Factorization for Nonleptonic Decays: pQCD, QCDF, SCET

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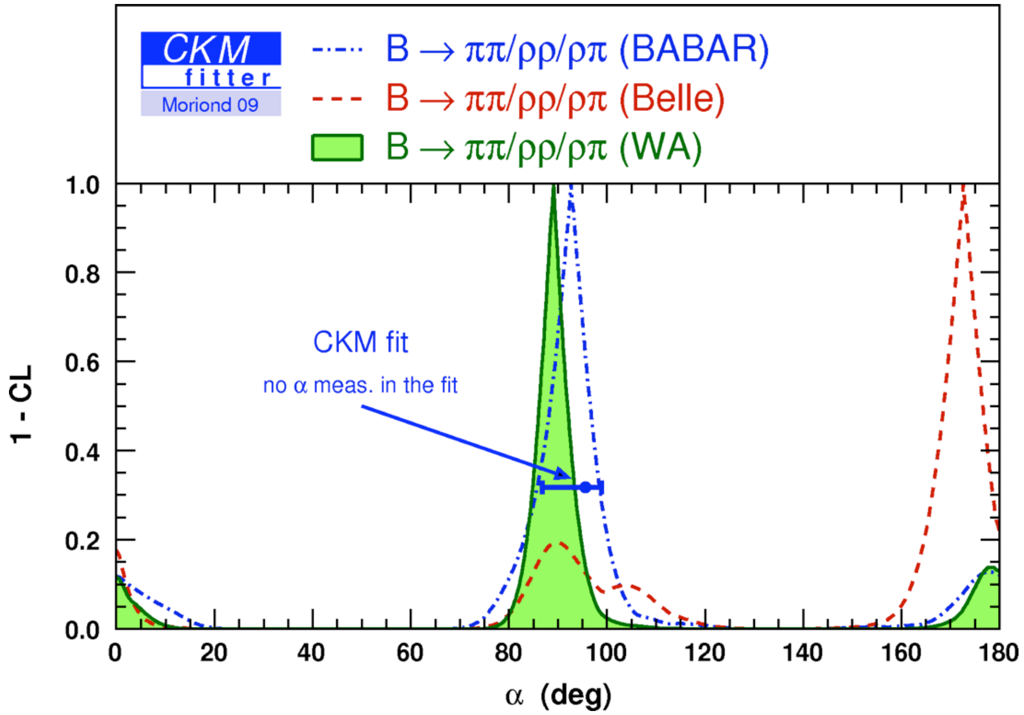
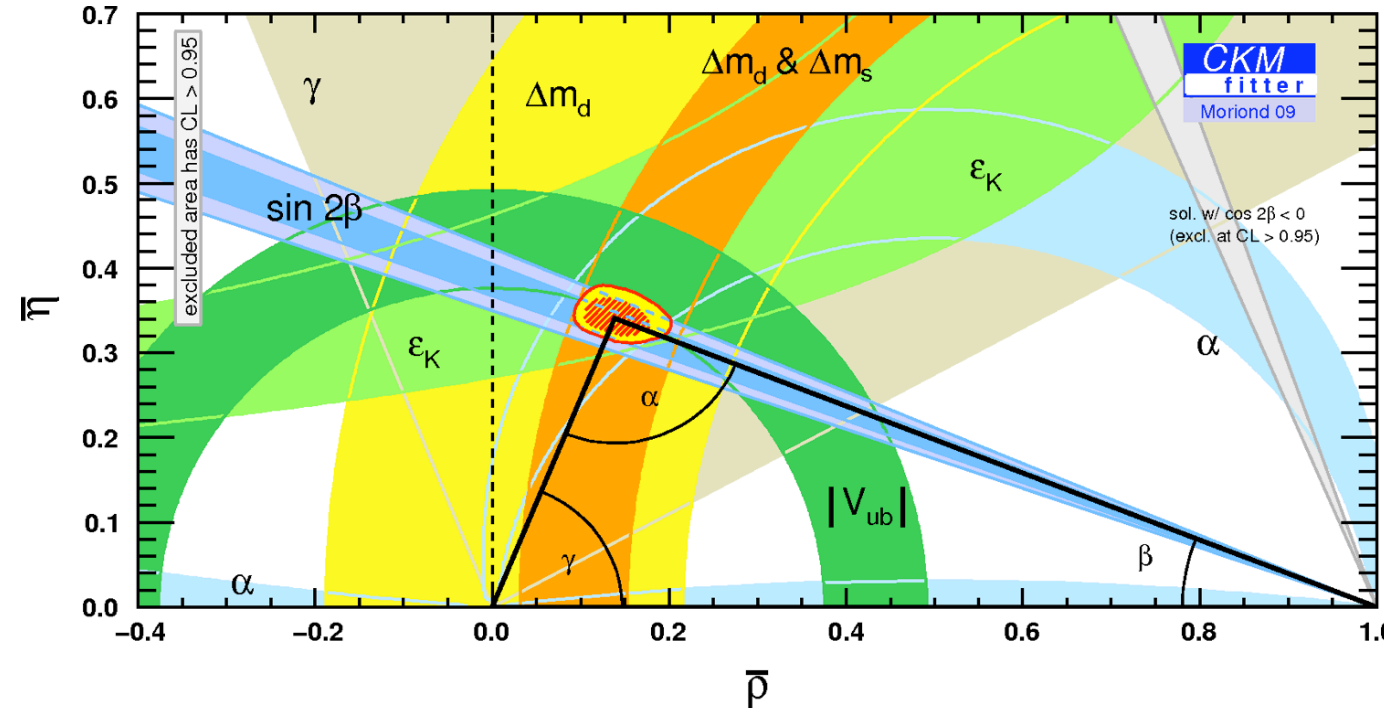
Outline:

- Why is Factorization Important?
- Theory Comparison: QCDF vs. pQCD vs. SCET
- Nonleptonic Predictions $B \to PP, \;\; B \to PV, \;\; B \to VV$
    - Global fits & uncertainties
    - Penguin-ology
    - $K\pi$
- Outlook

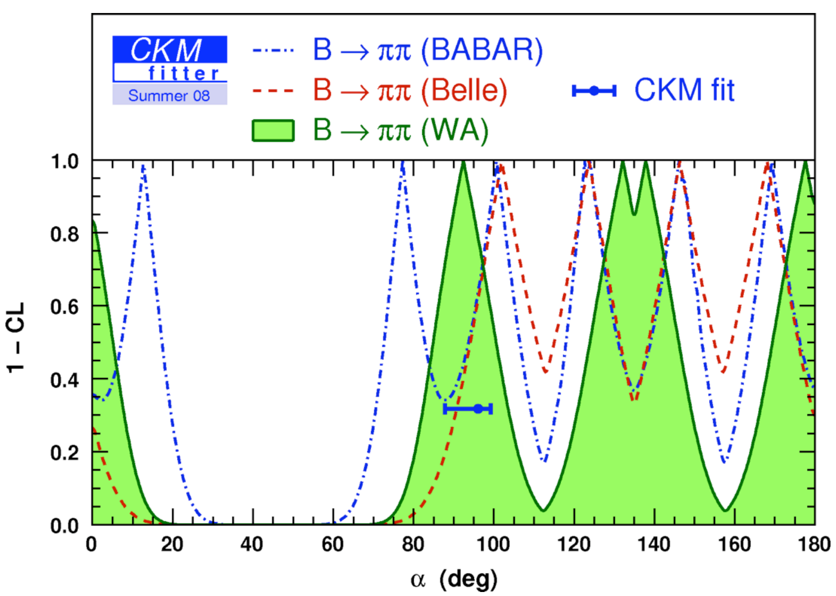
Isospin currently yields a *5%* measurement of α
when we combine $\rho\rho/\rho\pi/\pi\pi$: $\alpha = 89.0^\circ \pm 4.3^\circ$

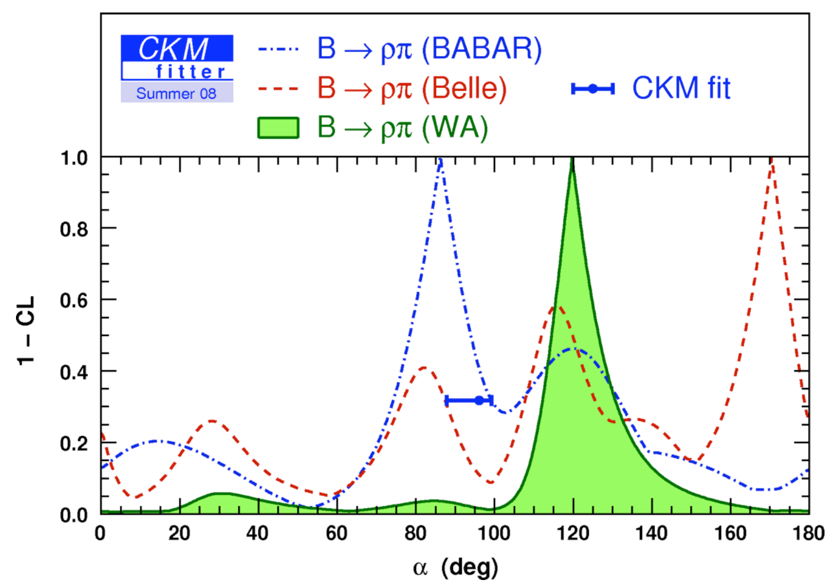


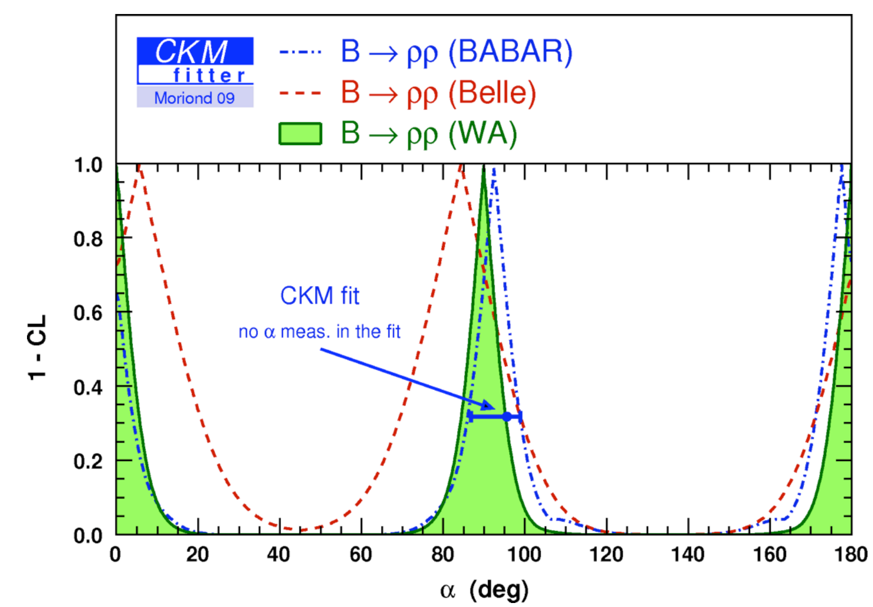


discrete ambiguities reduced significantly

agrees well with global CKM fit







Are we done?

What can be gained from other analyses?

What precisely are we testing when we make measurements of β or γ with different methods?

- Using CKM unitarity of the standard model we can write:

$$A^{SM}(\bar{B} \to M_1 M_2) = S_1 + S_2 e^{-i\gamma}$$

where $S_{1,2}$ are complex, CP even, "hadronic amplitudes".

- Consider an arbitrary new physics contribution to this channel, and write:

$$A^{NP}(\bar{B} \to M_1 M_2) = N e^{i\phi} = N_1 + N_2 e^{-i\gamma}$$

$$\& \quad N e^{-i\phi} = N_1 + N_2 e^{i\gamma}$$

Botella & Silva

$N_{1,2}$ are complex and CP even. eg. $\mathrm{Im} N_1 = \frac{\sin(\gamma + \phi)}{\sin(\gamma)} \mathrm{Im}(N)$

- Thus new physics in the decay simply shifts hadronic amplitudes:

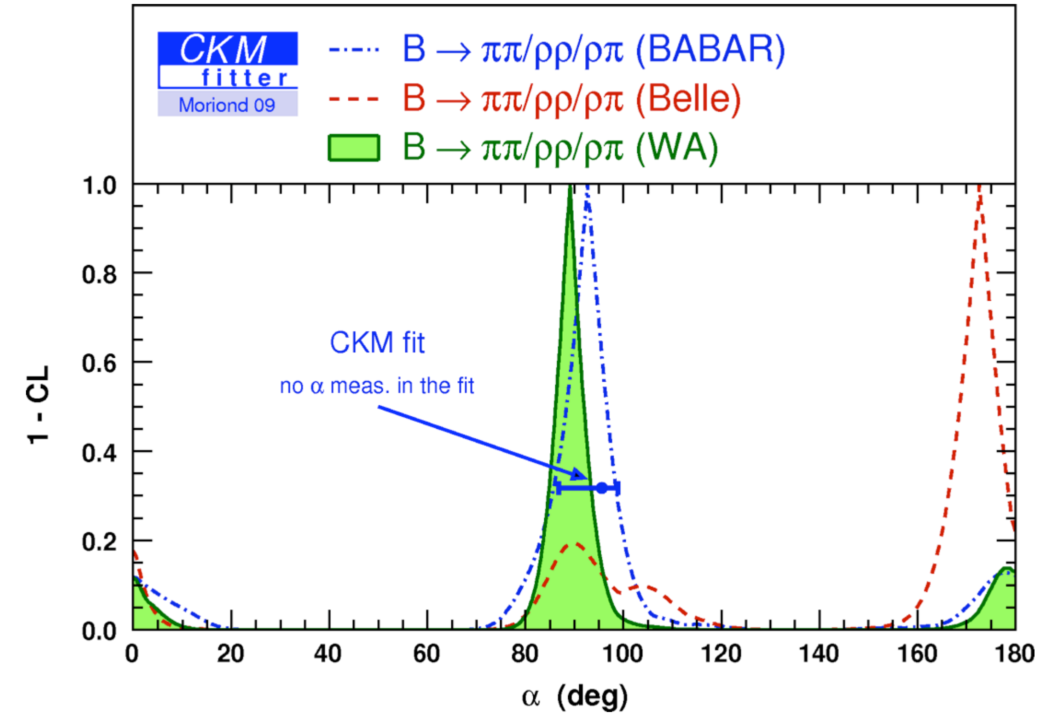
$$S_1 \to S_1 + N_1 \quad , \quad S_2 \to S_2 + N_2$$

Measurements test relations between SM amplitudes S_i which may be violated by new physics.

Applied to the Isospin Analysis:

5 amplitude parameters for $B \to \rho\rho$

5 amplitude parameters for $B \to \pi\pi$



Definitions:

$$A(\bar{B}^0 \to \pi^+\pi^-) = e^{-i\gamma}\,|\lambda_u|T - |\lambda_c|P$$

$$A(\bar{B}^0 \to \pi^0\pi^0) = e^{-i\gamma}\,|\lambda_u|C + |\lambda_c|P$$

$$\sqrt{2}A(B^- \to \pi^0\pi^-) = e^{-i\gamma}\,|\lambda_u|(T + C)$$

$$|\lambda_{c,u}| = \text{CKM factors}$$

P, "penguins", T "tree",

C, "color suppressed tree amplitude"

Applied to the Isospin Analysis:

5 amplitude parameters for $B \to \rho\rho$

- implies there are small penguins in

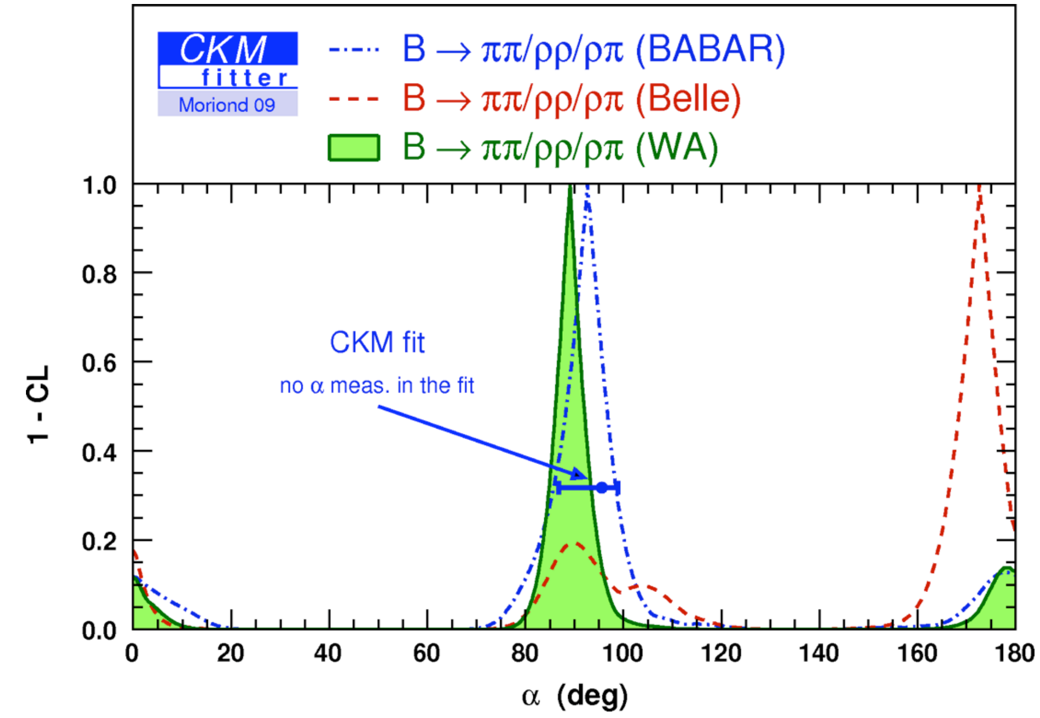
$$B \to \rho^0\rho^- \;,\; B \to \pi^0\pi^-$$

  and that electroweak penguins are not anomalously large

- does not untangle new physics that treats π and ρ differently

- can't see new physics in $I = 0$ amplitudes

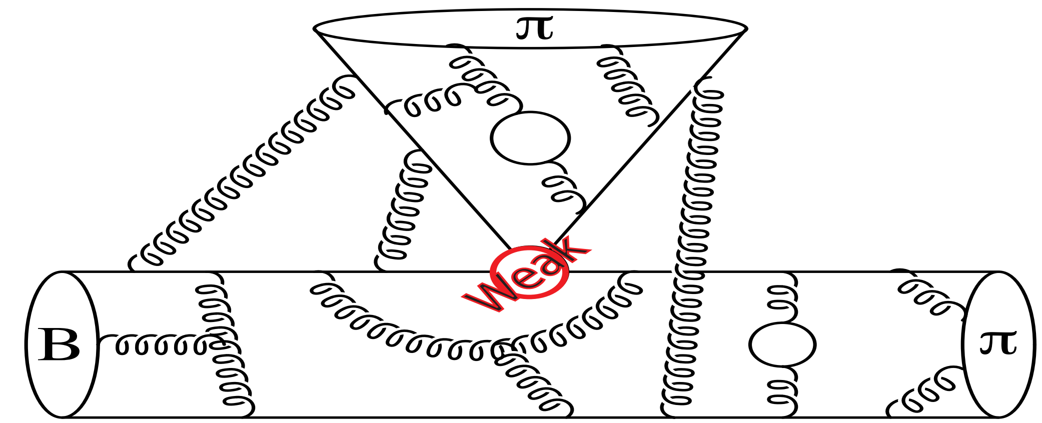
Baek, Botella, London, Silva



so we don't want to stop here!

Ideally we should test each measurable property of the nonleptonic amplitudes, and do so channel by channel. All amplitudes would be "related" by standard model Lagrangian parameters, but...

... Hadronic Uncertainties ...



In practice relations between SM amplitudes are approximate, and are always based on expansions of $\mathcal{L}^{\mathrm{SM}}$

$$\text{Observable} = O^{(0)} + \epsilon\, O^{(1)} + \epsilon^2 O^{(2)} + \ldots \qquad \epsilon \ll 1$$

The role of factorization is to yield new relations between SM amplitudes, and hence additional tests for new physics.

↑ More SM Input

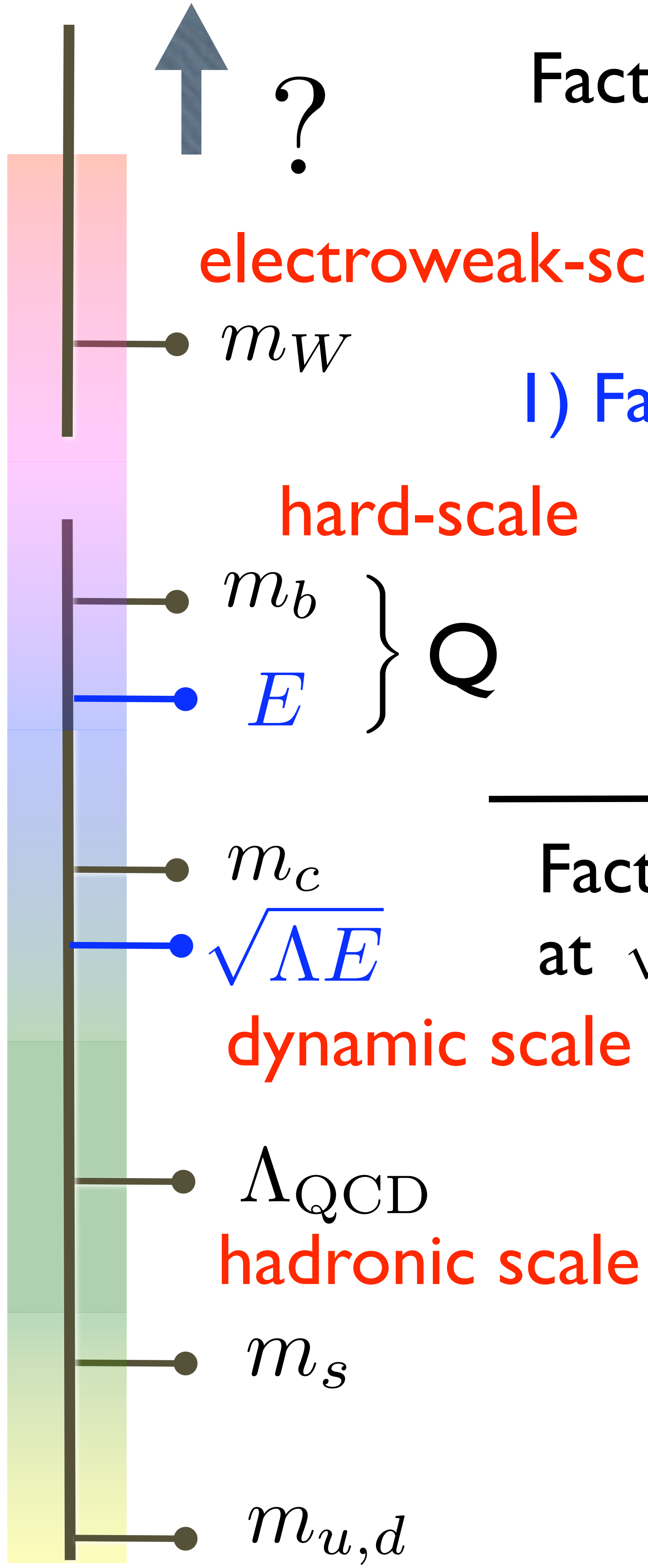
↑ More tests for new physics

↓ More precision (more trust) of SM results

It is worth testing every prediction from factorization, taking into account the expected precision.

Expansion		Parameter
• $m_W, m_t \gg m_b$	$H_{\text{weak}} = \frac{G_F}{\sqrt{2}} \sum_i \lambda^i C_i(\mu) O_i(\mu)$	$\epsilon^2 = \frac{m_b^2}{m_W^2} \sim 0.003$
• $\lambda^2 \ll 1$	$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$	$\epsilon^2 = \lambda^2 \sim 0.04$
• $\Lambda \gg m_{u,d}$	SU(2) ie. isospin	$\epsilon = \frac{m_{u,d}}{\Lambda} \sim 0.02$
• $m_b \gg \Lambda$	Heavy Quark Effective Theory	$\epsilon = \frac{\Lambda}{m_b} \sim 0.1$
• $E_\pi \gg \Lambda$	Factorization for Nonleptonic decays	$\epsilon = \frac{\Lambda}{E_\pi} \sim 0.2$
• $\Lambda \gg m_{s,d,u}$	SU(3) or U-spin	$\epsilon = \frac{m_s}{\Lambda} \sim 0.3$

Factorization is a Separation of Distance Scales



1) Factorization at mb

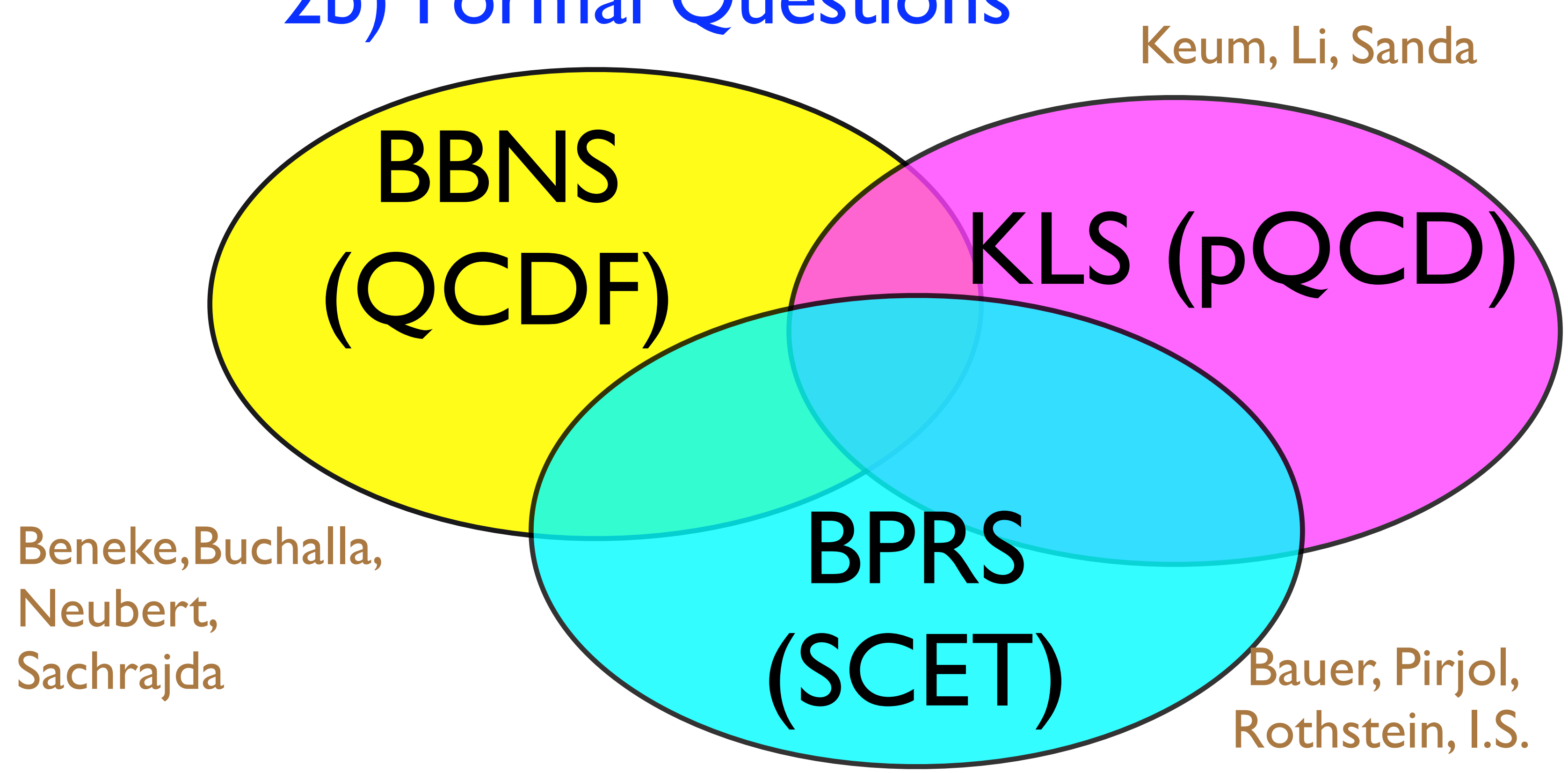
Approaches usually agree on predictions that only rely on factorization at this scale

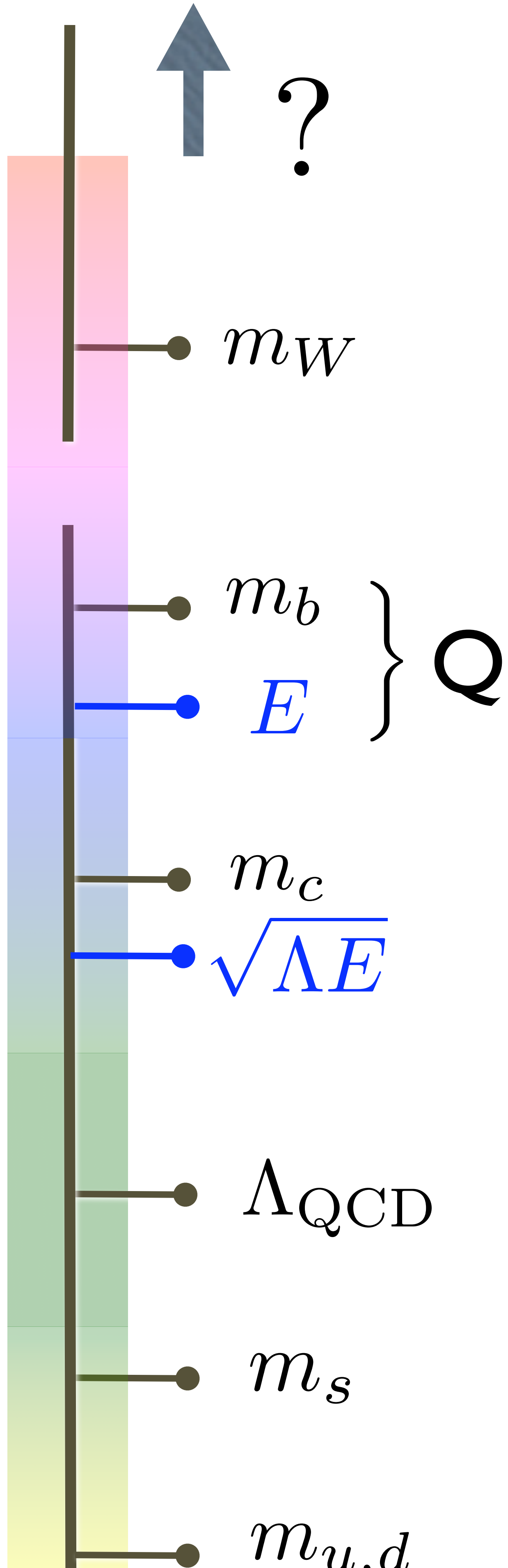
Annihilation

Factorization at $\sqrt{\Lambda E}$

2a) Input Parameters

2b) Formal Questions





Factorization at m_b

$\mathrm{SCET_I}$

expansion in $\alpha_s(m_b) \simeq 0.22$

hard-scale

↓

intermediate-scale

↓

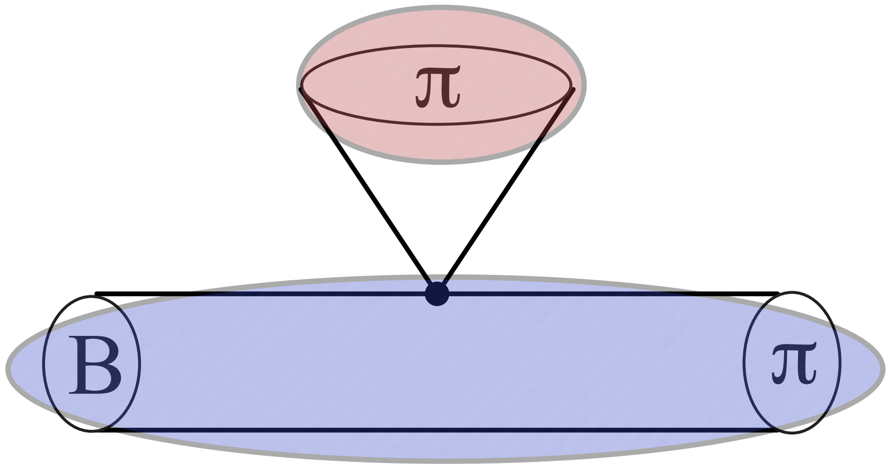
hadronic-scale

(intermediate-scale and hadronic-scale) treated as hadronic parameters in BPRS approach

$\mathcal{O}(\alpha_s(m_b))$ matching complete.

Beneke & Jager (tree & penguin)
Jain, Rothstein, I.S. (penguin)

Factorization at m_b



All the LO terms are factorized into two types of form factors

Nonleptonic $B \to M_1 M_2$

$$A(B \to M_1 M_2) = A^{c\bar{c}} + N\left\{ f_{M_2} \zeta^{BM_1} \int du T_{2\zeta}(u) \phi^{M_2}(u) + f_{M_2} \int du dz T_{2J}(u,z) \zeta_J^{BM_1}(z) \phi^{M_2}(u) + (1 \leftrightarrow 2) \right\}$$

no endpoint singularities here

soft form factor

twist-2 distn.

hard form factor

twist-2 distn.

Form Factors
$B \to$ pseudoscalar: $f_+,\ f_0,\ f_T$
$B \to$ vector: $V,\ A_0,\ A_1,\ A_2,\ T_1,\ T_2,\ T_3$

Same form factors at large E

$$f(E) = \int dz\, T(z,E)\, \zeta_J^{BM}(z,E) + C(E)\, \zeta^{BM}(E)$$

$B \to \pi \ell \bar{\nu}$,
$B \to K^* \ell^+ \ell^-$,
$B \to \rho\gamma$, ...

1) Nonleptonic data and β, γ, can be used to extract Tree amplitudes

(all approaches)

Tree amplitudes + Factorization yield form factors

$$\mathrm{B}\to \pi\pi : \qquad f_+(0) = \left(0.19 \pm 0.01\big|_{\rm exp} \pm 0.05\big|_{\rm thy}\right)\left(\frac{3.8\times 10^{-3}}{|V_{ub}|}\right)$$

$$\mathrm{B}\to \rho\rho : \qquad \text{-}\mathrm{A}_{\|}(0) = \left(0.31 \pm 0.02\big|_{\rm exp} \pm 0.06\big|_{\rm thy}\right)\left(\tfrac{3.8\times 10^{-3}}{|V_{ub}|}\right)$$

Agrees with Semileptonics:

$$f_+^{\rm FNAL}(0) = 0.23 \pm 0.03$$

(2008 Fermilab/MILC lattice +dispersion fit to expt. spectrum)

$$-A_0^{\|} = 0.30 \pm 0.03$$

(2005 Ball and Zwicky, Light Cone Sum Rules)

The simplest prediction from factorization works.

(LO, all approaches)

II) small strong phase between color suppressed and tree amplitudes

$$\mathrm{Im}\left(\frac{C}{T}\right) \sim \mathcal{O}\left(\alpha_s(m_b), \frac{\Lambda}{E_\pi}\right)$$

$B \to \pi\pi$ Can use this to do isospin analysis without $C_{\pi^0\pi^0}$

here we fit 4 amplitude parameters, $\mathrm{Br}(\pi^0\pi^0)$ fits fine

$$\gamma^{\pi\pi} = 73.9^{\circ}{}^{+7.5}_{-10.3}\Big|_{\mathrm{exp}}{}^{+1.0}_{-2.5}\Big|_{\mathrm{thy}}$$

(expt. and theory errors)

Bauer et.al. (2008 update)

there is a 2nd solution: $\gamma^{\pi\pi}_{\mathrm{2nd}} = 27.7^{\circ}{}^{+9.9}_{-7.3}\Big|_{\mathrm{exp}}{}^{+10}_{-4.5}\Big|_{\mathrm{thy}}$

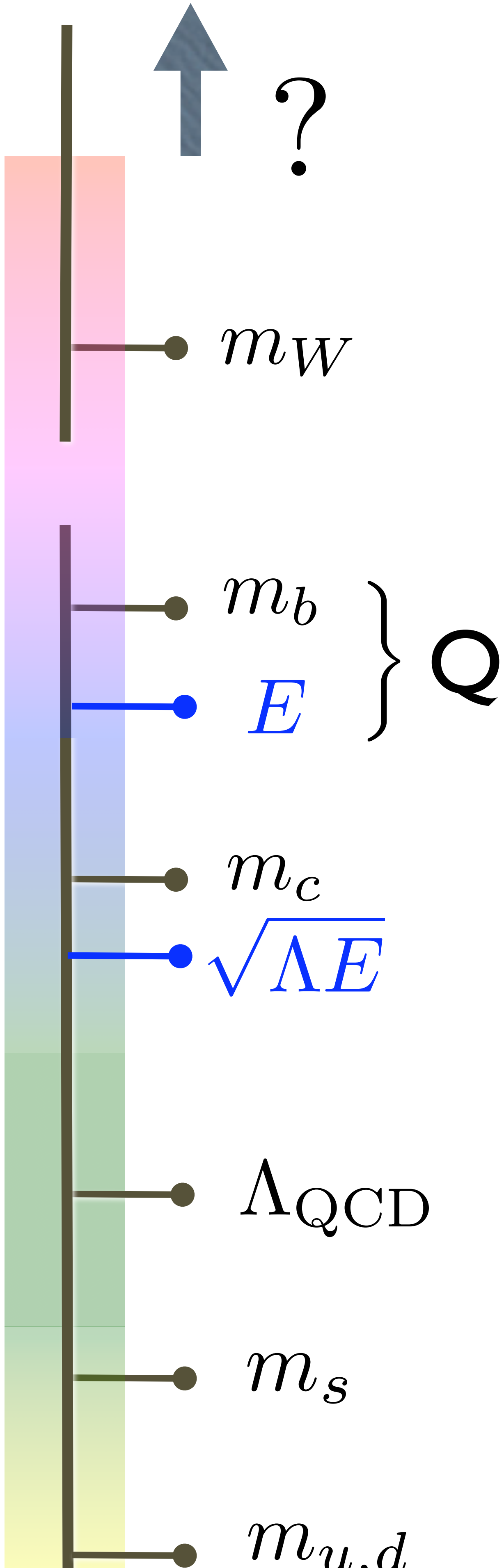
agrees at 1-σ with	$\gamma^{\mathrm{CKMfit.}}_{\mathrm{global}} = 67.8^{\circ}{}^{+4.2^{\circ}}_{-3.9^{\circ}}$	(2009)
	$\gamma^{\mathrm{UTfit.}}_{\mathrm{global}} = 65.6^{\circ} \quad 3.3^{\circ}$	(2008)

Agreement here further constrains ew. penguins & bounds imaginary terms from top/up penguins

but caution: $C^{\mathrm{here}}_{\pi^0\pi^0} = 0.5 \pm 0.3$ $\quad C^{\mathrm{expt.avg.}}_{\pi^0\pi^0} = -0.43 \pm 0.25$

if instead of fitting we use hadronic inputs, then $\mathrm{Br}(\pi^0\pi^0)$ is several σ low which is the situation for default parameters in BBNS and pQCD

analog: $B \to \rho\rho$ $\quad \gamma^{\rho\rho} = 77.5^{\circ}{}^{+7.4}_{-28}\Big|_{\mathrm{exp}}{}^{+1.0}_{-5.2}\Big|_{\mathrm{thy}}$ large errors



Intermediate scale μ_i

Factorization at $\sqrt{E\Lambda}$

SCET$_{\text{II}}$

used for trees and penguins
in BBNS & pQCD approaches

hard-scale

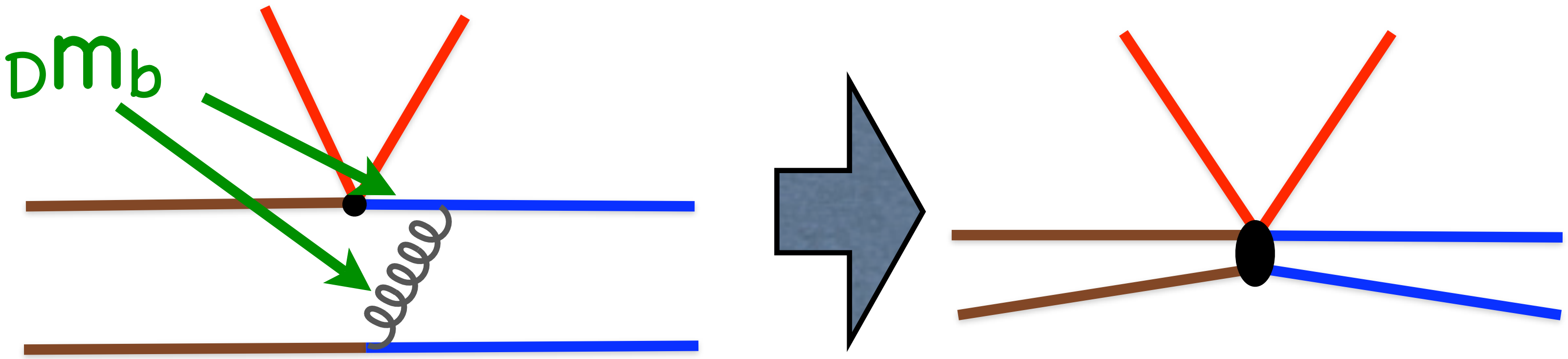
↓

intermediate-scale

↓

hadronic-scale

} treated as hadronic parameters



Factorization at $\sqrt{E\Lambda}$ is factorization of form factors

Beneke, Feldmann; Bauer, Pirjol, I.S.
Becher, Hill, Lange, Neubert

expansion in $\alpha_s(\sqrt{m_b\Lambda}) \simeq 0.35$

$$\zeta_J^{BM}(z) = f_M f_B \int_0^1 dx \int_0^\infty dk^+ J(z,x,k^+,E)\phi_M(x)\phi_B(k^+)$$

tree level $\zeta_J^{BM} = \int dz\, \zeta_J^{BM}(z) = 4\pi\alpha_s(\mu_i)\frac{f_B f_M}{m_b}\frac{\langle x^{-1}\rangle_{\phi_M}}{3}\frac{\langle k_+^{-1}\rangle_{\phi_B^+}}{3} > 0$

sign expectations can be used to remove discrete ambiguities in isospin analysis (eg. Buchalla, Safir; Lunghi et.al.)

$\zeta^{BM} = ?$ has endpoint singularities

-- BBNS: left as a form factor with counting $\zeta_J^{BM}/\zeta^{BM} \sim \alpha_s$

-- BPRS: left as a form factor, but counting is $\zeta_J^{BM}/\zeta^{BM} \sim 1$

-- in pQCD use $k_\perp$ dependence to factorize without singularities, get $\phi(x, k_\perp)$'s $\zeta_J^{BM}/\zeta^{BM} \sim 1$

Keum, Li, Sanda

Differences between Phenomenological Approaches to Applying Factorization

Fit Parameters:

	no expn.	SU(2)	SU(3)	BPRS/ SCET +SU(2)	BPRS/ SCET +SU(3)
$B \to \pi\pi$	11	7/5	15/13	4	4
$B \to K\pi$	15	11		+5(6)	
$B \to K\bar{K}$	11	11	+4/0	+3(4)	+0

Input Parameters:

BBNS: input model for $\phi_M(x)$, $\phi_B(k^+)$, ζ^{BM}

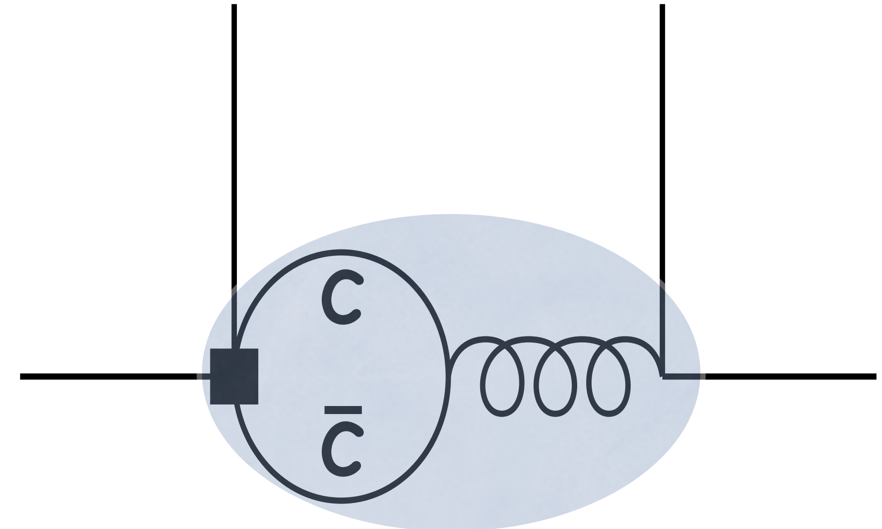
(use eg. light-cone sum rules for gegenbauer moments)

KLS: model wavefunctions

when pert. corrections are included, BPRS models shapes

Charm Loops

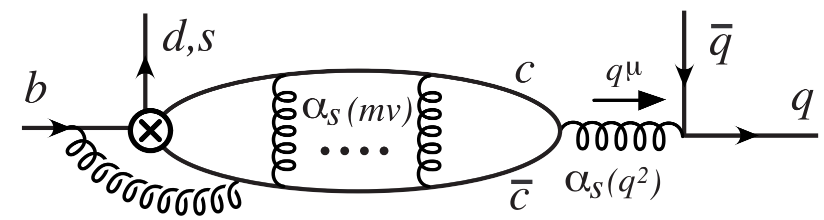
charm loops



BBNS	perturbative
KLS	perturbative
BPRS	nonperturbative fit parameters, may have large strong phases

as in: Ciuchini et al (charming penguin), Colangelo et al

charm is relatively heavy and may be more sensitive to nonperturbative effects



Recent work:

Beneke et.al. 0902.4446

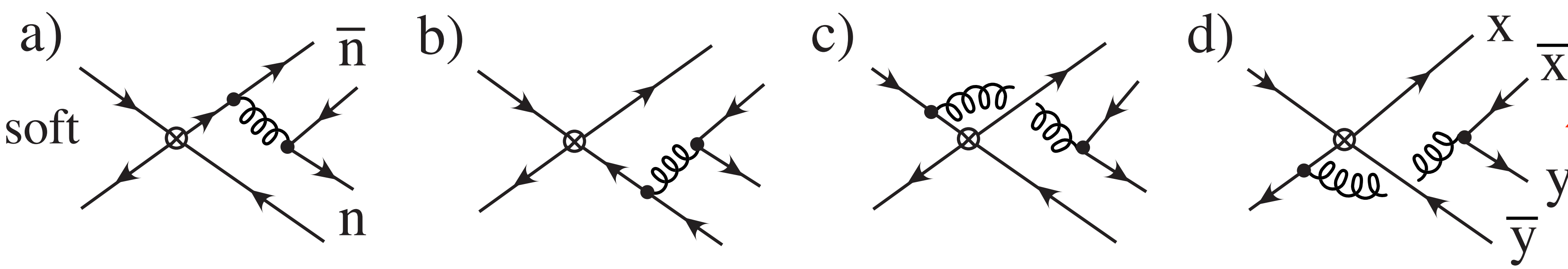
argue that duality violation in

$$B \to X_s \ell^+ \ell$$

does not apply for nonleptonics.

(Smearing argument assumes factorization.)

Endpoint Singularities   eg. annihilation



singular
$\bar{x} \to 0$

$$\int_0^1 dx \, \frac{\phi_\pi(x)}{\bar{x}^2}$$

pQCD: $$\frac{1}{m_b^2 \bar{x} - k_T^2 + i0}$$

singularity regulated by k_T

BBNS: Introduce hadronic parameters $\int_0^1 dx/x \to X_A$

$$X_A = (1 + \rho_A e^{i\phi_A}) \ln(m_B/500\,\mathrm{MeV})$$

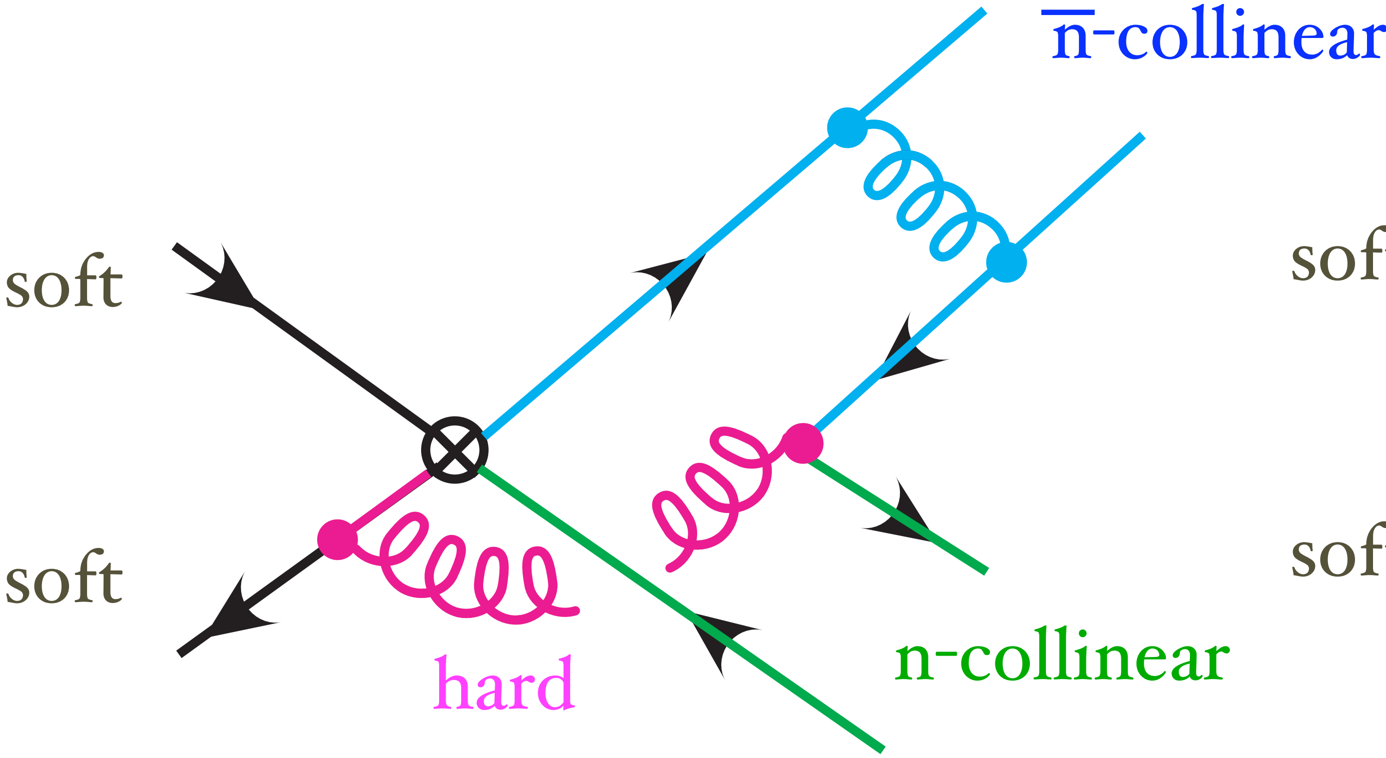
SCET:

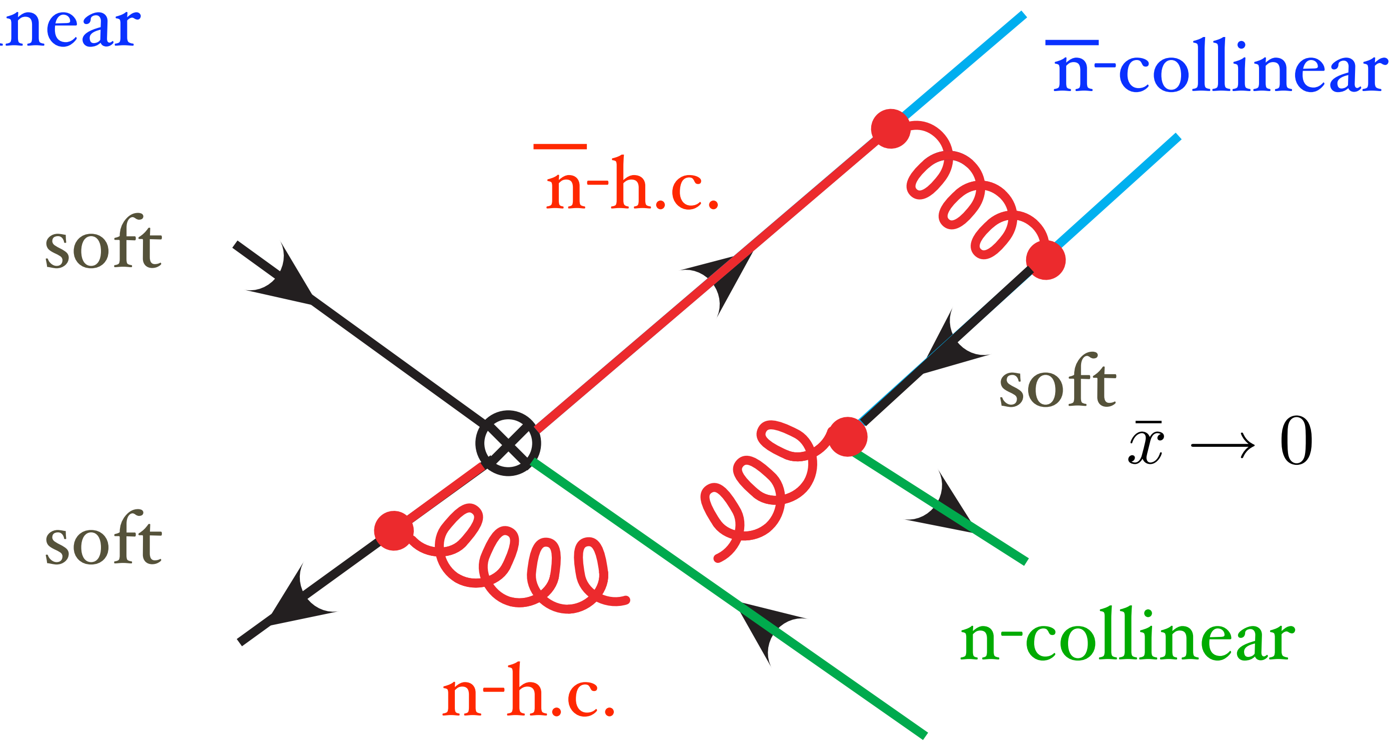
The annihilation singularity has to do with a potential double counting

Arnesen et.al.

Same QCD topology appears twice.

In SCET a rapidity cutoff is needed to distinguish these two terms (and zero bin subtractions)





This hard scattering term is real.

This soft rescattering term is complex.

Naive counting:

$$\sim \alpha_s(m_b)\,\frac{\Lambda}{m_b}$$

$$\sim \alpha_s^2(\sqrt{m\Lambda})\,\frac{\Lambda}{m_b}$$

conclude: "LO Annihilation is real"

Proper: the two graphs are factored at a high scale where all alphas' are equal. To determine the dominance one needs an RGE (which has not been derived for these rapidity cutoff amplitudes).

Comparison Summary

	BPRS	BBNS	KLS
Expansion in $\alpha_s(\mu_i)$?	No	Yes	Yes
T, P if Singular convolution	N/A	New parameters	uses k_T
Annihilation	Real at "LO", complex "NLO"	Complex, new parameters	perturbative, large phases
Charm Loop?	Non-perturbative	Perturbative	Perturbative
Number of fit parameters	Most	Middle	N/A

A few Applications

Counting parameters VP, VV modes

	no expn.	SU(2)	SU(3)	BPRS/ SCET +SU(2)	BPRS/ SCET +SU(3)
$B \to \pi\pi$	11	7/5	15/13	4	4
$B \to K\pi$	15	11		+5(6)	
$B \to K\bar{K}$	11	11	+4/0	+3(4)	+0

PP, PV with isosinglets

Wang, Wang, Yang, Lu (arXiv:0801.3123)

$\pi\eta, \eta\eta, K\eta', \ldots$	$+4$
$\rho\pi, \omega\pi, K^* K, \rho\eta, \ldots$	$+8$

Global Fit (2 solutions)

Comparison with pQCD and QCDF

Branching Ratios

Wang et.al.

Channel	Exp.	QCDF	PQCD	This work 1	This work 2
$B^- \to \rho^-\pi^0$	$10.9^{+1.4}_{-1.5}$	$14.0^{+6.5+5.1+1.0+0.8}_{-5.5-4.3-0.6-0.7}$	6-9	$8.8^{+0.2+1.0}_{-0.1-1.0}$	$11.0^{+0.6+1.0}_{-0.6-0.9}$
$B^- \to \rho^0\pi^-$	$8.7^{+1.0}_{-1.1}$	$11.9^{+6.3+3.6+2.5+1.3}_{-5.0-3.1-1.2-1.1}$	$10.4^{+3.3}_{-3.4} \pm 2.1$	$10.8^{+0.7+1.0}_{-0.7-0.9}$	$7.9^{+0.1+0.8}_{-0.0-0.8}$
$B^- \to \omega\pi^-$	6.9 ± 0.5	$8.8^{+4.4+2.6+1.8+0.8}_{-3.5-2.2-0.9-0.9}$	$11.3^{+3.3}_{-2.9} \pm 1.4$	$6.7^{+0.4+0.7}_{-0.3-0.6}$	$8.6^{+0.4+0.8}_{-0.3-0.8}$
$B^- \to K^{*0}K^-$	< 1.1	$0.30^{+0.11+0.12+0.09+0.57}_{-0.09-0.10-0.09-0.19}$	$0.31^{+0.12}_{-0.08}$	$0.48^{+0.25+0.09}_{-0.20-0.08}$	$0.51^{+0.18+0.07}_{-0.15-0.06}$
$B^- \to K^{*-}K^0$		$0.30^{+0.08+0.41+0.08+0.58}_{-0.07-0.18-0.07-0.17}$	$1.83^{+0.68}_{-0.47}$	$0.54^{+0.26+0.10}_{-0.21-0.08}$	$0.51^{+0.21+0.08}_{-0.17-0.07}$
$B^- \to \phi\pi^-$	< 0.24	≈ 0.005		≈ 0.003	0.003
$\bar{B}^0 \to \rho^-\pi^+$, $\bar{B}^0 \to \rho^+\pi^-$ }	24.0 ± 2.5	$36.5^{+18.2+10.3+2.0+3.9}_{-14.7-8.6-3.5-2.9}$	18-45	$13.1^{+0.6+1.2}_{-0.5-1.2}$	$16.8^{+0.5+1.6}_{-0.4-1.5}$
$B^0/\bar{B}^0 \to \rho^+\pi^-$			24-34	$12.5^{+1.9+1.2}_{-1.7-1.1}$	$16.0^{+1.6+1.5}_{-1.5-1.4}$
$B^0/\bar{B}^0 \to \rho^-\pi^+$			24-34	$13.8^{+1.9+1.3}_{-1.8-1.2}$	$17.7^{+1.6+1.6}_{-1.7-1.5}$
$\bar{B}^0 \to \rho^+\pi^-$ [a]	8.9 ± 2.5	$15.4^{+8.0+5.5+0.7+1.9}_{-6.4-4.7-1.3-1.3}$		$5.7^{+0.5+0.5}_{-0.5-0.5}$	$6.7^{+0.2+0.7}_{-0.1-0.7}$
$\bar{B}^0 \to \rho^-\pi^+$ [a]	13.9 ± 2.7	$21.2^{+10.3+8.7+1.3+2.0}_{-8.4-7.2-2.3-1.6}$		$7.4^{+0.2+0.8}_{-0.1-0.8}$	$10.1^{+0.4+0.9}_{-0.4-0.9}$
$\bar{B}^0 \to \rho^0\pi^0$	$1.8^{+0.6}_{-0.5}$	$0.4^{+0.2+0.2+0.9+0.5}_{-0.2-0.1-0.3-0.3}$	0.07-0.11	$2.6^{+0.2+0.2}_{-0.1-0.2}$	$1.4^{+0.1+0.1}_{-0.1-0.1}$
$\bar{B}^0 \to \omega\pi^0$	< 1.2	$0.01^{+0.00+0.02+0.02+0.03}_{-0.00-0.00-0.00-0.00}$	0.10-0.28	$0.003^{+0.047+0.000}_{-0.000-0.000}$	$0.025^{+0.036+0.002}_{-0.004-0.002}$
$\bar{B}^0 \to K^{*0}\bar{K}^0$		$0.26^{+0.08+0.10+0.08+0.46}_{-0.07-0.09-0.08-0.15}$		$0.45^{+0.24+0.09}_{-0.19-0.07}$	$0.47^{+0.17+0.06}_{-0.14-0.05}$
$\bar{B}^0 \to \bar{K}^{*0}K^0$	< 1.9	$0.29^{+0.10+0.39+0.08+0.60}_{-0.09-0.17-0.07-0.17}$		$0.51^{+0.24+0.09}_{-0.20-0.08}$	$0.48^{+0.20+0.07}_{-0.16-0.06}$
$\bar{B}^0 \to K^{*0}\bar{K}^0$, $\bar{B}^0 \to \bar{K}^{*0}K^0$ }			≈ 1.96	$0.96^{+0.34+0.18}_{-0.27-0.15}$	$0.95^{+0.26+0.14}_{-0.22-0.12}$
$B^0/\bar{B}^0 \to K^{*0}\bar{K}^0$				$0.96^{+0.34+0.18}_{-0.27-0.15}$	$0.95^{+0.26+0.14}_{-0.22-0.12}$
$B^0/\bar{B}^0 \to \bar{K}^{*0}K^0$				$0.96^{+0.34+0.18}_{-0.27-0.15}$	$0.95^{+0.26+0.14}_{-0.22-0.12}$
$\bar{B}^0 \to \phi\pi^0$	< 0.28	≈ 0.002		0.002	0.001
$B^- \to \rho^-\eta$	5.4 ± 1.2	$9.4^{+4.6+3.6+0.7+0.7}_{-3.7-3.0-0.4-0.7}$	$8.5^{+3.0+0.8+0.4+1.2}_{-2.1-0.7-0.4-0.2}$ [b]	$3.9^{+2.0+0.4}_{-1.7-0.4}$	$3.0^{+1.8+0.3}_{-1.5-0.3}$
$B^- \to \rho^-\eta'$	$9.1^{+3.7}_{-2.8}$	$6.3^{+3.1+2.4+0.5+0.5}_{-2.5-2.0-0.3-0.5}$	$8.7^{+3.0+0.7+0.5+1.1}_{-2.2-0.9-0.7-0.3}$ [b]	$0.37^{+2.51+0.08}_{-0.22-0.07}$	$0.36^{+2.59+0.06}_{-0.18-0.05}$
$\bar{B}^0 \to \rho^0\eta$	< 1.5	$0.03^{+0.02+0.16+0.02+0.05}_{-0.01-0.10-0.01-0.02}$	$0.024^{+0.012+0.004+0.002+0.102}_{-0.007-0.002-0.002-0.005}$ [b]	$0.03^{+0.18+0.00}_{-0.02-0.00}$	$0.17^{+0.36+0.02}_{-0.16-0.02}$
$\bar{B}^0 \to \rho^0\eta'$	< 1.3	$0.01^{+0.01+0.11+0.02+0.03}_{-0.00-0.06-0.00-0.01}$	$0.061^{+0.030+0.004+0.003+0.114}_{-0.018-0.003-0.003-0.009}$ [b]	$0.37^{+2.37+0.04}_{-0.11-0.05}$	$1.3^{+3.8+0.1}_{-1.1-0.1}$
$\bar{B}^0 \to \omega\eta$	< 1.9	$0.31^{+0.14+0.16+0.35+0.22}_{-0.12-0.11-0.14-0.16}$	$0.27^{+0.11}_{-0.10}$	$0.98^{+0.69+0.10}_{-0.51-0.10}$	$1.3^{+0.8+0.1}_{-0.6-0.1}$
$\bar{B}^0 \to \omega\eta'$	< 2.2	$0.20^{+0.10+0.15+0.25+0.15}_{-0.08-0.05-0.10-0.11}$	$0.075^{+0.037}_{-0.033}$	$0.20^{+1.46+0.04}_{-0.09-0.03}$	$3.1^{+4.8+0.3}_{-2.6-0.3}$
$\bar{B}^0 \to \phi\eta$	< 0.6	≈ 0.001	$0.0063^{+0.0033}_{-0.0019}$	0.0004	0.0008
$\bar{B}^0 \to \phi\eta'$	< 0.5	≈ 0.001	$0.0073^{+0.0035}_{-0.0026}$	0.0001	0.0007

Branching Ratios

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Channel	Exp.	QCDF	PQCD	This work 1	This work 2
$B^- \to K^{*-}\pi^0$	6.9 ± 2.3	$3.3^{+1.1+1.0+0.6+4.4}_{-1.0-0.9-0.6-1.4}$	$4.3^{+5.0}_{-2.2}$	$4.1^{+2.2+0.8}_{-1.7-0.7}$	$6.5^{+1.9+0.7}_{-1.6-0.7}$
$B^- \to \bar{K}^{*0}\pi^-$	10.7 ± 0.8	$3.6^{+0.4+1.5+1.2+7.7}_{-0.3-1.4-1.2-2.3}$	$6.0^{+2.8}_{-1.5}$	$8.5^{+4.6+1.7}_{-3.6-1.4}$	$9.9^{+3.4+1.3}_{-2.9-1.1}$
$B^- \to \rho^0 K^-$	$4.25^{+0.55}_{-0.56}$	$2.6^{+0.9+3.1+0.8+4.3}_{-0.9-1.4-0.6-1.2}$	$5.1^{+4.1}_{-2.8}$	$6.6^{+2.7+1.0}_{-2.2-0.9}$	$4.7^{+1.8+0.7}_{-1.5-0.6}$
$B^- \to \rho^- \bar{K}^0$	$8.0^{+1.5}_{-1.4}$	$5.8^{+0.6+7.0+1.5+10.3}_{-0.6-3.3-1.3-3.2}$	$8.7^{+6.8}_{-4.4}$	$9.3^{+4.7+1.7}_{-3.7-1.4}$	$10.0^{+4.0+1.5}_{-3.3-1.3}$
$B^- \to \omega K^-$	6.7 ± 0.5	$3.5^{+1.0+3.3+1.4+4.7}_{-1.0-1.6-0.9-1.6}$	$10.6^{+10.4}_{-5.8}$	$5.1^{+2.4+0.9}_{-1.9-0.8}$	$5.9^{+2.1+0.8}_{-1.7-0.7}$
$B^- \to \phi K^-$	8.30 ± 0.65	$4.5^{+0.5+1.8+1.9+11.8}_{-0.4-1.7-2.1-3.3}$	$7.8^{+5.9}_{-1.8}$	$9.7^{+4.9+1.8}_{-3.9-1.5}$	$8.5^{+3.2+1.2}_{-2.7-1.0}$
$\bar{B}^0 \to \bar{K}^{*0}\pi^0$	$0.0^{+1.3}_{-0.1}$	$0.7^{+0.1+0.5+0.3+2.6}_{-0.1-0.4-0.3-0.5}$	$2.0^{+1.2}_{-0.6}$	$4.6^{+2.3+0.9}_{-1.8-0.7}$	$3.6^{+1.4+0.5}_{-1.2-0.4}$
$\bar{B}^0 \to \bar{K}^{*-}\pi^+$	9.8 ± 1.1	$3.3^{+1.4+1.3+0.8+6.2}_{-1.1-1.2-0.8-1.6}$	$6.0^{+6.8}_{-2.6}$	$8.3^{+4.3+1.6}_{-3.4-1.3}$	$9.5^{+3.2+1.2}_{-2.7-1.1}$
$\bar{B}^0 \to \rho^0 \bar{K}^0$	$5.4^{+0.9}_{-1.0}$	$4.6^{+0.5+4.0+0.7+6.1}_{-0.5-2.1-0.7-2.1}$	$4.8^{+4.3}_{-2.3}$	$3.5^{+2.0+0.7}_{-1.5-0.6}$	$5.8^{+2.1+0.8}_{-1.8-0.7}$
$\bar{B}^0 \to \rho^+ K^-$	$15.3^{+3.7}_{-3.5}$	$7.4^{+1.8+7.1+1.2+10.7}_{-1.9-3.6-1.1-3.5}$	$8.8^{+6.8}_{-4.5}$	$9.8^{+4.5+1.7}_{-3.7-1.4}$	$10.2^{+3.8+1.5}_{-3.2-1.2}$
$\bar{B}^0 \to \omega \bar{K}^0$	5.0 ± 0.6	$2.3^{+0.3+2.8+1.3+4.3}_{-0.3-1.3-0.8-1.3}$	$9.8^{+8.6}_{-4.9}$	$4.1^{+2.1+0.8}_{-1.7-0.6}$	$4.9^{+1.9+0.7}_{-1.6-0.6}$
$\bar{B}^0 \to \phi \bar{K}^0$	$8.3^{+1.2}_{-1.0}$	$4.1^{+0.4+1.7+1.8+10.6}_{-0.4-1.6-1.9-3.0}$	$7.3^{+5.9}_{-1.8}$	$9.1^{+4.5+1.7}_{-3.6-1.4}$	$8.0^{+2.9+1.1}_{-2.5-0.9}$
$B^- \to K^{*-}\eta$	19.3 ± 1.6	$10.8^{+1.9+8.1+1.8+16.5}_{-1.7-4.4-1.3-5.5}$	$22.13^{+0.26}_{-0.27}$	$17.9^{+5.4+3.5}_{-5.3-2.9}$	$18.6^{+4.5+2.6}_{-4.6-2.2}$
$B^- \to K^{*-}\eta'$	$4.9^{+2.1}_{-1.9}$	$5.1^{+0.9+7.5+2.1+6.7}_{-1.0-3.8-3.0-3.3}$	6.38 ± 0.26	$4.4^{+6.5+0.9}_{-3.8-0.8}$	$4.1^{+4.9+0.7}_{-3.3-0.6}$
$\bar{B}^0 \to \bar{K}^{*0}\eta$	15.9 ± 1.0	$10.7^{+1.1+7.8+1.4+16.2}_{-1.0-4.3-1.2-5.5}$	$22.31^{+0.28}_{-0.29}$	$16.6^{+5.1+3.2}_{-5.0-2.7}$	$16.5^{+4.1+2.3}_{-4.2-2.0}$
$\bar{B}^0 \to \bar{K}^{*0}\eta'$	3.8 ± 1.2	$3.9^{+0.4+6.6+1.8+6.2}_{-0.4-3.3-2.5-2.9}$	$3.35^{+0.29}_{-0.27}$	$4.1^{+6.1+0.9}_{-3.6-0.7}$	$3.8^{+4.8+0.6}_{-3.3-0.5}$

Branching Ratios

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Channel	Exp.	QCDF	PQCD	This work 1	This work 2
$B^- \to K^{*-}\pi^0$	6.9 ± 2.3	$3.3^{+1.1+1.0+0.6+4.4}_{-1.0-0.9-0.6-1.4}$	$4.3^{+5.0}_{-2.2}$	$4.1^{+2.2+0.8}_{-1.7-0.7}$	$6.5^{+1.9+0.7}_{-1.6-0.7}$
$B^- \to \bar{K}^{*0}\pi^-$	10.7 ± 0.8	$3.6^{+0.4+1.5+1.2+7.7}_{-0.3-1.4-1.2-2.3}$	$6.0^{+2.8}_{-1.5}$	$8.5^{+4.6+1.7}_{-3.6-1.4}$	$9.9^{+3.4+1.3}_{-2.9-1.1}$
$B^- \to \rho^0 K^-$	$4.25^{+0.55}_{-0.56}$	$2.6^{+0.9+3.1+0.8+4.3}_{-0.9-1.4-0.6-1.2}$	$5.1^{+4.1}_{-2.8}$	$6.6^{+2.7+1.0}_{-2.2-0.9}$	$4.7^{+1.8+0.7}_{-1.5-0.6}$
$B^- \to \rho^- \bar{K}^0$	$8.0^{+1.5}_{-1.4}$	$5.8^{+0.6+7.0+1.5+10.3}_{-0.6-3.3-1.3-3.2}$	$8.7^{+6.8}_{-4.4}$	$9.3^{+4.7+1.7}_{-3.7-1.4}$	$10.0^{+4.0+1.5}_{-3.3-1.3}$
$B^- \to \omega K^-$	6.7 ± 0.5	$3.5^{+1.0+3.3+1.4+4.7}_{-1.0-1.6-0.9-1.6}$	$10.6^{+10.4}_{-5.8}$	$5.1^{+2.4+0.9}_{-1.9-0.8}$	$5.9^{+2.1+0.8}_{-1.7-0.7}$
$B^- \to \phi K^-$	8.30 ± 0.65	$4.5^{+0.5+1.8+1.9+11.8}_{-0.4-1.7-2.1-3.3}$	$7.8^{+5.9}_{-1.8}$	$9.7^{+4.9+1.8}_{-3.9-1.5}$	$8.5^{+3.2+1.2}_{-2.7-1.0}$
$\bar{B}^0 \to \bar{K}^{*0}\pi^0$	$0.0^{+1.3}_{-0.1}$	$0.7^{+0.1+0.5+0.3+2.6}_{-0.1-0.4-0.3-0.5}$	$2.0^{+1.2}_{-0.6}$	$4.6^{+2.3+0.9}_{-1.8-0.7}$	$3.6^{+1.4+0.5}_{-1.2-0.4}$
$\bar{B}^0 \to \bar{K}^{*-}\pi^+$	9.8 ± 1.1	$3.3^{+1.4+1.3+0.8+6.2}_{-1.1-1.2-0.8-1.6}$	$6.0^{+6.8}_{-2.6}$	$8.3^{+4.3+1.6}_{-3.4-1.3}$	$9.5^{+3.2+1.2}_{-2.7-1.1}$
$\bar{B}^0 \to \rho^0 \bar{K}^0$	$5.4^{+0.9}_{-1.0}$	$4.6^{+0.5+4.0+0.7+6.1}_{-0.5-2.1-0.7-2.1}$	$4.8^{+4.3}_{-2.3}$	$3.5^{+2.0+0.7}_{-1.5-0.6}$	$5.8^{+2.1+0.8}_{-1.8-0.7}$
$\bar{B}^0 \to \rho^+ K^-$	$15.3^{+3.7}_{-3.5}$	$7.4^{+1.8+7.1+1.2+10.7}_{-1.9-3.6-1.1-3.5}$	$8.8^{+6.8}_{-4.5}$	$9.8^{+4.5+1.7}_{-3.7-1.4}$	$10.2^{+3.8+1.5}_{-3.2-1.2}$
$\bar{B}^0 \to \omega \bar{K}^0$	5.0 ± 0.6	$2.3^{+0.3+2.8+1.3+4.3}_{-0.3-1.3-0.8-1.3}$	$9.8^{+8.6}_{-4.9}$	$4.1^{+2.1+0.8}_{-1.7-0.6}$	$4.9^{+1.9+0.7}_{-1.6-0.6}$
$\bar{B}^0 \to \phi \bar{K}^0$	$8.3^{+1.2}_{-1.0}$	$4.1^{+0.4+1.7+1.8+10.6}_{-0.4-1.6-1.9-3.0}$	$7.3^{+5.9}_{-1.8}$	$9.1^{+4.5+1.7}_{-3.6-1.4}$	$8.0^{+2.9+1.1}_{-2.5-0.9}$
$B^- \to K^{*-}\eta$	19.3 ± 1.6	$10.8^{+1.9+8.1+1.8+16.5}_{-1.7-4.4-1.3-5.5}$	$22.13^{+0.26}_{-0.27}$	$17.9^{+5.4+3.5}_{-5.3-2.9}$	$18.6^{+4.5+2.6}_{-4.6-2.2}$
$B^- \to K^{*-}\eta'$	$4.9^{+2.1}_{-1.9}$	$5.1^{+0.9+7.5+2.1+6.7}_{-1.0-3.8-3.0-3.3}$	6.38 ± 0.26	$4.4^{+6.5+0.9}_{-3.8-0.8}$	$4.1^{+4.9+0.7}_{-3.3-0.6}$
$\bar{B}^0 \to \bar{K}^{*0}\eta$	15.9 ± 1.0	$10.7^{+1.1+7.8+1.4+16.2}_{-1.0-4.3-1.2-5.5}$	$22.31^{+0.28}_{-0.29}$	$16.6^{+5.1+3.2}_{-5.0-2.7}$	$16.5^{+4.1+2.3}_{-4.2-2.0}$
$\bar{B}^0 \to \bar{K}^{*0}\eta'$	3.8 ± 1.2	$3.9^{+0.4+6.6+1.8+6.2}_{-0.4-3.3-2.5-2.9}$	$3.35^{+0.29}_{-0.27}$	$4.1^{+6.1+0.9}_{-3.6-0.7}$	$3.8^{+4.8+0.6}_{-3.3-0.5}$

CP Asymmetries

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Channel	Exp.	QCDF	PQCD	This work 1	This work 2
$B^- \to \rho^-\pi^0$	2 ± 11	$-4.0^{+1.2+1.8+0.4+17.5}_{-1.2-2.2-0.4-17.7}$	0-20	$8.3^{+17.8+0.8}_{-18.9-0.8}$	$5.4^{+9.7+0.4}_{-10.0-0.5}$
$B^- \to \rho^0\pi^-$	-7^{+12}_{-13}	$4.1^{+1.3+2.2+0.6+19.0}_{-0.9-2.0-0.7-18.8}$	-20-0	$-5.7^{+13.0+0.5}_{-12.8-0.4}$	$-8.4^{+15.6+0.8}_{-14.5-0.8}$
$B^- \to \omega\pi^-$	-4 ± 6	$-1.8^{+0.5+2.7+0.8+2.1}_{-0.5-3.3-0.7-2.2}$	~ 0	$-5.0^{+19.7+0.5}_{-19.3-0.5}$	$-5.8^{+13.7+0.5}_{-12.9-0.6}$
$B^- \to K^{*0}K^-$	...	$-23.5^{+6.9+7.8+5.5+25.2}_{-5.7-9.0-6.5-36.8}$	$-20\pm 5\pm 2$	$-0.8^{+5.8+0.1}_{-5.6-0.1}$	$-0.4^{+4.1+0.0}_{-4.1-0.0}$
$B^- \to K^{*-}K^0$	...	$-13.4^{+3.7+7.8+4.2+27.4}_{-3.0-3.5-4.7-36.7}$	-49^{+7+7}_{-3-7}	$-1.3^{+2.6+0.1}_{-2.4-0.1}$	$-1.1^{+1.7+0.1}_{-1.6-0.1}$
$\bar{B}^0 \to \rho^+\pi^-$	-53 ± 30	$0.6^{+0.2+1.3+0.1+11.5}_{-0.1-1.6-0.1-11.7}$		$-8.6^{+17.4+0.8}_{-17.0-0.6}$	$-11.0^{+17.4+1.0}_{-15.3-1.1}$
$\bar{B}^0 \to \rho^-\pi^+$	-15 ± 8	$-1.5^{+0.4+1.2+0.2+8.5}_{-0.4-1.3-0.3-8.4}$		$2.6^{+19.1+0.3}_{-19.7-0.2}$	$0.9^{+10.0+0.1}_{-10.1-0.1}$
$\bar{B}^0 \to \rho^0\pi^0$	-30 ± 38	$-15.7^{+4.8+12.3+11.0+19.8}_{-4.7-14.0-12.9-25.8}$	-75-0	$5.5^{+20.8+0.5}_{-21.8-0.5}$	$9.7^{+21.5+0.9}_{-22.5-0.9}$
$\bar{B}^0 \to \omega\pi^0$	...	...	-20-75	$-58.4^{+150.1+4.2}_{-0.0-4.1}$	$-72.9^{+179.1+4.7}_{-32.9-4.8}$
$\bar{B}^0 \to K^{*0}\bar{K}^0$	...	$-26.7^{+7.4+7.2+5.7+10.9}_{-5.7-9.0-6.9-13.4}$		$-0.8^{+5.8+0.1}_{-5.6-0.1}$	$-0.4^{+4.1+0.0}_{-4.1-0.0}$
$\bar{B}^0 \to \bar{K}^{*0}K^0$	...	$-13.1^{+3.8+5.4+4.5+5.8}_{-3.0-2.9-5.2-7.4}$		$-1.3^{+2.6+0.1}_{-2.4-0.1}$	$-1.1^{+1.7+0.1}_{-1.6-0.1}$
$B^- \to \rho^-\eta$	1 ± 16	$-2.4^{+0.7+6.3+0.4+0.2}_{-0.7-6.3-0.4-0.2}$	$-13^{+1.2+2}_{-0.5-14}$	$-11.7^{+22.0+1.1}_{-21.0-1.2}$	$9.1^{+17.7+0.9}_{-17.3-0.9}$
$B^- \to \rho^-\eta'$	-4 ± 28	$4.1^{+1.2+7.9+0.5+7.0}_{-1.1-6.9-0.8-7.0}$	$-18^{+3.0+1}_{-1.6-14}$	$-18.0^{+65.9+2.6}_{-44.1-2.9}$	$6.6^{+66.6+0.8}_{-119.9-0.9}$
$\bar{B}^0 \to \rho^0\eta$	...	...	$-13^{+1.2+2}_{-0.5-14}$	$-76.0^{+189.5+2.9}_{-33.3-4.5}$	$-28.2^{+55.0+2.4}_{-76.6-2.6}$
$\bar{B}^0 \to \rho^0\eta'$	...	...	$-18^{+3.0+1}_{-1.6-14}$	$-59.5^{+112.2+3.4}_{-40.1-4.2}$	$-57.5^{+68.6+4.4}_{-16.1-4.6}$
$\bar{B}^0 \to \omega\eta$	...	$-33.4^{+10.0+65.3+20.9+19.2}_{-\ 9.5-55.8-21.4-20.8}$	$-69.1^{+15.1}_{-13.4}$	$-16.1^{+30.2+1.5}_{-28.7-1.6}$	$9.5^{+18.3+0.9}_{-18.0-0.9}$
$\bar{B}^0 \to \omega\eta'$	...	$0.2^{+0.1+53.0+11.6+19.2}_{-0.1-76.5-11.5-20.1}$	$13.9^{+4.1}_{-3.5}$	$-55.4^{+104.1+4.9}_{-37.0-5.5}$	$35.6^{+38.9+2.9}_{-19.7-3.0}$

CP Asymmetries

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Channel	Exp.	QCDF	PQCD	This work 1	This work 2
$B^- \to K^{*-}\pi^0$	4 ± 29	$8.7^{+2.1+5.0+2.9+41.7}_{-2.6-4.3-3.4-44.2}$	-32^{+21}_{-28}	$-4.0^{+29.2+0.5}_{-27.8-0.5}$	$-1.1^{+11.8+0.1}_{-11.8-0.1}$
$B^- \to \bar{K}^{*0}\pi^-$	-8.5 ± 5.7	$1.6^{+0.4+0.6+0.5+2.5}_{-0.5-0.5-0.4-1.0}$	-1^{+1}_{-0}	0	0
$B^- \to \rho^0 K^-$	31^{+11}_{-10}	$-13.6^{+4.5+6.9+3.7+62.7}_{-5.7-4.4-3.1-55.4}$	71^{+25}_{-35}	$8.0^{+15.4+0.6}_{-16.1-0.6}$	$14.3^{+20.8+1.1}_{-22.5-1.4}$
$B^- \to \rho^- \bar{K}^0$	-12 ± 17	$0.3^{+0.1+0.3+0.2+1.6}_{-0.1-0.4-0.1-1.3}$	1 ± 1	0	0
$B^- \to \omega K^-$	2 ± 5	$-7.8^{+2.6+5.9+2.4+39.8}_{-3.0-3.6-1.9-38.0}$	32^{+15}_{-17}	$10.1^{+18.5+1.0}_{-20.5-0.9}$	$11.1^{+16.8+0.8}_{-17.3-1.0}$
$B^- \to \phi K^-$	3.4 ± 4.4	$1.6^{+0.4+0.6+0.5+3.0}_{-0.5-0.5-0.3-1.2}$	1^{+0}_{-1}	0	0
$\bar{B}^0 \to \bar{K}^{*0}\pi^0$	$\cdots$	$-12.8^{+4.0+4.7+2.7+31.7}_{-3.2-7.0-4.0-35.3}$	-11^{+7}_{-5}	$1.1^{+8.0+0.1}_{-8.3-0.1}$	$0.4^{+4.8+0.0}_{-4.8-0.0}$
$\bar{B}^0 \to \bar{K}^{*-}\pi^+$	-5 ± 14	$2.1^{+0.6+8.2+5.1+62.5}_{-0.7-7.9-5.8-64.2}$	-60^{+32}_{-19}	$-2.5^{+18.5+0.3}_{-17.8-0.3}$	$-1.0^{+11.4+0.1}_{-11.4-0.1}$
$\bar{B}^0 \to \rho^0 \bar{K}^0$	$-2 \pm 27 \pm 8 \pm 6$	$7.5^{+1.7+2.3+0.7+8.8}_{-2.1-2.0-0.4-8.7}$	7^{+8}_{-5}	$-5.9^{+11.9+0.7}_{-10.1-0.8}$	$-3.1^{+4.9+0.2}_{-4.8-0.2}$
$\bar{B}^0 \to \rho^+ K^-$	22 ± 23	$-3.8^{+1.3+4.4+1.9+34.5}_{-1.4-2.7-1.6-32.7}$	64^{+24}_{-30}	$6.0^{+11.1+0.6}_{-12.1-0.6}$	$8.7^{+13.1+0.6}_{-13.6-0.8}$
$\bar{B}^0 \to \omega \bar{K}^0$	21 ± 19	$-8.1^{+2.5+3.0+1.7+11.8}_{-2.0-3.3-1.4-12.9}$	-3^{+2}_{-3}	$4.7^{+8.4+0.5}_{-9.5-0.5}$	$3.4^{+5.2+0.3}_{-5.4-0.3}$
$\bar{B}^0 \to \phi \bar{K}^0$	1 ± 12	$1.7^{+0.4+0.6+0.5+1.4}_{-0.5-0.5-0.3-0.8}$	3^{+1}_{-2}	0	0
$B^- \to K^{*-}\eta$	2 ± 6	$3.5^{+0.9+1.9+0.8+20.7}_{-0.9-2.7-0.8-20.5}$	$-24.57^{+0.72}_{-0.27}$	$-0.9^{+5.3+0.1}_{-5.5-0.1}$	$-4.6^{+3.4+0.3}_{-3.4-0.3}$
$B^- \to K^{*-}\eta'$	30^{+33}_{-37}	$-14.2^{+4.7+8.5+4.9+27.5}_{-4.2-13.8-14.6-26.1}$	$4.60^{+1.16}_{-1.32}$	$2.6^{+29.1+0.3}_{-20.9-0.3}$	$-0.7^{+36.4+0.1}_{-34.5-0.1}$
$\bar{B}^0 \to \bar{K}^{*0}\eta$	19 ± 5	$3.8^{+0.9+1.1+0.2+3.8}_{-1.1-0.8-0.2-3.5}$	0.57 ± 0.011	$-0.4^{+2.3+0.0}_{-2.4-0.0}$	$-1.6^{+1.1+0.1}_{-1.1-0.1}$
$\bar{B}^0 \to \bar{K}^{*0}\eta'$	-8 ± 25	$-5.5^{+1.6+3.1+1.8+6.2}_{-1.3-5.1-5.9-7.0}$	-1.30 ± 0.08	$10.2^{+8.7+1.3}_{-10.3-1.3}$	$-9.8^{+4.5+0.9}_{-6.4-0.9}$

B_s Decays

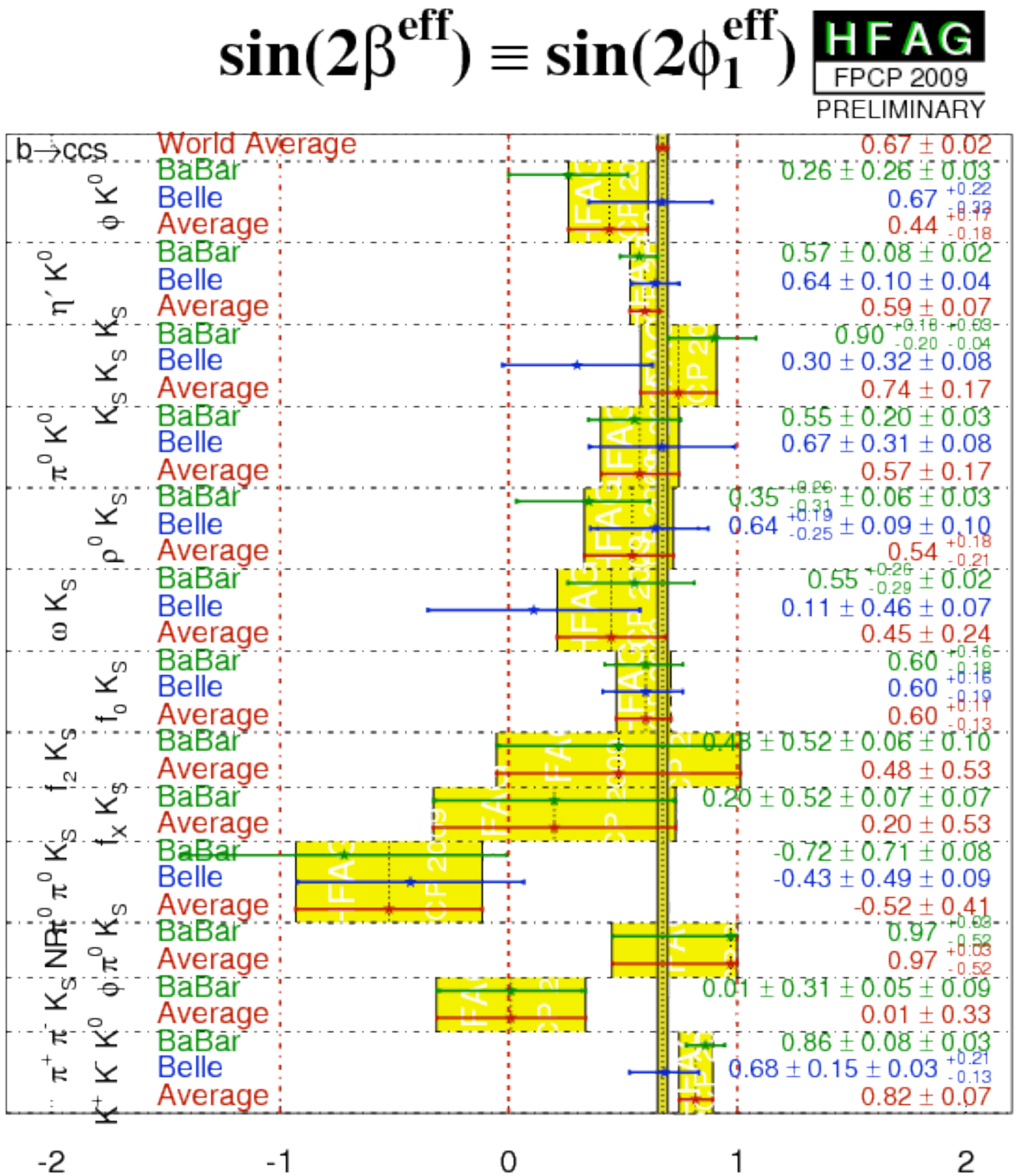
Branching Ratios

Wang et.al.

Modes	QCDF	PQCD	This work 1	This work 2
$\bar{B}_s^0 \to K^+ K^{*-}$	$4.1^{+1.7+1.5+1.0+9.2}_{-1.5-1.3-0.9-2.3}$	$6.0^{+1.7+1.7+0.7}_{-1.5-1.2-0.3}$	$8.3^{+4.3+1.6}_{-3.4-1.3}$	$9.5^{+3.2+1.2}_{-2.7-1.1}$
$\bar{B}_s^0 \to K^{*+} K^-$	$5.5^{+1.3+5.0+0.8+14.2}_{-1.4-2.6-0.7-\ 3.6}$	$4.7^{+1.1+2.5+0.0}_{-0.8-1.4-0.0}$	$9.8^{+4.6+1.7}_{-3.7-1.4}$	$10.3^{+3.8+1.5}_{-3.2-1.2}$
$\bar{B}_s^0 \to K^0 \overline{K}^{*0}$	$3.9^{+0.4+1.5+1.3+10.4}_{-0.4-1.4-1.4-\ 2.8}$	$7.3^{+2.5+2.1+0.0}_{-1.7-1.3-0.0}$	$7.9^{+4.3+1.6}_{-3.4-1.3}$	$9.3^{+3.2+1.2}_{-2.7-1.0}$
$\bar{B}_s^0 \to K^{*0} \overline{K}^0$	$4.2^{+0.4+4.6+1.1+13.2}_{-0.4-2.2-0.9-\ 3.2}$	$4.3^{+0.7+2.2+0.0}_{-0.7-1.4-0.0}$	$8.7^{+4.4+1.6}_{-3.5-1.3}$	$9.3^{+3.7+1.4}_{-3.1-1.2}$
$B_s^0/\bar{B}_s^0 \to K^+ K^{*-}$			$17.3^{+6.5+3.2}_{-5.1-2.7}$	$18.8^{+5.1+2.5}_{-4.5-2.2}$
$B_s^0/\bar{B}_s^0 \to K^{*+} K^-$			$18.8^{+6.8+3.3}_{-5.4-2.8}$	$20.8^{+5.3+2.7}_{-4.7-2.3}$
$\bar{B}_s^0 \to K^{*+} K^-$, $\bar{B}_s^0 \to K^{*-} K^+$			$18.1^{+6.3+3.3}_{-5.0-2.7}$	$19.8^{+4.9+2.6}_{-4.2-2.2}$
$B_s^0/\bar{B}_s^0 \to K^0 \overline{K}^{*0}$			$16.6^{+6.1+3.2}_{-4.8-2.7}$	$18.6^{+4.9+2.6}_{-4.1-2.2}$
$B_s^0/\bar{B}_s^0 \to K^{*0} \overline{K}^0$			$16.6^{+6.1+3.2}_{-4.8-2.7}$	$18.6^{+4.9+2.6}_{-4.1-2.2}$
$\bar{B}_s^0 \to K^{*0} \bar{K}^0$, $\bar{B}_s^0 \to \bar{K}^{*0} K^0$			$16.6^{+6.1+3.2}_{-4.8-2.7}$	$18.6^{+4.9+2.6}_{-4.1-2.2}$
$\bar{B}_s^0 \to \pi^0 \phi$	$0.12^{+0.03+0.04+0.01+0.02}_{-0.02-0.04-0.01-0.01}$	$0.16^{+0.06+0.02+0.00}_{-0.05-0.02-0.00}$	$0.07^{+0.00+0.01}_{-0.00-0.01}$	$0.09^{+0.00+0.01}_{-0.00-0.01}$
$\bar{B}_s^0 \to \pi^- K^{*+}$	$8.7^{+4.6+3.5+0.7+0.8}_{-3.7-2.9-1.0-0.7}$	$7.6^{+2.9+0.4+0.5}_{-2.2-0.5-0.3}$	$5.8^{+0.5+0.5}_{-0.5-0.5}$	$6.8^{+0.2+0.7}_{-0.1-0.7}$
$\bar{B}_s^0 \to \pi^0 K^{*0}$	$0.25^{+0.08+0.10+0.32+0.30}_{-0.08-0.06-0.14-0.14}$	$0.07^{+0.02+0.04+0.01}_{-0.01-0.02-0.01}$	$0.90^{+0.07+0.10}_{-0.00-0.11}$	$0.99^{+0.16+0.10}_{-0.15-0.08}$
$\bar{B}_s^0 \to \rho^- K^+$	$24.5^{+11.9+9.2+1.8+1.6}_{-9.7-7.8-3.0-1.6}$	$17.8^{+7.7+1.3+1.1}_{-5.6-1.6-0.9}$	$7.4^{+0.2+0.8}_{-0.1-0.8}$	$10.1^{+0.4+0.9}_{-0.4-0.9}$
$\bar{B}_s^0 \to \rho^0 K^0$	$0.61^{+0.33+0.21+1.06+0.56}_{-0.26-0.15-0.38-0.36}$	$0.08^{+0.02+0.07+0.01}_{-0.02-0.03-0.00}$	$2.1^{+0.2+0.2}_{-0.2-0.2}$	$0.79^{+0.02+0.08}_{-0.00-0.09}$
$\bar{B}_s^0 \to K^0 \omega$	$0.51^{+0.20+0.15+0.68+0.40}_{-0.18-0.11-0.23-0.25}$	$0.15^{+0.05+0.07+0.02}_{-0.04-0.03-0.01}$	$0.94^{+0.05+0.10}_{-0.00-0.11}$	$1.3^{+0.1+0.1}_{-0.1-0.1}$
$\bar{B}_s^0 \to K^0 \phi$	$0.27^{+0.09+0.28+0.09+0.67}_{-0.08-0.14-0.06-0.18}$	$0.16^{+0.04+0.09+0.02}_{-0.03-0.04-0.01}$	$0.44^{+0.23+0.08}_{-0.18-0.07}$	$0.54^{+0.21+0.08}_{-0.17-0.07}$
$\bar{B}_s^0 \to \rho^0 \eta$	$0.17^{+0.03+0.07+0.02+0.02}_{-0.03-0.06-0.02-0.01}$	$0.06^{+0.03+0.01+0.00}_{-0.02-0.01-0.00}$	$0.08^{+0.04+0.01}_{-0.03-0.01}$	$0.06^{+0.03+0.00}_{-0.02-0.00}$
$\bar{B}_s^0 \to \rho^0 \eta'$	$0.25^{+0.06+0.10+0.02+0.02}_{-0.05-0.08-0.02-0.02}$	$0.13^{+0.06+0.02+0.00}_{-0.04-0.02-0.01}$	$0.003^{+0.089+0.000}_{-0.000-0.000}$	$0.15^{+0.24+0.02}_{-0.12-0.01}$
$\bar{B}_s^0 \to \omega \eta$	$0.012^{+0.005+0.010+0.028+0.025}_{-0.004-0.003-0.006-0.006}$	$0.04^{+0.03+0.05+0.00}_{-0.01-0.02-0.00}$	$0.04^{+0.04+0.00}_{-0.02-0.00}$	$0.007^{+0.010+0.001}_{-0.002-0.001}$
$\bar{B}_s^0 \to \omega \eta'$	$0.024^{+0.011+0.028+0.077+0.042}_{-0.009-0.006-0.010-0.015}$	$0.44^{+0.18+0.15+0.00}_{-0.13-0.14-0.01}$	$0.002^{+0.108+0.000}_{-0.000-0.000}$	$0.22^{+0.35+0.02}_{-0.18-0.02}$
$\bar{B}_s^0 \to \phi \eta$	$0.12^{+0.02+0.95+0.54+0.32}_{-0.02-0.14-0.12-0.13}$	$3.6^{+1.5+0.8+0.0}_{-1.0-0.6-0.0}$	$0.40^{+1.40+0.08}_{-0.51-0.07}$	$1.2^{+2.1+0.2}_{-1.2-0.2}$
$\bar{B}_s^0 \to \phi \eta'$	$0.05^{+0.01+1.10+0.18+0.40}_{-0.01-0.17-0.08-0.04}$	$0.19^{+0.06+0.19+0.00}_{-0.01-0.13-0.00}$	$7.7^{+7.8+1.6}_{-5.5-1.3}$	$4.2^{+5.2+0.7}_{-3.5-0.6}$
$\bar{B}_s^0 \to K^{*0} \eta$	$0.26^{+0.15+0.49+0.15+0.57}_{-0.13-0.22-0.05-0.15}$	$0.17^{+0.04+0.10+0.03}_{-0.04-0.06-0.01}$	$1.7^{+0.3+0.2}_{-0.3-0.1}$	$0.55^{+0.13+0.07}_{-0.12-0.07}$
$\bar{B}_s^0 \to K^{*0} \eta'$	$0.28^{+0.04+0.46+0.23+0.29}_{-0.04-0.24-0.10-0.15}$	$0.09^{+0.02+0.03+0.01}_{-0.02-0.02-0.01}$	$0.66^{+0.34+0.12}_{-0.26-0.11}$	$0.77^{+0.33+0.09}_{-0.30-0.08}$

$b \to c\bar{c}s$ vs. $b \to q\bar{q}s$

Theory Uncertainty (from factorization)



Beneke	Buchalla, Hiller Nir, Raz	Zupan, Williamson
$+0.02^{+0.01}_{-0.01}$	$+0.02$	
$+0.01^{+0.01}_{-0.01}$	$+0.01^{+0.01}_{-0.02}$	-0.01 ± 0.02
$+0.07^{+0.05}_{-0.04}$	$+0.06^{+0.04}_{-0.03}$	$+0.07 \pm 0.03$
$+0.13^{+0.08}_{-0.08}$	$+0.19^{+0.06}_{-0.14}$	

- Constructive interference of penguins give a large $Br(B \to \eta' K^0)$ (to agree with data), and simultaneously a small uncertainty above
- Determination of hadronic parameters dominates factorization uncertainties

What does a Penguin Amplitude look like if we try to compute it?

Beneke, Jager; Jain et.al.

theory: $\alpha_s \equiv \alpha_s(m_b)$

LO terms

$$\hat{P}_0 \sim \Big(C_{3,4}+\frac{\alpha_s(m_b)C_{1,2,8g}}{\pi}\Big)\zeta^{BM}\phi^{M'} + \Big(C_{3,4}+\frac{\alpha_s(m_b)C_{1,2,8g}}{\pi}\Big)\zeta_J^{BM}\phi^{M'}$$

$$+\, C_{1,2}\,\alpha_s(2m_c)v\hat{A}_{c\bar{c}}^{BMM'}$$

Non.Pert. Charm Penguin

Ciuchini et al, Colangelo et al

$$+\Big(C_{5,6}+\frac{\alpha_s(m_b)C_{1,2,8g}}{\pi}\Big)\Big[\frac{\mu_{M'}}{m_b}\,\zeta^{BM}\phi_{pp}^{M'} + \frac{\mu_{M'}}{m_b}\,\zeta_J^{BM}\phi_{pp}^{M'}\Big] + \Big(C_{3,4}+\frac{\alpha_s(m_b)C_{1,2,8g}}{\pi}\Big)\frac{\mu_M}{m_b}\zeta_\chi^{BM}\phi^{M'}$$

$$+\frac{\alpha_s(m_b)}{m_b}\Big(C_{3,4}f_B\phi^M\phi^{M'} + C_{5,6}f_B\phi_B^+\phi^{3M}\phi^{M'}\Big)$$

$$+C_{5,6}\frac{\alpha_s(m_b)\mu_M}{m_b^2}f_B\phi_{pp}^M\phi^{M'},$$

Chiral Enh. terms

BBNS

$$\hat{P}_{\pi\pi}^{\zeta_J} + \hat{P}_{\pi\pi}^{\chi\zeta_J}\Big|_{C_{3-10}} \sim f_\pi\zeta_J^{B\pi}\Big(28 + 215\frac{\mu_\pi}{3m_B}\Big)$$

Arnesen et al.

$$+\ldots$$

singular

$$\int_0 \frac{dx}{x} = ?$$

Annihilation terms

Keum, Li, Sanda

$$\int dx\,\frac{\phi_\pi^{pp}(x)}{x(1-x)} \sim 6$$

$$+P^{\mathrm{new-physics}}$$

Jain et.al. (BPRS)

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	$\hat{P}^{LO}\times 10^4$	$\hat{P}^{\chi}\times 10^4$	$\hat{P}^{ann}\times 10^4$	$\hat{P}^{total}\times 10^4$	$\hat{P}^{expt}_{ispin}\times 10^4$ ($\gamma = 59°$)	$\hat{P}^{expt}_{ispin}\times 10^4$ ($\gamma = 74°$)	$\hat{P}^{expt}_{TF}\times 10^4$ ($\gamma = 59°$-$74°$)
$B \to \pi\pi$	(8.10 ± 0.63) $+i(1.61\pm0.21)$	(10.2 ± 2.9) $+i(1.10\pm0.39)$	-1.31 ± 5.08	(16.9 ± 5.9) $+i(2.71 \pm 0.45)$	(18 ± 9) $-i(29\pm6)$	(44 ± 6) $-i(29\pm6)$	
$B \to K\pi$	(9.34 ± 1.00) $+i(2.08 \pm 0.25)$	(13.8 ± 3.9) $+i(1.49 \pm 0.57)$	0.46 ± 8.03	(23.6 ± 9.0) $+i(3.57 \pm 0.62)$			$\pm(48 \pm 4 \pm 10)$ $-i(22 \pm 7 \pm 4)$
$B \to \rho\rho$	$22.4^{+3.7}_{-2.3}$ $+i\,5.68^{+2.45}_{-1.07}$	— —	$0.87^{+0.67}_{-0.29}$	$23.3^{+3.7}_{-2.4}$ $+i\,5.68^{+2.45}_{-1.07}$	$-(29 \pm 26)$ $-i(8 \pm 18)$	(38 ± 23) $-i(8 \pm 18)$	

phase relative to $T^{M_1^+ M_2^-}$

All terms directly related to Trees have SMALL imaginary parts

Possible Imaginary contributions:

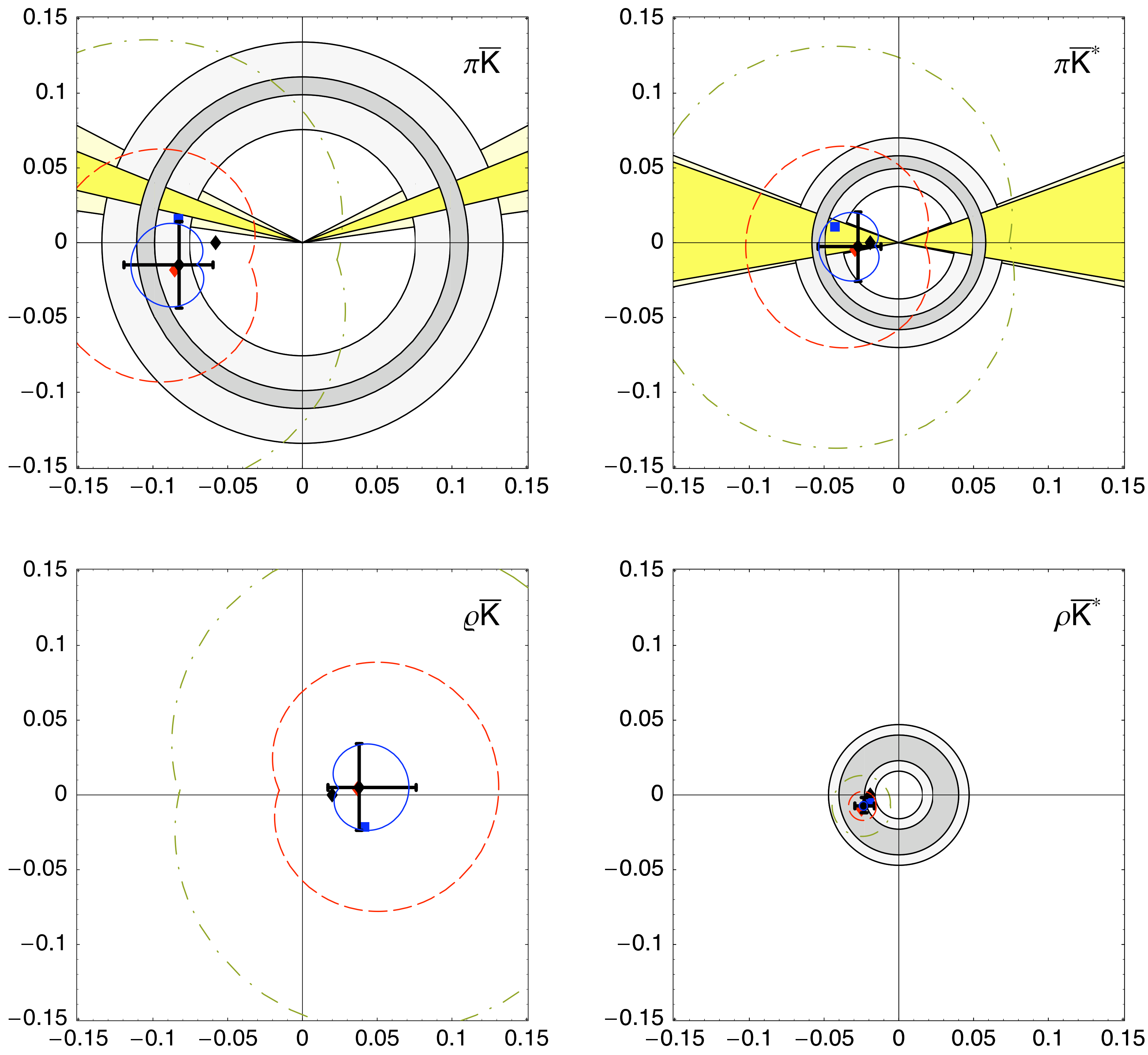
- new physics without long-distance penguins?
  very unlikely. A large imaginary part requires that the new physics have a large strong phase

$$|Im(N_{1,2})| \leq \frac{|\mathrm{Im}(N)|}{\sin\gamma}$$

$$N e^{-i\phi} = N_1 + N_2 e^{i\gamma}$$

- complex annihilation
- complex charm penguins

Beneke & Jager (BBNS): imaginary part is from annihilation



In all approaches, the terms used to bring the penguins into agreement with data depend on model parameters AND are the least well understood / agreed upon:

Charm Penguins or Annihilation

$B \to K\pi$ Br are reproduced IF penguin is reproduced

Within Factorization there is an interesting correlation in the CP-asymmetries: (any of: BBNS or KLS or BPRS or Williamson et.al.)

LO: $A_{K^+\pi^0} < A_{K^+\pi^-}$

$\sim 1.5 - 2.5\sigma$ deviation
(with theory error estimate from hadronic parameters and power corr.)

HFAG'08

$$A_{K^+\pi^-} = -0.098 \pm 0.012$$

$$A_{K^+\pi^0} = 0.050 \pm 0.025$$

The "usual largest" power corrections (chiral enhanced annihilation, chiral enhanced amplitudes, charming penguins) do not explain this, since they contribute equally to both amplitudes.

Ciuchini, Franco, Martinelli, Pierini, Silvestrini (arXiv: 0811.0341)

Power correction scan. Significant corrections to color suppressed amplitudes yield results compatible with the data.

Li and Mishima (arXiv: 0901.1272)

violations of kT factorization due to soft divergence can give a phase to color suppressed amplitude for pions, yielding a dynamical mechanism to explain the data (also is a power corr. in collinear fact). Simultaneously this improves the agreement for $Br(\pi^0\pi^0)$, $Br(\pi^0\rho^0)$, and $S_{\pi_0 K_S}$

Path to finding New Physics in the presence of Hadronic Parameters/Expansions (best we can do?)

I) use as much form factor information from semileptonic decays as possible (synergy is like $B \to X_s \gamma$ with $B \to X_u e \bar{\nu}$)

eg.

Lattice analysis of form factors (with fit to future $B \to \pi \ell \bar{\nu}$ spectra) can distinguish between

BBNS: $\zeta_J^{BM} / \zeta^{BM} \sim \alpha_s$ **BPRS:** $\zeta_J^{BM} / \zeta^{BM} \sim 1$

using

$$\delta \equiv 1 - \frac{(m_B^2 - m_\pi^2)}{f_+(0)} \left(\frac{df_+}{dq^2}\bigg|_{q^2=0} - \frac{df_0}{dq^2}\bigg|_{q^2=0} \right) = \frac{2\zeta_J^{B\pi}}{\zeta_J^{B\pi} + \zeta^{B\pi}} \left[1 + \mathcal{O}\Big(\alpha_s(m_b), \frac{\Lambda}{E} \Big) \right]$$

shape parameter Hill

Path to finding New Physics in the presence of Hadronic Parameters/Expansions (best we can do?)

I) use as much form factor information from semileptonic decays as possible (synergy is like $B \to X_s \gamma$ with $B \to X_u e \bar{\nu}$)

II) use global fits which combine Factorization and SU(3) to look for interesting channels with large deviations

III) use Factorization and SU(2) for individual channels, to obtain more precise predictions (at the expense of additional fit parameters)

IV) use SU(3) fits as a cross-check on the hadronic uncertainties (supplementing II and III)

V) include THEORY uncertainty when discussing any deviations (power corrections, model parameters, etc.)

VI) build a new-physics model that correlates and explains the deviations in several channels

The End