

General theoretical introduction

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Outlines

- Introduction
- Factorization assumption
- QCD-improved factorization
- Perturbative QCD factorization
- Soft-collinear effective theory
- (Light-cone) QCD sum rules
- Quark diagram parametrization
- Summary

In this talk...

- Simple ideas only
- For data, Smith's talk
- For numbers, Stewart's talk
- For new physics, Kou's talk

Introduction

- Why B physics?

Constrain standard-model parameters

CKM matrix elements, weak phases

Explore heavy quark dynamics

Form factors, penguins, strong phases

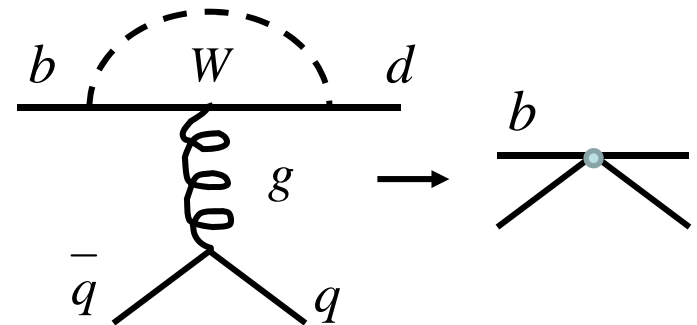
Search for new physics

SUSY, 4th generation, Z' ...

- Need critical comparison between data and QCD theories.

Hadronic B decays

- Involve three scales m_W , m_b , and Λ
- **Scale m_W is integrated out.** QCD dynamics is organized into weak effective Hamiltonian $H = V_{CKM} \sum_i C_i(\mu) O_i(\mu)$,
 $m_b < \mu < m_W$
- C_i : Wilson coefficients
- O_i : 4-fermion operators
- **Their μ dependence cancels.**



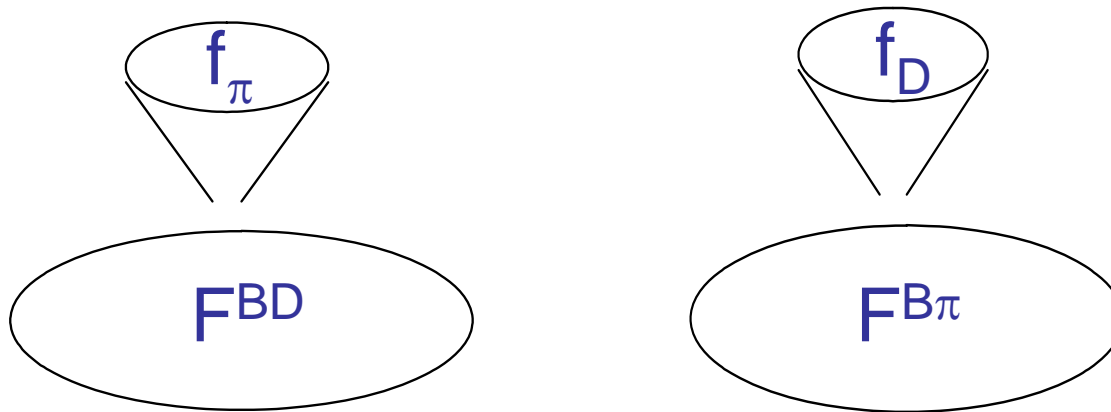
penguin operator

Factorization assumption

Decay amplitude

$$A = \langle D\pi | H_{\text{eff}} | B \rangle \sim C(\mu) \langle D\pi | O(\mu) | B \rangle$$

FA was proposed to deal with the hadronic matrix element (Bauer, Stech, Wirbel 85).



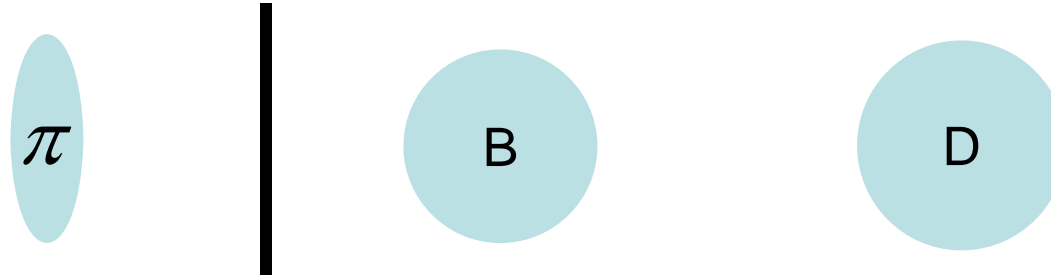
Color-allowed

Color-suppressed

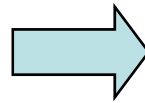
$$A(B \rightarrow D\pi) \propto a_1 f_\pi F^{BD} + a_2 f_D F^{B\pi}$$

a_1, a_2 : Wilson coefficients or fitting parameters

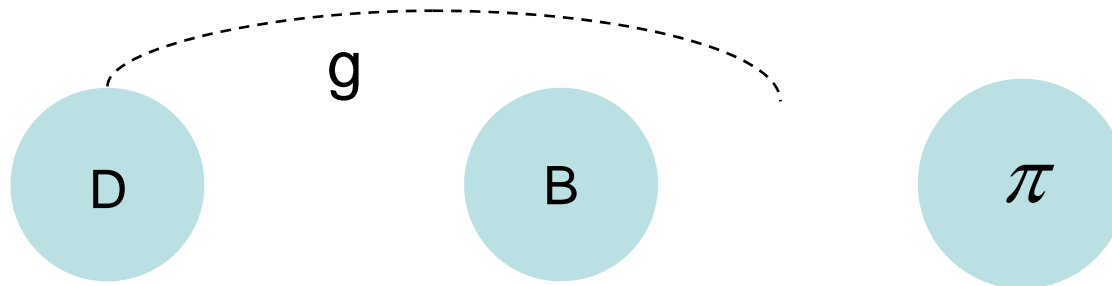
Color transparency



Lorentz contraction
small color dipole



decoupling in space-time
from the BD system



large corrections in color-suppressed modes due
to heavy D, large color dipole

Incompleteness of FA

- Form factor and decay constant are physical, independent of μ . Predictions depend on μ through $a_{1,2}(\mu)$.
- Nonfactorizable contributions must exist, especially in color-suppressed modes. They may be small in color-allowed decays, which are insensitive to μ .
- Power corrections, like strong phases, are crucial for CP violation.
- FA can not be a complete model.

Beyond FA?

- **Generalized naïve factorization**, beyond FA phenomenologically (Ali et al, Cheng et al)

- Parameterize nonfactorizable correction

$$a_1 \rightarrow a_1 + \chi_1, \quad a_2 \rightarrow a_2 + \chi_2$$

- Fine tune mode-dependent parameters χ

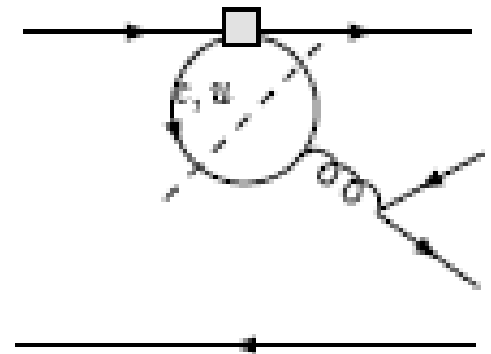
- Equivalently, effective number of colors in

$$a_{1(2)} = C_{2(1)} + C_{1(2)}/N_C \quad N_C \rightarrow N_C^{eff} = 2 \sim 6$$

- **Strong phase from BSS**

only and important source?

- Go beyond FA theoretically

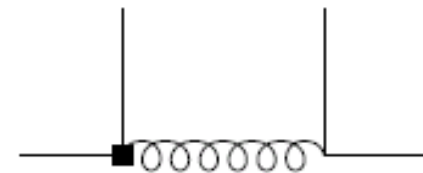
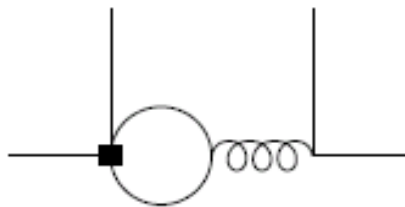
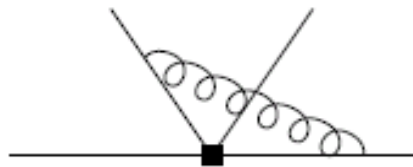
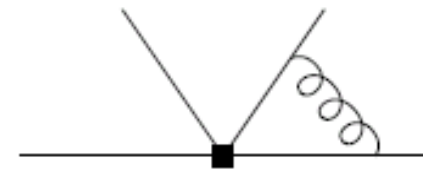
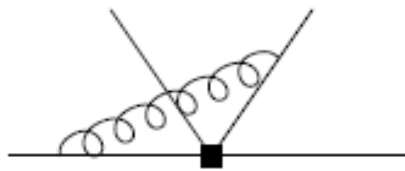
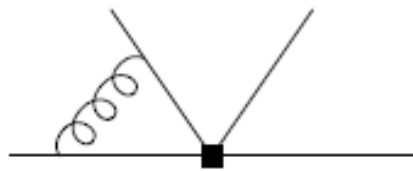
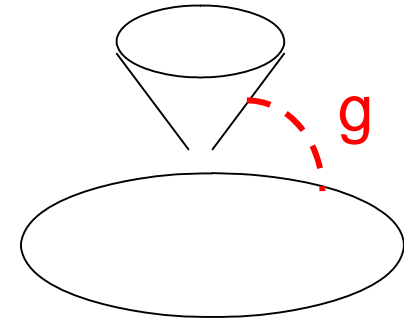


QCD-improved factorization

QCD corrections to FA

Higher-order corrections

- Color transparency hints addition of (nonfactorizable) hard gluons
- In terms of Feynman diagrams,



- Soft divergences cancel
- These diagrams weaken μ dependence in Wilson coefficients, and generate strong phases

More higher-order diagrams

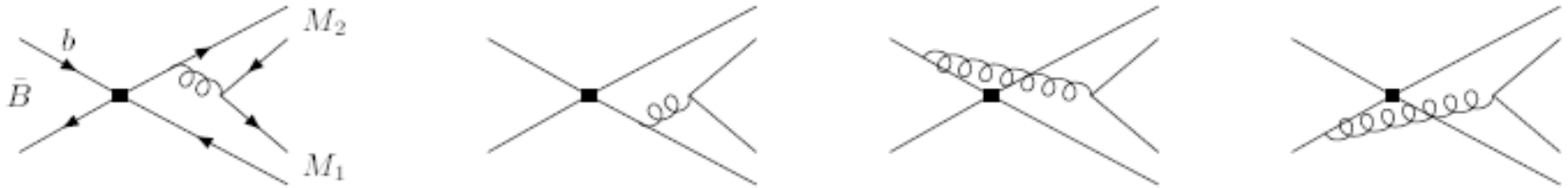
- Hard gluons can also be added to form the following nonfactorizable diagrams



- Feynman rules of these two diagrams (quark and anti-quark propagators) differ by a minus sign in soft region

Higher-power corrections in $1/m_B$

- Annihilation contribution



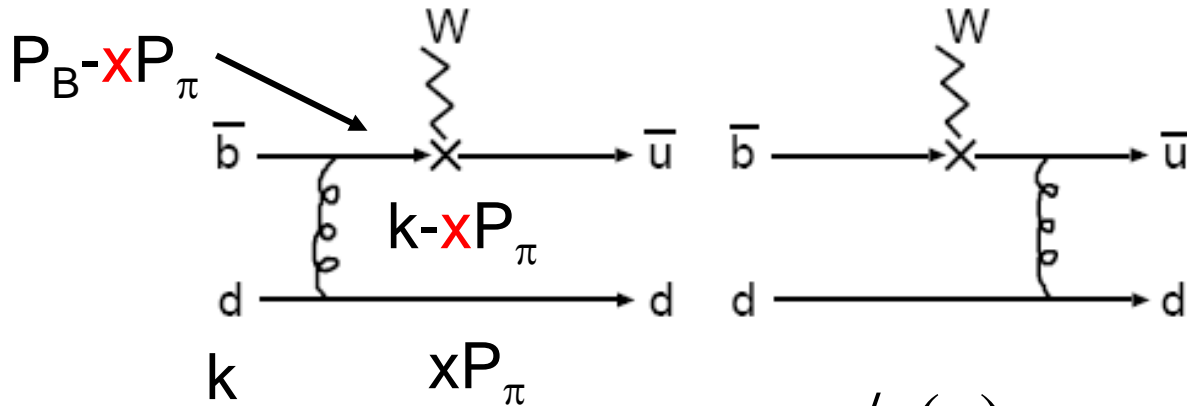
- Twist-3 nonfactorizable contribution
- They contain end-point singularity, not calculable, despite of the appearance of coupling constant in their parameterization

$$x_C = \alpha_S \ln \frac{m_B}{\Lambda} \left(1 + \rho_H e^{i\delta_H} \right), \quad \alpha_S \ln \frac{m_B}{\Lambda} \left(1 + \rho_A e^{i\delta_A} \right)$$

- Predictions (default) or fitting (S1,S2,...,G)?

End-point singularity

- Application of collinear factorization to B meson transition form factor gives end-point singularity



$$\int_0^1 dx \frac{\phi_\pi(x)}{x^2} \rightarrow \infty, \quad \phi_\pi(x) \propto x(1-x)$$

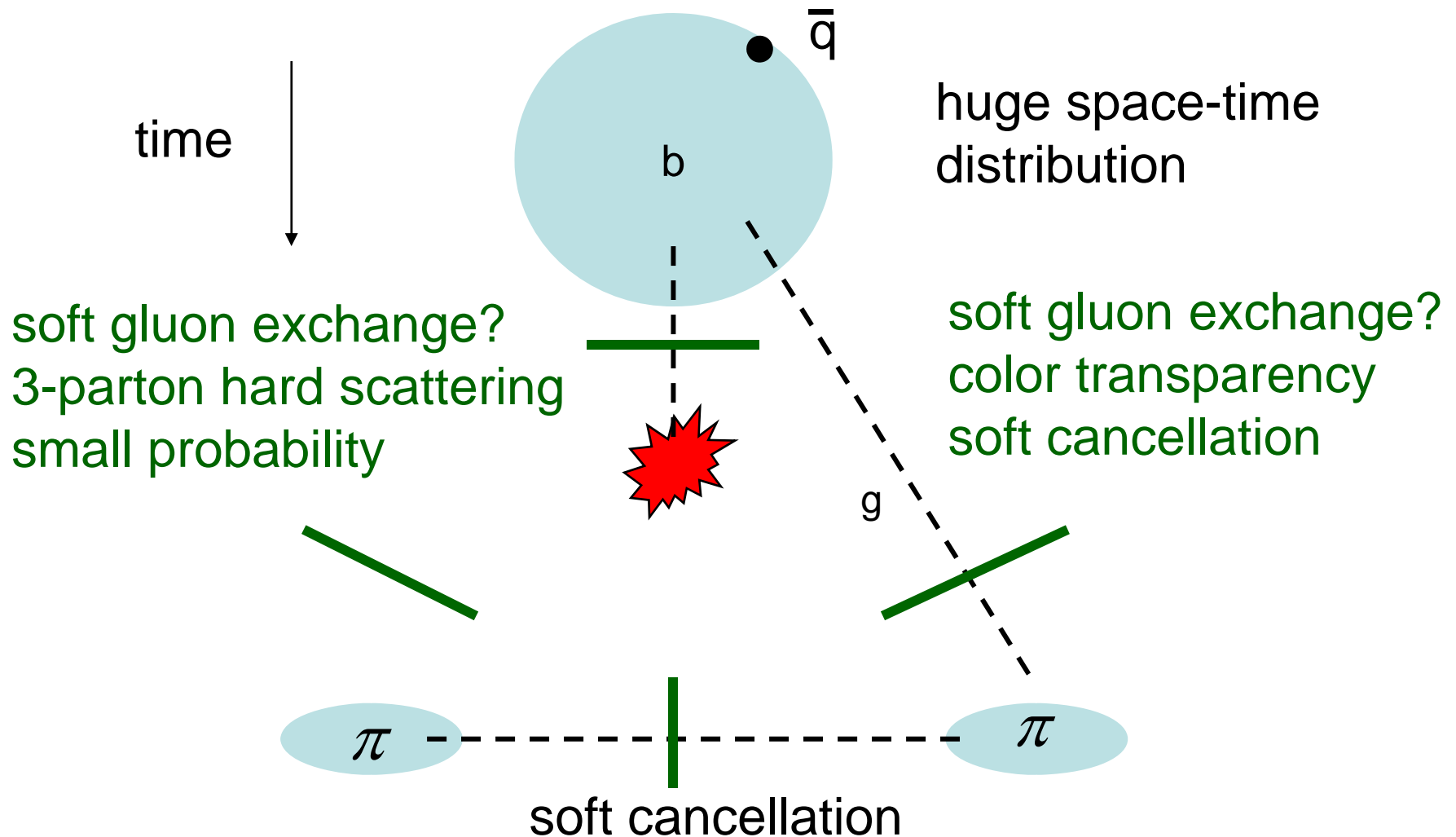
- End-point singularities in annihilation and nonfactorizable contributions have the same origin

PQCD factorization

starting from factorization theorem
motivated by removing end-point
singularity

Factorization theorem

- With hard scattering (large energy release)



Factorization assumption vs factorization theorem

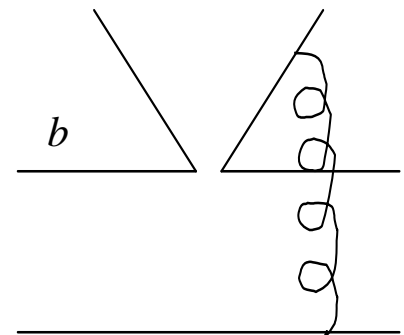
- FA
- of process
- $A(B \rightarrow M_1 M_2) \sim f_{M_2} F^{BM_1}$
- $Vac \rightarrow M_2, B \rightarrow M_1$
- Factorizable: in the above form
- Nonfactorizable: not in the above form
- This nonfactorizable amplitude is factorizable (calculable).

FT

of dynamics

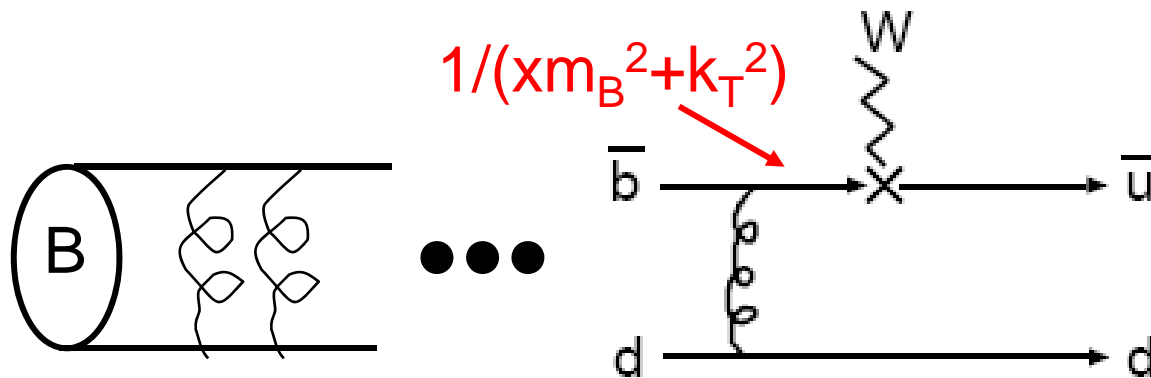
$$\sim \phi_B^* \phi_{M_1}^* \phi_{M_2}^* H$$

nonpert pert



k_T factorization

- Singularity implies small x . Parton k_T not negligible. Develop k_T factorization.
- Many gluon exchanges generate k_T



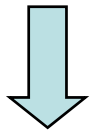
$$\int_0^1 dx \frac{\phi_\pi(x)}{x^2 m_B^2} \rightarrow \int_0^1 dx \frac{\phi_\pi(x)}{x(xm_B^2 + k_T^2)}$$

- k_T distribution governed by Sudakov factor

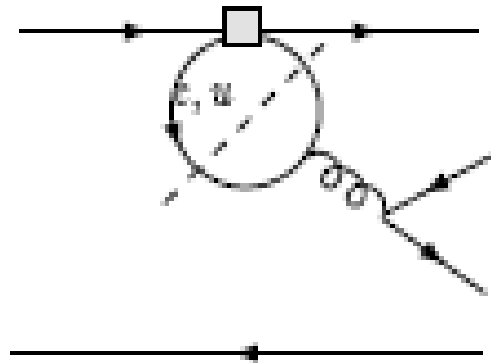
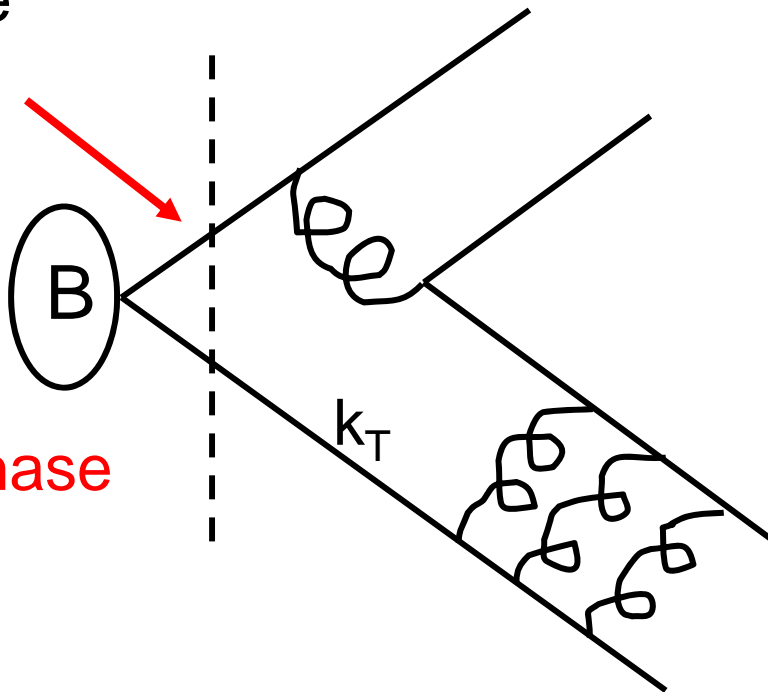
Imaginary annihilation

principle value $\frac{1}{xm_B^2 - k_T^2 + i\epsilon} = \frac{P}{xm_B^2 - k_T^2} - i\pi\delta(xm_B^2 - k_T^2).$

loop line
can go
on-shell



strong phase



Bander-Silverman-Soni
mechanism, strong
phase source in FA

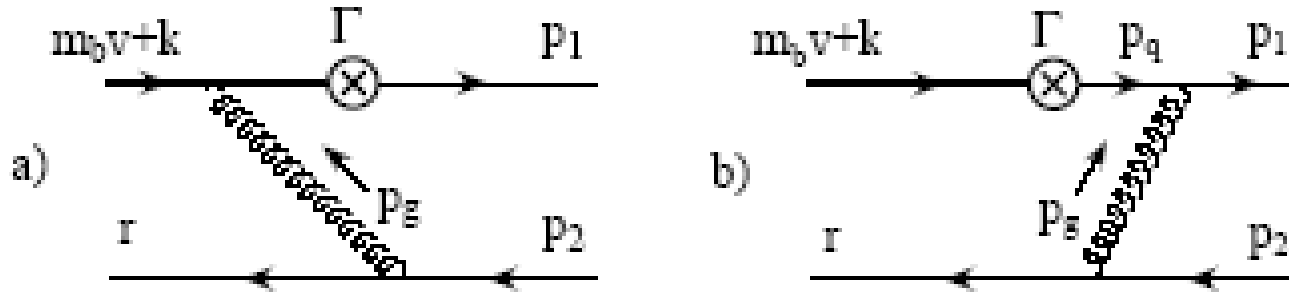
loop with the weight factor from
 k_T distribution

Soft-collinear effective theory

based on collinear factorization
but formulated via OPE

B \rightarrow π form factor

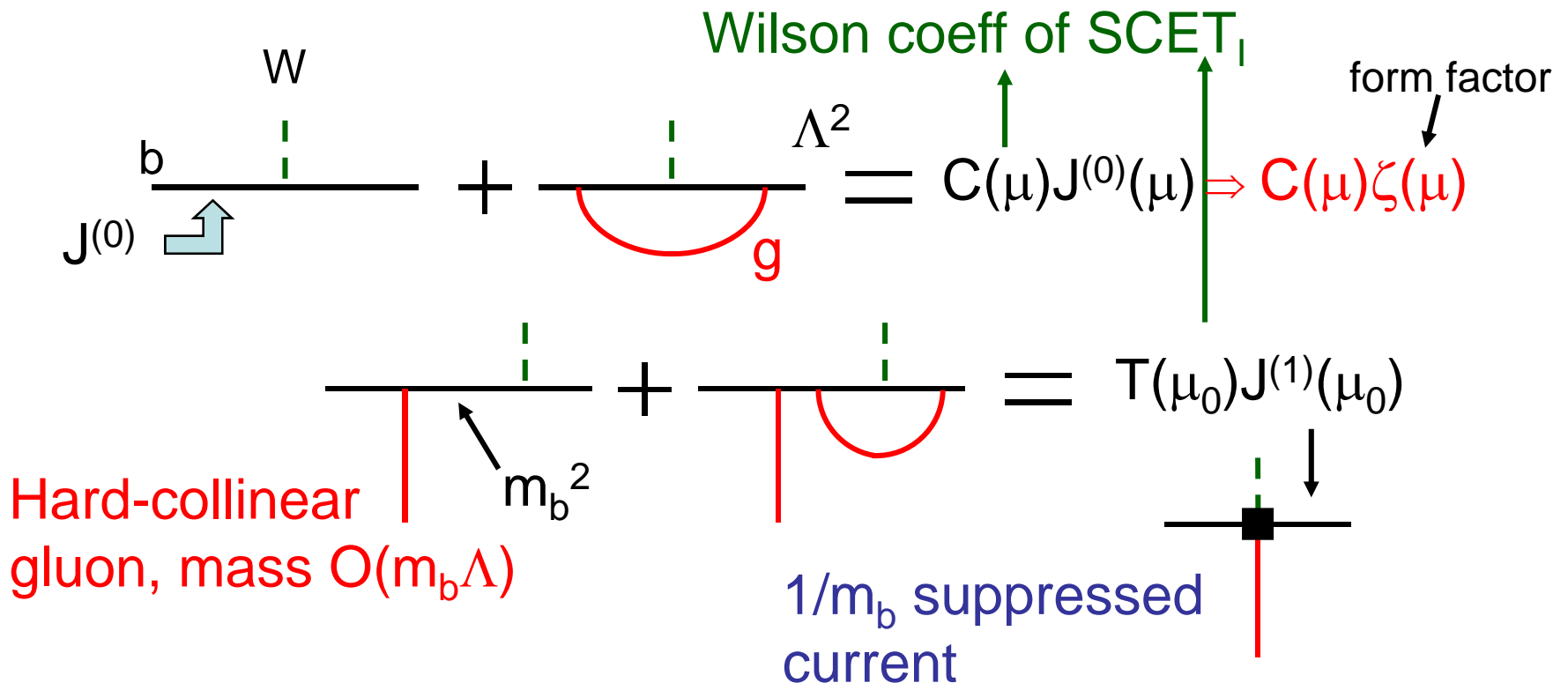
- Kinematics



- Soft spectator in B, $r \sim \Lambda$
- If $p_2 \sim m_b$, $p_g^2 = (p_2 - r)^2 = -2r \cdot p_2 \sim O(m_b \Lambda)$
- Then the internal quark is off-shell by $(m_b v + k + p_g)^2 - m_b^2 \sim O(m_b^2)$
- Two scales below m_w : $m_b \Lambda$ and m_b^2 , separate matching at these two scales

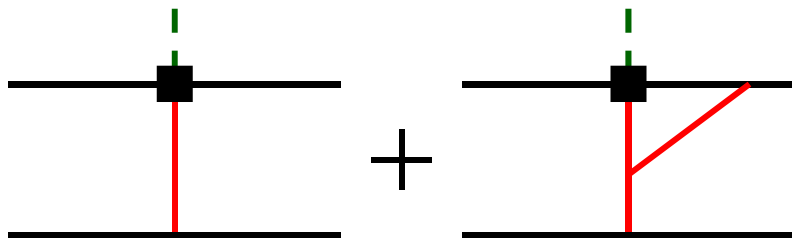
SCET_I

- Full theory \rightarrow SCET_I: integrate out the lines off-shell by m_b^2



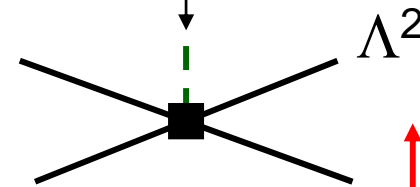
SCET_{II}

- SCET_I → SCET_{II}: integrate out the lines off-shell by $m_b \Lambda$



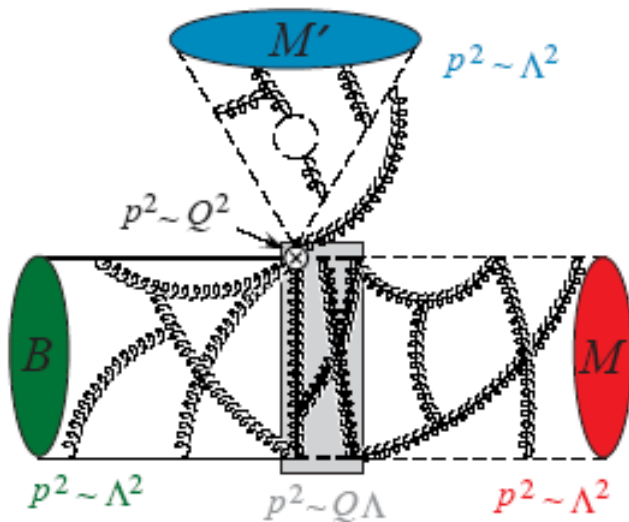
Jet=Wilson coeff of SCET_{II}

$$= J(\mu_0, \mu) O(\mu)$$



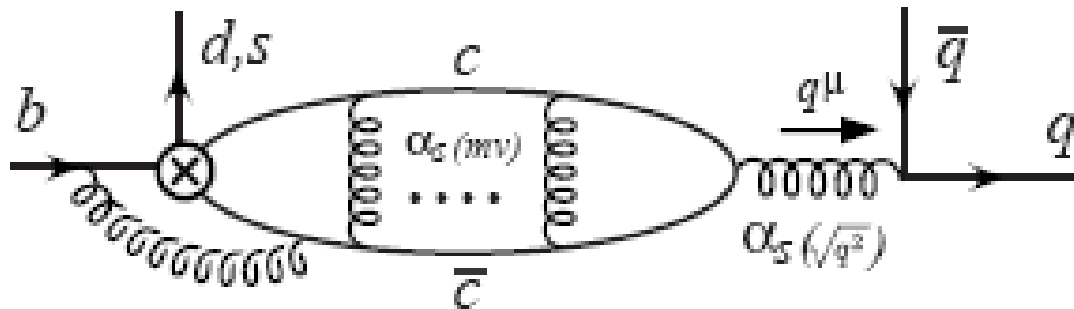
$$\Rightarrow T(\mu_0) J(\mu_0, \mu) \phi_M(\mu) \phi_B(\mu)$$

Sandwiched by the light meson and B meson states



Charming penguin (Bauer et al)

- At leading power, no large source of strong phases in SCET (no annihilation) .
- Long-distance $O(1/m_b)$ charming penguin is introduced, parameterized as Acc .



- With nonperturbative Acc , apply SCET as QCD-improved parametrization.

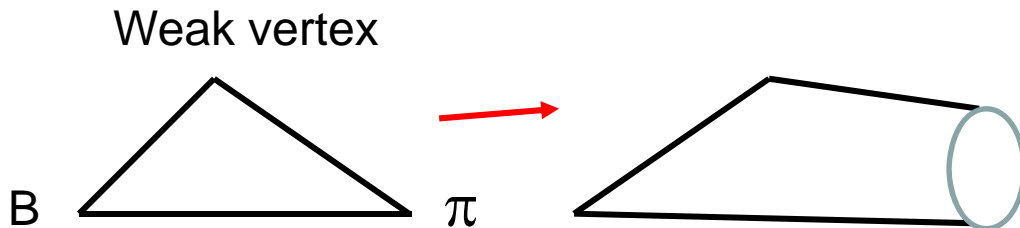
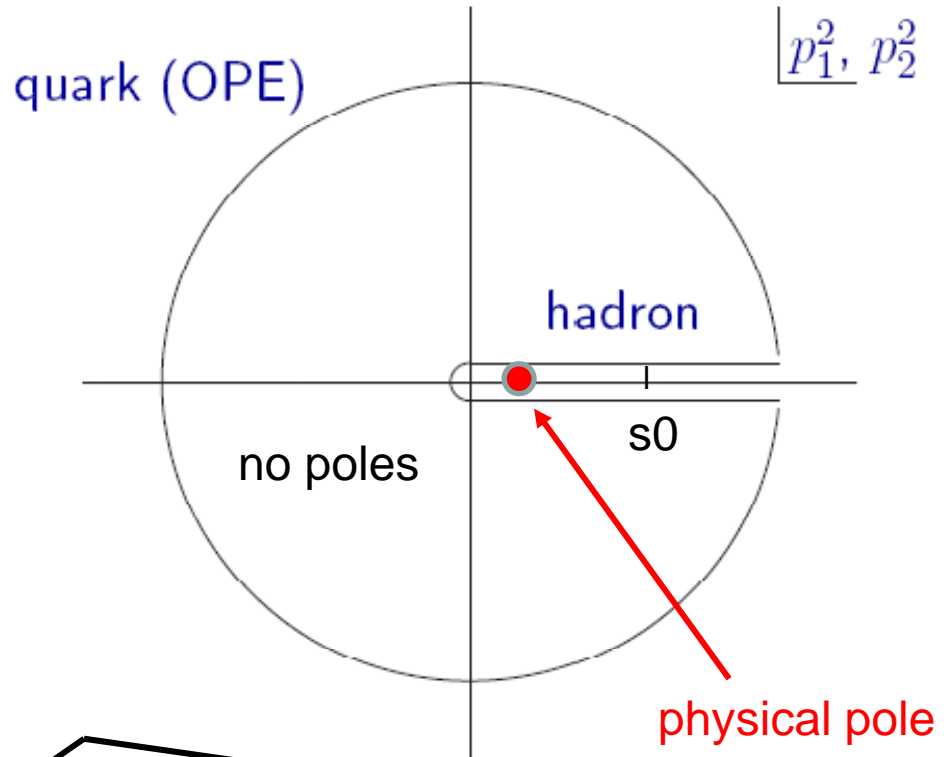
(Light-cone) QCD sum rules

based on quark-hadron duality
different from factorization

Quark-hadron duality

- “Nonperturbative” averaged over duality interval s_0 = “perturbative”

- Light-cone QSR approximates QSR by local current \Rightarrow nonlocal current on light cone



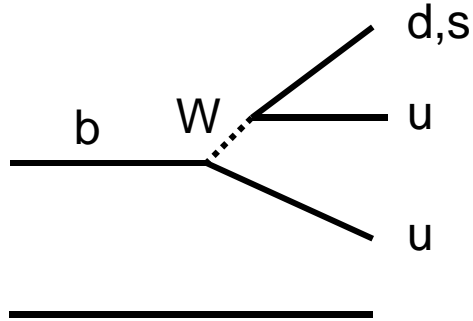
LC pion wave function

Quark-diagram parametrization

also different from factorization

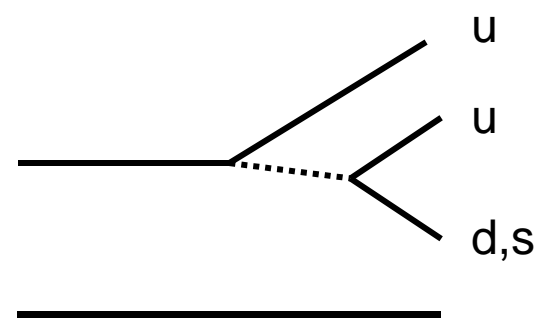
Quark amplitudes

2 color traces $\Rightarrow N_c^2$

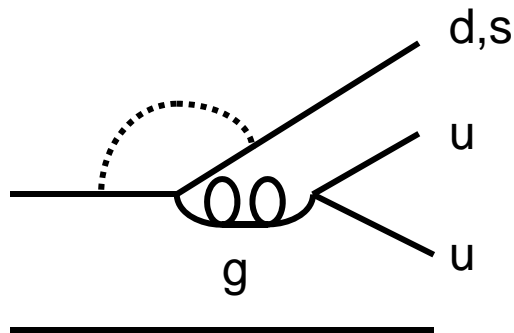


Color-allowed tree T

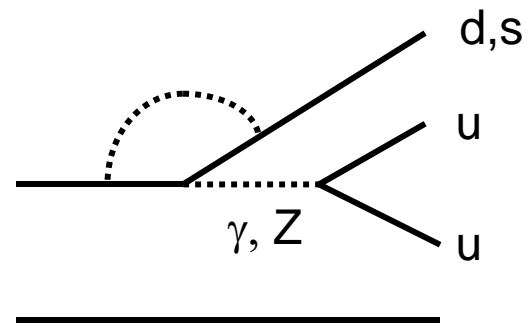
1 color trace $\Rightarrow N_c$



Color-suppressed tree C



QCD penguin P



Electroweak penguin P_{ew}

$\pi\pi$ parametrization

$$\begin{aligned}\sqrt{2}A(B^+ \rightarrow \pi^+\pi^0) &= -T \left[1 + \frac{C}{T} + \frac{P_{ew}}{T} e^{i\phi_2} \right], \\ A(B_d^0 \rightarrow \pi^+\pi^-) &= -T \left(1 + \frac{P}{T} e^{i\phi_2} \right), \\ \sqrt{2}A(B_d^0 \rightarrow \pi^0\pi^0) &= T \left[\left(\frac{P}{T} - \frac{P_{ew}}{T} \right) e^{i\phi_2} - \frac{C}{T} \right],\end{aligned}$$

$$\frac{P}{T} \sim \lambda, \quad \frac{C}{T} \sim \lambda, \quad \frac{P_{ew}}{T} \sim \lambda^2.$$



$$(C_4/C_2)(V_{td}V_{tb}/V_{ud}V_{ub})/1 \sim (\lambda^2/1)(\lambda^3/\lambda^4) \sim \lambda$$


$K\pi$ parametrization

$$A(B^+ \rightarrow K^0 \pi^+) = P',$$

$$A(B_d^0 \rightarrow K^+ \pi^-) = -P' \left(1 + \frac{T'}{P'} e^{i\phi_3} \right),$$

$$\sqrt{2}A(B^+ \rightarrow K^+ \pi^0) = -P' \left[1 + \frac{P'_{ew}}{P'} + \left(\frac{T'}{P'} + \frac{C'}{P'} \right) e^{i\phi_3} \right],$$

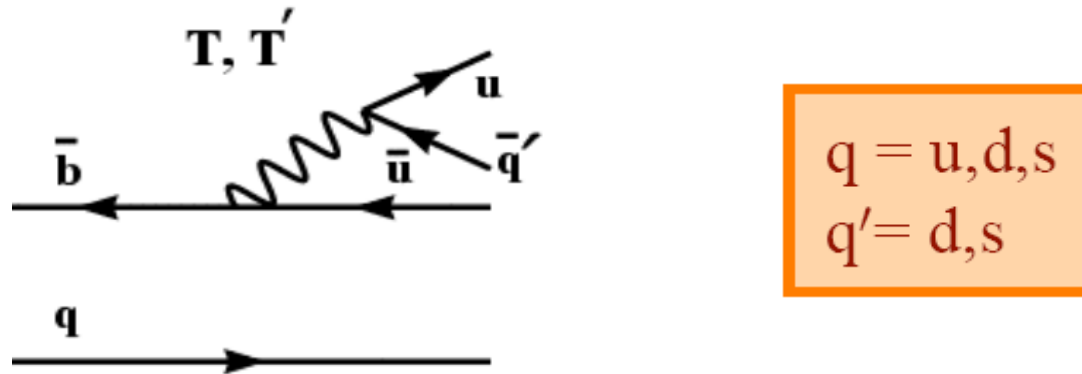
$$\sqrt{2}A(B_d^0 \rightarrow K^0 \pi^0) = P' \left(1 - \frac{P'_{ew}}{P'} - \frac{C'}{P'} e^{i\phi_3} \right),$$

$$\frac{T'}{P'} \sim \lambda, \quad \frac{P'_{ew}}{P'} \sim \lambda, \quad \frac{C'}{P'} \sim \lambda^2$$


$$(C_2/C_4)(V_{us}V_{ub}/V_{ts}V_{tb}) \sim (1/\lambda^2)(\lambda^5/\lambda^2) \sim \lambda$$

Flavor symmetry

- Predictive power from flavor symmetry
- SU(3) relates u,d,s interchange



- U-spin relates d,s interchange

$$B_d^0 \rightarrow K^+ \pi^- \quad \text{and} \quad B_s^0 \rightarrow \pi^+ K^-$$

$$B_s^0 \rightarrow K^+ K^- \quad \text{and} \quad B_d^0 \rightarrow \pi^+ \pi^-$$

- Difficult to estimate symmetry breaking

Summary

- Able to go beyond factorization assumption by including QCD corrections.
- No end-point singularity in PQCD (k_T factorization) and SCET (zero-bin subtraction), so transition form factors, and $1/m_B$ contributions are calculable.
- External lines are off-shell on OPE side of QSR. Soft contribution in QSR has a definition different from those in factorization approaches.

Summary

- Predictive power of factorization approaches comes from universality of nonperturbative inputs (meson wave functions), but that of quark diagram parametrization from flavor symmetry.

Back-up

Effective theory

- An effective theory by integrating out high energy (E) modes.
- Effective degrees of freedom: collinear fields, soft fields,...
- Express an amplitude in $1/E$ in terms of the effective operators.
- The Wilson coefficients of these operators are the hard kernels.
- The matrix element of the (nonlocal) operators are DAs or form factors.
- Convenient for deriving factorization theorem.

Wilson coefficients

- Is $J(\alpha_s(m_b\Lambda))$, the **Wilson coefficient of SCET_{II}**, calculable?
 - No from Bauer, Pirjol, Rothstein, Stewart, but yes from Beneke and Yang.
- ⇒ Different phenomenological applications of SCET
- **C, T (Wilson coefficients of SCET_I) and J have been computed up to NLO. See Beneke and Yang, hep-ph/0508250.**

Decay amplitude

- SCET factorization formula for $B \rightarrow M_1 M_2$

Wilson coeff of SCET_I

$$\begin{aligned}
 A(\bar{B} \rightarrow M_1 M_2) = & \lambda_c^{(J)} A_{\alpha\bar{\alpha}}^{M_1 M_2} + \frac{G_F m_B^2}{\sqrt{2}} \left\{ \begin{array}{l} \text{Color-allowed} \\ f_{M_2} \zeta^{B M_1} \\ \text{Color-suppressed} \\ f_{M_1} \zeta^{B M_2} \end{array} \int_0^1 du T_{2\zeta}(u) \phi^{M_2}(u) + \int_0^1 du T_{1\zeta}(u) \phi^{M_1}(u) \right. \\
 & + \frac{f_B f_{M_1} f_{M_2}}{m_b} \int_0^1 du \int_0^1 dx \int_0^1 dz \int_0^\infty dk_+ J(z, x, k_+) \left[T_{2J}(u, z) \right. \\
 & \left. \left. \times \phi^{M_1}(x) \phi^{M_2}(u) + T_{1J}(u, z) \phi^{M_2}(x) \phi^{M_1}(u) \right] \phi_B^+(k_+) \right\}, \\
 & \text{factorizable}
 \end{aligned}$$

Jet function

Wilson coeff of SCET_I

Annihilation in QCDF, SCET₀, PQCD

- Applied to charmless nonleptonic decays, **scalar penguin annihilation** is treated as:
- **Parameter in QCDF** due to end-point singularity (nonfactorizable),
$$X_A = \ln(m_B/\Lambda)[1 + \rho_A \exp(i\phi_A)]$$
- What mechanism generates ϕ_A ?
- **Factorizable in SCET₀, but real (ALRS 06).**
Strong phase appears at $\alpha_s^2 \Lambda/m_b$.
- Phase generated without k_T expansion (Chay, Li, Mishima)