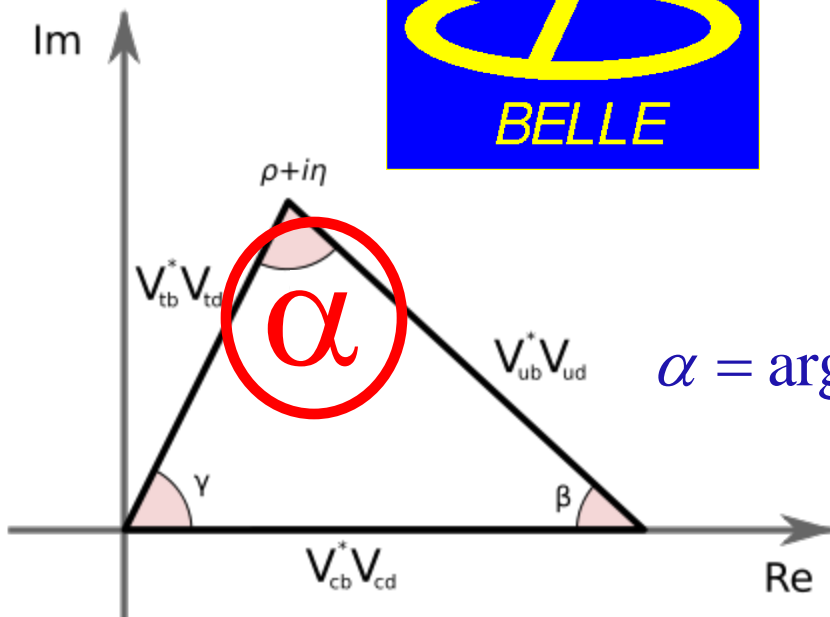


Measurement of the CKM angle α (or ϕ_2)

J. William Gary, U. California, Riverside



$$\alpha = \arg \left(- \frac{V_{td} (V_{tb})^*}{V_{ud} (V_{ub})^*} \right)$$



Outline

$b \rightarrow u\bar{u}d$ processes:

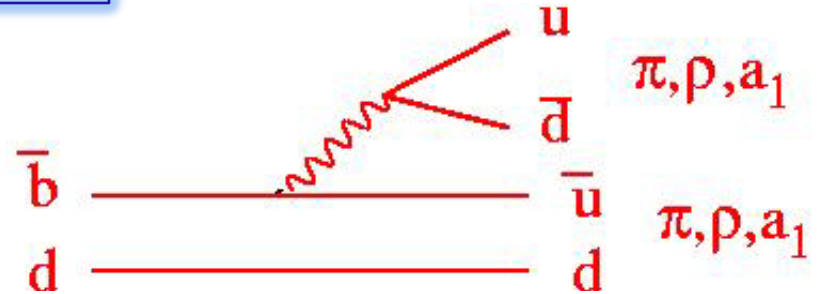
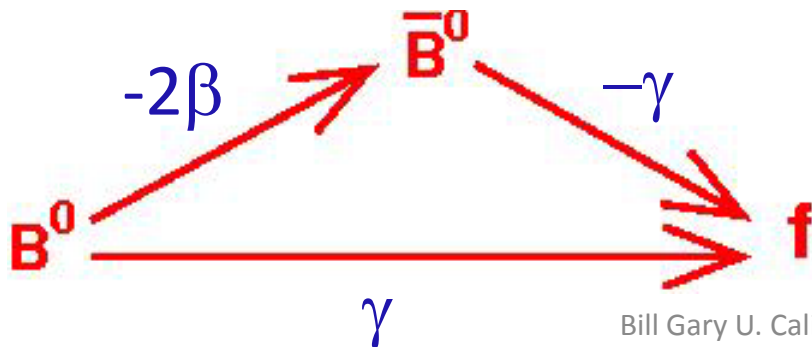
$B \rightarrow \pi\pi$ $[2\pi]$

$B \rightarrow \rho\rho$ $[4\pi]$

$B \rightarrow \rho\pi$ $[3\pi]$

$B \rightarrow a_1(1260)\pi$ $[4\pi]$

$[\rightarrow \text{all-}\pi \text{ final states}]$



CKM phase $V_{ub} \sim e^{-i\gamma}$
in tree amplitude

Mixing-induced CP asymmetry:
Interference between direct B^0 decay and decay after B^0 - \bar{B}^0 mixing $\sim -(2\beta+2\gamma) \sim 2\alpha$
 $[\alpha+\beta+\gamma=\pi]$

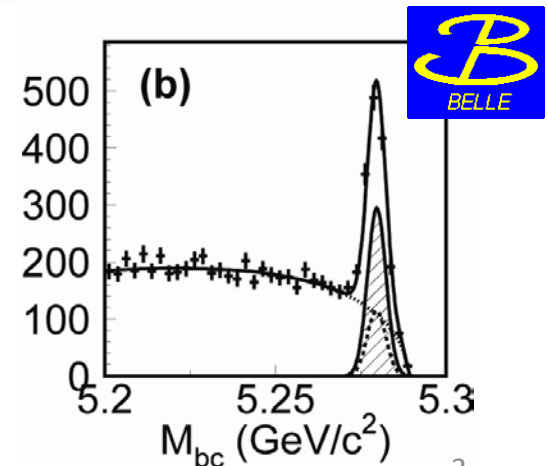
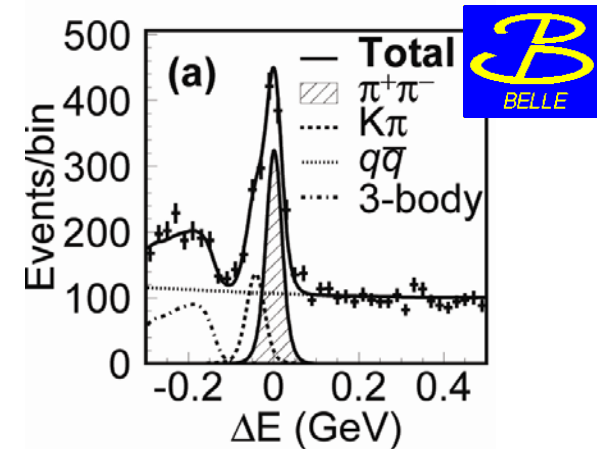
π identification & event selection

- π^\pm : Cherenkov detectors; dE/dx from tracking chambers
[detection efficiency $\geq 95\%$; mis-ID $\approx 10\%$]
- $\pi^0 \rightarrow \gamma\gamma$ [some use of $\gamma \rightarrow e+e^-$, merged $\gamma\gamma$]
[efficiency $\approx 50\%$, purity $\sim 70\%$]

Key variables for B reconstruction:

- Energy difference $\Delta E = (E_B)^* - (E_{beam})^*$
(peaks at zero)
- Beam energy-constrained (-substituted)

mass $m_{bc} = m_{ES} = \sqrt{(E_{beam})^* - (p_B)^*}$
(peaks at m_B)



Time-dependent CP asymmetries in $B^0(\bar{B}^0) \rightarrow f_{CP} (= \pi^+\pi^-, \rho^+\rho^-, \rho^0\rho^0)$

$$a_{CP}(t) = \frac{\Gamma[B^0(t) \rightarrow f_{CP}] - \Gamma[\bar{B}^0(t) \rightarrow f_{CP}]}{\Gamma[B^0(t) \rightarrow f_{CP}] + \Gamma[\bar{B}^0(t) \rightarrow f_{CP}]}$$

$$= S \sin \Delta m t - C \cos \Delta m t$$

$$S = -\frac{2 \operatorname{Im} \lambda}{1 + |\lambda|^2} \quad C = \frac{1 - |\lambda|^2}{1 + |\lambda|^2} \quad \lambda = e^{-2i\beta} \frac{\bar{A}}{A} \quad A = \langle f_{CP} | H | B^0 \rangle$$

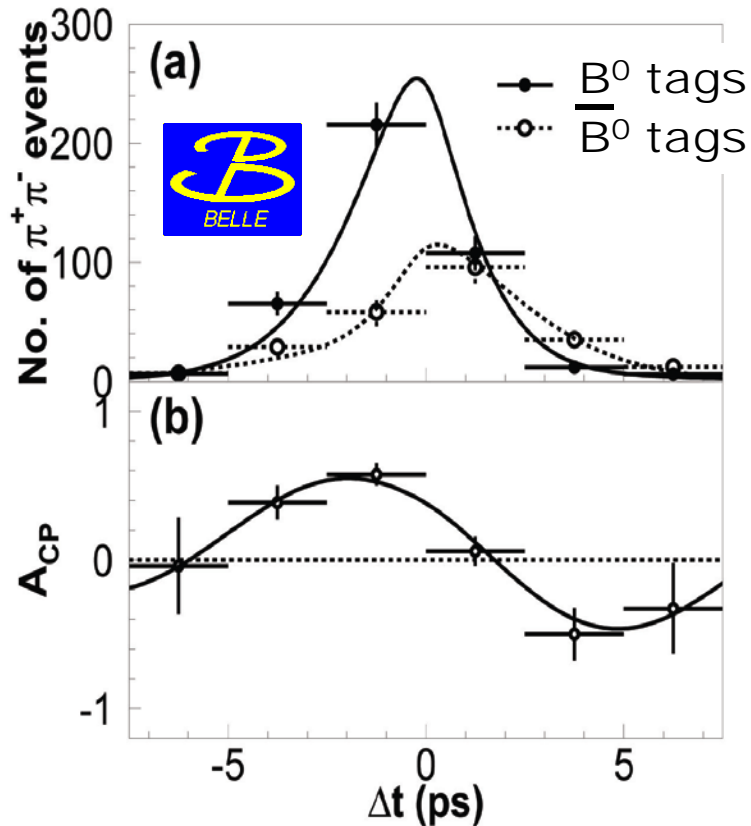
$$\bar{A} = \langle f_{CP} | H | \bar{B}^0 \rangle$$

If one amplitude (tree) dominates, $|A| = |\bar{A}|$ and $\bar{A}/A = e^{-2i\gamma}$

$$\rightarrow |\lambda| = 1 \quad C = 0 \quad S = \sin 2\alpha$$

$C \neq 0$ implies $|A| \neq |\bar{A}|$, i.e., more than one amplitude (penguins)

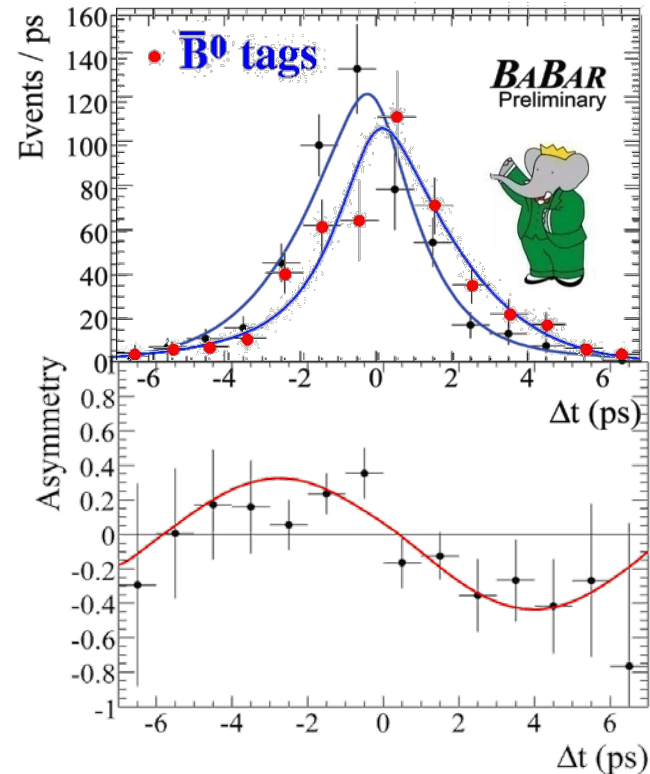
$a_{cp}(t)$ in $B \rightarrow \pi^+\pi^-$



[PRL 98 (2007) 211801, 535×10^6 events]

$$S = -0.61 \pm 0.10 \pm 0.04$$

$$C = -0.55 \pm 0.08 \pm 0.05$$

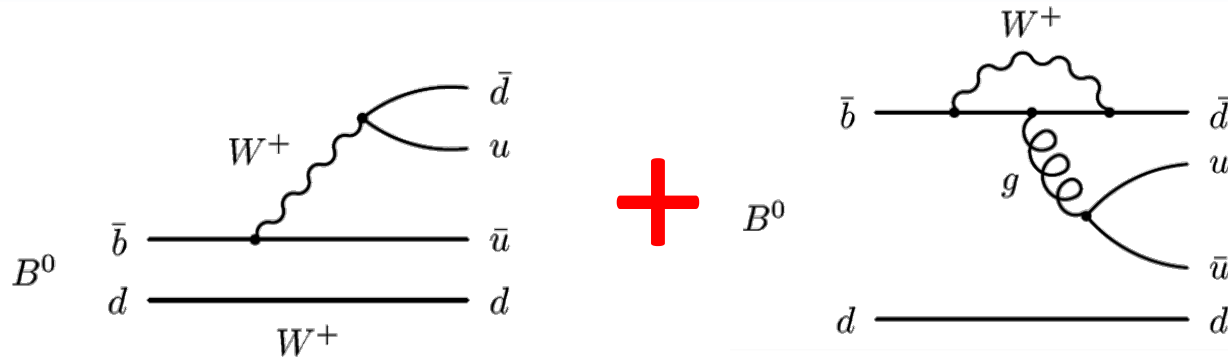


[arXiv:0807.4226 (2008), 467×10^6 events (the final event sample)]

$$S = -0.68 \pm 0.10 \pm 0.03$$

$$C = -0.25 \pm 0.08 \pm 0.02$$

$C_{\pi^+\pi^-} \neq 0 \rightarrow$ more than 1 amplitude



$$a_{CP}(t) = S \sin \Delta m t - C \cos \Delta m t \quad \text{as before}$$

but now a_{CP} determines an effective angle α_{eff} , defined by

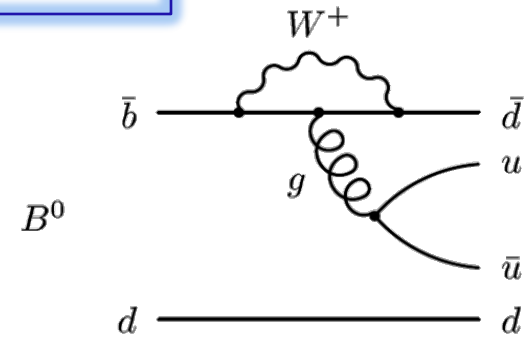
$$S \equiv \sqrt{1 - C^2} \sin 2\alpha_{\text{eff}}$$

Gronau & London (GL) [PRL 65 (1990)3381]: use isospin symmetry to disentangle the contributions of tree and penguin amplitudes,

measure $\delta\alpha = \alpha - \alpha_{\text{eff}}$

Isospin analysis

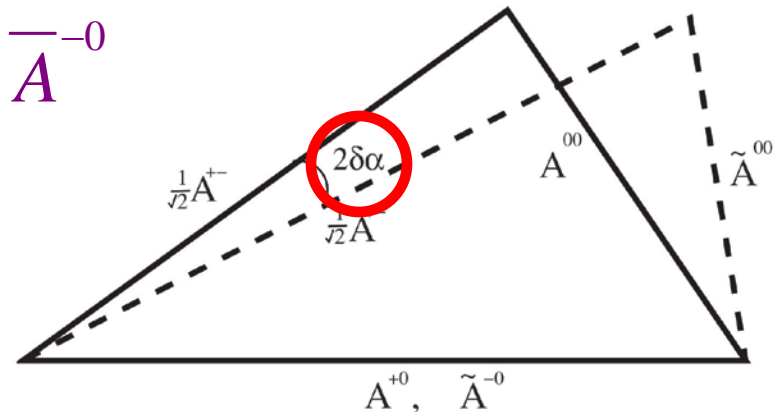
- $B^+ \rightarrow \pi^+ \pi^0$: Isospin $I=1,2$ ($I_3=1$)
- Penguin: $I=0,1$
- $I=1$ forbidden by BE statistics
 $\rightarrow B^+ \rightarrow \pi^+ \pi^0$ pure tree



Three $B \rightarrow \pi\pi$ charge states ($+0, +-, 00$) \rightarrow triangles

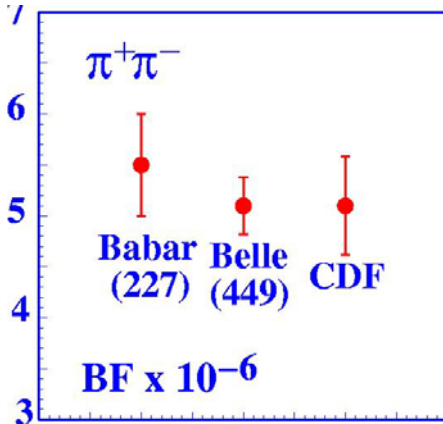
$$(1) A^{+0} = \frac{A^{+-}}{\sqrt{2}} + A^{00} \quad A^{+0} = \bar{A}^{-0}$$

$$(2) \bar{A}^{-0} = \frac{A^{-+-}}{\sqrt{2}} + \bar{A}^{-00}$$

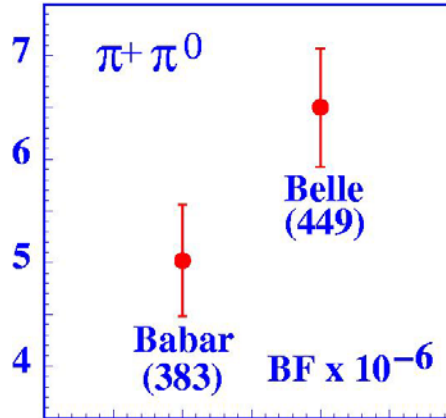


\rightarrow 8-fold discrete ambiguity: (4 relative orientations of 2 triangles) \times (2 solutions to $\alpha_{\text{eff}} \sim \sin^{-1} S$) [α_{eff} or $90^\circ - \alpha_{\text{eff}}$]

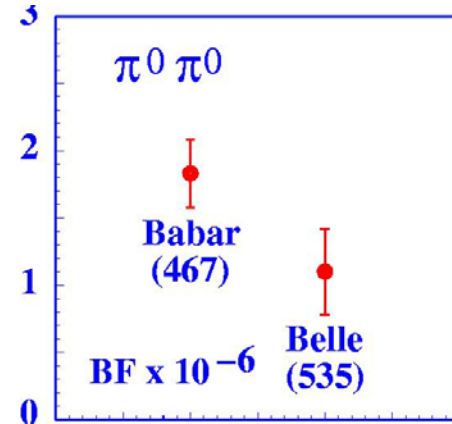
$B \rightarrow \pi\pi$: branching fractions and A_{CP} 's



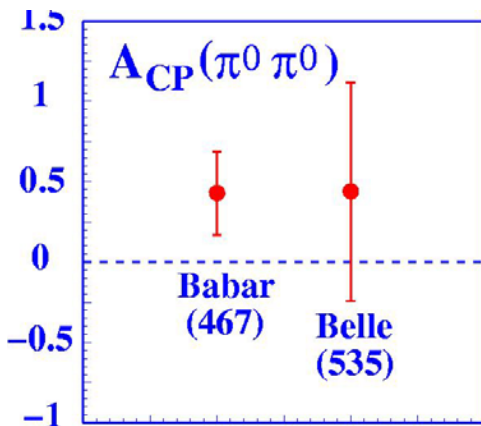
Babar PRD75(2007)012008
 Belle PRL99(2007)121601
 CDF hep-ex/0612018 (2007)



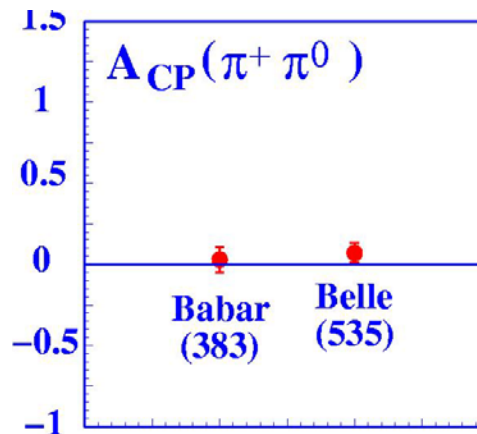
Babar PRD76(2007)091102
 Belle PRL99(2007)121601



Babar arXiv:0807.4226 (2008)
 Belle hep-ex/0610065 (2006)



Babar arXiv:0807.4226 (2008)
 Belle hep-ex/0610065 (2006)

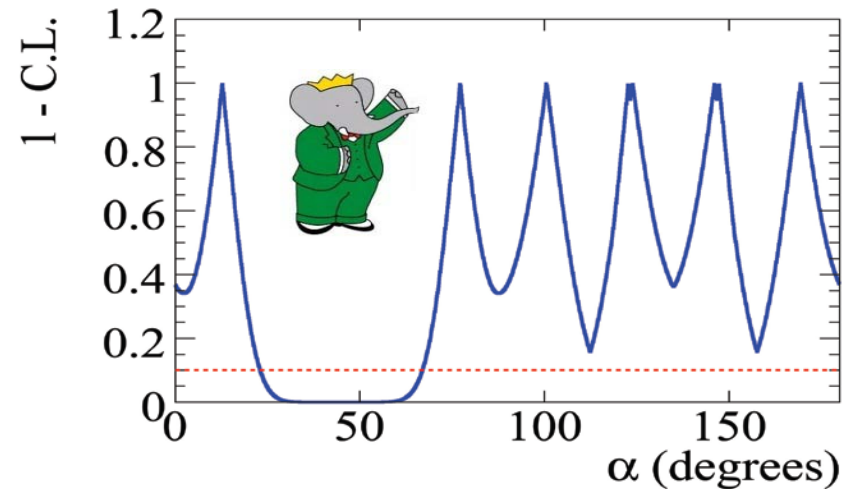
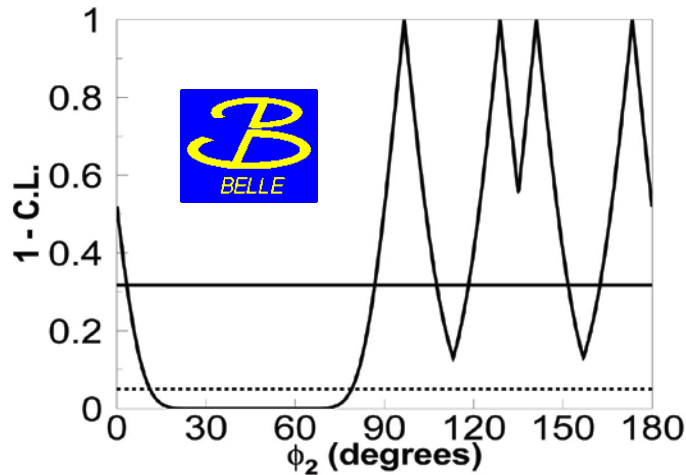


Babar PRD76(2007)091102
 Belle Nature 452(2008)332

Bill Gary U. California, Riverside; FPCP
 2009, Lake Placid NY ; May 28, 2009

- $BF(\pi^0\pi^0)/BF(\pi^+\pi^-) \approx 30 \pm 4\%$
 \rightarrow large penguin contribution (tree color-suppressed in $\pi^0\pi^0$)
- $A_{CP}(\pi^+\pi^0) \approx 0$
 \rightarrow EW penguins small (negligible)

α from $B \rightarrow \pi\pi$: Babar and Belle



- 6 inputs: 3 BF's, S, C, A_{CP} for $\pi^0\pi^0$
- α scan; find minimum χ^2 in fit of isospin triangle to the measurements, convert to C.L. (frequentist)
- Multiple solutions from discrete ambiguities

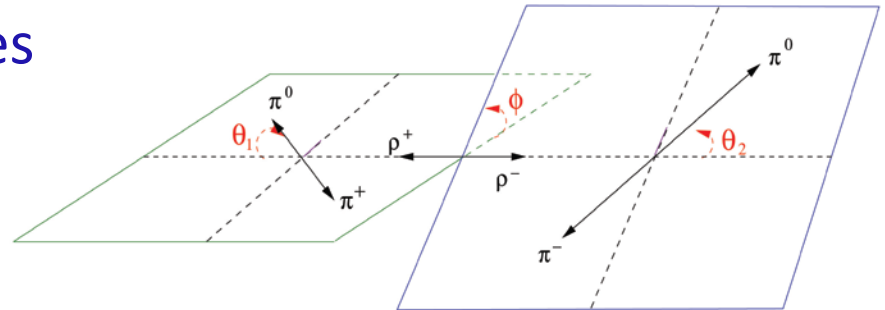
Belle: $11 < \alpha < 79^\circ$ excluded at 95% C.L.

Babar: $23 < \alpha < 67^\circ$ excluded at 90% C.L.



- Spin $0 \rightarrow 1 + 1$; $L = 0,1,2$; $CP = (-1)^L$
- Helicity basis: Superposition of 3 polarization amplitudes:
 - \rightarrow 1 longitudinal (helicity=0); $L=0,2$ $CP = +1$
 - \rightarrow 2 transverse (helicity= ± 1); $L=0,1,2$ $CP = \text{mixed}$
- Separate polarization states with an angular analysis

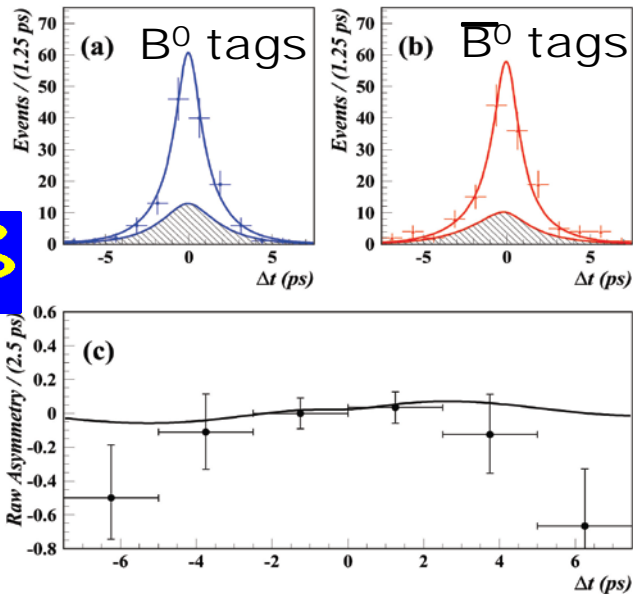
Helicity angles $\theta_{1,2}$:



Integrate over ϕ :
$$\frac{1}{\Gamma} \frac{d^2\Gamma}{d \cos \theta_1 d \cos \theta_2} = \frac{9}{16} [4f_L \cos^2 \theta_1 \cos^2 \theta_2 + (1-f_L) \sin^2 \theta_1 \sin^2 \theta_2]$$

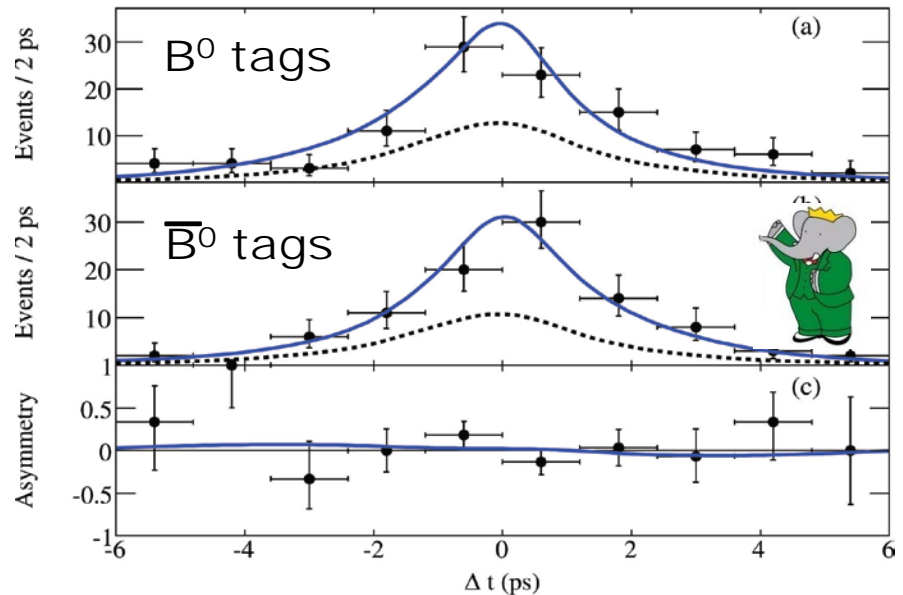
- Simultaneously determine longitudinal polarization fraction f_L
- Apply $a_{CP}(t)$ & GL isospin analysis to the longitudinal amplitudes

$a_{cp}(t)$ in $B \rightarrow \rho^+\rho^-$



[PRD 76 (2007) 011104, 535×10^6 events]

Assume $f_L = 1.0$
 $S = 0.19 \pm 0.30 \pm 0.08$
 $C = -0.16 \pm 0.21 \pm 0.08$

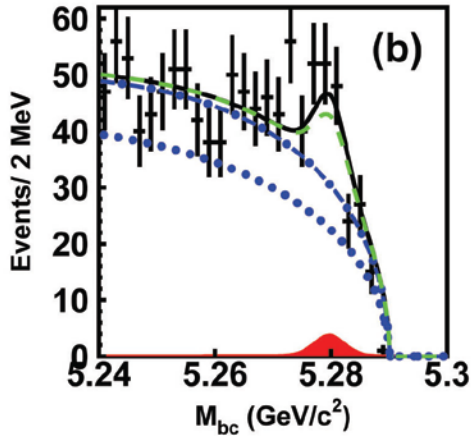


[PRD76(2007)052007, 384×10^6 events]

$f_L = 0.992 \pm 0.024^{+0.026}_{-0.013}$
 $S_{\text{Long}} = -0.17 \pm 0.20 \pm 0.06$
 $C_{\text{Long}} = -0.01 \pm 0.15 \pm 0.06$

$C_{\rho^+\rho^-} \approx 0 \rightarrow$ small penguin contribution $\rightarrow \alpha_{\text{eff}} \approx \alpha$ (≈ 0 or 90°)

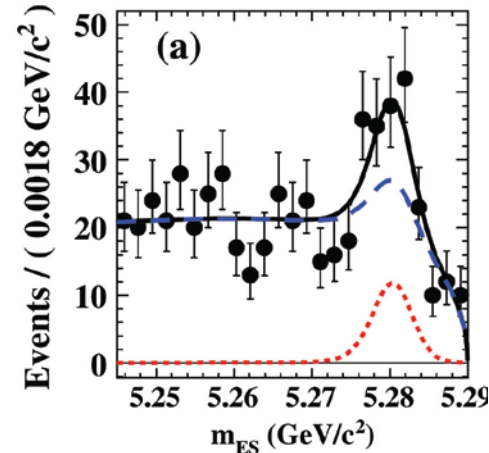
$a_{cp}(t)$ in $B \rightarrow \rho^0 \rho^0$



[PRD 78 (2008) 111102, 657×10^6 events]

$$BF(B \rightarrow \rho^0 \rho^0) = (0.4 \pm 0.4^{+0.2}_{-0.3}) \times 10^{-6}$$

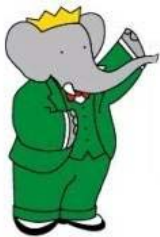
1 σ significance [signal eff. = 9.2%]



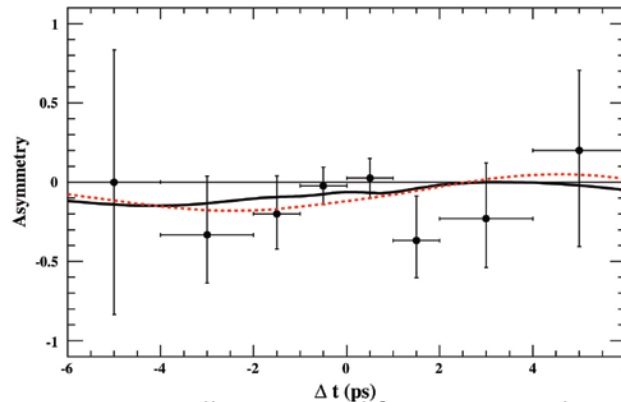
[PRD78(2008)071104, 465×10^6 events]

$$BF = (0.92 \pm 0.32 \pm 0.14) \times 10^{-6}$$

3.1 σ significance [signal eff.=23.3%]



Asymmetry $a_{CP}(t)$



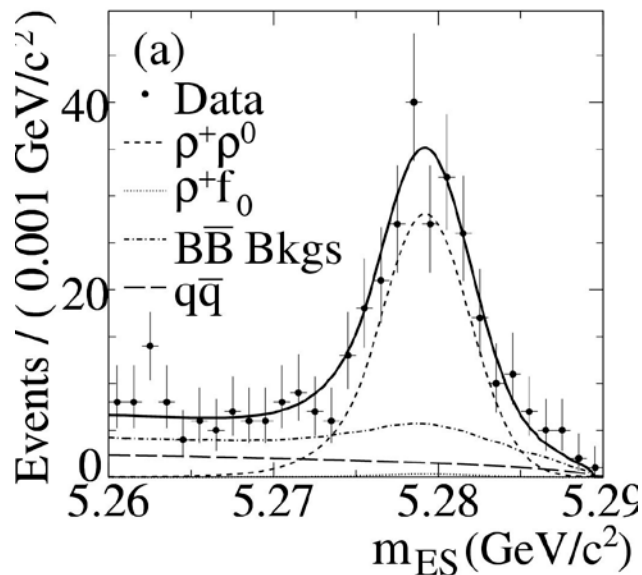
$$f_L = 0.75^{+0.11}_{-0.14} \pm 0.05$$

$$S_{Long} = 0.3 \pm 0.7 \pm 0.2$$

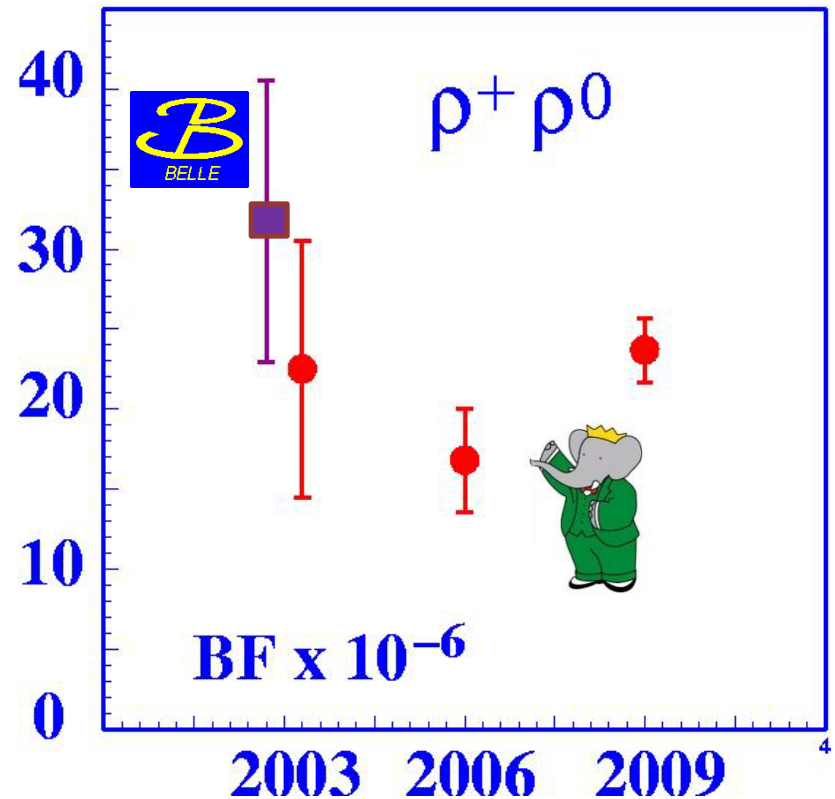
$$C_{Long} = 0.2 \pm 0.8 \pm 0.3$$

New $B^+ \rightarrow \rho^+ \rho^0$ study

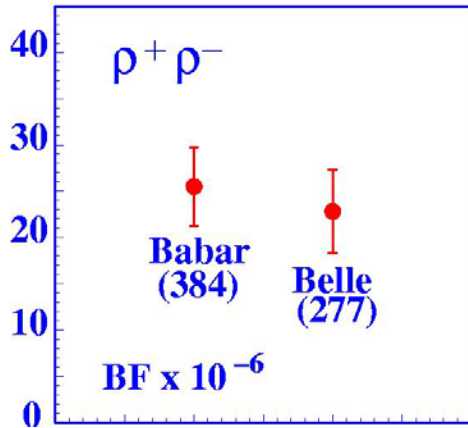
Babar PRL102 (2009) 141802



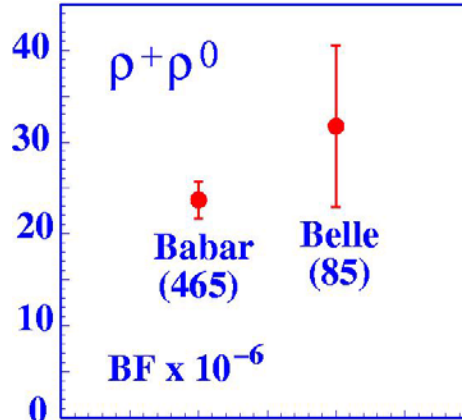
- $f_L = 0.950 \pm 0.015 \pm 0.006$
- Improved methods (3D PDFs) to account for correlations in the main (continuum & combinatoric BB) backgrounds



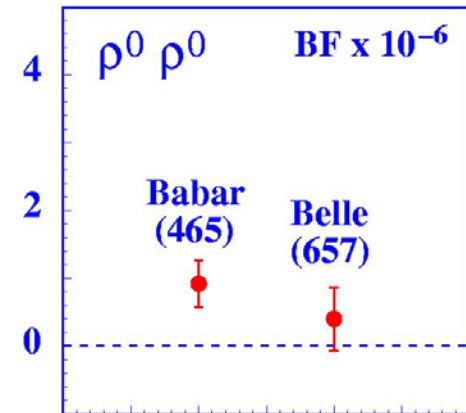
$B \rightarrow \rho\rho$: branching fractions and A_{CP} 's



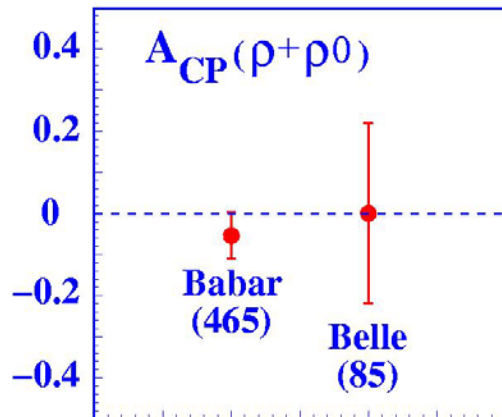
Babar PRD76(2007)052007
Belle PRL96(2006)171801



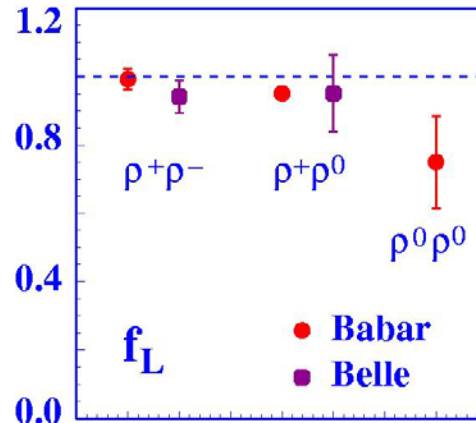
Babar PRL102(2009)141802
Belle PRL91(2003)221801



Babar PRD78(2008)071104
Belle PRD78(2008)111102

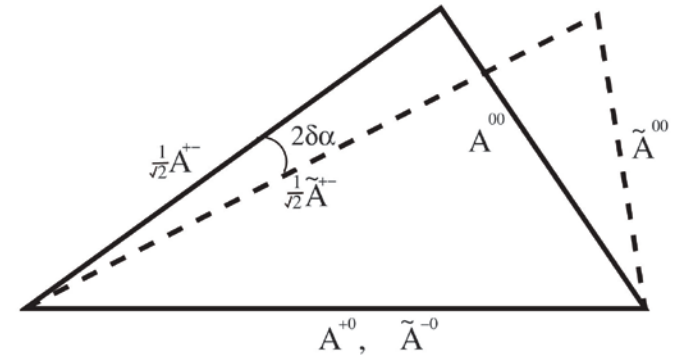


Babar PRL102(2009)141802
Belle PRL91(2003)221801

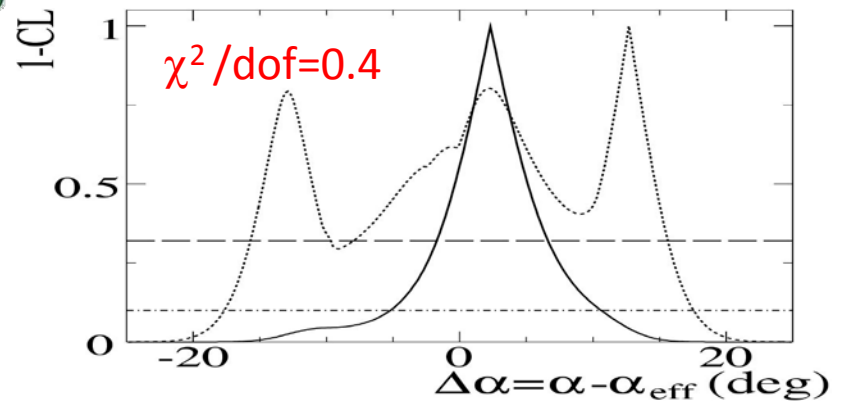
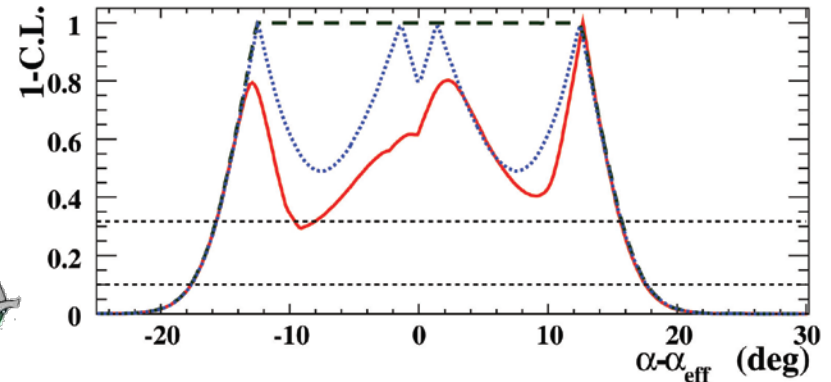
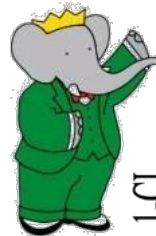


- $BF(\rho^0 \rho^0)/BF(\rho^+ \rho^-) \approx 3.6 \pm 1.5\%$
→ small penguin contribution
- $A_{CP}(\rho^+ \rho^0) \approx 0$
→ contribution of EW penguins small (negligible)

Isospin analysis: $B \rightarrow (\rho\rho)_{\text{Long}}$



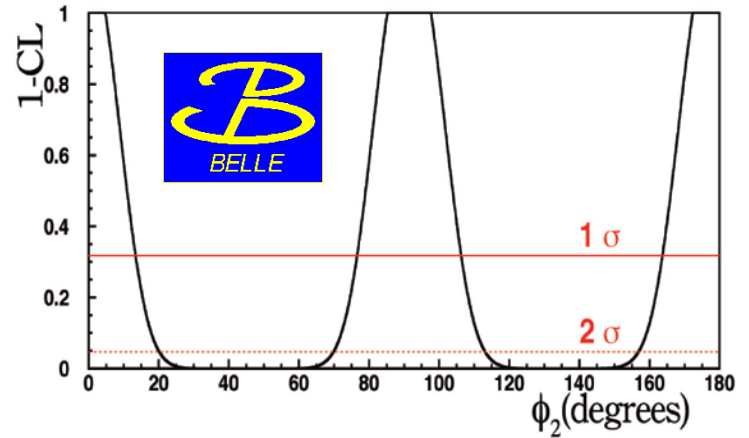
- 10 inputs: 3 BF's, 3 f_L 's, 2 S 's & 2 A 's
- Perform α scan, minimize χ^2
- Impact of S^{00} and C^{00} :
[Babar PRD78(2008)071104]
 - - - without S^{00} or C^{00}
 without S^{00}
- Impact of $\text{BF}(B^+ \rightarrow \rho^+\rho^0)$:
[Babar PRL102 (2009) 141802]
 with 2006 result
 ——— with 2009 result



Belle $\rho^0\rho^0$: $\alpha=(91.7\pm 14.9)^\circ$

[PRD 78 (2008) 111102]

- 657×10^6 events
- Old (2006) result for $\rho^+\rho^0$



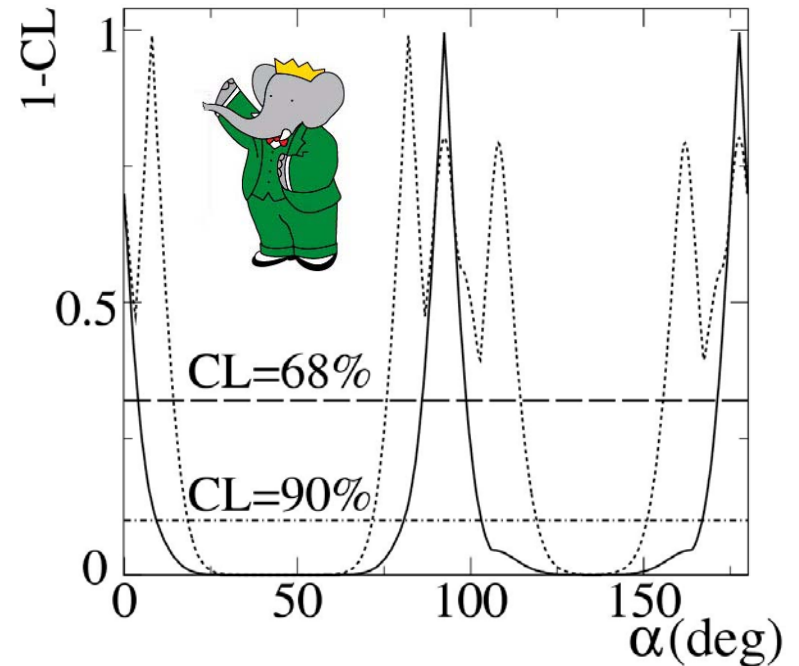
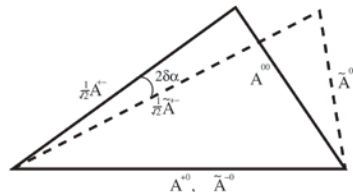
Babar $\rho^+\rho^0$: $\alpha=(92.4^{+6.0}_{-6.5})^\circ$

[PRL102 (2009) 141802]

- 465×10^6 events, $\rho^0\rho^0$ & $\rho^+\rho^0$
- 384×10^6 events, $\rho^+\rho^-$
- Includes new S, C, f_L from $\rho^0\rho^0$

Discrete ambiguities (except for $\pi-\alpha$) reduced by larger $\rho^+\rho^0$ BF

→ flattens the isospin triangle



B \rightarrow $\rho\pi$

- Not a CP eigenstate
- Need to simultaneously consider $B^0(\bar{B}^0) \rightarrow \rho^+\pi^-$ and $\rho^-\pi^+$
- Two methods proposed to eliminate penguin contributions:

1. Isospin analysis: [Lipkin et al., PRD44(1991)1454, Gronau, PLB265(1991)389]

\rightarrow Extend isospin $\pi\pi/\rho\rho$ isospin method

\rightarrow 5 charge states:

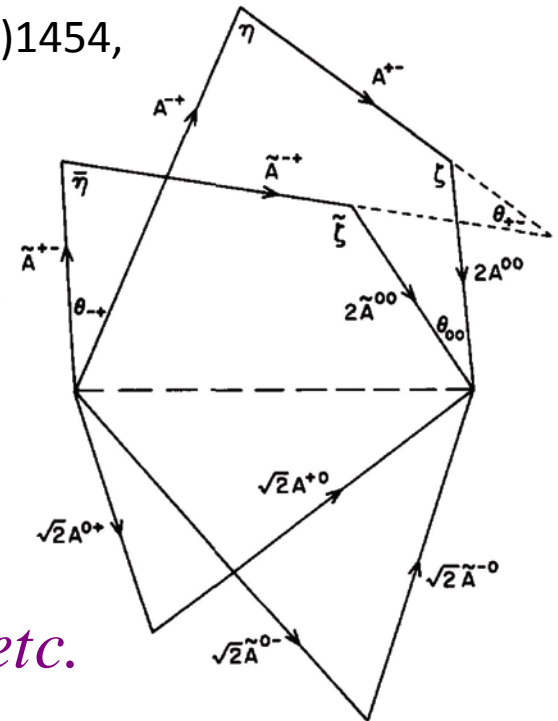
$$B^+ \rightarrow \rho^+\pi^0, \rho^0\pi^+ \quad + \text{c.c.}$$

$$B^0 \rightarrow \rho^+\pi^-, \rho^-\pi^+, \rho^0\pi^0 \quad + \text{c.c.}$$

\rightarrow isospin pentagons

$$A^{-+} + A^{+-} + 2A^{00} = \sqrt{2}(A^{0+} + A^{+0}), \text{ etc.}$$

Precision expected to be small



2. Time-dependent Dalitz plot analysis of the 3 $(\rho\pi)^0$

modes [Snyder & Quinn, PRD48(1993)2139; Quinn & Silva, PRD62(2000)054002]

→ Determine relative phases & moduli of six decay amplitudes

$$A^+(\bar{A}^+) = A [B^0(\bar{B}^0) \rightarrow \rho^+ \pi^-]$$

$$A^-(\bar{A}^-) = A [B^0(\bar{B}^0) \rightarrow \rho^- \pi^+]$$

$$A^0(\bar{A}^0) = A [B^0(\bar{B}^0) \rightarrow \rho^0 \pi^0]$$

through interference in the $B^0(\bar{B}^0) \rightarrow \pi^+ \pi^- \pi^0$ Dalitz plot

$$A^k = T^k e^{-i\alpha} + P^k ; \quad (q/p) \bar{A}^{\bar{k}} = T^{\bar{k}} e^{+i\alpha} + P^{\bar{k}} ; \quad k = +, -, 0 = \rho \text{ charge}$$

→
$$e^{2i\alpha} = \frac{\bar{A}^+ + \bar{A}^- + 2\bar{A}^0}{A^+ + A^- + 2A^0}$$

- Assumes isospin symmetry
- No discrete ambiguities

B \rightarrow $\rho\pi$: time-dependent Dalitz analysis

$$A_{3\pi}(s) \equiv A[B^0 \rightarrow \pi^+ \pi^- \pi^0] = \sum_{k=+,-,0} f_k(s) A^k \quad ; \quad s = m_{\pi^+ \pi^0}^2, m_{\pi^- \pi^0}^2$$

$$\bar{A}_{3\pi}(s) \equiv A[\bar{B}^0 \rightarrow \pi^+ \pi^- \pi^0] = \sum_{k=+,-,0} f_k(s) \bar{A}^k \quad A^k = A[B^0 \rightarrow \rho^k \pi^-]$$

- ρ lineshape: $f_k(s) = [F_{\rho(770)}(s) + a_1 e^{i\phi_1} F_{\rho(1450)}(s) + a_2 e^{i\phi_2} F_{\rho(1700)}(s)]_k$
- F_ρ = Breit-Wigner functions (Gounaris-Sakurai form)
- $a_1, \phi_1, a_2, \phi_2 \rightarrow$ determined in a fit

Time-dependent
decay rate
 $B \rightarrow (\pi\pi\pi)^0$

$$\frac{d\Gamma^\pm(s)}{dt} \propto \left(|A_{3\pi}|^2 + |\bar{A}_{3\pi}|^2 \right) \mp \left(|A_{3\pi}|^2 - |\bar{A}_{3\pi}|^2 \right) \cos \Delta m t$$

$$\pm 2 \operatorname{Im} \left(\frac{q}{p} \bar{A}_{3\pi} A_{3\pi}^* \right) \sin \Delta m t \quad \left(\begin{array}{l} + = B^0 \text{ tags} \\ - = \bar{B}^0 \text{ tags} \end{array} \right)$$

B \rightarrow $\rho\pi$: time-dependent Dalitz, cont.

$\frac{d\Gamma^\pm(s)}{dt}$ depends on 27 real-valued coefficients: U & I

$$U_k^\pm = |A^k|^2 \pm |\bar{A}^k|^2 \quad ; \quad k = +, -, 0$$

$$I_k = \text{Im} \bar{A}^k A^{k*}, \quad \text{etc.}$$

The U & I specify the phases and moduli of the A^k and \bar{A}^k , and thus α

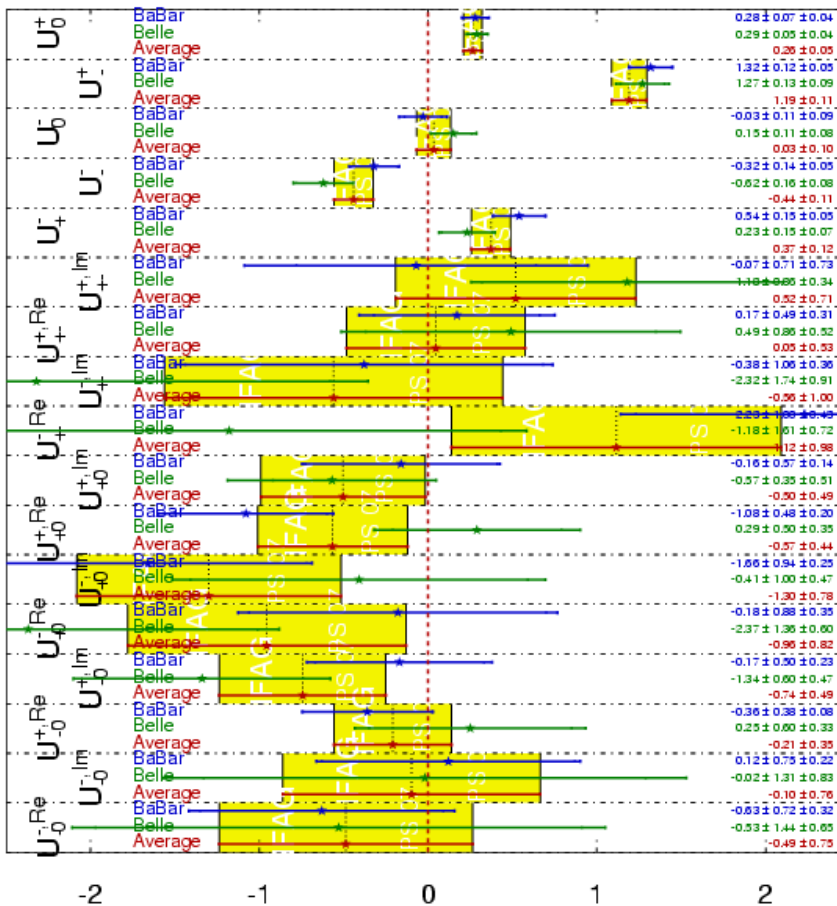
$$U_+^+ \sim \text{BF}(B^0 \rightarrow \rho^+ \pi^-) + \text{BF}(\bar{B}^0 \rightarrow \rho^+ \pi^-) \quad [\text{total BF's}] \quad A^k = A[B^0 \rightarrow \rho^k \pi^-]$$

- Set $U_+^+ = 1$ (fixes overall normalization)

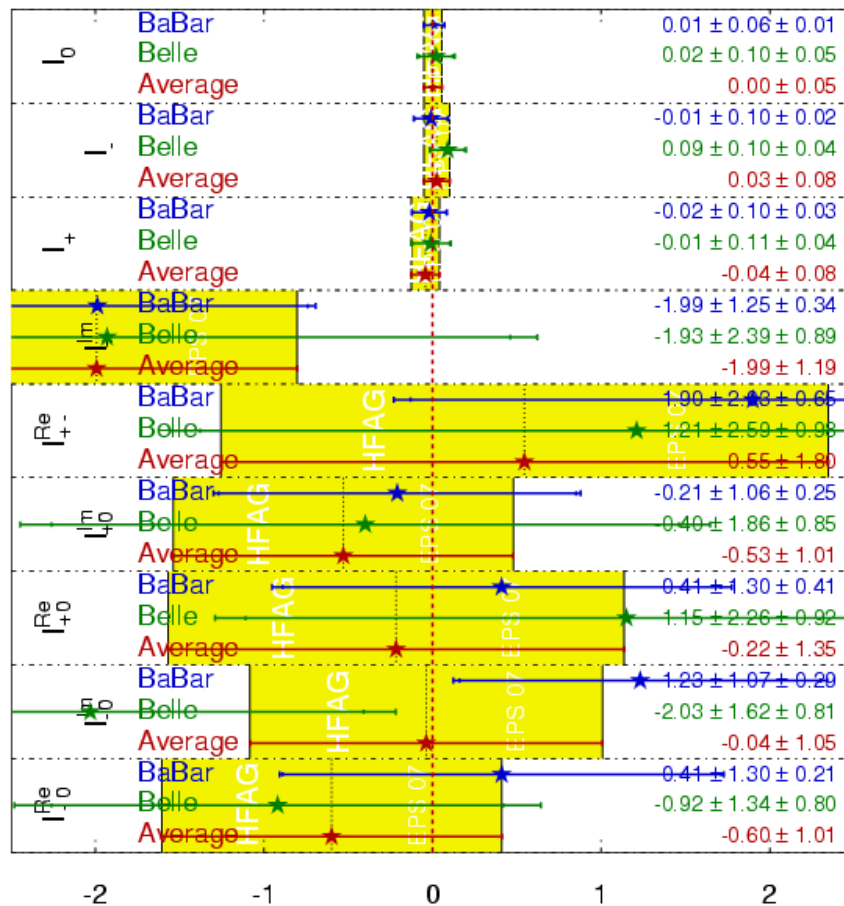
- Fit $\frac{d\Gamma^\pm(s)}{dt}$ to the time-sliced Dalitz plot with 26 free U & I coefficients (+ other parameters)

U & I: consistent results between Babar and Belle

$\pi^+\pi^-\pi^0$ U parameters
HFAG
 EPS 07
 PRELIMINARY

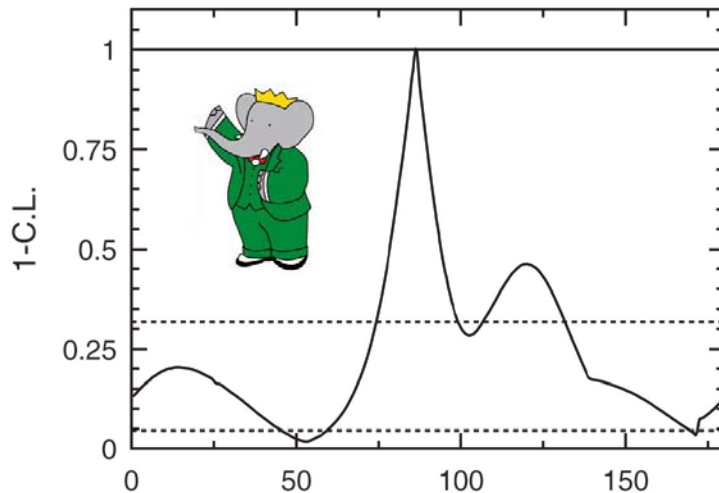


$\pi^+\pi^-\pi^0$ I parameters
HFAG
 EPS 07
 PRELIMINARY



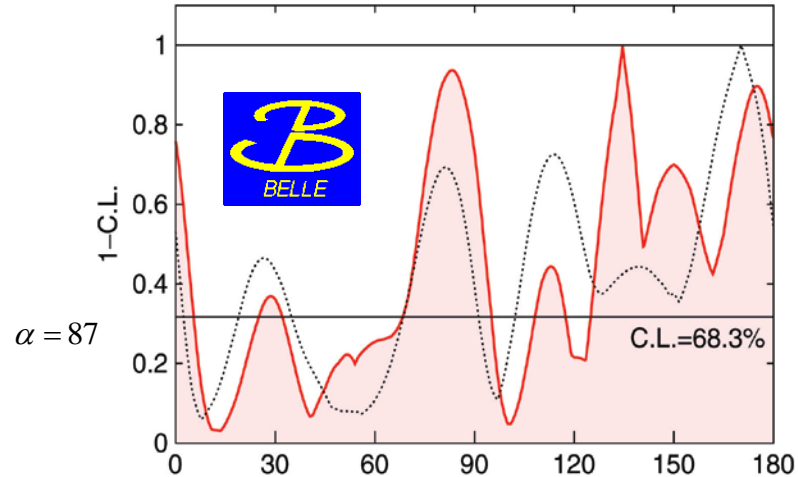
B \rightarrow $\rho\pi$: α scans

PRD76 (2007) 012004; 375×10^6 events;
 2067 ± 86 $B^0 \rightarrow \pi^+ \pi^- \pi^0$ events



$\alpha = (87^{+45}_{-13})^\circ$ at 68% C.L.

PRL98 (2007) 221602; 449×10^6 events;
 971 ± 42 $B^0 \rightarrow \pi^+ \pi^- \pi^0$ events



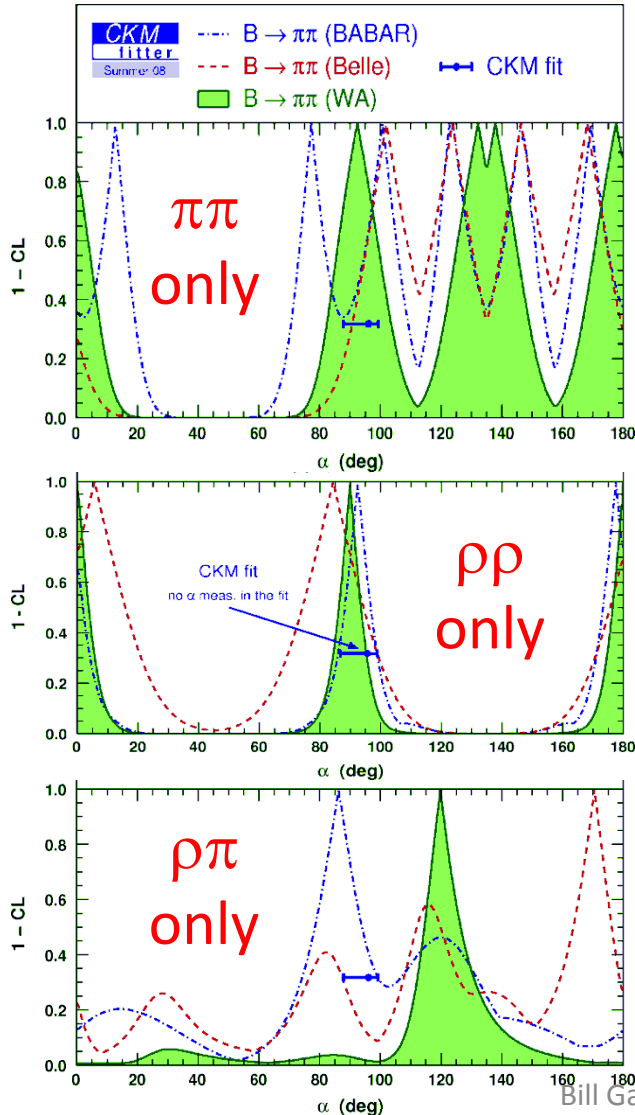
$68^\circ < \alpha < 95^\circ$ at 68% C.L.

Belle: Dotted \rightarrow Dalitz analysis only

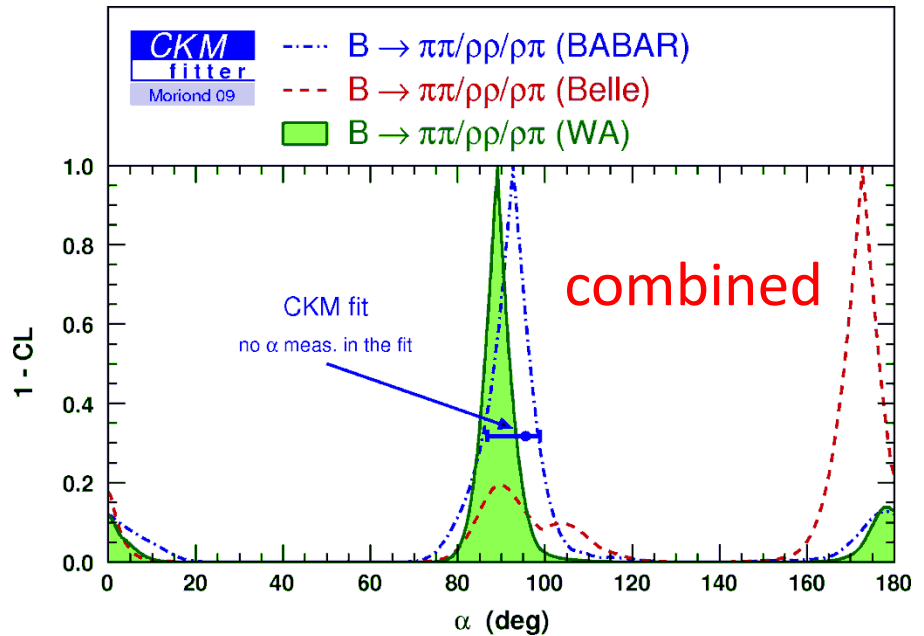
Solid \rightarrow include $B^+ \rightarrow (\rho\pi)^+$ modes [pentagon relations]

Babar: $\rho\pi$ results help resolve the discrete ambiguity
 from $B \rightarrow \rho\rho$ at $\alpha \approx 0^\circ/180^\circ$

Combined $\pi\pi$, $\rho\rho$, $\rho\pi$ results



CKMfitter group, ckmfitter.in2p3.fr
(frequentist construction of C.L.)

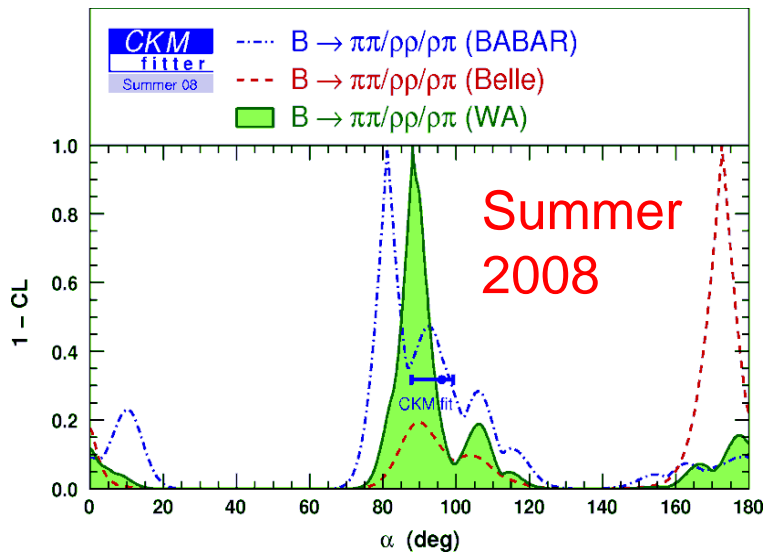


$$\alpha = 89.0 \pm 4.3^\circ \text{ (4.8\% precision)}$$

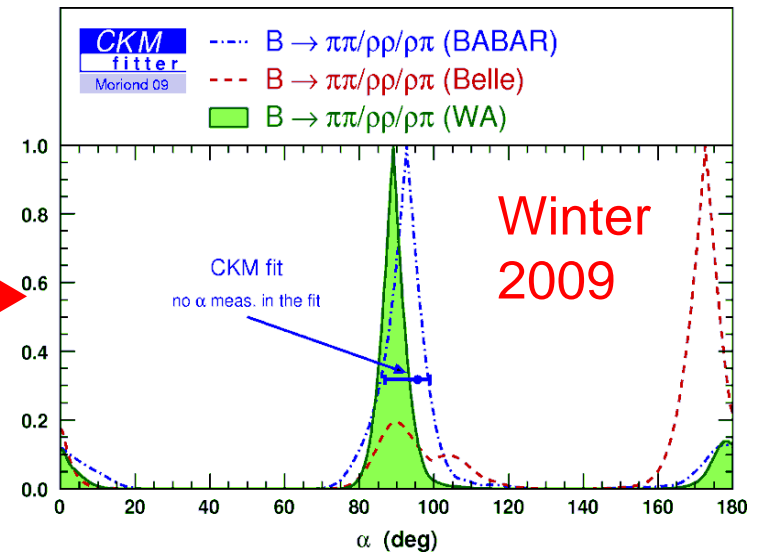
CKM fit: indirect SM constraint on α :

$|V_{us}|$, $|V_{ud}|$, $|V_{ub}|$, $|V_{cb}|$, CPV in K^0 , $B_{(s)}-B_{(s)}$ mixing

Improvement since Summer 2008



$$\alpha = (88.2^{+6.1}_{-4.8})^\circ \text{ (6.2\% precision)}$$



$$(89.0 \pm 4.3)^\circ \text{ (4.8\% precision)}$$

2 σ limits: 4 allowed regions

$$0^{+7}_{-0} \quad 88^{+24}_{-11} \quad 114.4^{+1.2}_{-1.2} \quad 176.9^{+3.1}_{-13.9}$$



Only 3, with much tighter bounds

$$0^{+5}_{-0} \quad 89.0^{+9.1}_{-8.3} \quad 178.3^{+1.7}_{-5.6}$$

B \rightarrow $a_1(1260)\pi$ with $a_1^+ \rightarrow \pi^+\pi^+\pi^-$

\rightarrow Quasi-two-body approach [Gronau & Zupan, PRD73(2006) 057502]

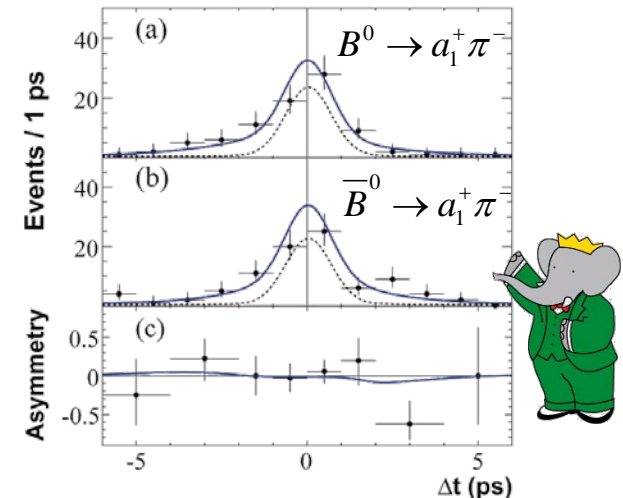
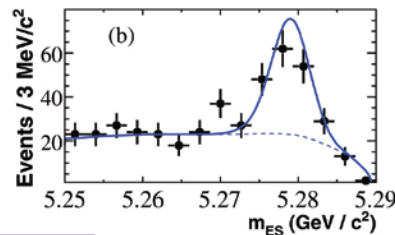
\rightarrow $\alpha_{\text{eff}} = (78.6 \pm 7.3)^\circ$ [4-fold ambiguity]

[PRL98(2007)181803, 384×10^6 BB]

\rightarrow SU(3)-flavor to constrain penguin:

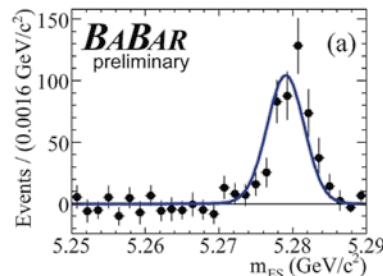
• $B \rightarrow a_1 K$ [PRL100 (2008)051803, 383×10^6 events]

$B^+ \rightarrow a_1 K^0$



• $B \rightarrow K_{1A}(1270)\pi$ [arXiv:0807.4760 (2008), 454×10^6 BB]

$B^0 \rightarrow K_1(1270)\pi^+$
 $+ K_1(1400)\pi^-;$
 $[K_1 \rightarrow K\pi\pi]$



$B^0 \rightarrow K_1 \pi^-$

$B \rightarrow K_1 \pi$ being finalized;
 unofficial naïve
 expectation is

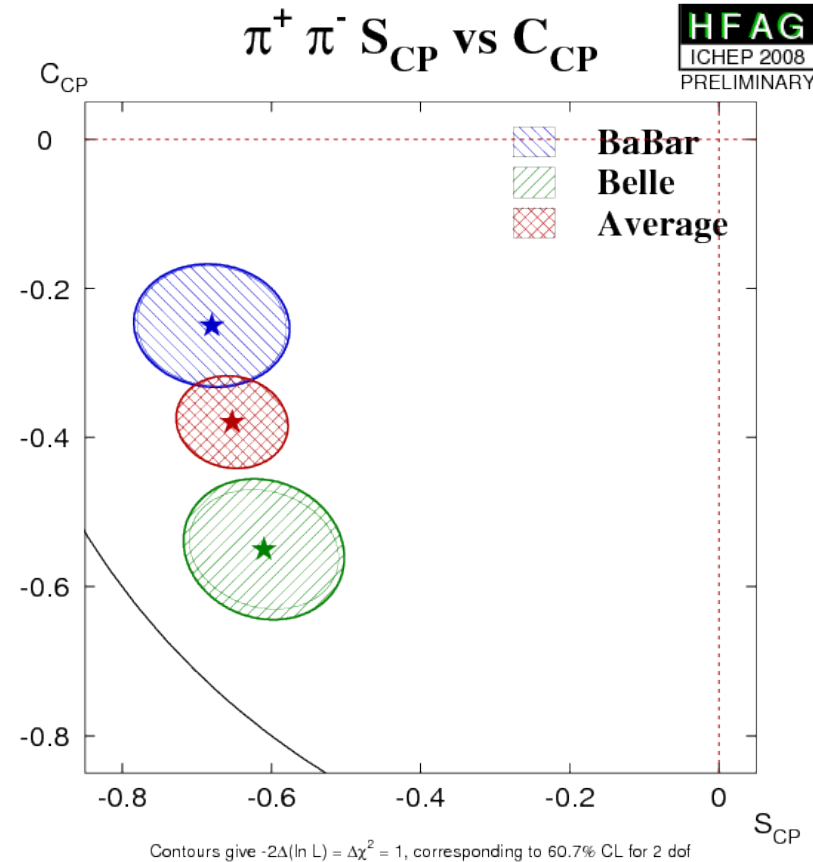
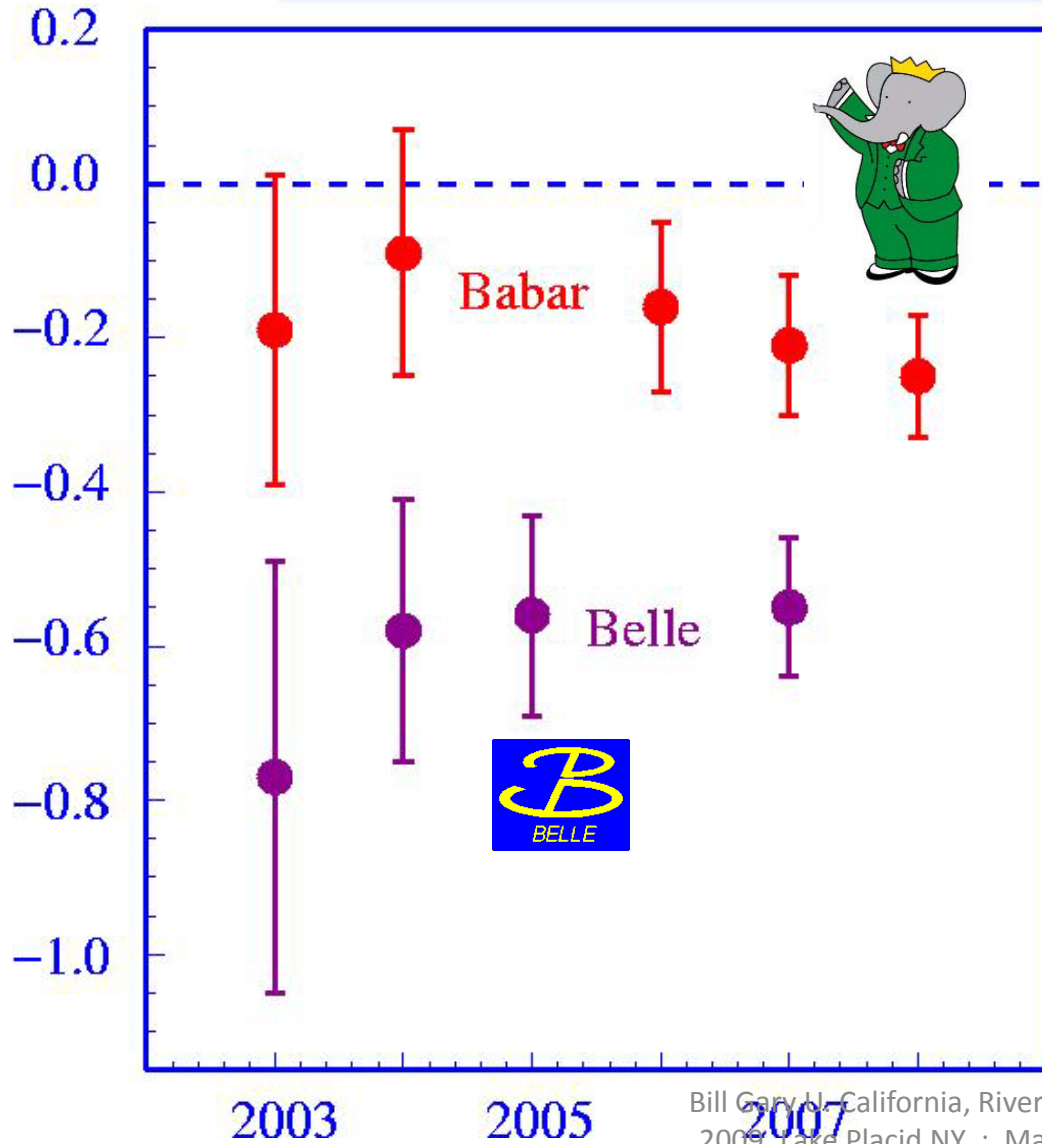
$$\delta\alpha = |\alpha - \alpha_{\text{eff}}| \leq 16^\circ$$

Summary and Conclusions

- α/ϕ_2 precision has improved considerably since summer 2008
 - new $\rho^0\rho^0$ results from Belle and Babar
 - new $\rho^+\rho^0$ results from Babar
- The $B\rightarrow\rho\pi$ Dalitz analysis important to resolve the discrete ambiguities from $\rho\rho$ at $(\alpha/\phi_2) \approx 0,180^\circ$
- α/ϕ_2 now known to better than 5% precision
 $\alpha/\phi_2 = 89.0 \pm 4.3^\circ$ (CKMfitter), $= 92 \pm 7^\circ$ (UTfit)
[the new $\rho^+\rho^0$ results not yet included in Ufit]
- Independent constraint from $B\rightarrow a_1\pi$ expected soon
- Babar results either final or nearly so
- Room for Belle to improve ϕ_2 with updated $\rho^+\rho^0$ BF, possibly $\rho\pi$ Dalitz results with a lot more data

BACKUP

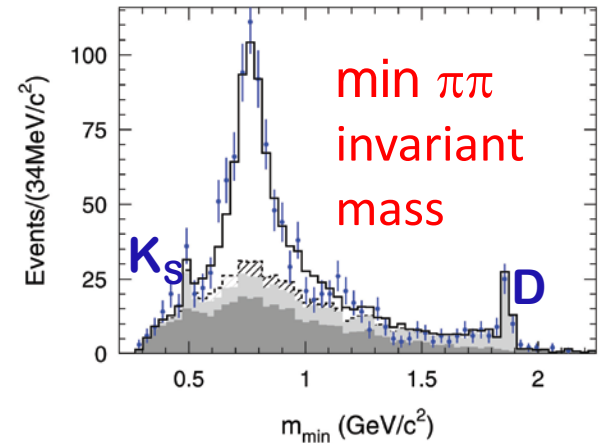
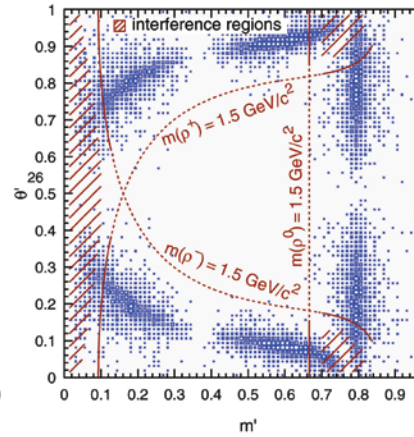
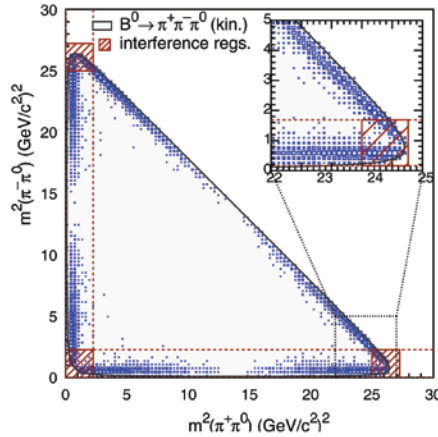
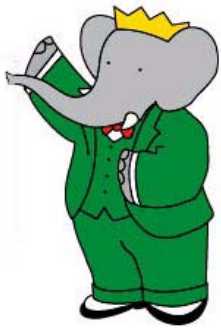
$C_{\pi^+\pi^-}$: Babar versus Belle



1.9 σ discrepancy
CL = 0.055 (HFAG)

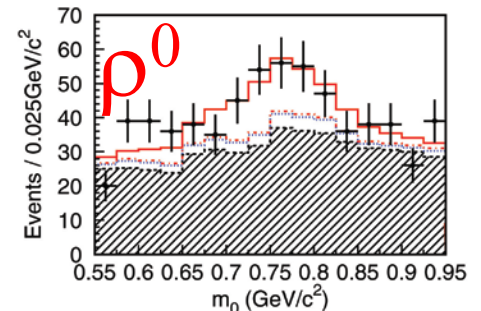
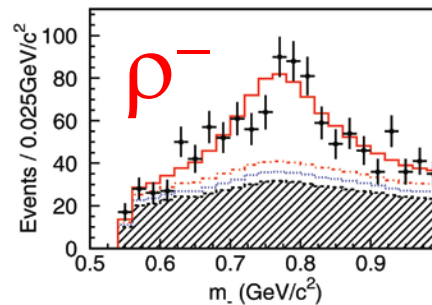
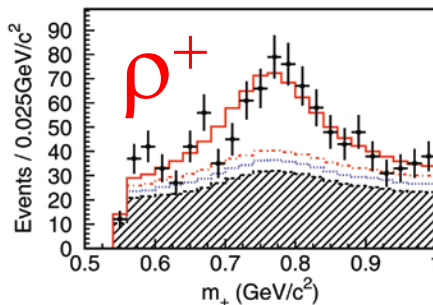
B \rightarrow $\rho\pi$: time-dependent Dalitz

Babar, PRD76 (2007) 012004; 375×10^6 events; 2067 ± 86 $B^0 \rightarrow \pi^+ \pi^- \pi^0$ events

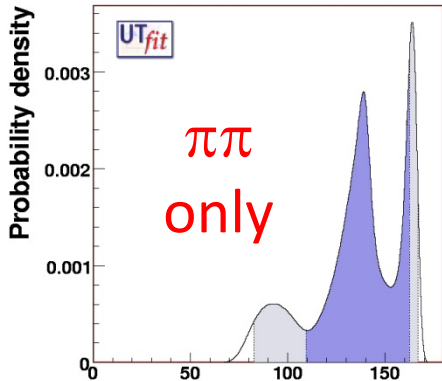


$\pi^+ \pi^- \pi^0$ -Dalitz plot, signal MC

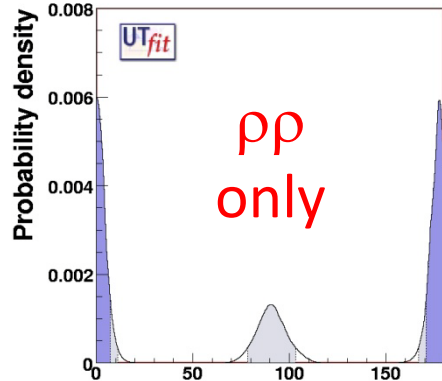
Belle, PRL98 (2007) 221602; 449×10^6 events; 971 ± 42 $B^0 \rightarrow \pi^+ \pi^- \pi^0$ events



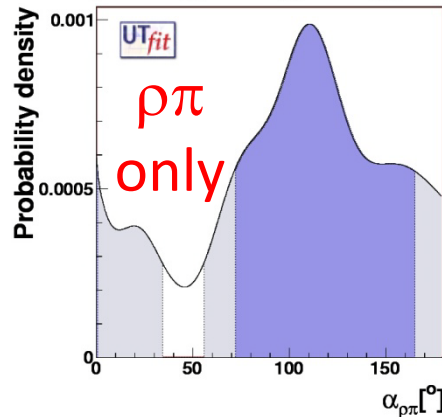
Combined $\pi\pi$, $\rho\rho$, $\rho\pi$ results, UTfit



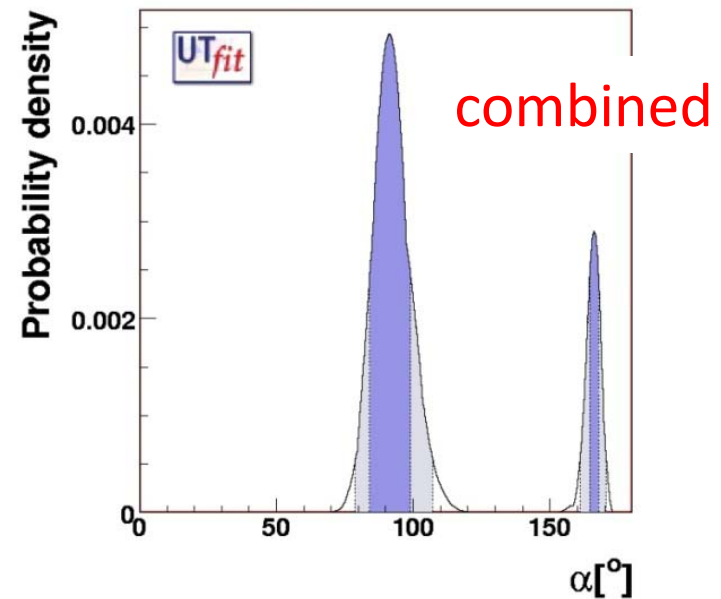
Require penguin contributions finite & physical:
 \rightarrow suppresses α at 0 and 180°



Does not yet include 2009 Babar $\rho^+\rho^0$ update



UTfit group, www.utfit.org
 (Bayesian construction of prob.)



$\alpha = (92 \pm 7)^\circ$ OR $(166 \pm 2)^\circ$
 (7.6% precision for SM result)