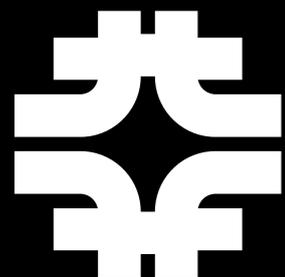


BOOSTED GGF H(BB)

# FIRST SEARCH FOR BOOSTED HIGGS $\rightarrow$ BB WITH CMS

LHC HIGGS XS WG WORKSHOP  
CERN  
GENEVA, SWITZERLAND

JULY 14, 2017

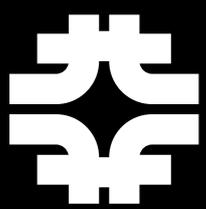
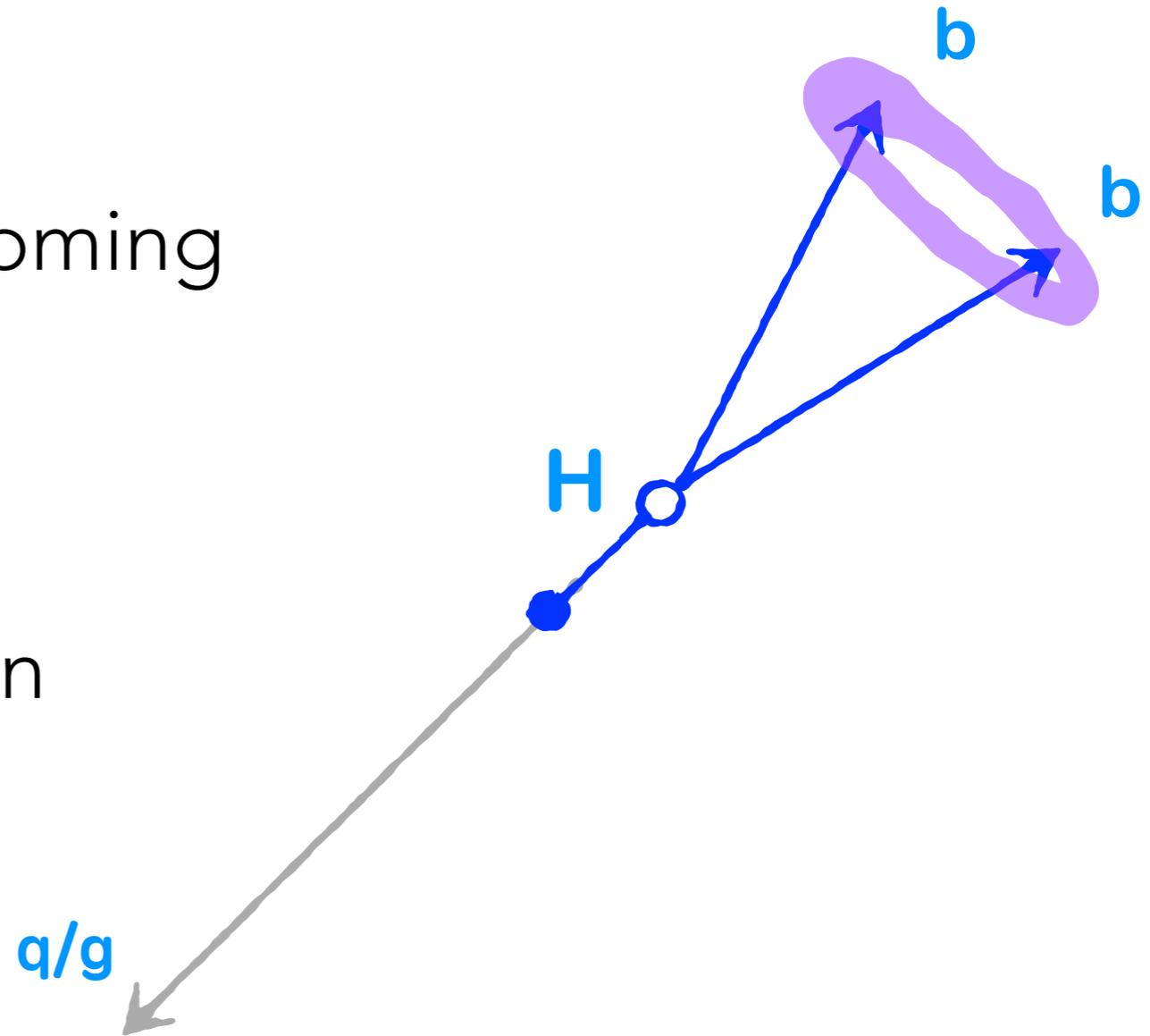


Javier Duarte  
Fermilab



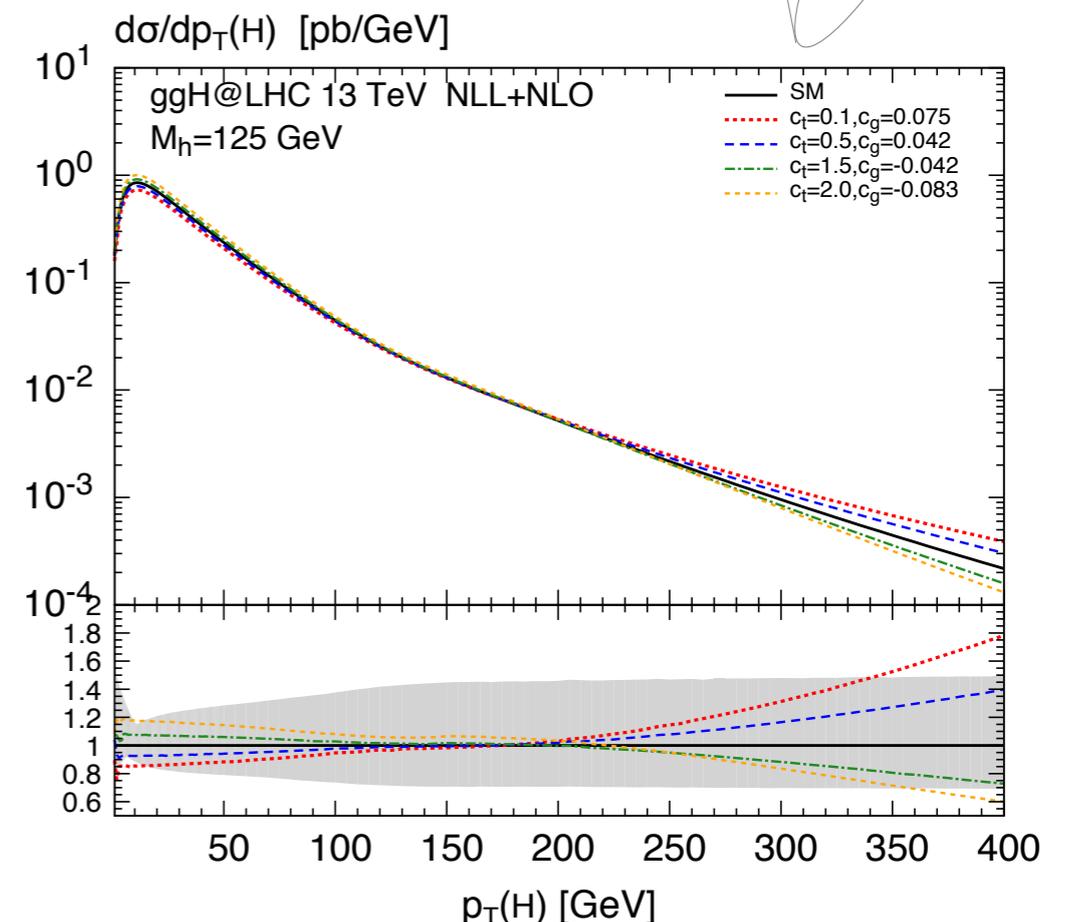
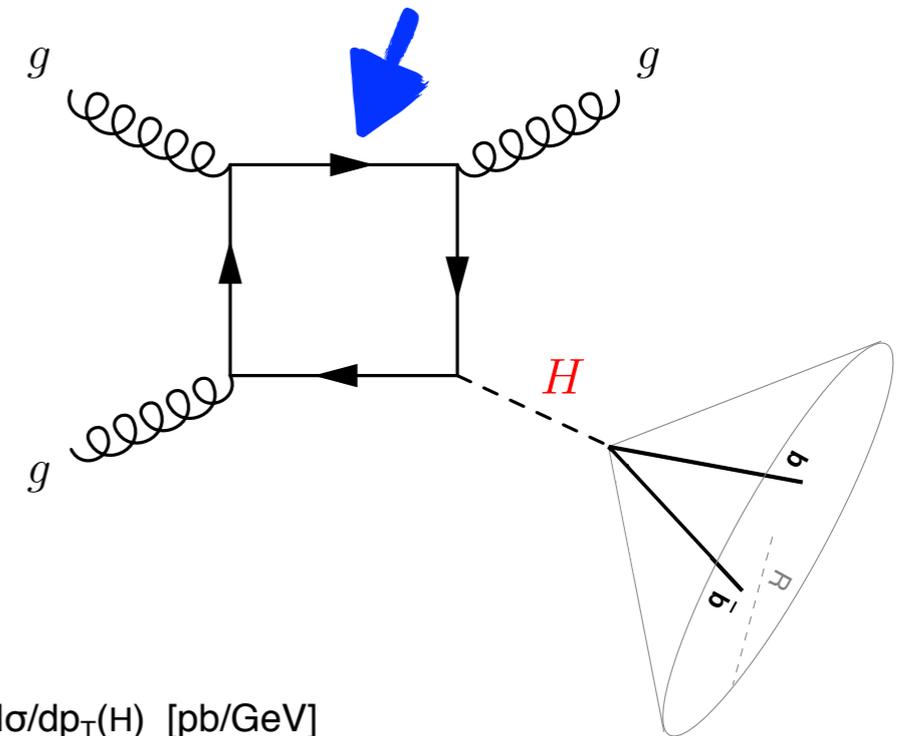
# OUTLINE

- Motivation
- Experimental techniques
  - Jet substructure and grooming
  - Double-b-tagging
- Event selection
- Data-driven QCD estimation
- Higgs  $p_T$  modeling
- Results
- Summary and outlook



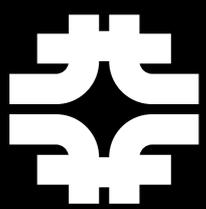
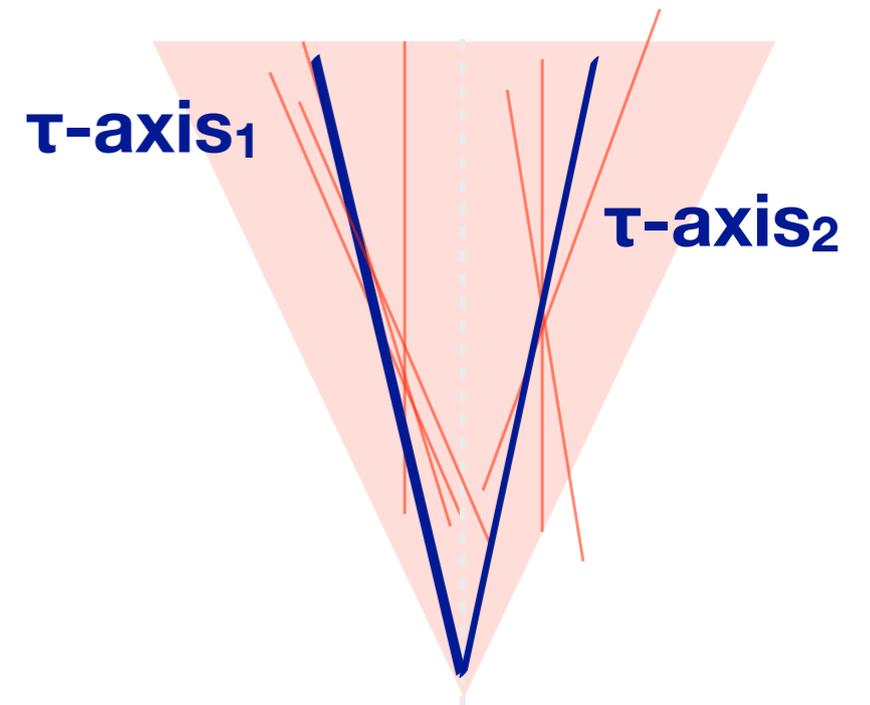
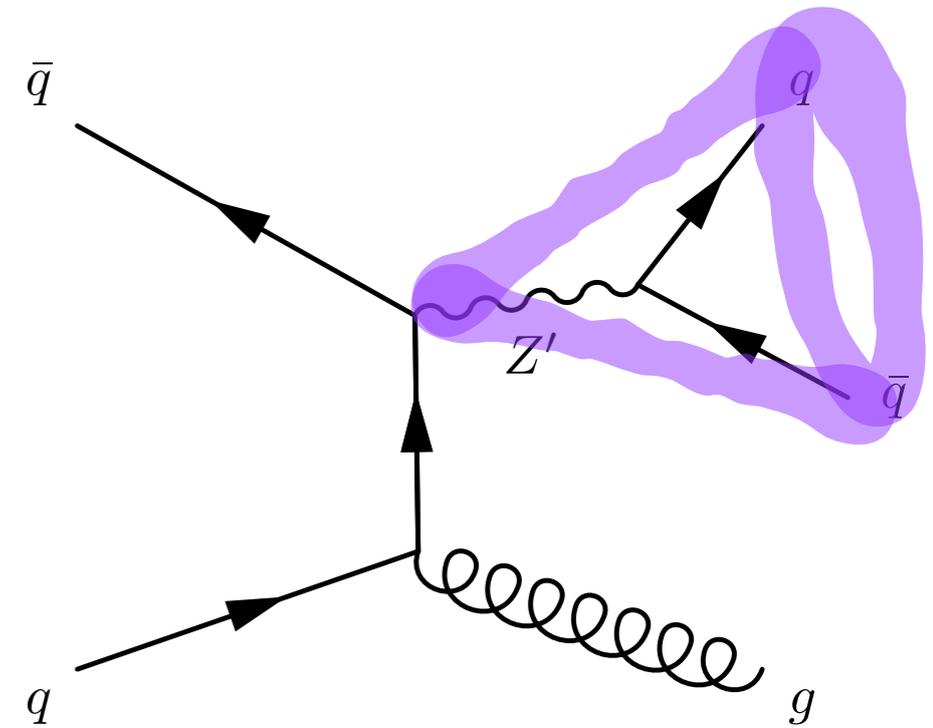
# MOTIVATION $t, \tilde{t}, X?$

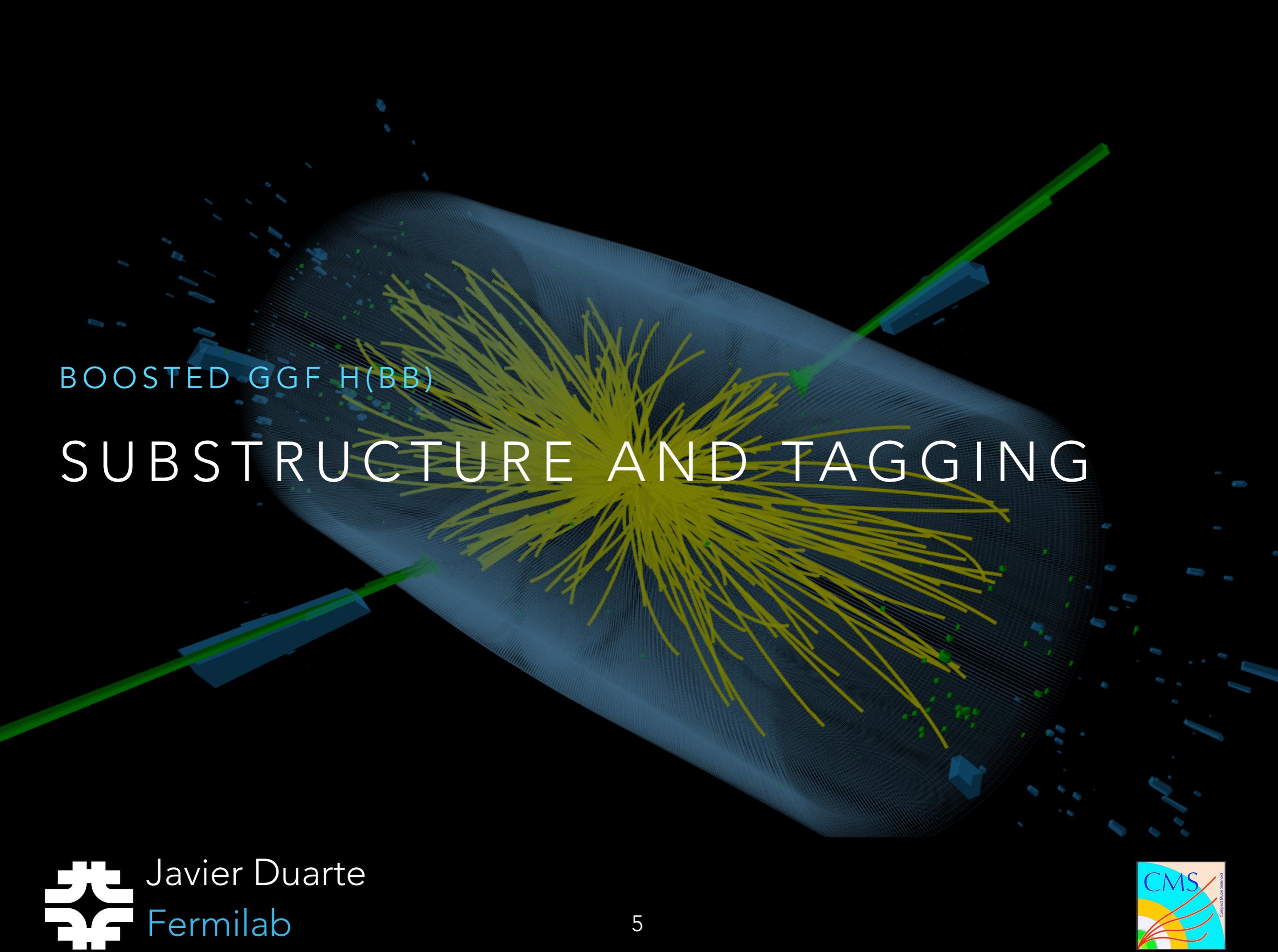
- Search for  $gg \rightarrow H \rightarrow bb$   
historically thought impossible  
due to overwhelming and  
difficult to predict QCD  
background
- We can access this process in  
the boosted dijet topology
- Probing Higgs couplings at high  
momentum transfer ( $Q$ )  
accesses large new physics  
energy scale ( $\Lambda$ )



# SO HOW CAN WE DO IT?

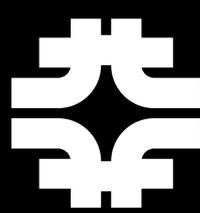
- Inspiration from boosted  $Z'$ +jet search?
  - ✓ Use ISR jet to get you above the trigger threshold
  - ✓ Requires one boosted fat jet
  - ✓ Substructure and jet grooming to enhance S/B
  - ✓ Data-driven background estimate
- Inspiration from machine learning and b-tagging?
  - ✓ Double b-tagger selects fat jets containing two b-quarks





BOOSTED GGF H(BB)

# SUBSTRUCTURE AND TAGGING

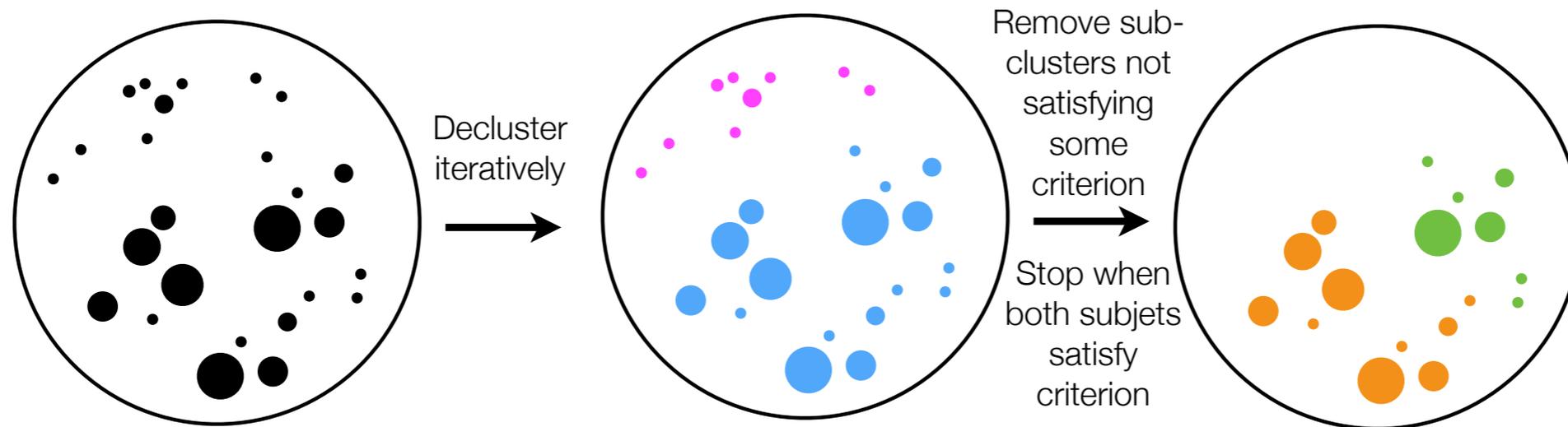


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Fermilab



# JET MASS

- Provides good separation between W/Z/H-jets from q/g jets
- Grooming removes soft and wide-angle radiation (soft drop / modified mass soft drop)



Soft Drop Condition: 
$$\frac{\min(p_{T1}, p_{T2})}{p_{T1} + p_{T2}} > z_{\text{cut}} \left( \frac{\Delta R_{12}}{R_0} \right)^\beta$$

CMS:  $z_{\text{cut}} = 0.1, \beta = 0$

# JET SUBSTRUCTURE

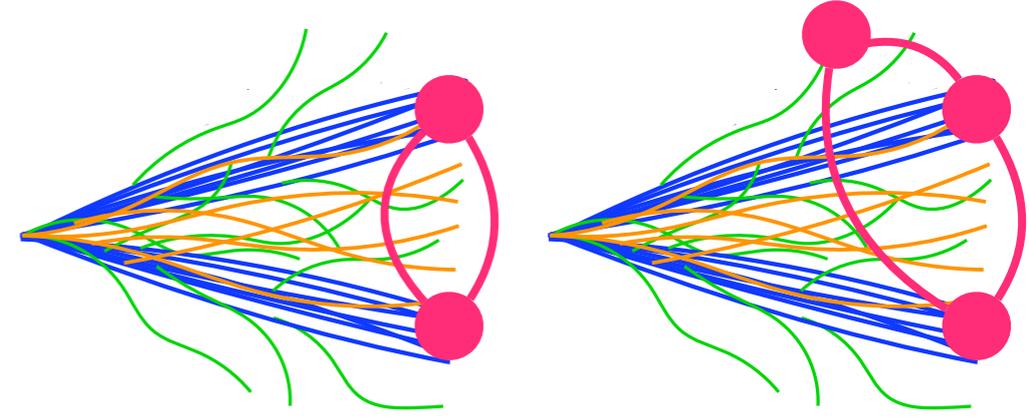
- How many “prongs” are in the jet?
- Generalized energy correlation functions are sensitive to N-point correlations within a jet

$$N_2^\beta = \frac{2e_3^\beta}{(1e_2^\beta)^2}$$

- A two-pronged jet has  $2e_3 < (1e_2)^2$
- Taking a ratio gives  $N_2^1$

2-point

3-point

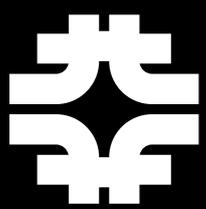
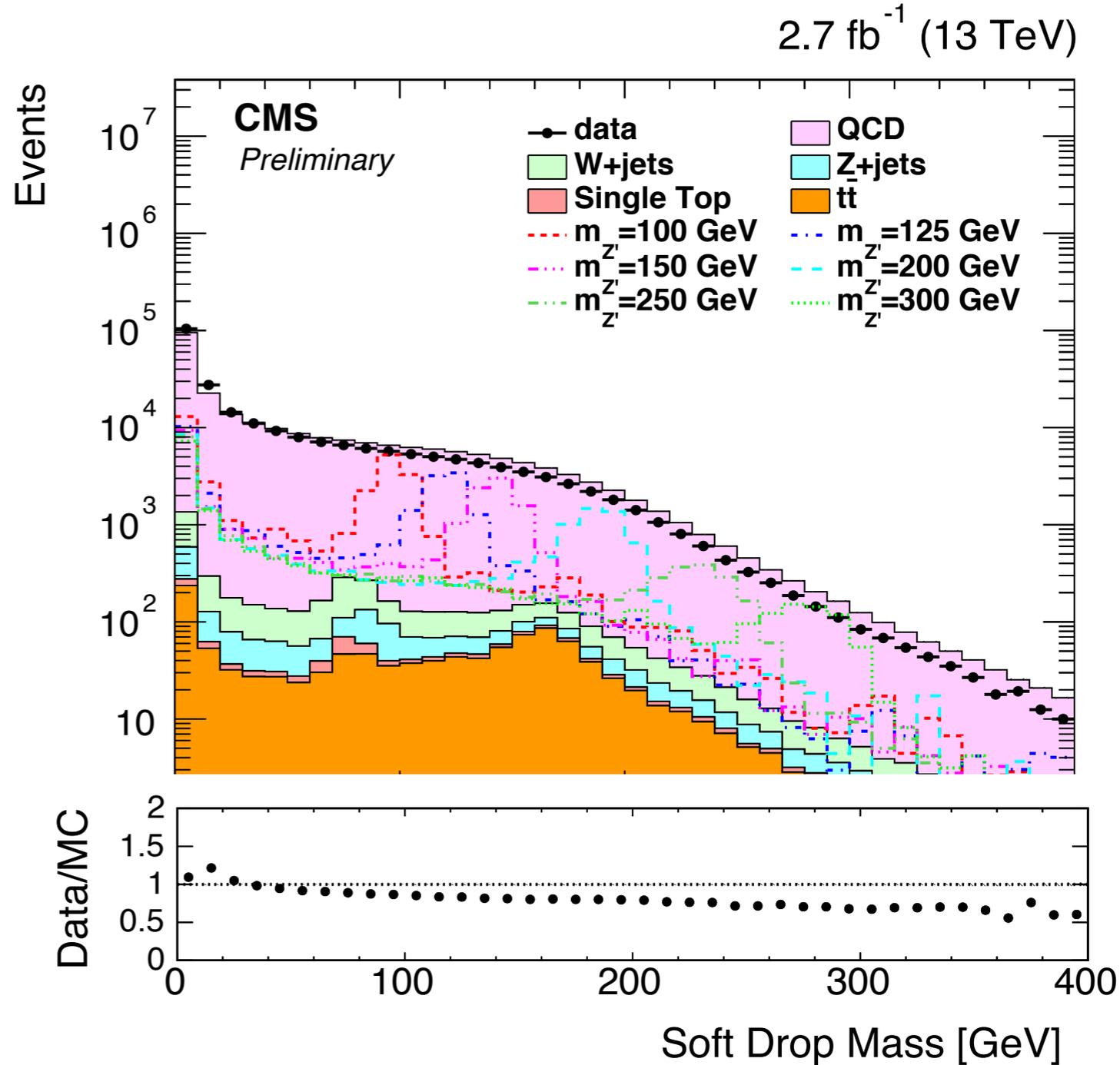


$$1e_2^\beta = \sum_{1 \leq i < j \leq n_J} z_i z_j \Delta R_{ij}^\beta$$

$$2e_3^\beta = \sum_{1 \leq i < j < k \leq n_J} z_i z_j z_k \min\{\Delta R_{ij}^\beta \Delta R_{ik}^\beta, \Delta R_{ij}^\beta \Delta R_{jk}^\beta, \Delta R_{ik}^\beta \Delta R_{jk}^\beta\}$$

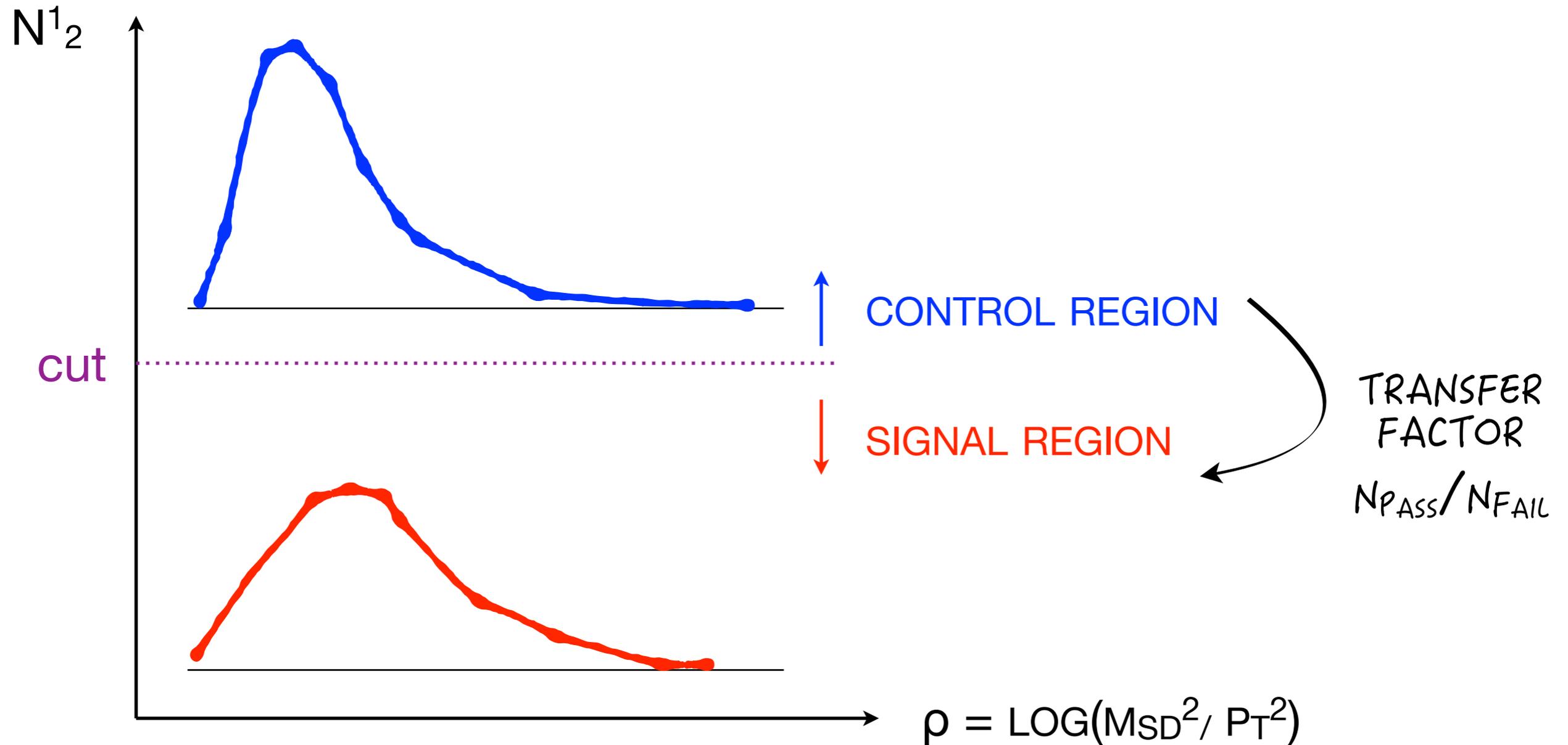
$$\beta = 1$$

- Here's what the boosted Z'+jet analysis looks like after kinematic selection
- Difficult to use the QCD Monte Carlo to predict the background in this phase space
- Fitting this mass distribution directly requires high order polynomial → large background uncertainties
- Can we try a data-driven sideband prediction?



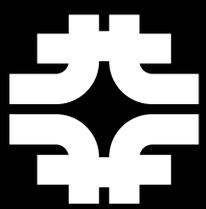
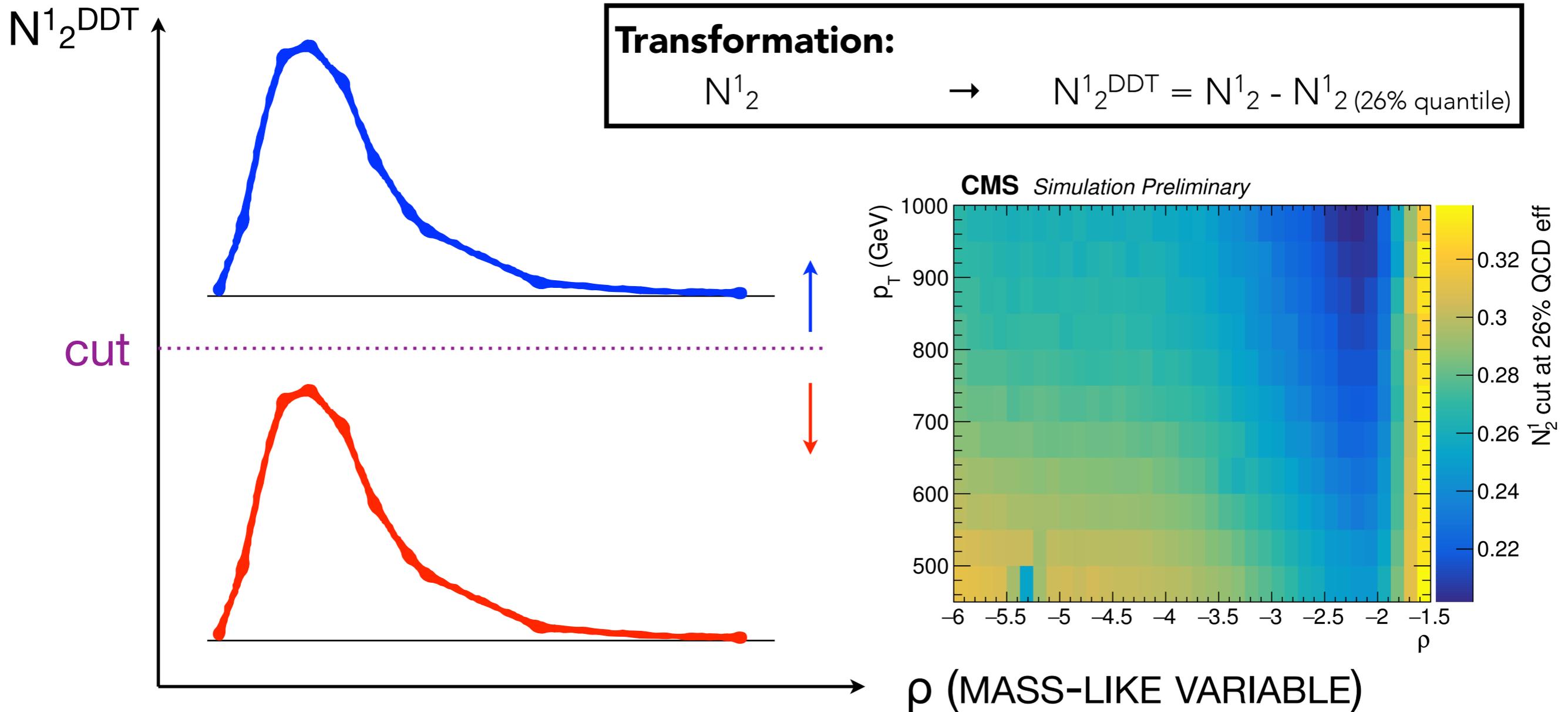
# SIDEBAND QCD PREDICTION

- Core idea: predict QCD jet mass distribution from region failing the tagger
- Possible problem: does tagger sculpt jet mass distribution?



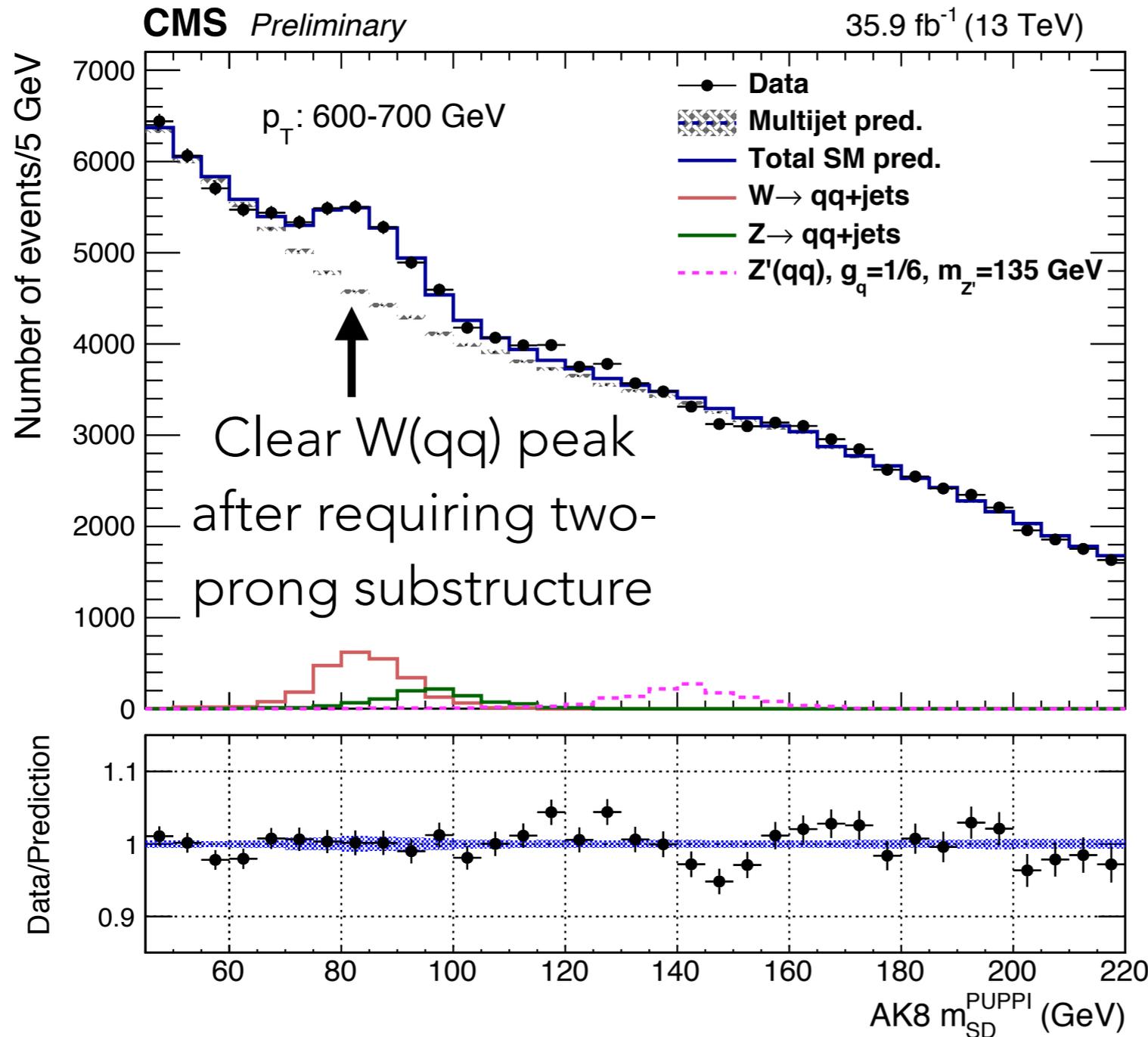
# SIDEBAND QCD PREDICTION

- Solution: define new substructure variable intended to be decorrelated from jet mass



# Z' RESULTS

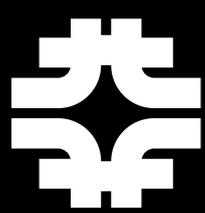
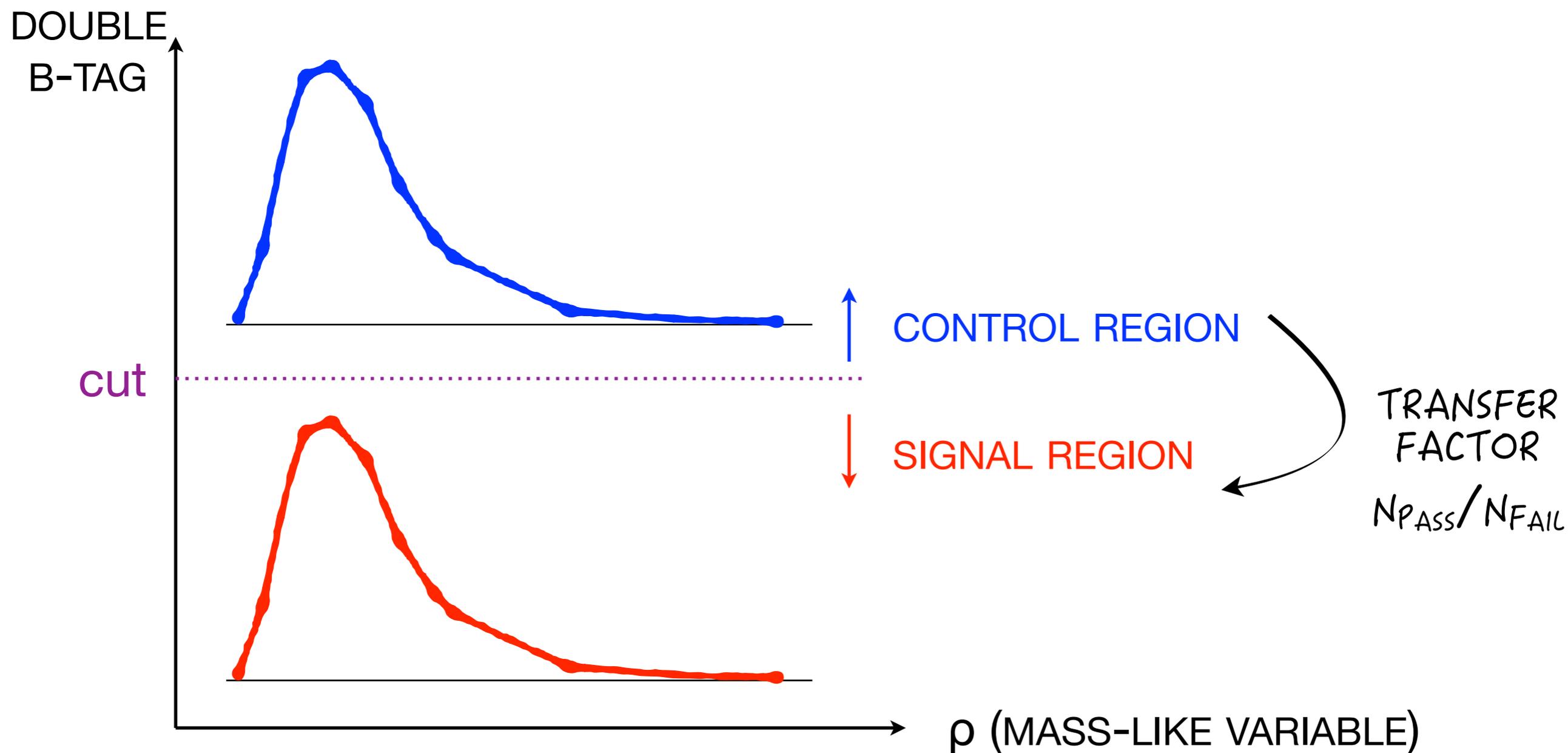
- Jet mass distribution is fit down to 40 GeV



← background prediction from failing region allows good precision

# SIDEBAND QCD PREDICTION (REDUX)

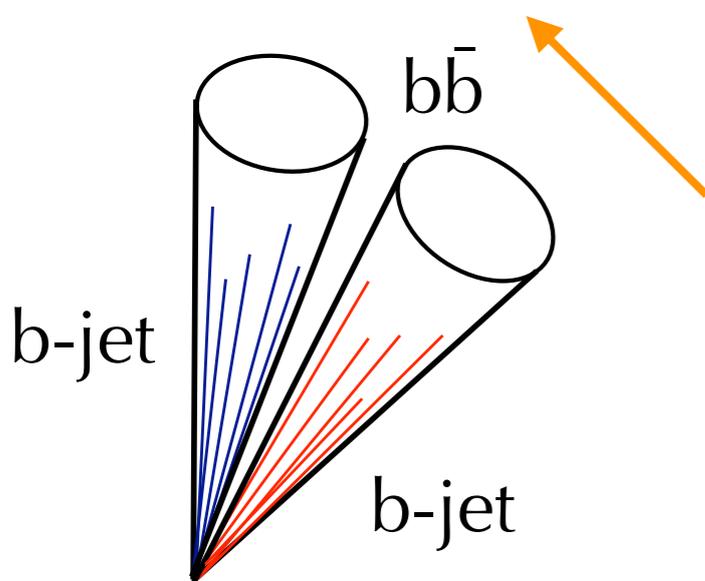
- Can we use the same QCD prediction when using a double-b tagger?
  - Yes — if it's sufficiently decorrelated from jet mass and  $p_T$



# BOOSTED H(BB)

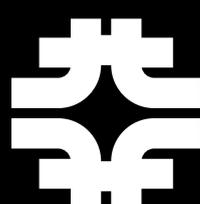
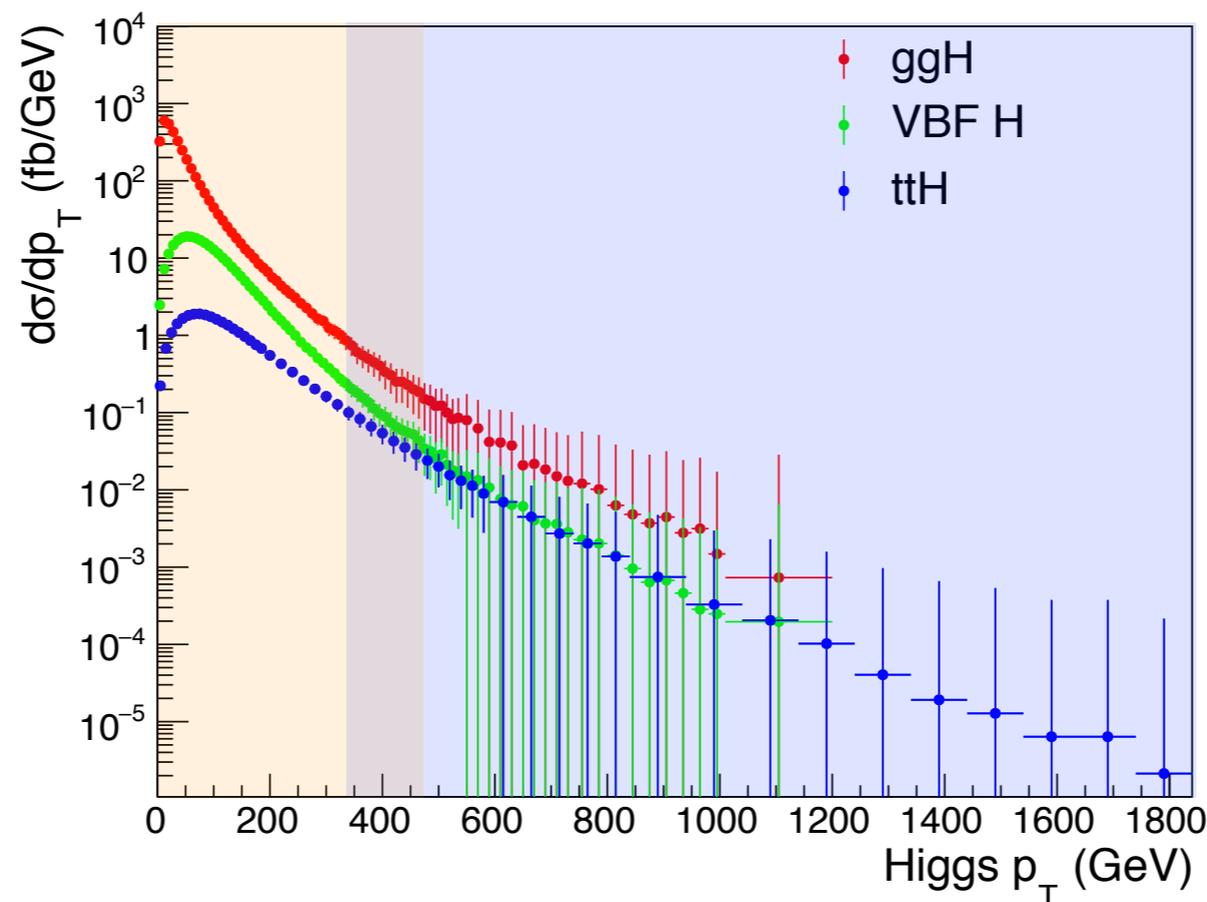
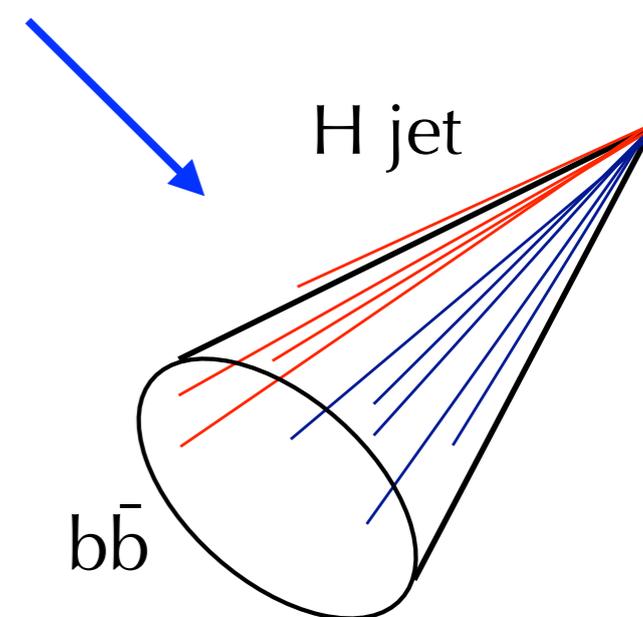
- With large boost, both b quarks merge into a single large radius jets
- How can we best exploit the presence of the b-quarks in the jet in a tagger?

two separated  
b-jets (R=0.4)

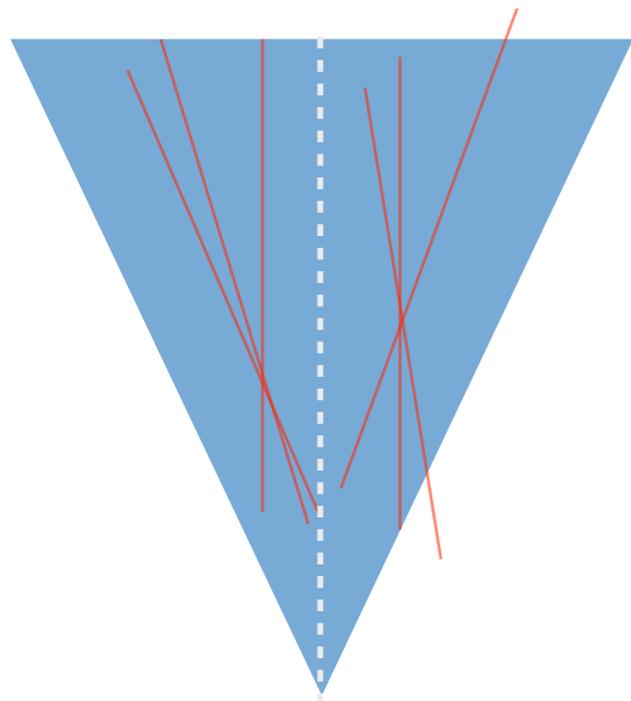


$$\Delta R(b\bar{b}) \sim 2m_H/p_T$$

one merged double  
b-jet (R=0.8)

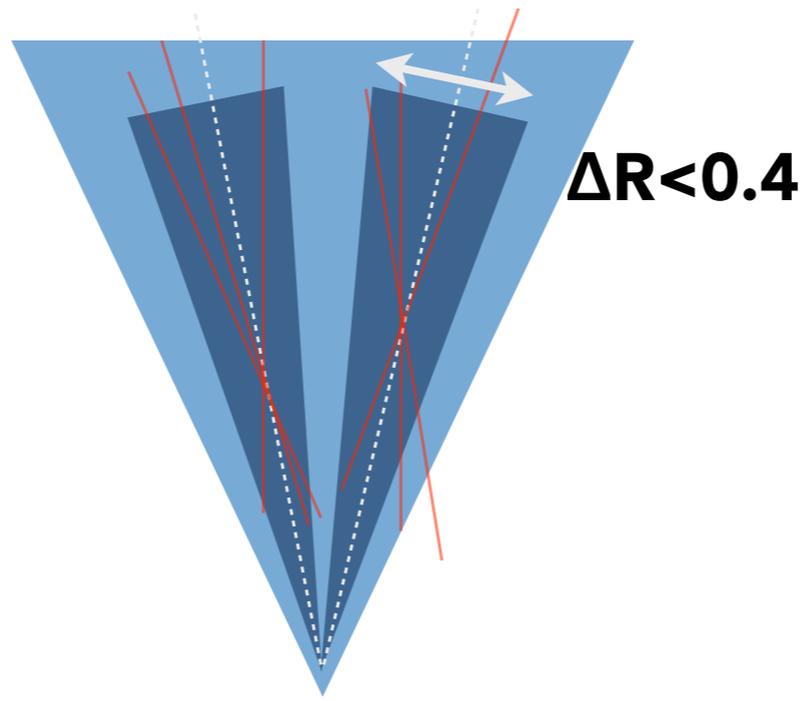


# MULTIPLE APPROACHES



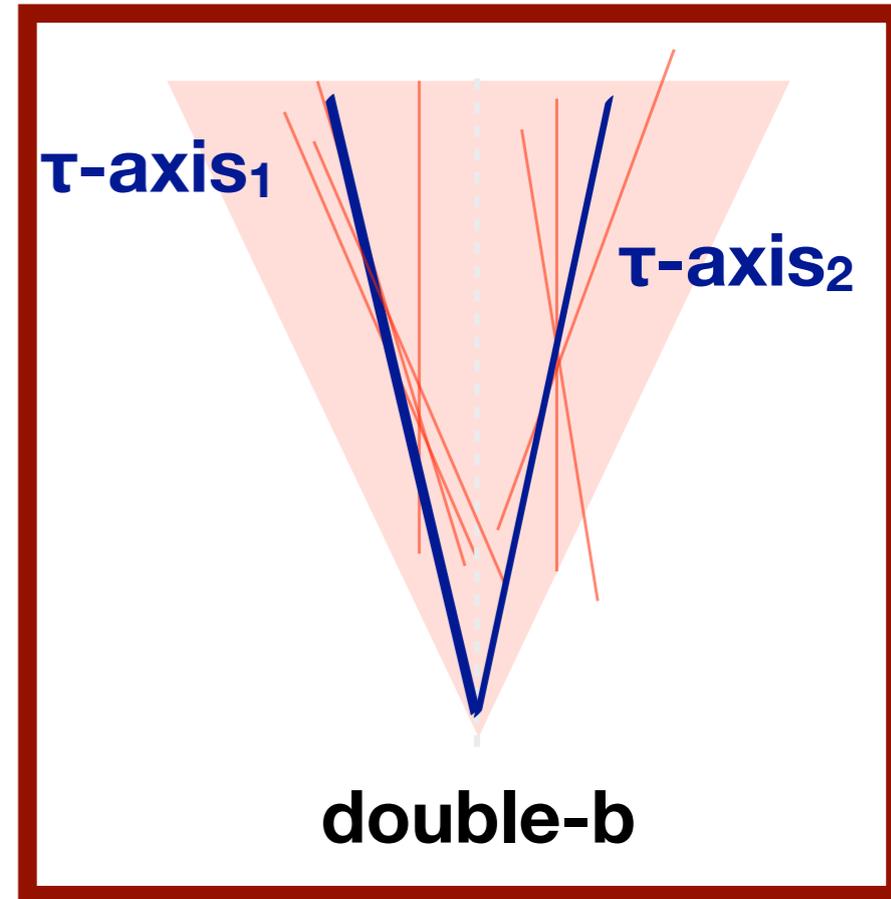
**fatjet**

- Based on standard b-tagging algorithm
- Not designed for two b's in the same jet



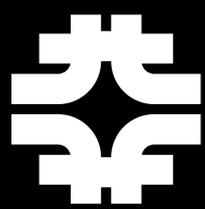
**subjets**

- Defines sub-jets
- Standard b-tagging applied to each subject



**double-b**

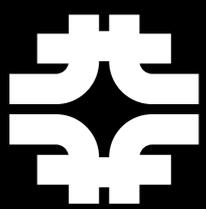
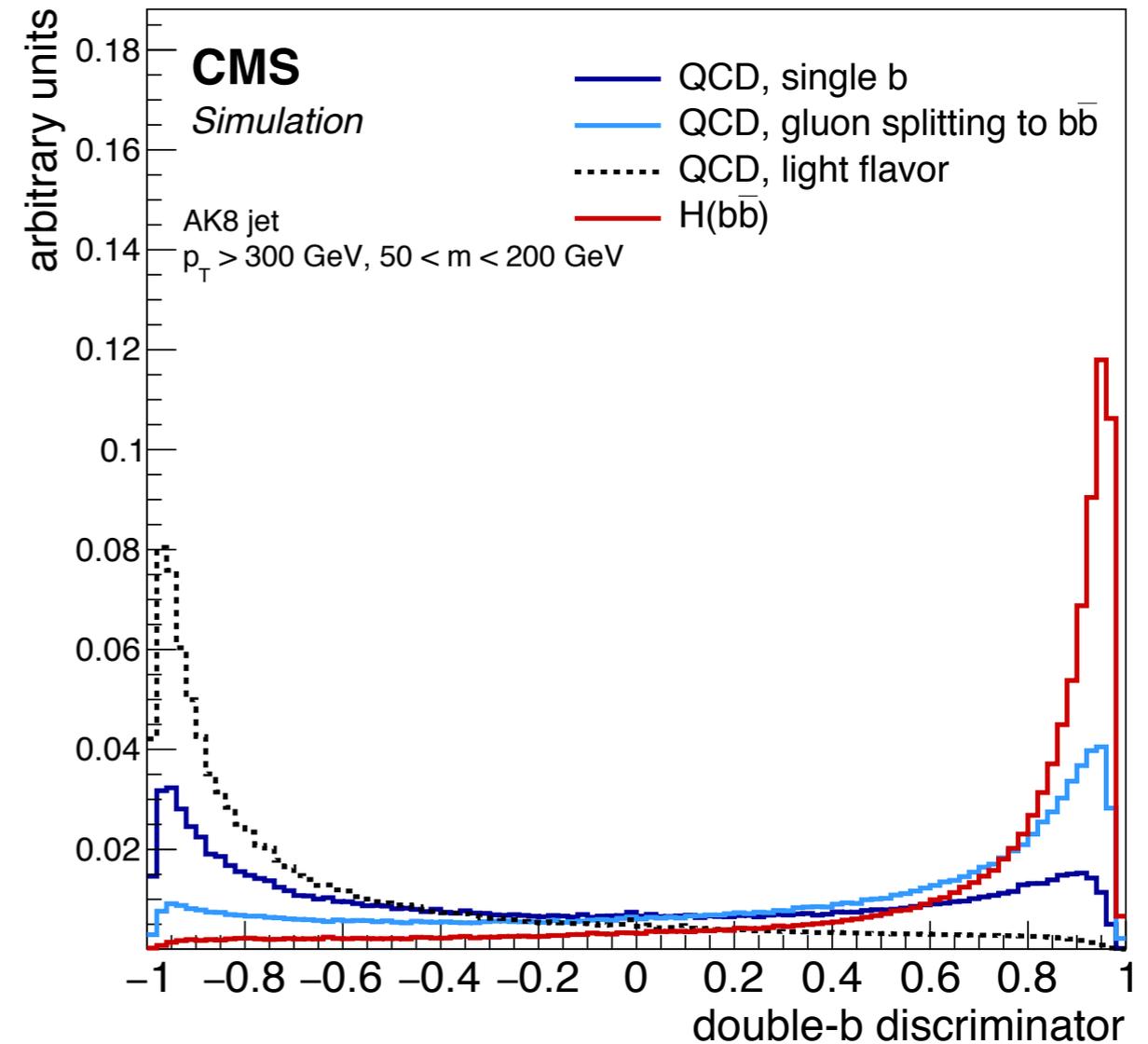
- Identifies two b hadron decay chains in the same fat jet
- Does not define subjects, but uses N-subjettiness axes



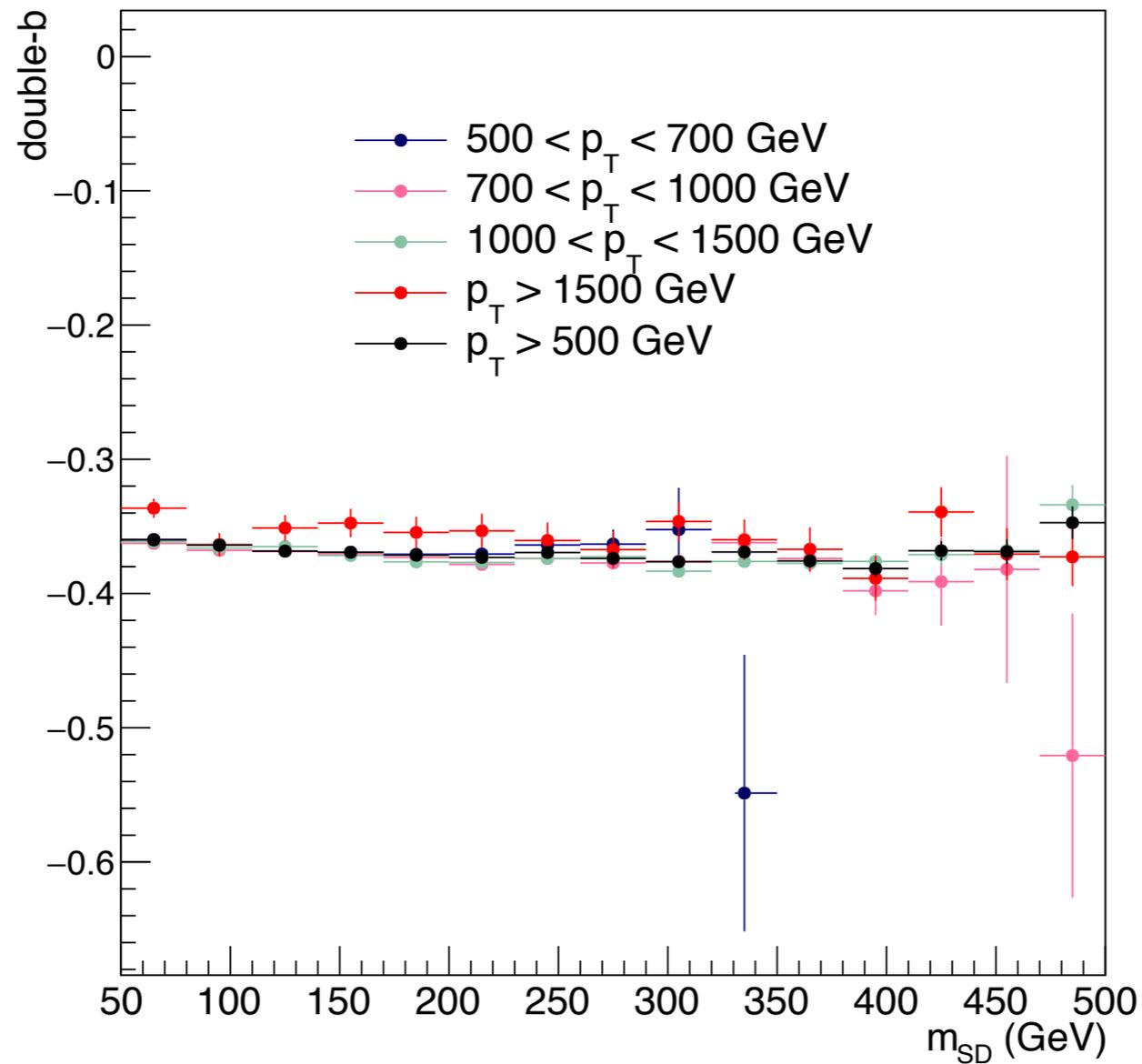
# DOUBLE B-TAGGER

- Combines tracking and vertexing information in a multivariate classifier with 27 observables
- Targets the  $b\bar{b}$  signal with additional aims:
  - jet mass and  $p_T$  independent
  - cover a very wide  $p_T$  range
  - inputs are chosen to avoid  $p_T$  correlation
    - e.g. no  $\Delta R$ -like variables, no substructure info

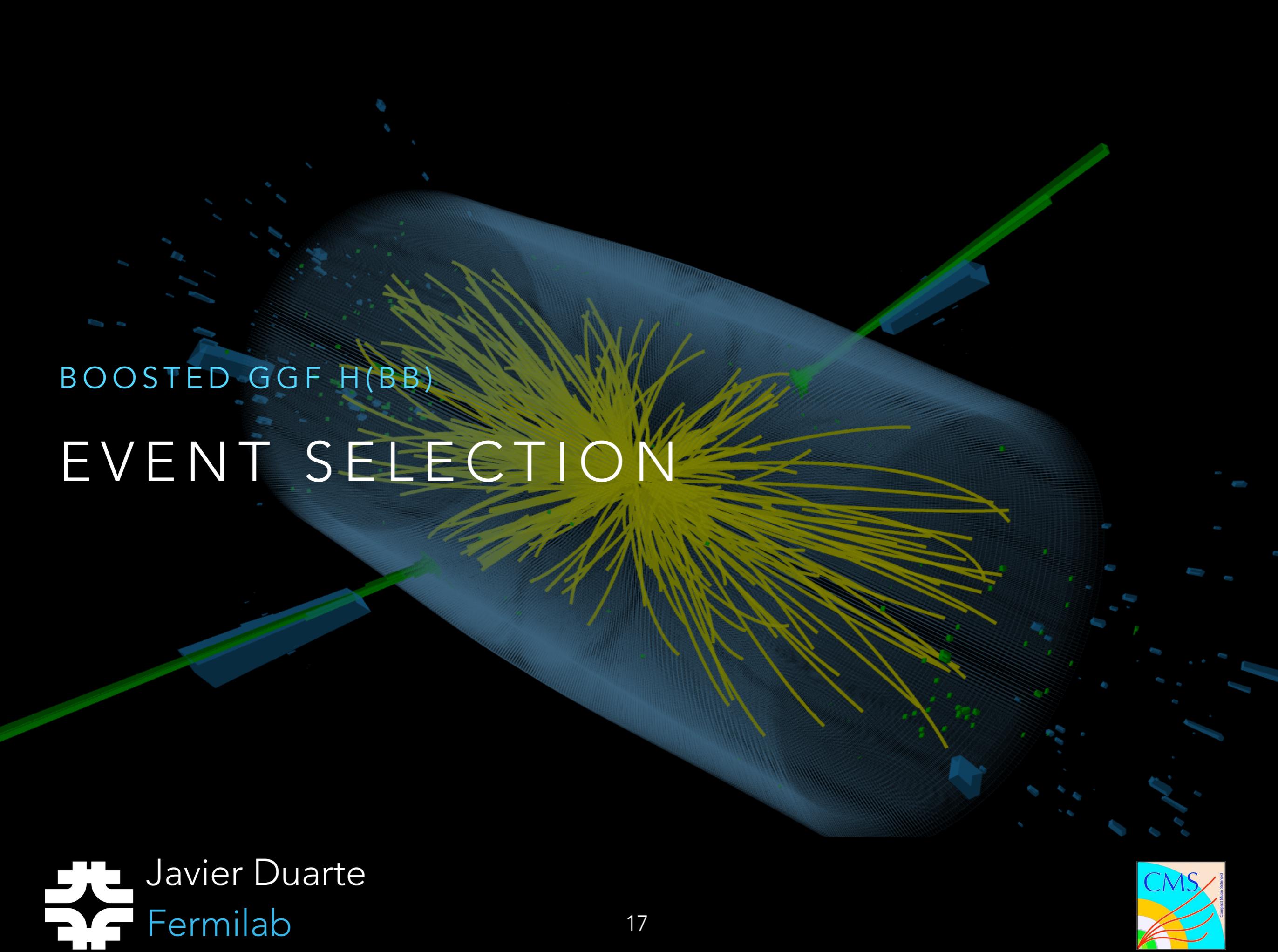
13 TeV, 2016



# CORRELATIONS?

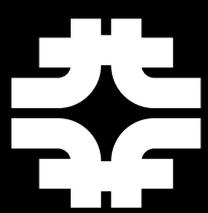


- No strong correlations in double-b tagger versus  $m_{SD}$  or  $p_T$  in QCD background

A 3D visualization of a particle detector event. The central feature is a dense, starburst-like cluster of yellow lines radiating from a central point, representing particle tracks. This cluster is contained within a blue, cylindrical volume that represents the detector's sensitive region. Two green lines, representing the incoming beams, enter from the left and right sides of the detector. The background is dark blue with scattered small blue and green particles, suggesting a complex event environment.

BOOSTED GGF H(BB)

# EVENT SELECTION

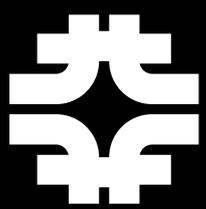
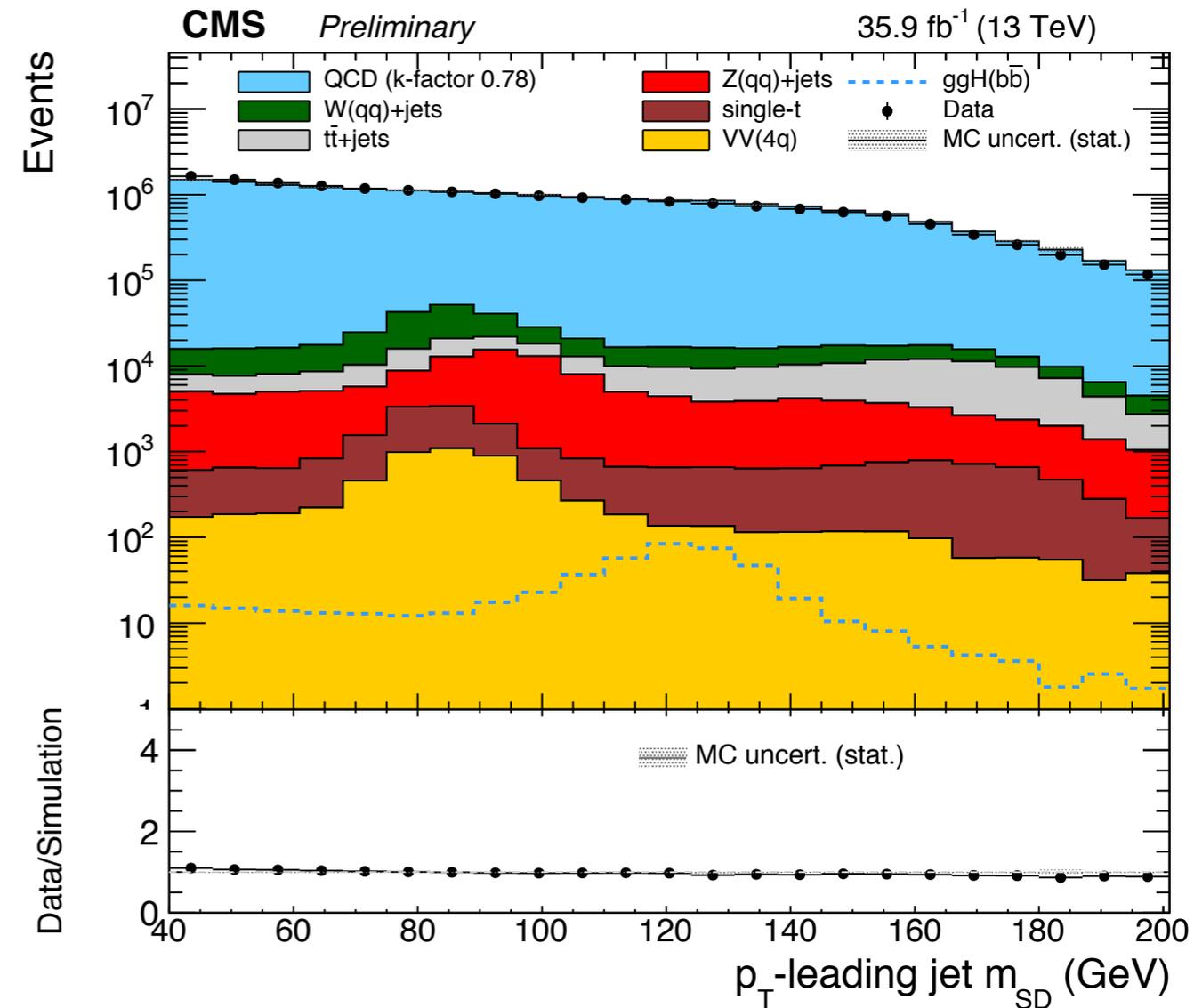


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Fermilab



## EVENT SELECTION

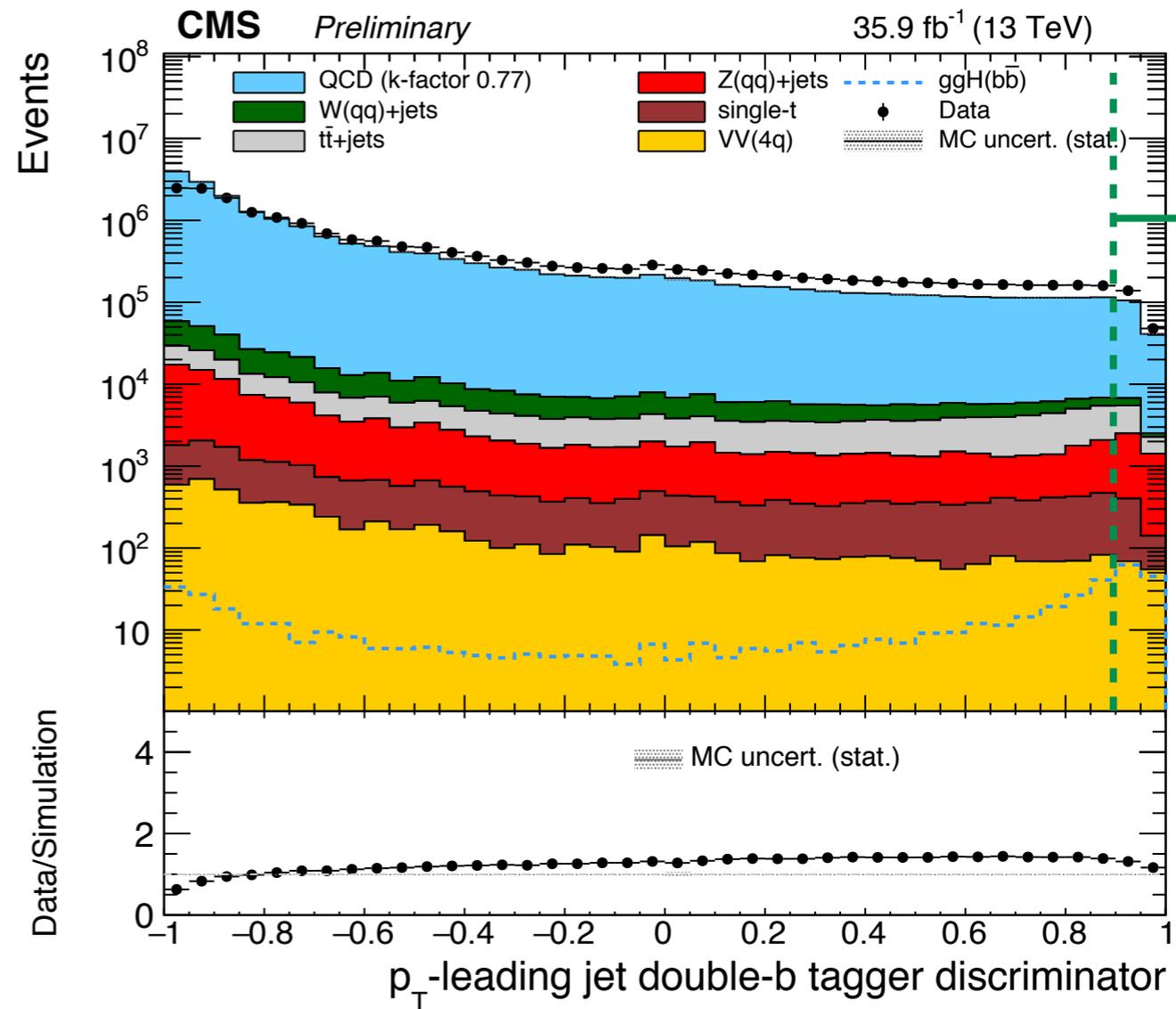
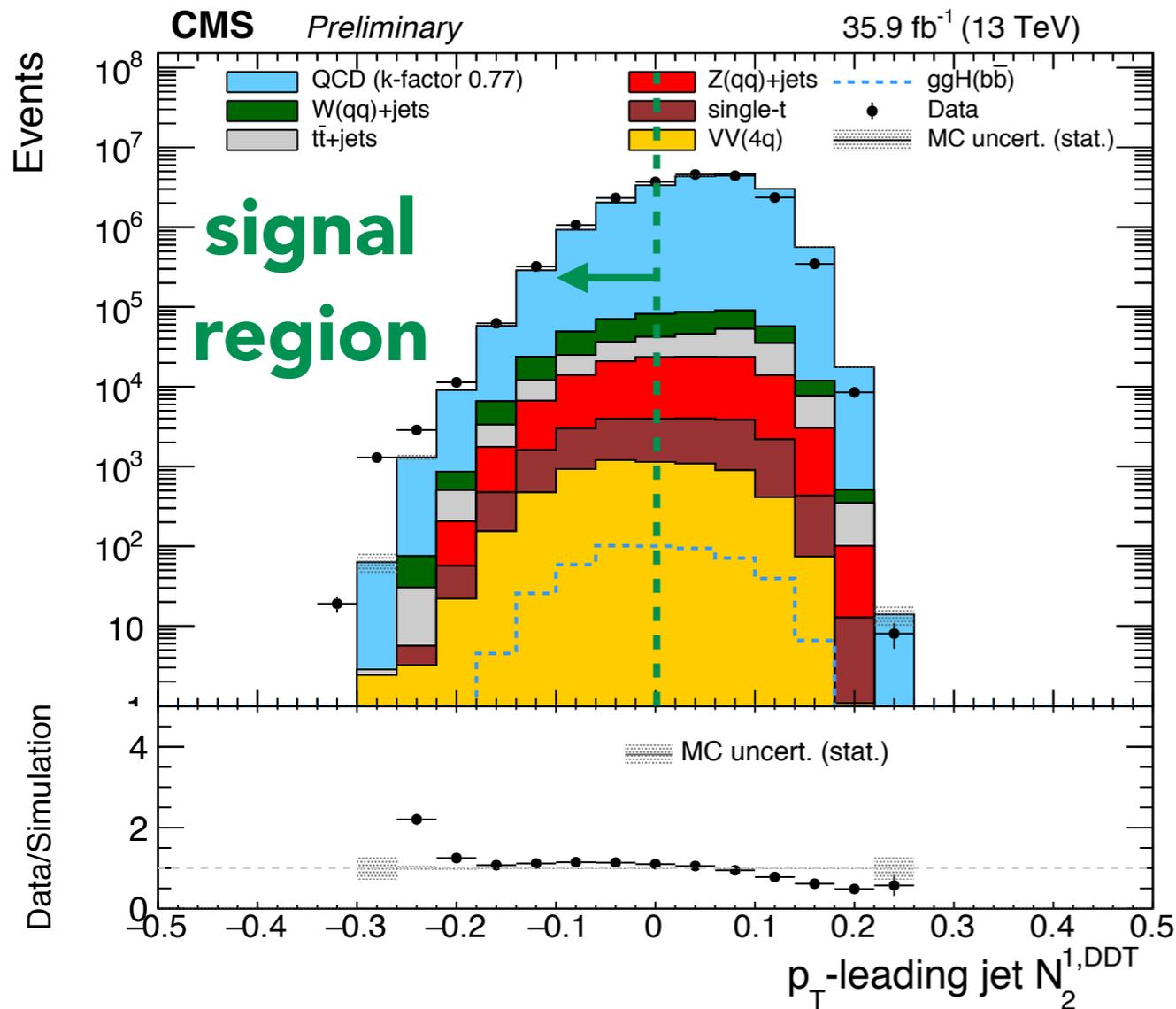
- Online selection asks for a high  $p_T$  single jet or large hadronic activities
  - $p_T > 360$  GeV ( $m > 30$ ) or  $\Sigma p_T > 900$  GeV
- Offline: Highest  $p_T$  jet
  - $p_T > 450$  GeV,  $|\eta| < 2.5$
  - jet mass  $m_{SD} > 40$  GeV
- lepton veto,  $p_T^{\text{miss}}$  veto
- $-6.0 < \rho = \log(m_{SD}^2 / p_T^2) < -2.1$



# EVENT SELECTION

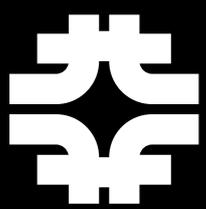
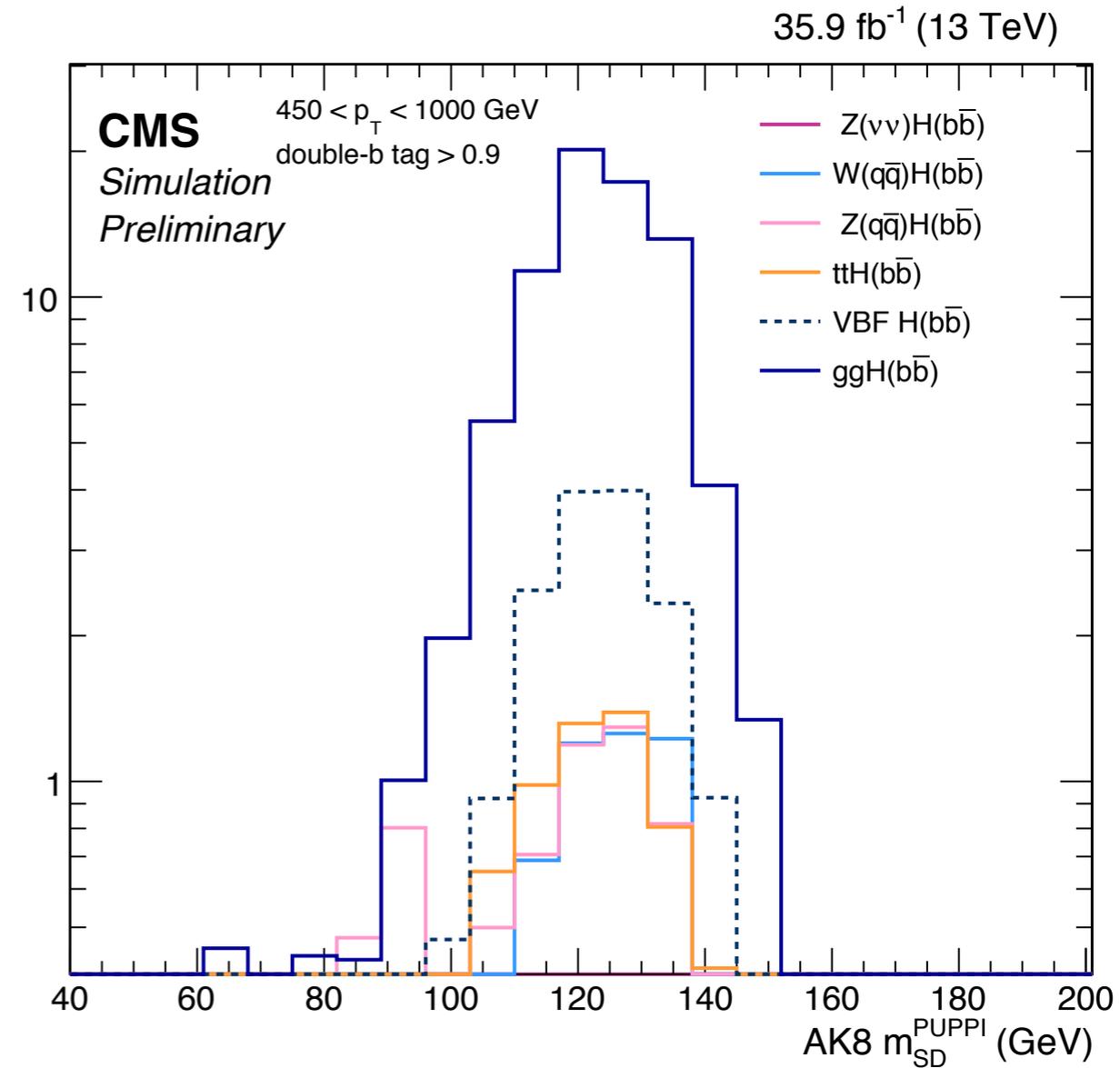
**Substructure:** two prong discrimination, ~50% sig. efficiency, 26% bkg. efficiency

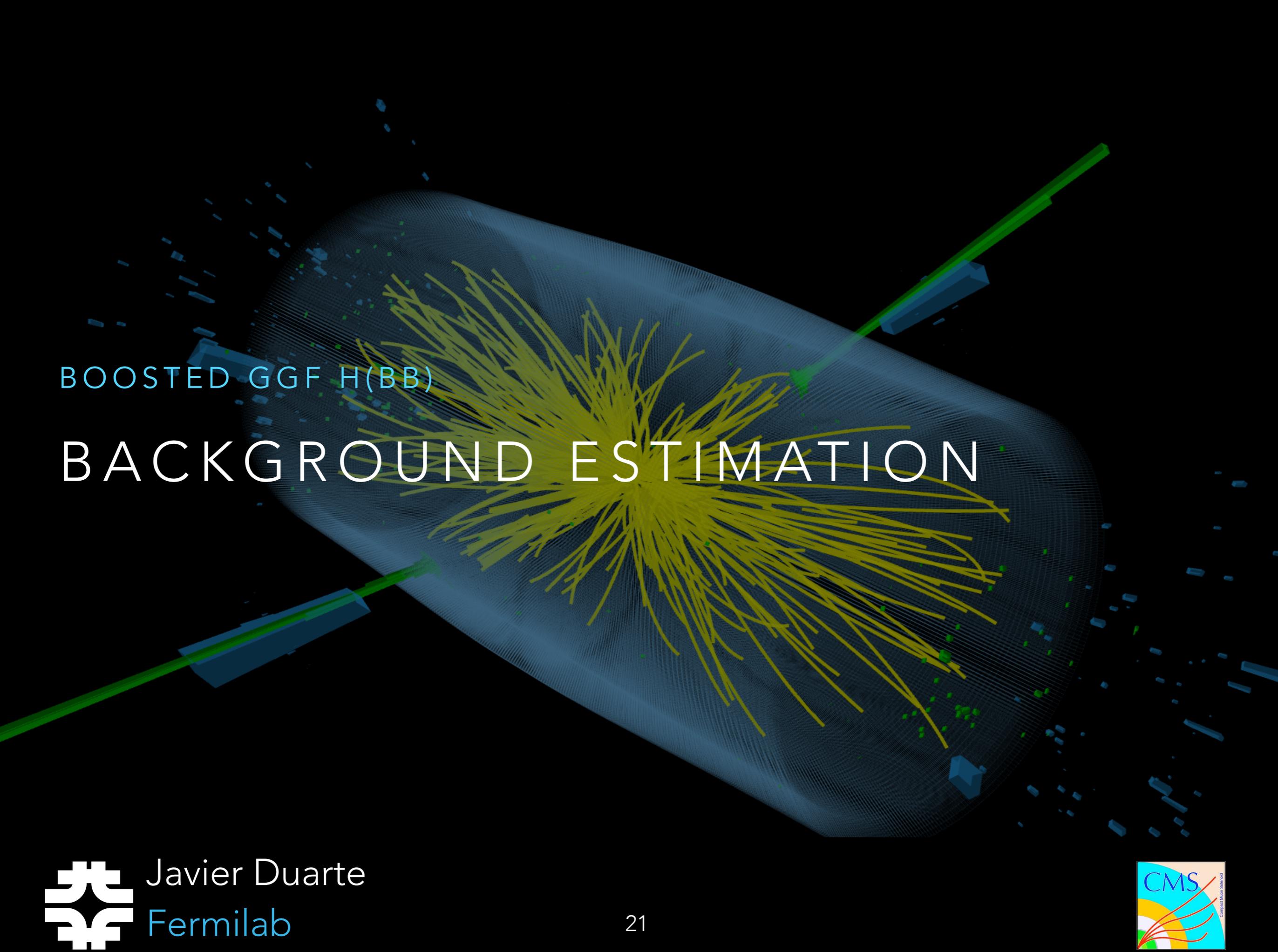
**Double-b tagger:** ~30% sig. efficiency, 1% bkg. efficiency (tight working point)



# SIGNAL COMPOSITION

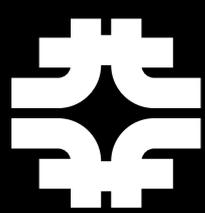
- Analysis is inclusive in Higgs production mode
- Dominant contribution is ggF (74%)
  - 12% VBF
  - 8% VH
  - 6% ttH





BOOSTED GGF H(BB)

# BACKGROUND ESTIMATION

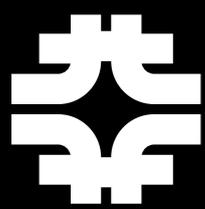
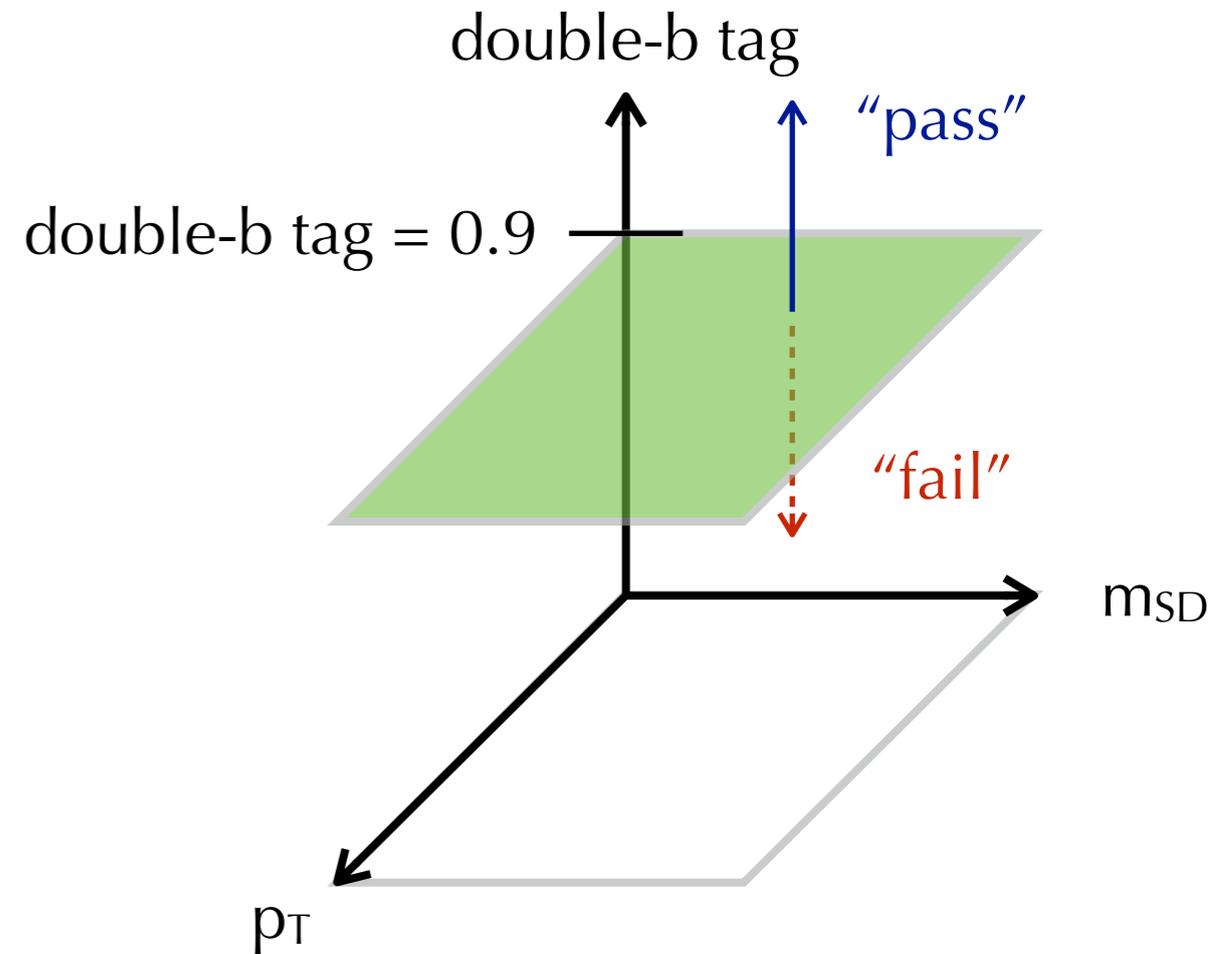


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Fermilab



# BACKGROUND STRATEGY

- Backgrounds estimated from data
  - QCD (90%): from failing double b-tag x transfer factor
  - tt+jets (3%): from  $1\mu$  control region
- Backgrounds estimated from MC including NLO QCD + EWK corrections and jet mass, resolution, and substructure tagging scale factors
  - W/Z+jets (5%)
  - single-t, VV (<1%)

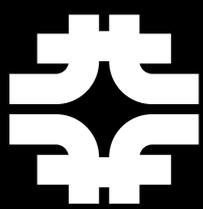


# QCD TRANSFER FACTOR

- If the double b-tagger were completely uncorrelated from jet mass and  $p_T$ , the transfer factor would be flat
- Taylor expand as a polynomial in  $\rho$  and  $p_T$  to parameterize any small correlations
- F-test determined 2<sup>nd</sup> order in  $\rho$  and 1<sup>st</sup> order in  $p_T$  is sufficient to fit the ratio

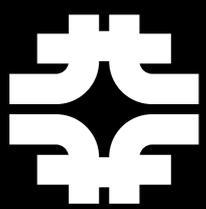
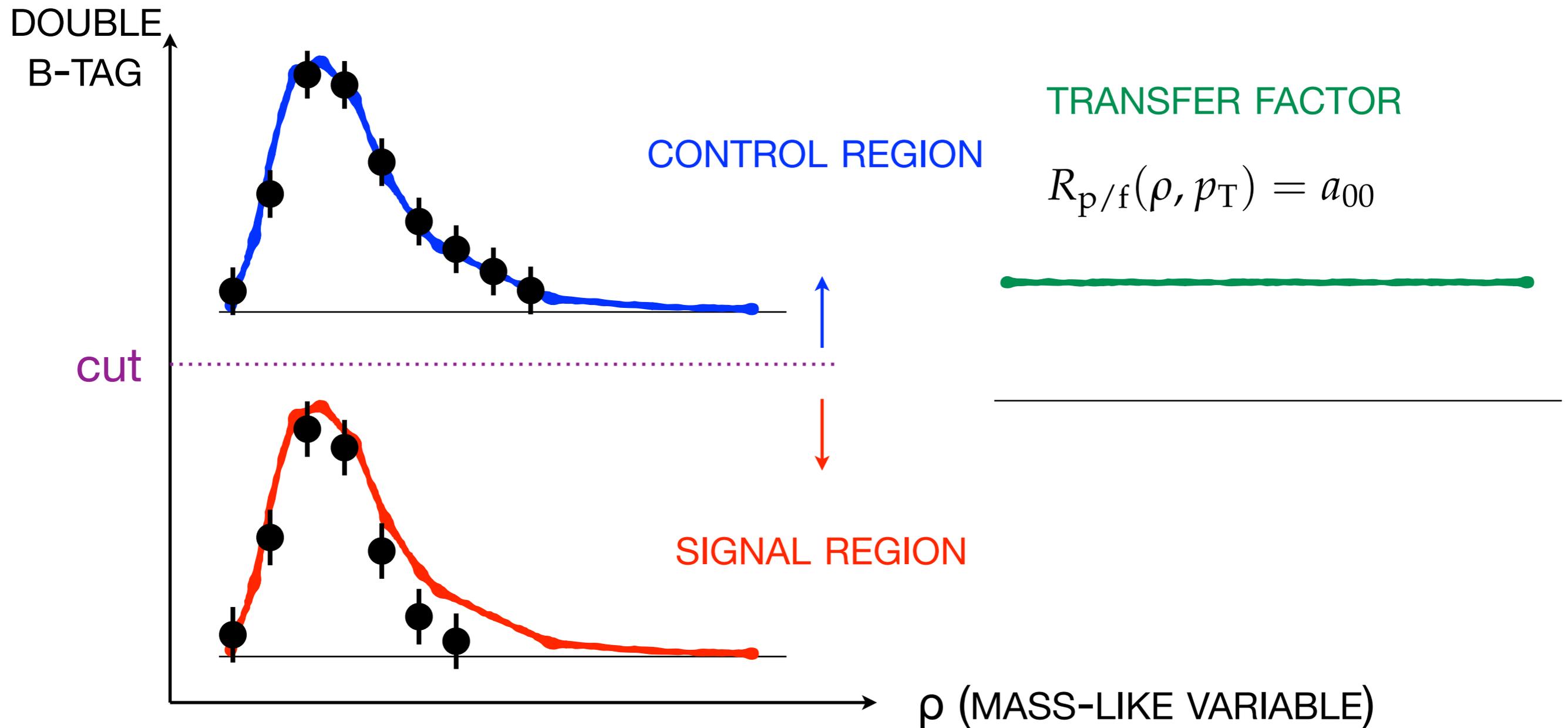
$$N_{\text{pass}}^{\text{QCD}}(m_{\text{SD}}, p_T) = R_{\text{p/f}}(\rho, p_T) \cdot N_{\text{fail}}^{\text{QCD}}(m_{\text{SD}}, p_T)$$

$$N_{\text{pass}}^{\text{QCD}}(m_{\text{SD}i}, p_{Tj}) = \left( \sum_{k,l} a_{kl} \rho_{ij}^k p_{Tj}^l \right) \cdot N_{\text{fail}}^{\text{QCD}}(m_{\text{SD}i}, p_{Tj})$$



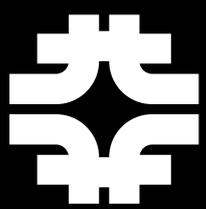
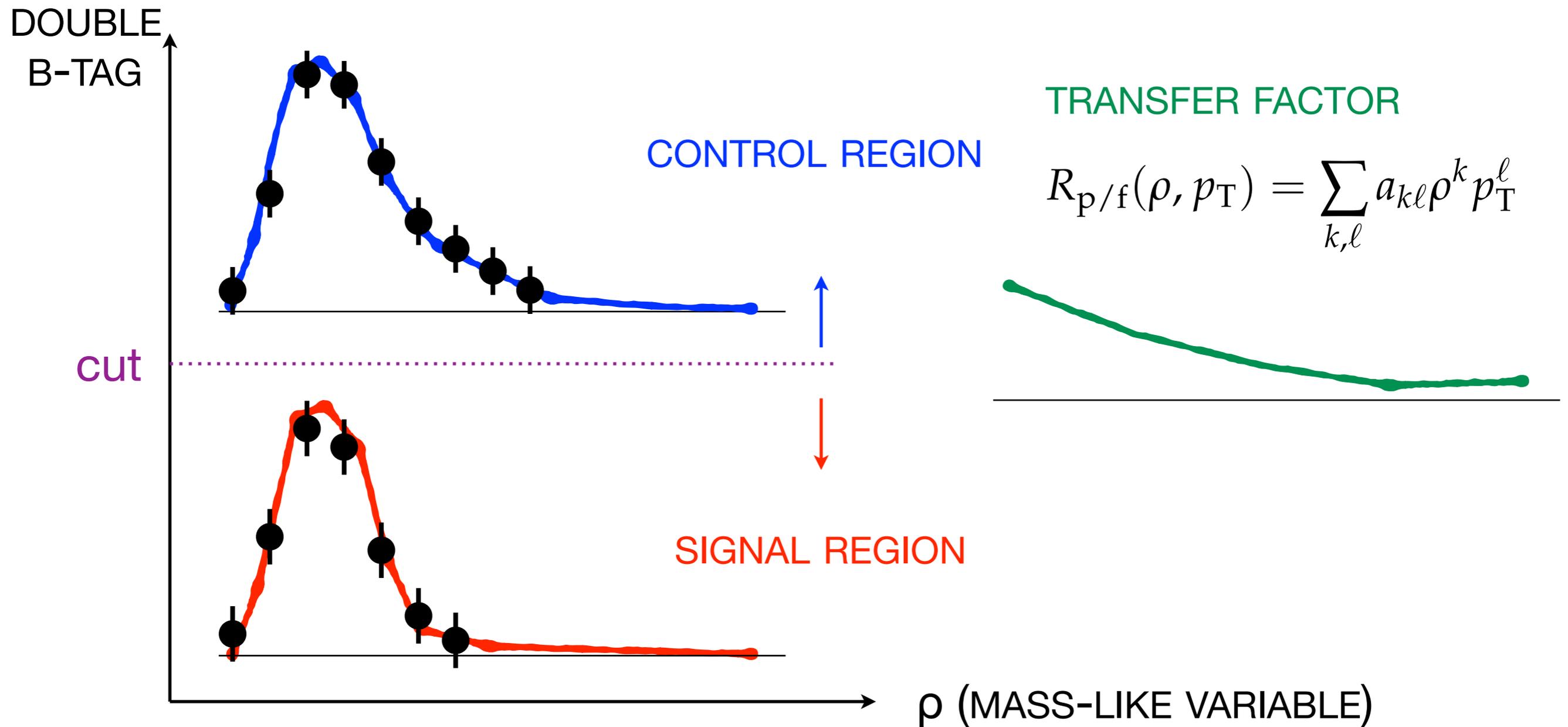
# FITTING TRANSFER FACTOR

- Pre-fit both regions have the same predicted shape



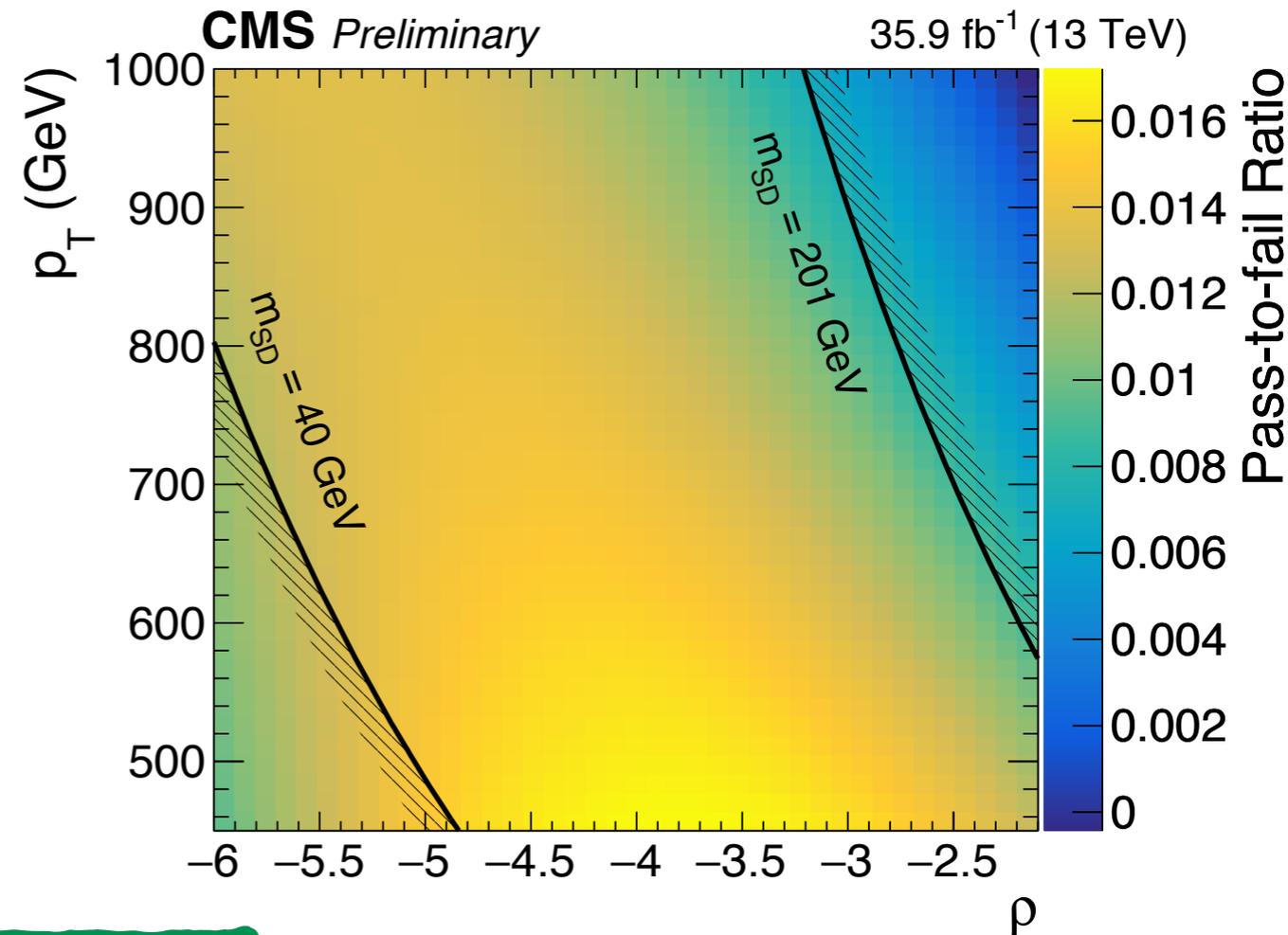
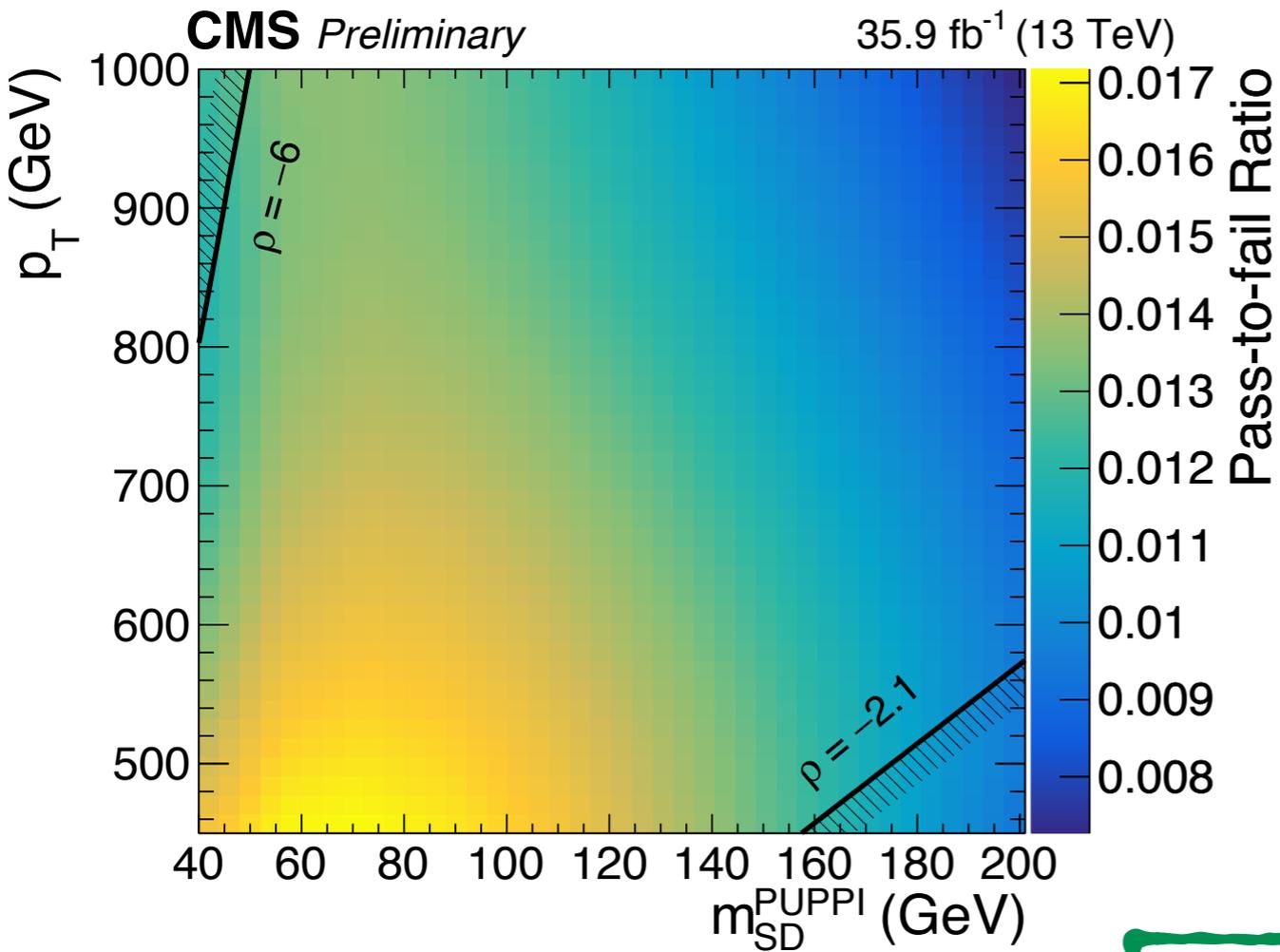
# FITTING TRANSFER FACTOR

- Post-fit signal region has slightly different shape with the ratio given by the polynomial transfer factor

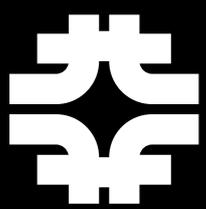


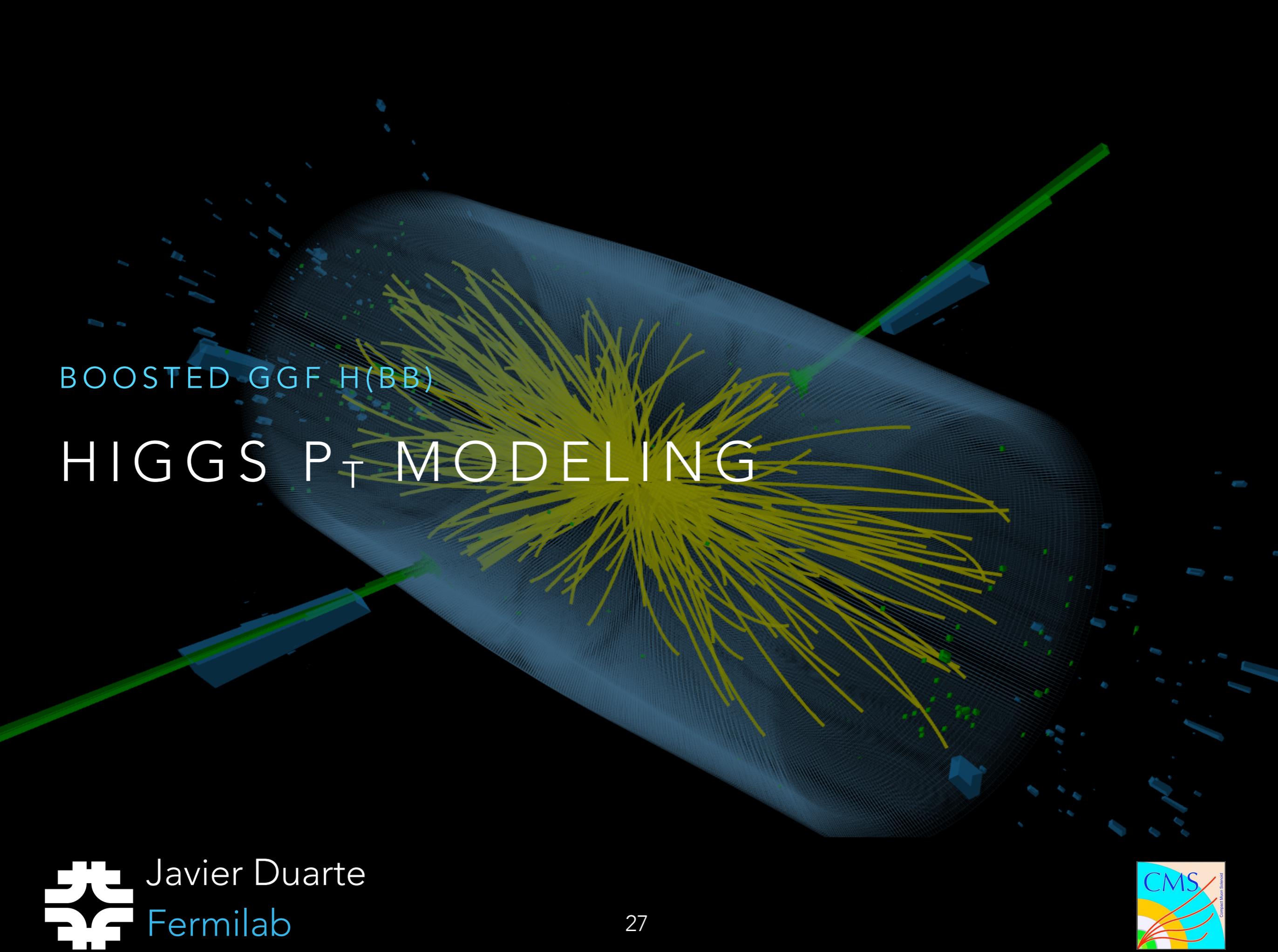
# FINAL TRANSFER FACTOR

- Two views of the same transfer factor function



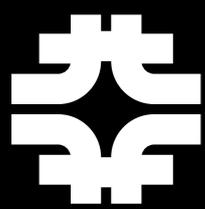
$$N_{\text{pass}}^{\text{QCD}}(m_{SDi}, p_{Tj}) = \left( \sum_{k,l} a_{kl} \rho_{ij}^k p_{Tj}^l \right) \cdot N_{\text{fail}}^{\text{QCD}}(m_{SDi}, p_{Tj})$$





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# HIGGS $P_T$ MODELING



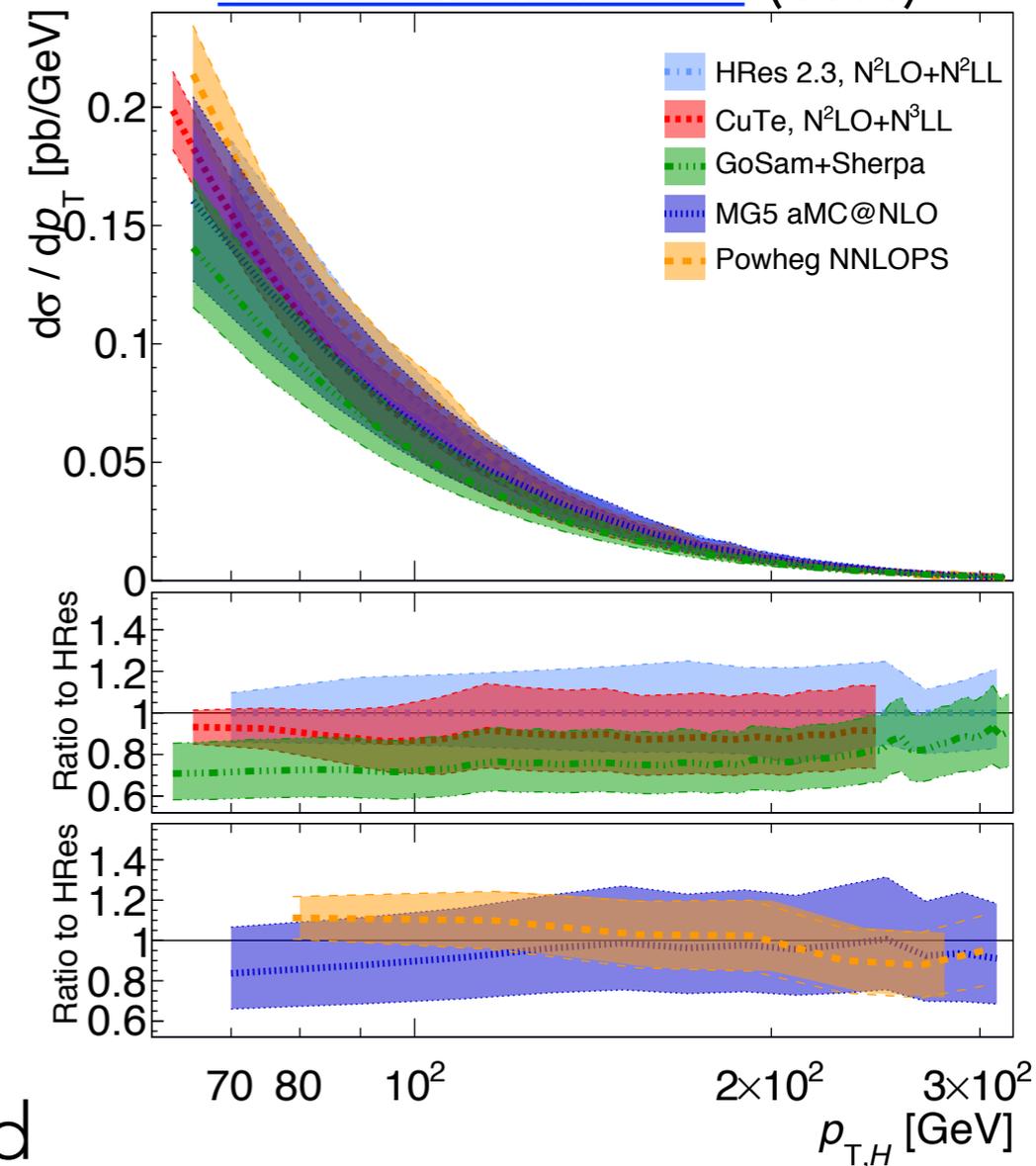
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Fermilab



# HIGGS PT SPECTRUM

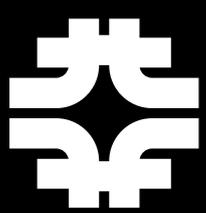
[arXiv:1610.07922](https://arxiv.org/abs/1610.07922) (YR4)

- Other CMS Higgs results use Powheg: 1 jet +  $m_t = \infty$ , [arXiv:1111.2854](https://arxiv.org/abs/1111.2854)
- We want to account for both effects of **higher order corrections** and **finite top mass**
  - No real NLO + finite top mass calculation available in the literature



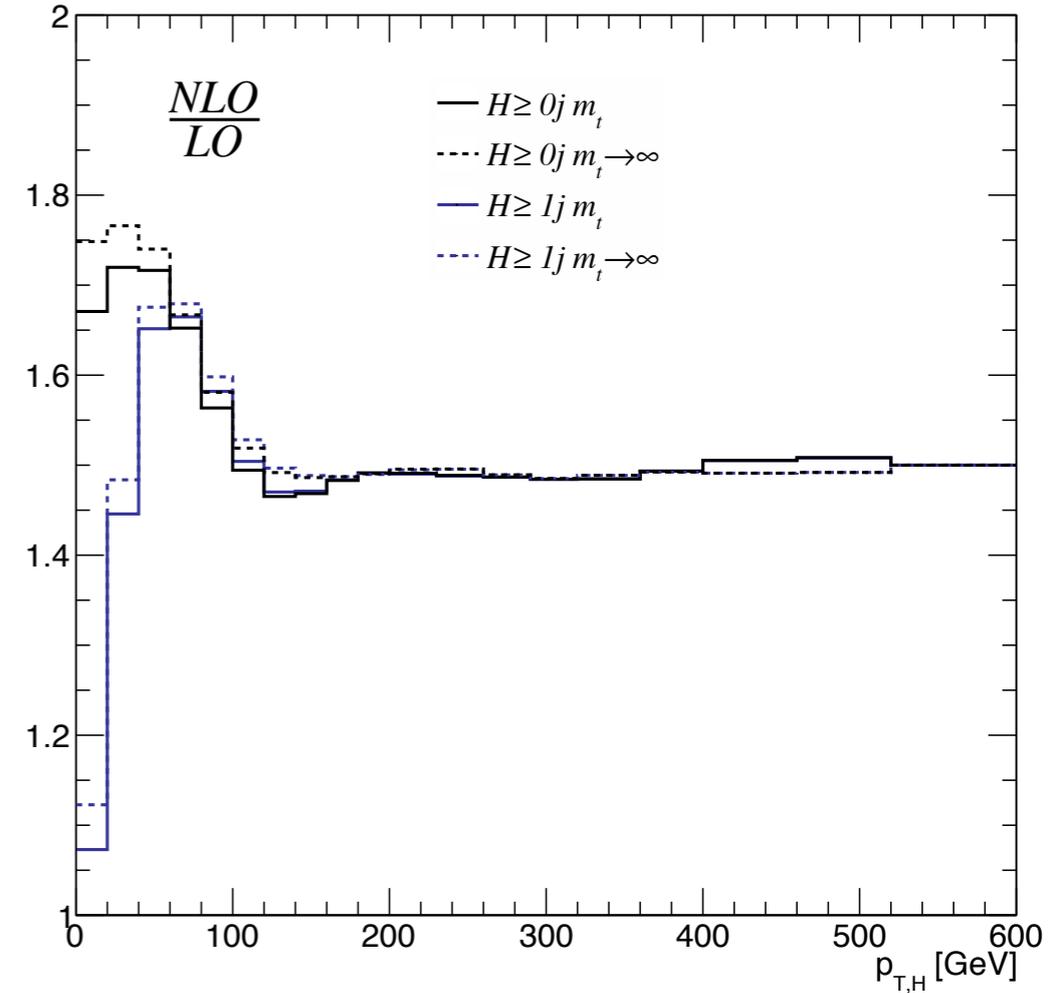
Disclaimer: we made an educated choice and assigned a reasonable uncertainty, but it's not the only possible choice. We also provide results with CMS standard Powheg sample

$\ll 1$  TeV



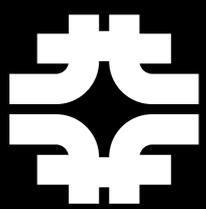
# HIGGS PT SPECTRUM

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  - No real NLO + finite top mass calculation available in the literature
- Adopt a factorized approach:
- LO H+0-2jet, finite  $m_t$ ,  $p_t^H$  up to 600 GeV, including WW acceptance cuts [arXiv:1410.5806](https://arxiv.org/abs/1410.5806) → We build on this
- 
- 



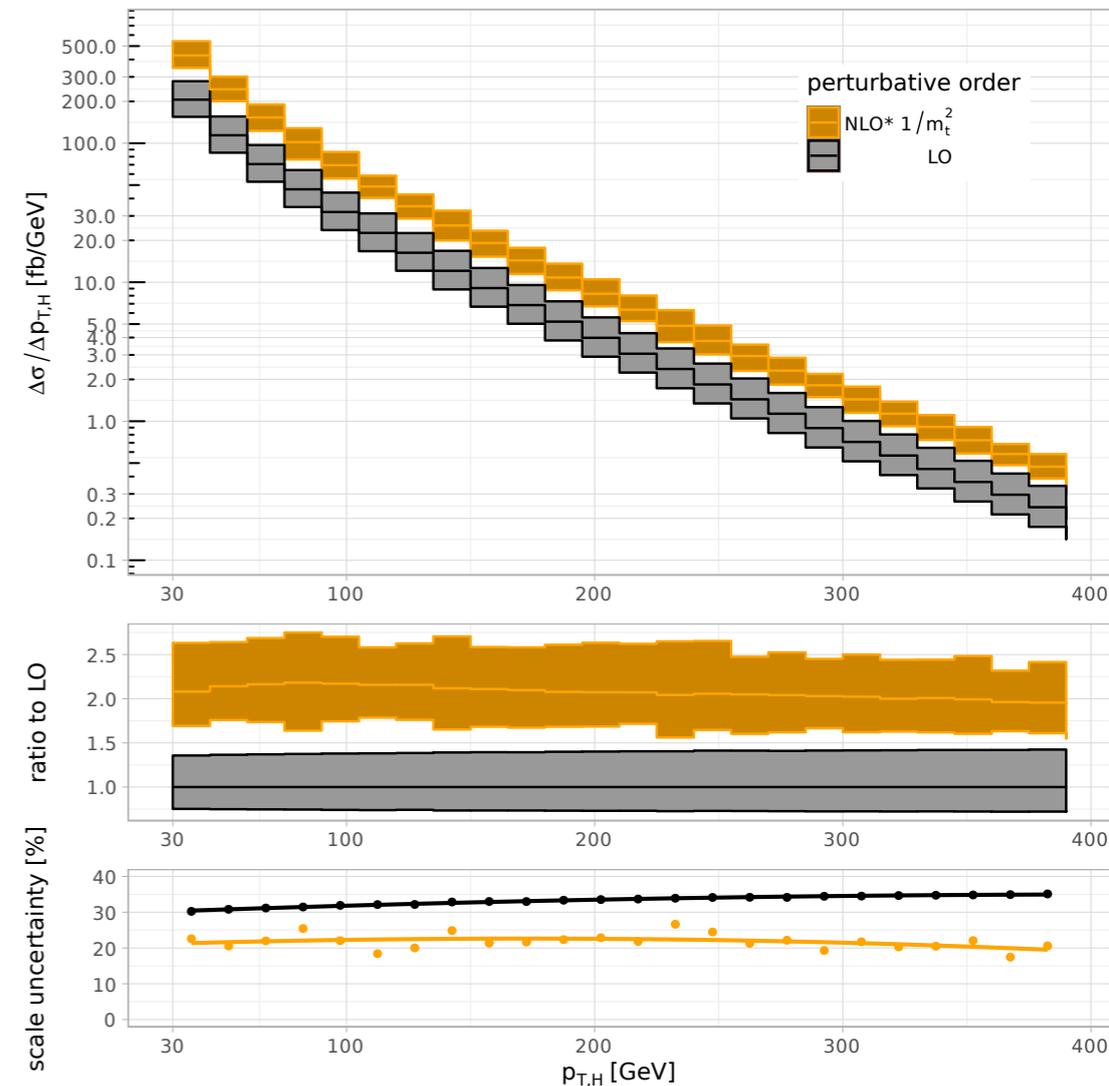
$$\text{GF H(NNLO} + m_t) = (1 \text{ jet } m_t \rightarrow \infty) \times \frac{\text{MG LO 0 - 2 jet } m_t}{(1 \text{ jet } m_t \rightarrow \infty)}$$

CKKW merged



# HIGGS PT SPECTRUM

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- NLO H+1jet finite  $m_t$  up to  $1/m_t^4$  expansion: [arXiv:1609.00367](https://arxiv.org/abs/1609.00367)
- 

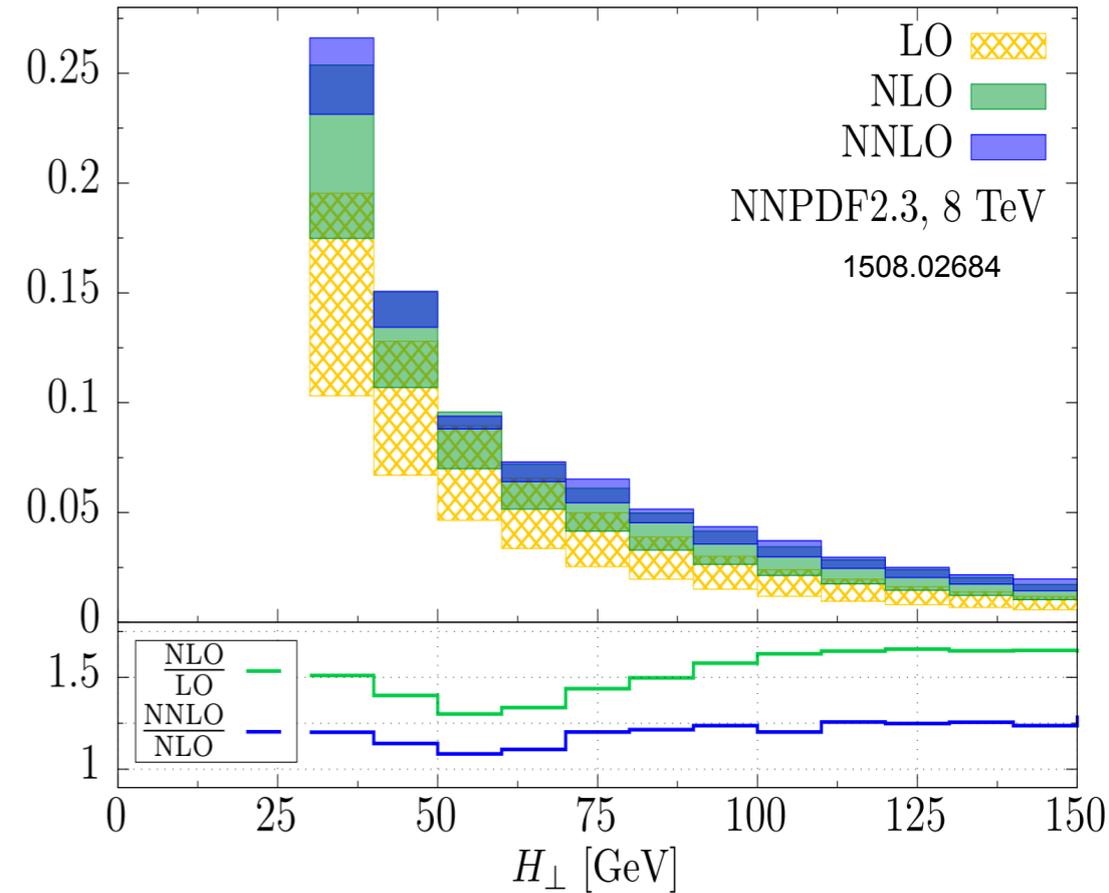


$$\text{GF H(NNLO} + m_t) = (1 \text{ jet } m_t \rightarrow \infty) \times \frac{\text{MG LO 0 - 2 jet } m_t}{(1 \text{ jet } m_t \rightarrow \infty)} \times \frac{\text{NLO 1 jet } m_t}{\text{LO 1 jet } m_t}$$

CKKW merged
factor of 2

# HIGGS PT SPECTRUM

- Other CMS Higgs results use Powheg: 1 jet +  $m_t = \infty$ , [arXiv:1111.2854](https://arxiv.org/abs/1111.2854)
- We want to account for both effects of higher order corrections and finite top mass
  - No real NLO + finite top mass calculation available in the literature
- Adopt a factorized approach:
- LO H+0-2jet, finite  $m_t$ ,  $p_t^H$  up to 600 GeV, including WW acceptance cuts [arXiv:1410.5806](https://arxiv.org/abs/1410.5806) → We build on this
- NLO H+1jet finite  $m_t$  up to  $1/m_t$  expansion: [arXiv:1609.00367](https://arxiv.org/abs/1609.00367)
- NNLO H+1jet,  $m_t = \infty$ ,  $p_T^H$  up to ~200 GeV, [arXiv:1408.5325](https://arxiv.org/abs/1408.5325), [arXiv:1302.6216](https://arxiv.org/abs/1302.6216), [arXiv:1504.07922](https://arxiv.org/abs/1504.07922), [arXiv:1505.03893](https://arxiv.org/abs/1505.03893), [arXiv:1508.02684](https://arxiv.org/abs/1508.02684)

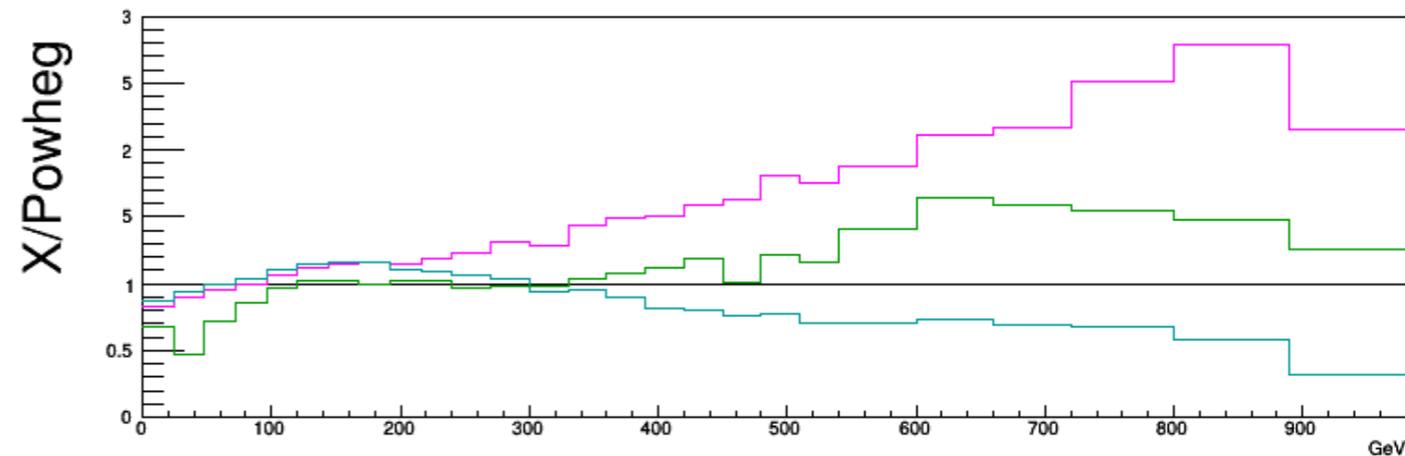
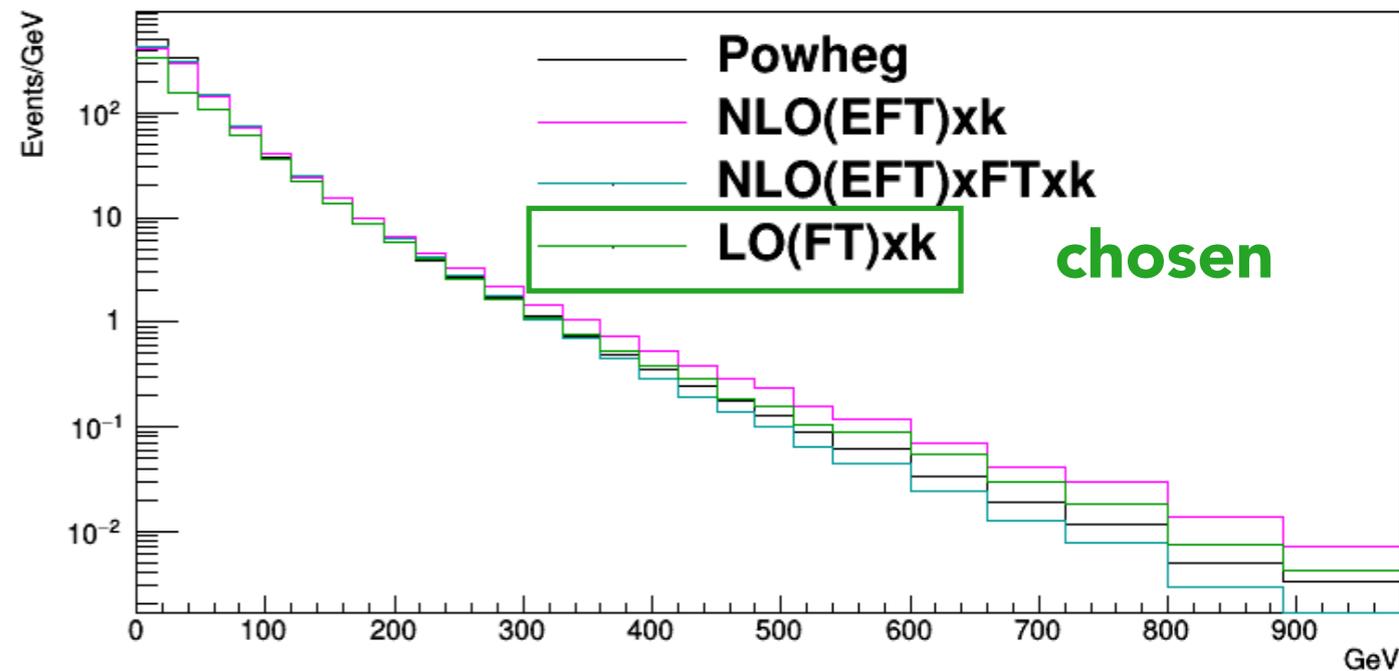


$$\text{GF H(NNLO} + m_t) = (1 \text{ jet } m_t \rightarrow \infty) \times \frac{\text{MG LO 0 - 2 jet } m_t}{(1 \text{ jet } m_t \rightarrow \infty)} \times \frac{\text{NLO 1 jet } m_t}{\text{LO 1 jet } m_t} \times \frac{\text{NNLO 1 jet } m_t \rightarrow \infty}{\text{NLO 1 jet } m_t \rightarrow \infty}$$

CKKW merged
factor of 2
factor of 1.25

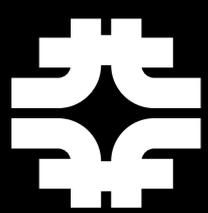
# HIGGS PT SPECTRUM

- Pythia version of CKKW-L merged 0,1,2jet LO finite top mass
- ME generation in aMC@NLO (ptj > 20) with xqcut = 30 GeV
- CKKW shower is extended down to a merging scale of TMS = 20 GeV
- Two factorized systematic uncertainties:
  - 30% overall normalization
  - 30% linear change in slope (no effect on overall norm.)



BOOSTED GGF H(BB)

# RESULTS

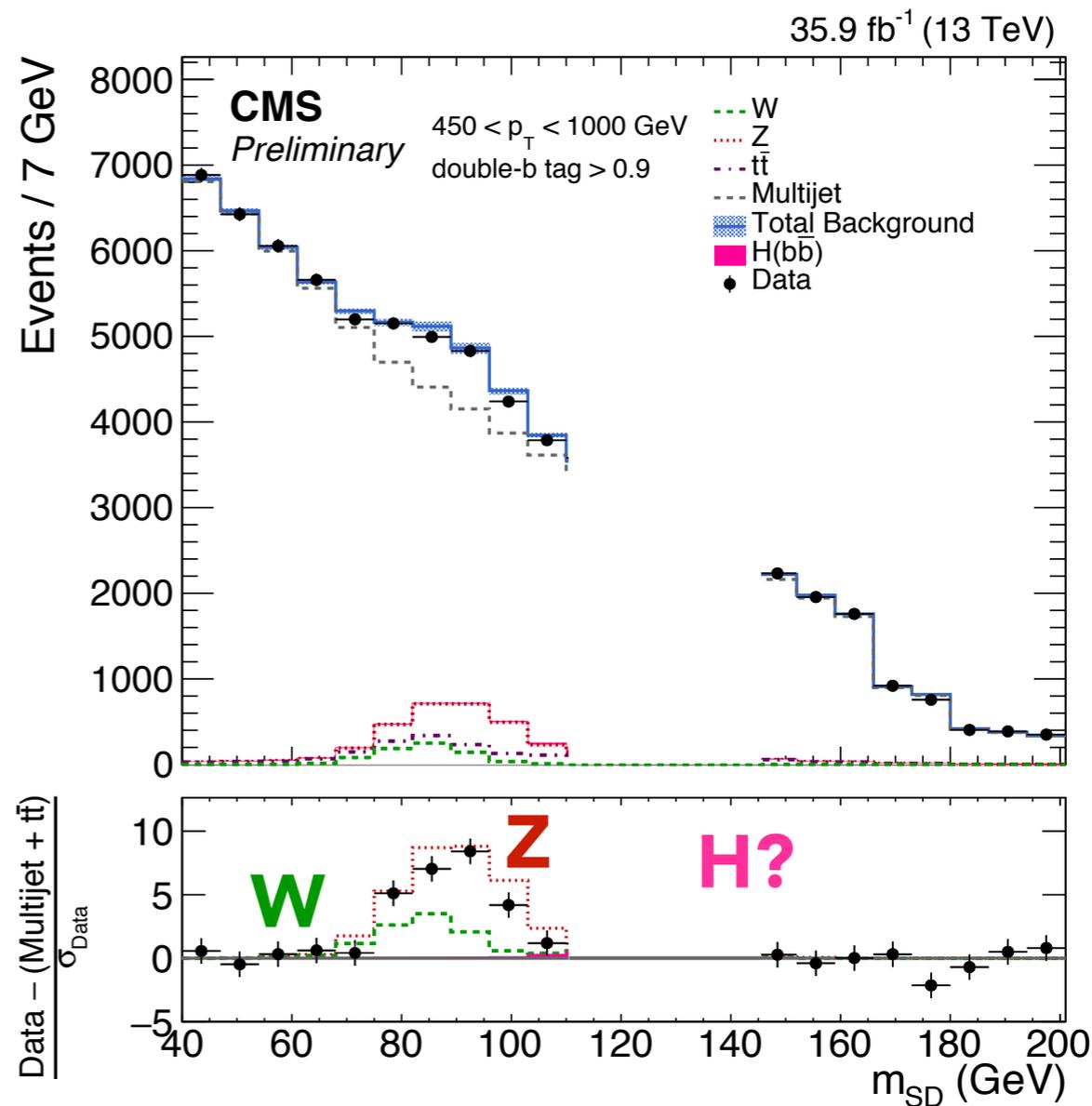
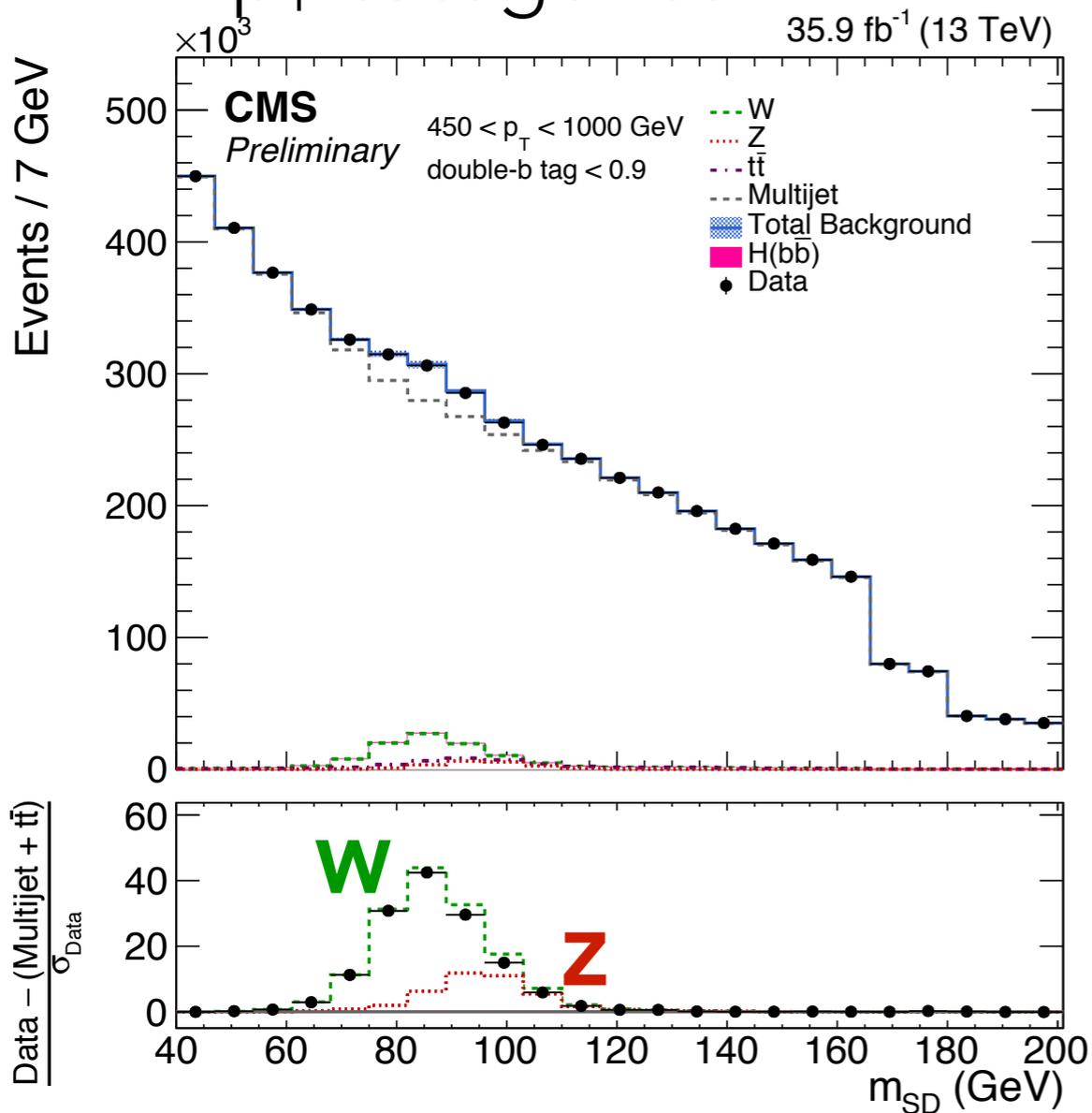


Javier Duarte  
Fermilab



# FIT RESULTS

- Simultaneous fit for Z(bb) and H(bb)
- All  $p_T$  categories

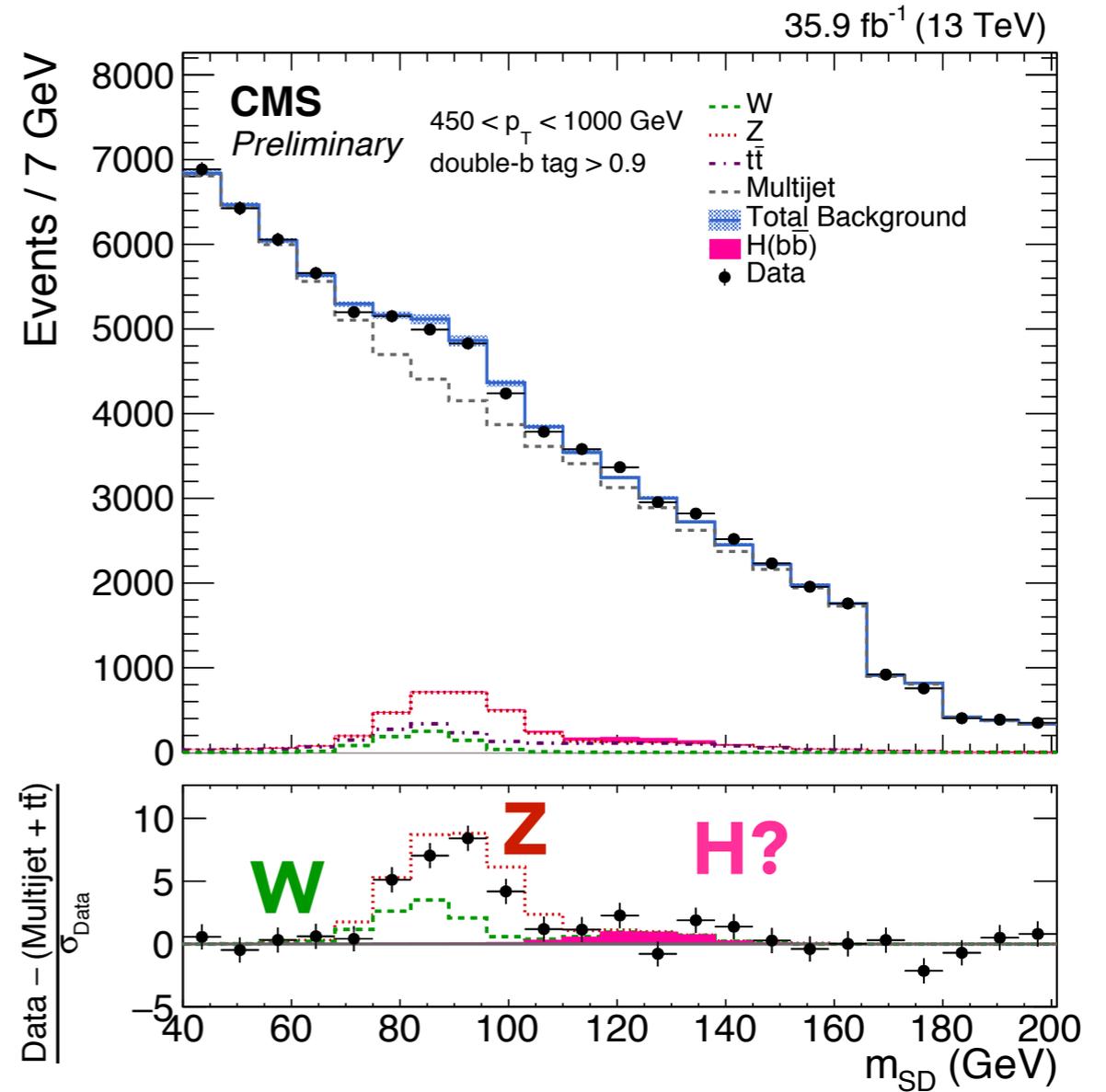
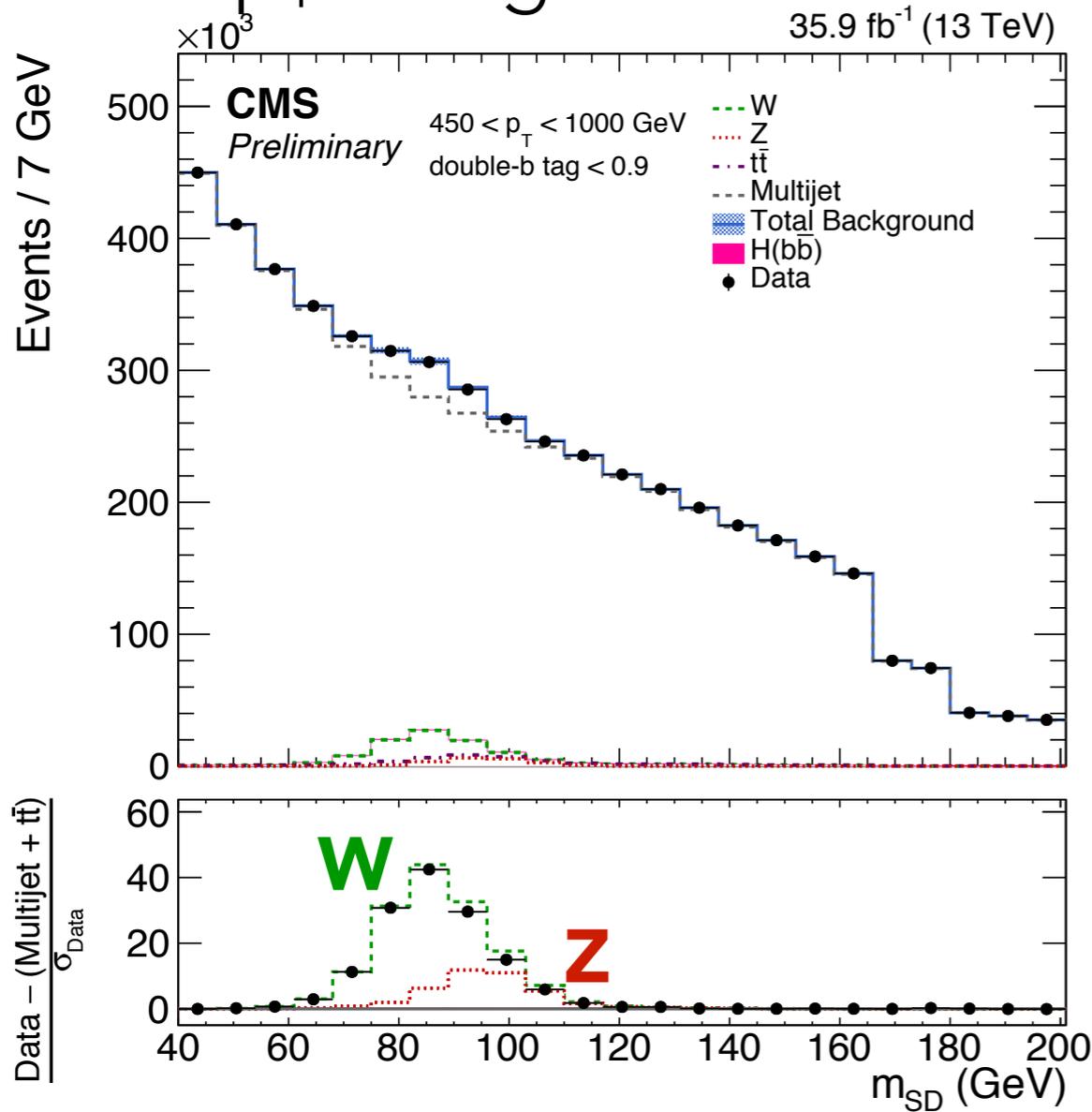


SM candles: Z(bb) peak provides in-situ constraint of H(bb) signal systematics

**observed Z(bb) significance:**  
 **$5.1\sigma$ ,  $\mu_Z = 0.78^{+0.23}_{-0.19}$**

# FIT RESULTS

- Simultaneous fit for Z(bb) and H(bb)
- All  $p_T$  categories



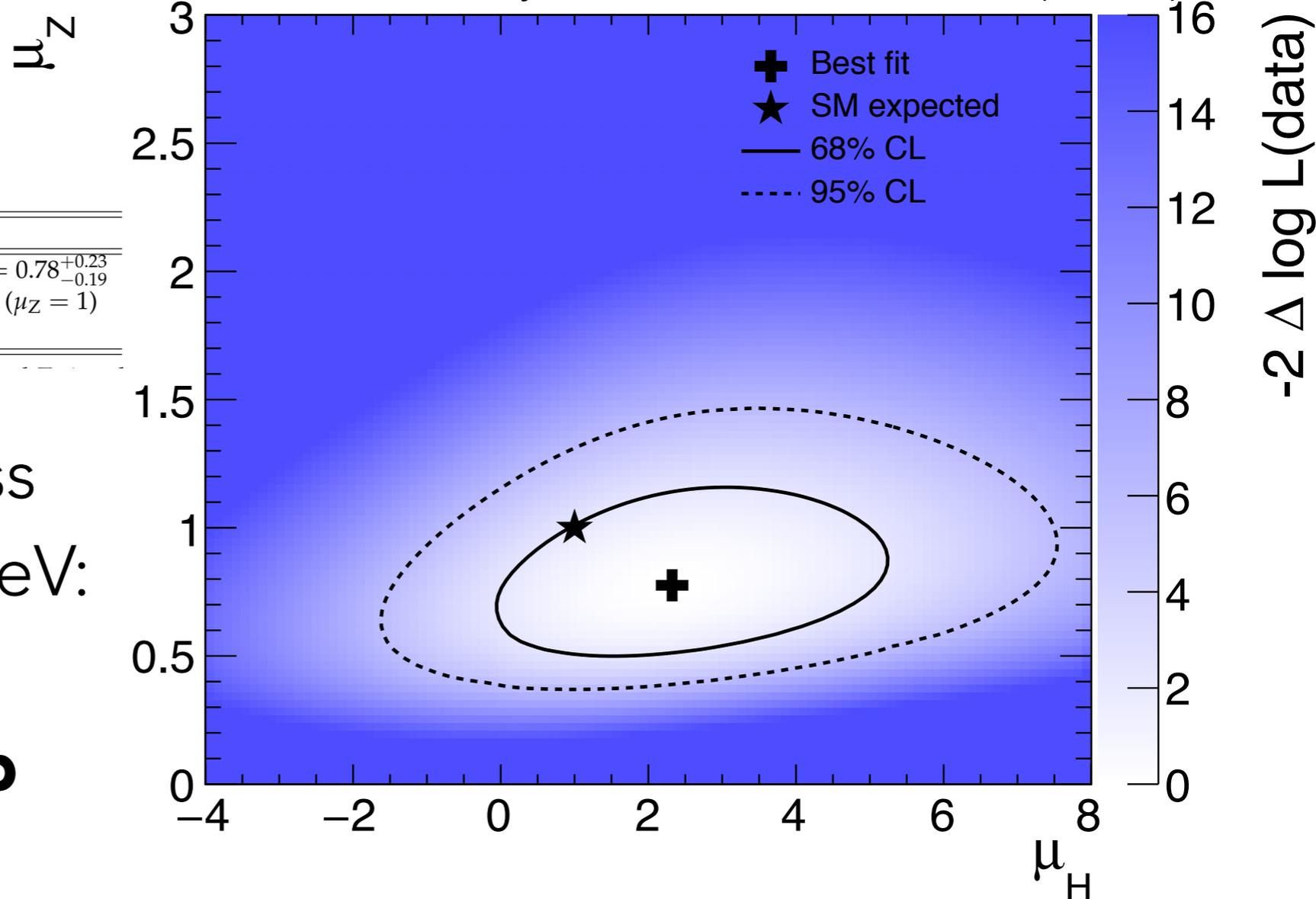
**observed H(bb) significance:**

$$1.5\sigma, \mu_H = 2.3^{+1.8}_{-1.6}$$

# FIT RESULTS

- Two dimensional likelihood scan

CMS Preliminary 35.9 fb<sup>-1</sup> (13 TeV)



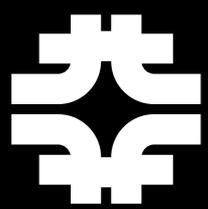
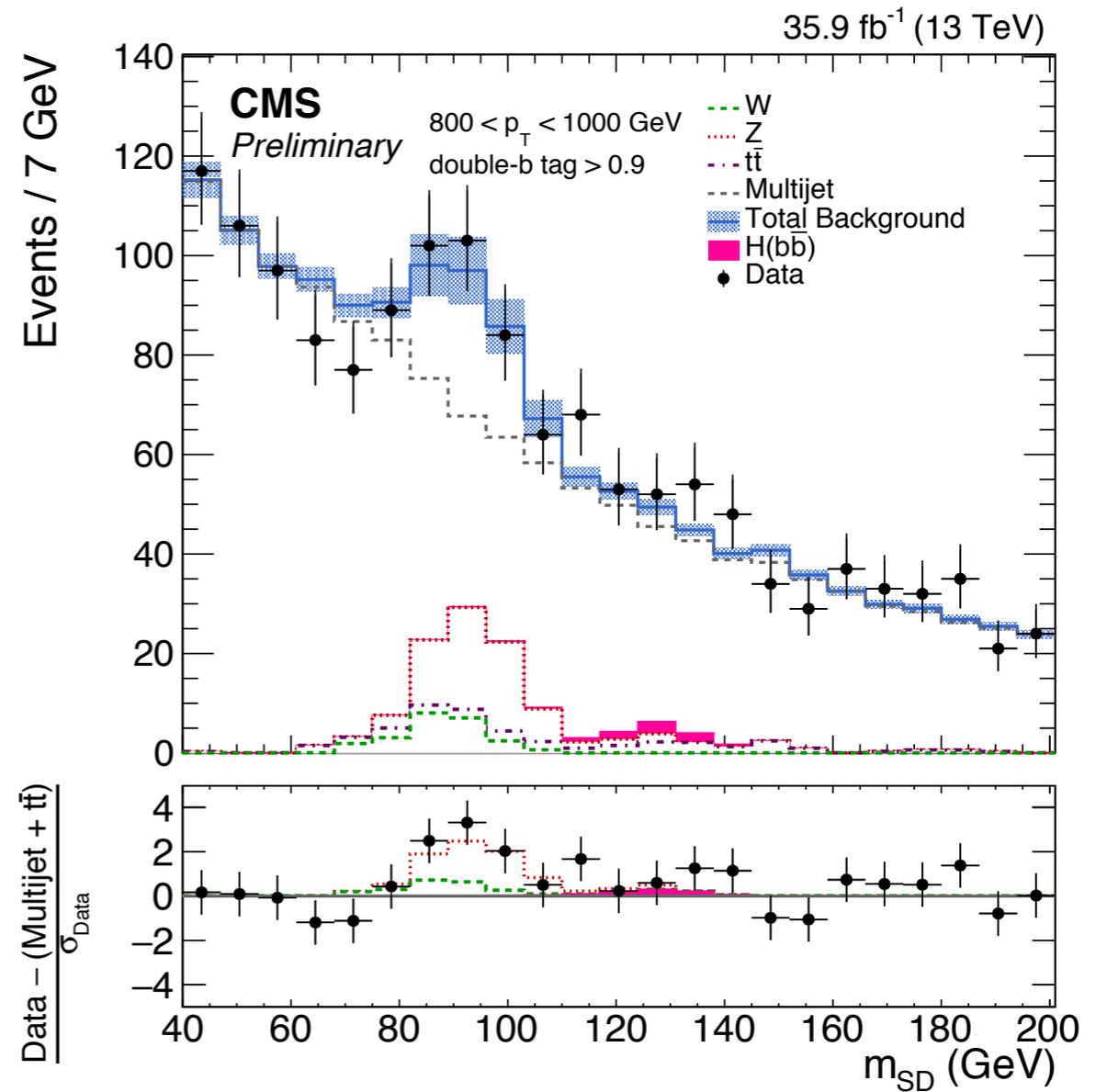
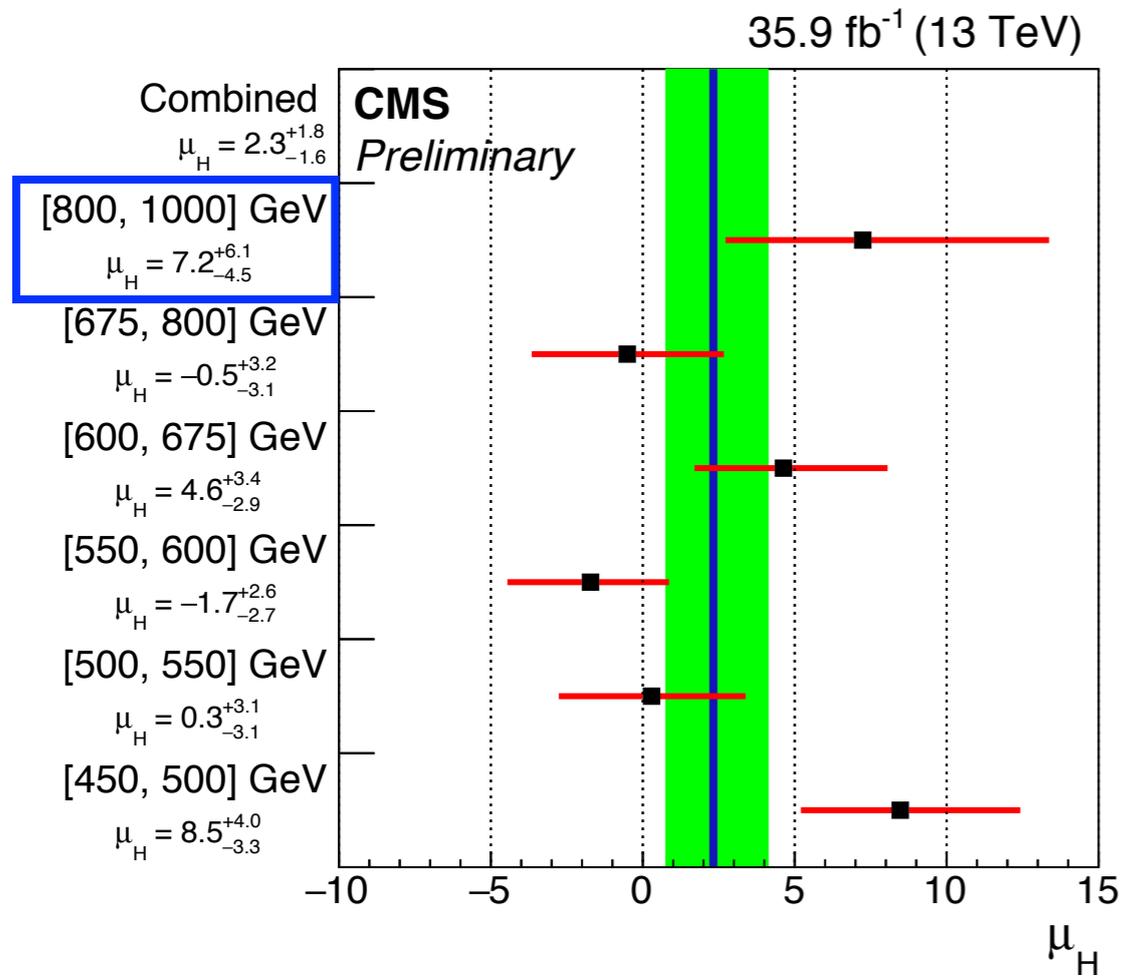
|                       | H                           | H no $p_T$ corrections       | Z                              |
|-----------------------|-----------------------------|------------------------------|--------------------------------|
| Observed best fit     | $\mu_H = 2.3_{-1.6}^{+1.8}$ | $\mu'_H = 3.2_{-2.0}^{+2.2}$ | $\mu_Z = 0.78_{-0.19}^{+0.23}$ |
| Expected significance | $0.7\sigma$ ( $\mu_H = 1$ ) | $0.5\sigma$ ( $\mu'_H = 1$ ) | $5.8\sigma$ ( $\mu_Z = 1$ )    |
| Observed significance | $1.5\sigma$                 | $1.6\sigma$                  | $5.1\sigma$                    |

measured visible cross sections for  $p_T > 450$  GeV:

$$\sigma_H = 74^{+51}_{-49} \text{ fb}$$

$$\sigma_Z = 0.85^{+0.26}_{-0.21} \text{ pb}$$

# P<sub>T</sub> CATEGORIES

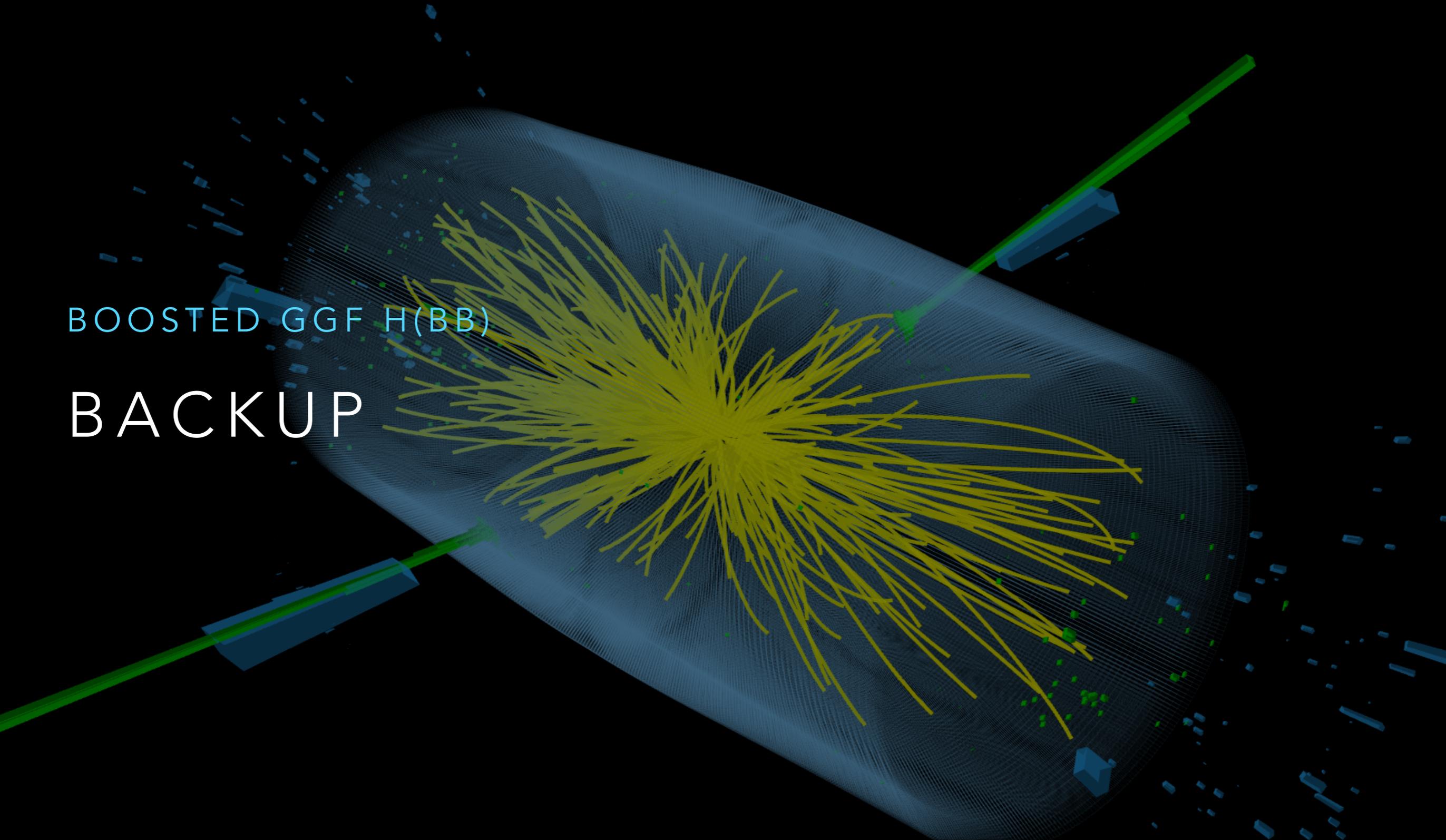


# SUMMARY AND OUTLOOK

- First LHC search for  $gg \rightarrow H \rightarrow bb$  in boosted topology
  - First observation of  $Z(bb)$  in single-jet topology,  $5.1\sigma$  observed ( $5.8\sigma$  expected)
  - Observed significance of  $H(bb)$  is  $1.5\sigma$
- Measured cross sections agree with SM
- Search probes **previously unexplored** regions of Higgs phase space
- **New** and **generic** strategy to search for boosted hadronic Higgs decays
  - Future prospects are bright
  - Means we need help from LHC H XS WG for best possible theory prediction in boosted Higgs regime —  $p_T^H$  up to **1 TeV** and beyond...

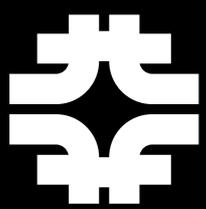
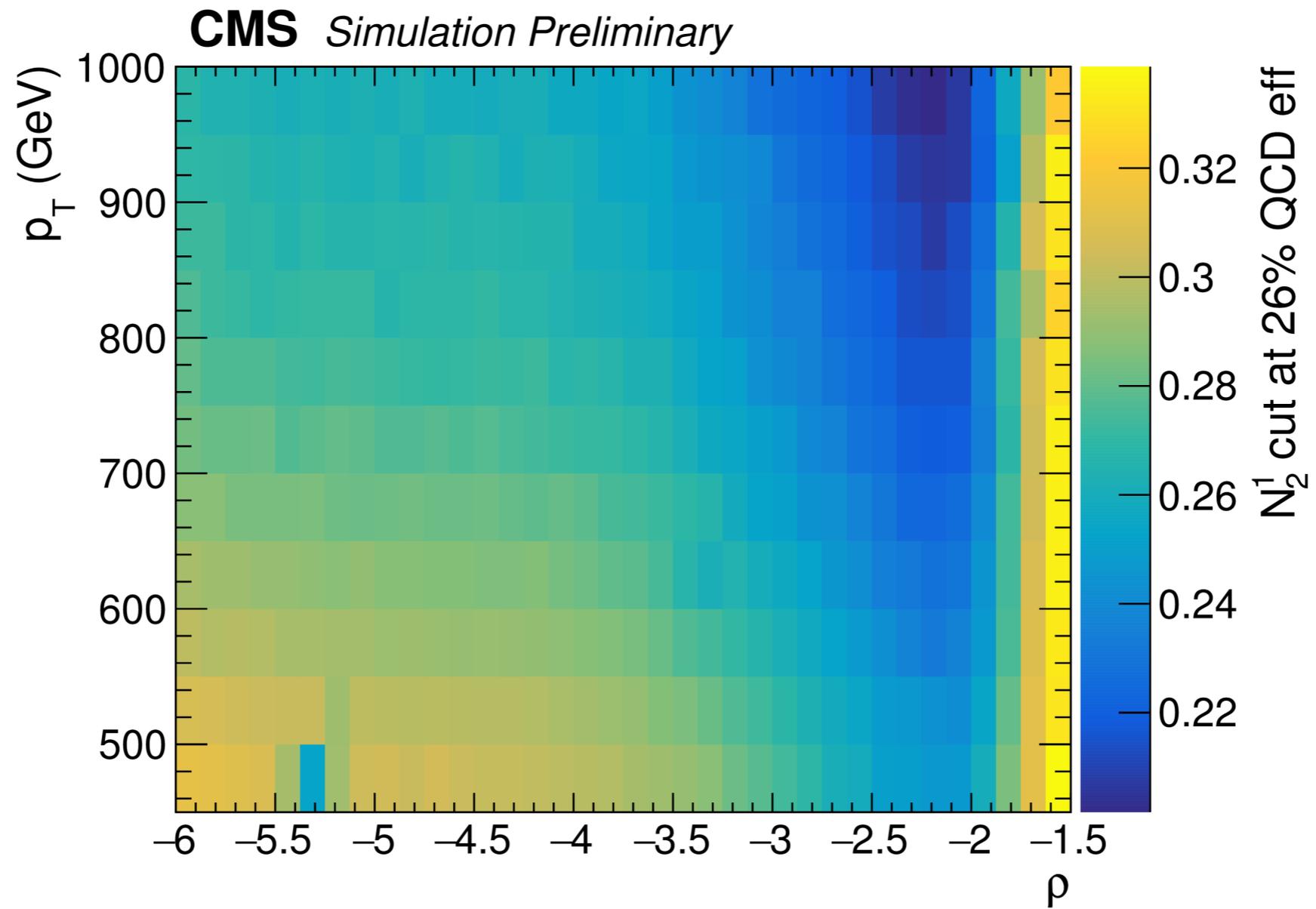
BOOSTED GGF H(BB)

BACKUP



$$N^1_2^{DDT}$$

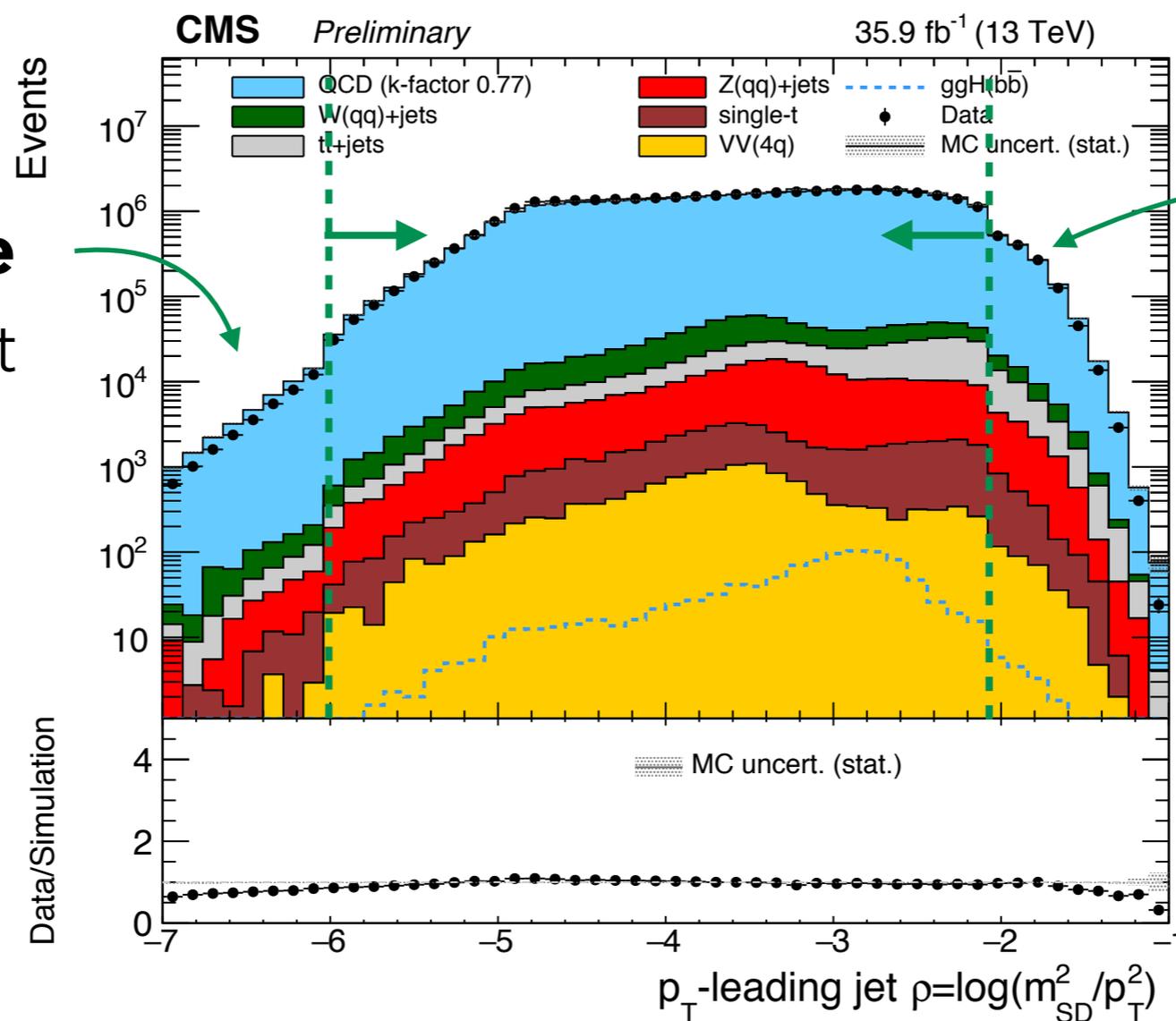
- Cut value map used to transform  $N^1_2$



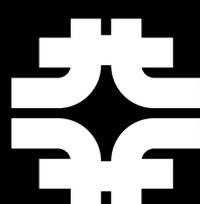
# EVENT SELECTION

$$-6.0 < \rho < -2.1$$

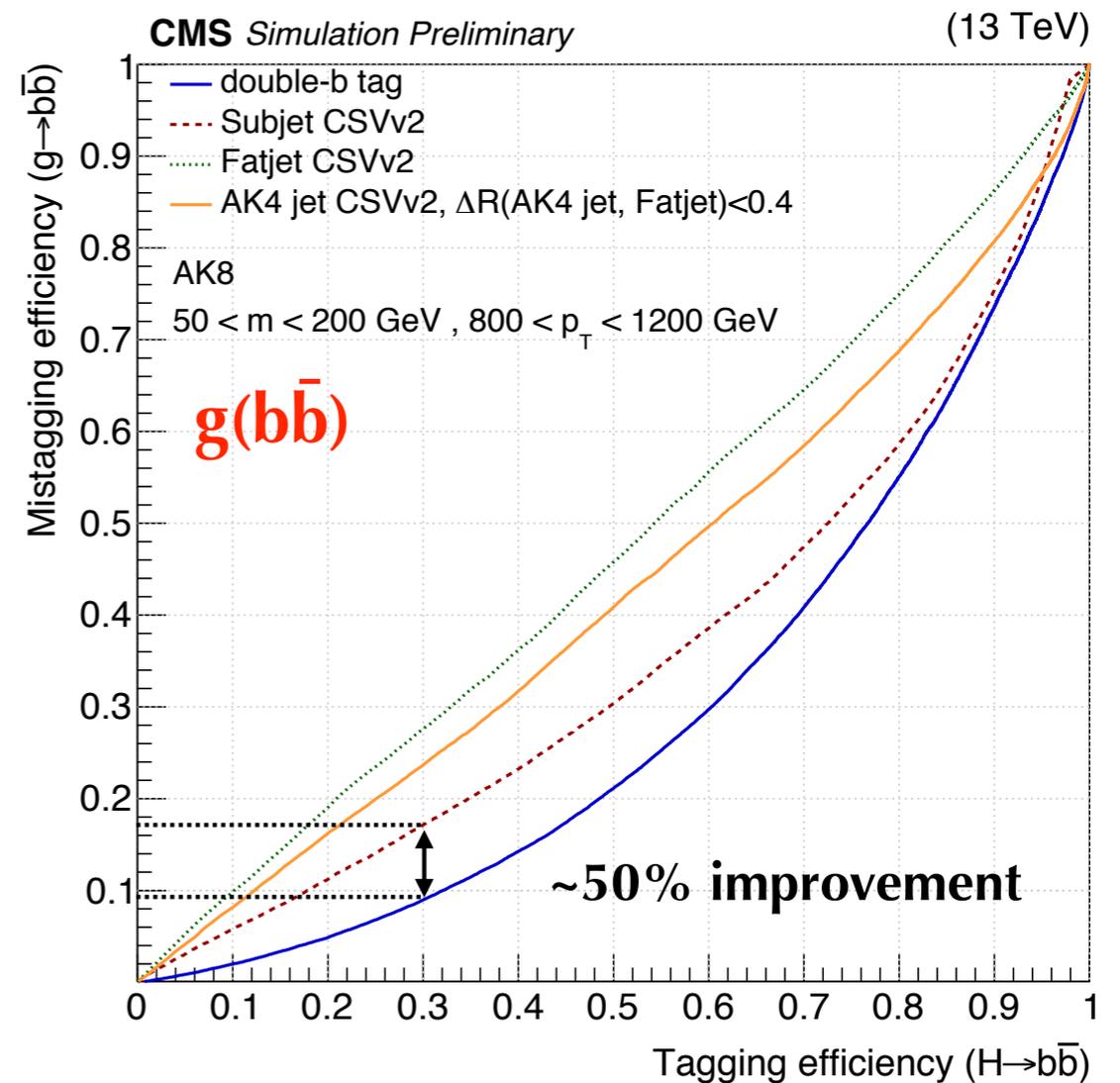
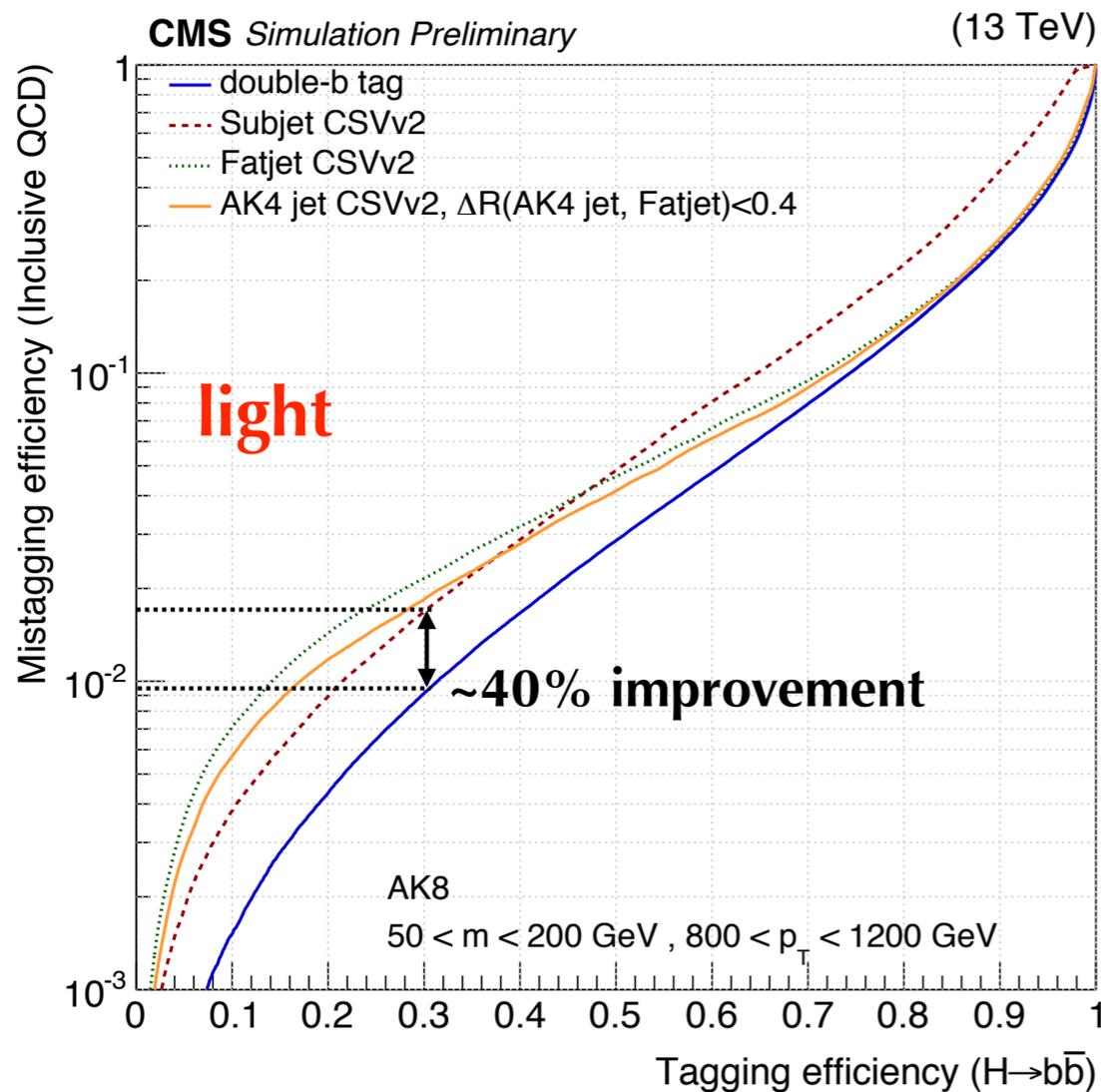
**non-perturbative regime** of the soft drop mass calculation



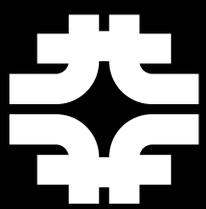
**finite cone effects** from the AK8 jet clustering



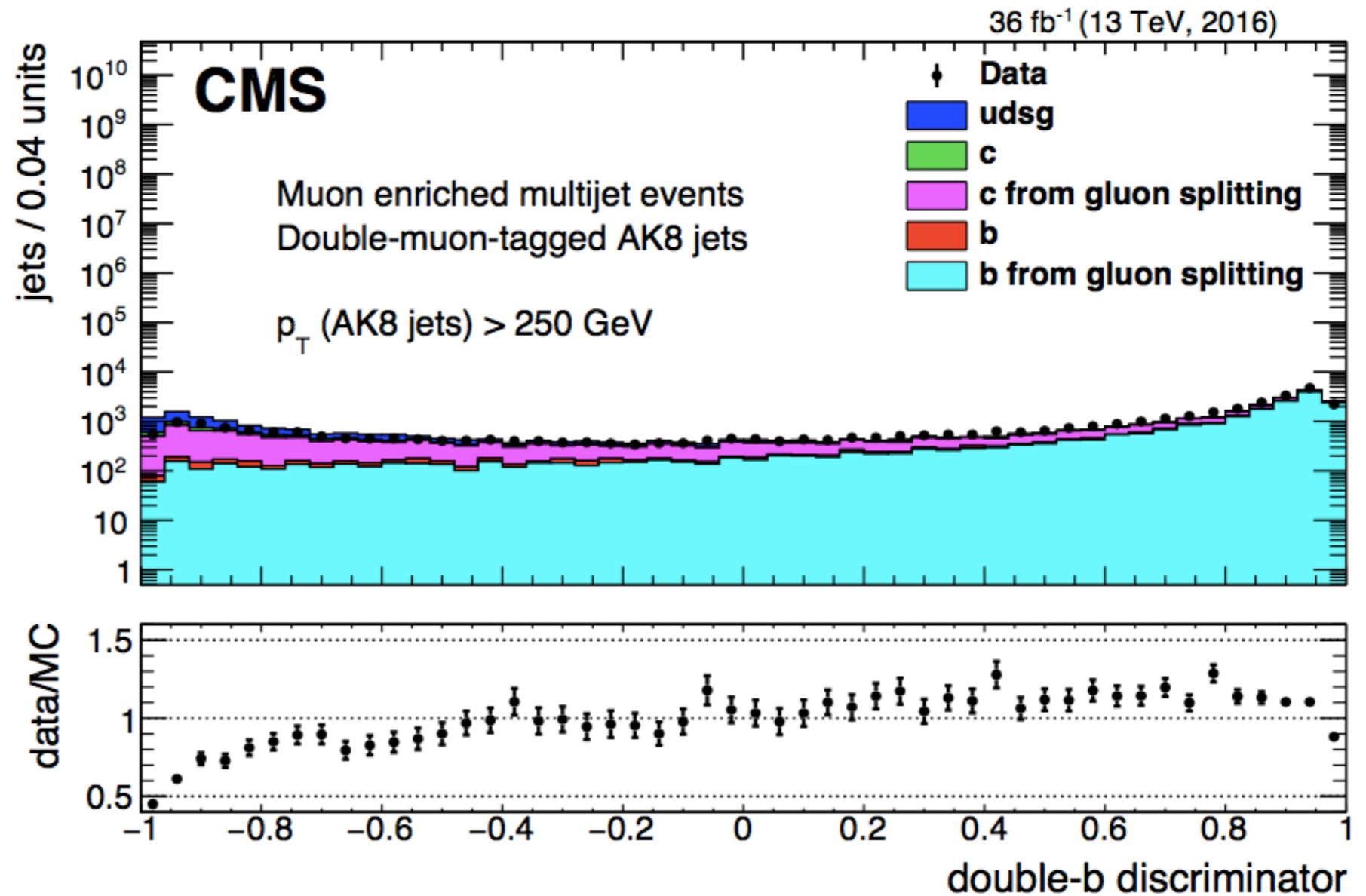
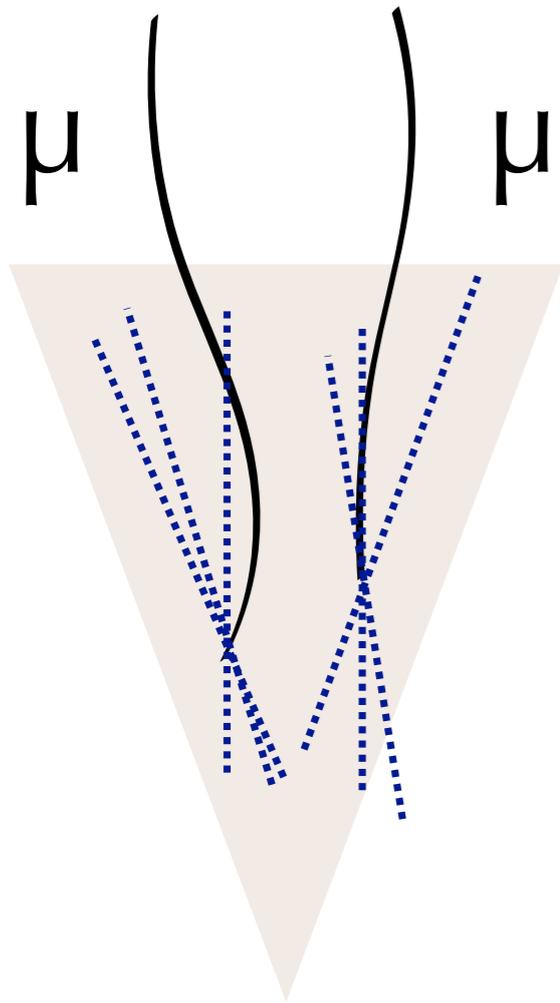
# EFFICIENCY AND MIS-TAG



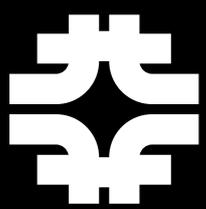
- Mis-tag is reduced by more than 40% at 30% signal efficiency for a tight working point



# EFFICIENCY IN DATA

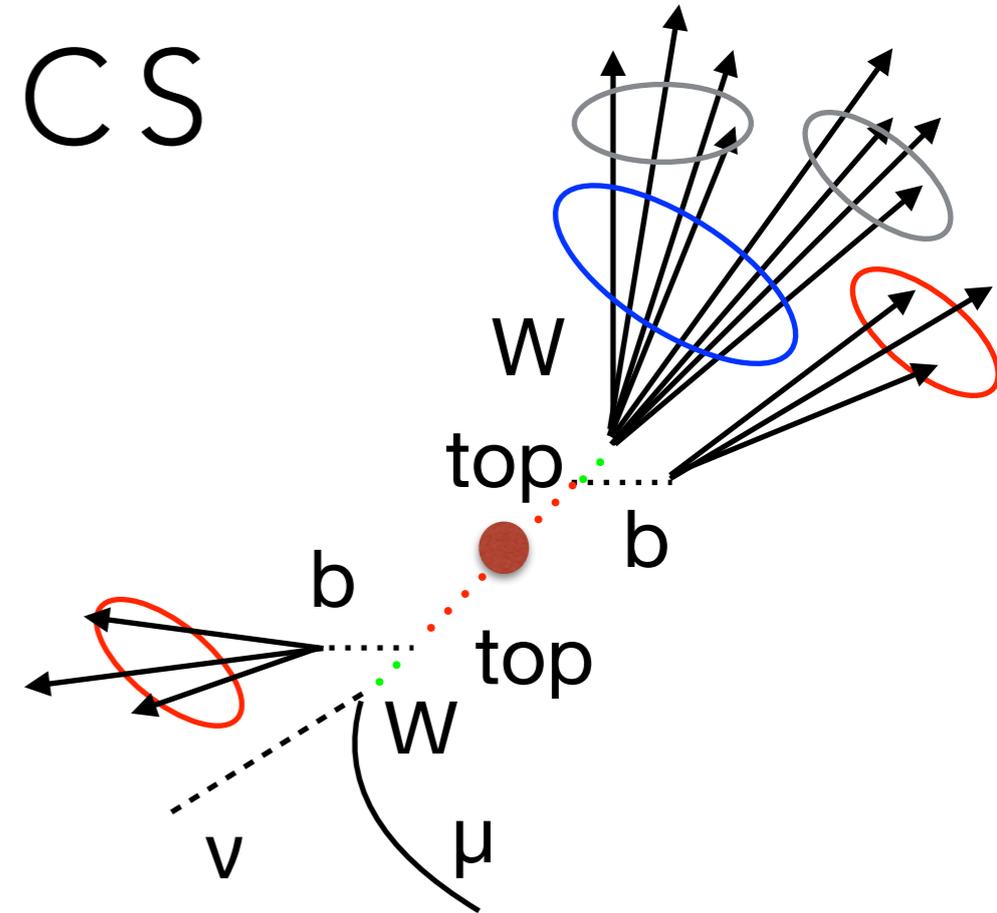


- Using g(bb) jet as proxy in double muon tagged jet sample
- Associated data/MC uncertainty 3-5%

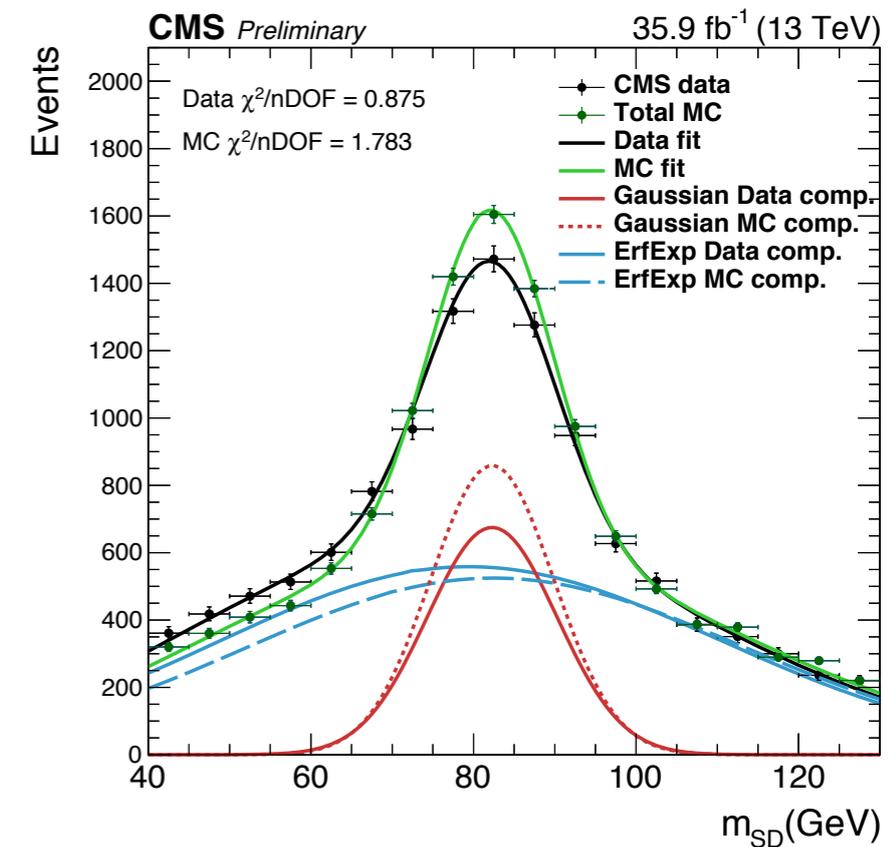


# SYSTEMATICS

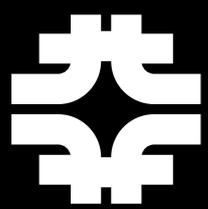
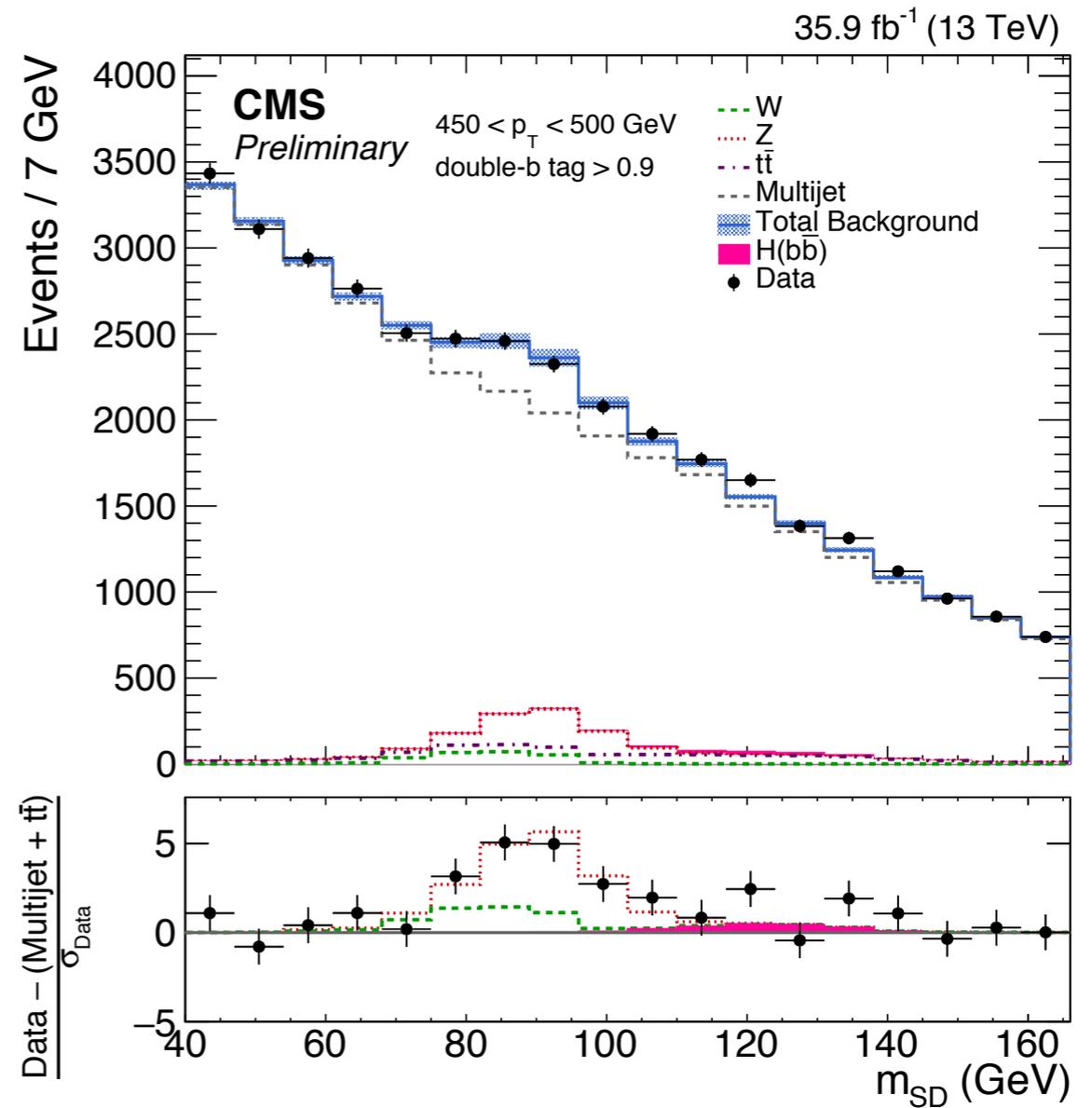
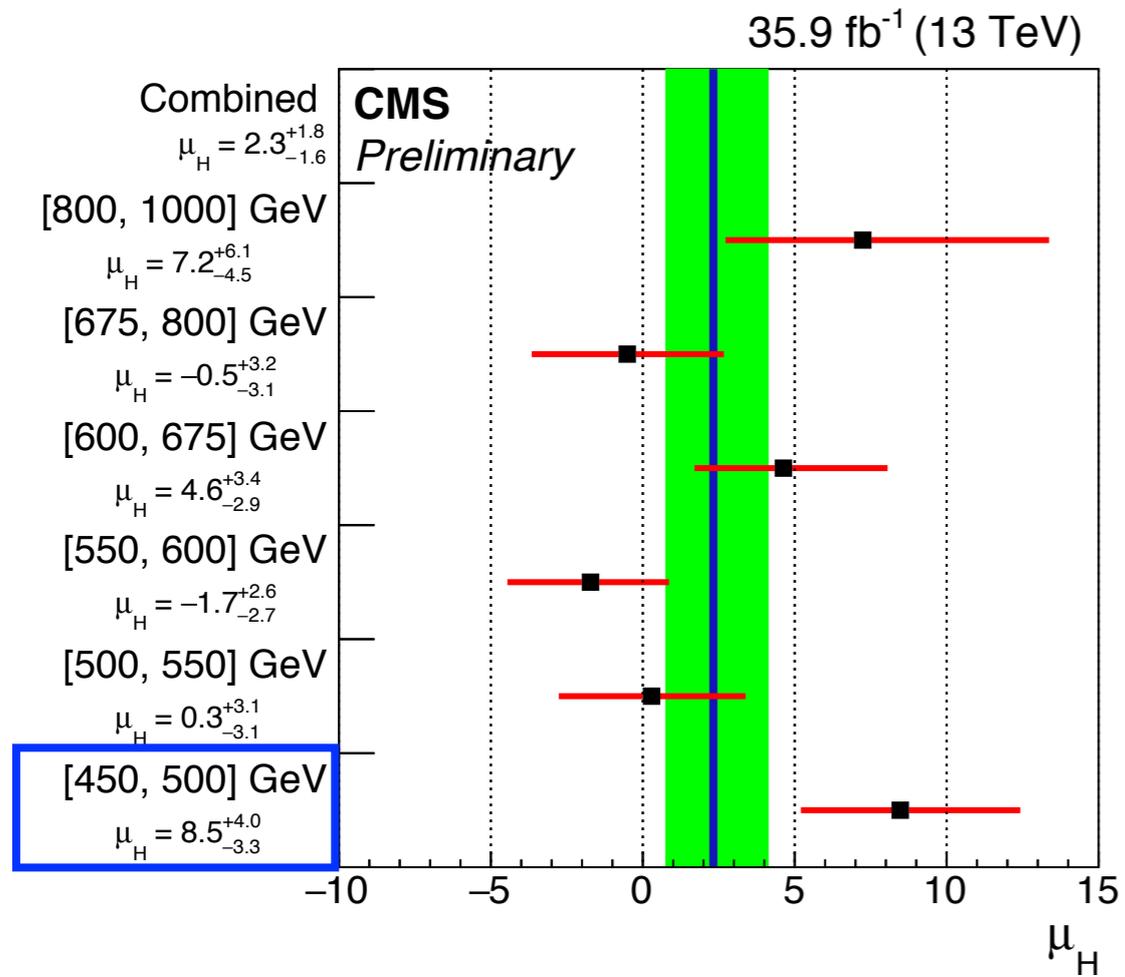
| Systematic uncertainty source      | Type (shape or normalization) | Relative size (or description)            |
|------------------------------------|-------------------------------|---|
| QCD transfer factor                | both                          | profile $a_{k\ell}$ and QCD normalization |
| Luminosity                         | normalization                 | 2.5%                                      |
| V-tag ( $N_2^{1,DDT}$ ) efficiency | normalization                 | 4.3%                                      |
| Muon veto efficiency               | normalization                 | 0.5%                                      |
| Electron veto efficiency           | normalization                 | 0.5%                                      |
| Trigger efficiency                 | normalization                 | 4%  |
| Muon ID efficiency                 | shape                         | up to 0.2%                                |
| Muon isolation efficiency          | shape                         | up to 0.1%                                |
| Muon trigger efficiency            | shape                         | up to 8%                                  |
| $t\bar{t}$ normalization SF        | normalization                 | from $1\mu$ CR: 8%                        |
| $t\bar{t}$ double-b mis-tag SF     | normalization                 | from $1\mu$ CR: 15%                       |
| W/Z NLO QCD corrections            | normalization                 | 10%                                       |
| W/Z NLO EWK corrections            | normalization                 | 15% – 35%                                 |
| W/Z NLO EWK ratio decorrelation    | normalization                 | 5% – 15%                                  |
| double-b tagging efficiency        | normalization                 | 4%  |
| Jet energy scale                   | normalization                 | up to 10%                                 |
| Jet energy resolution              | normalization                 | up to 15%                                 |
| Jet mass scale                     | shape                         | shift $m_{SD}$ peak by $\pm 0.4\%$        |
| Jet mass resolution                | shape                         | smear $m_{SD}$ distribution by $\pm 9\%$  |
| Jet mass scale $p_T$               | normalization                 | 0.4%/100 GeV ( $p_T$ )                    |
| Monte Carlo statistics             | normalization                 | -   |
| H $p_T$ correction (gluon fusion)  | both                          | 30%                                       |



- Signal systematic uncertainties from merged W sample in semi-leptonic  $t\bar{t}b$  events (external constraint)
- SM candles: presence of W/Z in final jet mass distribution provides additional in-situ constraint

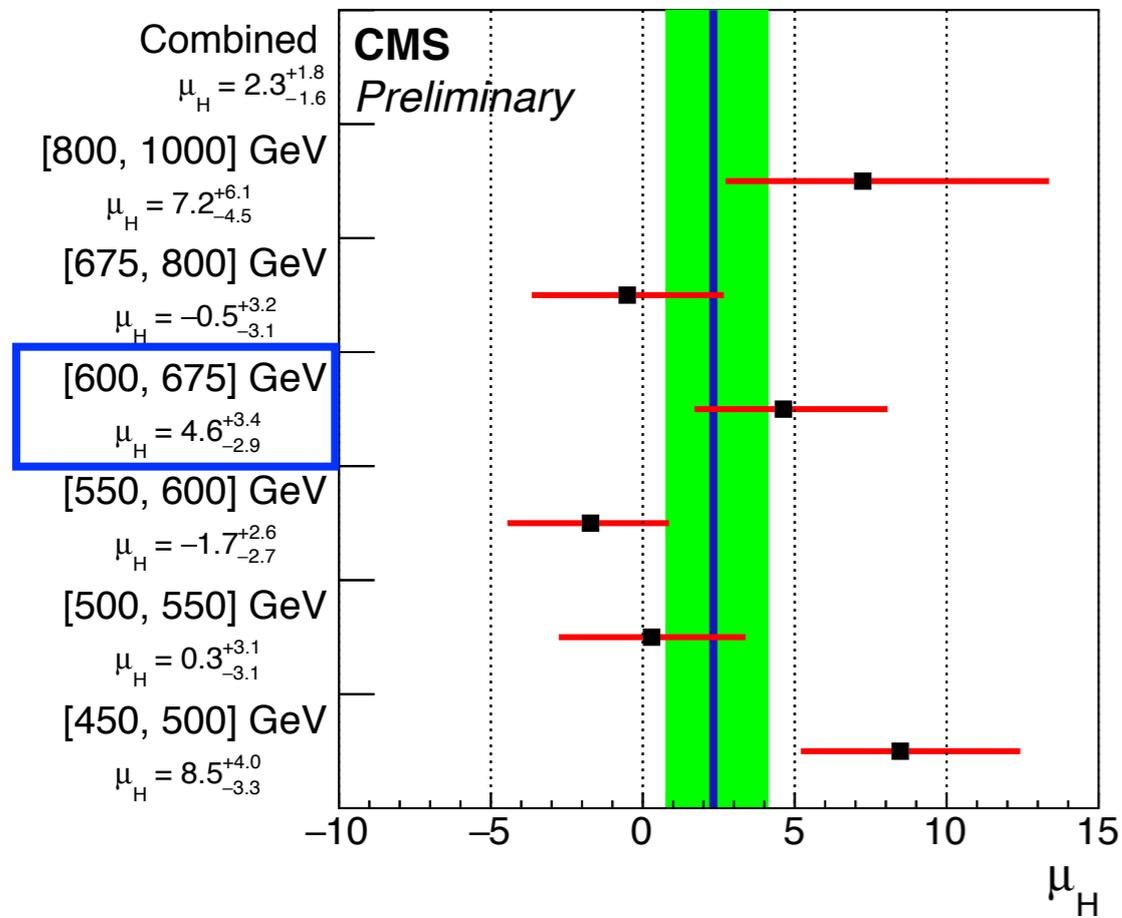


# P<sub>T</sub> CATEGORIES

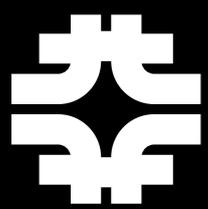
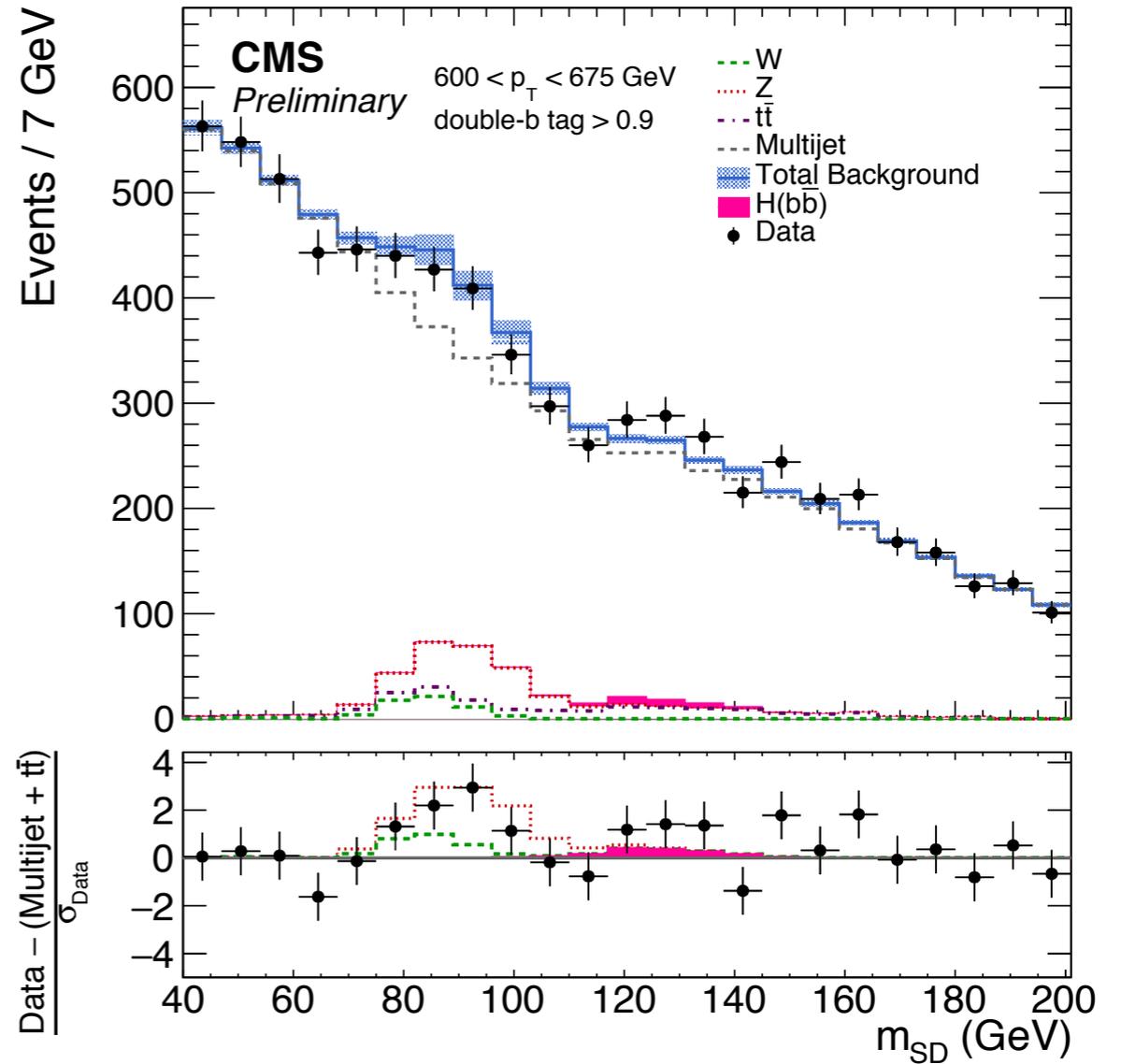


# P<sub>T</sub> CATEGORIES

35.9 fb<sup>-1</sup> (13 TeV)



35.9 fb<sup>-1</sup> (13 TeV)



# LARGE HADRON COLLIDER

Lake Geneva

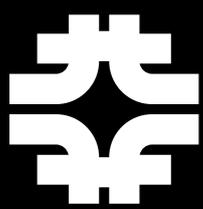
★ CMS



CERN

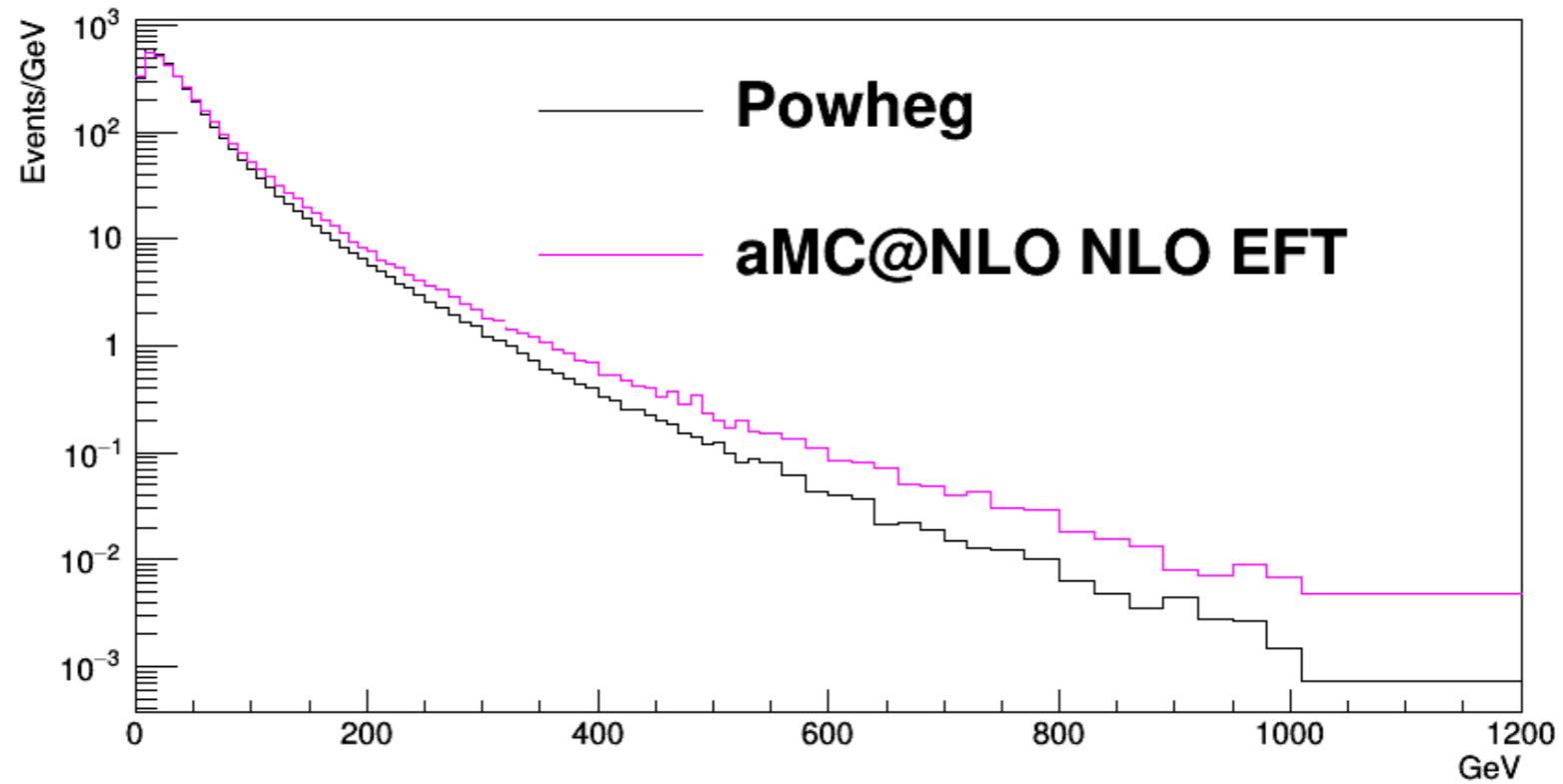
# What is the best Higgs $p_T$ : Options

- The key is to identify two different effects
  - Finite top mass effect
  - NNLO differential corrections
- What are the orders known:
  - Differential EFT : NNLO H+1jet production
  - Finite top mass : almost NLO
  - At MC level EFT : NLO H+0/1/2jet
  - At MC level finite top mass : LO 0/1/2

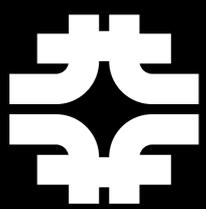
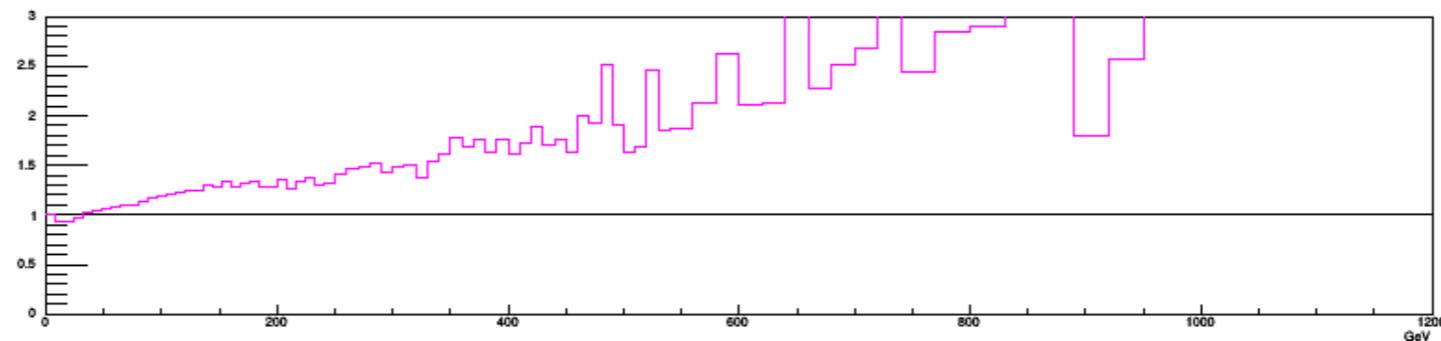


# Going to EFT

- When going to EFT large gain

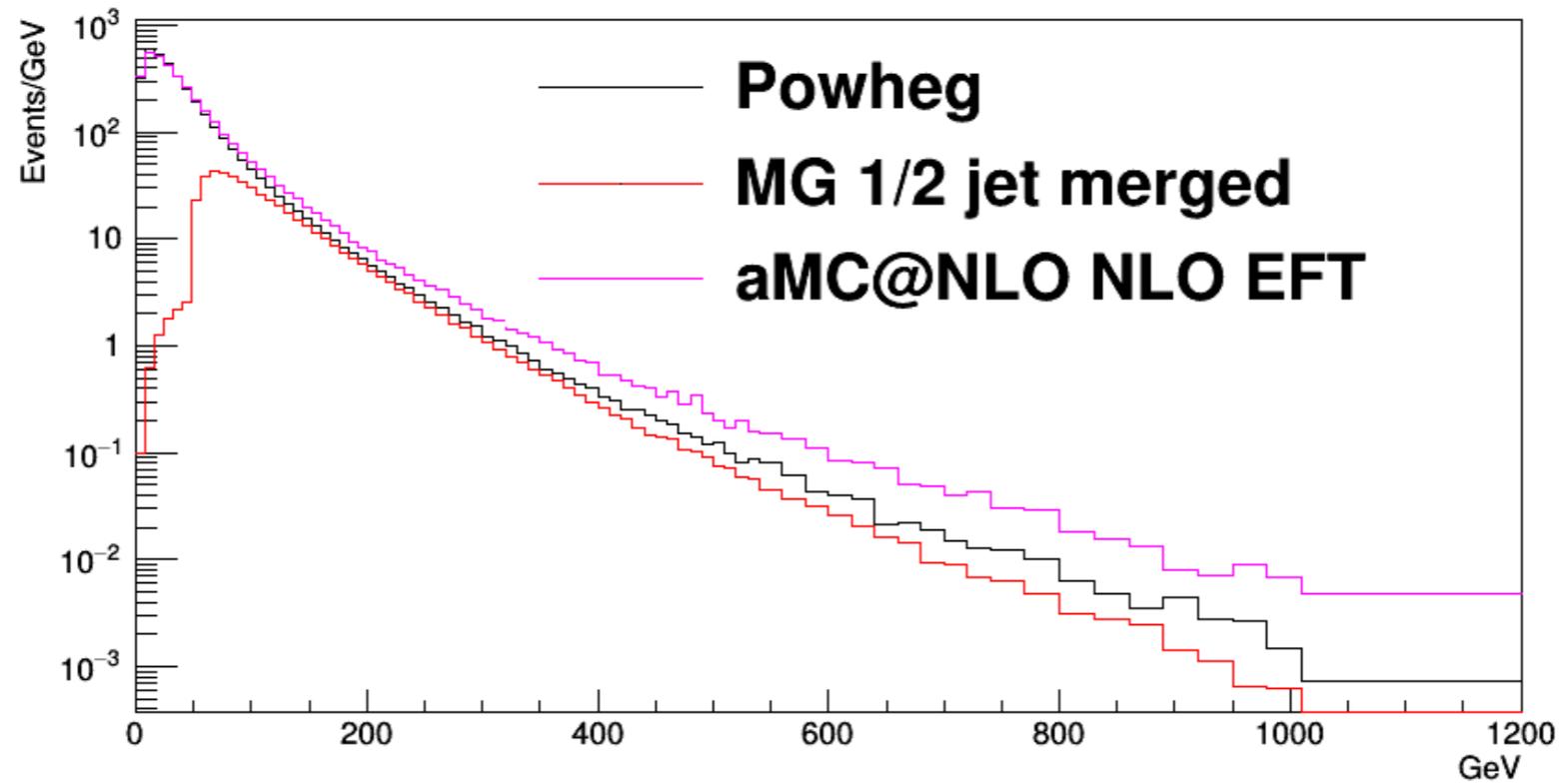


X/Powheg

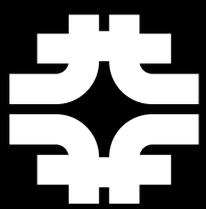
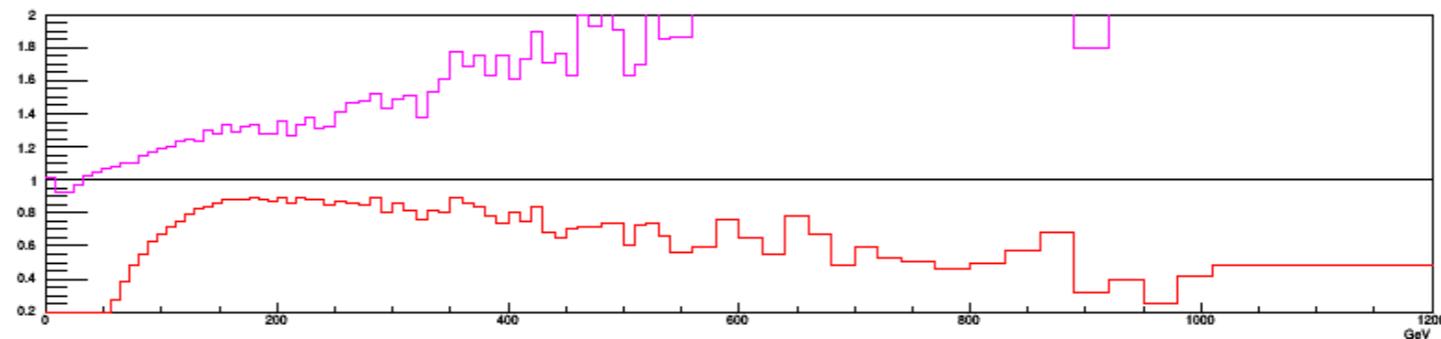


# Going to EFT

- Adding the finite top mass merged LO its lower

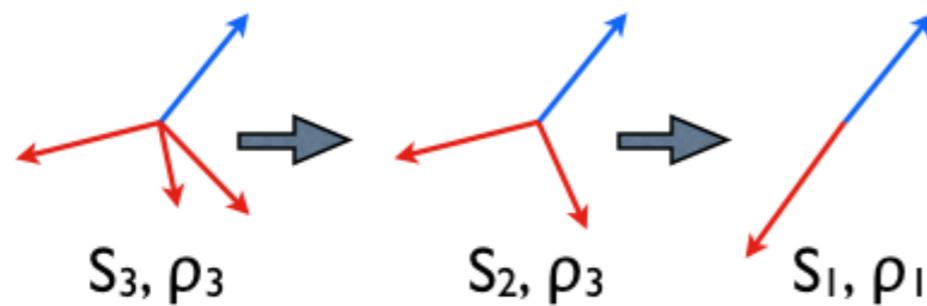


X/Powheg



Idea: Reduce the dependence to the merging scale  $MS$ .

- Start by generate events with  $N_1..N_2$  ME partons, hard and well separated
- Assume an event with  $n$  ME partons, reconstruct the possible shower histories, pick one according to the occurrence probabilities



- Each clustering step  $i$  is characterized by the emission scale  $\rho_i$ , reweight by the product of  $\alpha_s(\rho_i)/\alpha_s(ME)$
- For  $i=2..n$  ( $\neq N_2$ ):
  - Generate one emission  $\rho$  with  $\rho_i$  as starting scale.
  - If  $\rho > \rho_{i+1} \Rightarrow$  reject the event.
    - This is equivalent to the product of Sudakovs  $\Pi(\rho_i, \rho_{i+1})$ ,  $i=2..n-1$ .
- if not HME: generate an emission at  $\rho < \rho_n$ , if  $\rho > MS \Rightarrow$  reject the event.
- if HME, accept the event and start shower with  $\rho_n$ .

# Practical use: Madgraph 5

## Essential parameters for matching/merging:

- `ickkw`
  - Applies the  $\alpha_s$  reweighting at each QCD vertex in the ME calculation.  $K_T$ -MLM, Shower- $K_T$ :1  
CKKW-L, UMEPS:0
- `xqcut`
  - Defines the minimal  $K_T$  between the partons (+beam) at ME level.
- `auto_ptj_mjj`
  - Set to False: leaves the xqcut be the only cut applied to ME partons  $\Rightarrow$  `ptj`, `mmjj=0`
- `maxjetflavor`
  - QCD partons with `pdgId`  $\leq$  `maxjetflavor` are affected by xqcut ptj,etc... Otherwise, affected by `ptb`, `mmbb`, etc... That means that for a n-Flavour prediction, `maxjetflavor = n`

# Practical use: main89.cc

- main89cckwl.cmnd: CKKWL. Essential parameters are
  - `Merging:TMS = XXX.`
    - The merging scale
  - `Merging:Process = UUU`
    - Type of process, e.g. `pp>LEPTONS,NEUTRINOS`
  - `Merging:nJetMax = WWW`
    - Maximal number of additional jets in the matrix element
  - `Merging:doPTLundMerging = on`
    - Set the merging scale definition to  $P_{T,evol}$  (cfr definition in the manual)
- main89umeps.cmnd: UMEPS. Essential parameters are
  - `Merging:TMS = XXX.`
  - `Merging:Process = (e.g.) pp>LEPTONS,NEUTRINOS`
  - `Merging:nJetMax = WWW`
  - `Merging:doUMEPSTree = on`
    - Reweight events according to the UMEPS prescription for tree-level configurations)