Status of CKM matrix elements from B-physics on the lattice

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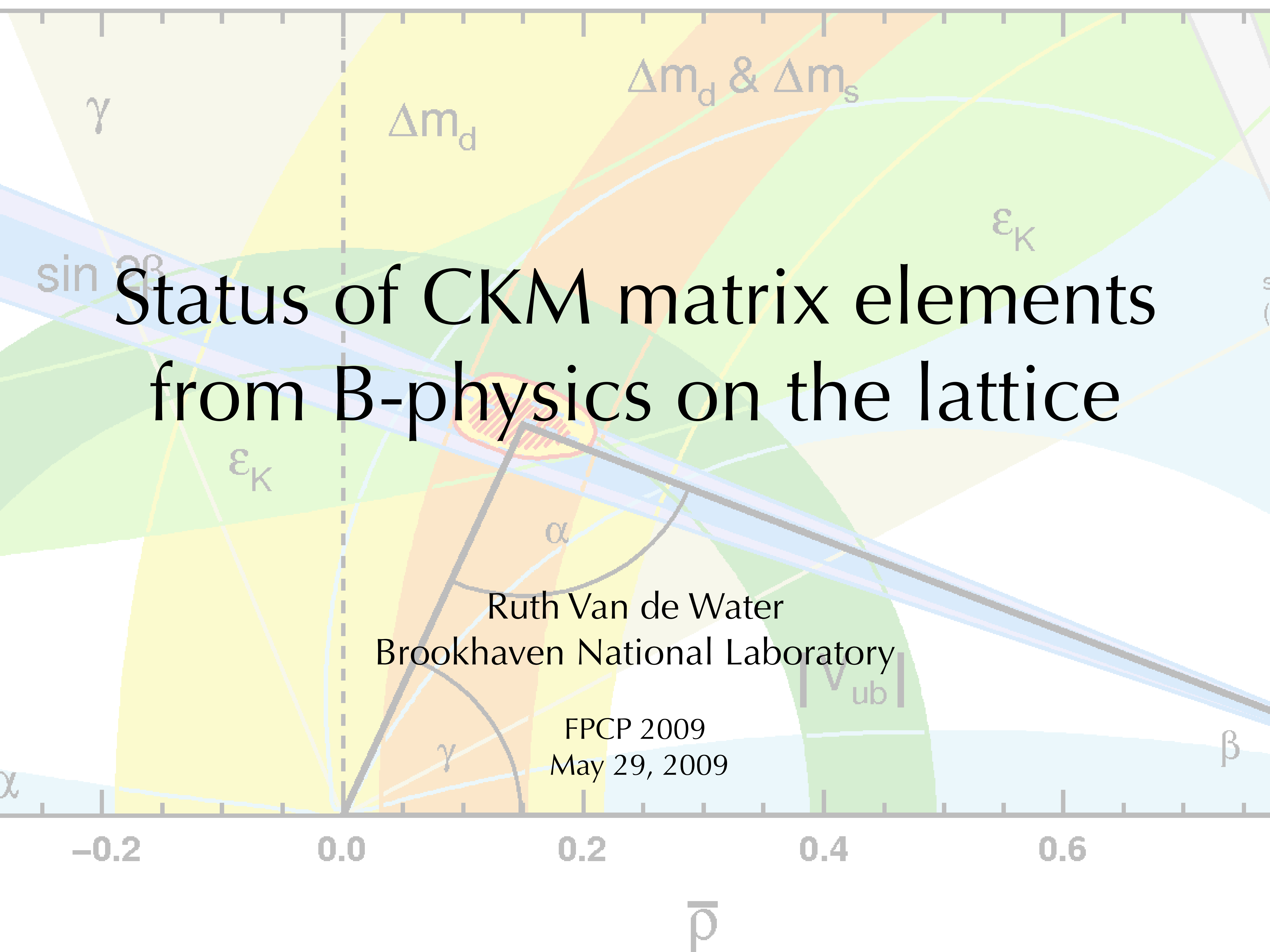
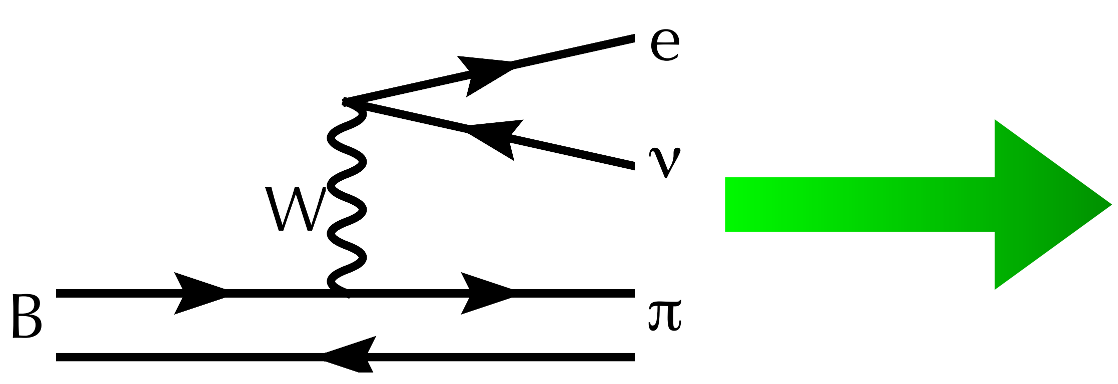
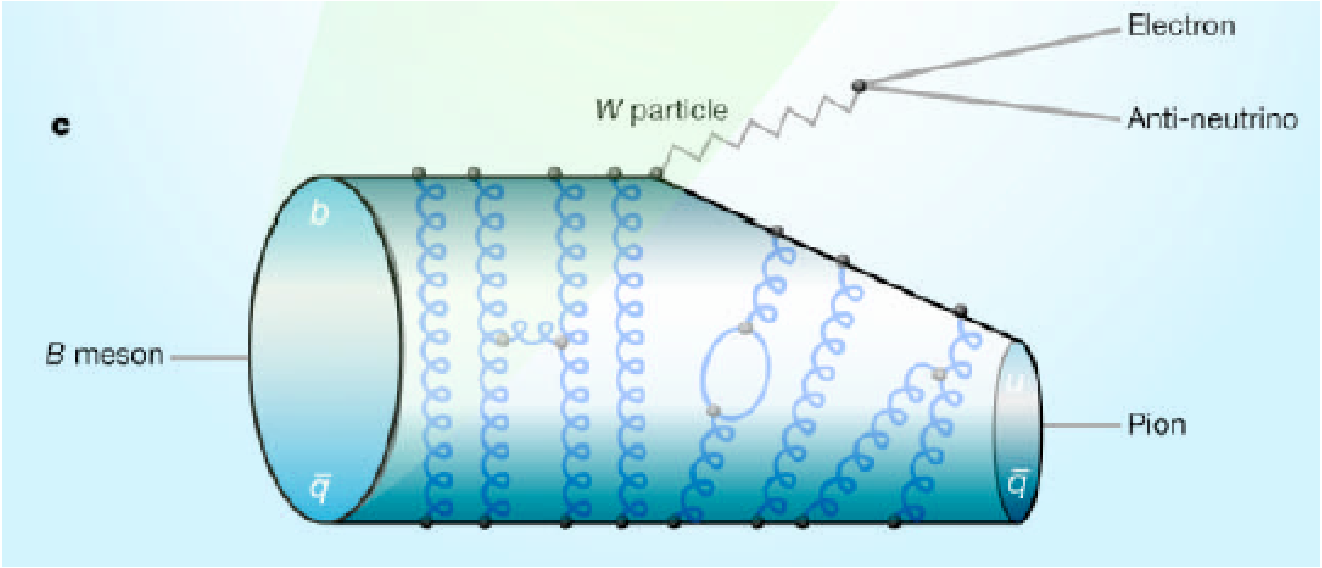


Why compute B-decays in lattice QCD?

- B-factories and Tevatron have been pouring out data to pin down the CKM matrix elements -- lattice QCD calculations are needed to interpret may of their results

- In order to accurately describe weak interactions involving quarks, must include effects of confining quarks into hadrons:



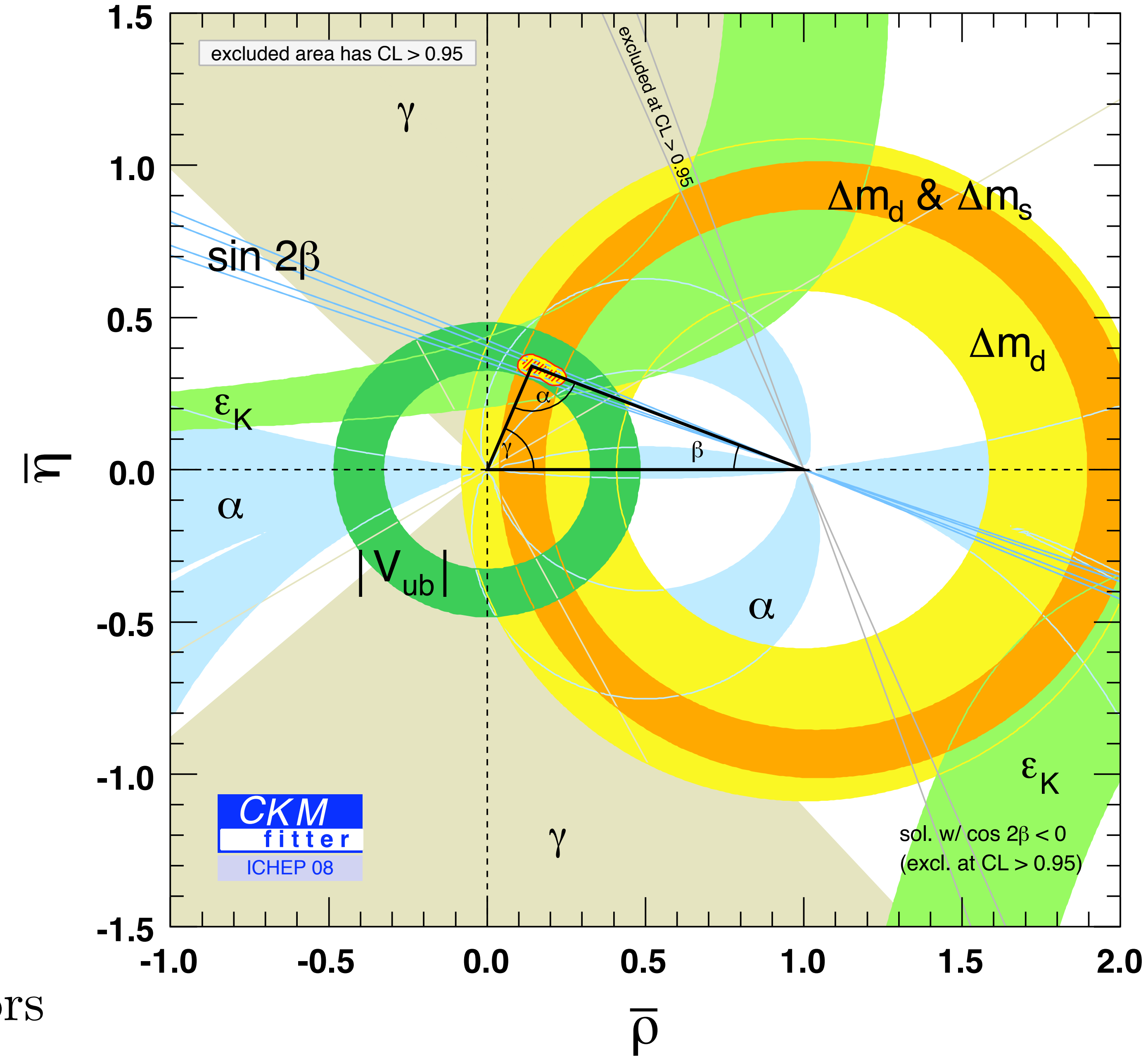


- Absorb non-perturbative QCD effects into quantities such as decay constants, form factors, and bag-parameters

- Only way to **calculate hadronic weak matrix elements with all systematic uncertainties under control** is numerically using lattice QCD

Lattice QCD and the CKM unitarity triangle

- In the Standard Model, the CKM matrix is unitary
- Leads to relationships among matrix elements that can be expressed as the CKM unitarity triangle
- New quark flavor-changing interactions & CP-violating phases would manifest themselves as apparent inconsistencies between experimental measurements that are predicted to be the same within the Standard Model framework



- Schematically,

$$\text{expt.} = \text{CKM} \times \text{lattice} \times \text{known factors}$$

- ⇒ To test the Standard Model and observe new physics, **need precise (few % or better) lattice QCD calculations**

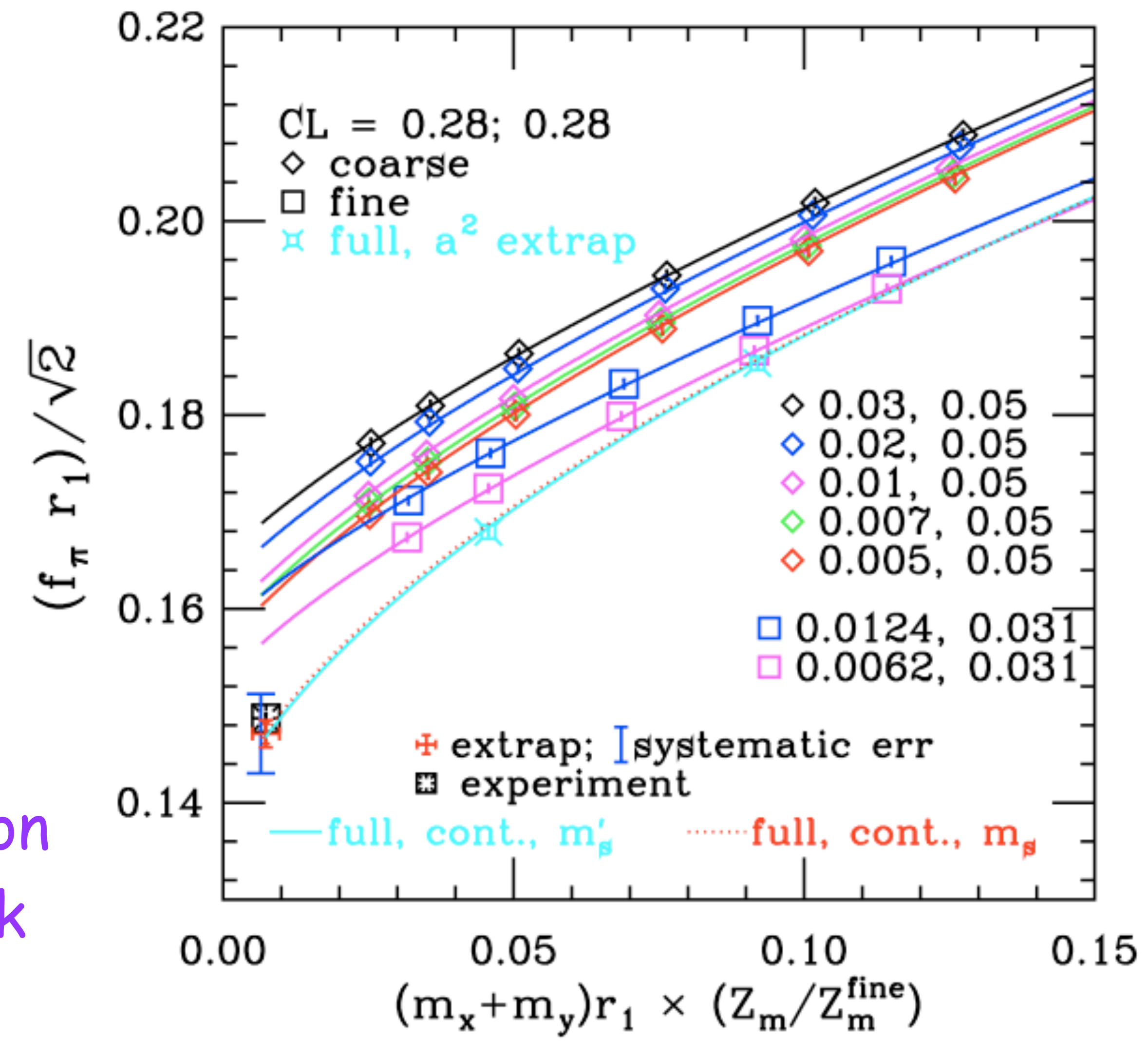
Systematics in lattice calculations

- Lattice calculations typically quote the following sources of error:
  - **(1)** Monte carlo statistics & fitting
  - **(2)** Tuning lattice spacing, a, and quark masses
  - **(3)** Matching lattice gauge theory to continuum QCD
    - (Sometimes split up into relativistic errors, discretization errors, perturbation theory, ...)
  - **(4)** Chiral extrapolation to physical up, down quark masses
  - **(5)** Extrapolation to continuum
    - (Often combined with chiral extrapolation)
- In order to verify understanding and control of systematic uncertainties in lattice calculations, **COMPARE RESULTS FOR KNOWN QUANTITIES WITH EXPERIMENT**
- Two such examples are the pion decay constant and the $D \to K \ell \nu$ form factor . . .

The pion decay constant

- Tests:
  - Dynamical (sea) quark effects
  - Light quark formalism
  - Chiral and continuum extrapolation

[MILC Lat'07 arXiv:0710.1118 [hep-lat]]

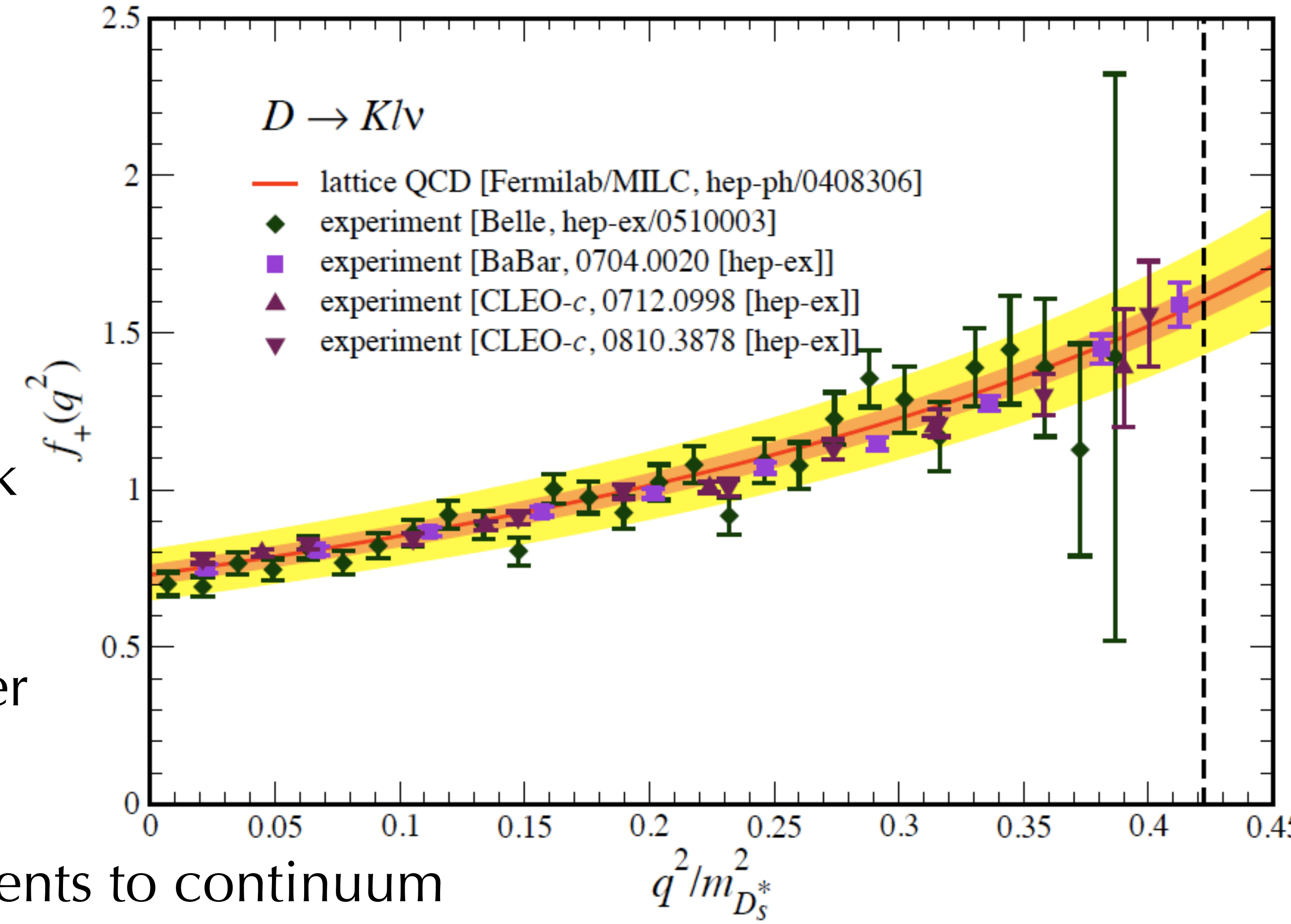


- Because of limited computing resources, quark masses in lattice simulations are higher than those in the real world
- Must extrapolate lattice results to physical values of up, down quark mass
- Use expressions derived in chiral perturbation theory to extrapolate to the physical quark masses in a controlled way
- Can also use symmetries of lattice action to incorporate discretization errors and extrapolate to the continuum
- **Can compute f_π to ~2% accuracy** and result agrees with experiment!

The D→K$\ell\nu$ form factor

- Also tests:
  - Heavy-quark formalism
  - Lattice operator matching
- Generic lattice quark action will have discretization errors $\propto (am_Q)^n$
- Can use knowledge of the heavy quark or nonrelativistic quark limits of QCD to systematically eliminate HQ discretization errors order-by-order
- Requires tuning parameters of lattice action and matching lattice weak currents to continuum
  - Typically calculate matching coefficients in lattice perturbation theory
- Estimate errors using knowledge of short-distance coefficients and power-counting
- Successfully **predicted the shape and normalization of the D→Kl ν form factor!**

[Fermilab/MILC; Phys.Rev.Lett.94:011601,2005]

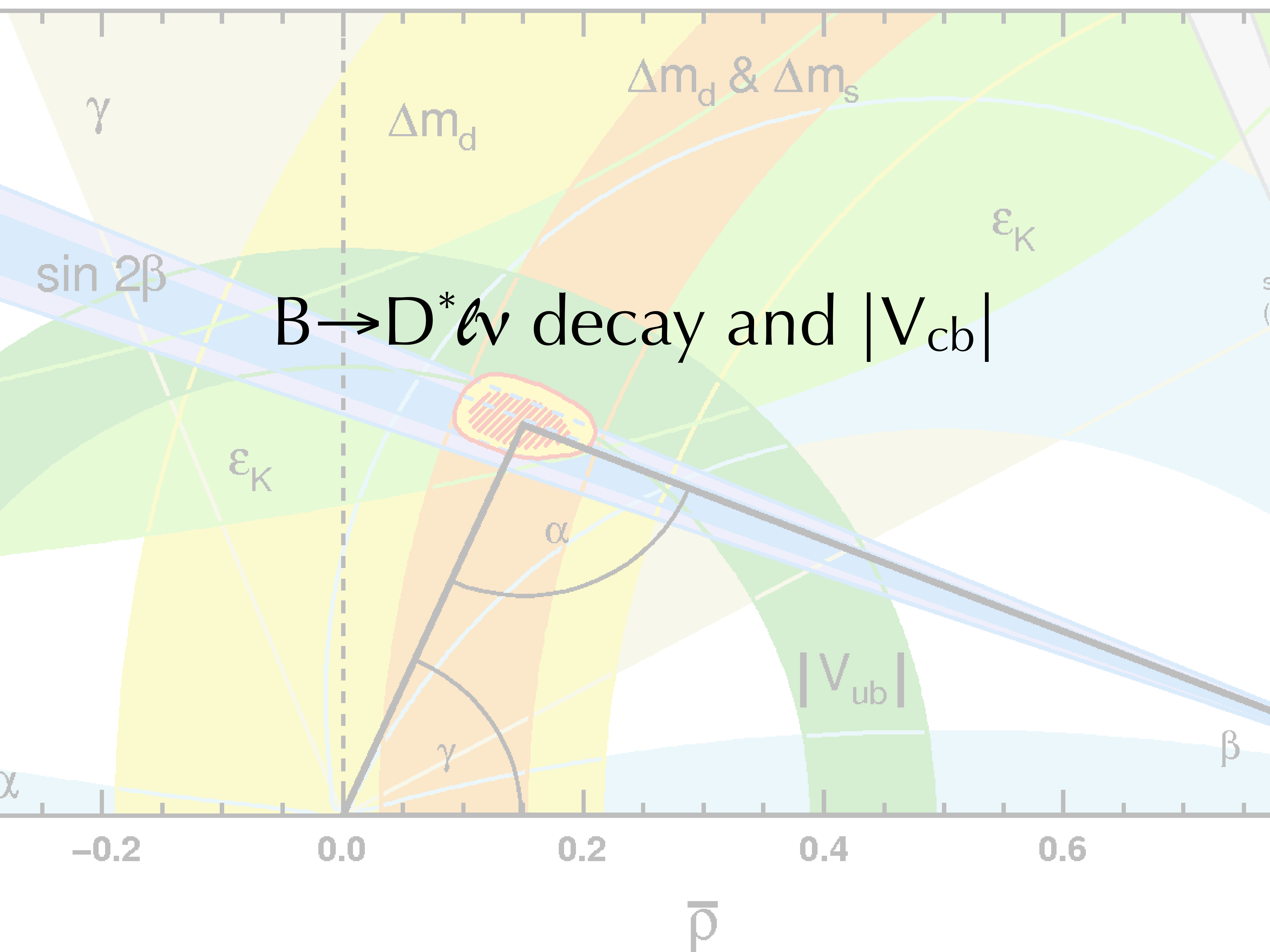


Lattice calculations of B-meson quantities

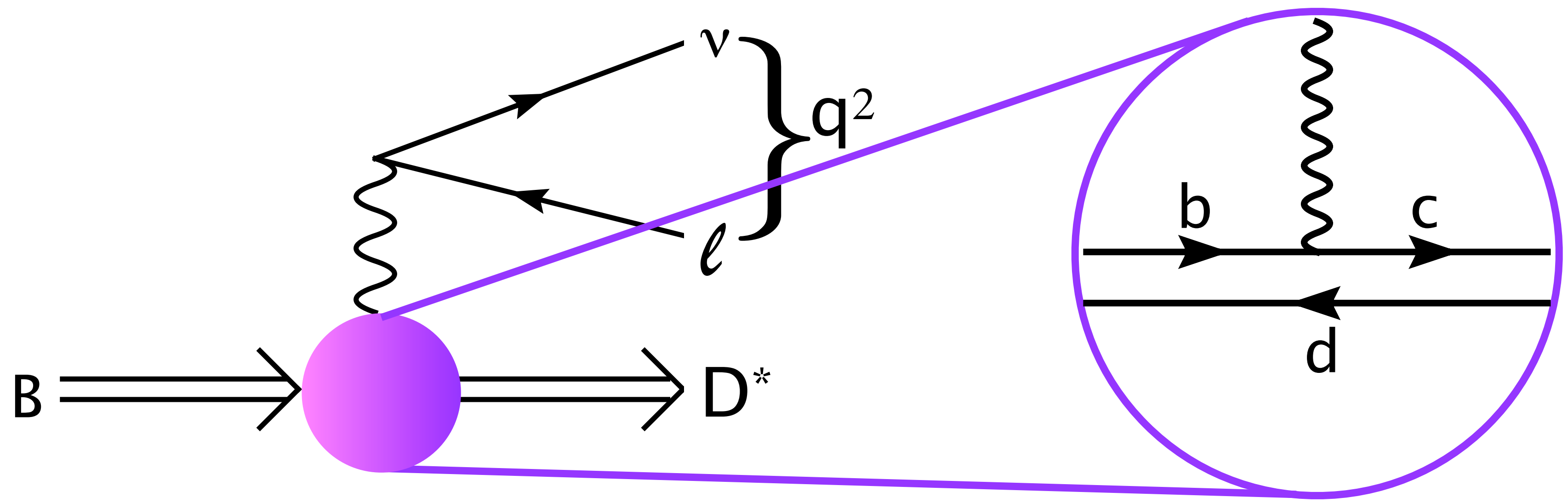
CAVEAT: This talk will be restricted to three-flavor unquenched lattice calculations

- Currently two groups calculating heavy-light meson quantities with three dynamical quark flavors: Fermilab/MILC & HPQCD
- Both use the publicly available "2+1 flavor" **MILC configurations** [**Phys.Rev.D70:114501,2004**] which have three flavors of improved staggered quarks:
  - Two degenerate light quarks and one heavy quark ($\approx m_s$)
  - Light quark mass ranges from $m_s/10 \leq m_l \leq m_s$ (minimum $m_\pi \approx$ 240-330 MeV)
  - Two or more lattice spacings with minimum a $\approx$ 0.09 fm
- Groups use different heavy quark discretizations:
  - Fermilab/MILC uses Fermilab quarks
  - HPQCD uses nonrelativistic (NRQCD) heavy quarks

$B \to D^* \ell \nu$ decay and $|V_{cb}|$



$B \to D^* \ell \nu$ semileptonic decay



- Experiments can only measure the product (form factor) x $|V_{cb}|$

$$\frac{d\Gamma(B \to D l \nu)}{dw} = \frac{G_F^2}{48\pi^3} m_D^3 (m_B + m_D)^2 (w^2 - 1)^{3/2} |V_{cb}|^2 |\mathcal{F}_{B\to D}(w)|^2$$

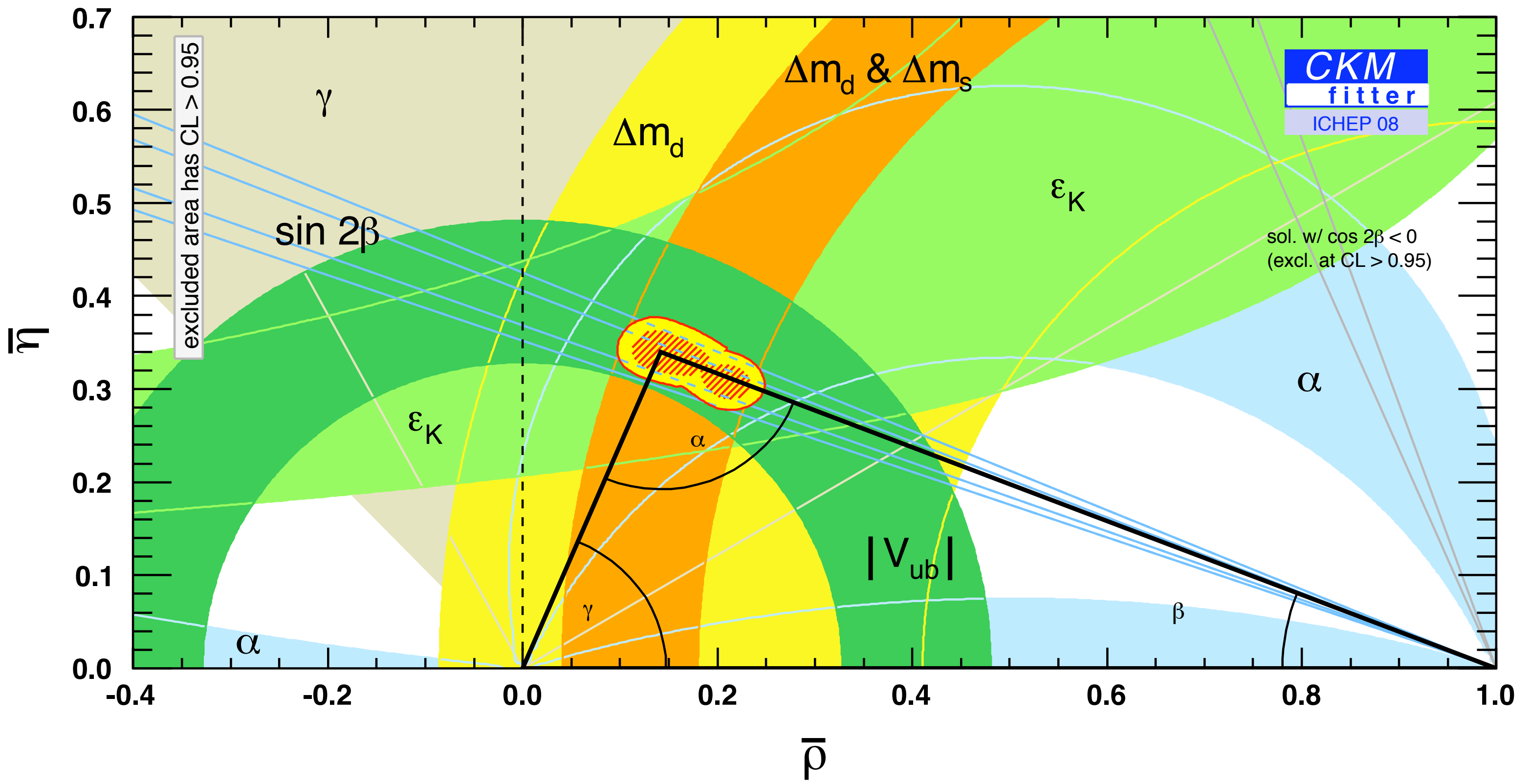
} $w \equiv v' \cdot v$
$w = 1$
at zero recoil

- Lattice QCD calculations needed to determine normalization and extract the CKM matrix element $|V_{cb}|$

- Only need one q^2 point from lattice -- choose **w=1** because easiest to calculate

$|V_{cb}|$ normalizes the CKM unitarity triangle

- In order to make the base of the CKM triangle have unit length, the convention is to divide everything by $|V_{cd} V_{cb}^*|$



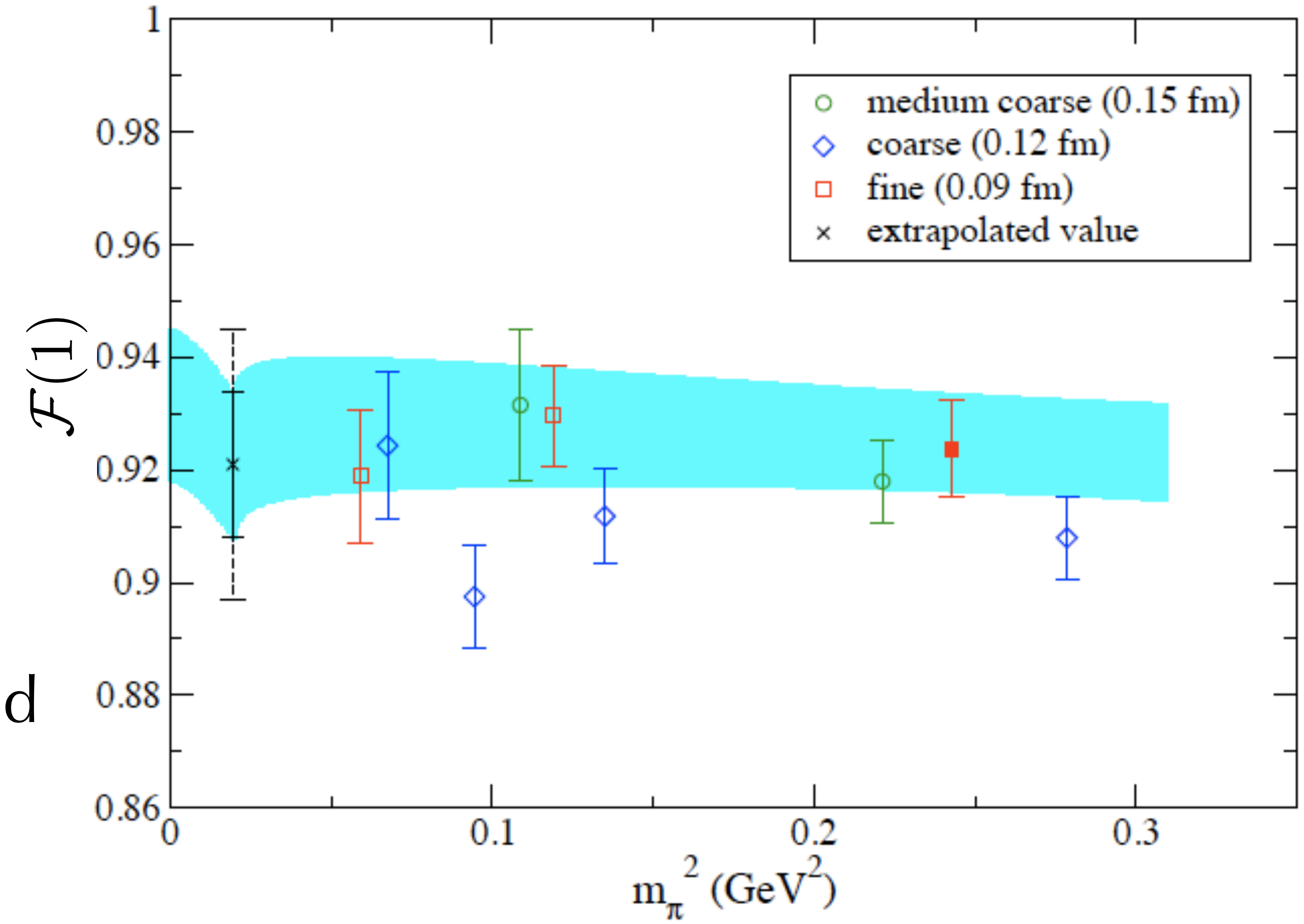
- ⇒ **$|V_{cb}|$ enters all constraints on the apex of CKM unitarity triangle** (not the angles) except for those from ratios

- ~2% error in $|V_{cb}|$ already limits the constraint from neutral kaon mixing (the ε_K band) will ultimately limit other constraints if it is not reduced . . .

Calculation of the $B \to D^* \ell\nu$ form factor and $|V_{cb}|$

$$\mathcal{F}(1) = 0.927(13)(20)$$

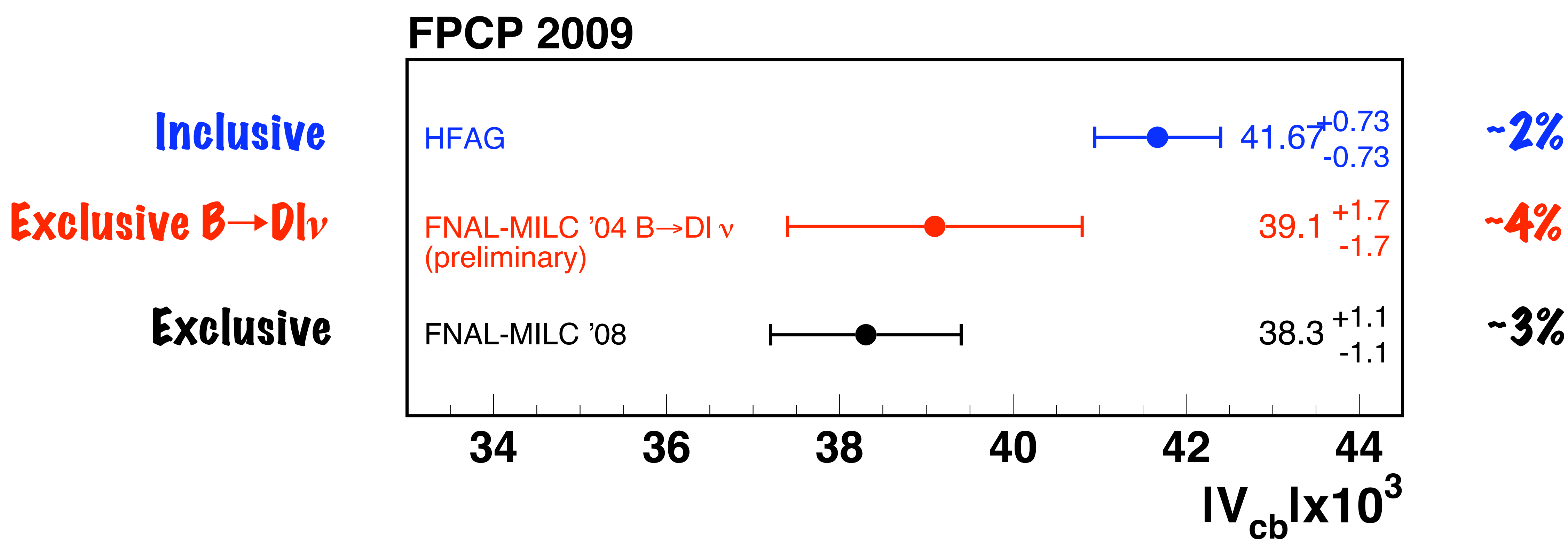
[**Fermilab/MILC;**
Phys. Rev. D 79, 014506 (2009)]



- *Mild quark mass dependence*
- Largest uncertainties from statistics and discretization errors, and can be reduced in a straightforward manner:
  - MILC has recently generated 4× the configurations on the a ≈ 0.12 fm lattices
  - Configurations with a ≈ 0.06 fm, a ≈ 0.045 fm still need to be analyzed
- Using the most recent experimental value of F(1) x $|V_{cb}|$ from the Heavy Flavor Averaging Group gives

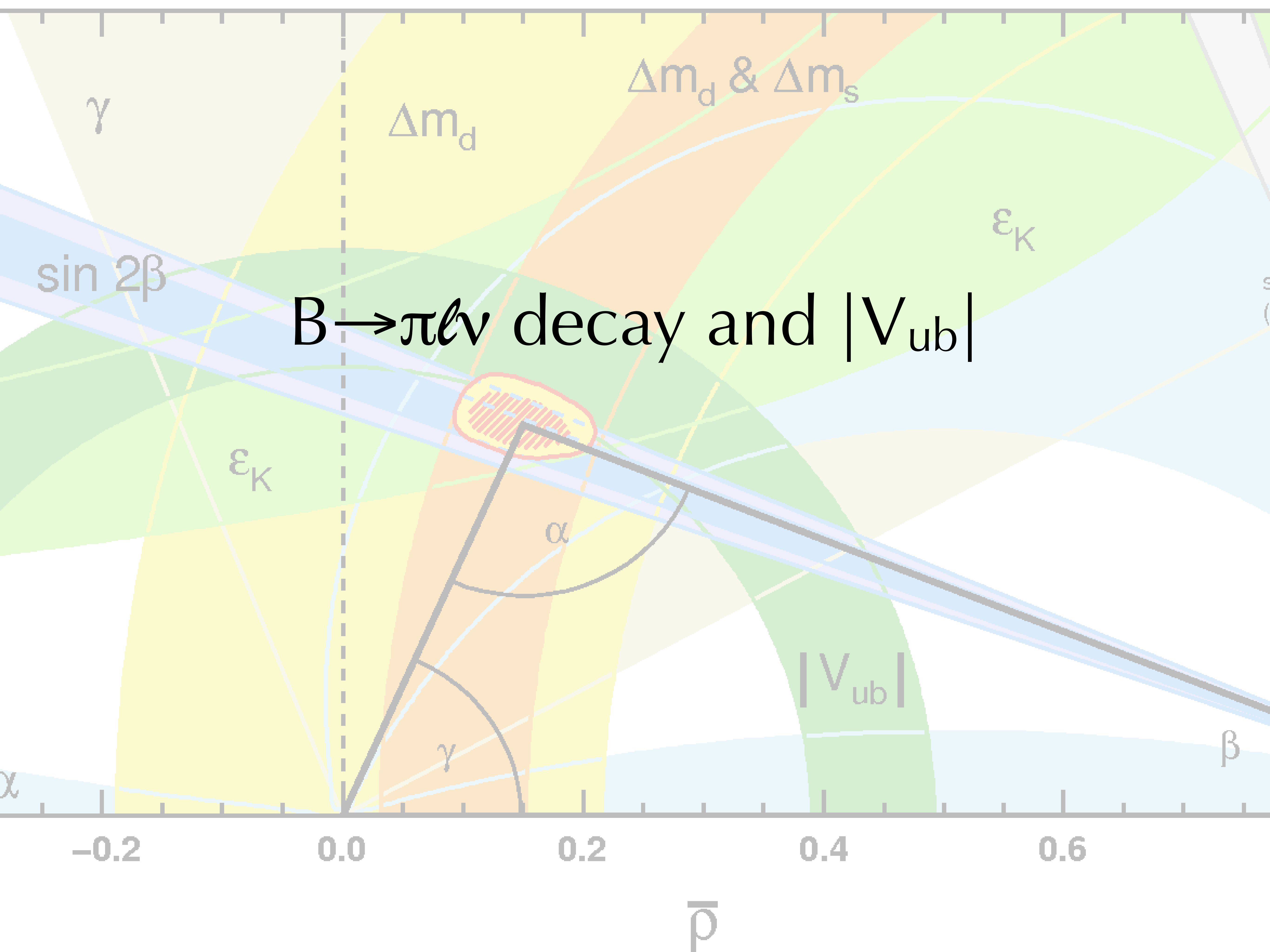
$$|V_{cb}| \times 10^3 = 38.3 \pm 0.5_{\text{exp.}} \pm 1.0_{\text{theo.}}$$

Comparison with other determinations

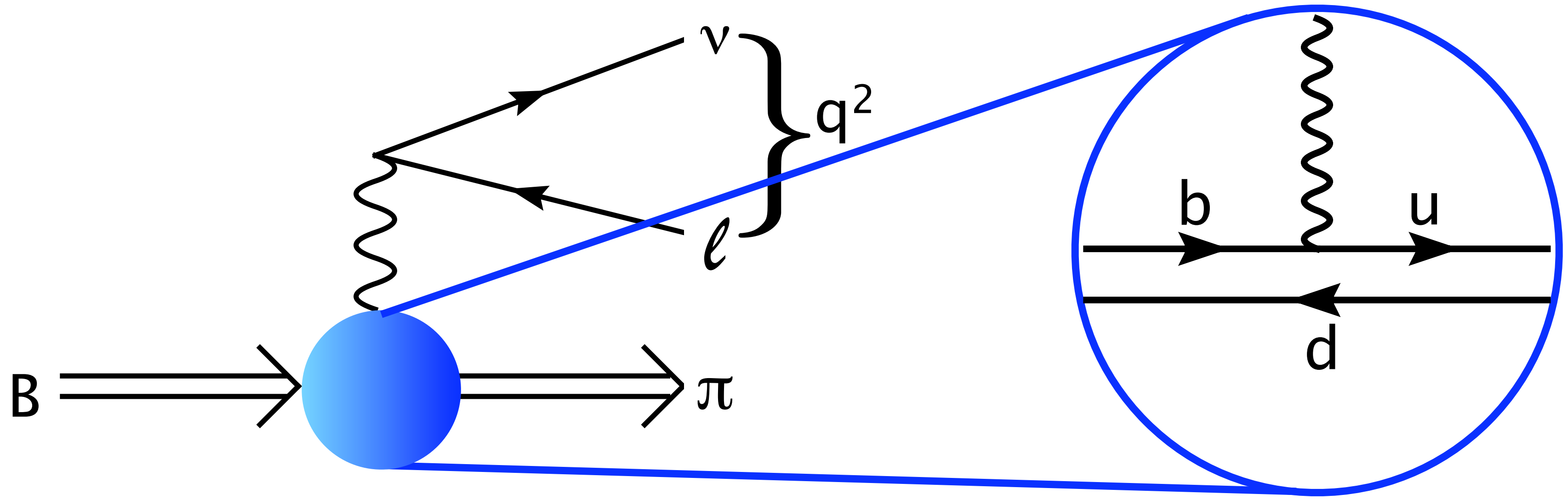


- Experiment updated since publication, with only slight change in $|V_{cb}|$
- Exclusive $|V_{cb}|$ approximately **2- σ lower than inclusive determinations** (see talks by **Schwanda, Tackmann**)
- *Experiments not consistent* for $B\to D^*\ell\nu$:
  - Confidence level of HFAG global fit is 0.01%
- Calculation of $B\to D^*\ell\nu$ form factor at non-zero recoil could perhaps shed some light . . .

$B \to \pi \ell \nu$ decay and $|V_{ub}|$



$B\to\pi\ell\nu$ semileptonic decay



- Experiments can only measure the product $f_+(q^2)$ x $|V_{ub}|$

$$\frac{d\Gamma(B^0\to\pi^-\ell^+\nu)}{dq^2} = \frac{G_F^2}{192\pi^3 m_B^3}\left[(m_B^2+m_\pi^2-q^2)^2-4m_B^2m_\pi^2\right]^{3/2}|V_{ub}|^2|f_+(q^2)|^2$$

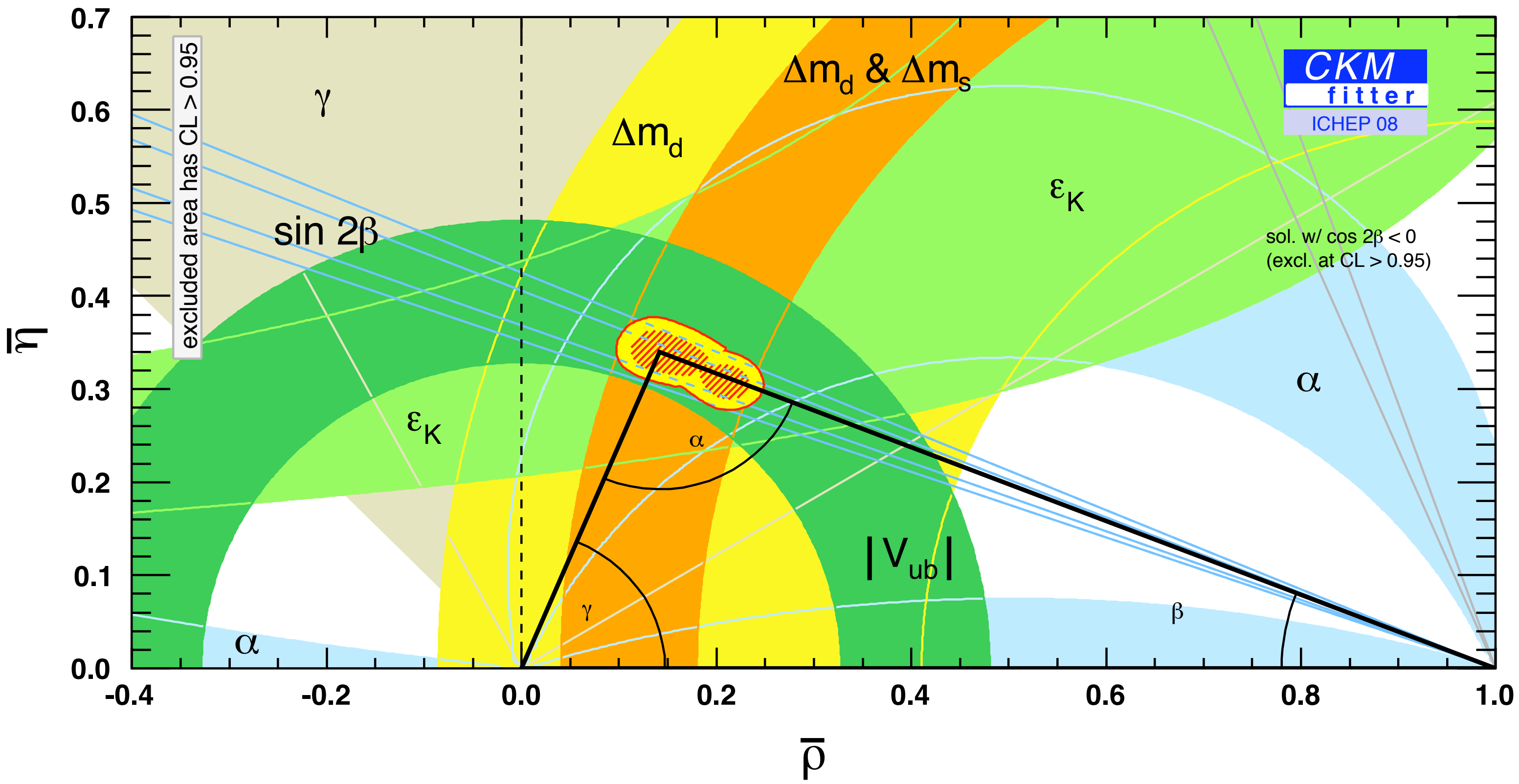
- Need lattice calculation of the $B\to\pi\ell\nu$ form factor to determine $|V_{ub}|$
- Few percent determination of $|V_{ub}|$ difficult because errors in experimental branching fraction smallest at low q^2, whereas errors in lattice form factor determination smallest at high q^2

$|V_{ub}|$ and the CKM unitarity triangle

- $|V_{ub}|$ constrains the apex $(\bar{\rho}, \bar{\eta})$ of the unitarity triangle:

$$\frac{|V_{ub}|}{|V_{cb}|} = \frac{\lambda}{1 - \frac{\lambda^2}{2}} \sqrt{\bar{\rho}^2 + \bar{\eta}^2}$$

  - $\lambda = |V_{us}|$ known to ~1%
  - $|V_{cb}|$ known to ~2%



- Width of green error ring dominated by uncertainty in $|V_{ub}|$

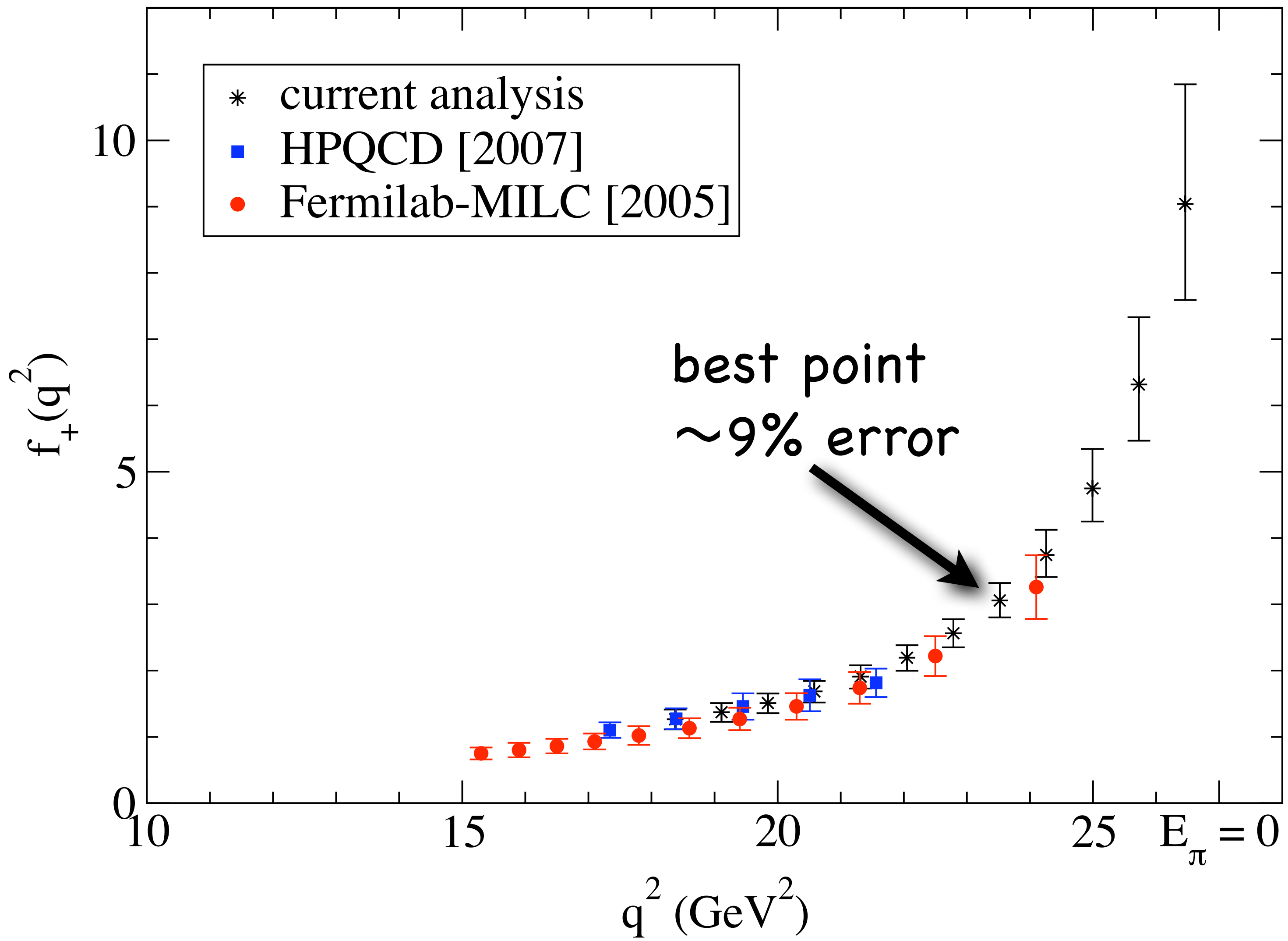
- $\sin(2\beta)$ currently constrains the height to better than 4% and is still improving

- ∴ A **precise determination of $|V_{ub}|$ will allow a strong test of CKM unitarity**

Calculation of the $B \to \pi \ell \nu$ form factor $f_+(q^2)$

- Compute the form factor at 12 q^2 values from ≈18 GeV^2 to q^2_{max} = 26.5 GeV^2
  - Shape and normalization consistent with other 2+1 flavor determinations
  - *Errors smaller and more reliable due to use of second lattice spacing*

[Fermilab/MILC; arXiv:0811.3640 [hep-lat]]



- Largest uncertainty from statistics and chiral extrapolation, and can be reduced with the following:
  - MILC has recently generated 4× the configurations on the $a \approx 0.12$ fm lattices
  - Configurations with larger spatial volumes exist and will allow lighter pion masses

Exclusive determination of $|V_{ub}|$ from $B\to\pi\ell\nu$

- Standard method is to combine lattice form factor experimentally-measured $B\to\pi\ell\nu$ branching fraction and B-meson lifetime and integrate over q^2:

$$\frac{\Gamma(q_{\rm min})}{|V_{ub}|^2} = \frac{G_F^2}{192\pi^3 m_B^3}\int_{q^2_{\rm min}}^{q^2_{\rm max}} dq^2 \left[(m_B^2+m_\pi^2-q^2)^2-4m_B^2m_\pi^2\right]^{3/2}|f_+(q^2)|^2$$

  - **Requires analytic parameterization** of lattice form factor $f_+(q^2)$

- Standard functional form used to interpolate/extrapolate form factor data is the Becirevic-Kaidalov parameterization:

$$f_+(q^2) = \frac{f(0)}{(1-q^2/m^2_{B^*})(1-\alpha\, q^2/m^2_{B^*})} \qquad f_0(q^2) = \frac{f(0)}{(1-\frac{1}{\beta}\, q^2/m^2_{B^*})}$$

properly incorporates
B* pole

α and β parameterize
physics above threshold
(other poles and cuts)

  - Easy to use, but **introduces hard-to-estimate model dependence due to choice of fit ansatz**

z-expansion of semileptonic form factors

[**Arnesen *et. al.* Phys. Rev. Lett. 95, 071802 (2005) and refs. therein**]

- Consider mapping the variable q^2 onto a new variable, z, in the following way: $z = \frac{\sqrt{1-q^2/t_+} - \sqrt{1-t_0/t_+}}{\sqrt{1-q^2/t_+} + \sqrt{1-t_0/t_+}}$

- Choose the free parameter t_0 to make the maximum |z| in the region as small as possible -- choosing 0.65 t_- maps z in the $B\to\pi\ell\nu$ decay region onto $-0.34 < z < 0.22$

- In terms of z, semileptonic form factors have simple form:

$$P(t)\,\phi(t,t_0)\,f(t) = \sum_{k=0}^{\infty} a_k(t_0) z(t,t_0)^k$$

Accounts for subthreshold (e.g. B*) poles [$P(t)$]

"Arbitrary" analytic function -- choice only affects particular values of coefficients (a_k's) [$\phi(t,t_0)$]

- Unitarity constrains the size of the coefficients: $\sum_{k=0}^{N} a_k^2 \leq 1$

Constraint holds for any value of N

- Thus, in combination with the small range of |z|, **one needs only a small number of parameters** to obtain the form factors to a high degree of accuracy

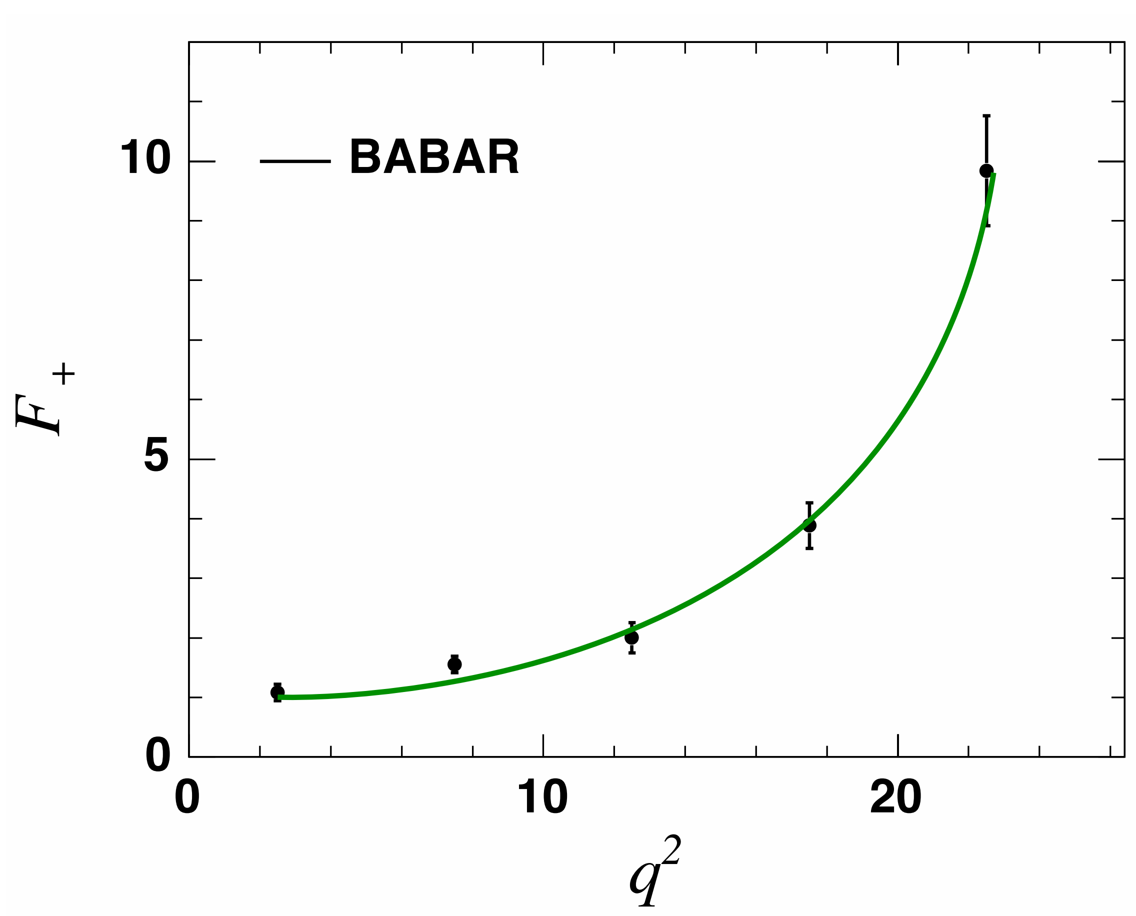
Heavy quark constraint on coefficients

- Unitarity bound on coefficients come from fact that the decay rate to the exclusive channel $B \to \pi \ell \nu$ must be less than the inclusive B-meson decay rate
- It is also true that, as the mass of B-meson increases, its branching fraction to any particular exclusive channel decreases
- The branching fraction for the semileptonic decay $B \to \pi \ell \nu$ as a power of Λ_{QCD}/m_B has been calculated by **Becher and Hill**
- It can be used to place an even **tighter constraint** on the coeffiecents of the z-expansion for the form factors:

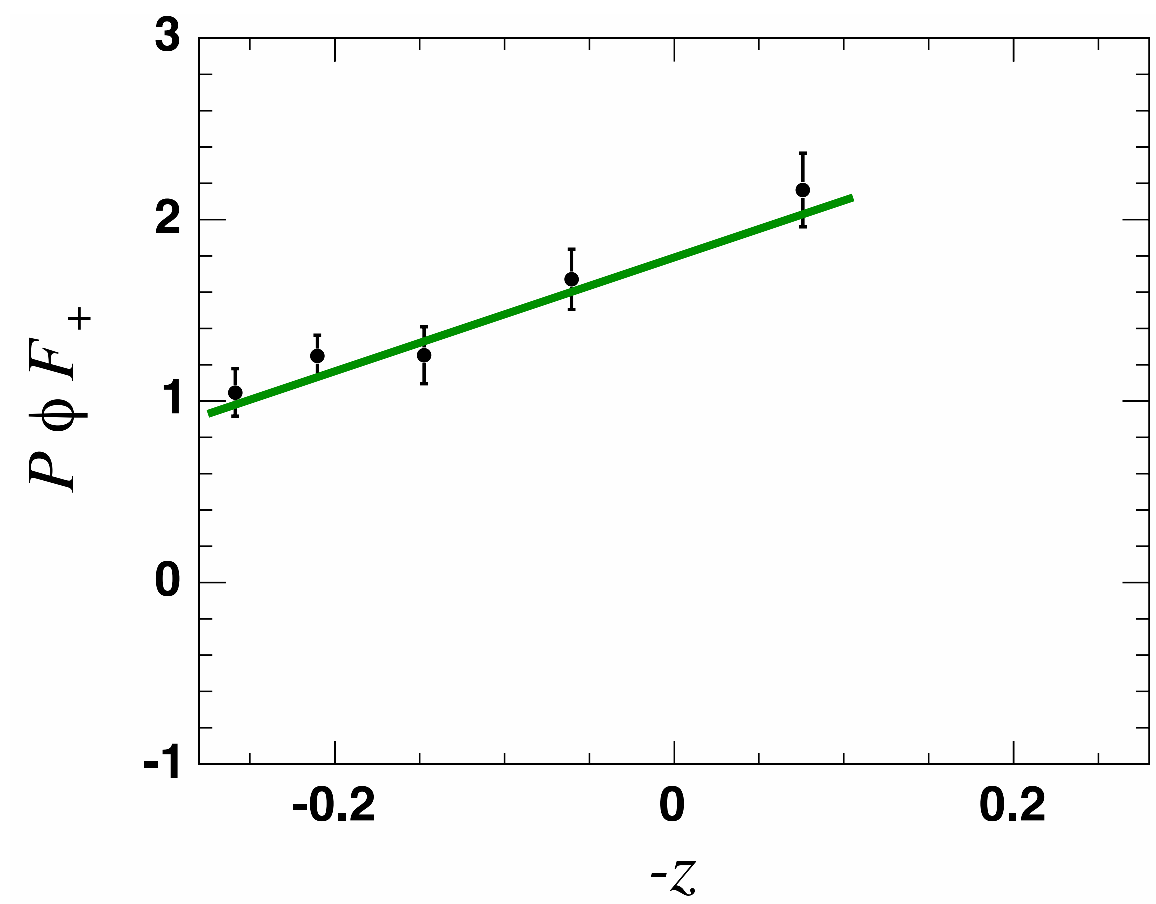
$$\sum_{k=0}^{N} a_k^2 \sim \left(\frac{\Lambda}{m_B}\right)^3 \approx 0.001$$

- Implies that the unitarity bound is far from saturated, i.e. that the coefficients will be much less than one

Effect of z-remapping on B→πℓν form factor



Striking curvature in B→πℓν form factor data versus q^2



No visible curvature after remapping

- Curvature in data due to well-understood perturbative QCD effects
- Data completely described by a *normalization and a slope*, and constrains the size of possible curvature

The program for lattice and experiment

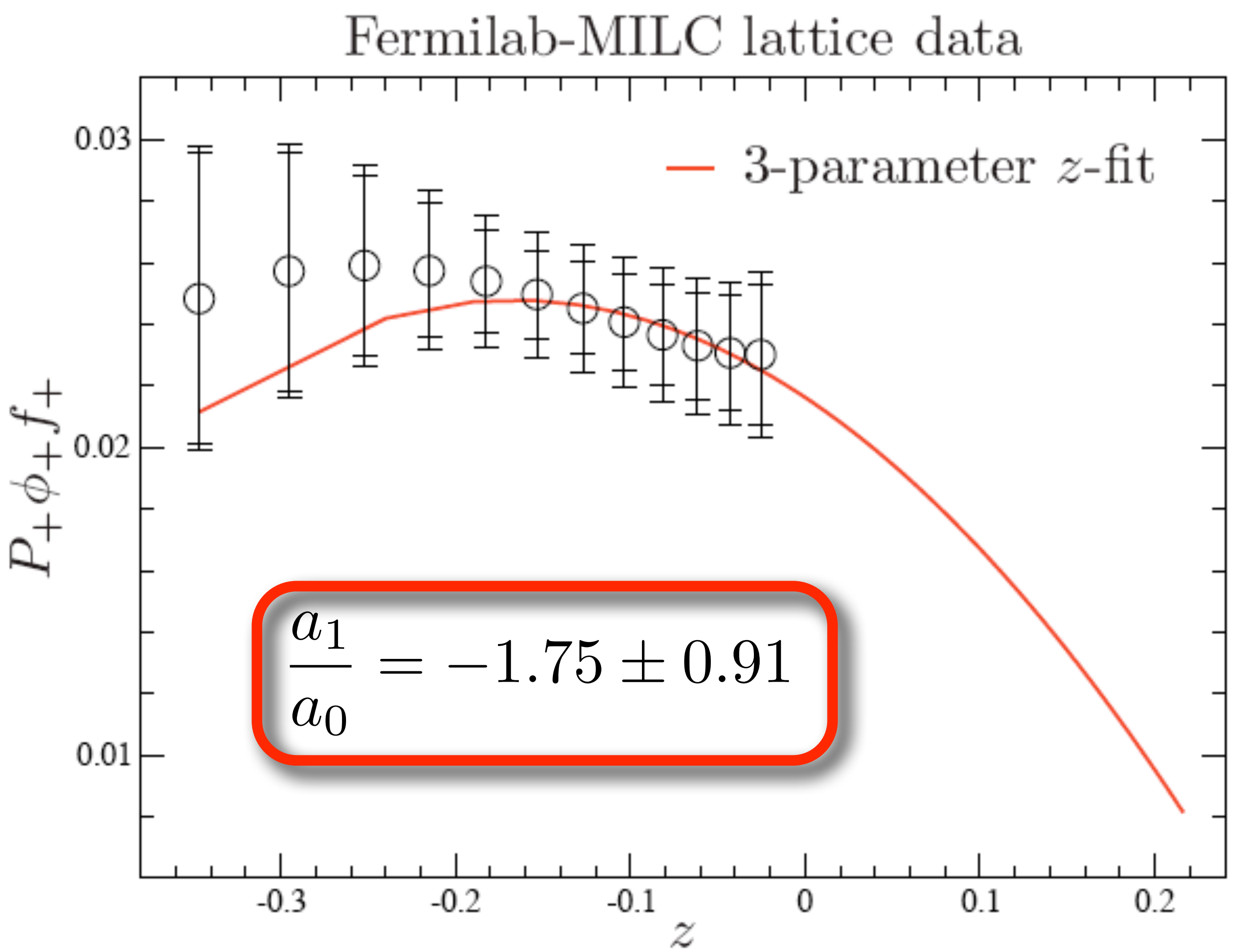
1. Fit experimental and lattice data in terms of z expansion
2. Determine and compare the slopes (and curvature) in z
3. If consistent, fit lattice and experimental data simultaneously with an unknown relative offset to determine $|V_{ub}|$

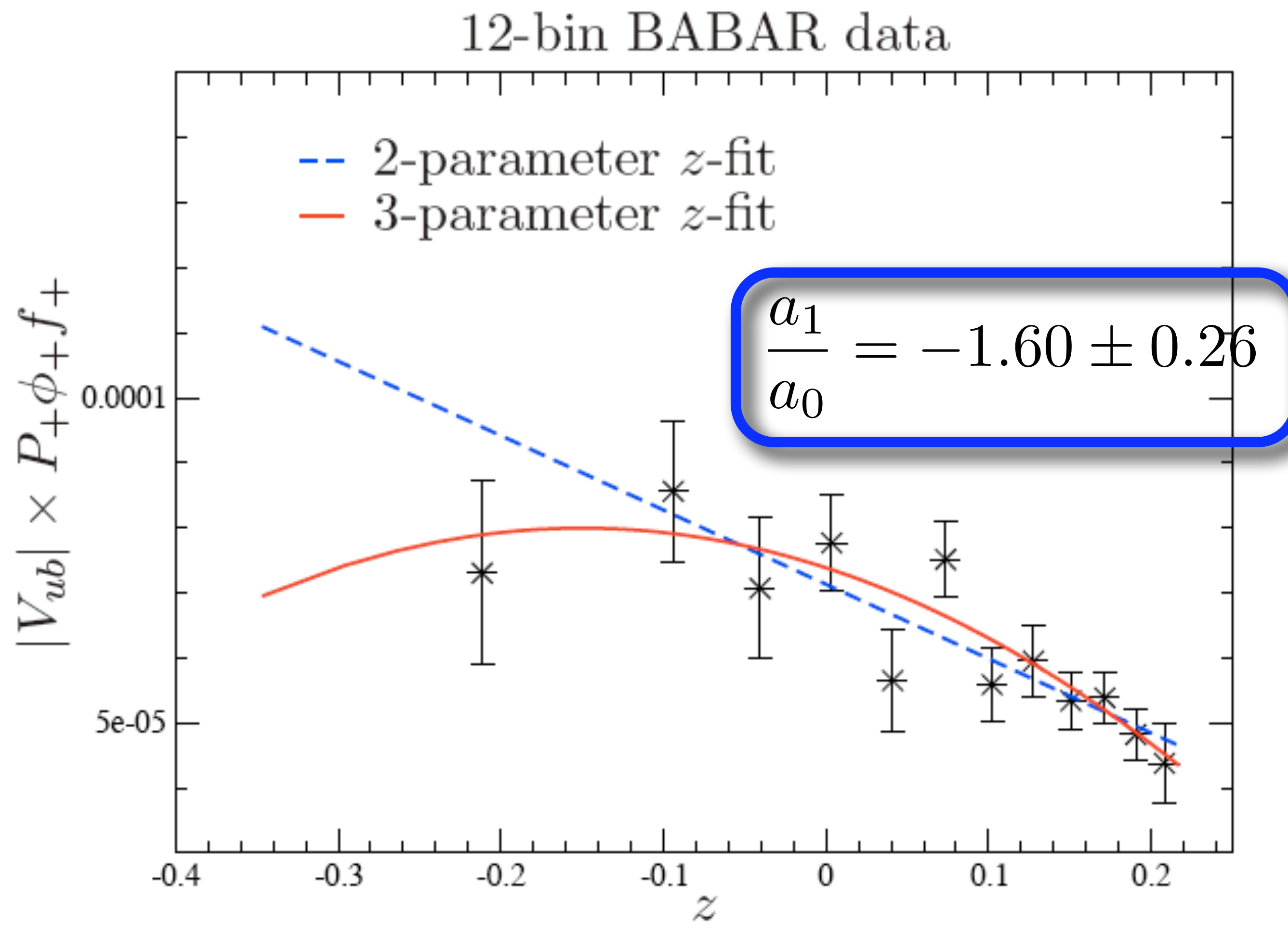
Advantages to this approach:

- **Model-independent**
- Can quantify the agreement between lattice and experiment using slope measurements
- **Systematically improvable** -- as data gets more precise can add more terms in z
- **Minimizes error in $|V_{ub}|$** by using all of the lattice and experimental data in a single fit

Hope is that this method will be more generally adopted by HFAG and others in the future!

Consistency check: separate z-fits



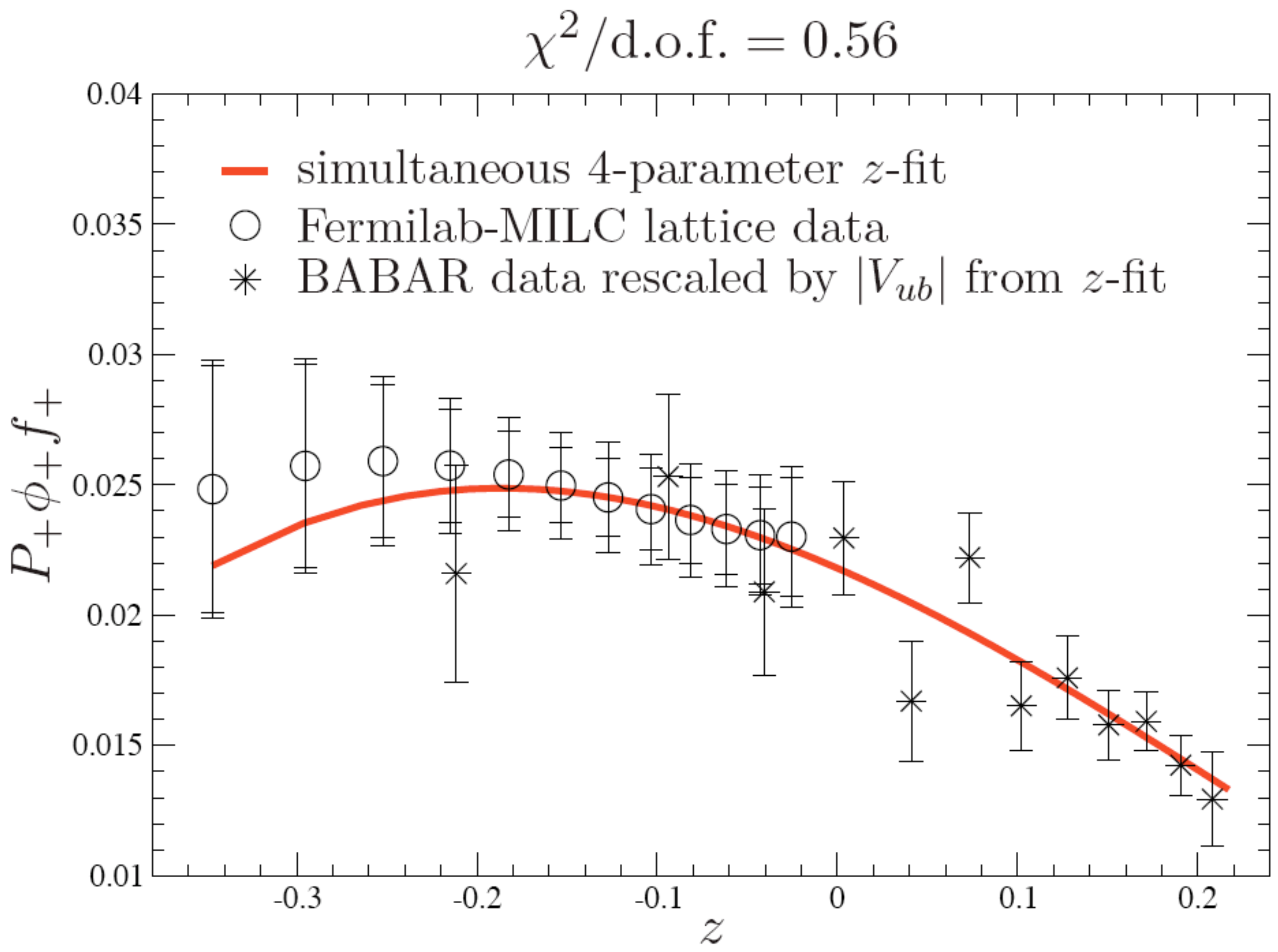


- Lattice data determines both the slope and curvature
- Experimental data consistent with zero curvature
- Lattice and experimental slope and curvature **agree within uncertainties**

⇒ Proceed to simultaneous fit of lattice and experimental data

Simultaneous z-fit to determine $|V_{ub}|$

- Fit lattice and 12-bin BABAR experimental data [**Phys. Rev. Lett. 98, 091801 (2007)**] together to z-expansion leaving relative normalization factor ($|V_{ub}|$) as a free parameter



Fit results

- The result of the 4-parameter combined z-fit is:

$$\begin{aligned} |V_{ub}| \times 10^3 &= 3.38 \pm 0.36 \\ a_0 &= 0.0218 \pm 0.0021 \\ a_1 &= -0.0301 \pm 0.0063 \\ a_2 &= -0.059 \pm 0.032 \\ a_3 &= 0.079 \pm 0.068 \end{aligned}$$

- Coefficients are much smaller than 1, as expected from heavy-quark power-counting

$$\sum a_k^2 \sim 0.01$$

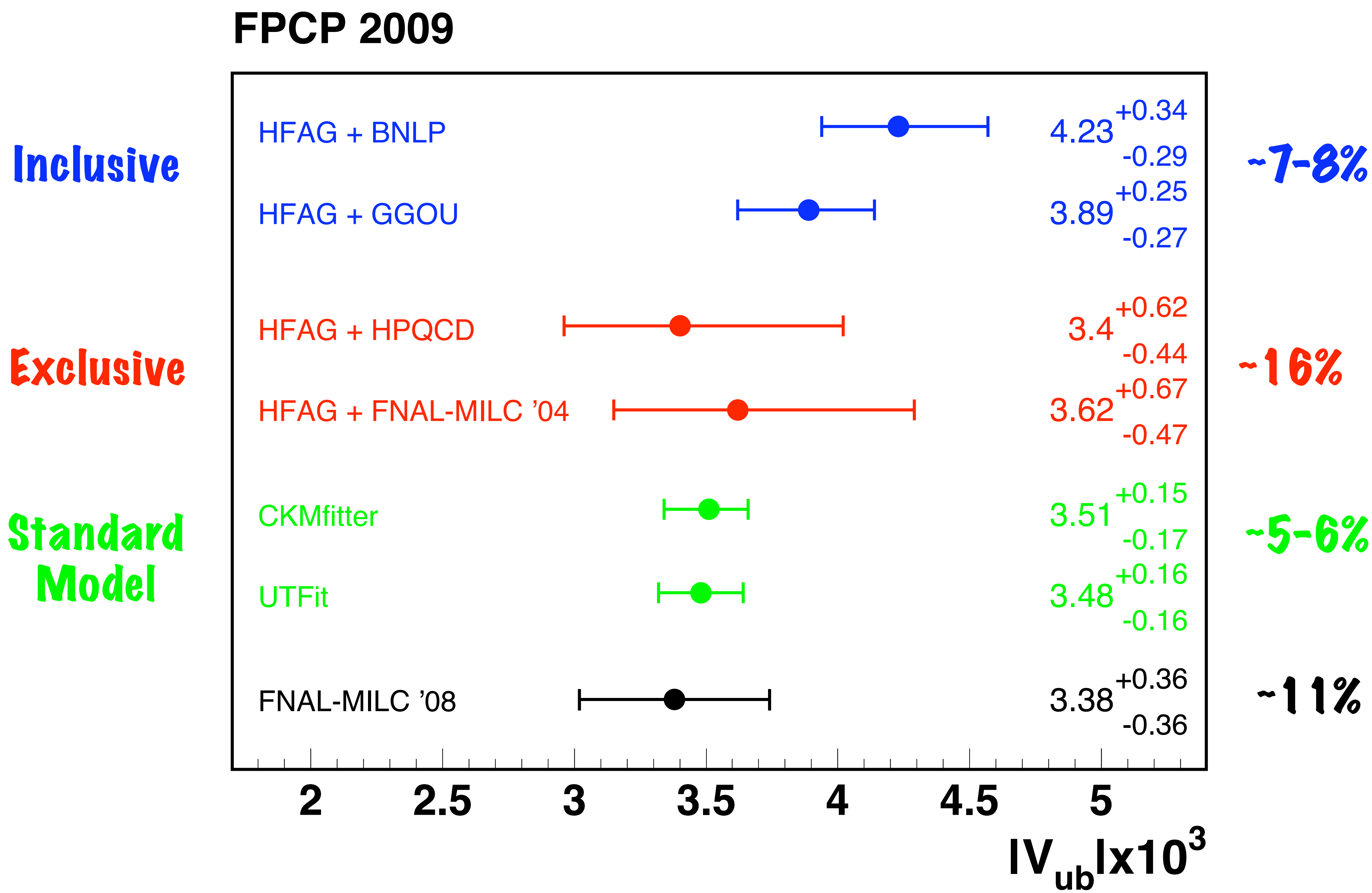
  - Result independent of constraint on coefficients

- $|V_{ub}|$ determined to **~11% accuracy**

- Improved uncertainty largely due to combined z-fit method:

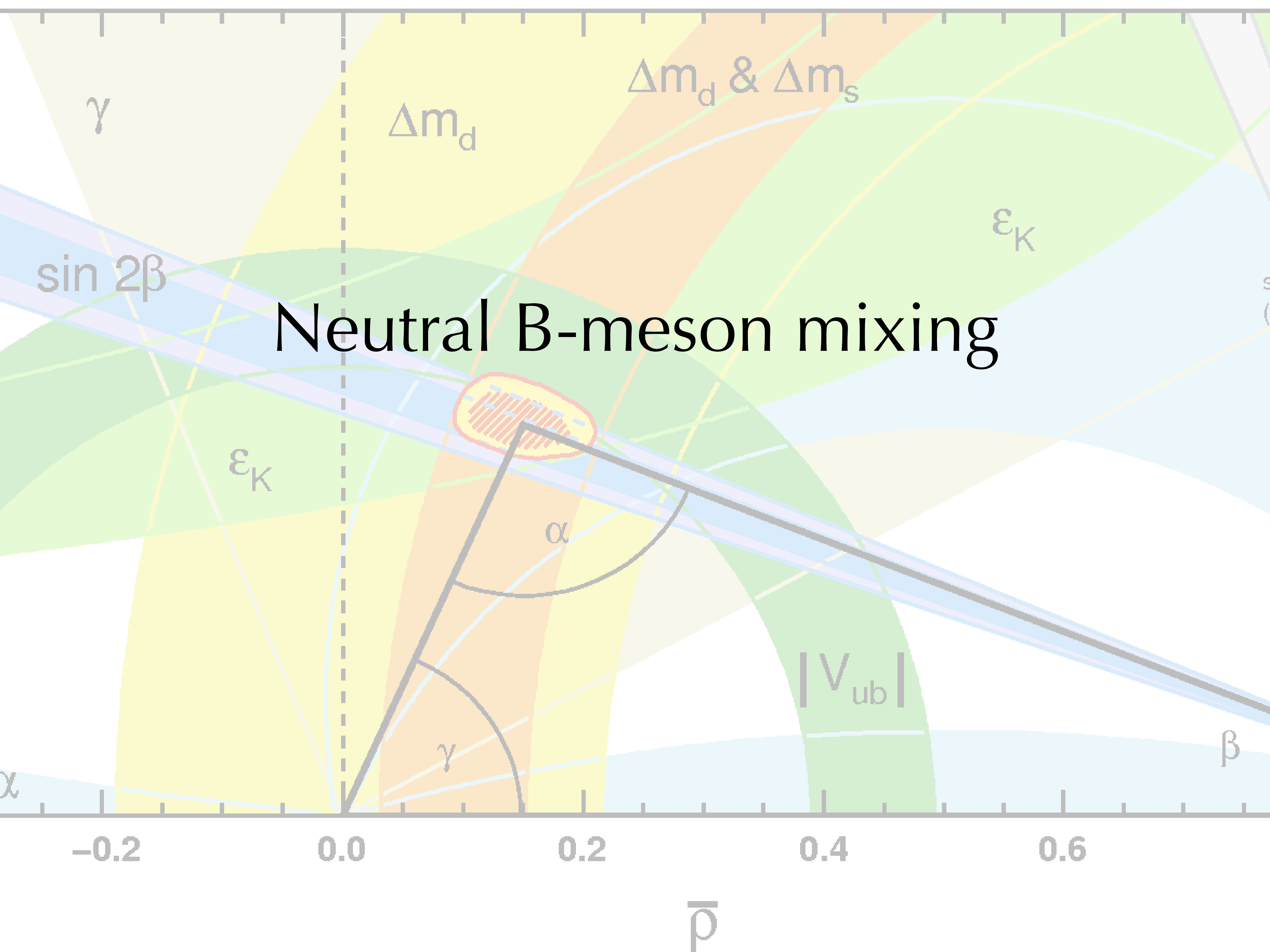
  - If perform separate z-fits of lattice and experimental data and take ratio of normalizations, only determine $|V_{ub}|$ to ~16%

Comparison with other determinations



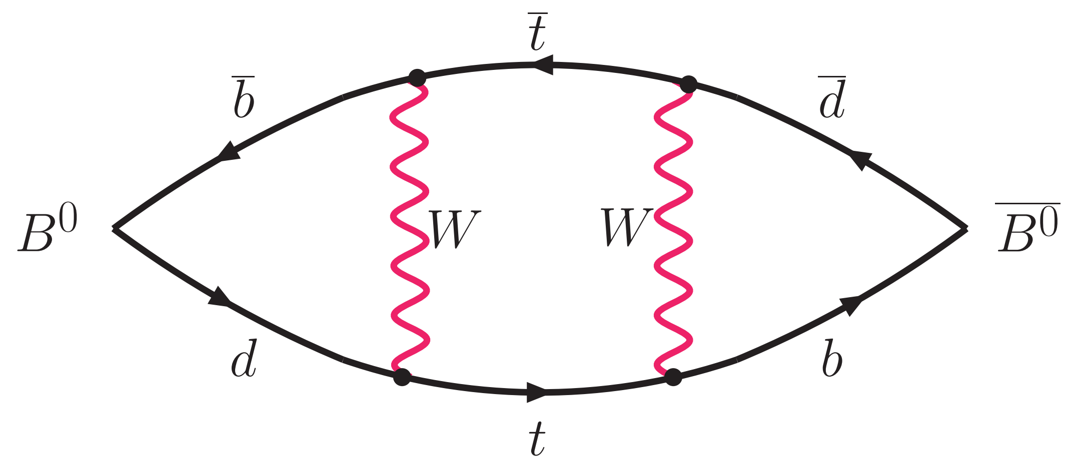
- Exclusive $|V_{ub}|$ **~1-2 - σ below inclusive determinations** (see talks by **Barberio, Tackmann**)
- Consistent with preferred values from unitarity triangle analyses

Neutral B-meson mixing



B-mixing constraint on the unitarity triangle

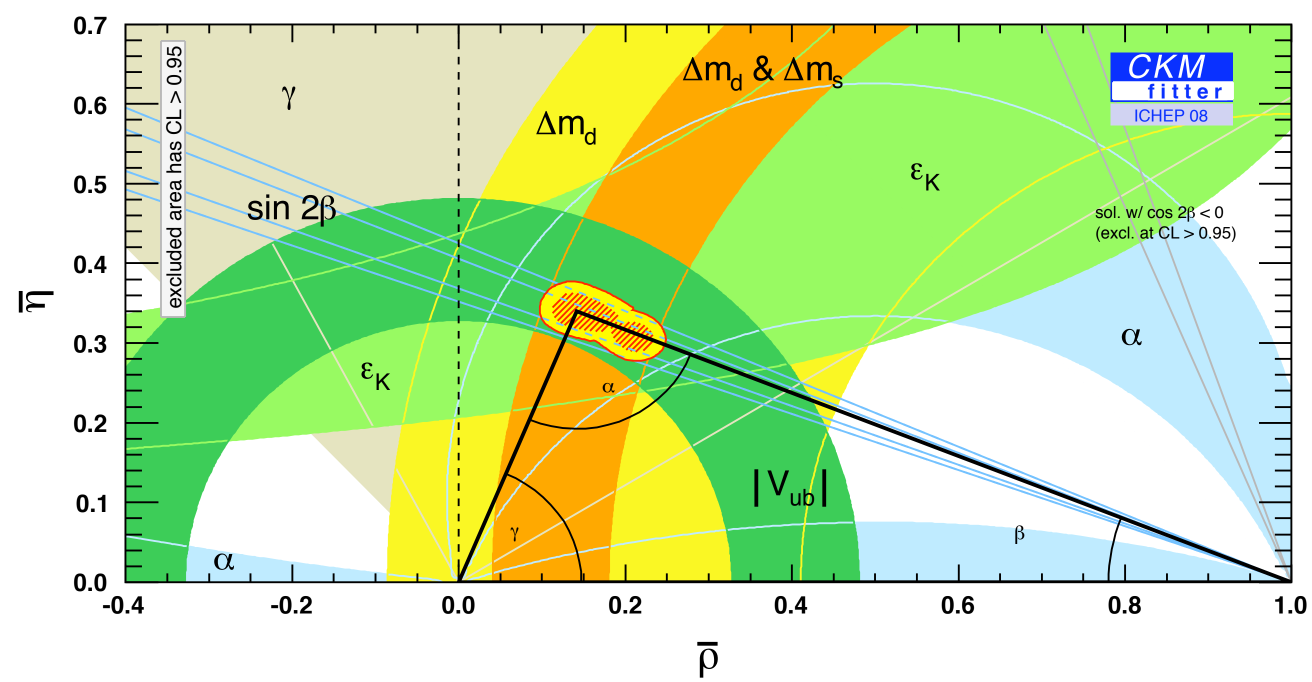
- Underlying quark flavor-changing weak interaction is proportional to:
  - $|V^*_{td}\, V_{tb}|$ for B_d-mixing
  - $|V^*_{ts}\, V_{tb}|$ for B_s-mixing



- The ratio of B_d to B_s oscillation frequencies (Δm_q) constrains the apex of the CKM unitarity triangle:

$$\frac{\Delta m_d}{\Delta m_s} = \left(\frac{f_{B_d}\sqrt{\hat{B}_{B_d}}}{f_{B_s}\sqrt{\hat{B}_{B_s}}}\right)^2 \frac{m_{B_d}}{m_{B_s}} \frac{|V_{td}|^2}{|V_{ts}|^2} = \frac{1}{\xi^2}\frac{m_{B_d}}{m_{B_s}} \left(\frac{\lambda}{1-\lambda^2/2}\right)^2 \frac{((1-\bar{\rho})^2+\bar{\eta}^2)}{\left(1+\frac{\lambda^2}{1\quad\lambda^2/2}\bar{\rho}\right)+\lambda^4\bar{\eta}^2}$$

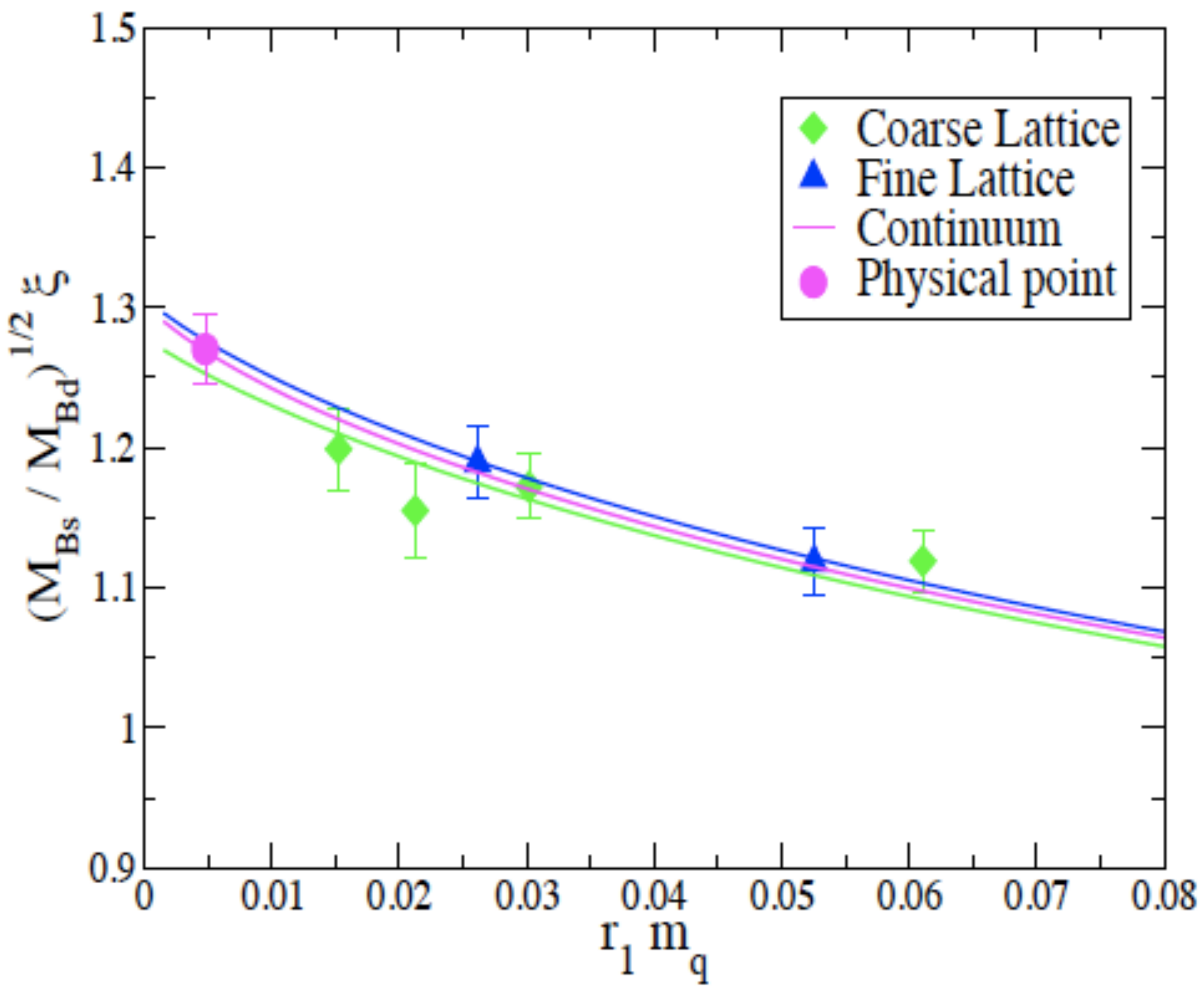
- Δm_q measured to better than 1%
- $\lambda = |V_{us}|$ known to ~1%
- Dominant error currently from **uncertainty in lattice QCD calculation of the ratio ξ**



Calculation of B-meson mixing parameters

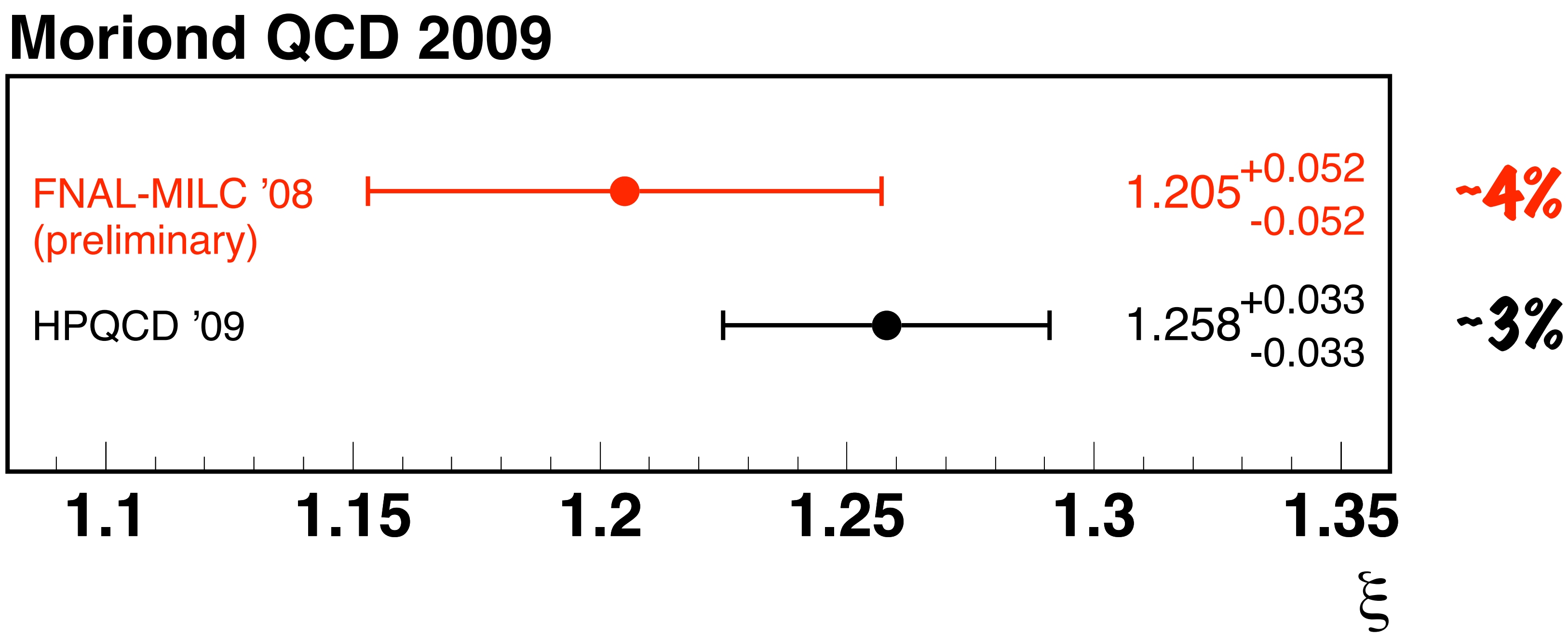
[HPQCD; arXiv:0902.1815 [hep-lat]]

$$\xi = 1.258(33)$$
$$f_{B_d}\sqrt{\hat{B}_{B_d}} = 216(15)\ \text{MeV}$$
$$f_{B_s}\sqrt{\hat{B}_{B_s}} = 266(18)\ \text{MeV}$$



- *Almost no lattice spacing dependence* in ξ
- Largest uncertainty in ξ (2%) from statistics and chiral extrapolation and can be reduced:
  - MILC has recently generated 4× the configurations on the a ≈ 0.12 fm lattices
  - Configurations with larger spatial volumes exist and allow lighter pion masses

Comparison with other determinations



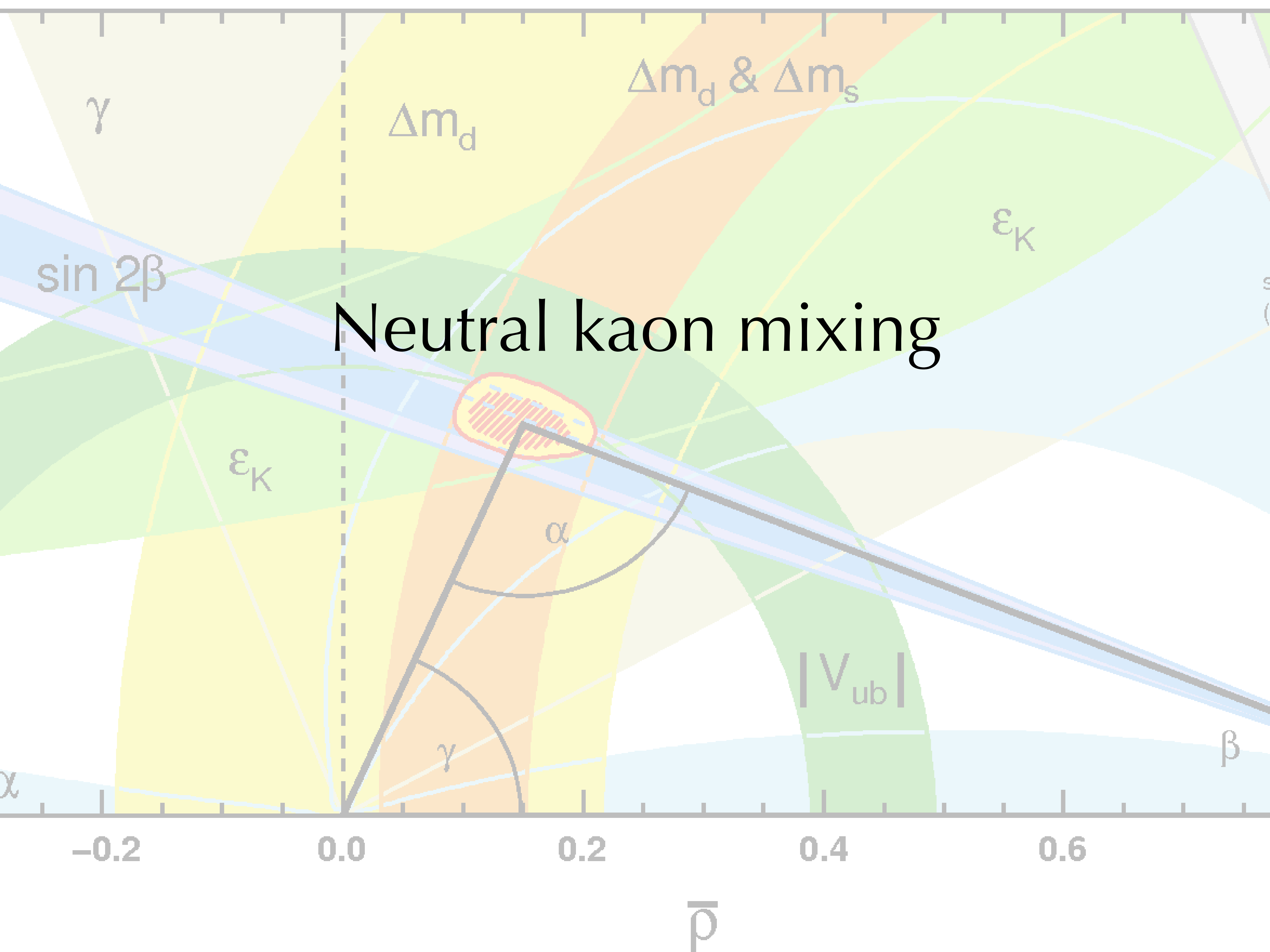
- Value of ξ consistent with preliminary 2+1 flavor determination of Fermilab/MILC from Lattice 2008

- Leads to the following ratio of CKM matrix elements:

$$\frac{|V_{td}|}{|V_{ts}|} = 0.214(1)_{\text{exp.}}(5)_{\text{theo.}}$$

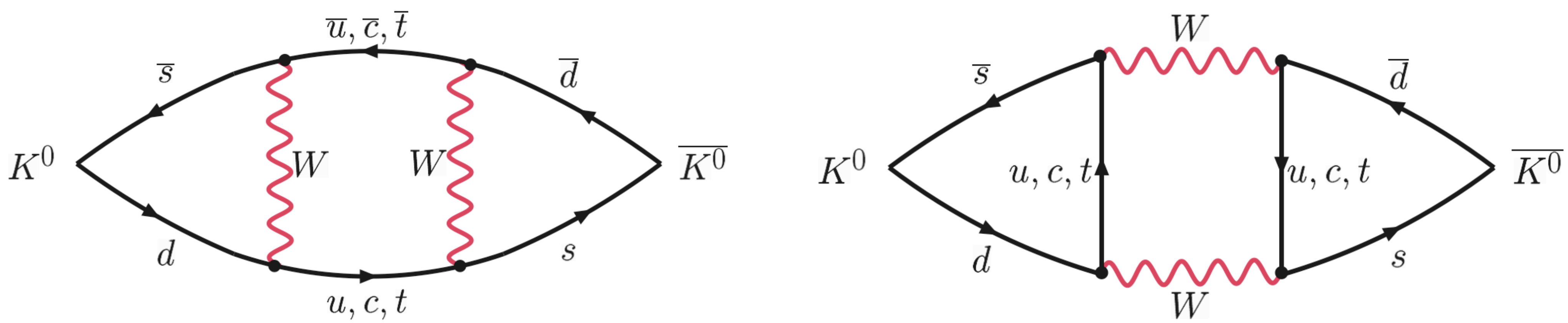
- Also consistent with less precise determination from $B \to \rho\gamma$ / $B \to K^*\gamma$: $|V_{td}/V_{ts}| = 0.203(20)$ (see talk by **E. Salvati**)

Neutral kaon mixing



Kaon mixing constraint on the unitarity triangle

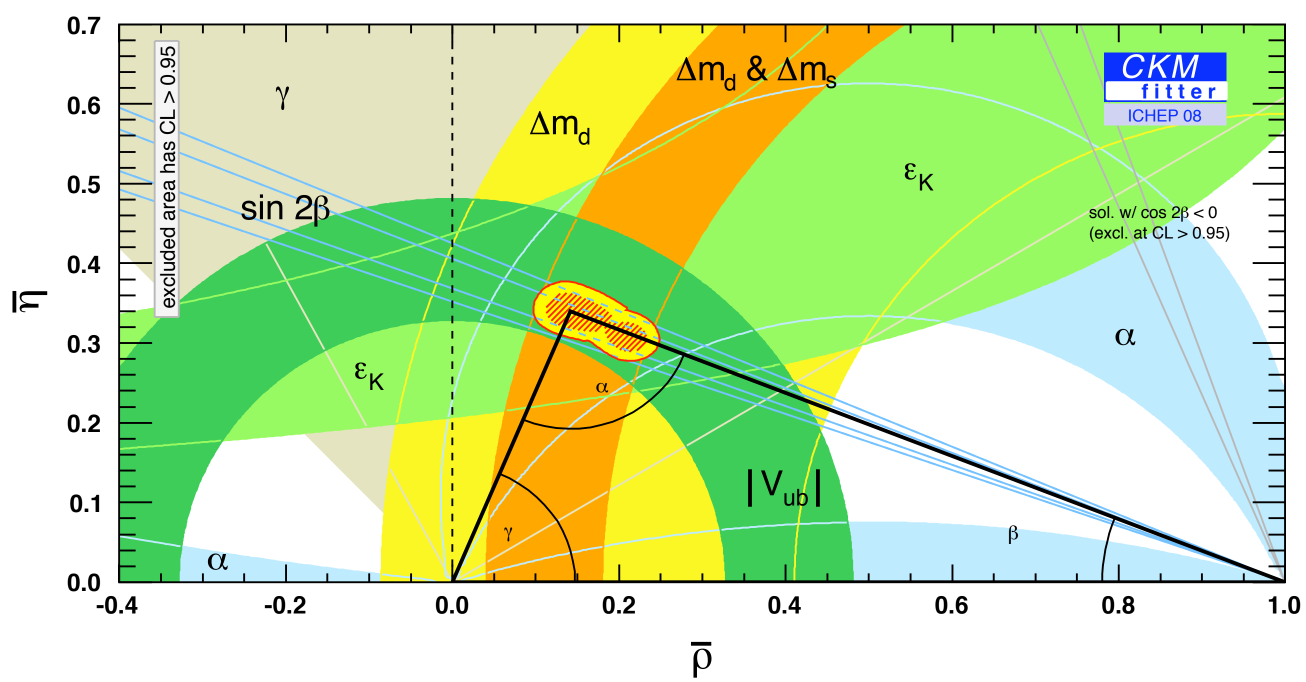
- Underlying quark flavor-changing interaction proportional to $|V^*_{td} V_{ts}|$



- Experimental measurement of direct CP-violation in the neutral kaon system (ε_K) constrains the apex of the CKM unitarity triangle:

$$|\epsilon_K| = C_\epsilon B_K A^2 \overline{\eta}\{-\eta_1 S_0(x_c)(1-\lambda^2/2) + \eta_3 S_0(x_c, x_t) + \eta_2 S_0(x_t) A^2 \lambda^2 (1-\overline{\rho})\}$$

- ε_K measured to better than 1%
- $A=|V_{cb}|$ known to ~2%
- The **hadronic matrix element B_K must be computed with lattice QCD**

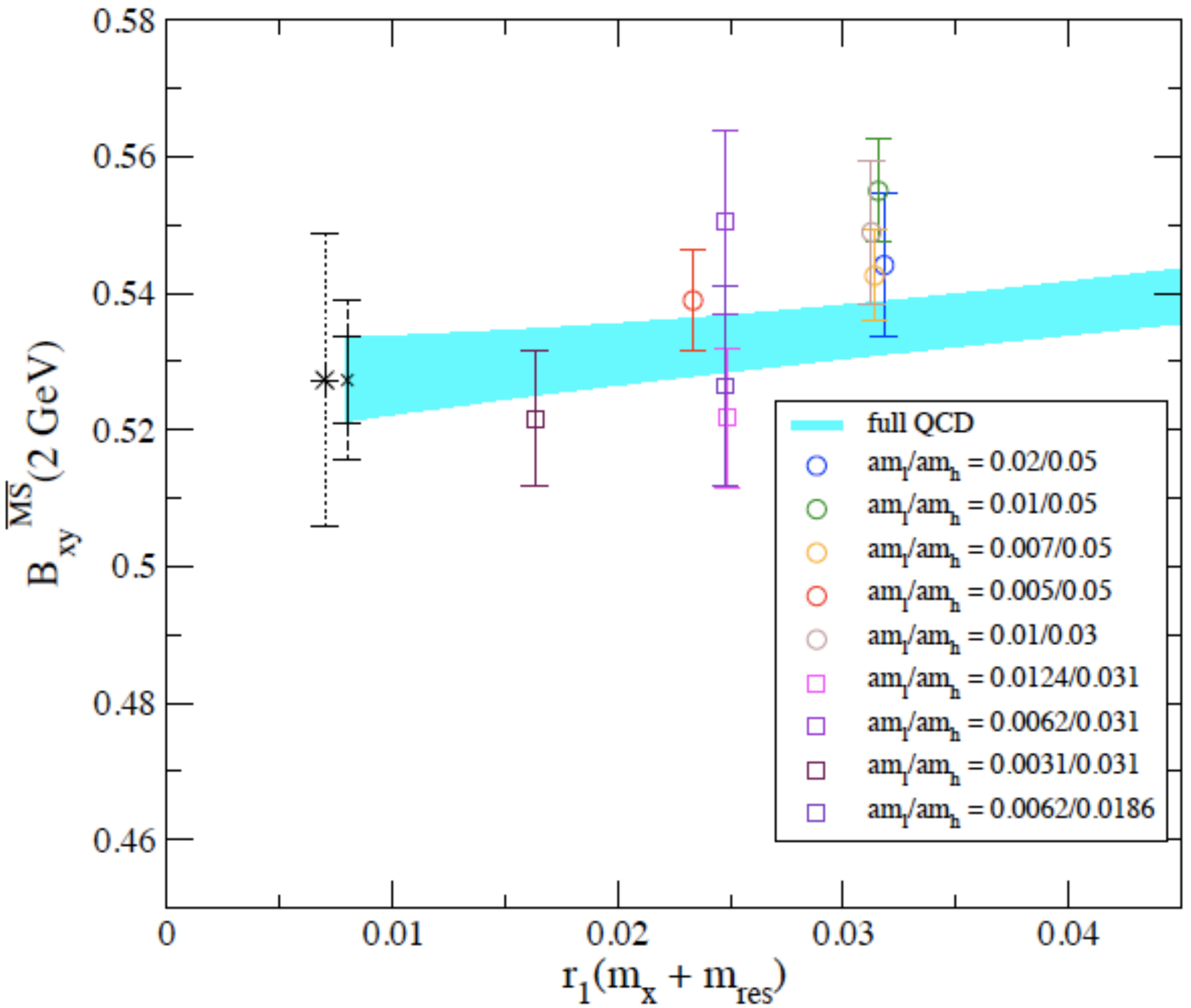


Calculation of B_K

[Aubin, Laiho, RV; arXiv:0905.3947 [hep-lat]]

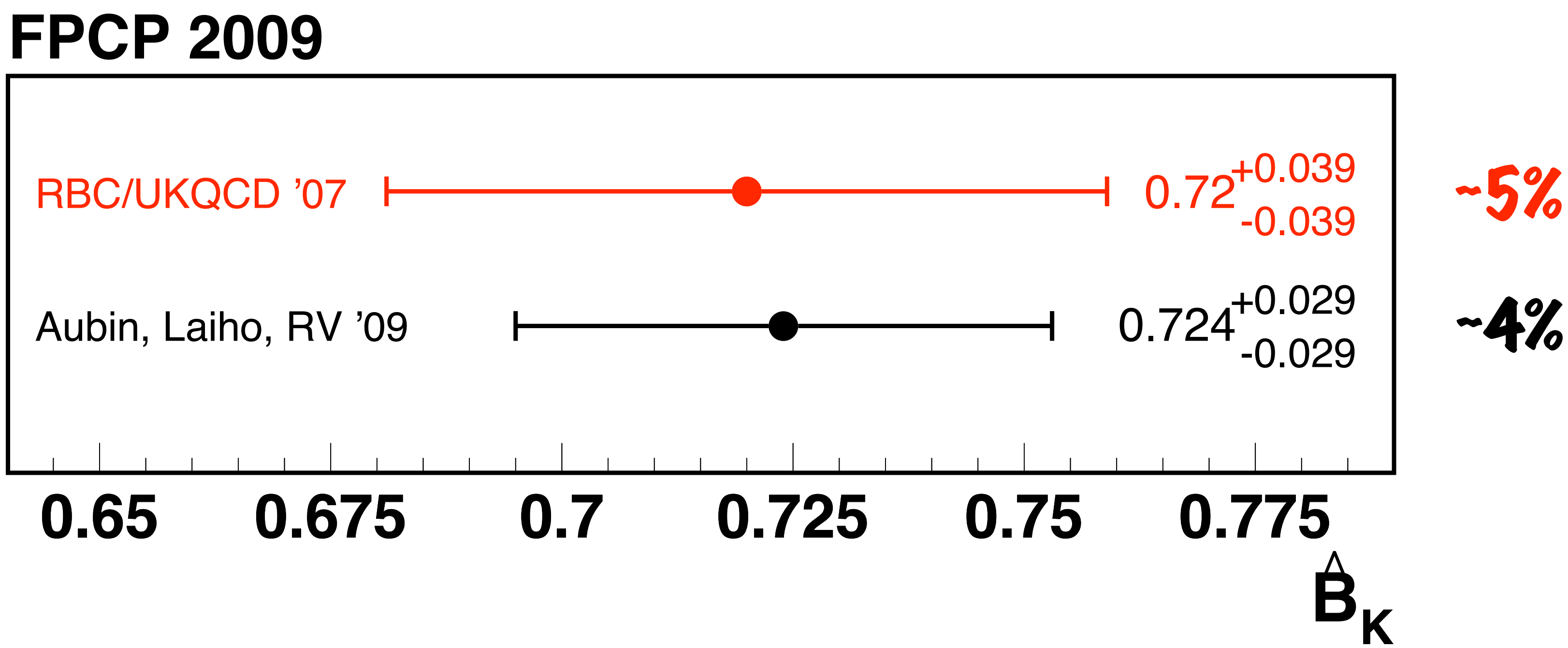
$$B_K^{\overline{\rm MS},{\rm NDR}}(2\ {\rm GeV}) = 0.527(6)(20)$$

- First unquenched lattice determination of B_K with data at two lattice spacings

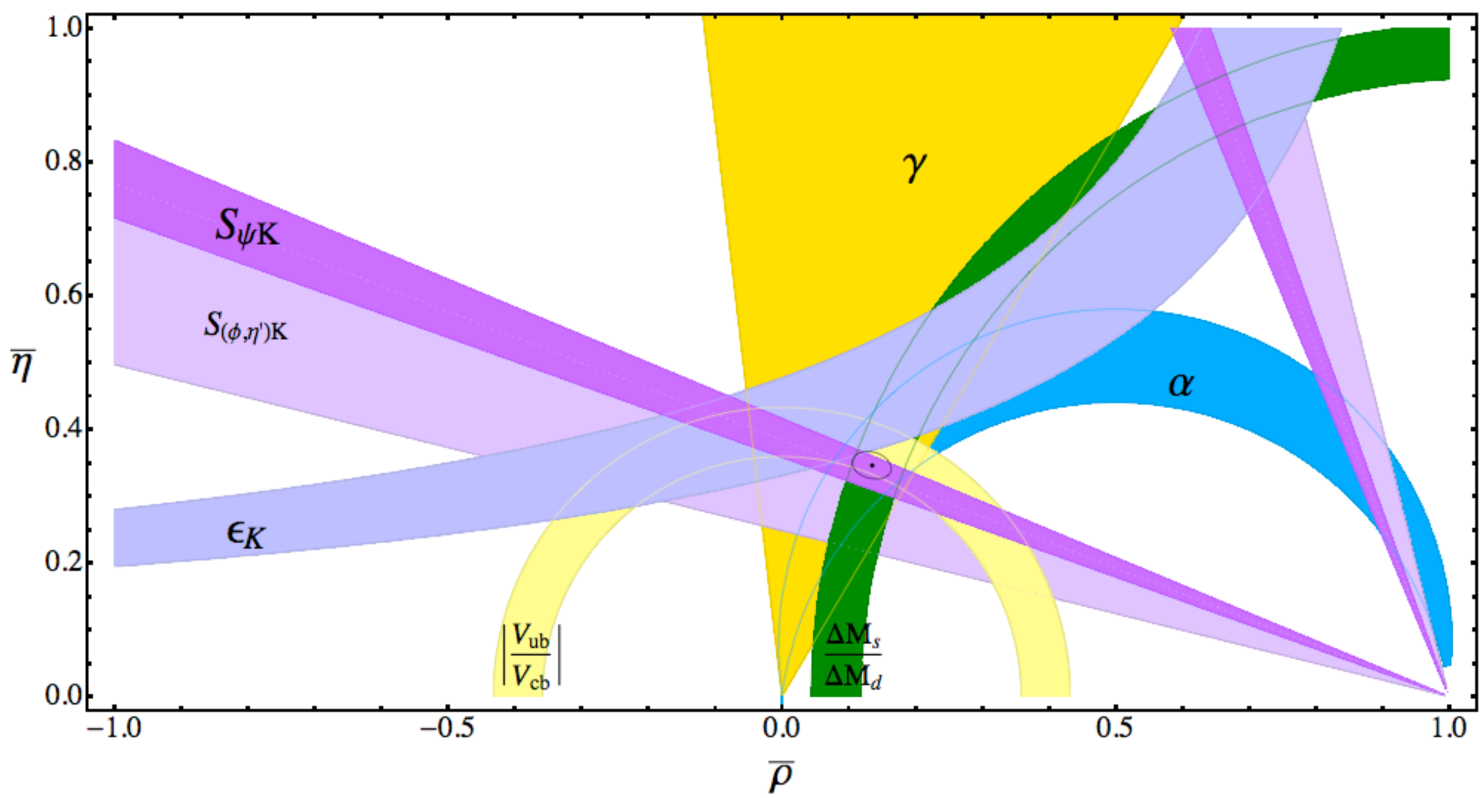


- Mild lattice spacing dependence
- Largest uncertainty from matching lattice operator to continuum (3%)
  - Calculation of the 2-loop continuum perturbation theory formulae needed to match from the lattice RI/MOM scheme to the continuum MS-bar scheme critical for a more reliable estimate of the truncation error

Comparison with other determinations



[courtesy of E. Lunghi]



- Both results higher than value of $\hat{B}_K = 0.92 \pm 0.10$ preferred by the unitarity triangle fit including all other inputs
- Leads to 1.8-σ tension in global fit
- *Indication of new physics in the quark flavor sector?*

Summary and outlook

- Lattice QCD calculations of B-meson decays and mixing now allow **reliable determinations of CKM matrix elements**

- In the past year lattice QCD has produced:

  (1) First 2+1 flavor calculation of the $B \to D^* \ell \nu$ form factor and $|V_{cb}|$ exclusive

  (2) Best 2+1 flavor calculation of the $B \to \pi \ell \nu$ form factor and $|V_{ub}|$ exclusive

  (3) First 2+1 flavor calculation of neutral B-meson mixing parameters and their ratio ξ

- Lattice QCD results will continue to improve with:
  - Higher statistics, finer lattice spacings
  - Improved heavy-quark actions
  - Improved form factor data at nonzero q^2

- Lattice QCD will soon allow *percent-level tests of the Standard Model* in the quark flavor sector and may eventually reveal new physics