of states

diverges



NEAR $(0,\pi)$, $(\pi,0)$

$$E = t(k_x^2 - k_y^2) = (k_+ k_-) +$$

Break up Fermi surface

$$\psi = \psi_{vH} + \psi_{NvH}$$

$$\epsilon = p_x^2 - p_y^2 \qquad \epsilon = v_F \cdot p$$

Moreover:

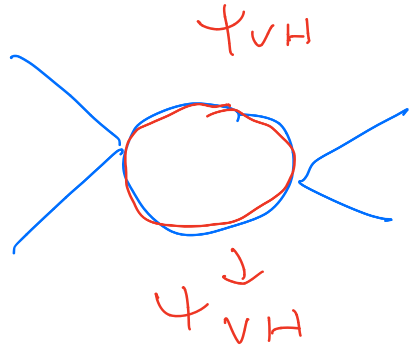
$$\psi_{vH} = \psi_n + \psi_{\bar{n}} + \psi_s$$

$$\boxed{(1,\lambda)} \quad \boxed{(\lambda,1)} \quad \boxed{(\sqrt{\lambda},\sqrt{\lambda})}$$

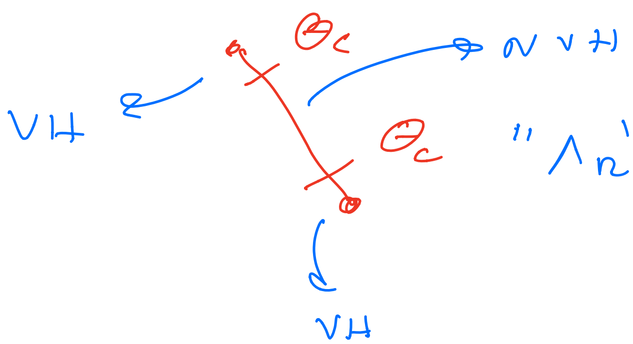
Analagous to

SCET II

Expect Rapidity R.G!



energy cut-off not sufficient to regulate. (Rapidity regulate)



V-H loop:

$$\sim \ln\left(\frac{\Lambda_E}{E}\right)\ln\left(\frac{\Lambda_E}{\Lambda_R}\right)$$

N-VH loop: $\sim \ln\left(\frac{\Lambda_E}{E}\right)\ln\left(\frac{\Lambda_R}{t}\right)$

TOTAL: $\ln\left(\frac{\Lambda_E}{\epsilon}\right)\ln\left(\frac{\Lambda_E}{t}\right)$

β function:

$$\frac{dg}{d\ln\Lambda_E} \sim \frac{g^2}{4\pi}\ln\left(\frac{t}{\Lambda_E}\right)$$

note: no
longer 1-loop
EXACT

$$g = \frac{g(\Lambda_0)}{1+\frac{g(\Lambda_0)}{8\pi^2}\left(\ln^2\frac{\Lambda}{t}-\ln^2\frac{\Lambda_0}{t}\right)}$$

enhanced running

T_c is raised as
cooper pairs form
earlier.

GAP EQUATION:

Calculate T_c:

$$T_c = \frac{2e^{\gamma}}{\pi} \Lambda_*$$

STRONG COUPLING SCALE

$$\Lambda_* = t\, e^{-\sqrt{\ln^2 \frac{t}{\Lambda_0} + \frac{8\pi^2}{g_0}}}$$

$\boxed{g_0 < 0}$

EFFECT OF Singularity

IS TO RAISE T_c.

NOTE: DID NOT ACCOUNT

FOR $g = g(\theta)$

IN VH REGION $g = g$

IGNORED g RUNNING AWAY

FROM HOT SPOTS.

- High T_c compound known to be p-wave superconductors

$$\langle \psi^\dagger \partial^i \psi \rangle \neq 0$$

- EXISTENCE OF "PSEUDO-GAPS"

① Below Tc gap DOES NOT CLOSE.

② "VERY FEW" STATE EXIST BETWEEN GROUND STATE AND 'PSEUDO-GAP'.

D.O.S $\longrightarrow 0$.

- EXISTENCE OF FERMI-ARCS

'LOCALIZED" GAPS, ON FERMI SURFACE.